Mixing angle of hadrons in QCD: A new view

T. M. Aliev, $1,*,^{\dagger}$ $1,*,^{\dagger}$ $1,*,^{\dagger}$ A. Ozpineci, $1,^{\ddagger}$ and V. Zamiralov^{2, §}

¹Middle East Technical University, Ankara, Turkey

2 Institute of Nuclear Physics, M. V. Lomonosov Moscow State University, Moscow, Russia

(Received 7 September 2010; published 19 January 2011)

A new method for calculation of the mixing angle between the hadrons within QCD sum rules is proposed. In this method, the mixing is expressed in terms of quark and gluon degrees of freedom. As an application, the detailed calculation of the mixing angle between heavy cascade baryons Ξ_Q and Ξ_Q' , $Q = c$, b is presented and it is found that the mixing angle between Ξ_b (Ξ_c) and Ξ_b' (Ξ_c') is given by $\rho = 6.4^\circ + 1.8^\circ$ ($\rho = 5.5^\circ + 1.8^\circ$) $\theta_b = 6.4^{\circ} \pm 1.8^{\circ}$ ($\theta_c = 5.5^{\circ} \pm 1.8^{\circ}$).

DOI: [10.1103/PhysRevD.83.016008](http://dx.doi.org/10.1103/PhysRevD.83.016008) PACS numbers: 11.55.Hx, 14.20.Lq, 14.20.Mr

I. INTRODUCTION

During the last years, many new and unexpected experimental data appeared on heavy hadron spectroscopy [[1\]](#page-4-0). The study of spectroscopy and decays of heavy hadrons can give essential information on the quark structure of these hadrons, in particular, of the properties of the cascade baryons.

The impressive progress on the experimental physics stimulated comprehensive theoretical studies for understanding dynamics of heavy flavored hadrons at the hadronic scale. This scale belongs to the nonperturbative sector of QCD and therefore for the calculations of different characteristics of the baryons, a nonperturbative approach is needed. Among existing nonperturbative methods, the QCD sum rules method [[2\]](#page-4-1) is one of the most predictive in studying properties of hadrons. The main ingredient of the QCD sum rules method is the choice of the interpolating currents which is directly related to the quark content of the corresponding baryons.

Approximate flavor symmetries are useful tools in classifying hadrons. Breaking of these symmetries might lead to mixing of hadrons which differ only in the flavor quantum numbers. For example, two baryons that mix due to flavor symmetry violation are Ξ_Q and Ξ_Q' baryons, both of which are made up of $q (q = u, d)$, s and $Q (Q = b, c)$ quarks. This mixing of charmed Ξ_c baryons has been calculated using the quark model in [\[3](#page-4-2),[4](#page-4-3)] and using heavy quark effective theory with $1/m_Q$ corrections, in [\[5\]](#page-4-4). (More about the mixing problem in the meson sector can be found in [\[6](#page-4-5)].) In [\[7](#page-4-6)], it was shown that this mixing can be important in determining properties of these baryons and a framework has been proposed to calculate this mixing within the QCD sum rules method. In the present work, we demonstrate the new approach for the calculation of the mixing angle between hadrons in the QCD sum rules framework and an application of the proposed method to study the mixing angle between the Ξ_Q and Ξ_Q' baryons carried out. The central assumption of this work is that the physical states are related to the ''pure'' states by an orthogonal transformation and hence are also orthogonal. As will be shown below, considering the nondiagonal correlation function, one can obtain the mixing angle in terms of the quark and gluon degrees of freedom.

II. MIXING OF THE INTERPOLATING CURRENTS

Approximate $SU(3)_f$ flavor symmetry of QCD allows us to identify the observed hadrons with a multiplet of the $SU(3)_f$ group. If this symmetry were exact, the observed hadrons all would have definite flavor quantum numbers under this flavor symmetry. The mass differences between the light u , d and s quarks, breaks this symmetry explicitly and hence the mass eigenstates need not be definite eigenstates of the $SU(3)_f$ symmetry, i.e. definite flavor eigenstates can mix to form the physically observed particles (unless there is another symmetry which prevents this mixing).

The mass sum rules can be obtained by considering the following two-point correlation function (for more information about the mass sum rules for baryons, see e.g. [[8](#page-4-7)])

$$
\Pi = i \int d^4x e^{ipx} \langle 0| \mathcal{T} \eta_H(x) \bar{\eta}_H |0\rangle \tag{1}
$$

where T is the time ordering operator and η_H is an operator that can create the hadron H from the vacuum. If the pure H_1^0 and H_2^0 states mix, then the physical states, i.e. states that have definite mass, should be represented as a linear combination of these states. In such a case, currents corresponding to physical states should also be written as a superposition of the pure operators:

$$
\eta_{H_1} = \cos\theta \eta_{H_1^0} + \sin\theta \eta_{H_2^0}
$$

$$
\eta_{H_2} = -\sin\theta \eta_{H_1^0} + \cos\theta \eta_{H_2^0}
$$
 (2)

where θ is the mixing angle between H_1 and H_2 .

[^{*}P](#page-0-3)ermanent institute: Institute of Physics, Baku, Azerbaijan [†](#page-0-3) taliev@metu.edu.tr

[[‡]](#page-0-3) ozpineci@metu.edu.tr

x zamir@depni.sinp.msu.ru

T. M. ALIEV, A. OZPINECI, AND V. ZAMIRALOV PHYSICAL REVIEW D 83, 016008 (2011)

Consider the following two-point correlation function:

$$
\Pi_{H_1H_2}(p) = i \int d^4x e^{ipx} \langle 0| \mathcal{T} \{ \eta_{H_1}(x) \bar{\eta}_{H_2}(0) \} | 0 \rangle.
$$
 (3)

It is natural to expect that if $\eta_{H_{1(2)}}$ creates only $H_{1(2)}$ and not
the other are this completion function should be zero. the other one, this correlation function should be zero. Hence, the angle θ should be chosen in such a way as to give $\Pi_{H_1H_2} = 0$. That is, the eventual mixing mass sum rules should vanish rules should vanish.

The general form of the correlation function can be written as

$$
\Pi_{H_1 H_2}(p) = \hat{p} \Pi_1(p^2) + \Pi_2(p^2)
$$
 (4)

where $p = \gamma_{\mu} p^{\mu}$, and $\Pi_i(p^2)$, $(i = 1 \text{ or } i = 2)$, are some
Lorenz inverient functions. Generally speaking, any of the Lorenz invariant functions. Generally speaking, any of the functions Π_i ($i = 1, 2$) can be used for the sum rules
analysis. In this work, the coefficient of the *b* structure analysis. In this work, the coefficient of the \dot{p} structure has been chosen to obtain a prediction for the mixing angle.

Using the notation

$$
\Pi_{H_i H_j}^0 = i \int d^4x e^{ipx} \langle 0| \mathcal{T} \{ \eta_{H_i^0}(x) \bar{\eta}_{H_j^0}(0) \} |0 \rangle, \quad (5)
$$

it is straightforward to show that the angle θ that makes $\Pi_{H_1H_2} = 0$ can be expressed as:

$$
\tan 2\theta = \frac{-ac \pm b\sqrt{b^2 + a^2 - c^2}}{-bc \mp a\sqrt{b^2 + a^2 - c^2}} \tag{6}
$$

where $a = \frac{1}{2}(\Pi_{H_2H_2}^0 - \Pi_{H_1H_1}^0)$, $b = \frac{1}{2}(\Pi_{H_1H_2}^0 + \Pi_{H_2H_1}^0)$ and $c = \frac{1}{2}(\Pi_{H_1H_2}^0 - \Pi_{H_2H_1}^0)$. This expression can be simplified further by noting that as explicit coloulation beginning. plified further by noting that, as explicit calculation has shown, $\Pi_{H_2H_1}^0 = \Pi_{H_1H_2}^0$, i.e. $c = 0$, which yields

$$
\tan 2\theta = -\frac{b}{a} = \frac{2\Pi_{H_2H_1}^0}{\Pi_{H_1H_1}^0 - \Pi_{H_2H_2}^0}
$$
(7)

Note that, Eq. [\(7\)](#page-1-0) has two solutions for $0^{\circ} < \theta < 180^{\circ}$ that differ by 90° . In this work, we present the solution that is close to 0° . The other solution corresponds to exchanging the identification of the dominant part of $H_{1(2)}$ with $H_{2(1)}^0$. $2(1)$ Further studies of other properties of $H_{1(2)}$ baryons is necessary to remove this ambiguity.

After setting up the framework for the study of the mixing angle between the hadrons, let us concentrate on this mixing angle of the heavy cascade baryons. In $SU(3)_f$ classification, baryons containing two light and one heavy quarks can be grouped into an antitriplet and a sextet representation. It is well known that the dominant components of Ξ_{Q} and Ξ_{Q}^{\prime} belong to the antitriplet and sextet representations of $SU(3)_f$ respectively, i.e. $\mathbb{E}_{\mathcal{Q}}(\mathbb{E}'_Q)$ is
contrarium at the original continuous commutation in the original contrary theory approximately antisymmetric (symmetric) under the exchange of light quarks. The reason why they are not exactly (anti)symmetric is the mixing between the $SU(3)_f$ representations. Note that, in the infinite heavy quark mass limit, there is an additional conserved quantity: the total angular

momentum of the light degrees of freedom, s_l . Since Ξ_Q^0 and $\Xi_0^{\prime 0}$ correspond to different values of s_l , this additional symmetry prevents the mixing of these two states.

As we have already noted, the unmixed Ξ_Q and Ξ_Q' belong to the antitriplet and the sextet representations of $SU(3)_f$ respectively. Therefore, corresponding interpolating currents should be antisymmetric and symmetric. For this reason the interpolating currents for the unmixed states can be chosen as:

$$
\eta_{\Xi_Q}^0 = \frac{1}{\sqrt{6}} \epsilon^{abc} [2(s_a^T C q_b) \gamma_5 Q_c + 2t(s_a^T C \gamma_5 q_b) Q_c \n+ (s_a^T C Q_b) \gamma_5 q_c + t(s_a^T C \gamma_5 Q_b) q_c \n- (q_a^T C Q_b) \gamma_5 s_c - t(q_a^T C \gamma_5 Q_b) s_c]\n\eta_{\Xi_Q'}^0 = \frac{1}{\sqrt{2}} \epsilon^{abc} [(s_a^T C Q_b) \gamma_5 q_c + t(s_a^T C \gamma_5 Q_b) q_c \n+ (q_a^T C Q_b) \gamma_5 s_c + t(q_a^T C \gamma_5 Q_b) s_c]
$$
\n(8)

Here a, b and c are the color indices, $q = u$ or d and t is an arbitrary auxiliary parameter. Note that the general form of the interpolating currents for the octet baryons were intro-duced first in [\[9\]](#page-4-8). The $t = -1$ case, corresponds to the Ioffe current. The notation η^0 is used to denote that these operators are pure $\bar{3}$ or 6 operators.

The correlation function, Eq. ([1](#page-0-4)), can be calculated in terms of quark and gluon degrees of freedom in the deep Euclidean region, $p^2 \ll 0$, using operator product expansion. The analytical results for the corresponding correlators are presented in the appendix.

III. NUMERICAL ANALYSIS

In this section, we present the numerical analysis of our results. The numerical values for the input parameters are $\langle \bar{q}q \rangle = (-0.243 \text{ GeV})^3, \frac{\langle \bar{s}s \rangle}{\langle \bar{q}q \rangle} = 0.8, \langle g_s^2 G^2 \rangle = 0.47 \text{ GeV}^4,$
 $m_0^2 = 0.8 \text{ GeV}^2, \quad m_b = 4.8 \text{ GeV}, \quad m_c = 1.4 \text{ GeV}$ and
 $m_1 = 0.14 \text{ GeV}$ $m_s = 0.14$ GeV.

The sum rules results depend on three auxiliary parameters: the continuum threshold s_0 , the Borel parameter M^2 and the arbitrary parameter t in the interpolating current. The continuum threshold should be close to the first excited state that can couple to the current. In our analysis, it is chosen to be near $s_0 = (m_B + 0.5)^2$, where $m_B =$
2.5 GeV for the $\Xi^{(')}$ how top and $m_A = 5.8$ GeV for the 2.5 GeV for the $\Xi_c^{\text{()}\text{}}$ baryons and $m_B = 5.8$ GeV for the $\Xi^{(\text{()}}\text{)}$ baryons. The layer limit of the working region of the Ξ_b^{C} baryons. The lower limit of the working region of the Borel parameter can be found by requiring that the contribution of the highest dimensional operator is less than 20% of the perturbative term. Its upper limit is determined in such a way that the contribution of the higher states and the continuum is less than the contribution of the first pole.

In Figs. [1](#page-2-0) and [2](#page-2-1), the dependence of θ_h (θ_c) on the Borel parameter is plotted, at $t = -3$. For Ξ_b , the continuum threshold is chosen to be $s_0 = 40 \text{ GeV}^2$, and 42 GeV², and for Ξ it is chosen to be $s_0 = 9 \text{ GeV}^2$ and 10 GeV². for Ξ_c , it is chosen to be $s_0 = 9 \text{ GeV}^2$ and 10 GeV². It is

FIG. 1. The dependence of θ_b on the Borel parameter for $t = -3$ and for two different values of the continuum threshold s_0 .

FIG. 2. The same as Fig. [1,](#page-2-0) but for θ_c .

seen that although the Borel parameter is allowed to change in a wide range of values, there is practically no dependence of the predictions of the sum rules on the value of the Borel parameter.

In Figs. [3](#page-2-2) and [4,](#page-2-3) the mixing angle θ_b (θ_c) is plotted as the function of cos α , where α is defined through $t = \tan \alpha$ at $M^2 = 10 \text{ GeV}^2$ and 15 GeV² ($M^2 = 5$ and 8 GeV²). It is seen that in both graphs, the mixing angle takes very large values at certain values of $cos \alpha$. This enhancement can be understood by noting that the tangent of the mixing angle, Eq. [\(7](#page-1-0)), is a ratio of two sum rules. In principal, both the numerator and the denominator should become zero at the same value of t. But due to approximations in the sum rules, their zeros are shifted. Hence, when the denominator becomes zero, and the numerator is nonzero (although small), tan θ diverges. Near these points the sum rules are not reliable, hence in obtaining a prediction for the mixing angle, one should keep away from these points. From the figures, it is seen that, sum rule predictions on the mixing angle are almost independent of the value of t chosen.

Finally, the predictions on the mixing angle between Ξ_0 and Ξ_Q' baryons of QCD sum rules are:

$$
\theta_b = 6.4^\circ \pm 1.8^\circ \tag{9}
$$

FIG. 3. The dependence of θ_b on cos α , where $t = \tan \alpha$, for two different values of the continuum threshold, s_0 and the Borel parameter M^2 .

FIG. 4. The same as Fig. [3,](#page-2-2) but for θ_c .

for the $\Xi_b - \Xi'_b$ mixing and

$$
\theta_c = 5.5^\circ \pm 1.8^\circ \tag{10}
$$

for the $\Xi_c - \Xi_c'$ mixing. In these predictions, the uncer-
tainties are due to the neglected higher dimensional operatainties are due to the neglected higher dimensional operators and the uncertainties in the auxiliary parameters of the sum rules. The largest source of uncertainty is the variations in the continuum threshold s_0 . [I](#page-2-4)n Table I, sum rules predictions and the predictions of [\[3–](#page-4-2)[5](#page-4-4)] are presented. It is seen that, within errors, the predictions for θ_c are in agreement, whereas, our prediction on θ_h is in disagreement with the prediction of [\[4\]](#page-4-3).

As also mentioned previously, in the heavy quark limit, the mixing angle should be zero. The heavy quark limit of the sum rules can be obtained by taking the $m_Q \rightarrow \infty$ limit

TABLE I. The predictions of QCD sum rules on the mixing angle along with the prediction of quark model [\[3,](#page-4-2)[4\]](#page-4-3) and heavy quark effective theory [[5\]](#page-4-4).

θ_{O}	This Work	131	141	151
	$5.5^{\circ} \pm 1.8^{\circ}$	3.8°	3.8°	$14^{\circ} \pm 14^{\circ}$
θ_b	$6.4^{\circ} \pm 1.8^{\circ}$		10°	-

T. M. ALIEV, A. OZPINECI, AND V. ZAMIRALOV PHYSICAL REVIEW D 83, 016008 (2011)

after setting $M^2 \to m_Q^2 + 2m_Q T$, $s \to m_Q^2 + 2m_Q \nu$ and $s_0 \rightarrow m_Q^2 + 2m_Q \nu_0$, where T is the new Borel parameter, ν is the four velocity of the heavy baryon, and ν_0 is the threshold in the heavy quark limit. Using the expressions in the Appendix, it can be shown that, in the heavy quark limit $\tan\theta_Q \propto \frac{1}{m_Q^2}$. But the numeric results for the b and c quarks show that the suppression of the mixing angle in the heavy quark limit is not realized at the physical b quark mass. The suppression starts at much larger values of the heavy quark mass.

In conclusion, a new method is presented for the determination of the mixing angle between the hadrons in QCD sum rules. This method is applied to the calculation of the mixing angle between the heavy cascade hyperons Ξ_Q and Ξ'_Q .

ACKNOWLEDGMENTS

One of the authors (V. Z.) is grateful to B. L. Ioffe for useful discussion at the early stages of the work.

APPENDIX: ANALYTICAL RESULTS

In this appendix, we present the explicit expression for the coefficient of the ρ structure in the correlation function

$$
\Pi_{\Xi\Xi}^{0} = -\frac{1}{288M^{4}} e^{-(m_{Q}^{2}/M^{2})} m_{0}^{2} \langle \bar{q}q \rangle \langle \bar{s}s \rangle m_{Q}(m_{s}(-1+t)^{2} + 2m_{Q}(-13+2t+11t^{2})) \n+ \frac{1}{144M^{2}} e^{-(m_{Q}^{2}/M^{2})} m_{s} m_{Q} \langle \bar{q}q \rangle \langle \bar{s}s \rangle (1+4t-5t^{2}) \Big(1 - \frac{5}{12} \frac{m_{Q}^{2} m_{0}^{2}}{M^{4}} \Big) \n+ \frac{1}{288M^{2}} e^{-(m_{Q}^{2}/M^{2})} m_{0}^{2} \langle \bar{q}q \rangle \langle \bar{s}s \rangle (25-2t-23t^{2}) - \frac{1}{72} e^{-(m_{Q}^{2}/M^{2})} \langle \bar{q}q \rangle \langle \bar{s}s \rangle (13-2t-11t^{2}) \n+ \frac{1}{768\pi^{2}} e^{-(m_{Q}^{2}/M^{2})} m_{s} m_{0}^{2} (3 \langle \bar{s}s \rangle (1+t)^{2} + 2 \langle \bar{q}q \rangle (-13+2t+11t^{2})) + \int_{m_{Q}^{2}}^{\infty} dse^{-(s/M^{2})} \rho_{11}(s) \tag{A1}
$$

where

$$
\rho_{11}(s) = \frac{3m_{Q}^{4}}{512\pi^{4}}(5+2t+5t^{2})\ln(1+\hat{s}) + \frac{3m_{Q}^{3}}{384\pi^{2}}\hat{s}\frac{(2+\hat{s})}{(1+\hat{s})^{2}}\left[\frac{m_{Q}}{16\pi^{2}}(-6-6\hat{s}+\hat{s}^{2}) + \frac{m_{s}}{m_{Q}^{3}}\langle\bar{s}s\rangle\right](5+2t+5t^{2}) + \frac{1}{1536\pi^{4}}m_{Q}^{3}m_{s}(-1-4t+5t^{2})\left[6\ln(1+\hat{s})-\hat{s}\frac{6+9\hat{s}+2\hat{s}^{2}}{(1+\hat{s})^{2}}\right] - \frac{1}{192\pi^{2}}m_{Q}(\langle\bar{q}q\rangle) + \langle\bar{s}s\rangle\right)\frac{\hat{s}^{2}}{(1+\hat{s})^{2}}(-1-4t+5t^{2}) - \frac{1}{192\pi^{2}}m_{s}\hat{s}\frac{2+\hat{s}}{(1+\hat{s})^{2}}\langle\bar{q}q\rangle(-13+2t+11t^{2}) - \frac{1}{768m_{Q}\pi^{2}}m_{0}^{2}(\langle\bar{q}q\rangle) + \langle\bar{s}s\rangle)\frac{1}{(1+\hat{s})^{2}}(-1+t)(-1-5t+6\hat{s}(1+t)) + \frac{1}{768m_{Q}^{2}\pi^{2}}m_{s}m_{0}^{2}(\langle\bar{q}q\rangle + \langle\bar{s}s\rangle)\frac{1}{(1+\hat{s})^{2}}(-1+t)^{2} + \frac{1}{16m_{Q}^{2}\pi^{2}}m_{s}m_{0}^{2}\langle\bar{q}q\rangle\frac{1}{(1+\hat{s})^{3}}(1-t^{2})\ln\hat{s} - \frac{1}{32\pi^{2}M^{2}}m_{s}m_{0}^{2}\langle\bar{q}q\rangle(-1+t^{2})\left[\ln\frac{m_{Q}^{2}}{\Lambda^{2}} + \left(1+\frac{1}{(1+\hat{s})^{2}}\right)\ln\hat{s}\right] \quad (A2)
$$

$$
\Pi_{\Xi'\Xi'}^0 = -\frac{1}{48M^4} e^{-(m_Q^2/M^2)} m_0^2 \langle \bar{q}q \rangle \langle \bar{s}s \rangle m_Q (-1+t)(m_s(1+t) + m_Q(-1+t)) \n+ \frac{1}{16M^2} e^{-(m_Q^2/M^2)} m_s m_Q \langle \bar{q}q \rangle \langle \bar{s}s \rangle (1-t^2) \left(1 - \frac{5}{12} \frac{m_Q^2 m_0^2}{M^4}\right) - \frac{1}{96M^2} e^{-(m_Q^2/M^2)} m_0^2 \langle \bar{q}q \rangle \langle \bar{s}s \rangle (-1+t)^2 \n+ \frac{1}{24} e^{-(m_Q^2/M^2)} \langle \bar{q}q \rangle \langle \bar{s}s \rangle (-1+t)^2 + \frac{1}{768\pi^2} e^{-(m_Q^2/M^2)} m_s m_0^2 (-\langle \bar{s}s \rangle (13+10t+13t^2) \n+ 6\langle \bar{q}q \rangle (-1+t)^2)) + \int_{m_Q^2}^{\infty} ds e^{-(s/M^2)} \rho_{22}(s)
$$
\n(A3)

where

MIXING ANGLE OF HADRONS IN QCD: A NEW VIEW

$$
\rho_{22}(s) = \frac{3m_{\mathcal{Q}}^4}{512\pi^4} (5 + 2t + 5t^2) \ln(1 + \hat{s}) + \frac{m_{\mathcal{Q}}^3}{128\pi^2} \hat{s} \frac{(2 + \hat{s})}{(1 + \hat{s})^2} \left[\frac{m_{\mathcal{Q}}}{16\pi^2} (-6 - 6\hat{s} + \hat{s}^2) + \frac{m_s}{m_{\mathcal{Q}}^3} \langle \bar{s}s \rangle \right] (5 + 2t + 5t^2)
$$

+
$$
\frac{3}{512\pi^4} m_{\mathcal{Q}}^3 m_s (-1 + t^2) \left[6 \ln(1 + \hat{s}) - \hat{s} \frac{6 + 9\hat{s} + 2\hat{s}^2}{(1 + \hat{s})^2} \right] - \frac{3}{64\pi^2} m_{\mathcal{Q}} (\langle \bar{q}q \rangle + \langle \bar{s}s \rangle) \frac{\hat{s}^2}{(1 + \hat{s})^2} (-1 + t^2)
$$

-
$$
\frac{1}{64\pi^2} m_s \hat{s} \frac{2 + \hat{s}}{(1 + \hat{s})^2} \langle \bar{q}q \rangle (-1 + t)^2 - \frac{1}{256m_{\mathcal{Q}} \pi^2} m_0^2 (\langle \bar{q}q \rangle + \langle \bar{s}s \rangle) \frac{6\hat{s} - 7}{(1 + \hat{s})^2} (-1 + t^2)
$$

+
$$
\frac{1}{256m_{\mathcal{Q}}^2 \pi^2} m_s m_0^2 \frac{1}{(1 + \hat{s})^2} [-\langle \bar{q}q \rangle (-1 + t)^2 + \langle \bar{s}s \rangle (3 + 2t + 3t^2)] \tag{A4}
$$

$$
\sqrt{3}\Pi_{\Xi\Xi'}^{0} = \sqrt{3}\Pi_{\Xi'\Xi}^{0} = -\frac{1}{192M^{4}}e^{-(m_{Q}^{2}/M^{2})}m_{s}m_{Q}m_{0}^{2}\langle\bar{q}q\rangle\langle\bar{s}s\rangle(-3+2t+t^{2})
$$

$$
-\frac{1}{24M^{2}}e^{-(m_{Q}^{2}/M^{2})}m_{s}m_{Q}\langle\bar{q}q\rangle\langle\bar{s}s\rangle(-2+t+t^{2})\left(1-\frac{5}{12}\frac{m_{Q}^{2}m_{0}^{2}}{M^{4}}\right)+\int_{m_{Q}^{2}}^{\infty}dse^{-(s/M^{2})}\rho_{12}(s) \tag{A5}
$$

where

$$
\rho_{12}(s) = -\frac{1}{256\pi^4} m_0^3 m_s (-2 + t + t^2) \left[6\ln(1 + \hat{s}) - \hat{s} \frac{6 + 9\hat{s} + 2\hat{s}^2}{(1 + \hat{s})^2} \right] - \frac{1}{32\pi^2} m_Q(\langle \bar{q}q \rangle - \langle \bar{s}s \rangle) \frac{\hat{s}^2}{(1 + \hat{s})^2} (-2 + t + t^2) \n- \frac{1}{128m_Q \pi^2} m_0^2(\langle \bar{q}q \rangle - \langle \bar{s}s \rangle) \frac{1}{(1 + \hat{s})^2} (-1 + t)(-3 - 2t + 3\hat{s}(1 + t)) \n- \frac{1}{128m_Q^2 \pi^2} m_s m_0^2 \frac{1}{(1 + \hat{s})^2} (\langle \bar{q}q \rangle (-1 + t^2) + 2\langle \bar{s}s \rangle (1 + t + t^2))
$$
\n(A6)

where $\hat{s} = \frac{s}{m_o^2} - 1$.

The contribution of the higher states and the continuum is subtracted using quark hadron duality. It amounts to replacing the upper limit of integration in s with s_0 , i.e.

$$
\int_{m_Q^2}^{\infty} ds \to \int_{m_Q^2}^{s_0} ds \tag{A7}
$$

- [1] M. Nielsen, F. S. Navarra, and S.-H. Lee, Phys. Rep. 497, 41 (2010).
- [2] M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Nucl. Phys. B147, 385 (1979).
- [3] J. Franklin, Phys. Rev. D 55, 425 (1997).
- [4] J. Franklin, D.B. Lichtenberg, W. Namgung, and D. Carydas, Phys. Rev. D 24, 2910 (1981).
- [5] T. Ito and Y. Matsui, Prog. Theor. Phys. 96, 659 (1996).
- [6] Th. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D 58, 114006 (1998).
- [7] T.M. Aliev, A. Ozpineci, and V.S. Zamiralov, Phys. At. Nucl. 73, 1754 (2010).
- [8] V.M. Belyaev and B.L. Ioffe, J. Exp. Theor. Phys. 83, 876 $(1982).$
- [9] Y. Chung, H. G. Dosch, M. Kremer, and D. Schall, Nucl. Phys. B197, 55 (1982).