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# Nature of the $X(3872)$ from QCD

S. Narison*

*Laboratoire de Physique Théorique et Astroparticules, CNRS-IN2P3 and Université de Montpellier II, Case 070, Place Eugène Bataillon, 34095 Montpellier Cedex 05, France*

F. S. Navarra† and M. Nielsen‡

*Instituto de Física, Universidade de São Paulo, C.P. 66318, 05389-970 São Paulo, SP, Brazil*

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We have studied some possible four-quark and molecule configurations of the $X(3872)$ using double ratios of sum rules, which are more accurate than the usual simple ratios often used in the literature to obtain hadron masses. We found that the different structures ($\bar{3}-3$ and $\bar{6}-6$ tetraquarks and $D-D^{(*)}$ molecule) lead to the same prediction for the mass (within the accuracy of the method), indicating that the alone prediction of the $X$ mass may not be sufficient to reveal its nature. In doing these analyses, we also find that (within our approximation) the use of the $\overline{\rm MS}$ running $\bar{m}_c(m_c^2)$, rather than the on-shell mass, is more appropriate to obtain the $J/\psi$ and $X$ meson masses. Using vertex sum rules to roughly estimate the $X(3872)$ hadronic and radiative widths, we found that the available experimental data does not exclude a $\lambda - J/\psi$-like molecule current.

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## I. INTRODUCTION

The nature of the narrow ($\leq 2.3$ MeV width) $X(3872)$ decaying to $J/\psi\pi^+\pi^-$ [1] discovered by BELLE in $B$ decays [2] and confirmed by *BABAR* [3], CDF [4], and D0 [5] in hadronic productions remains puzzling. Different scenarios (four-quark state, molecule, large mixing with conventional $\bar{c}c$ states) have been evoked in the literature [6,7]. In this work we use QCD spectral sum rules (QSSR) (the Borel/Laplace sum rules [8–10]) in order to test the previous four-quark and molecule scenarios.

In a previous calculation [11] some of us and our collaborators have considered the $X(3872)$ as being a tetraquark state where the diquark-antidiquark pairs are in the $\bar{3}-3$ color configuration. *A priori*, the diquark-antidiquark pairs could also be in a $\bar{6}-6$ color configuration. This system is expected to be too weakly bound by a two-body potential but it could be bound by a four-body potential, such as the one of the Steiner model [12]. In this work we shall, for the first time, investigate this configuration using QSSR.

Using QSSR, we shall, also for the first time, analyze the mass and hadronic width of a $\lambda - J/\psi$-like molecule,[1] which we shall compare with the ones of the four-quark states.

## II. THE INTERPOLATING $X$ CURRENTS

In order to study the two-point functions of the $X(3872)$ meson assumed to be an $1^{++}$ axial-vector meson, the interpolating current which describes the $X(3872)$ as a diquark-antidiquark system in the $\bar{3}-3$ color configuration with total $J^{PC}=1^{++}$ is [11]

$$j_3^\mu = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}}[(q_a^T C\gamma_5 c_b)(\bar{q}_d\gamma^\mu C\bar{c}_e^T) + (q_a^T C\gamma^\mu c_b) \times(\bar{q}_d\gamma_5 C\bar{c}_e^T)], \quad (1)$$

while for a diquark-antidiquark in the color sextet ($\bar{6}-6$) configuration, the interpolating current is

$$j_6^\mu = \frac{i}{\sqrt{2}}[(q_a^T C\gamma_5\lambda_{ab}^S c_b)(\bar{q}_d\gamma^\mu C\lambda_{de}^S\bar{c}_e^T) + (q_a^T C\gamma^\mu\lambda_{ab}^S c_b)(\bar{q}_d\gamma_5 C\lambda_{de}^S\bar{c}_e^T)], \quad (2)$$

where $a, b, c, \ldots$ are color indices, $C$ is the charge conjugation matrix, $q$ denotes a $u$ or $d$ quark, and $\lambda^S$ stands for the six symmetric Gell-Mann matrices: $\lambda^S = (\lambda_0, \lambda_1, \lambda_3, \lambda_4, \lambda_6, \lambda_8)$.

These tetraquark currents can be compared with the one describing the $X$ state as a $D^* - D$ molecule:

$$j_{\rm mol}^\mu(x) = \left(\frac{g}{\Lambda}\right)^2_{\rm eff}\frac{1}{\sqrt{2}}[(\bar{q}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma^\mu q_b(x)) - (\bar{q}_a(x)\gamma^\mu c_a(x)\bar{c}_b(x)\gamma_5 q_b(x))], \quad (3)$$

and as a $\lambda - J/\psi$-like molecule current:

$$j_\lambda^\mu = \left(\frac{g'}{\Lambda}\right)^2_{\rm eff}(\bar{c}\lambda^a\gamma^\mu c)(\bar{q}\lambda_a\gamma_5 q), \quad (4)$$

where $\lambda_a$ is the color matrix.

In the molecule assignment it is assumed that there is an effective local current and the meson pairs are weakly bound by a van der Vaals force in a Fermi-like theory

*snarison@yahoo.fr
†navarra@if.usp.br
‡mnielsen@if.usp.br

[1] An analogous configuration has been studied within QSSR for light four-quark states in [13].

 

with a strength $(g/\Lambda)^2_{\rm eff}$ which has nothing to do with the quarks and gluons inside each meson.

## III. THE TWO-POINT CORRELATOR AND FORM OF THE SUM RULES

The two-point correlation function associated to the axial-vector current is defined as

$$\Pi_i^{\mu\nu}(q) = i\int d^4x e^{iq.x}\langle 0|T[j_i^\mu(x)j_i^{\nu\dagger}(0)]|0\rangle = -\Pi_{1i}(q^2)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \Pi_{0i}(q^2)\frac{q^\mu q^\nu}{q^2}, \tag{5}$$

where $i = 3, 6$, mol, $\lambda$ according to the currents in Eqs. (1)–(4). The two functions, $\Pi_1$ and $\Pi_0$, appearing in Eq. (5) are independent and have, respectively, the quantum numbers of the spin 1 and 0 mesons. Because of its analyticity, the correlation function, $\Pi_{1i}$, obeys a dispersion relation:

$$\Pi_{1i}(q^2) = \int_{4m_c^2}^\infty ds\frac{\rho_i(s)}{s-q^2} + \cdots, \tag{6}$$

where $\pi\rho_i(s) \equiv \mathrm{Im}[\Pi_{1i}(s)]$ is the spectral function. After making an inverse-Laplace (or Borel) transform on both sides, the sum rule and its ratio read

$$\mathcal{F}_i(\tau) = \int_{4m_c^2}^\infty dse^{-s\tau}\rho_i(s) \qquad \mathcal{R}_i(\tau) = -\frac{d}{d\tau}\log\mathcal{F}_i(\tau), \tag{7}$$

where $\tau \equiv 1/M^2$ is the sum rule variable with $M$ being the inverse-Laplace (or Borel) mass. In the following, we shall work with the double ratio of sum rules (DRSR):

$$r_{ij} = \sqrt{\frac{\mathcal{R}_i}{\mathcal{R}_j}} : i = 3, 6, \ldots \tag{8}$$

to obtain the $X$-meson mass. Defining the coupling of the current with the state through

$$\langle 0|j_i^\mu|X\rangle = \sqrt{2}f_X M_i^4\epsilon^\mu, \tag{9}$$

and using the minimal duality ansatz: "one resonance" $\oplus$ "QCD continuum", where the QCD continuum comes from the discontinuity of the QCD diagrams from a continuum threshold $t_c$, the phenomenological side of Eq. (5) can be written as

$$\Pi_{\mu\nu}^{\rm phen}(q^2) = \frac{2f_X^2 M_i^8}{M_i^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_i^2}\right) + \cdots, \tag{10}$$

where the Lorentz structure projects out the $1^{++}$ state. The dots denote higher axial-vector resonance contributions that will be parametrized, as usual, by the QCD continuum. Transferring the continuum contribution to the QCD side, the sum rules can be written in a finite energy form as

$$\begin{aligned}\mathcal{F}_i(\tau) &\equiv 2f_X^2 M_i^8 e^{-M_i^2\tau} = \int_{4m_c^2}^{t_c} dse^{-s\tau}\rho_i(s)\\ \mathcal{R}_i(\tau) &\equiv -\frac{d}{d\tau}\log\mathcal{F}_i(\tau) \simeq M_i^2,\\ r_{ij} &\equiv \sqrt{\frac{\mathcal{R}_i}{\mathcal{R}_j}} \simeq \frac{M_i}{M_j} : i = 3, 6, \ldots.\end{aligned} \tag{11}$$

## IV. THE QCD EXPRESSIONS OF THE TWO-POINT CORRELATORS

The QCD expressions of the spectral densities of the two-point correlator associated to the currents in Eqs. (1) and (3) have been obtained, respectively, in [11,14] and will not be reported here. The expression associated to the current in Eqs. (2) and (4) is new. Up to dimension-six condensates, we can write

$$\rho_i(s) = \rho_i^{\rm pert}(s) + \rho_i^{m_q}(s) + \rho_i^{\langle\bar{q}q\rangle}(s) + \rho_i^{\langle G^2\rangle}(s) + \rho_i^{\rm mix}(s) + \rho_i^{\langle\bar{q}q\rangle^2}(s). \tag{12}$$

The renormalization improved perturbative expression of the sum rule is given by

$$\mathcal{F}_i(\tau)|_{\rm pert} = (\alpha_s(\tau))^{-(\gamma_i/\beta_1)}A_i\left[1 + K_i\frac{\alpha_s}{\pi} + \cdots\right], \tag{13}$$

where $\gamma_i$ is the anomalous dimension of the corresponding correlator, $-\beta_1 = (1/2)(11 - 2n/3)$ is the first coefficient of the $\beta$ function for SU($n$) flavors, $A_i$ is the known LO expression, and $K_i$ is the radiative correction. By inspection we observe that in the ratio of moments $\mathcal{R}$ defined in Eq. (11), the $\alpha_s$ corrections disappear and only the radiative corrections induced by the anomalous dimensions of the currents survive. In the double ratios of sum rules (DRSR) which we shall use in this paper, this induced radiative correction will also disappear to $\mathcal{O}(\alpha_s)$ as the different currents studied (which have all the same Lorentz structure) have the same anomalous dimensions. Therefore, we expect that, although we work in leading order of the QCD expressions, our results for the ratios of masses are accurate up to order $\alpha_s$ for the perturbative contributions.

### A. 6-6 four-quark current

For the 6-6 current in Eq. (2), we get to lowest order in $\alpha_s$:

$$\rho_6^{\rm pert}(s) = \frac{1}{2^9\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)(1+\alpha+\beta)[(\alpha+\beta)m_c^2-\alpha\beta s]^4,$$

$$\rho_6^{m_q}(s) = -\frac{m_q}{2^2\pi^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha}\bigg\{-\frac{\langle\bar qq\rangle}{2^2}\frac{[m_c^2-\alpha(1-\alpha)s]^2}{(1-\alpha)}+\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}[(\alpha+\beta)m_c^2-\alpha\beta s]$$
$$\times\bigg[-m_c^2\langle\bar qq\rangle+\frac{\langle\bar qq\rangle}{2^2}[(\alpha+\beta)m_c^2-\alpha\beta s]+\frac{m_c}{2^5\pi^2\alpha\beta^2}(3+\alpha+\beta)(1-\alpha-\beta)[(\alpha+\beta)m_c^2-\alpha\beta s]^2\bigg]\bigg\},$$

$$\rho_6^{\langle\bar qq\rangle}(s) = -\frac{m_c\langle\bar qq\rangle}{2^4\pi^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha^2}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta}(1+\alpha+\beta)[(\alpha+\beta)m_c^2-\alpha\beta s]^2,$$

$$\rho_6^{\langle G^2\rangle}(s) = \frac{\langle g^2G^2\rangle}{2^8 3\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\int_{\beta_{min}}^{1-\alpha}\frac{d\beta}{\beta^2}[(\alpha+\beta)m_c^2-\alpha\beta s]\bigg[\frac{m_c^2(1-(\alpha+\beta)^2)}{\beta}+\frac{(1-2\alpha-2\beta)}{4\alpha}[(\alpha+\beta)m_c^2-\alpha\beta s]\bigg].$$

$$\rho_6^{\rm mix}(s) = \frac{m_c\langle\bar qg\sigma\cdot Gq\rangle}{2^5\pi^4}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\bigg[-\frac{2}{\alpha}(m_c^2-\alpha(1-\alpha)s)+\int_{\beta_{\rm min}}^{1-\alpha}d\beta[(\alpha+\beta)m_c^2-\alpha\beta s]\bigg(\frac{1}{\alpha}-\frac{\alpha+\beta}{2\beta^2}\bigg)\bigg].$$

$$\rho_6^{\langle\bar qq\rangle^2}(s) = \frac{m_c^2\rho\langle\bar qq\rangle^2}{6\pi^2}\sqrt{\frac{s-4m_c^2}{s}}, \tag{14}$$

where $m_c$, $\langle g^2G^2\rangle$, $\langle\bar qq\rangle$, $\langle\bar qg\sigma\cdot Gq\rangle$ are, respectively, the charm quark mass, gluon condensate, light quark, and mixed condensates; $\rho$ indicates the violation of the four-quark vacuum saturation. The integration limits are given by

$$\alpha_{\rm min} = \tfrac{1}{2}(1-v), \qquad \alpha_{\rm max} = \tfrac{1}{2}(1+v)$$
$$\beta_{\rm min} = \alpha m_c^2/(s\alpha - m_c^2), \tag{15}$$

where $v$ is the $c$-quark velocity:

$$v \equiv \sqrt{1-4m_c^2/s}. \tag{16}$$

### B. $\lambda - J/\psi$-like molecule current

For the current in Eq. (4), the corresponding spectral functions read

$$\rho_\lambda^{\rm pert}(s) = \frac{1}{2^7 3\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}\frac{d\alpha}{\alpha^3}\int_{\beta_{\rm min}}^{1-\alpha}\frac{d\beta}{\beta^3}(1-\alpha-\beta)((\alpha+\beta)m_c^2-\alpha\beta s)^3[(1+\alpha+\beta)((\alpha+\beta)m_c^2-\alpha\beta s)$$
$$-4m_c^2(1-\alpha-\beta)],$$

$$\rho_\lambda^{\langle\bar qq\rangle}(s) = \mathcal{O}(m_q),$$

$$\rho_\lambda^{\langle G^2\rangle}(s) = -\frac{\langle g^2G^2\rangle}{2^6 3\pi^6}\int_{\alpha_{\rm min}}^{\alpha_{\rm max}}d\alpha\bigg\{\int_{\beta_{\rm min}}^{1-\alpha}d\beta\bigg[\frac{m_c^2(1-\alpha-\beta)}{3\alpha^3}\bigg[m_c^2(1-\alpha-\beta)-[(\alpha+\beta)m_c^2-\alpha\beta s]$$
$$\times\bigg(4+\alpha+\beta+\frac{3}{\beta}(1-\alpha)\bigg)\bigg]-\frac{[(\alpha+\beta)m_c^2-\alpha\beta s]}{16\alpha\beta}((2+\alpha+\beta)m_c^2-\alpha\beta s)$$
$$-\frac{(1-\alpha-\beta)}{96\alpha^2\beta^2}[(\alpha+\beta)m_c^2-\alpha\beta s](3-\alpha-\beta)\bigg]+\frac{(m_c^2-\alpha(1-\alpha)s)^2}{16\alpha(1-\alpha)}\bigg\},$$

$$\rho_\lambda^{\rm mix}(s) = \mathcal{O}(m_q),$$

$$\rho_\lambda^{\langle\bar qq\rangle^2}(s) = \frac{2}{27\pi^2}\rho\langle\bar qq\rangle^2(s+2m_c^2)\sqrt{1-\frac{4m_c^2}{s}}, \tag{17}$$

where the integration limits have been defined in Eq. (15).

## V. CALIBRATION OF THE METHOD FROM $M_\psi$ AND CHOICE OF $m_c$

Using the QSSR method, one usually estimates the $J/\psi$ mass, from the ratio

$$\mathcal{R}_\psi = \frac{\int_{4m_c^2}^{t_c} ds\, s\rho_\psi(s)e^{-s\tau}}{\int_{4m_c^2}^{t_c} ds\rho_\psi(s)e^{-s\tau}} \simeq M_\psi^2, \tag{18}$$

where $\rho_\psi$ is the spectral density associated to the vector current:

$$J_\psi^\mu = \bar c\gamma^\mu c. \tag{19}$$

TABLE I. QCD input parameters. For the heavy quark masses, we use the range spanned by the running $\overline{\mathrm{MS}}$ mass $\bar{m}_Q(M_Q)$ and the on-shell mass from QSSR compiled in pages 602 and 603 of the book in [10]. The values of $\Lambda$ and $\hat{\mu}_d$ have been obtained from $\alpha_s(M_\tau) = 0.325(8)$ [18] and from the running masses: $(\bar{m}_u + \bar{m}_d)(2) = 7.9(3)$ MeV [19]. The original errors have been multiplied by 2 for a conservative estimate of the errors.

| Parameters | Values | Ref. |
|---|---|---|
| $\Lambda(n_f = 4)$ | $(324 \pm 15)$ MeV | [1,18] |
| $\hat{\mu}_d$ | $(263 \pm 7)$ MeV | [10,19] |
| $M_0^2$ | $(0.8 \pm 0.2)$ GeV$^2$ | [20–22] |
| $\langle \alpha_s G^2 \rangle$ | $(6 \pm 2) \times 10^{-2}$ GeV$^4$ | [15,18,23–29] |
| $\rho \alpha_s \langle \bar{d} d \rangle^2$ | $(4.5 \pm 0.3) \times 10^{-4}$ GeV$^6$ | [18,20,23] |
| $m_c$ | $(1.26 \sim 1.47)$ GeV | [1,10,19,28,30,31] |

The QCD expression of the vector correlator is known in the literature [8] including the $d = 8$ condensates [10]. The full expression of the exponential moments $\mathcal{R}_\psi$ is given in [15] and its expansion in $1/m_c$ can be found in [16]. For the numerical analysis we shall introduce the renormalization group invariant quantities $\hat{\mu}_q$ [17]:

$$\langle \bar{q} q \rangle(\tau) = -\hat{\mu}_q^3 (-\log \sqrt{\tau} \Lambda)^{2/-\beta_1}$$
$$\langle \bar{q} g \sigma \cdot G q \rangle(\tau) = -\hat{\mu}_q^3 (-\log \sqrt{\tau} \Lambda)^{1/-3\beta_1} M_0^2, \tag{20}$$

We have used the quark mass and condensate anomalous dimensions reported in [10]. We shall use the QCD parameters in Table I. At the scale where we shall work, and using the parameters in Table I, we deduce

$$\rho = 2.1 \pm 0.2, \tag{21}$$

which controls the deviation from the factorization of the four-quark condensates. We shall not include the $1/q^2$ term discussed in [32,33], which is consistent with the LO approximation used here as the latter has been motivated by a phenomenological parametrization of the larger order terms of the QCD series.

Including the $d = 4$ gluon condensate, we show in Fig. 1(a) the $\tau$- behavior of $M_\psi = \sqrt{\mathcal{R}_\psi}$, for a given $\sqrt{t_c} = 4.6$ GeV, from which the pQCD expression of the spectral density starts to be seen experimentally. From Fig. 1(a) we see that the gluon contribution plays an important role in stabilizing the result. In Fig. 1(b) we show the $t_c$ behavior of $M_\psi$ for two values of $\tau$. We see that the results are very stable against $t_c$. One can deduce from Figs. 1(a) and 1(b) that one can better reproduce the experimental value of $M_{J/\psi}$ using the running mass $\bar{m}_c(m_c)$ rather than the on-shell mass $m_c^{\mathrm{os}}$. This feature had already been noticed in [28,31], where a better convergence of the QCD perturbative series was found when working with the $\overline{\mathrm{MS}}$ mass. Therefore, in the following we shall only consider the running mass. We have checked that the use of the on-shell mass does not affect our result from the double ratio of sum rules as it was intuitively expected.

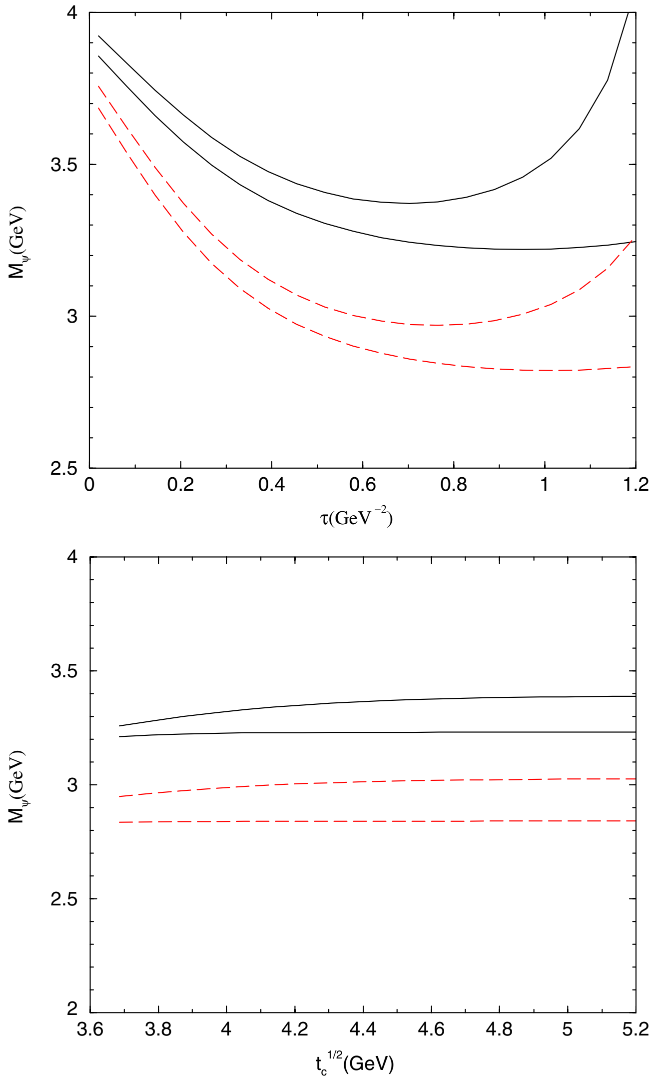


FIG. 1 (color online). The $J/\psi$ mass, $M_\psi = \sqrt{\mathcal{R}_\psi}$, as a function of (a) $\tau$ for $\sqrt{t_c} = 4.6$ GeV for two values of $m_c$. Solid line $m_c = 1.47$ GeV: upper line, LO + $\langle G^2 \rangle$; lower line, LO + NLO + $\langle G^2 \rangle$. Dashed line: the same as the solid line but for $m_c = 1.26$ GeV. (b) $t_c$ behavior of $M_\psi = \sqrt{\mathcal{R}_\psi}$, for two different values of $m_c$. Solid line for $m_c = 1.47$ GeV: $\tau = 0.4$ GeV$^{-2}$ (upper line), and $\tau = 0.8$ GeV$^{-2}$ (lower line). Dashed line for $m_c = 1.26$ GeV: $\tau = 0.4$ GeV$^{-2}$ (upper line) and $\tau = 0.8$ GeV$^{-2}$ (lower line).

## VI. $M_X$ FROM THE DRSR

### A. The $\bar{3} - 3$ tetraquark

Using QSSR, one can usually estimate the mass of the $X$ meson, from the ratio $\mathcal{R}_i$ analogue to the one in Eq. (18), where $i = 3, 6$ is related to the spectral densities obtained from the currents (1) and (2), respectively. The $\bar{3} - 3$

component of the $X$ mass has been studied with the help of the current (1). At the sum rule stability point and using a slightly different (though consistent) set of QCD parameters than in Table I, one obtains with a good accuracy [11]

$$M_3 \simeq \sqrt{\mathcal{R}_3} = (3925 \pm 127)\ \mathrm{MeV}, \tag{22}$$

and the correlated continuum threshold value fixed simultaneously by the Laplace and finite energy sum rules:

$$\sqrt{t_c|_3} \simeq (4.15 \pm 0.03)\ \mathrm{GeV}. \tag{23}$$

$M_3$ is in good agreement (within the errors) with the experimental candidate [1]:

$$M_X|_{\mathrm{exp}} \simeq \sqrt{\mathcal{R}_3} = (3872.2 \pm 0.8)\ \mathrm{MeV}, \tag{24}$$

while the relative low value of $t_c$ indicates that the next radial excitation of the $X$ meson can be in the range

$$M_{X'} \approx M_X + (225 \pm 127)\ \mathrm{MeV}. \tag{25}$$

This low value of $t_c$ suggests that the $\bar{3} - 3$ resonance may be difficult to separate from the QCD continuum and suggests also that it can be a wide resonance. Although the agreement with the experimental data is remarkable, the result may not be sufficient to provide a definite statement on the quark substructure of the $X$ meson.

### $\bar{6} - 6$ *over* $\bar{3} - 3$ *tetraquark*

A better understanding of the nature of the $X$, for discriminating different proposals, requires a more precise determination of $M_X$. This can be reached by considering the DRSR [16,22,34–37]:

$$r_{6/3} = \sqrt{\frac{\mathcal{R}_6}{\mathcal{R}_3}} \simeq \frac{M_6}{M_3}. \tag{26}$$

These quantities are less sensitive to the choice of the heavy quark masses, to the perturbative radiative corrections, and to the value of the continuum threshold than the simple ratios $\mathcal{R}_\psi$ and $\mathcal{R}_3$ in Eqs. (18) and (22). Fixing $\sqrt{t_c} = 4.15$ GeV [11], we show in Fig. 2(a) the $\tau$ behavior of $r_{6/3}$ (continuous line) for two values of $m_c$. One can notice that the result is very stable against the $\tau$ variation in a large range for $\tau \leq 0.8\ \mathrm{GeV}^{-2}$. We show in Fig. 2(b) its $t_c$ behavior (continuous line) for a given $\tau = 0.4\ \mathrm{GeV}^{-2}$ and $m_c = 1.26$ GeV. We deduce

$$r_{6/3} \simeq 1.00, \tag{27}$$

with a negligible error, which shows that, from a QCD spectral sum rules approach, the $X(3872)$ can be equally described by the currents in Eqs. (1) and (2).

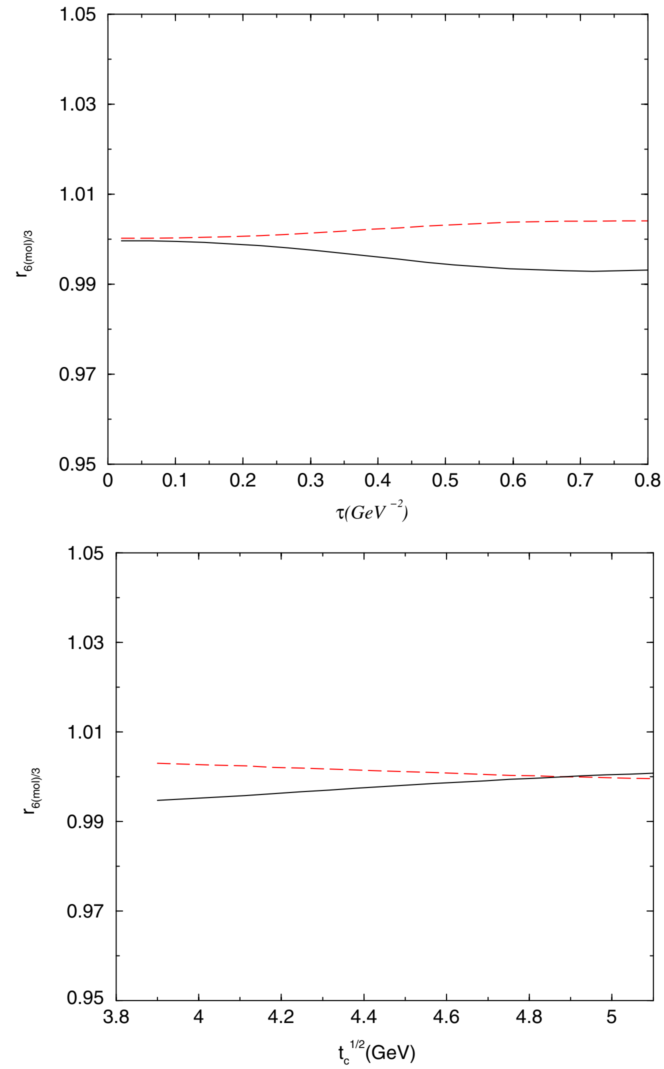


FIG. 2 (color online). The double ratio $r_{6/3}$ (solid line) defined in Eq. (26) and $r_{\mathrm{mol}/3}$ (dashed line) defined in Eq. (28): (a) as a function of $\tau$ for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c = 1.26$ and $1.47$ GeV; (b) as a function of $t_c$ for $\tau = 0.4\ \mathrm{GeV}^{-2}$ and $m_c = 1.26$ GeV.

## B. $D^* - D$ molecule over $\bar{3} - 3$ tetraquark

We can also work with the double ratio:

$$r_{\mathrm{mol}/3} = \sqrt{\frac{\mathcal{R}_{\mathrm{mol}}}{\mathcal{R}_3}}, \tag{28}$$

by using the spectral densities for the current (3). In Fig. 2(a) we also show the double ratio $r_{\mathrm{mol}/3}$ (dashed line) for $\sqrt{t_c} = 4.15$ GeV and for two values of $m_c$, while we show in Fig. 2(b) its $t_c$ behavior (dashed line) for a given $\tau = 0.4\ \mathrm{GeV}^{-2}$ and $m_c = 1.26$ GeV. One can deduce from the previous analysis

$$r_{\mathrm{mol}/3} \simeq 1.00, \tag{29}$$

also with a negligible error.

### C. $\lambda - J/\psi$-like molecule over the $\bar{3} - 3$ tetraquark

Using approaches similar to the previous ones, we study the ratio of the $\lambda - J/\psi$-like molecule over the tetraquark $\bar{3} - 3$ one. We show the analysis in Fig. 3 from which one can deduce at the $\tau$ and $t_c$ stability regions:

$$r_{\lambda/3} \equiv \frac{M_\lambda}{M_3} = 0.96 \pm 0.03, \tag{30}$$

where the errors come from the stability regions and $m_c$.[2]

### D. Comments on the results

(i) Our analysis has shown that the three substructure assignments for the $X$ meson ($\bar{3} - 3$ and $\bar{6} - 6$ tetraquarks and $D - D^{(*)}$ molecule) lead to (almost) the same mass predictions within the accuracy of the approach. Therefore, *a priori*, the alone study of the $X$ mass cannot reveal its nature if it is mainly composed by these substructures. From the previous analysis we observe that the distance between the continuum threshold (about 4 GeV) and the resonance masses (see, e.g., the ratio $r_{6/3}$ in Fig. 2) is relatively small. This indicates that the separation between the resonance and the continuum may be difficult to achieve. This feature is also signaled by the (almost) absence of the so-called sum rule window (a compromise region where the resonance dominates over the continuum contribution and where the QCD OPE is convergent) when one extracts the absolute mass of the $\bar{6} - 6$ mass. Then, as in the analysis of the wide $\sigma$ [38] and hybrid or some other large width states [10,39], we expect that the $\bar{6} - 6$[3] and, to a lesser extent, the $\bar{3} - 3$ four-quark or molecule $D - D^{(*)}$ states can be wide or/and weakly bound.

(ii) The analysis of the $\lambda - J/\psi$-like molecule mass in Eq. (30) shows that it can be lower than the other configurations studied previously.

(iii) In order to get a deeper understanding of the properties of these states, we shall, in what follows, compute their hadronic widths.

[2]The analysis of the ratio between the $\lambda$-molecule $J/\psi$-like current and $J/\psi$ mass is not conclusive within our approximation due to the absence of a stability region. The appearance of an inflexion point favors a lower value of the $\lambda$-molecule mass. However, analyzing the ratio of the 4-quark over the 2-quark correlators which do not necessarily optimize at the same $\tau$ values may be inappropriate.

[3]In a particular two-body potential model, one might expect that the $\bar{6} - 6$ tetraquark state can be weakly bound due to the repulsive force between the two quarks, but this may not necessarily be true for a more general potential [12].

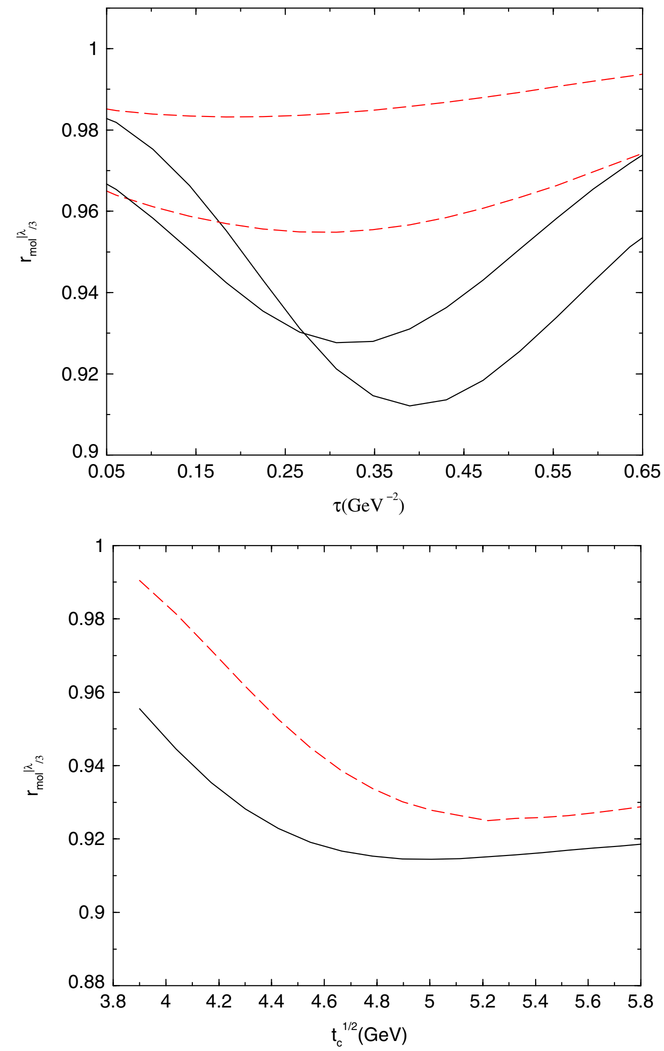


FIG. 3 (color online). The double ratios $r_{\lambda/3}$ of the $\lambda - J/\psi$-like molecule over the $\bar{3} - 3$ tetraquark masses defined in (30): (a) as a function of $\tau$ for $\sqrt{t_c} = 3.9$ GeV (dashed line) and 5 GeV (solid line). The upper and lower minima correspond, respectively, to $m_c = 1.47$ and 1.26 GeV; (b) as a function of $\sqrt{t_c}$ for $\tau = 0.35$ GeV$^{-2}$ and $m_c = 1.26$ GeV (solid line) and for $\tau = 0.3$ GeV$^{-2}$ and $m_c = 1.47$ GeV (dashed line).

## VII. CAN THE $X$-MESON HADRONIC WIDTH REVEAL ITS NATURE?

One can study the decays $X \to J/\psi + 3\pi$ and $X \to J/\psi + 2\pi$ using vertex sum rules [40], where the $2\pi$ and $3\pi$ can be assumed to come from the $\rho$ and $\omega$ mesons using vector meson dominance .[4] In so doing, one works with the three-point function:

[4]This approach assumes implicitly that the decay occurs through a direct coupling of the $X$ meson to $J/\psi$ and $\rho$, $\omega$ mesons where some eventual rescattering contributions (which could be important) have been neglected.

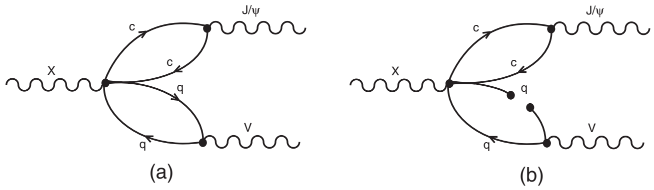


FIG. 4. Vertex diagrams contributing to the $X$ width for the diquark currents (1) and (2) and the molecular current (3).

$$\Pi^{\mu\nu\alpha}(p, p', q) \equiv \int d^4x d^4y e^{i(p'x+qy)} \times \langle 0|\mathcal{T} J^{\mu}_{\psi}(x) J^{\nu}_{V}(y) J^{\alpha\dagger}_{X}(0)|0\rangle, \quad (31)$$

associated to the $J/\psi$-meson $J^{\mu}_{\psi}$, to the vector mesons $J^{\nu}_{V}$, and to the $X$-meson $J_X$.

### A. The tetraquarks and $D^* - D$ molecule

In the case of the three $X$ currents ($\bar{3} - 3$, $\bar{6} - 6$ tetraquarks and molecule) discussed previously, the lowest order and lowest dimension correction (fall apart) QCD diagrams are shown in Fig. 4. An estimate of the $X - J/\psi - V$ coupling in [14,40] indicates that, if the $X$ is a pure $\bar{3} - 3$ tetraquark or a molecule state, one would obtain

$$g^{3,\mathrm{mol}}_{X\psi\omega} \simeq 14 \pm 2, \quad (32)$$

which would correspond to a width:

$$\Gamma^{3,\mathrm{mol}}_{X\to J/\psi+n\pi} \approx 50\ \mathrm{MeV}. \quad (33)$$

Doing an analogous analysis if the $X$ is a $\bar{6} - 6$ tetraquark state, one also obtains a similar value.

These previous results are too big compared with the data upper bound [2]:

$$\Gamma_{X\to\mathrm{all}} \leq 2.3\ \mathrm{MeV}. \quad (34)$$

### B. The $\lambda - J/\psi$-like molecule

Another possibility is to study the $\lambda - J/\psi$-like molecule current. In contrast to the case of previous currents, the leading order contribution to the three-point function is due to one gluon exchange in Fig. 5. The exact evaluation of these diagrams is technically involved. However, a rough approximation by including loop factors[5] leads to the coupling:

$$g^{\lambda}_{X\psi\omega} \approx \left(\frac{\alpha_s}{\pi}\right) g^{\mathrm{mol}}_{X\psi\omega} \approx 1, \quad (35)$$

where we have used $\alpha_s(M_X) \simeq 0.26$. This would correspond to a width

[5] A similar estimate has been done in [13] for explaining the too small $\gamma\gamma$ width of the $a_0(980)$ if it is a four-quark or molecule state.

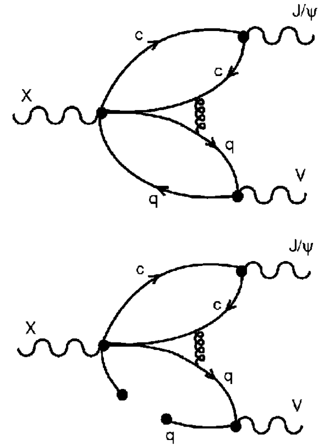


FIG. 5. Lowest order and lowest dimension vertex diagrams contributing to the $X$ width for the $\lambda$-molecule current in Eq. (4).

$$\Gamma^{\lambda}_{X\to J/\psi+n\pi} \approx 0.3\ \mathrm{MeV}, \quad (36)$$

which satisfies the previous experimental upper bound. Because of the rough approximation used in the estimate, we may expect that the result is known within a factor 2.

A similar rough approximation can be made to evaluate the radiative decay width $X(3872) \to J/\psi\gamma$. This decay was studied in Ref. [41] considering the $X(3872)$ as having charmonium ($c\bar{c}$) and molecular ($D\bar{D}^*$) components. In the case that $X$ is a pure $\bar{3} - 3$ tetraquark or a molecule state, one would obtain

$$\Gamma^{3,\mathrm{mol}}_{X\to J/\psi\gamma} \approx 3.4\ \mathrm{MeV}. \quad (37)$$

Therefore, using also in this case the rough approximation

$$\Gamma^{\lambda}_{X\to J/\psi\gamma} \approx \left(\frac{\alpha_s}{\pi}\right)^2 \Gamma^{\mathrm{mol}}_{X\to J/\psi\gamma}, \quad (38)$$

we get within a factor 2:

$$\Gamma^{\lambda}_{X\to J/\psi\gamma} \approx 0.02\ \mathrm{MeV}, \quad (39)$$

which also satisfies the experimental upper bound. From the results in Eqs. (36) and (39) we would get

$$\frac{\Gamma^{\lambda}_{X\to J/\psi\gamma}}{\Gamma^{\lambda}_{X\to J/\psi\pi\pi}} \approx 0.07. \quad (40)$$

Taking into account the rough approximation of a factor 2 used to estimate each width, this result can be consistent with the experimental value [42]:

$$\frac{\Gamma^{\mathrm{exp}}_{X\to J/\psi\gamma}}{\Gamma^{\mathrm{exp}}_{X\to J/\psi\pi\pi}} = 0.14 \pm 0.05. \quad (41)$$

However, our results do not exclude the possibility of $\bar{6} - 6$, $\bar{3} - 3$ tetraquark or $D - D^{(*)}$ molecule components and charmonium $c\bar{c}$ components in the $X(3872)$.

## VIII. CONCLUSIONS

(i) We have studied the mass of the $X(3872)$ using double ratios of sum rules, which are more accurate than the usual simple ratios used in the literature. We found that the different proposed configurations ($\bar{3} - 3$ and $\bar{6} - 6$ tetraquarks and $D - D^{(*)}$ molecule) lead to (almost) the same mass predictions within the accuracy of the method [see Eqs. (27) and (29)], indicating that the predictions of the $X$-meson mass is not enough to reveal its nature. However, the (relatively) small distance between the resonance mass and the continuum threshold in the QSSR analysis and also the (almost) absence of the sum rule window, indicate that these $\bar{3} - 3$ and $\bar{6} - 6$ tetraquarks and $D - D^{(*)}$ molecule states can be wide or weakly bound. These observations are also supported by their large hadronic decay widths from vertex sum rules analysis given in Eq. (33) [40].

(ii) Among these different proposals, the only eventual possibility which can lead to a $X(3872)$ with narrow hadronic and radiative widths consistent within the errors with the present data, is the choice of the $\lambda - J/\psi$-like molecule current given in Eq. (4).

(iii) Sharper tests of the previous results can be done from an explicit evaluation of the QCD vertex function and from a more precise experimental measurement of the ratio in Eq. (41). In this case, some mixing among different currents (see e.g. [14,41]) may help to improve the agreement between theory and experiment.

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