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# Mixed dark matter from technicolor

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<span id="page-0-5"></span>We study natural composite cold dark matter candidates which are pseudo-Nambu-Goldstone bosons (pNGB) in models of dynamical electroweak symmetry breaking. Some of these can have a significant thermal relic abundance, while others must be mainly asymmetric dark matter. By considering the thermal abundance alone we find a lower bound of  $m<sub>W</sub>$  on the pNGB mass when the (composite) Higgs is heavier than 115 GeV. Being pNGBs, the dark matter candidates are in general light enough to be produced at the LHC.

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## I. INTRODUCTION

A second strongly coupled sector in Nature akin to QCD is a likely possibility. The new strong interaction may naturally break the electroweak (EW) symmetry through the formation of a chiral condensate, thus making the standard model (SM) Higgs a composite particle. Models of this type are called ''technicolor'' (TC) [\[1,](#page-7-0)[2](#page-7-1)] and several new variants have been proposed recently [[3](#page-7-2)[–11\]](#page-7-3) with interesting dynamics relevant for collider phenomenology  $[12–15]$  $[12–15]$  as well as cosmology  $[10,16–36]$  $[10,16–36]$  $[10,16–36]$ . A review of these models and the phase diagram of strongly coupled theories can be found in Ref. [[37](#page-7-9)]. A relevant point is that the technicolor dynamics is strongly modified by the new interactions necessary to give masses to SM fermions [\[38\]](#page-7-10) and the interplay between these two sectors leads to an entirely new class of models, constraints on which were discussed in Ref. [[39\]](#page-7-11).

We discuss different possibilities for dark matter (DM) candidates within this rich framework and show that some of these composite states can be thermal relics while being sufficiently light to be produced at the LHC.

We call dark matter candidates composed of technicolor fields ''technicolor interacting massive particles'' (TIMPs) and focus on those which are pseudo-Nambu-Goldstone bosons (pNGB). Our analysis is general since we use a low energy effective description for the TIMPS which can easily be adapted for specific models. We will discuss some of these models [\[8](#page-7-12),[10](#page-7-6),[20](#page-7-13),[30](#page-7-14)] which provide particularly interesting candidates for dark matter.

# II. THE SIMPLEST TIMPS FROM PARTIALLY GAUGED TECHNICOLOR

An interesting class of TIMPs arise from partially gauged technicolor models [[6](#page-7-15),[40](#page-7-16)] in which only part of the TC group is gauged under the EW interactions. The EW gauged technifermions are organized in doublets in the usual way while the other technifermions are collectively denoted  $\lambda^f$ , with f counting these flavors only:

<span id="page-0-6"></span>
$$
Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \qquad U_R, \qquad D_R; \qquad \lambda^f. \tag{1}
$$

These models were introduced originally in order to yield the smallest naïve EW S parameter, while still being able to achieve walking dynamics.<sup>1</sup> The nonminimal flavor symmetry of the resulting model allows for a number of light states accessible at colliders. A similar scenario is envisaged in so-called conformal technicolor [\[43,](#page-7-17)[44\]](#page-7-18). The technifermions not gauged under the EW interactions essentially constitute a strongly interacting hidden sector.

To be specific we consider a scalar TIMP,  $\phi \sim \lambda \lambda$ , made of the SM gauge singlet technifermions  $\lambda^f$  and possessing a global  $U(1)$  symmetry protecting the lightest state against decay. Moreover we take  $\phi$  to be a pNGB from the breaking of chiral symmetry in the hidden sector, which leaves this  $U(1)$  unbroken. This constitutes the simplest type of

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<sup>&</sup>lt;sup>1</sup>The naïve S-parameter from a loop of technifermions counts the number of fermion doublets transforming under weak  $SU(2)_L$ , while walking dynamics is required to reduce nonperturbative contributions to the full S-parameter. The naïve S-parameter has recently been conjectured to be the absolute lower bound of the full S-parameter [\[41,](#page-7-19)[42](#page-7-20)], making the TC models presented here optimal with respect to satisfying the LEP precision data.

TIMP from the point of view of its interactions so we study this first and consider later TIMPs with constituents charged under the SM. An explicit model of partially gauged technicolor featuring this kind of TIMP is ''ultra minimal technicolor'' (UMT) [[8\]](#page-7-12). We therefore identify our DM candidate with a complex scalar  $\phi$ , singlet under SM interactions and charged under a new  $U(1)$  symmetry (not the usual technibaryon symmetry), which makes it stable.

In addition to the TIMP we consider a light (composite) Higgs boson. A general effective Lagrangian to describe this situation is presented below and can be derived, for any specific model, from the UMT Lagrangian [[8](#page-7-12)]. At low energies we can describe the interactions of  $\phi$  through a chiral Lagrangian. The (composite) Higgs  $H$  couples to the TIMP as

<span id="page-1-0"></span>
$$
\mathcal{L} = \partial_{\mu} \phi^* \partial_{\mu} \phi - m_{\phi}^2 \phi^* \phi + \frac{d_1}{\Lambda} H \partial_{\mu} \phi^* \partial_{\mu} \phi
$$
  
+ 
$$
\frac{d_2}{\Lambda} m_{\phi}^2 H \phi^* \phi + \frac{d_3}{2\Lambda^2} H^2 \partial_{\mu} \phi^* \partial_{\mu} \phi
$$
  
+ 
$$
\frac{d_4}{2\Lambda^2} m_{\phi}^2 H^2 \phi^* \phi.
$$
 (2)

The interactions between technihadrons such as  $\phi$  made of EW singlet constituents and states with EW charged constituents (e.g.  $H$ ) are due mainly to TC dynamics and, as such, the couplings between these two sectors are *not* suppressed [[8](#page-7-12)]. However, since  $\phi$  is a pNGB it must have either derivative couplings or the nonderivative couplings must vanish in the limit  $m_{\phi} \rightarrow 0$ . The mass  $m_{\phi}$  is assumed to come from interactions beyond the TC sector, e.g. ''extended technicolor'' (ETC) [\[45](#page-8-0)[,46\]](#page-8-1), which can provide masses for the TC Nambu-Goldstone bosons, as well as for SM fermions. The couplings  $d_1, \ldots, d_4$  are dimensionless and expected to be of  $O(1)$ , while  $\Lambda$  is the scale  $\Lambda \sim 4\pi F_{\pi}$  below which the derivative expansion is sensible.

We emphasize the differences between a composite scalar TIMP and fundamental scalar dark matter considered earlier  $[47-49]$  $[47-49]$  $[47-49]$ : (i) the  $U(1)$  is natural, i.e. it is identified with a global symmetry (not necessarily the technibaryon one), (ii) its pNGB nature makes the DM candidate naturally light with respect to the EW scale and influences the structure of its couplings, (iii) compositeness requires the presence around the EW scale of spin-1 resonances in addition to the TIMP and the (composite) Higgs; their interplay can lead to striking collider signatures [[10](#page-7-6),[30](#page-7-14)].

## III. THERMAL VERSUS ASYMMETRIC DARK MATTER

When the TIMP is a composite state made of particles charged under the EW interactions it becomes a good candidate for asymmetric dark matter, i.e. its present abundance is due to a relic asymmetry between the particle and its antiparticle, just as for baryons. This has been the case usually considered when discussing TC DM candidates [\[16](#page-7-7)[,50\]](#page-8-4) since the technibaryon self-annihilation cross section, obtained by scaling the proton-antiproton annihilation cross section up to the EW scale, is high enough to essentially erase any symmetric thermal relic abundance. Hence an asymmetry between technibaryons and antitechnibaryons is invoked, especially as this can be generated quite naturally in the same manner as for baryons. However, scaling up the proton-antiproton annihilation cross section is not applicable to generic TIMPs, in particular, not to pNGBs, hence they may have an interesting symmetric (thermal) relic abundance.

<span id="page-1-1"></span>Let us solve for the thermal relic abundance of TIMPs  $\phi$ with singlet constituents using the Boltzman continuity equation  $[51,52]$  $[51,52]$  $[51,52]$  $[51,52]$  $[51,52]$ :

$$
\frac{d}{dt}(n_{\phi}R^3) = -\langle \sigma_{\text{ann}}\nu \rangle [n_{\phi}^2 - (n_{\phi}^{\text{eq}})^2]R^3,\tag{3}
$$

where R is the cosmological scale-factor,  $n_{\phi}$  the TIMP number density and  $\sigma_{\text{ann}}$  the TIMP-anti-TIMP annihilation cross section (given in the Appendix along with the relevant interaction vertices). From the Lagrangian ([2](#page-1-0)) we see that annihilations proceed via the (composite) Higgs into SM fermions and gauge bosons pairs, as well as into a pair of (composite) Higgs particles. As discussed in Refs. [[53](#page-8-7)–[55\]](#page-8-8), we can rewrite the continuity Eq. ([3\)](#page-1-1) in terms of the dimensionless quantities  $Y = n_{\phi}/s$ ,  $Y^{eq} =$  $n^{eq}/s$ , and  $x \equiv m_{\phi}/T$ , where  $s \equiv g_s T^3$  is the specific entropy determining the value of the adiabat RT:

<span id="page-1-2"></span>
$$
\frac{dY}{dx} = \lambda x^{-2} [(Y^{eq})^2 - Y^2],
$$
  
where 
$$
\lambda \equiv \left(\frac{g_s^4}{180\pi}\right)^{1/6} m_\phi m_\text{P} \langle \sigma_{\text{ann}} v \rangle g_\rho^{1/2},
$$
 (4)

and  $g_{\rho} \equiv \rho/T^4$  counts the number of relativistic degrees of freedom contributing to the energy density, which determines the Hubble expansion rate  $R/R$ . The values of  $g<sub>o</sub>(T)$ and  $g_s(T)$  have been computed in the SM [\[56\]](#page-8-9) and are modified to account for the additional particle content of TC models.

In the hot early universe, the particle abundance initially tracks its equilibrium value but when the temperature falls below its mass and it becomes nonrelativistic, its equilibrium abundance falls exponentially due to the Boltzmann factor. Hence so does the annihilation rate, eventually becoming sufficiently small that the (comoving) particle abundance becomes constant. Defining the parameter  $\Delta \equiv$  $(Y - Y<sup>eq</sup>)/Y<sup>eq</sup>$ , the freeze-out temperature is given by [\[53\]](#page-8-7):

$$
x_{\rm fr} = \frac{1}{b_s} \ln[\Delta_{\rm fr}(2 + \Delta_{\rm fr})\delta_s]
$$
  
\n
$$
-\frac{1}{2b_s} \ln\left[\frac{1}{b_s} \ln(\Delta_{\rm fr}(2 + \Delta_{\rm fr})\delta_s)\right]
$$
  
\nwhere  $b_s \equiv \left(\frac{2\pi^2 g_s}{45}\right)^{1/3}$ ,  
\n
$$
\delta_s = \frac{g}{(2\pi)^{3/2}} b_s^{-5/2} \lambda,
$$
\n(5)

and g counts the internal degrees of freedom, e.g.  $g = 2$  for the TIMP. This gives a good match to the exact numerical solution of Eq. [\(4](#page-1-2)) for the choice  $\Delta_{\text{fr}} = 1.5$  which corresponds to the epoch when the annihilation rate,  $n_{\phi}^{\text{eq}} \langle \sigma_{\text{ann}} v \rangle$ , equals the logarithmic rate of change of the particle abundance itself:  $d \ln n^{\text{eq}}/dt = x_{\text{fr}}\dot{R}/R$  [[51](#page-8-5)]. Note that the usual criterion of equating the annihilation rate to the Hubble expansion rate  $R/R$  would give an *erroneous* answer when there is an asymmetry [\[53\]](#page-8-7).

We calculate the TIMP freeze-out parameter  $x_{\text{fr}}$  as a function of the TIMP mass  $m_{\phi}$  taking  $\Lambda = 1$  TeV, for three values of the (composite) Higgs mass  $m_H = 250$ , 500, 1000 GeV. We also take the dimensionless effective couplings to the (composite) Higgs to be of  $\mathcal{O}(1)$  and define  $d_{12} \equiv d_1 + d_2$ ,  $d_{34} \equiv d_3 + d_4$ , since at low energies the  $d_1$  and  $d_2$  terms contribute very nearly equally, as do the  $d_3$  and  $d_4$  terms. There are spikes in  $x_{\text{fr}}$  at the (composite) Higgs resonance when  $2m_{\phi} = m_H$ , however the simple approximation above is not reliable near such a resonance [\[57\]](#page-8-10) and we must then solve the full continuity equation including the (composite) Higgs width. We do this using the program MICROMEGAS [[58](#page-8-11)–[60\]](#page-8-12) which computes the full annihilation cross section of the model using CALCHEP [\[61](#page-8-13)]. We also use LANHEP [\[62\]](#page-8-14) for the model implementation.

After freeze-out, only annihilations are important since the temperature is now too low for the inverse creations to proceed; the asymptotic abundance is then

$$
Y_{\infty} \equiv Y(t \to \infty) \sim \frac{x_{\text{fr}}}{\lambda_{\text{fr}}}.
$$
 (6)

The resulting cosmological energy density of relic TIMPs is shown in Fig. [1.](#page-2-0) We have checked explicitly with the numerical code that the contributions from  $d_1$  and  $d_2$  terms are (very nearly) identical, as are the contributions from  $d_3$ and  $d_4$  terms.

Figure [2](#page-2-1) shows the region in the (composite) Higgs versus TIMP mass plane where the TIMP relic abundance matches the DM abundance  $\Omega h^2 = 0.11 \pm 0.01$  (2 $\sigma$ ) inferred from WMAP-7 [\[63\]](#page-8-15).

From Figs. [1](#page-2-0) and [2](#page-2-1) we observe first that the relic energy density drops significantly for  $m_\phi \sim 2m_H$  as expected (because  $\phi$  can then decay resonantly through the Higgs) and, second, that if  $m_H$  is greater than about 115 GeV then for TIMPs to be dark matter requires  $m_{\phi} > m_W$ .

<span id="page-2-0"></span>

FIG. 1 (color online). The relic TIMP  $(\phi)$  abundance vs its mass. The thick lines show the MICROMEGAS computation taking into account the (composite) Higgs decay width.

As discussed below, in the presence of an asymmetry the total relic abundance always increases relative to the same model with no asymmetry, so  $m_W$  provides a general *lower* bound for the mass of the pNGB TIMPs we consider. It follows that in the interesting region  $m_W \le m_\phi \le 1$  TeV, symmetric relic TIMPs with singlet constituents could make up a significant fraction of the dark matter in the universe. However when the TIMP is heavier than about a TeV, the strength of the interaction is similar to that of an ordinary (scalar) technibaryon, and a relic abundance large enough to account for dark matter now does require an

<span id="page-2-1"></span>

FIG. 2 (color online). Regions corresponding to  $\Omega h^2 =$  $0.11 \pm 0.01$  for the relic TIMP ( $\phi$ ) abundance in the (composite) Higgs vs TIMP mass plane. The dashed box shows that given  $m_H > 115$  GeV, we require  $m_\phi > m_W$  for TIMPs to be dark matter.

initial asymmetry similar to that of baryons as discussed earlier [\[16](#page-7-7)[,17\]](#page-7-21). We note that recently a different type of OCD-like pions (which do not carry a  $U(1)$  quantum number) was also considered as dark matter candidates [\[64](#page-8-16)[,65\]](#page-8-17).

## A. Adding an asymmetry

To study the relic abundance in the presence of both a thermal component and an initial asymmetry we follow Ref. [\[53\]](#page-8-7) and define the asymmetry as  $\alpha = (Y_+ - Y_-)/2$ where  $Y_+$  are the abundances of the majority and minority species (TIMP and anti-TIMP), respectively. The abundance in thermal and chemical equilibrium is

$$
Y_{-}^{\text{eq}} = e^{-\mu/T} Y^{\text{eq}} \sim e^{-\mu/T} g \left(\frac{x}{2\pi b_s}\right)^{3/2} e^{-b_s x}, \qquad (7)
$$

where  $\mu$  is the chemical potential. The continuity equation in the presence of an asymmetry is [[53](#page-8-7)]:

$$
\frac{dY_{-}}{dx} = \lambda x^{-2} [Y_{-}^{\text{eq}} (Y_{-}^{\text{eq}} + 2\alpha) - Y_{-} (Y_{-} + 2\alpha)], \quad (8)
$$

and the total asymptotic abundance of TIMPs and anti-TIMPs is

$$
\Omega_{\phi}h^2 = 5.5 \times 10^8 (Y_{-\infty} + \alpha) \frac{m_{\phi}}{\text{GeV}}.
$$
 (9)

In Fig. [3](#page-3-0) we show the minority species abundance  $Y_{-}$  as a function of  $x \equiv m_{\phi}/T$  for  $m_{\phi} = 100$  GeV and  $\alpha = 9.8 \times 10^{-9}$ ,  $5.6 \times 10^{-10}$ ,  $10^{-11}$ , taking the Higgs mass to be  $m_H = 250, 500, 1000$  GeV.

We see from the figure that the *symmetric* component of the relic abundance is  $\sim$ 10% of the asymmetric component, when the asymmetry is comparable to the would-be symmetric abundance (in the absence of an asymmetry). In the limit where  $\alpha \ll Y_{\infty}$ , the symmetric component is unchanged and the asymmetry provides a small addition to the total relic abundance. When  $\alpha \ge Y_{\infty}$ , the abundance of the minority species is exponentially suppressed in  $\alpha$ and provides a negligible addition to the asymmetric

<span id="page-3-0"></span>

FIG. 3 (color online). TIMP  $(\phi)$  abundance when there is no asymmetry (thick lines), compared with the abundance of the *minority* species  $Y_$  (thin lines) when an asymmetry  $\alpha$  is present.

component. Adding an asymmetry will always increase the relic abundance relative to its value in the same model in the absence of an asymmetry. This implies a nontrivial constraint on the mass of TIMPs with uncharged constituents discussed above, such as appear in e.g. the UMT model. The constraint is nontrivial since such states would evade direct detection at colliders unless the (composite) Higgs is very light [[30](#page-7-14)], as well as the direct detection experiments discussed below.

Figure [4](#page-3-1) shows the contour in the  $m_H - m_\phi$  plane where the relic abundance of  $\phi$  agrees with the DM abundance inferred from WMAP-7 [[63](#page-8-15)] for a fixed value of the  $d$ coefficients. Just as in Figs. [1](#page-2-0) and [2](#page-2-1) we observe that if  $m_H$  is greater than  $\sim$ 115 GeV we require  $m_{\phi} > m_W$  to avoid an excessive TIMP relic abundance. However, due to the asymmetry, for TIMP masses above  $m_W$  there is now a much broader range of  $m_H$  and  $m_\phi$  where the relic abundance of  $\phi$  matches the observed DM abundance.

## B. TIMPs with charged constituents

We consider now pNGB TIMPs with charged constituents of the form  $T \sim UD$  arising from the TC sector carrying EW interactions (see Eq. ([1\)](#page-0-6)). These states carry an  $U(1)$  quantum number which makes them stable and it is natural to identify this global symmetry with the technibaryon number. Such particles arise generally in TC models with the technifermions transforming in either real or pseudoreal representations of the gauge group. Explicit examples are furnished again in the UMT scheme in which the composite  $T$  is a SM singlet and the "orthogonal" minimal technicolor" (OMT) model in which the  $T$  state is the isospin-0 component of a complex triplet [\[10](#page-7-6)].

<span id="page-3-1"></span>

FIG. 4 (color online). Regions in the (composite) Higgs versus TIMP mass plane corresponding to  $\Omega h^2 = 0.11 \pm 0.01$  for the TIMP  $(\phi)$  relic abundance, for different values of the relic asymmetry  $\alpha$ . The dashed box shows that given  $m_H$  > 115 GeV, we require  $m_{\phi} > m_W$  for TIMPs to be dark matter.

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We demonstrate that similarly to the case of TIMPs with neutral constituents, TIMPs with charged constituents have a significant symmetric component in only a small region of parameter space. However, as opposed to the TIMPs with SM neutral constituents, this region is essentially independent of the parameters of the (composite) Higgs interactions. In addition to the interactions investigated above, the scalar TIMPs containing charged constituents will also have an effective interaction with the photon, due to a nonzero electromagnetic charge radius of  $T$  [\[18](#page-7-22)[,30\]](#page-7-14):

$$
\mathcal{L}_B = ie \frac{d_B}{\Lambda^2} T^* \overleftrightarrow{\partial}_{\mu} T \partial_{\nu} F^{\mu \nu}.
$$
 (10)

<span id="page-4-1"></span>The corresponding charge radius of the TIMP is  $r_T \sim \sqrt{d_B}/\Lambda$ . For our choice  $\Lambda = 1$  TeV we consider the range  $|d_B| = 0$  to 0.3 while for a higher cutoff  $\Lambda$ , a larger  $d_B \sim O(1)$  would be expected.

In the case of the TIMP  $T$  there are also contact interactions with two SM vector bosons V, arising from the kinetic term of the chiral Lagrangian, which can significantly affect the symmetric relic density. In general these can be written as

$$
L_{VV} = \frac{1}{2} T^* T V_{\mu} V^{\mu} \operatorname{Tr}[[\Lambda_S, [\Lambda_S, X_T]] X_{T^*} - [\Lambda_S, [\Lambda_S, X_T]] X_{T^*}],
$$
\n(11)

where  $X_T$  is the generator of the broken (techni)flavor direction corresponding to the TIMP and  $\Lambda_S$  are the, appropriately normalized, EW generators imbedded in the TC chiral group [\[66–](#page-8-18)[68](#page-8-19)]. The resulting TTWW and TTZZ contact interactions (assuming that the EW symmetry is broken already) are

<span id="page-4-2"></span>
$$
L_{WW,ZZ} = -\frac{T^*T}{2} \operatorname{Tr}[d_W W_\mu W^\mu + d_Z Z_\mu Z^\mu],\qquad(12)
$$

with  $d_w = g^2$  and  $d_z = (g^2 + g^2)/2$  for the TIMPs T of both the UMT and the OMT models.

In Fig. [5](#page-4-0) we display the effect of the  $W$  and  $Z$  contact interactions, as well as of the charge-radius interaction, on the thermal relic abundance. It is seen that for  $m<sub>T</sub>$  significantly below  $m_W$ , the interactions in Eq. ([10](#page-4-1)) do affect TIMP annihilations significantly, even so the TIMP relic abundance is too large. As  $m<sub>T</sub>$  increases towards  $m<sub>W</sub>$ , annihilations due to the  $TTVV$  interactions in Eq. [\(12\)](#page-4-2) begin to dominate and reduce the TIMP relic abundance to the observationally acceptable level only when the TIMP is a few GeV lighter than the W.

Again, once we include an asymmetry, the range of TIMP masses where a cosmologically acceptable symmetric relic abundance is achieved, can be much broader. This is demonstrated in Fig. [6](#page-4-3) which shows the contours of  $\Omega h^2$  for different values of the asymmetry and  $m_T$ . There is a crossover in the small TIMP mass region mentioned above, from just below  $m_W$  where the relic abundance is dominated by the asymmetry, to just above  $m_W$  where it is dominated by the symmetric component.

<span id="page-4-0"></span>

FIG. 5 (color online). The TIMP  $(T)$  relic energy density as a function of its mass when W and Z contact interactions are present, calculated taking into account their off-shell decays. The horizontal band corresponds to  $\Omega h^2 = 0.11 \pm 0.01$ .

<span id="page-4-3"></span>

FIG. 6 (color online). The region in the asymmetry  $(\alpha)$  vs TIMP  $(T)$  mass plane which yields the marked relic energy density (consistent with WMAP) when  $W$  and  $Z$  contact interactions are present.

## IV. DIRECT DETECTION

Direct detection of the TIMPs  $\phi$  with SM singlet constituents will be challenging. The exchange of the (composite) Higgs leading to a scattering cross section on nuclei is the most relevant interaction here, and we will assume

that the (composite) Higgs couples to SM fermions with ordinary Yukawa couplings. (In fact this represents an upper bound and so the actual cross section could be lower). Again, since the TIMP is a pNGB the couplings to the Higgs are suppressed at low masses. The TIMP nucleon scattering cross section from the (composite) Higgs exchange is given by

$$
\sigma_{\text{nucleon}}^H = \frac{\mu^2}{2\pi} \left[ \frac{d_H f m_N}{m_H^2 m_\phi v} \right]^2, \qquad d_H = (d_1 + d_2) \frac{m_\phi^2}{\Lambda},\tag{13}
$$

where  $\mu$  is the nucleon-TIMP reduced mass,  $\nu$  the electroweak vev and f parametrizes the (composite) Higgsnucleon coupling. We refer to Refs. [[69](#page-8-20)[,70](#page-8-21)] for recent discussions on the strange quark contribution to  $f$  which we take to be  $f = 0.3$ .

For TIMPs T with charged constituents there is an additional contribution to the scattering on nuclei via the charge-radius operator [\[18](#page-7-22)[,30\]](#page-7-14)

$$
\sigma_p^{\gamma} = \frac{\mu^2}{4\pi} \left[ \frac{8\pi \alpha d_B}{\Lambda^2} \right]^2.
$$
 (14)

To take into account the possible interference between the composite Higgs and photon exchange [[30\]](#page-7-14) we write an averaged scalar cross section per nucleon as

<span id="page-5-0"></span>

FIG. 7 (color online). Direct detection cross section (per nucleon) for TIMP scattering off nuclei. The full line is for Higgs exchange only (with  $m_H = 200$  GeV) while the short- and longdashed lines show the additional effect of the charge-radius operator (with  $d_B = -0.3$  and  $d_B = +0.3$  respectively). The shaded region is experimentally excluded.

<span id="page-5-1"></span>

FIG. 8 (color online). Contours of the direct detection cross section (per nucleon) for TIMP scattering off nuclei in the TIMP mass vs (composite) Higgs mass plane. The top, middle and bottom frames are for  $d_B = 0$ ,  $-0.3$ ,  $+0.3$  respectively. The dashed curves show the upper limits from CDMS-II (red), XENON-10 (blue) and XENON-100 (green).

$$
\sigma_{\text{nucleon}} = \frac{\mu^2}{4\pi A^2} (f_p Z + f_n (A - Z))^2
$$
  
where  $f_n = -\frac{\sqrt{2} d_H f m_N}{m_H^2 m_\phi v}$ ,  

$$
f_p = f_n + \frac{8\pi \alpha d_B}{\Lambda^2}.
$$
 (15)

The direct detection cross section per nucleon as a function of the TIMP mass is shown in Fig. [7,](#page-5-0) where we also indicate, (following Ref. [\[71\]](#page-8-22)) the limits from the CDMS-II [\[72\]](#page-8-23) and XENON-100 [[73](#page-8-24)] experiments.

For Higgs exchange only  $(d_B = 0)$  the direct detection cross section increases with TIMP mass since the TIMPs are pNGBs (c.f. Ref. [\[30\]](#page-7-14) where the TIMPs were not derivatively coupled to the (composite) Higgs). However, the charge-radius interaction can significantly alter the cross section, by up to 2 orders of magnitude for  $|d_B| \approx 0.3$ , thus greatly affecting the discovery potential of direct detection experiments. For example, for  $d_B = -0.3$  and  $d_{12} = 3$  the signals from the TIMP T with SM charged constituents would have been observed already by CDMS-II and XENON-100. On the other hand, for *positive* values of  $d_B$ , there is a destructive interference between (composite) Higgs and photon exchange, lowering the cross section to  $10^{-47}$  cm<sup>2</sup> at  $m<sub>T</sub> \approx 200$  GeV for a particular choice of the parameters.

Since the direct detection rate depends strongly on the (composite) Higgs mass, we present in Fig. [8](#page-5-1) results in the  $(M_H - m_T)$  plane for different values of the  $d_B$  parameter. The figure also shows the exclusion limits from the XENON-10 [[74](#page-8-25)], CDMS-II [\[72\]](#page-8-23) and XENON-100 [\[73\]](#page-8-24) experiments. One can see that for  $d_B = 0$  and  $d_{12} = 3$ (top frame), XENON-100 and CDMS-II can cover essentially the whole range of  $m<sub>T</sub>$  for  $M<sub>H</sub>$  below 150 GeV where the cross section always exceeds  $10^{-42}$  cm<sup>2</sup>. Our results trivially scale as  $d_{12}^2$  for  $d_B = 0$ . Negative  $d_B = -0.3$ significantly enhances the cross section (middle frame) while positive  $d_B = 0.3$  (bottom frame) brings the negative interference effect into play, resulting in a deep valley.

#### V. CONCLUSIONS

We have shown that models of dynamical EW symmetry breaking can provide symmetric (thermal) DM, as well as asymmetric (nonthermal) DM. This is true, in particular, for partially gauged technicolor [[6](#page-7-15),[40](#page-7-16)] which can satisfy constraints from EW precision measurements.

From our analysis we conclude that for pNGB TIMPs:

- (1) The TIMP cannot be significantly lighter than the W if its relic abundance is to be acceptable, unless the (composite) Higgs mass is below 115 GeV—this holds whether it is symmetric, asymmetric or a combination.
- (2) If the TIMP is made of constituents charged under EW interactions, it can be symmetric DM only in a

narrow mass range close to  $m_W$ . Above this mass an initial asymmetry is required for TIMPs to be dark matter.

- (3) If the constituents of the TIMP are neutral with respect to the EW interactions then it can be symmetric dark matter for a range of masses tied to the (composite) Higgs mass. This does not exclude the possibility that it has an asymmetry as well.
- (4) Direct detection of light pNGB TIMPs with neutral constituents is challenging due to its mass suppressed couplings to the (composite) Higgs. However, for TIMPs with charged constituents there is an additional charge-radius interaction which, if sizeable, can bring such TIMPs within the reach of current nuclear recoil detection experiments.

Both types of TIMPs considered here—with charged or neutral constituents—can coexist and contribute to the dark matter (as in e.g. the UMT model [\[8\]](#page-7-12)). These models provide interesting signals for direct dark matter detection experiments which are already sensitive enough to exclude TIMPs in certain regions of parameter space or even discover them in the near future.

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### APPENDIX: ANNIHILATION CROSS SECTION

The relevant vertex factors at low energies (if we keep only the light (composite) Higgs in the spectrum) are

$$
\phi^* \phi H: i\left(\frac{p_{\phi^*} p_{\phi}}{\Lambda} d_1 + d_2 \frac{m_{\phi}^2}{\Lambda}\right) \to i d_{12} \frac{m_{\phi}^2}{\Lambda}
$$
\n
$$
\phi^* \phi H H: i\left(\frac{p_{\phi^*} p_{\phi}}{\Lambda^2} d_3 + d_4 \frac{m_{\phi}^2}{\Lambda^2}\right) \to i d_{34} \frac{m_{\phi}^2}{\Lambda^2}
$$
\n(A1)

where  $d_{12} = d_1 + d_2$  and  $d_{34} = d_3 + d_4$  are the only independent parameters at low energies. Hence the (composite) Higgs mediated contributions to the cms annihilation cross section  $\langle \sigma v_{rel} \rangle$  in the limit  $v_{rel} \rightarrow 0$  are

 $\phi \phi^* \to HH$ :

$$
\frac{1}{64\pi m_{\phi}^2} \left[ \frac{3d_{12}m_H^2 m_{\phi}^2}{\nu \Lambda (4m_{\phi}^2 - m_H^2)} - \frac{2d_{12}^2 m_{\phi}^4}{\Lambda^2 (m_H^2 - 2m_{\phi}^2)} + \frac{d_{34}m_{\phi}^2}{\Lambda^2} \right]^2
$$
  
 
$$
\times \left( 1 - \frac{m_H^2}{m_{\phi}^2} \right)^{1/2}, \tag{A2}
$$

 $\phi \phi^* \to W^+ W^-$ 

$$
2\left[1+\frac{1}{2}\left(1-\frac{2m_{\phi}^2}{m_W^2}\right)^2\right] \frac{d_{12}^2 m_{\phi}^2 m_W^4}{8\pi v^2 \Lambda^2 \left[(4m_{\phi}^2 - m_h^2)^2 + m_h^2 \Gamma_h^2\right]} \times \left(1-\frac{m_W^2}{m_{\phi}^2}\right)^{1/2}
$$
\n(A3)

 $\phi \phi^* \rightarrow ZZ$ 

$$
2\left[1+\frac{1}{2}\left(1-\frac{2m_{\phi}^2}{m_Z^2}\right)^2\right]\frac{d_{12}^2m_{\phi}^2m_Z^4}{16\pi v^2\Lambda^2[(4m_{\phi}^2-m_h^2)^2+m_h^2\Gamma_h^2]}
$$

$$
\times\left(1-\frac{m_Z^2}{m_{\phi}^2}\right)^{1/2}
$$
(A4)

$$
\phi \phi^* \to \bar{f} f
$$

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$$
\frac{c_f}{4\pi v^2 \Lambda^2} \frac{\lambda_f^2 d_{12}^2 m_{\phi}^4}{[(4m_{\phi}^2 - m_h^2)^2 + m_h^2 \Gamma_h^2]} \left(1 - \frac{m_f^2}{m_{\phi}^2}\right)^{3/2}.
$$
 (A5)

Here the fermion Yukawa coupling is  $\lambda_f = m_f/v$  where  $v \approx 246$  GeV and  $m_f$  is the fermion mass, while  $c_f = 1, 3$ for leptons and quarks, respectively. The contributions from the photon mediated annihilations from the chargeradius operator are negligible and we do not include these.

We implement the Lagrangian in Eq. ([2\)](#page-1-0) in CALCHEP [\[61\]](#page-8-13) and in MICROMEGAS [\[59\]](#page-8-26) (using the LANHEP module [\[62\]](#page-8-14) to check the above implementation), in order to compute the full  $2 \rightarrow 2$  annihilation cross section including finite widths and to study the collider phenomenology.

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