Generalized Cahn effect and parton 3D motion in a covariant approach

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(Received 18 September 2009; published 27 January 2011)

The Cahn effect and the unintegrated unpolarized parton distribution function $f_1^q(x, \mathbf{p}_T)$ are studied in a covariant approach. The Cahn effect is compared with some other effects due to the parton intrinsic motion. The comparison suggests that the present understanding of parton transverse momenta and intrinsic motion in general is still rather incomplete. The new relation for $f_1^q(x, \mathbf{p}_T)$ is obtained in the framework of the covariant parton model from which a prediction for this distribution function follows.

DOI: 10.1103/PhysRevD.83.014022 PACS numbers: 12.39.−x, 11.55.Hx, 13.60.−r, 13.88.+e

I. INTRODUCTION

Studies of the transverse momentum dependent (or ''unintegrated'') parton distribution functions (TMDs) [1] open a new way to a better understanding of the partonic quark-gluon structure of the nucleon. At the same time, it is evident that the explanation of some experimental observations could be hardly possible without a more accurate and realistic 3D picture of the nucleon, which naturally includes transverse motion. The azimuthal asymmetry in the distribution of hadrons produced in deep-inelastic lepton-nucleon scattering (DIS), known as the Cahn effect [2,3], is a classical example. The role of the quark (transversal) intrinsic motion is crucial also for the explanation of some spin effects, like the asymmetries in particle production related to the direction of proton polarization [4–10].

In our previous study, we proposed a covariant parton model in which the 3D picture of parton momenta with rotational symmetry in the nucleon rest frame represents a basic input [11–17]. At the same time, the model is based on the assumption that for sufficiently large momentum transfer Q^2, the quarks can be considered as almost free due to the asymptotic freedom. It appears that the main potential of this approach is the implication of some old and new sum rules and relations among various parton distribution functions. The sum rules which relate the structure functions g_1 and g_2 in a Wandzura-Wilczek approximation and some others are proved in [12]. Assuming the $SU(6)$ symmetry (in addition to the covariance and rotational symmetry) we have proved relations between polarized and unpolarized structure functions [13], which agree very well with the experimental data. In [14], we studied transversity in the framework of this model and we derived relations between transversity and helicity. Recently, we generalized the model to include also the pretzelosity distribution [16] and derived relations which connect helicity, transversity, and pretzelosity. Finally, with the same model we studied the TMDs and a set of relations among them [17]. Further, in the framework of the model we demonstrated that the 3D picture of parton momenta inside the nucleon is a necessary input for a consistent accounting for quark orbital angular momentum (OAM). The dominating contribution of the OAM to the nucleon spin is a consequence of the quark relativistic motion inside the nucleon, i.e. when quark mass $\ll$ momentum in the nucleon rest frame. In this case, only the total angular momentum $J_z^q = L_z^q + S_z^q$ is a good quantum number and we obtained mean values of the quark orbital and spin components: $\langle L_z^q \rangle = 2\langle S_z^q \rangle = \Delta\Sigma$ or $\langle J_z^q \rangle = \langle S_z^q \rangle + \langle L_z^q \rangle = \frac{3}{2}\Delta\Sigma$ [15].

A comparison of the obtained relations and predictions with experimental data is very important and interesting from phenomenological point of view. It allows to judge to which extent the experimental observation can be interpreted in terms of simplified, intuitive notions. The obtained picture of the nucleon can be a useful complement to the exact but more complicated theory of the nucleon structure based on the QCD. For example, the covariant parton model can be a useful tool for separating QCD effects from effects of relativistic kinematics.

In this work, we study further aspects of the intrinsic motion of quarks. The Cahn effect is due to transverse momentum of quarks, and in Sec. II we analyze the conditions, which induce this effect in more detail. We show that the azimuthal asymmetry is a general consequence of the intrinsic motion of constituents inside the composite target. We obtain the corresponding asymmetry as a function of parton transverse momentum in the covariant approach. In Sec. III, we make a comparison of the data on average transverse momenta of the quarks obtained by the method based on the Cahn effect with the data obtained by other also model-dependent methods. In Sec. IV, we analyze the unpolarized TMD defined in our previous study [17] and, as a result, we obtain the relation between this unintegrated distribution and its integrated counterpart. This relation allows us to make a prediction for the TMD using the known parton distribution function. We also make a detailed comparison with the recent approach by U. D'Alesio, E. Leader, and F. Murgia [18], in which an equivalent relation has been obtained. Finally, in Sec. V, we summarize the obtained results.

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II. CAHN EFFECT: MANIFESTATION OF THE INTRINSIC MOTION

The Cahn effect, which is related to azimuthal asymmetry of struck quarks in DIS, is due to the nonzero transverse momentum of quarks inside the nucleon. The probability $W = |M_{fi}|^2$ of the elementary lepton-quark scattering in one photon exchange approximation is given by the expression

$$W(\hat{s}, \hat{u}) \propto \hat{s}^2 + \hat{u}^2, \tag{1}$$

where the Mandelstam variables depend on the azimuthal angle φ (angle between leptonic and hadronic planes) as:

$$\hat{s}^{\,2} = \frac{Q^4}{y^2}\left[1 - 4\frac{p_T}{Q}\sqrt{1-y}\cos\varphi\right] + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right), \tag{2}$$

$$\hat{u}^{\,2} = \frac{Q^4}{y^2}(1-y)^2\left[1 - 4\frac{p_T}{Q}\frac{\cos\varphi}{\sqrt{1-y}}\right] + \mathcal{O}\left(\frac{p_T^2}{Q^2}\right), \tag{3}$$

where p_T is the quark momentum component transverse to the photon momentum $\mathbf{q}$, $Q^2 \equiv -q^2$ [19]. Apparently, the dependence on φ disappears for $p_T \to 0$. The intrinsic motion of the constituents creating the composite target is a necessary condition for the appearance of the effect. The Cahn effect is a kinematical effect accompanying the QED scattering of leptons on quarks inside the nucleon and its origin is different from that of the QCD higher-twist effects [20–22]. At the same time it is evident, that the intrinsic quark motion in itself is due to nonperturbative QCD. The Mandelstam variables in terms of the lepton and quark momenta (l, p) read

$$\hat{s} = (l+p)^2 = 2pl + m_l^2 + m_q^2, \tag{4}$$

$$\hat{u} = (p - l')^2 = -2pl + Q^2 + m_l^2 + m_q^2, \tag{5}$$

where m_l, m_q are the corresponding masses. Obviously, one can substitute the variables of the probability (1):

$$\hat{s}, \hat{u} \to pl, Q^2; \qquad W(\hat{s}, \hat{u}) \to W(pl, Q^2). \tag{6}$$

The probability W expressed in terms of the new variables clearly demonstrates an azimuthal symmetry of $\mathbf{p}$ with respect to the lepton beam direction $\boldsymbol{l}$, which represents the axis of the azimuthal symmetry. It follows that the photon direction $\mathbf{q}$, being different from the direction $\boldsymbol{l}$, in general cannot be the second axis of the azimuthal symmetry. In fact, this is the essence of the Cahn effect, see Fig. 1(a). Let us consider two reference frames:

A. The nucleon rest frame, where the first axis is directed along $\mathbf{q}$ and the projection of $\boldsymbol{l}$ on the plane transversal to $\mathbf{q}$ defines a second axis. The azimuthal angle φ and the transverse momentum p_T are defined equally as above (p_T and φ do not change under any Lorentz boost along $\mathbf{q}$), so the quark momentum $\mathbf{p}$ in this frame has the components:

$$\mathbf{p}_A = (p_1, p_T\cos\varphi, p_T\sin\varphi). \tag{7}$$

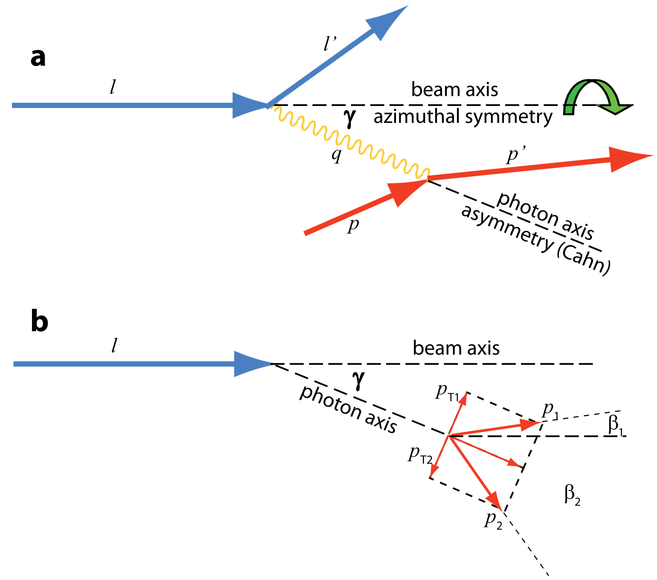

FIG. 1 (color online). (a) The interaction of a lepton with a quark defines two axes of different symmetry. (b) The azimuthal asymmetry as a result of variable collision energy, see text.

B. The nucleon rest frame, where the first axis is directed along $\boldsymbol{l}$ and projection of $-\mathbf{q}$ on the plane transversal to $\boldsymbol{l}$ defines the second axis. This reference frame is obtained by a rotation of the frame A by an angle γ around a third axis, so the quark momentum has the new components:

$$\mathbf{p}_B = (p_1\cos\gamma - p_T\sin\gamma\cos\varphi, p_T\cos\gamma\cos\varphi + p_1\sin\gamma, p_T\sin\varphi). \tag{8}$$

The angle γ is defined as

$$\cos\gamma = \frac{q_L}{|\mathbf{q}|}, \qquad \sin\gamma = \frac{q_T}{|\mathbf{q}|}, \tag{9}$$

where q_L and q_T are the longitudinal and transversal components of the photon momentum in the frame B, $\mathbf{q}_B = (q_L, q_T, 0)$. For the lepton energy l_0 (the lepton mass will be neglected in the following), one can obtain [11]:

$$\frac{|q_L|}{\nu} = 1 + \frac{M}{l_0}x_B, \qquad \frac{|\mathbf{q}|}{\nu} = \sqrt{1 + \frac{4M^2}{Q^2}x_B^2} \tag{10}$$

and

$$\frac{q_T}{\nu} = \sqrt{\left(\frac{4M^2}{Q^2} - \frac{M^2}{l_0^2}\right)x_B^2 - \frac{2M}{l_0}x_B}, \tag{11}$$

where the standard notation is used:

$$x_B = \frac{Q^2}{2M\nu}, \qquad \nu = l_0 - l_0'. \tag{12}$$

Now the variable pl can be expressed as

$$pl = (p_0 - p_1\cos\gamma - p_T\sin\gamma\cos\varphi)l_0. \tag{13}$$

This variable, after inserting into the relations (4) and (5), allows to exactly calculate the azimuthal dependence of the probability (1).

If one assumes

$$Q^2 \gg 4M^2x_B^2, \qquad l_0 \gg Mx_B, \tag{14}$$

then the relations (9) and (10) give

$$|\mathbf{q}| \approx |q_L| \approx \nu, \qquad \cos\gamma \approx 1. \tag{15}$$

Now, since

$$p_1 = \frac{\mathbf{pq}}{|\mathbf{q}|} = \frac{p_0\nu - pq}{|\mathbf{q}|}, \tag{16}$$

the relation (13) is modified as

$$pl \approx \left(\frac{pq}{\nu} - \frac{p_T q_T}{\nu}\cos\varphi\right) l_0. \tag{17}$$

Further, Eq. (11) is rearranged as

$$\frac{q_T}{\nu} = \frac{2Mx_B}{Q}\sqrt{1 - \frac{\nu}{l_0} - \frac{Q^2}{4l_0^2}}. \tag{18}$$

Since the complete expression for the probability $W(\hat{s}, \hat{u})$ involves the δ–function term

$$\delta((p+q)^2 - m_q^2) = \delta(2pq + q^2) = \frac{1}{2Pq}\delta\left(\frac{pq}{Pq} - x_B\right), \tag{19}$$

where P is the nucleon momentum, one can replace the product pq in (17) by $Mx_B\nu$. Then, assuming $4l_0^2 \gg Q^2$, after inserting (18) into (17) one gets

$$pl \approx \frac{Q^2}{2y}\left(1 - \frac{2p_T\sqrt{1-y}}{Q}\cos\varphi\right), \tag{20}$$

where

$$y = \frac{\nu}{l_0} = \frac{Pq}{Pl}, \qquad \frac{Q^2}{2y} = x_B Pl.$$

Now, the term

$$\lambda = \frac{2p_T\sqrt{1-y}}{Q}\cos\varphi \tag{21}$$

represents a "small" correction and one can check, that the Mandelstam variables (4) and (5) in which the term pl is replaced by the expression (20) and the quark masses are neglected, give the relations (2) and (3).

Now the probability $W(pl, Q^2)$ can be expanded as

$$\begin{aligned} W(pl, Q2) &= W(pl, Q2)|_{\lambda=0} - \frac{\partial W(pl, Q2)}{\partial(pl)} pl \bigg|_{\lambda=0} \lambda + \ldots \\ &\approx W(pl, Q2)|_{\lambda=0}\left(1 - \frac{\partial \ln W(pl, Q2)}{\partial \ln(pl)}\bigg|_{\lambda=0}\lambda\right). \end{aligned} \tag{22}$$

Let us make some remarks on this relation:

i) The relation implies, that the azimuthal asymmetry of the recoiled quark is described by the distribution

$$P(\varphi) = (1 - a\cos\varphi), \tag{23}$$

where

$$a = \frac{2\sqrt{1-y}}{Q} \cdot \left[\frac{\partial \ln W(pl, Q^2)}{\partial \ln(pl)}\right]_{\lambda=0} \cdot \langle p_T \rangle. \tag{24}$$

From the analysis of experimental data one can obtain the parameter a. Obviously for obtaining $\langle p_T \rangle$, one has to know also the term involving differentiation of W. This term can be estimated either from the model (Eq. (1)) or from the experiment, if the data for a few lepton energies are available.

ii) The azimuthal asymmetry generated by the probability $W(pl, Q^2)$ has a simple geometrical interpretation. In Fig. 1(b), the two momenta $\mathbf{p}_1$, $\mathbf{p}_2$ with opposite transverse components $\mathbf{p}_{T1}$, $\mathbf{p}_{T2}$ correspond to different collision energies $\hat{s}_1$, $\hat{s}_2$ since

$$\hat{s} = 2pl = 2(p_0 l_0 - |\mathbf{p}||\boldsymbol{l}|\cos\beta), \tag{25}$$

where β is an angle between the lepton and quark momenta. Obviously, $\hat{s}_1 < \hat{s}_2$ in this figure, and because W depends on $\hat{s}$, then the two corresponding momenta $\mathbf{p}_1$, $\mathbf{p}_2$ give different probabilities. In this way the asymmetry is generated. The figure reflects the necessary conditions for the asymmetry:

$$\sin\gamma > 0, \qquad \frac{\partial W}{\partial s} > 0, \qquad \langle p_T \rangle > 0, \tag{26}$$

which correspond to the three factors in the asymmetry parameter (24).

iii) In fact, we have shown that this asymmetry can be expected in *any* process $l + p \rightarrow l' + p'$ described by the probability $W(\hat{s}, \hat{u})$, which is defined only by the incoming particle vector l, momentum transfer q and by the parton vector p (or another constituent of composite target having some distribution of intrinsic $\mathbf{p}_T$).

In our case, the probability W is related to the individual lepton-quark scattering, which is only one stage of the Cahn effect. For complete phenomenology of the effect in lepton-nucleon DIS one needs further inputs:

a) 3D distribution $G(p)d^3p$ of parton momenta in the nucleon. The covariant approach will be studied in Sec. IV.

b) Fragmentation of recoiled quark and transfer of azimuthal asymmetry to hadrons. It is a complex stage containing both perturbative and nonperturbative QCD aspects, but some standard parameterization of the fragmentation function can be used, like in [19].

III. WHAT DO WE KNOW ABOUT INTRINSIC MOTION?

In the lepton-quark scattering the distributions of the scattered lepton and the recoiled quark are controlled by the initial quark distribution $G_q(p)$. And vice versa, from the knowledge of the distributions of the scattered leptons or quarks (in real analysis hadrons from the quark fragmentation), one can obtain information about the initial distribution by two independent ways. The comparison of the results can serve as a consistency check. So we can analyze two sets of data:

A. Leptonic data

The nucleon structure function $F_2(x, Q^2)$ is obtained from the analysis of lepton data from DIS experiments.

i) The interpretation of this function in the framework of the usual, *noncovariant* parton model suggests, that (valence + sea) quarks carry approximately only 50% of the nucleon energy-momentum. It follows that the one valence quark can carry less than roughly 15% (more exactly $\langle x\rangle = \int xq_{\rm val}(x)dx/\int q_{\rm val}(x)dx = 0.155(0.118)$ for the *u(d)* valence quarks at $Q^2 = 4\ \mathrm{GeV}^2$ [23]). This estimate follows from the approach in the nucleon infinite momentum frame, where the transversal momentum of the quarks is neglected.

ii) The analysis of the function $F_2(x, Q^2)$ in the framework of the *covariant* parton models gives the following results. The model [24] gives the prediction for the dependence $\langle p_T^2/M^2\rangle$ on x: the ratio vanishes at $x = 0$ and $x = 1$ and reaches the peak value 0.04–0.05 at $x \approx 0.5$. A very similar picture is obtained also in [18]. Since $p_T/M \approx 0.2$ at the peak, the mean value averaged over x must be smaller. These results are quite consistent with those obtained in the covariant model in which we obtained for massless quarks the relation [11]

$$p_T^2 \leq M^2x(1-x) \equiv p_{T\,\max}^2(x) \tag{27}$$

and for average momentum of the valence quarks in the nucleon *rest frame* we get [15]

$$\langle|\mathbf{p}_{\rm val}|\rangle \approx 0.1\ \mathrm{GeV}, \qquad \langle p_{T\rm val}\rangle = \frac{\pi}{4}\langle|\mathbf{p}_{\rm val}|\rangle. \tag{28}$$

iii) The statistical model [25] of the nucleon gives a very good description of the unpolarized ($F_2^{p,n}$) and polarized ($g_1^{p,n}$) structure functions in a broad kinematical region. The temperature, one of the free parameters of the model, is fixed to the value $T \approx 0.06$ GeV. Similar estimates follow also from a statistical model [26,27]. Let us remark that lattice QCD calculations suggest that the temperature corresponding to the transition of the nuclear matter to the quark-gluon plasma is around $T \approx 0.175$ GeV [28]. Naively, one could expect that the average quark momenta (or temperature) in the nucleon rest frame are less than this transition temperature. The estimates above do not contradict this expectation. Further, despite the variety of applied models, the analysis of structure functions gives compatible results on the measure of intrinsic motion of quarks inside the nucleon. Roughly speaking, the average momentum of the quark, if "measured" by the scattered lepton should not exceed ≈ 0.15 GeV in the nucleon rest frame, or $\approx 15\%$ of the nucleon energy-momentum regardless of the reference frame. One can add that the leptonic information is straightforward, since after interaction with a quark, the lepton state is not affected by other processes (final state interaction).

B. Hadronic data (quark line)

The Cahn effect is a method for measuring transverse quark momenta by means of produced hadrons. This process has two stages:

(1) The lepton-quark interaction generates an azimuthal asymmetry on the level of the recoiled quarks, which is defined by the relations (1)–(3) and by the distribution of their transverse momentum.

(2) The fragmentation of the recoiled quark—the asymmetry is partially smeared in this stage. The inclusion of this effect requires additional free parameters, so this method of evaluating the quark intrinsic motion is less direct.

The p_T dependence of the quark distribution function is usually parameterized as

$$f_1^q(x, p_T) = f_1^q(x)\frac{1}{\pi\langle p_T^2\rangle}\exp\left(-\frac{p_T^2}{\langle p_T^2\rangle}\right), \tag{29}$$

where

$$\int \frac{1}{\pi\langle p_T^2\rangle}\exp\left(-\frac{p_T^2}{\langle p_T^2\rangle}\right)d^2p_T = 1. \tag{30}$$

One can calculate

$$\langle p_T\rangle = \int \frac{p_T}{\pi\langle p_T^2\rangle}\exp\left(-\frac{p_T^2}{\langle p_T^2\rangle}\right)d^2p_T = \frac{\sqrt{\pi\langle p_T^2\rangle}}{2} \tag{31}$$

and from the transverse momentum, one estimates the total momentum in the nucleon rest frame as

$$\langle|\mathbf{p}|\rangle = \sqrt{\frac{3\langle p_T^2\rangle}{2}} = \sqrt{\frac{6}{\pi}}\langle p_T\rangle. \tag{32}$$

The analysis of the experimental data on the azimuthal asymmetry suggests the following. In the paper [19], the value $\langle p_T^2\rangle \approx 0.25\ \mathrm{GeV}^2$ (i.e. $\langle p_T\rangle \approx 0.44$ GeV) is obtained (note different notation). This result is close to the estimate $\langle p_T\rangle \approx 0.5$–$0.6$ GeV following from the analyses [29,30]. Using the latest information on transverse hadron momenta measured in semi-inclusive DIS, similar numbers were obtained in an independent approach [31]. These figures suggest that the corresponding average energy-momentum of a quark in the nucleon rest frame amounts to ≈ 0.6–0.8 GeV, i.e. ≈ 64–85% of the nucleon mass. They are also substantially higher than the QCD transition temperature mentioned above.

Obviously, two questions arise:

(a) Why do the results related to the intrinsic quark momentum obtained by the methods *A* and *B*, differ by a factor greater than four?

(b) Why does the method B lead to a paradox, that total energy of quarks in the nucleon rest frame can considerably exceed the nucleon mass and related temperature is higher than the QCD transition temperature?

We do not know the answer, but we realize that the contradiction is related to the parton model, which has its limits of validity. Nevertheless, the questions are legitimate and require further discussion. In fact, the inconsistency can originate in an arbitrary stage of the process. For example, the approximation of the probability W by only the one photon exchange (1) can be insufficient without further QCD corrections. Or, another function W can generate a different degree of azimuthal asymmetry in the general expression (24), which means, that fitting the data with the false W can give false $\langle p_T \rangle$ even though the corresponding χ^2 is good. Further, the quark fragmentation into hadrons is a complex stage containing both perturbative and nonperturbative QCD aspects. So the present estimates of its impact on the smearing of primordial quark azimuthal asymmetry can be also rather approximate. Actually, the same inconsistency is discussed also in [18].

IV. INTRINSIC 3D MOTION IN COVARIANT PARTON MODEL

This section follows from our previous study [15,17]. In the present paper, we again assume the quark mass $m \to 0$. This assumption substantially simplifies the calculation and seems be in a good agreement with experimental data—in all model relations and sum rules where such a comparison can be done. But, in principle, a more complicated calculation with $m > 0$ is possible [13]. After fixing the quark mass, there are no free parameters and the construction of the model is based only on the two symmetry requirements: covariance and rotational symmetry. The formulation of the model in terms of the light-cone formalism is suggested in [17] and allows to define the unpolarized leading-twist TMDs f_1 and $f_{1T}^{\perp}$ by means of the light-front correlators $\phi(x, \mathbf{p}_T)_{ij}$ as:

$$\frac{1}{2}\,\mathrm{tr}[\gamma^+ \phi(x, \mathbf{p}_T)] = f_1(x, \mathbf{p}_T) - \frac{\varepsilon^{jk} p_T^j S_T^k}{M_N} f_{1T}^{\perp}(x, \mathbf{p}_T). \tag{33}$$

The corresponding expressions for the integrated and unintegrated distributions f_1 are given by Eqs. (5) and (25) in the cited paper and can be equivalently rewritten as:

$$f_1^q(x) = Mx \int G_q(p_0)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{dp_1 d^2\mathbf{p}_T}{p_0}, \tag{34}$$

$$f_1^q(x, \mathbf{p}_T) = Mx \int G_q(p_0)\delta\left(\frac{p_0 + p_1}{M} - x\right)\frac{dp_1}{p_0}. \tag{35}$$

Now we shall study these expressions in more detail. Because of rotational symmetry in the nucleon rest frame, the distribution G_q depends on one variable p_0; in the manifestly covariant representation the p_0 is replaced by the variable pP/M. In this way, the relation (34) defines the transformation

$$G_q \to f_1^q, \tag{36}$$

where both functions depend on one variable. In [15] we showed that the integral (34) can be inverted

$$G_q(p) = -\frac{1}{\pi M^3}\left(\frac{f_1^q(x)}{x}\right)', \tag{37}$$

where

$$x = \frac{2p}{M}, \qquad p \equiv p_0 = \sqrt{p_1^2 + p_T^2}.$$

In this way, the distributions G_q can be obtained from the distributions f_1^q, which are extracted from the structure functions by global analysis. Apparently, there is a one-to-one mapping

$$G_q(p) \leftrightarrows f_1^q(x) \tag{38}$$

so both distributions represent equivalent descriptions.

Now, we will calculate the TMD integral (35). First we calculate roots of the expression in the $\delta-$ function for the variable p_1:

$$\frac{p_0 + p_1}{M} - x = 0, \tag{39}$$

there is just one root

$$\tilde{p}_1 = \frac{Mx}{2}\left(1 - \left(\frac{p_T}{Mx}\right)^2\right). \tag{40}$$

At the same time the corresponding variable p_0 reads:

$$\tilde{p}_0 = \frac{Mx}{2}\left(1 + \left(\frac{p_T}{Mx}\right)^2\right). \tag{41}$$

The δ-function term can be modified as

$$\begin{aligned}\delta\left(\frac{p_0 + p_1}{M} - x\right)dp_1 &= \frac{\delta(p_1 - \tilde{p}_1)dp_1}{|\frac{d}{dp_1}(\frac{p_0+p_1}{M} - x)_{p_1=\tilde{p}_1}|} \\ &= \frac{\delta(p_1 - \tilde{p}_1)dp_1}{x/p_0},\end{aligned} \tag{42}$$

then after inserting into Eq. (35), one gets:

$$f_1^q(x, \mathbf{p}_T) = M\int G_q(p_0)\delta(p_1 - \tilde{p}_1)dp_1 = MG_q(\tilde{p}_0). \tag{43}$$

One can observe that $f_1^q(x, \mathbf{p}_T)$ depends on x, $\mathbf{p}_T$ via one variable $\tilde{p}_0$ defined by Eq. (41). It is due to fact that this variable in $G_q(\tilde{p}_0)$ reflects rotational symmetry in the rest frame. Obviously, x, $\mathbf{p}_T$ are not independent variables at fixed p_0 or p_1. Also in Eq. (43), both functions represent equivalent description. Further, if we define

$$\xi = x\left(1 + \left(\frac{p_T}{Mx}\right)^2\right), \tag{44}$$

then

$$f_1^q(x, \mathbf{p}_T) = MG_q\left(\frac{M}{2}\xi\right). \tag{45}$$

Since Eq. (37) implies

$$G_q\left(\frac{M}{2}\xi\right) = -\frac{1}{\pi M^3}\left(\frac{f_1^q(\xi)}{\xi}\right)', \tag{46}$$

after inserting to Eq. (45) we get the result

$$f_1^q(x, \mathbf{p}_T) = -\frac{1}{\pi M^2}\left(\frac{f_1^q(\xi)}{\xi}\right)'; \qquad \xi = x\left(1 + \left(\frac{p_T}{Mx}\right)^2\right). \tag{47}$$

This equation represents a new relation which connects integrated and unintegrated unpolarized distribution functions. Before further discussion, one can verify the compatibility with Eqs. (34) and (35):

$$f_1^q(x) = \int f_1^q(x, \mathbf{p}_T) d^2\mathbf{p}_T. \tag{48}$$

Equation (47) implies

$$\int f_1^q(x, \mathbf{p}_T) d^2\mathbf{p}_T = -\frac{2}{M^2}\int_0^{p_{T\max}(x)}\left(\frac{f_1^q(\xi)}{\xi}\right)' p_T dp_T, \tag{49}$$

where we replaced $d^2\mathbf{p}_T = 2\pi p_T dp_T$. From Eq. (44) we have

$$d\xi = \frac{2p_T dp_T}{M^2 x}. \tag{50}$$

and Eqs. (27) and (44) imply

$$x \le \xi \le 1. \tag{51}$$

Now the Eq. (49) can be modified as

$$\int f_1^q(x, \mathbf{p}_T) d^2\mathbf{p}_T = -x\int_x^1\left(\frac{f_1^q(\xi)}{\xi}\right)' d\xi, \tag{52}$$

from which Eq. (48) follows easily.

We can make two remarks about the obtained results:

i) Because of covariance and rotational symmetry (which follows from the invariant variable pP/M in the rest frame), all the following distributions used in our approach involve equivalent information

$$f_1^q(x, \mathbf{p}_T) \Leftrightarrow G_q(\mathbf{p}) \Leftrightarrow G_q(p_0) \Leftrightarrow G_q\left(\frac{pP}{M}\right) \Leftrightarrow f_1^q(x) \tag{53}$$

and also the two sets of variables are equivalent:

$$\mathbf{p} \Leftrightarrow (x, \mathbf{p}_T); \qquad d^3\mathbf{p} = \frac{p_0}{x} dx d^2\mathbf{p}_T. \tag{54}$$

ii) All the functions (53) are assumed to depend also on Q^2, although the evolution is not involved in the present version of the model. Nevertheless, due to this equivalence, in the *present* approach, the evolution of $f_1^q(x, \mathbf{p}_T, Q^2)$ can be obtained from $f_1^q(x, Q^2)$, which is evolved in standard way and similarly for the distribution $G_q(\mathbf{p}, Q^2)$.

Now, we can apply the obtained relations for corresponding numerical calculation. The transverse momentum dependent distribution functions $f_1^q(x, \mathbf{p}_T)$ are calculated from Eq. (47), for input distributions $f_1^q(x)$ we used the standard parameterization [23] (LO at the scale 4 GeV^2). In Fig. 2 we have results for u and d-quarks. The left panel demonstrates, that x and p_T are not independent variables. In accordance with the relation (27), in the sample of partons with fixed p_T, the region of low x is effectively suppressed. For larger p_T, the effect is getting more pronounced. The right panel of the figure demonstrates, that the typical value of p_T in this approach corresponds to the estimates based on the leptonic data in Sec. III.

Further, let us compare our model giving the relation (47) with the approach described in the recent paper [18]. The corresponding relation (57) in the cited paper reads:

$$q(x, \mathbf{k}_T^2) = -\frac{1}{\pi M^2}\frac{d}{dx}\left[\frac{q(x)}{x}\right]_{x=\eta}\theta[x(1-x)M^2 - \mathbf{k}_T^2], \tag{55}$$

where

$$\eta = x + \frac{\mathbf{k}_T^2}{xM^2}.$$

We agree with the authors of the cited paper, that both relations are equivalent (the authors refer to a first version

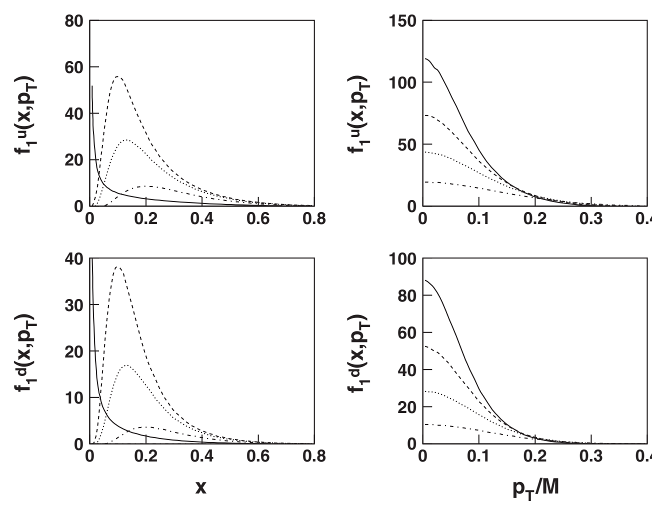

FIG. 2. Transverse momentum dependent unpolarized distribution functions for u (*upper panel*) and d-quarks (*lower panel*). *Left panel*: dependence on x for $p_T/M = 0.10$, 0.13, 0.20 is indicated by dash, dotted and dash-dotted curved lines; solid curved line corresponds to the integrated distribution $f_q(x)$. *Right panel*: dependence on p_T/M for $x = 0.15$, 0.18, 0.22, 0.30 is indicated by solid, dash, dotted and dash-dotted curved lines.

of our paper). The θ-function term corresponds to the constraint (27) valid in our approach, and the correspondence of other symbols is obvious. The relation (55) follows from a previous relation (55) in [18]

$$q(x, \mathbf{k}_T^2) = \frac{1}{\pi M^2}\varphi_3\left(x + \frac{\mathbf{k}_T^2}{xM^2}\right)\theta[x(1-x)M^2 - \mathbf{k}_T^2]. \tag{56}$$

If we integrate this equation, then the left-hand side (lhs) represents the definition (54) in [18]

$$\int q(x, \mathbf{k}_T^2) d^2\mathbf{k}_T = q(x) \tag{57}$$

and after substitutions $d^2\mathbf{k}_T \to \pi d\mathbf{k}_T^2$, $\mathbf{k}_T^2 \to \eta = x + \mathbf{k}_T^2/xM^2$, the right-hand side (rhs) gives

$$\frac{1}{\pi M^2}\int_0^{x(1-x)M^2}\varphi_3\left(x + \frac{\mathbf{k}_T^2}{xM^2}\right)d^2\mathbf{k}_T = x\int_x^1 \varphi_3(\eta)d\eta. \tag{58}$$

The last two equations imply the relation

$$q(x) = x\int_x^1 \varphi_3(\eta)d\eta, \tag{59}$$

which after differentiation and inserting into Eq. (56) gives the final relation (55). The approach developed in [18] is motivated by the classic papers [32,33] from which also the starting Eq. (56) is adopted. The corresponding model is represented by the handbag diagram, in which the incoming line is put on-mass-shell $k^2 = 0$ but has nonzero transverse momentum [32], Fig. 1(a). Let us also remark, that the form of the expression (56) is dictated by Lorentz invariance. Further, comparing this expression with Eq. (45) allows to identify

$$\frac{1}{\pi M^3}\varphi_3(\xi) = G_q\left(\frac{M}{2}\xi\right), \tag{60}$$

where

$$\xi = x + \frac{\mathbf{p}_T^2}{xM^2} = \frac{2P\cdot p}{M^2}, \tag{61}$$

see e.g. Eq. (28) in [18]. The last equality means, that in the nucleon rest frame $\xi = 2p_0/M$, which implies rotational symmetry of both functions φ_3 and G_q in this frame.

So, we can conclude that both approaches have a common basis represented by the requirements:

(i) Lorentz invariance, which, in fact, implies also rotational symmetry of the quark momentum distribution in the nucleon rest frame.

(ii) Quarks are on-mass-shell: $p^2 = 0$.

The equivalent results, like Eqs. (47) and (55) are just a consequence of these conditions. The Wandzura-Wilczek relation obtained equally in [12,18] is a further example. At the same time, it is apparent that despite a common input, the procedures applied in both approaches are substantially different. Other distribution functions like transversity or pretzelosity require additional assumptions to be included in the approach, so the corresponding results from both approaches may differ depending on the chosen method of generalization.

V. SUMMARY AND CONCLUSION

We studied some questions related to the distribution of quark transverse momenta in the framework of the covariant approach. From this point of view, this distribution is a projection of a more general 3D motion of quarks inside the nucleon with respect to the plane transverse to the momentum of the probing particle. Because of general arguments, the 3D motion of quarks in the nucleon rest frame has rotational symmetry. We suggested that in our approach, this rotational symmetry follows from covariance (Lorentz invariance). It follows that in both pictures, 2D and 3D momenta distributions involve equivalent information. The main results obtained in this paper can be summarized as follows.

i) We analyzed the conditions generating the Cahn effect, which represents an important tool for measuring of the quark transverse motion. We suggested that the effect has a more general origin than it is currently considered. We obtained a general expression for azimuthal asymmetry, which depends on intrinsic transverse momentum of the quarks and on the probability $W(\hat{s}, \hat{u})$ of the lepton-quark scattering. At the same time, we presented arguments why the analysis of data on azimuthal asymmetry due to Cahn effect requires caution.

ii) We have done a comparison which suggests that the data on transverse motion based on Cahn effect disagree with the data based on analysis of the structure functions (F_2) in the framework of various models. Both methods differ in estimation of $\langle p_T\rangle$ by factor ≈ 4.

iii) We studied the unpolarized parton distribution functions $f_1^q(x, \mathbf{p}_T)$ in the framework of the 3D covariant parton model. We obtained a new relation which relates this TMD to its integrated counterpart $f_1^q(x)$. Using this relation with the input on the integrated distribution obtained from global analysis, we calculated $f_1^q(x, \mathbf{p}_T)$ also numerically.

iv) We have done a detailed comparison with the recent approach by U. D'Alesio, E. Leader, and F. Murgia [18], in which an equivalent relation and other results coincident with our approach have been obtained. We have proved that both approaches have a common general basis consisting in Lorentz invariance (covariance), and in the on-mass-shell condition $p^2 = 0$. That is why, despite substantially different procedures and formalism applied in both approaches, some results are identical.

v) We confirmed that the requirement of relativistic covariance combined with the nucleon rotational symmetry represents a powerful tool for revealing new relations connecting various parton distribution functions, including the relations between the unintegrated distributions $f_1^q(x, \mathbf{p}_T)$ and their integrated counterparts $f_1^q(x)$.

ACKNOWLEDGMENTS

This work was supported by the project AV0Z10100502 of the Academy of Sciences of the Czech Republic. I am grateful to Anatoli Efremov, Peter Schweitzer and Oleg Teryaev for many useful discussions and valuable comments. I would like to thank also to Jacques Soffer and Claude Bourrely for helpful comments on an earlier version of the manuscript.

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