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Tests of factorization and $SU(3)$ relations in B decays into heavy-light final states

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Using data from the B factories and the Tevatron, we perform tests of how well nonleptonic B decays of the kind $B \to D_{(s)}^{(*)}P$, where P is a pion or kaon, are described within the factorization framework. We find that factorization works well—as is theoretically expected—for color-allowed, tree-diagram-like topologies. Moreover, also exchange topologies, which have a nonfactorizable character, do not show any anomalous behavior. We discuss also isospin triangles between the $B \to D^{(*)}\pi$ decay amplitudes, and determine the corresponding amplitudes in the complex plane, which show a significant enhancement of the color-suppressed tree contribution with respect to the factorization picture. Using data for $B \to D^{(*)}K$ decays, we determine $SU(3)$-breaking effects and cannot resolve any nonfactorizable $SU(3)$-breaking corrections larger than $\sim$5%. In view of these results, we point out that a comparison between the $\bar{B}_d^0 \to D^+\pi^-$ and $\bar{B}_s^0 \to D_s^+\pi^-$ decays offers an interesting new determination of f_d/f_s. Using CDF data, we obtain the most precise value of this ratio at CDF, and discuss the prospects for a corresponding measurement at LHCb.

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I. INTRODUCTION

Nonleptonic weak decays of B mesons play an outstanding role for the exploration of flavor physics and strong interactions. The key challenge of their theoretical description is related to the fact that the corresponding low-energy effective Hamiltonian contains local four-quark operators. Consequently, in the calculation of the transition amplitude, we have to deal with nonperturbative ''hadronic'' matrix elements of these operators. For decades we have applied the ''factorization'' hypothesis, i.e. to estimate the matrix element of the four-quark operators through the product of the matrix elements of the corresponding quark currents [1]. In the 1980s, the $1/N_C$ expansion of QCD [2] and ''color transparency'' arguments [3,4] were used to justify this concept, while it could be put on a rigorous theoretical basis in the heavy-quark limit for a variety of B decays about ten years ago [5,6]. A very useful approach to deal with nonleptonic decays is provided by the decomposition of their amplitudes in terms of different decay topologies and to apply the $SU(3)$ flavor symmetry of strong interactions to derive relations between them [7]. We shall use the same notation as introduced in that paper to distinguish between color-allowed (T), color-suppressed (C), and exchange (E) topologies, which are shown in Fig. 1. For a detailed discussion of the connection between this diagrammatic approach and the low-energy effective Hamiltonian description, the reader is referred to Ref. [8].

Factorization is not a universal feature of nonleptonic B decays and there are cases where it is not expected to work. In fact, nonfactorizable effects are also required to cancel the renormalization-scale dependence in the calculation of the transition amplitude by means of the low-energy effective Hamiltonian. The B-factory data also have shown that nonfactorizable effects can indeed play a significant role, in particular for large CP-conserving strong phases and direct CP violation. In the framework developed in Refs. [5,6], such effects are described by $\Lambda_{\rm QCD}/m_b$ corrections, which are nonperturbative quantities and can therefore only be estimated theoretically with large uncertainties.

Prime examples where factorization is expected to work well are given by the decays $\bar{B}_d^0 \to D^{(*)+}K^-$, which receive only contributions from color-allowed tree-diagram-like topologies. In Ref. [9], we have exploited this feature to propose a new strategy to determine the ratio f_d/f_s of the fragmentation functions, which describe the probability that a b quark will fragment in a $\bar{B}_{d,s}$ meson. It uses the decays $\bar{B}_d^0 \to D^+K^-$ and $\bar{B}_s^0 \to D_s^+\pi^-$. Since the ultimate precision is limited by nonfactorizable U-spin-breaking corrections, which are theoretically expected at the few-percent level in these decays, it is interesting to get experimental insights into factorization and $SU(3)$-breaking corrections. The ratio f_d/f_s enters the measurement of any B_s branching ratio at LHCb and is—in particular—the major limiting factor for the search of new-physics signals through $\mathrm{BR}(B_s^0 \to \mu^+\mu^-)$.

In this paper, we would like to use the currently available B-factory data to check how well factorization works. Factorization tests in B decays into heavy-light final states have been studied before, but the precision of the corresponding input data has now reached a level to obtain a significantly sharper picture.

The outline is as follows: in Sec. II, we discuss factorization tests for the color-allowed amplitude T. In Sec. III, we constrain the impact of exchange topologies, E, which do *not* factorize, and determine their relative orientation with respect to T. In Sec. IV, we use an isospin triangle construction to determine also the color-allowed amplitude C, while we focus on tests of the $SU(3)$ flavor symmetry in

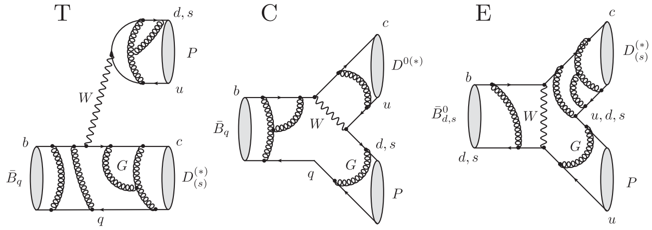

FIG. 1. The color-allowed (tree) (T), color-suppressed (C), and exchange (E) topologies contributing to heavy-light decays ($q \in \{u, d, s\}$).

Sec. V. In Sec. VI, we propose an application of these studies, which is a determination of f_d/f_s by means of the ratio of the branching ratios of the $\bar{B}^0_d \to D^+\pi^-$ and $\bar{B}^0_s \to D^+_s\pi^-$ decays, and discuss the implications of CDF data and the prospects for the corresponding measurement at LHCb. Finally, we summarize our conclusions in Sec. VII. The input parameters for our numerical analysis are collected in Table I.

II. INFORMATION ON T

Let us start our discussion by having a closer look at the decays $\bar{B}^0_d \to D^{(*)+}K^-$, which receive only contributions from color-allowed tree-diagram-like topologies $T^{(*)}$. The Particle Data Group (PDG) gives the branching ratios $\mathrm{BR}(\bar{B}^0_d \to D^+K^-) = (2.0 \pm 0.6) \times 10^{-6}$ and $\mathrm{BR}(\bar{B}^0_d \to D^{*+}K^-) = (2.14 \pm 0.16) \times 10^{-4}$ [10]. Using the differential rates of semileptonic decays, we can actually probe nonfactorizable terms [3]. The corresponding expression can be written as follows [5]:

$$R_P^{(*)} \equiv \frac{\mathrm{BR}(\bar{B}^0_d \to D^{(*)+}P^-)}{d\Gamma(\bar{B}^0_d \to D^{(*)+}\ell^-\bar{\nu}_\ell)/dq^2|_{q^2=m_P^2}} = 6\pi^2\tau_{B_d}|V_P|^2 f_P^2 |a_1(D_qP)|^2 X_P, \qquad (1)$$

where τ_{B_d} is the B_d lifetime, q^2 the four-momentum transfer to the lepton pair, $|V_P|$ the corresponding element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, f_P is the decay constant of the P meson, and X_P deviates from 1 below the percent level. The quantity $a_1(D_qP)$ describes the deviation from naive factorization. As discussed in detail in Ref. [5], this parameter is found in "QCD factorization" as a quasiuniversal quantity $|a_1| \simeq 1.05$ with very small process-dependent "nonfactorizable" corrections.

A first implementation of the factorization test in (1) for the $\bar{B}^0_d \to D^{*+}\pi^-$ channel was performed in Ref. [11]. In the past decade, we have seen a lot of progress with the measurements of the semileptonic $\bar{B}^0_d \to D^{(*)+}\ell^-\bar{\nu}_\ell$ decays, which play a key role for the determination of $|V_{cb}|$, and of the nonleptonic $B \to D^{(*)}_{(s)}P$ decays. The averages of the total exclusive semileptonic branching fractions amount to $\mathrm{BR}(\bar{B} \to D\ell^-\bar{\nu}_\ell) = (2.17 \pm 0.12)\%$ and $\mathrm{BR}(\bar{B}^0_d \to D^{*+}\ell^-\bar{\nu}_\ell) = (5.05 \pm 0.12)\%$ [10].

To parametrize the form factors, usually the variable

$$w \equiv v \cdot v' = \frac{m_B^2 + m_D^2 - q^2}{2m_Bm_D} \qquad (2)$$

is used, which is the product of the four-velocities v and v' of the B and $D^{(*)}_{(s)}$ mesons, respectively. The correspondence between the differential rates is given by

$$\frac{d\Gamma}{dq^2} = \frac{1}{2m_Bm_D}\frac{d\Gamma}{dw}. \qquad (3)$$

In order to determine the differential semileptonic decay rate at the appropriate momentum transfer for the factorization test in (1), we use the form-factor parametrization proposed by Caprini, Lellouch, and Neubert [12], with parameters summarized in Table II, yielding the rates shown in Fig. 2.

In the values of the semileptonic decay rates, the systematic uncertainty is estimated by propagating the uncertainties from the parameters in Table II to the appropriate value of w, taking the correlations into account.

Using the numerical values from Table III and the branching ratios for the nonleptonic decays given by the

TABLE I. Parameters used in the numerical analysis.

m_{B^0}	5279.17 MeV	$m_{B^0_s}$	5336.3 MeV
m_{D^+}	1869.60 MeV	$m_{D^+_s}$	1968.47 MeV
m_{D^0}	1864.83 MeV	$m_{D^{*0}}$	2006.96 MeV
$m_{D^{*+}}$	2010.25 MeV	$m_{D^{*+}_s}$	2112.3 MeV
m_{K^+}	497.61 MeV	m_{K^0}	493.68 MeV
m_{π^+}	139.57 MeV	m_{π^0}	134.98 MeV
f_π	130.41 MeV	f_K	156.1 MeV
f_ρ	215 MeV	f_D	206.7 MeV
f_{D^*}	206.7 MeV	f_{D_s}	257.5 MeV
τ_{B^0}	1.525 ps	$\tau_{B^\pm}/\tau_{B^0}$	1.071
$\lvert V_{ud}\rvert$	0.974 25	$\lvert V_{us}\rvert$	0.2252

TABLE II. The parameters for the form-factor parametrization of Ref. [12], as determined by the Belle and *BABAR* collaborations, and the world average given by the Heavy Flavour Averaging Group (HFAG). The recent precise determination of the form-factor parametrization for $\bar{B}^0_d \to D^{*+}\ell^-\bar{\nu}_\ell$ presented by the Belle collaboration [13] is not taken into account in the world average yet.

$\bar{B}^0_d \to D^+\ell^-\bar{\nu}_\ell$	HFAG [14]	Belle [14,15]	*BABAR* [16]
$F(1)\lvert V_{cb}\rvert[10^{-3}]$	42.3 ± 1.48	40.85 ± 7.0	43.0 ± 2.15
ρ^2	1.18 ± 0.06	1.12 ± 0.26	1.20 ± 0.10
$\bar{B}^0_d \to D^{*+}\ell^-\bar{\nu}_\ell$	HFAG [14]	Belle [13]	*BABAR* [17]
$F(1)\lvert V_{cb}\rvert[10^{-3}]$	36.04 ± 0.52	34.6 ± 1.0	34.4 ± 1.2
ρ^2	1.24 ± 0.04	1.214 ± 0.034	1.191 ± 0.056
R_1	1.410 ± 0.049	1.401 ± 0.034	1.429 ± 0.075
R_2	0.844 ± 0.027	0.864 ± 0.024	0.827 ± 0.043

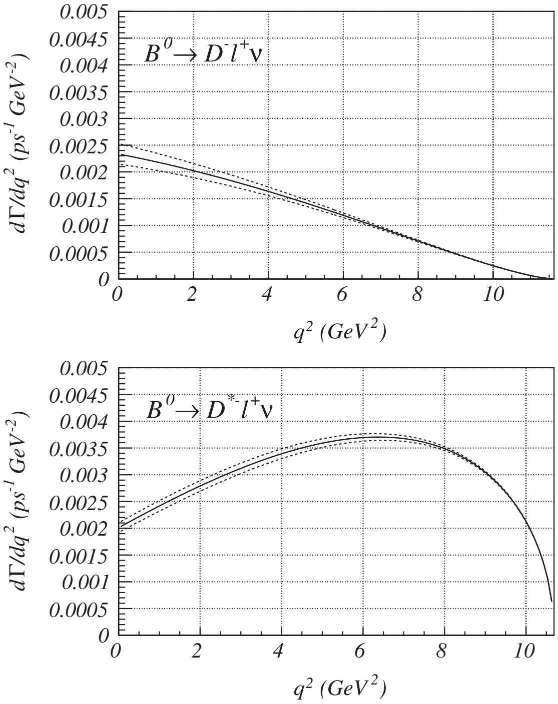

FIG. 2. $d\Gamma/dq^2$ for the form-factor parametrization of Ref. [12] and the HFAG parameters as given in Table II. The uncertainty on ρ^2 is illustrated by the dotted curves.

PDG [10], we arrive at the values for $|a_1(D_qP)|$ collected in Table IV and compiled in Fig. 3. In naive factorization, we have $|a_1(D_qP)| = 1$, while the QCD factorization analysis of Ref. [5] results in an essentially universal value of $|a_1| \simeq 1.05$. It is interesting to note that the current experimental values of the $|a_1|$ favor a central value that is smaller than 1, around $|a_1| \simeq 0.95$. Within the errors, we cannot resolve nonfactorizable effects. Only the $\bar{B}_d^0 \to D^+\pi^-$ decay shows a value of a_1 that is about 2σ away from factorization.

We encourage the B factories to determine the ratios of the semileptonic differential decay rates and the relevant hadronic branching ratios directly. Correlated systematic uncertainties, such as the $D^{(*)}$ reconstruction efficiencies and the $D^{(*)}$ branching fractions, would cancel so that the B-factory results could be fully exploited. These correlations are not considered in the errors estimated in Table IV.

TABLE III. The semileptonic differential decay rates at the values of the relevant four-momentum transfers entering the factorization test in Eq. (1). The parameters from Table II are used in the form-factor parametrization, and the full correlations are taken into account in the uncertainty of $d\Gamma = dq^2$.

Corresponding hadronic decay	w	$d\Gamma/dq^2(\times 10^3)[\mathrm{GeV}^{-2}\,\mathrm{ps}^{-1}]$ *BABAR*	Belle
$\bar{B}_d^0 \to D^+\pi^-$	1.588	2.34 ± 0.13	2.36 ± 0.42
$\bar{B}_d^0 \to D^+K^-$	1.577	2.31 ± 0.13	2.32 ± 0.42
$\bar{B}_d^0 \to D^{*+}\pi^-$	1.503	1.99 ± 0.13	1.86 ± 0.09
$\bar{B}_d^0 \to D^{*+}K^-$	1.492	2.08 ± 0.14	1.95 ± 0.10

Recently, calculations became available that estimate electromagnetic corrections to two-body B-meson decays into two light hadrons [18]. They can be as large as 5% for $B_d^0 \to \pi^+\pi^-$ for a $E_{\gamma,\max} = 250$ MeV, but we do not know to what extend these corrections are accounted for in the measurements of heavy-light decays.

As noted in Ref. [5], further tests of factorization are offered by the measurement of the ratios of nonleptonic decay rates [5]:

$$\frac{\mathrm{BR}(\bar{B}_d^0 \to D^+\pi^-)}{\mathrm{BR}(\bar{B}_d^0 \to D^{*+}\pi^-)} = \frac{(m_B^2 - m_D^2)^2|\vec{q}|}{4m_B^2|\vec{q}|^3}\left(\frac{F_0(m_\pi^2)}{A_0(m_\pi^2)}\right)^2 \times \left|\frac{a_1(D\pi)}{a_1(D^*\pi)}\right|^2, \tag{4}$$

$$\frac{\mathrm{BR}(\bar{B}_d^0 \to D^+\rho^-)}{\mathrm{BR}(\bar{B}_d^0 \to D^+\pi^-)} = \frac{4m_B^2|\vec{q}|^3}{(m_B^2 - m_D^2)^2|\vec{q}|}\frac{f_\rho^2}{f_\pi^2}\left(\frac{F_+(m_\rho^2)}{F_0(m_\pi^2)}\right)^2 \times \left|\frac{a_1(D\rho)}{a_1(D\pi)}\right|^2. \tag{5}$$

Using the branching ratios from Table IV gives

$$\left|\frac{a_1(D\pi)}{a_1(D^*\pi)}\right|\frac{F_0(m_\pi^2)}{A_0(m_\pi^2)} = 0.95 \pm 0.03 \tag{6}$$

$$\left|\frac{a_1(D\rho)}{a_1(D\pi)}\right|\frac{F_+(m_\rho^2)}{F_0(m_\pi^2)} = 1.07 \pm 0.10, \tag{7}$$

so that there is—within the errors—no evidence for any deviation from naive factorization.

It is worth noticing that, in the case where the pseudoscalar is replaced by a vector meson, the structure is much richer. In this case factorization can be tested through the longitudinal polarization of the D^* mesons [19]. This feature was exploited in the decay $\bar{B}_d^0 \to D^{*+}\rho^-$ in Ref. [20], where the measured polarization was found in excellent agreement with the factorization prediction within 2%. Other final states such as $D^{*+}D^{*-}$, $D^{*+}D_s^{*-}$, and $D^{*+}\omega\pi^-$ further strengthen the agreement with factorization [21].

Finally, the large value for the longitudinal polarization of $B_s^0 \to D_s^{-*}\rho^+$ as reported in Ref. [22] not only agrees with factorization, but is also—within the errors—in agreement with the value for $B_d \to D^{*-}\rho^+$, thereby supporting the $SU(3)$ flavor symmetry:

$$f_L(B^0 \to D^{-*}\rho^+) = 0.885 \pm 0.02 \tag{8}$$

$$f_L(B_s^0 \to D_s^{-*}\rho^+) = 1.05 \pm 0.09. \tag{9}$$

Here $f_L = \Gamma_L/\Gamma = |H_0|^2/(|H_{-1}|^2 + |H_0|^2 + |H_{+1}|^2)$.

TABLE IV. Determination of the $|a_1(D_qP)|$ from the current data. The error is estimated by adding the uncertainties of the hadronic branching ratio and the semileptonic rate in quadrature. The correlations between the form-factor parameters for the semileptonic decay rate are taken into account.

Topology	Decay	BR [10] ($\times 10^4$)	$\lvert a_1(D_qP)\rvert$ *BABAR*	$\lvert a_1(D_qP)\rvert$ Belle
T'	$\bar{B}^0_d \to D^+K^-$	2.0 ± 0.6	0.89 ± 0.13	0.88 ± 0.16
T'^*	$\bar{B}^0_d \to D^{*+}K^-$	2.14 ± 0.16	0.96 ± 0.05	0.99 ± 0.05
$T + E$	$\bar{B}^0_d \to D^+\pi^-$	26.8 ± 1.3	0.88 ± 0.04	0.88 ± 0.09
$T^* + E^*$	$\bar{B}^0_d \to D^{*+}\pi^-$	27.6 ± 1.3	0.98 ± 0.04	1.01 ± 0.04

III. INFORMATION ON E

Exchange topologies E (see Fig. 1), which are naively expected to be significantly suppressed with respect to the color-allowed T amplitudes, are examples where factorization is not expected to be a good approximation [5]. In contrast to the $D^{(*)}K$ decays considered in the previous section, the $\bar{B}^0_d \to D^{(*)+}\pi^-$ modes receive contributions from a color-allowed tree and an exchange topology so that their decay amplitudes take the following form:

$$\mathrm{BR}\,(\bar{B}^0_d \to D^{(*)+}\pi^-) = |A(\bar{B}^0_d \to D^{(*)+}\pi^-)|^2 \Phi^d_{D^{(*)}\pi}\tau_{B_d},$$

where $\Phi^d_{D^{(*)}\pi}$ is a phase-space factor and

$$A(\bar{B}^0_d \to D^{(*)+}\pi^-) = T^{(*)} + E^{(*)}. \tag{10}$$

The current experimental averages for their branching ratios are given in the lower half of Table IV.

We will distinguish the $D^{(*)}\pi$ amplitudes from the $D^{(*)}K$ amplitudes by the prime symbol. This will be relevant in Sec. V, where the validity of the $SU(3)$ flavor symmetry is further discussed.

The $E'^{(*)}$ amplitudes can actually be probed in three ways, namely, by comparing the hadronic branching

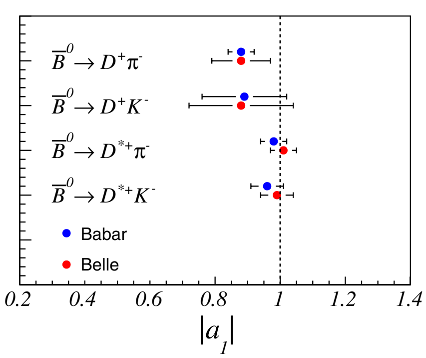

FIG. 3 (color online). The values for $|a_1(D_qP)|$ as obtained with the Belle and *BABAR* parameters from Table II. The errors represent the error from the hadronic branching ratio [10] with the uncertainty of the semileptonic decay rate added in quadrature. The full correlation matrix of the uncertainties in the determination of the form-factor parametrization of both the Belle and *BABAR* result is taken into account. No uncertainty on the decay constants is included.

fractions to the semileptonic decay rates as was done in the previous section, by using the ratios of branching ratios governed by the $T'^{(*)}$ and $T^{(*)} + E^{(*)}$ amplitudes, and by probing $E'^{(*)}$ directly through the branching ratios of decays that originate only from exchange topologies.

The comparison to the semileptonic rates is shown in the lower half of Table IV, and shows no sign of an enhancement of the $E'^{(*)}$ amplitudes with respect to the naive expectation [5].

Let us next probe the $E'^{(*)}$ topologies through the ratios of branching ratios, $\mathrm{BR}(\bar{B}^0_d \to D^{(*)+}\pi^-)/\mathrm{BR}(\bar{B}^0_d \to D^{(*)+}K^-)$. In the following, we will correct the $T'^{(*)}$ amplitudes from $\bar{B}^0_d \to D^{(*)+}K^-$ for factorizable $SU(3)$-breaking corrections, to allow for a direct comparison with the $T^{(*)} + E^{(*)}$ amplitude from $\bar{B}^0_d \to D^{(*)+}\pi^-$. The factorizable $SU(3)$-breaking corrections contain the pion and kaon decay constants f_π and f_K, respectively, and the corresponding form factors, which we discussed in the previous section:

$$\left|\frac{T'^{(*)}}{T^{(*)}}\right|_{\mathrm{fact}} = \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} \frac{F^{B\to D^{(*)}}(m_K^2)}{F^{B\to D^{(*)}}(m_\pi^2)}. \tag{11}$$

In the case of the decays involving D^{*+} mesons, the ratio of the branching ratios has been measured with impressive precision [23]:

$$\frac{\mathrm{BR}(\bar{B}^0_d \to D^{*+}K^-)}{\mathrm{BR}(\bar{B}^0_d \to D^{*+}\pi^-)} = (7.76 \pm 0.34 \pm 0.29)\%, \tag{12}$$

which allows us to extract the ratio of $|T + E|$ and $|T'|$ amplitudes:

$$\left|\frac{T'^*}{T^* + E^*}\right| \xrightarrow{\mathrm{fact}} \left|\frac{T^*}{T^* + E^*}\right| = 0.983 \pm 0.028. \tag{13}$$

The consistency of the numerical value with 1 is remarkable and shows both a small impact of the exchange topology and of nonfactorizable $SU(3)$-breaking effects.

Unfortunately, the $SU(3)$-counterpart $\bar{B}^0_d \to D^+K^-$ of $\bar{B}^0_d \to D^+\pi^-$ still suffers from large uncertainties that are introduced by the experimental value of $\mathrm{BR}(\bar{B}^0_d \to D^+K^-)$, yielding

$$\left|\frac{T'}{T+E}\right| \overset{\text{fact}}{\to} \left|\frac{T}{T+E}\right| = 0.99 \pm 0.15. \qquad (14)$$

The CDF collaboration has quoted the ratio $\text{BR}(\bar{B}_d^0 \to D^+K^-)/\text{BR}(\bar{B}_d^0 \to D^+\pi^-) = 0.086 \pm 0.005(\text{stat})$ [24], but has unfortunately not yet assigned a systematic error. This result would lead to a numerical value of $1.07 \pm 0.03(\text{stat})$ for the ratio in Eq. (14). It would be interesting to get also an assessment of the corresponding systematic uncertainty.

Finally, we can also probe the exchange topologies directly through $\bar{B}_d^0 \to D_s^{(*)+}K^-$ decays:

$$A(\bar{B}_d^0 \to D_s^{(*)+}K^-) = E'^{(*)}. \qquad (15)$$

As in Eq. (11) we take differences in the final state into account through

$$\left|\frac{E'^{(*)}}{E^{(*)}}\right|_{\text{fact}} = \frac{f_K f_{D_s^{(*)}}}{f_\pi f_{D^{(*)}}}, \qquad (16)$$

where the $f_{D^{(*)}}$ and $f_{D_s^{(*)}}$ are the decay constants of the $D^{(*)}$ and $D_s^{(*)}$ mesons, respectively. In our numerical analysis, we use $f_{D_s^{(*)}}/f_{D^{(*)}} = 1.25 \pm 0.06$ [10]. The branching ratios are already well measured, as can be seen in Table V [10], and yield

$$\left|\frac{E}{T+E}\right| = 0.073 \pm 0.006 \qquad (17)$$

$$\left|\frac{E^*}{T^*+E^*}\right| = 0.066 \pm 0.006, \qquad (18)$$

where we have rescaled the E' amplitude to the E amplitude according to Eq. (16).

It is instructive to illustrate the triangle relation between the $E^{(*)}$, $T^{(*)}$, and $E^{(*)} + T^{(*)}$ amplitudes in the complex plane. In Fig. 4, we show the situations emerging from the current data for the $B \to D^*P$ decays. While the $B \to DP$ decays still suffer from large uncertainties due to (14), we arrive at a significantly sharper picture for the $B \to D^*P$ modes. In particular, we can also determine the strong phase δ_* between the E'^* and T'^* amplitudes, which is given by $\delta_* \sim (77 \pm 30)°$. The favored large value of this phase explains the small impact of the E^* amplitude on the total $\bar{B}_d^0 \to D^{*+}\pi^-$ branching ratio.

TABLE V. Predictions for the branching ratios of B_s decays that occur only through exchange topologies.

Decay	Measured BR [10] ($\times 10^6$)	Predicted BR ($\times 10^6$)
$\bar{B}_d^0 \to D_s^+K^-$	30 ± 4	
$\bar{B}_s^0 \to D^+\pi^-$		1.19 ± 0.16
$\bar{B}_d^0 \to D_s^{*+}K^-$	21.9 ± 3	
$\bar{B}_s^0 \to D^{*+}\pi^-$		0.90 ± 0.12

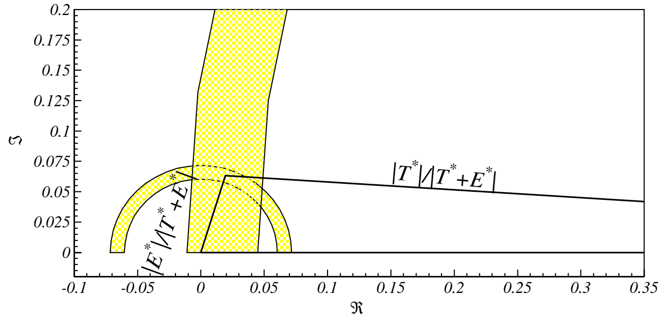

FIG. 4 (color online). The E^*, T^*, and $E^* + T^*$ amplitudes in the complex plane for the $B^0 \to D_{(s)}^*\pi/K$ decays. The E'^* and T'^* amplitudes are rescaled by factorizable $SU(3)$ corrections to match the E^* and T^* amplitudes for the $B \to D^*\pi$ case.

Other potentially interesting decays to obtain insights into the exchange topologies are the $\bar{B}_s^0 \to D^{(*)+}\pi^-$ modes. Using the U-spin flavor symmetry, we expect

$$\frac{\text{BR}(\bar{B}_s^0 \to D^{(*)+}\pi^-)}{\text{BR}(\bar{B}_d^0 \to D_s^{(*)+}K^-)} = \left|\frac{V_{us}}{V_{ud}}\right|^2 \left[\frac{f_{D^{(*)}} f_\pi f_{B_s}}{f_{D_s^{(*)}} f_K f_{B_d}}\right]^2 \times \frac{\tau_{B_s} \Phi^s_{D^{(*)}\pi}}{\tau_{B_d} \Phi^d_{D_s^{(*)}K}}. \qquad (19)$$

The predictions for the B_s branching ratios using this relation are given in Table V. Unfortunately, it will be challenging for LHCb to measure this small branching ratio accurately since only a dozen of $\bar{B}_s^0 \to D^{(*)+}\pi^-$ events are expected to be selected within the 1 fb^{-1} data sample, which should be available by the end of 2011. However, for a luminosity of (5–10) fb^{-1}, LHCb has the potential to discover these strongly suppressed decays. A future measurement of the ratios in Eq. (19) would be an interesting probe of nonfactorizable U-spin-breaking effects.

IV. ISOSPIN TRIANGLES AND INFORMATION ON C

The amplitudes for the three $B \to D^{(*)}\pi$ decays can be expressed in terms of color-allowed and color-suppressed tree as well as exchange topologies. Alternatively, the system can also be decomposed in terms of two isospin amplitudes, $A_{1/2}$ and $A_{3/2}$, which correspond to the transition into $D^{(*)}\pi$ final states with isospin $I = 1/2$ and $I = 3/2$, respectively [25]. The ratio

$$\frac{A_{1/2}}{\sqrt{2}A_{3/2}} = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_b) \qquad (20)$$

is a measure of the departure from the heavy-quark limit [5], and has been measured by the CLEO [26] and *BABAR* collaborations [27].

Using updated information on the nonleptonic branching ratios, we will repeat this isospin analysis. The corresponding isospin relations read as

$$A(\bar{B}^0_d \to D^+\pi^-) = \sqrt{\tfrac{1}{3}}A_{3/2} + \sqrt{\tfrac{2}{3}}A_{1/2} = T + E \quad (21)$$

$$\sqrt{2}A(\bar{B}^0_d \to D^0\pi^0) = \sqrt{\tfrac{4}{3}}A_{3/2} - \sqrt{\tfrac{2}{3}}A_{1/2} = C - E \quad (22)$$

$$A(B^- \to D^0\pi^-) = \sqrt{3}A_{3/2} = T + C, \quad (23)$$

so that

$$A_{1/2} = \frac{2T - C + 3E}{\sqrt{6}} \quad (24)$$

$$A_{3/2} = \frac{T + C}{\sqrt{3}}, \quad (25)$$

which leads to the following expression:

$$\frac{A_{1/2}}{\sqrt{2}A_{3/2}} = 1 - \frac{3}{2}\left(\frac{C - E}{T + C}\right). \quad (26)$$

The $(T + E)$, $(C - E)$, and $(T + C)$ amplitudes can be depicted in the complex plane, and related to the ratio of isospin amplitudes, as shown in Fig. 5.

The absolute values of the amplitudes $A_{1/2}$ and $A_{3/2}$ can also be obtained directly from the measured decay rates:

$$|A_{1/2}|^2 = |A(D^+\pi^-)|^2 + |A(D^0\pi^0)|^2 - \tfrac{1}{3}|A(D^0\pi^-)|^2$$
$$|A_{3/2}|^2 = \tfrac{1}{3}|A(D^0\pi^-)|^2, \quad (27)$$

which can be expressed in terms of partial decay widths through $|A(D\pi)|^2 = \Gamma(D\pi)/\Phi^d_{D\pi}$, i.e. corrected for the small differences in phase space, which mainly leads to a small but measurable correction for the $D^0\pi^0$ final

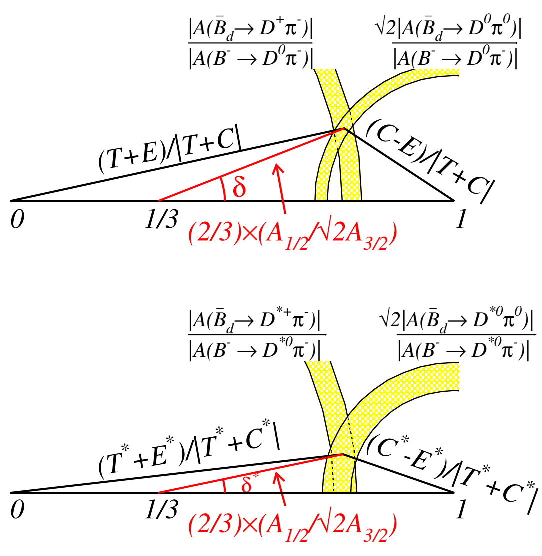

FIG. 5 (color online). Sketch of the $T + E$ and $C - E$ amplitudes, normalized to $|T + C|$ (and corrected for differences in phase space) in the complex plane for the $B \to D\pi$ decays (top) and $B \to D^*\pi$ decays (bottom). The ratio of isospin amplitudes in Eq. (26) is also drawn.

state. The relative strong phase between the $I = 3/2$ and $I = 1/2$ amplitudes can be calculated with

$$\cos\delta = \frac{3|A(D^+\pi^-)|^2 + |A(D^0\pi^-)|^2 - 6|A(D^0\pi^0)|^2}{6\sqrt{2}|A_{1/2}||A_{3/2}|}, \quad (28)$$

and similar for δ^*.

We find the following numerical results:

$$\left|\frac{A_{1/2}}{\sqrt{2}A_{3/2}}\right|_{D\pi} = 0.676 \pm 0.038 \quad (29)$$

$$\left|\frac{A_{1/2}}{\sqrt{2}A_{3/2}}\right|_{D^*\pi} = 0.639 \pm 0.039, \quad (30)$$

which are complemented by

$$\cos\delta = 0.930^{+0.024}_{-0.022}, \quad (31)$$

$$\cos\delta^* = 0.979^{+0.048}_{-0.043}. \quad (32)$$

The nominal value is calculated from the central values of the branching fractions, whereas the $\pm 1\sigma$ confidence interval is defined as the integral of 68.3% of the total area of its likelihood function, similar to the procedure followed in Ref. [27]. The corresponding central values for the strong phases then become $\delta = 21.6°$ and $\delta^* = 11.9°$ for the $D\pi$ and the $D^*\pi$ case, respectively.

Comparing with (20), we observe that the isospin-amplitude ratio shows significant deviations from the heavy-quark limit. In view of our analysis of the exchange topologies in Sec. III and the expression in (26), we can trace this feature back to the color-suppressed C topologies.

V. TESTS OF $SU(3)$ SYMMETRY

Let us next probe the impact of nonfactorizable $SU(3)$-breaking corrections in B-meson decays into heavy-light final states. To this end, we compare the three $B \to D^{(*)}\pi$ decays with their $SU(3)$-related $B \to D^{(*)}_{(s)}K$ channels, which have decay amplitudes of the following structure:

$$A(\bar{B}^0_d \to D^0K^0) = C' \quad (33)$$

$$A(\bar{B}^0_d \to D^+_sK^-) = E' \quad (34)$$

$$A(\bar{B}^0_d \to D^+K^-) = T' \quad (35)$$

$$A(B^- \to D^0K^-) = T' + C'. \quad (36)$$

Here the notation is as above and the primes remind us again that we are dealing with $b \to c\bar{c}s$ quark-level transitions in this case. In order to quantify the validity of the $SU(3)$ flavor symmetry, we can perform the following four experimental tests:

(i) consistency between E'^*, T'^*, and $T^* + E^*$;
(ii) consistency between $E'^{(*)}$, $C'^{(*)}$, and $C^{(*)} - E^{(*)}$;
(iii) ratio of $|T^{(*)} + C^{(*)}|$ and $|T'^{(*)} + C'^{(*)}|$;
(iv) prediction for $E'^{(*)}$, based on all amplitudes listed in Table VI apart from $A(\bar{B}_d^0 \to D_s^+ K^-)$.

Tests (ii)–(iv) can be performed with both the $B \to D_{(s)}P$ and the $B \to D_{(s)}^* P$ systems. On the other hand, due to the large uncertainty affecting $\mathrm{BR}(\bar{B}_d^0 \to D^+ K^-)$, test (i) can currently only be applied to the D^* case. We will use the values for the branching fractions as listed in Table VI. The size of the E'^*, T'^*, and $T^* + E^*$ amplitudes are internally consistent, as is shown by the overlapping circles in Fig. 5. As we noted already in Sec. III, this also indicates that there are no large nonfactorizable $SU(3)$-breaking effects in the E'^* or T'^* amplitudes.

Similarly to Eq. (13) we can check the consistency between the $E'^{(*)}$, $C'^{(*)}$, and $C^{(*)} - E^{(*)}$ amplitudes. As before we will correct the $C'^{(*)}$ amplitudes from $\bar{B}_d^0 \to D^{(*)0}K^0$ for factorizable $SU(3)$-breaking corrections, to allow for a direct comparison with the $C - E$ amplitude,

$$\left|\frac{C'^{(*)}}{C^{(*)}}\right|_{\mathrm{fact}} = \left|\frac{V_{us}}{V_{ud}}\right| \frac{F^{B\to K}(m_{D^{(*)}}^2)}{F^{B\to\pi}(m_{D^{(*)}}^2)}, \quad (37)$$

where we use the parametrization for $F^{B\to\pi/K}$ from Ref. [28]. We extract the following ratio of $|C - E|$ and $|C'|$ amplitudes:

$$\left|\frac{C - E}{C}\right| = 0.913 \pm 0.074 \quad (38)$$

$$\left|\frac{C^* - E^*}{C^*}\right| = 0.89 \pm 0.18, \quad (39)$$

where the factorizable $SU(3)$-breaking corrections are taken into account. Again, the ratio is close to 1, indicating that the contribution of E is small, and that there are no unexpected nonfactorizable $SU(3)$ violating effects, in addition to the factorizable $SU(3)$ corrections. This is remarkable in view of the nonfactorizable character of the color-suppressed decays.

A direct measure of the size of $SU(3)$-breaking effects in $B \to D^{(*)}P$ decays is provided by the ratio of the $|T^{(*)} + C^{(*)}|$ and $|T'^{(*)} + C'^{(*)}|$ amplitudes:

$$\frac{\mathrm{BR}(\bar{B}^- \to D^{(*)0}\pi^-)}{\mathrm{BR}(\bar{B}^- \to D^{(*)0}K^-)} = \left|\frac{T^{(*)} + C^{(*)}}{T'^{(*)} + C'^{(*)}}\right|^2 \frac{\Phi_{D^{(*)}\pi}}{\Phi_{D^{(*)}K}}, \quad (40)$$

where the ratio of branching ratios has been measured for the D^0 case as follows [29]:

$$\frac{\mathrm{BR}(B^- \to D^0 K^-)}{\mathrm{BR}(B^- \to D^0 \pi^-)} = (7.7 \pm 0.5 \pm 0.6)\%. \quad (41)$$

If we include factorizable $SU(3)$-breaking effects through the corresponding decay constants and form factors, the numerical values of the relevant amplitude ratios are given as follows:

$$\left|\frac{T + C}{T' + C'}\right| \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} \frac{F^{B\to D}(m_K^2)}{F^{B\to D}(m_\pi^2)} = 0.997 \pm 0.047, \quad (42)$$

$$\left|\frac{T^* + C^*}{T'^* + C'^*}\right| \left|\frac{V_{us}}{V_{ud}}\right| \frac{f_K}{f_\pi} \frac{F^{B\to D^*}(m_K^2)}{F^{B\to D^*}(m_\pi^2)} = 0.995 \pm 0.048. \quad (43)$$

The factorizable $SU(3)$-breaking effects for the C amplitudes (37) are numerically close to the ones for the T amplitudes (11), and since the T amplitude is the dominant amplitude here, we rescale in the same way as in Eq. (11).

The consistency with 1 is remarkable. In particular, we find that nonfactorizable $SU(3)$-breaking effects are smaller than 5%, even in decays that have a large contribution from color-suppressed amplitudes where factorization does not work well, as we have seen in the previous section.

Finally, we can—in analogy to Fig. 5—construct a second amplitude triangle, which involves now the T' and C' amplitudes, as shown in Fig. 6. If we rescale the primed amplitudes involving a kaon in the final state to the amplitudes with a pion in the final state by correcting for the factorizable $SU(3)$-breaking corrections, the distance between the apexes of Figs. 5 and 6 shows graphically how $|E|$ can be predicted. The consistency between the corresponding value and the measured value for $|E'|$ from $\mathrm{BR}(\bar{B}_d^0 \to D_s^- \pi^+)$ is a direct probe for nonfactorizable $SU(3)$-breaking effects in nonleptonic decays of the type $B \to D^{(*)}P$. The numerical picture for the $D\pi$ and $D^*\pi$ cases is still not precise enough to predict the measured value:

$$\left|\frac{E}{T + C}\right|_{\mathrm{meas}} = 0.056 \pm 0.004 \quad (44)$$

and

TABLE VI. Branching fractions used in the various tests of the $SU(3)$ flavor symmetry.

Topology	Final state	Measured BR($\times 10^4$) [10] $\bar{B}_d^0 \to DP$	$\bar{B}_d^0 \to D^*P$
Isospin amplitudes			
$T^{(*)} + E^{(*)}$	$D^{(*)+}\pi^-$	26.8 ± 1.3	27.6 ± 1.3
$\frac{1}{\sqrt{2}}(C^{(*)} - E^{(*)})$	$D^{(*)0}\pi^0$	2.61 ± 0.24	1.7 ± 0.4
$T^{(*)} + C^{(*)}$	$D^{(*)0}\pi^-$	48.4 ± 1.5	51.9 ± 2.6
Amplitudes used to probe $SU(3)$ symmetry			
$E'^{(*)}$	$D_s^{(*)+}K^-$	0.30 ± 0.04	0.219 ± 0.03
$C'^{(*)}$	$D^{(*)0}K^0$	0.52 ± 0.07	0.36 ± 0.12
$T'^{(*)}$	$D^{(*)+}K^-$	2.0 ± 0.6	2.14 ± 0.16
$T'^{(*)} + C'^{(*)}$	$D^{(*)0}K^-$	3.68 ± 0.33	4.21 ± 0.35

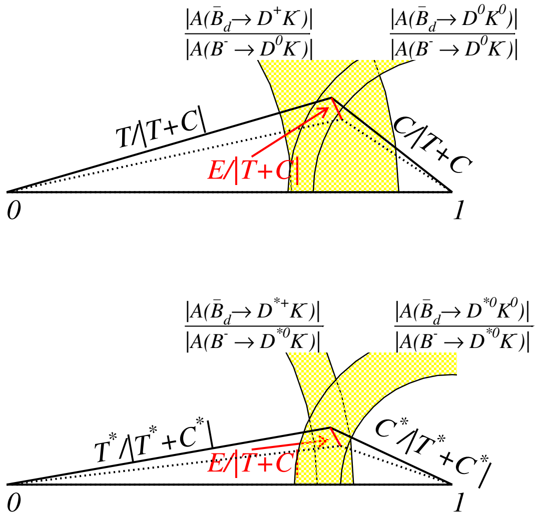

FIG. 6 (color online). Sketch of the T' and C' amplitudes, normalized to $|T' + C'|$ [and corrected for factorizable $SU(3)$-breaking effects] in the complex plane for the $B \to DK$ decays (top) and $B \to D^*K$ decays (bottom). The predicted E amplitude, assuming $SU(3)$ symmetry is also drawn.

$$\left|\frac{E^*}{T^* + C^*}\right|_{\rm meas} = 0.047 \pm 0.004, \tag{45}$$

respectively, where $E'^{(*)}$ is rescaled to $E^{(*)}$ according to Eq. (16). The knowledge of $T'^{(*)}$ and $C'^{(*)}$ will probably be improved in the near future, which will provide another interesting test of the validity of the $SU(3)$ flavor symmetry.

In the factorization tests discussed above, we did not consider B_s decays. In this context, interesting information on $SU(3)$-breaking effects can be obtained from the comparison between the polarization amplitudes of $B_s \to J/\psi\phi$ and $B_d \to J/\psi K^{*0}$ decays, which are found in excellent agreement with one another [30,31]. Moreover, also analyses of $B_s \to K^+K^-$ and $B_s \to \pi^{\mp}K^{\pm}$ modes and their comparison with $B \to \pi K$, $\pi\pi$ decays do not show any indications of large nonfactorizable $SU(3)$-breaking corrections [32]. A similar comment applies to the $B_s^0 \to J/\psi K_{\rm S}$ mode [33], which has recently been observed by the CDF collaboration [34].

VI. APPLICATION: EXTRACTION OF f_d/f_s

As we have seen in Sec. III, the impact of the exchange topology on the $\bar{B}_d^0 \to D^{*+}\pi^-$ decays is small. Consequently, this channel looks at first sight also interesting for another implementation of the method for the determination of f_d/f_s at LHCb proposed by us in Ref. [9]. Here we have to compare it with the $\bar{B}_s^0 \to D_s^{*+}\pi^-$ channel. Unfortunately, the reconstruction of the D_s^{*+} is challenging at LHCb. However, the $\bar{B}_d^0 \to D^+\pi^-$ and $\bar{B}_s^0 \to D_s^+\pi^-$ modes are nicely accessible at this experiment. In view of the similar patterns of the modes involving D^* and D mesons discussed above, we expect that also in the $\bar{B}_d^0 \to D^+\pi^-$ channel the exchange amplitude plays a minor role. The expression for the extraction of f_d/f_s from these channels reads as follows:

$$\frac{f_d}{f_s} = 1.018\frac{\tau_{B_s}}{\tau_{B_d}}\left[\tilde{\mathcal{N}}_a\mathcal{N}_F\mathcal{N}_E\frac{\epsilon_{D_s\pi}}{\epsilon_{D_d\pi}}\frac{N_{D_d\pi}}{N_{D_s\pi}}\right], \tag{46}$$

where the numerical factor takes phase-space effects into account,

$$\tilde{\mathcal{N}}_a \equiv \left|\frac{a_1(D_s\pi)}{a_1(D_d\pi)}\right|^2, \qquad \mathcal{N}_F \equiv \left[\frac{F_0^{(s)}(m_\pi^2)}{F_0^{(d)}(m_\pi^2)}\right]^2, \tag{47}$$

describe $SU(3)$-breaking effects, and

$$\mathcal{N}_E \equiv \left|\frac{T}{T+E}\right|^2 \tag{48}$$

takes into account the effect of the exchange diagram, which was absent in the $\bar{B}_d^0 \to D^+K^-$ decay [9].

The difference of $|a_1|$ from unity at the order of 5% discussed in Sec. II leads to an uncertainty of about 10% on the theoretical prediction of the hadronic branching ratio. Assuming an $SU(3)$ suppression in the $\mathcal{N}_a$ factor introduced in Ref. [9] and the $\tilde{\mathcal{N}}_a$ by a factor $\sim$5, which is still generous in view of the analysis of the $SU(3)$-breaking effects in Sec. V, we arrive at an uncertainty of about 2% for $\mathcal{N}_a$ and $\tilde{\mathcal{N}}_a$. This experimentally constrained error is fully consistent with the theoretical discussion given in Ref. [9].

Unfortunately, the $B_s \to D_s$ form factors entering $\mathcal{N}_F$ have so far received only small theoretical attention. In Ref. [35], such effects were explored using heavy-meson chiral perturbation theory, while QCD sum-rule techniques were applied in Ref. [36]. The numerical value given in the latter paper yields $\mathcal{N}_F = 1.24 \pm 0.08$.

Finally, in contrast to the determination of f_s/f_d by means of the $\bar{B}_d^0 \to D^+K^-$, $\bar{B}_s^0 \to D_s^+\pi^-$ system [9], we have to deal with the $\mathcal{N}_E$ factor in (46). Using (13) and adding an additional 5% uncertainty to account for possible differences between the D and D^* cases, we obtain $\mathcal{N}_E = 0.966 \pm 0.056 \pm 0.05$.

Interestingly, the CDF collaboration has already published the ratio [37]:

$$\frac{\epsilon_{D_d\pi}}{\epsilon_{D_s\pi}}\frac{N(D_s^-(\phi\pi^-)\pi^+)}{N(D^-(K^+\pi^-\pi^-)\pi^+)} = 0.067 \pm 0.04. \tag{49}$$

After taking the branching fractions of the D mesons into account, $\mathrm{BR}(D^- \to K^+\pi^-\pi^-) = (9.4 \pm 0.4)\%$ and $\mathrm{BR}(D_s^- \to \phi\pi^-) = (2.32 \pm 0.14)\%$, we obtain

$$\frac{\epsilon_{D_d\pi}}{\epsilon_{D_s\pi}}\frac{N_{D_s\pi}}{N_{D_d\pi}} = 0.271 \pm 0.016 \pm 0.020(\mathrm{BR}(D)). \tag{50}$$

If we use now Eq. (46), we can convert this number into a value of f_s/f_d. Assuming $\tilde{\mathcal{N}}_a = 1.00 \pm 0.02$ and $\mathcal{N}_E = 0.966 \pm 0.056 \pm 0.05$, we obtain the nonleptonic result,

$$\left(\frac{f_s}{f_d}\right)_{\rm NL} = 0.285 \pm 0.036, \qquad (51)$$

for $\mathcal{N}_F = 1$, where all errors have been added in quadrature. Here we have a theoretical error of 8.2% on top of an experimental error of 9.4% from (50) and $\tau_{B_s}/\tau_{B_d} = 0.965 \pm 0.017$. As discussed in Ref. [9], we expect $\mathcal{N}_F \geq 1$, which may result in a decrease of f_s/f_d. Lattice results of the form-factor ratio entering $\mathcal{N}_F$ will hopefully be available soon. In order to surpass the possible future experimental uncertainty, knowledge on the corresponding $SU(3)$-breaking corrections would be needed at the 20% level.

It is interesting to compare the result in (51) with the published ratio of fragmentation functions extracted from semi-inclusive $\bar{B} \to D\ell^- \bar{\nu}_\ell X$ decays [38]. The reconstructed $D\ell^-$ signal yields are related to the number of produced b hadrons by assuming the $SU(3)$ flavor symmetry and neglecting $SU(3)$-breaking corrections [i.e. $\Gamma(\bar{B}^0_d \to \ell^- \bar{\nu}_\ell D^+) = \Gamma(\bar{B}^0_s \to \ell^- \bar{\nu}_\ell D^+_s)$, which corresponds to $\mathcal{N}_F = 1$]. Together with an earlier result using double semileptonic decays (containing two muons and either a K^* or a ϕ meson) [39], the average value $f_s/(f_d + f_u) = 0.142 \pm 0.019$ was obtained [10], which can be written as

$$\left(\frac{f_s}{f_d}\right)_{\rm SL} = 0.284 \pm 0.038. \qquad (52)$$

The consistency of this result with (51) is remarkable. Note that the uncertainties on the $\mathrm{BR}(D_{(s)})$ and $D_{(s)}$-meson reconstruction efficiencies are expected to be correlated in (51) and (52).

As the mode $\bar{B}^0_d \to D^+\pi^-$ is Cabibbo favored with respect to the $\bar{B}^0_d \to D^+K^-$ channel, it allows an early measurement of the hadronization fraction at LHCb. Moreover, the possibility of using $\bar{B}^0_d \to D^+\pi^-$ offers a useful experimental handle to keep systematic uncertainties due to particle identification under control. These decays can be reconstructed with $D^+ \to K^-\pi^+\pi^+$ and $D^+_s \to K^+K^-\pi^+$ final states. The uncertainty on the ratio of the two efficiencies $\epsilon_{D_s\pi}/\epsilon_{D\pi}$ is expected to be small since the topology is the same and the main difference is the particle identification of one of the kaons coming from the charmed meson. The number of events in 10 pb^{-1} [40] is expected to be $\sim$3000 for $\bar{B}^0_d \to D^+\pi^-$ and about $10\times$ smaller (depending on the value of f_d/f_s) for $\bar{B}^0_s \to D^+_s\pi^-$, with $D^+ \to K^-\pi^+\pi^+$ and $D^+_s \to K^-K^+\pi^+$, respectively. Therefore this would allow LHCb to make a precise measurement of f_d/f_s with a few tens pb^{-1}.

VII. CONCLUSIONS

We have considered nonleptonic B-meson decays of the kind $B \to D^{(*)}_{(s)}P$ and have performed tests of how well these channels are described by factorization and $SU(3)$ flavor-symmetry relations. Using data from semileptonic B decays to determine the relevant $B \to D^{(*)}$ form factors, we could not resolve nonfactorizable effects within the current experimental precision, which is as small as about 5% in the most fortunate cases. Using data on nonleptonic decays to probe exchange topologies, we obtained a picture with amplitudes as naively expected, i.e. without any enhancement due to long-distance effects. However, in an isospin analysis of the $B \to D^{(*)}\pi$ system, we found significant corrections to the heavy-quark limit, which could be traced back to nonfactorizable contributions to color-suppressed tree contributions. Concerning the $SU(3)$ flavor symmetry, we did not find any indication for nonfactorizable $SU(3)$-breaking corrections, with a resolution as small as 5%.

These results support—from an experimental point of view—the intrinsic theoretical errors for a determination of the ratio f_s/f_d of fragmentation functions from a simultaneous measurement of the $\bar{B}^0_d \to D^+K^-$ and $\bar{B}^0_s \to D^+_s\pi^-$ modes, as proposed and discussed in Ref. [9].

We found an interesting variant of this method, which arises if we replace the $\bar{B}^0_d \to D^+K^-$ channel by $\bar{B}^0_d \to D^+\pi^-$. In this case, we have then also to deal with a contribution from an exchange topology, which we constrain experimentally. Interestingly, the CDF collaboration has already published a ratio of the corresponding event numbers, which we can convert into $(f_s/f_d)_{\rm NL} = 0.285 \pm 0.036$, with a smaller error than and in excellent agreement with the result from analyses of semileptonic decays at CDF. It should be noted that in these values $SU(3)$-breaking effects in the ratio of the relevant $B \to D$ and $B_s \to D_s$ form factors were neglected, which could reduce the value of f_s/f_d. In the near future, precise lattice calculations of this quantity should become available. The extraction of f_s/f_d from the $\bar{B}^0_s \to D^+_s\pi^-$, $\bar{B}^0_d \to D^+\pi^-$ system, as proposed in this paper, is interesting for the early data taking at LHCb.

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