Baryon fields with $U_L(3) \times U_R(3)$ chiral symmetry. III. Interactions with chiral $[(3, \bar{3}) \oplus (\bar{3}, 3)]$ spinless mesons

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Three-quark nucleon interpolating fields in QCD have well-defined $SU_L(3) \times SU_R(3)$ and $U_A(1)$ chiral transformation properties, viz. $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$, $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$, $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$, and their mirror images; see [H. X. Chen, V. Dmitrašinović, A. Hosaka, K. Nagata, and S. L. Zhu, Phys. Rev. D **78**, 054021 (2008)]. It has been shown (phenomenologically) in [H. X. Chen, V. Dmitrašinović, and A. Hosaka, Phys. Rev. D **81**, 054002 (2010)] that mixing of the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ chiral multiplet with one ordinary (naive) and one mirror field belonging to the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$, $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ multiplets can be used to fit the values of the isovector ($g_A^{(3)}$) and the flavor-singlet (isoscalar) axial coupling ($g_A^{(0)}$) of the nucleon and then predict the axial F and D coefficients, or vice versa, in reasonable agreement with experiment. In an attempt to derive such mixing from an effective Lagrangian, we construct all $SU_L(3) \times SU_R(3)$ chirally invariant nonderivative one-meson-baryon interactions and then calculate the mixing angles in terms of baryons' masses. It turns out that there are (strong) selection rules: for example, there is only one nonderivative chirally symmetric interaction between $J = \frac{1}{2}$ fields belonging to the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ and the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ chiral multiplets, that is also $U_A(1)$ symmetric. We also study the chiral interactions of the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ and $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ nucleon fields. Again, there are selection rules that allow only one off-diagonal nonderivative chiral $SU_L(3) \times SU_R(3)$ interaction of this type, that also explicitly breaks the $U_A(1)$ symmetry. We use this interaction to calculate the corresponding mixing angles in terms of baryon masses and fit two lowest-lying observed nucleon (resonance) masses, thus predicting the third ($J = \frac{1}{2}$, $I = \frac{3}{2}$) Δ resonance, as well as one or two flavor-singlet Λ hyperon(s), depending on the type of mixing. The effective chiral Lagrangians derived here may be applied to high density matter calculations.

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I. INTRODUCTION

Axial current coupling constants of the baryon flavor-octet are well known; see Ref. [1]. The zeroth (timelike) components of these axial currents are generators of the $SU_L(3) \times SU_R(3)$ chiral symmetry that is one of the fundamental symmetries of QCD. The general flavor $SU_F(3)$ symmetric form of the nucleon axial current contains two free parameters, called F and D couplings, that are empirically determined as $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$, see Ref. [1]. Another, perhaps separate, yet equally important piece of information is the flavor-singlet axial coupling $g_A^{(0)} = 0.33 \pm 0.08$ of the nucleon [2,3].

Recent studies [4,5] point toward baryon chiral mixing (of $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ with the $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$, $[(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})]$ chiral multiplets [6]) as a possible mechanism underlying the baryons' axial couplings. This finding is in line with the old current algebra results of Gerstein and Lee [7] and of Harari [8,9], updated to include recently measured values of F and D couplings, Ref. [1], and extended to include the flavor-singlet coupling $g_A^{(0)}$ of the nucleon, which was not considered in the mid-1960s at all, presumably due to the lack of data. Our own starting point was the study of the QCD interpolating fields' chiral properties [10–12].

The next step is to try and reproduce this phenomenological mixing starting from a chiral effective model interaction, rather than *per fiat*. As the first step in that direction we must look for a dynamical source of mixing. One such mechanism is the simplest chirally symmetric *nonderivative* one-(σ, π)-meson interaction Lagrangian; nonderivative because that induces baryon masses via the σ-baryon coupling.

We construct all $SU_L(3) \times SU_R(3)$ chirally invariant nonderivative one-meson-baryon interactions and then use them to calculate the mixing angles in terms of baryons' masses. It turns out that there are severe chiral selection rules at work here. For example, we show that only the mirror field $[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$ can be coupled to the $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ baryon chiral multiplet by nonderivative terms; whereas the ordinary (naive) multiplet $[(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})]$ requires one (or generally an odd number of) derivative(s). Moreover, this interaction also conserves the $U_A(1)$ symmetry. This is interesting, as the mixing with a mirror baryon field of this type seems preferable from the point of view of the two-flavor phenomenological study, Ref. [4].

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We note that all but one of the $SU_L(3) \times SU_R(3)$ symmetric interactions, viz. the $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})] - [(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$, also conserve the $U_A(1)$ symmetry. This means that explicit $U_A(1)$ symmetry breaking may occur in baryons only in so far as the $SU_L(3) \times SU_R(3)$ symmetry is explicitly broken, with the exception mentioned above. This is in stark contrast with the $SU_L(2) \times SU_R(2)$ case, where all of the interaction terms have both the $U_A(1)$ symmetry-conserving and the $U_A(1)$ symmetry-breaking version [4,13]. In this sense, the three-flavor chiral symmetry is more restrictive and consequently more instructive than the two-flavor one.

The conventional models of (linearly realized) chiral $SU_L(3) \times SU_R(3)$ symmetry, Refs. [14–19], on the other hand appear to fix the F and D parameters at either ($F = 0$, $D = 1$), which case goes by the name of $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$, or at ($F = 1$, $D = 0$), which case goes by the name of $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$ chiral representation. Both of these chiral representations suffer from the shortcoming that $F + D = 1, \neq g_A^{(3)} = 1.267$ without derivative couplings. But, even with derivative interactions, one cannot change the value of the vanishing coupling, i.e. of $F = 0$, in $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$, or of $D = 0$, in $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$. Rather, one can only renormalize the nonvanishing coupling to 1.267. This is perhaps the most troublesome problem of the linear realization chiral $SU_L(3) \times SU_R(3)$ symmetric Lagrangians as it has far-reaching consequences for the kaon and hyperon interactions, hypernuclear physics, and nuclear astrophysics of collapsed stars [20,21].

Another perhaps equally important and difficult problem is that of the flavor-singlet axial coupling of the nucleon [2,3]. This is widely thought of as being disconnected from the F, D problem, but we have already shown (see Refs. [4,5]) that the chiral mixing of three-quark interpolating fields casts some new light on this problem. Namely, the flavor-singlet axial coupling turns out to be $g_A^{(0)} = (3F - D)$, i.e., a function of the flavor $SU(3)$ octet (F, D) coefficients and thus proportional to the eighth flavor component of the $SU(3)$ symmetric axial coupling $g_A^{(8)} = \frac{1}{\sqrt{3}}(3F - D)$, so long as one mixes only three-quark interpolating fields. In other words, the ratio of these two measured quantities is fixed at $\sqrt{3}$ in the three-quark assumption, so one must go beyond this approximation in order to break the deadlock.

Even though an awareness of this mixing has been around for more than 40 years [14–16,22], the $SU_L(3) \times SU_R(3)$ chiral interactions necessary to describe such chiral mixing(s) have not been considered in print [23], let alone derived. The present paper serves to provide a dynamical model of chiral mixing that is the "best" approximation to the phenomenological solution of both the (F, D) and the flavor-singlet axial coupling problems, assuming only three-quark baryon interpolating fields. We found two simple solutions/fits [26]: one that conserves the $U_A(1)$ symmetry and another one that does not. This goes to show that the "QCD $U_A(1)$ anomaly" may, but need not, be the underlying source of the "nucleon spin problem" [2,3], as was once widely thought [18]. In all likelihood the $U_A(1)$ anomaly provides only a (relatively) small part of the solution, the largest part coming from the chiral structure ("mixing") of the nucleon.

One immediate application of our results ought to be in high density matter calculations, where only one baryon chiral multiplet ($[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$) and its interaction with mesons have been used for some time now [20,21].

The present paper consists of five parts: after the present introduction, in Sec. II we define the $SU(3) \times SU(3)$ chiral transformations of three-quark baryon fields and of the spinless mesons, with special emphasis on the $SU(3)$ phase conventions. In Sec. III we construct the $SU_L(3) \times SU_R(3)$ chirally invariant interactions. In Sec. IV we apply chiral mixing formalism to the hyperons' axial currents and then use the chiral interactions to reproduce the mixing angles. In this way we determine the masses of the admixed states. Finally, in Sec. V we discuss the results and offer a summary and an outlook on future developments.

II. PRELIMINARIES: CHIRAL TRANSFORMATIONS OF MESONS AND BARYONS

A. Chiral transformations of $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$ spinless mesons

We follow the same definition of chiral transformation in Ref. [12]:

$$\begin{aligned}
&\mathbf{U(1)_V}\colon\ q \to \exp\left(i\frac{\lambda^0}{2}a_0\right)q = q + \delta q,\\
&\mathbf{SU(3)_A}\colon\ q \to \exp\left(i\frac{\vec{\lambda}}{2}\cdot\vec{a}\right)q = q + \delta^{\vec{a}} q,\\
&\mathbf{U(1)_A}\colon\ q \to \exp\left(i\gamma_5\frac{\lambda^0}{2}b_0\right)q = q + \delta_5 q,\\
&\mathbf{SU(3)_A}\colon\ q \to \exp\left(i\gamma_5\frac{\vec{\lambda}}{2}\cdot\vec{b}\right)q = q + \delta_5^{\vec{b}} q.
\end{aligned} \tag{1}$$

We define the scalar and pseudoscalar mesons in the $SU(3)$ space as

$$\sigma^a = \bar{q}_A \lambda^a_{AB} q_B, \tag{2}$$

$$\pi^a = \bar{q}_A \lambda^a_{AB} i\gamma_5 q_B, \tag{3}$$

where the index a goes from 0 to 8, and the zero component of Gell-Mann matrices is $\lambda^0 = \sqrt{\frac{2}{3}}\mathbf{1}$.

The nucleon fields belong to the chiral representation of $(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})$, and their combination transforms as

$$\delta_5^{\vec{b}}(\sigma^b + i\gamma_5\pi^b) = -i\gamma_5 b^a d_{abc}(\sigma^c + i\gamma_5\pi^c),$$
$$\delta_5^{\vec{b}}(\sigma^b - i\gamma_5\pi^b) = i\gamma_5 b^a d_{abc}(\sigma^c - i\gamma_5\pi^c), \tag{4}$$

where d_{abc} and f_{abc} are defined to contain the 0 index,

$$\{\lambda^a, \lambda^b\} = 2d^{abc}\lambda^c, \qquad [\lambda^a, \lambda^b] = 2if^{abc}\lambda^c. \tag{5}$$

We note here that in these equations we do not have the δ^{ab} factors which are necessary in the usual equation

$$\{\lambda^a, \lambda^b\} = 2d^{abc}\lambda^c + \frac{4}{3}\delta^{ab}, \tag{6}$$

where

$$(a, b = 1, \ldots, 8). \tag{7}$$

The nonzero f and d coefficients are

abc	f^{abc}
123	1
147	1/2
156	−1/2
246	1/2
257	1/2
345	1/2
367	−1/2
458	$\sqrt{3}/2$
678	$\sqrt{3}/2$

abc	d^{abc}	abc	d^{abc}	abc	d^{abc}
000	$\sqrt{2/3}$	118	$1/\sqrt{3}$	355	1/2
011	$\sqrt{2/3}$	146	1/2	366	−1/2
022	$\sqrt{2/3}$	157	1/2	377	−1/2
033	$\sqrt{2/3}$	228	$1/\sqrt{3}$	448	$-1/(2\sqrt{3})$
044	$\sqrt{2/3}$	247	−1/2	558	$-1/(2\sqrt{3})$
055	$\sqrt{2/3}$	256	1/2	668	$-1/(2\sqrt{3})$
066	$\sqrt{2/3}$	338	$1/\sqrt{3}$	778	$-1/(2\sqrt{3})$
077	$\sqrt{2/3}$	344	1/2	888	$-1/\sqrt{3}$
088	$\sqrt{2/3}$				

To simplify our calculations sometimes we use the "physical" basis, whose definitions are

$$\begin{pmatrix} M^1 \\ M^2 \\ M^3 \\ M^4 \\ M^5 \\ M^6 \\ M^7 \\ M^8 \\ M^9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sigma^0 + i\gamma_5\pi^0 \\ \sigma^1 + i\gamma_5\pi^1 \\ \sigma^2 + i\gamma_5\pi^2 \\ \sigma^3 + i\gamma_5\pi^3 \\ \sigma^4 + i\gamma_5\pi^4 \\ \sigma^5 + i\gamma_5\pi^5 \\ \sigma^6 + i\gamma_5\pi^6 \\ \sigma^7 + i\gamma_5\pi^7 \\ \sigma^8 + i\gamma_5\pi^8 \end{pmatrix}. \tag{8}$$

In this basis,

$$M^1 = \sigma_0 + i\gamma_5\eta_0, \qquad M^2 = a_0^+ + i\gamma_5\pi^+,$$
$$M^3 = a_0^0 + i\gamma_5\pi^0, \qquad M^4 = a_0^- + i\gamma_5\pi^-,$$
$$M^5 = \kappa^+ + i\gamma_5 K^+, \qquad M^6 = \kappa^- + i\gamma_5 K^-,$$
$$M^7 = \kappa^0 + i\gamma_5 K^0, \qquad M^8 = \bar{\kappa}^0 + i\gamma_5\bar{K}^0,$$
$$M^9 = f_0 + i\gamma_5\eta_8. \tag{9}$$

We have classified the baryon interpolating fields in our previous paper [12]. We found that the baryon interpolating fields $N_+^a = N_1^a + N_2^a$ belong to the chiral representation $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$; Λ and $N_-^a = N_1^a - N_2^a$ belong to the chiral representation $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$; N_μ^a and Δ_μ^P belong to the chiral representation $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$; and $\Delta_{\mu\nu}^P$ belong to the chiral representation $(\mathbf{10}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{10})$. Here N_1^a and N_2^a are the two independent kinds of nucleon fields. N_1^a contains the "scalar diquark" and N_2^a contains the "pseudoscalar diquark." Moreover, we calculated their chiral transformations in Ref. [12]. In the following sections, we will use these baryon fields together with one meson field to construct the chiral invariant Lagrangians.

B. Chiral transformations of baryons

1. Chiral transformations of $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ baryons

The baryon field $N_{(18)} = (N_\mu, \Delta_\mu)^T$ belongs to the chiral representation $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$,

$$N^1 = p, \qquad N^2 = n, \qquad N^3 = \Sigma^+,$$
$$N^4 = \Sigma^0, \qquad N^5 = \Sigma^-, \qquad N^6 = \Xi^0,$$
$$N^7 = \Xi^-, \qquad N^8 = \Lambda_8, \qquad N^9 = \Delta^{++},$$
$$N^{10} = \Delta^+, \qquad N^{11} = \Delta^0, \qquad N^{12} = \Delta^-,$$
$$N^{13} = \Sigma^+, \qquad N^{14} = \Sigma^0, \qquad N^{15} = \Sigma^-,$$
$$N^{16} = \Xi^0, \qquad N^{17} = \Xi^-, \qquad N^{18} = \Omega, \tag{10}$$

and we can write out their chiral transformation as

$$\delta_5^{\vec{b}} N_{(18)} = i\gamma_5 b^a \mathbf{F}_{(18)}^a N_{(18)}$$
$$= i\gamma_5 b^a \begin{pmatrix} \mathbf{D}_{(8)}^a + \frac{2}{3}\mathbf{F}_{(8)}^a & \frac{2}{\sqrt{3}}\mathbf{T}_{(8/10)}^a \\ \frac{2}{\sqrt{3}}\mathbf{T}_{(8/10)}^{\dagger a} & \frac{1}{3}\mathbf{F}_{(10)}^a \end{pmatrix} \begin{pmatrix} N_\mu \\ \Delta_\mu \end{pmatrix}, \tag{11}$$

where the matrices $\mathbf{D}_{(8)}^a$, $\mathbf{F}_{(8)}^a$, $\mathbf{F}_{(10)}^a$, and $\mathbf{T}_{(8/10)}^a$ are calculated in our previous paper [5].

2. Chiral transformations of $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ baryons

This chiral representation contains the flavor-octet and singlet representations $\bar{\mathbf{3}}\otimes\mathbf{3}=\mathbf{8}\oplus\mathbf{1}\sim N_{(9)}=(\Lambda,N_-)^T$:

$$\begin{aligned} &N^1=\Lambda_0, && N^2=p, && N^3=n,\\ &N^4=\Sigma^+, && N^5=\Sigma^0, && N^6=\Sigma^-,\\ &N^7=\Xi^0, && N^8=\Xi^-, && N^9=\Lambda_8, \end{aligned} \tag{12}$$

and their chiral transformations are

$$\begin{aligned} \delta_5^{\vec{b}} N_{(9)} &= i\gamma_5 b^a \mathbf{F}^a_{(9)} N_{(9)} \\ &= i\gamma_5 b^a \begin{pmatrix} 0 & \sqrt{\frac{2}{3}}\mathbf{T}^a_{(1/8)} \\ \sqrt{\frac{2}{3}}\mathbf{T}^{\dagger a}_{(1/8)} & \mathbf{D}^a_{(8)} \end{pmatrix} \begin{pmatrix} \Lambda_1 \\ N_- \end{pmatrix}. \end{aligned} \tag{13}$$

3. Chiral transformations of $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ baryons

This chiral representation $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ contains the flavor-octet representation $\mathbf{8}\otimes\mathbf{1}=\mathbf{8}\ \sim N_{(8)}=N_+$. The chiral transformation is

$$\delta_5^{\vec{b}} N_{(8)} = i\gamma_5 b^a \mathbf{F}^a_{(8)} N_{(8)}. \tag{14}$$

4. Chiral transformations of $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ baryons

This chiral representation $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ contains the flavor decuplet representation $\mathbf{10}\otimes\mathbf{1}=\mathbf{10}\sim N_{(10)}=\Delta_{\mu\nu}$. The chiral transformation is

$$\delta_5^{\vec{b}} N_{(10)} = i\gamma_5 b^a \mathbf{F}^a_{(10)} N_{(10)}. \tag{15}$$

III. CHIRAL INTERACTIONS

In this section we propose a new method for the construction of $N_f=3$ chiral invariants that differs from the one proposed for $N_f=2$ in Ref. [18] and used in Refs. [4,13].

A. Diagonal interactions: Mass terms

1. Chiral $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ baryons' diagonal interactions

Our aim is to construct a chiral invariant Lagrangian,

$$\bar{N}^{\,a}_{(18)} M^c N^b_{(18)} \mathbf{C}^{abc}_{(18)}, \tag{16}$$

where the indices a and b run from 1 to 18, and the index c just runs from 1 to 9. By performing the chiral transformation to this Lagrangian, we can obtain many equations. For example we have

$$\begin{aligned} \delta_5^1(\bar{p}M^2 n\mathbf{C}^{122}_{(18)}) &= \frac{5}{6}\mathbf{C}^{122}_{(18)}\bar{n}M^2(i\gamma_5 b_1)n+\cdots,\\ \delta_5^1(\bar{\Delta}M^2 n\mathbf{C}^{10,2,2}_{(18)}) &= -\frac{\sqrt{2}}{3}\mathbf{C}^{10,2,2}_{(18)}\bar{n}M^2(i\gamma_5 b_1)n+\cdots,\\ \delta_5^1(\bar{n}M^2\Delta^-\mathbf{C}^{2,12,2}_{(18)}) &= \sqrt{\frac{2}{3}}\mathbf{C}^{2,12,2}_{(18)}\bar{n}M^2(i\gamma_5 b_1)n+\cdots,\\ \delta_5^1(\bar{n}M^1 n\mathbf{C}^{221}_{(18)}) &= \frac{1}{\sqrt{3}}\mathbf{C}^{221}_{(18)}\bar{n}M^2(i\gamma_5 b_1)n+\cdots,\\ \delta_5^1(\bar{n}M^9 n\mathbf{C}^{229}_{(18)}) &= \frac{1}{\sqrt{6}}\mathbf{C}^{229}_{(18)}\bar{n}M^2(i\gamma_5 b_1)n+\cdots. \end{aligned} \tag{17}$$

These are all the fields that are transformed to $\bar{n}M^2(i\gamma_5 b_1)n$. If the Lagrangian (16) is chiral invariant, this sum should be zero,

$$\frac{5}{6}\mathbf{C}^{122}_{(18)}-\frac{\sqrt{2}}{3}\mathbf{C}^{10,2,2}_{(18)}+\sqrt{\frac{2}{3}}\mathbf{C}^{2,12,2}_{(18)}+\frac{1}{\sqrt{3}}\mathbf{C}^{221}_{(18)}+\frac{1}{\sqrt{6}}\mathbf{C}^{229}_{(18)}=0. \tag{18}$$

Solving these equations for $\mathbf{C}^{abc}_{(18)}$ together with the Hermiticity condition, we find that there is only one solution. The uniqueness of the solution is guaranteed by the fact that there is only one way to form the chiral singlet combination out of the baryon field $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ and the meson field $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$. This solution can be written out much more easily using $\mathbf{D}^c_{(18)}$ in the following form:

$$g_{(18)}\bar{N}^a_{(18)}(\sigma^c+i\gamma_5\pi^c)(\mathbf{D}^c_{(18)})_{ab}N^b_{(18)}, \tag{19}$$

where $g_{(18)}$ is the coupling constant, and the matrices $\mathbf{D}_{(18)}$ are solved to be

$$\begin{aligned} \mathbf{D}^0_{(18)} &= \frac{1}{\sqrt{6}}\begin{pmatrix} \mathbf{1}_{8\times8} & 0 \\ 0 & -2\times\mathbf{1}_{10\times10} \end{pmatrix},\\ \mathbf{D}^a_{(18)} &= \begin{pmatrix} \mathbf{D}^a_{(8)}+\frac{2}{3}\mathbf{F}^a_{(8)} & -\frac{1}{\sqrt{3}}\mathbf{T}^a_{(8/10)} \\ -\frac{1}{\sqrt{3}}\mathbf{T}^{\dagger a}_{(8/10)} & -\frac{2}{3}\mathbf{F}^a_{(10)} \end{pmatrix}. \end{aligned} \tag{20}$$

Besides the Lagrangian (16), its mirror part,

$$g_{(18)}\bar{N}^a_{(18m)}(\sigma^c-i\gamma_5\pi^c)(\mathbf{D}^c_{(18)})_{ab}N^b_{(18m)}, \tag{21}$$

is also chiral invariant. Using these solutions, and performing the chiral transformation, we can obtain the following relation:

$$\mathbf{F}^{a\dagger}_{(18)}\mathbf{D}^b_{(18)}+\mathbf{D}^b_{(18)}\mathbf{F}^a_{(18)}-d_{abc}\mathbf{D}^c_{(18)}=0, \tag{22}$$

where $\mathbf{F}^a_{(18)}$ and $\mathbf{D}^b_{(18)}$ are defined in the previous Eqs. (11) and (20).

The solution in the physical basis $(\bar{N}^a_{(18)}M^cN^b_{(18)}\mathbf{C}^{abc}_{(18)})$ can be obtained by the following relations:

$$
\begin{aligned}
\mathbf{C}^{ab1}_{(18)} &= (\mathbf{D}^{0}_{(18)})_{ab},\\
\mathbf{C}^{ab3}_{(18)} &= (\mathbf{D}^{3}_{(18)})_{ab},\\
\mathbf{C}^{ab9}_{(18)} &= (\mathbf{D}^{8}_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab2}_{(18)} + \mathbf{C}^{ab4}_{(18)}) &= (\mathbf{D}^{1}_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab2}_{(18)} + \mathbf{C}^{ab4}_{(18)}) &= (\mathbf{D}^{2}_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab5}_{(18)} + \mathbf{C}^{ab6}_{(18)}) &= (\mathbf{D}^{4}_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab5}_{(18)} + \mathbf{C}^{ab6}_{(18)}) &= (\mathbf{D}^{5}_{(18)})_{ab},\\
\frac{1}{\sqrt{2}}(\mathbf{C}^{ab7}_{(18)} + \mathbf{C}^{ab8}_{(18)}) &= (\mathbf{D}^{6}_{(18)})_{ab},\\
\frac{i}{\sqrt{2}}(-\mathbf{C}^{ab7}_{(18)} + \mathbf{C}^{ab8}_{(18)}) &= (\mathbf{D}^{7}_{(18)})_{ab}.
\end{aligned}
\tag{23}
$$

2. Chiral $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ baryons' diagonal interactions

Following the same procedure of the previous section, we find that the Lagrangian $\bar{N}^{a}_{(9)}M^{c}N^{b}_{(9)}\mathbf{C}^{abc}_{(9)}$ cannot be chiral invariant, which means that their is no solution for $\mathbf{C}^{abc}_{(9)}$. However, we can still get a chiral invariant Lagrangian through different fields. There are two possible ways:

(1) We use the meson field $\sigma^{a} - i\gamma_5\pi^{a}$,

$$
\delta^{\vec{b}}_{5}(\sigma^{b} - i\gamma_5\pi^{b}) = i\gamma_5 b^{a} d_{abc}(\sigma^{c} - i\gamma_5\pi^{c}). \tag{24}
$$

(2) We use the mirror field of $N_{(9)}$,

$$
\begin{aligned}
\delta^{\vec{b}}_{5}N_{(9m)} &= -i\gamma_5 b^{a}\mathbf{F}^{a}_{(9)}N_{(9m)}\\
&= i\gamma_5 b^{a}\begin{pmatrix} 0 & -\sqrt{\frac{2}{3}}\mathbf{T}^{a}_{(1/8)}\\ -\sqrt{\frac{2}{3}}\mathbf{T}^{\dagger a}_{(1/8)} & -\mathbf{D}^{a}_{(8)}\end{pmatrix}N_{(9m)}.
\end{aligned}
\tag{25}
$$

Here we just give the Lagrangian for the simplest case 1, which is $M^{+a}M^{a}\bar{N}^{b}_{(8)}\gamma_5 N^{b}_{(8m)}$ + H.c. The others can be obtained by using M, M^{+}, $N_{(8)}$, and $N_{(8m)}$ as well as related coefficients d_{abc} and f_{abc}.

Then we can construct the chiral invariant Lagrangian,

$$
\bar{N}^{a}_{(9m)}M^{c}N^{b}_{(9m)}\mathbf{C}^{abc}_{(9)} \tag{26}
$$

or its mirror part

$$
\bar{N}^{a}_{(9)}(M^{+})^{c}N^{b}_{(9)}\mathbf{C}^{abc}_{(9)}. \tag{27}
$$

Assuming that they are Hermitian, we find that there is only one solution for $\mathbf{C}^{abc}_{(9)}$. The solution for the coefficients $\mathbf{C}^{abc}_{(9)}$ in these two Lagrangians is the same, and it can be written out in the following form:

$$
g_{(9)}\bar{N}^{a}_{(9m)}(\sigma^{c} + i\gamma_5\pi^{c})(\mathbf{D}^{c}_{(9)})_{ab}N^{b}_{(9m)}, \tag{28}
$$

where the solution is

$$
\begin{aligned}
\mathbf{D}^{0}_{(9)} &= \frac{1}{\sqrt{6}}\begin{pmatrix} -2 & \mathbf{0}_{1\times 8}\\ \mathbf{0}_{8\times 1} & \mathbf{1}_{8\times 8}\end{pmatrix},\\
\mathbf{D}^{a}_{(9)} &= \begin{pmatrix} 0 & \frac{1}{\sqrt{6}}\mathbf{T}^{a}_{(1/8)}\\ \frac{1}{\sqrt{6}}\mathbf{T}^{\dagger a}_{(1/8)} & -\mathbf{D}^{a}_{(8)}\end{pmatrix}.
\end{aligned}
\tag{29}
$$

The uniqueness of the solution is guaranteed by the fact that there is only one way to form the chiral singlet combination out of the baryon field $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ and the meson field $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$. The coefficients $\mathbf{C}^{abc}_{(9)}$ can be similarly obtained like Eq. (23). From this Lagrangian, we can obtain another relation,

$$
\mathbf{F}^{a\dagger}_{(9)}\mathbf{D}^{b}_{(9)} + \mathbf{D}^{b}_{(9)}\mathbf{F}^{a}_{(9)} + d_{abc}\mathbf{D}^{c}_{(9)} = 0. \tag{30}
$$

3. Chiral $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ baryons' diagonal interactions

Simply adding one $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ meson field to two $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ baryon fields cannot produce a chirally invariant Lagrangian. By adding two $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ meson fields, however, there are several possible ways to construct chirally invariant Lagrangians [25]. First we can write out the group structures

$$
\begin{aligned}
&((\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8}))^{2}\otimes((\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3}))^{2}\\
&\rightarrow((\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1}))\otimes((\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1}))\rightarrow((\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1})) \quad (1)\\
&\rightarrow(2\times((\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})))\otimes((\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8}))\rightarrow 2\times((\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1})) \quad (2)\\
&\rightarrow(4\times((\mathbf{8},\mathbf{8})\oplus(\mathbf{8},\mathbf{8})))\otimes((\mathbf{8},\mathbf{8})\oplus(\mathbf{8},\mathbf{8}))\rightarrow 4\times((\mathbf{1},\mathbf{1})\oplus(\mathbf{1},\mathbf{1})) \quad (3).
\end{aligned}
\tag{31}
$$

4. Chiral $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ baryons' diagonal interactions

We find that simply adding one $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ meson field to two $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ baryon fields cannot produce a chirally invariant Lagrangian.

B. Chiral mixing interactions

1. Chiral mixing interaction $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$

The mixing of $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ with $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ (we note that this is a mirror baryon) together with a meson field can be a chiral singlet. So from this section we will study the five nontrivial off-diagonal Lagrangians.

The simple form made from the naive baryons $N_{(18)}\sim[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ and $N_{(9)}\sim[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$, $N^a_{(9)}M^cN^b_{(18)}\mathbf{C}^{abc}_{(9/18)}+\text{H.c.}$ cannot be chiral invariant. We need to use the mirror field $N_{(9m)}\sim[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$ (mir), and find the following form of field

$$\bar{N}^{\,a}_{(9m)}M^cN^b_{(18)}\mathbf{C}^{abc}_{(9/18)}+\text{H.c.} \tag{32}$$

as well as its mirror part can be chiral invariant. Again we turn to the following form:

$$g_{(9/18)}\bar{N}^a_{(9m)}(\sigma^c+i\gamma_5\pi^c)(\mathbf{T}^c_{(9/18)})_{ab}N^b_{(18)}+\text{H.c.} \tag{33}$$

We find that the only solution is

$$\mathbf{T}^{\,0}_{(9/18)}=\frac{1}{\sqrt{6}}\begin{pmatrix}\mathbf{0}_{1\times8} & \mathbf{0}_{1\times10}\\ \mathbf{1}_{8\times8} & \mathbf{0}_{8\times10}\end{pmatrix}, \tag{34}$$

$$\mathbf{T}^{\,a}_{(9/18)}=\begin{pmatrix}-\frac{1}{\sqrt{6}}\mathbf{T}^a_{(1/8)} & \mathbf{0}_{1\times10}\\ \frac{1}{3}\mathbf{F}^a_{(8)} & \frac{1}{\sqrt{3}}\mathbf{T}^a_{(8/10)}\end{pmatrix}. \tag{35}$$

The coefficients $\mathbf{C}^{abc}_{(9/18)}$ can be similarly obtained as in Eq. (23), and we have the following relation:

$$-\mathbf{F}^{a\dagger}_{(9)}\mathbf{T}^b_{(9/18)}+\mathbf{T}^b_{(9/18)}\mathbf{F}^a_{(18)}-d_{abc}\mathbf{T}^c_{(9/18)}=0. \tag{36}$$

2. Chiral mixing interaction $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$

The mixing of a mirror baryon $[(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})]$ (mir) with $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ together with a meson field can be a chiral singlet, and we find the following form of field

$$\bar{N}^{\,a}_{(8)}M^cN^b_{(18m)}\mathbf{C}^{abc}_{(9/18)}+\text{H.c.} \tag{37}$$

and its mirror part can be chiral invariant. Again we turn to the basis

$$g_{(8/18)}\bar{N}^a_{(8)}(\sigma^c+i\gamma_5\pi^c)(\mathbf{T}^c_{(8/18)})_{ab}N^b_{(18m)}+\text{H.c.} \tag{38}$$

and the only solution is

$$\mathbf{T}^{\,0}_{(8/18)}=\frac{1}{\sqrt{6}}(\mathbf{1}_{8\times8},\mathbf{0}_{8\times10}), \tag{39}$$

$$\mathbf{T}^{\,a}_{(8/18)}=\left(-\frac{1}{2}\mathbf{D}^a_{(8)}+\frac{1}{6}\mathbf{F}^a_{(8)},-\frac{1}{\sqrt{3}}\mathbf{T}^a_{(8/10)}\right). \tag{40}$$

The coefficients $\mathbf{C}^{abc}_{(8/18)}$ can be similarly obtained as in Eq. (23). And we have the following relation:

$$-\mathbf{F}^{a\dagger}_{(8)}\mathbf{T}^b_{(8/18)}+\mathbf{T}^b_{(8/18)}\mathbf{F}^a_{(18)}+d_{abc}\mathbf{T}^c_{(8/18)}=0. \tag{41}$$

3. Chiral mixing interaction $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$-$[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$

The mixing of $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ with $[(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})]$ together with a meson field can be a chiral singlet, and we find that there are two possibilities. One is the following form of Lagrangian:

$$\bar{N}^{\,a}_{(8)}M^cN^b_{(9)}\mathbf{C}^{abc}_{(8/9)}+\text{H.c.} \tag{42}$$

and its mirror part can be chiral invariant. Again we turn to the basis

$$g_{(8/9)}\bar{N}^a_{(8)}(\sigma^c+i\gamma_5\pi^c)(\mathbf{T}^c_{(8/9)})_{ab}N^b_{(9)}+\text{H.c.} \tag{43}$$

and the only solution is

$$\mathbf{T}^{\,0}_{(8/9)}=\frac{1}{\sqrt{6}}(\mathbf{0}_{8\times1},\mathbf{1}_{8\times8}), \tag{44}$$

$$\mathbf{T}^{\,a}_{(8/9)}=\left(\frac{1}{\sqrt{6}}\mathbf{T}^{\dagger a}_{(1/8)},\frac{1}{2}\mathbf{D}^a_{(8)}+\frac{1}{2}\mathbf{F}^a_{(8)}\right). \tag{45}$$

The coefficients $\mathbf{C}^{abc}_{(8/9)}$ can be similarly obtained like Eq. (23). and we have the following relation:

$$-\mathbf{F}^{a\dagger}_{(8)}\mathbf{T}^b_{(8/9)}-\mathbf{T}^b_{(8/9)}\mathbf{F}^a_{(9)}+d_{abc}\mathbf{T}^c_{(8/9)}=0. \tag{46}$$

The other possibility is the following form of Lagrangian, and the mixing of $[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$ with $[(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})]$ (mir)

$$\bar{N}^{\,a}_{(8m)}M^cN^b_{(9)}\mathbf{C}^{abc}_{(8/9)}+\text{H.c.} \tag{47}$$

This and its mirror image part can both be chiral invariant. Again we turn to the particle basis

$$g_{(B)}\bar{N}^a_{(8m)}(\sigma^c+i\gamma_5\pi^c)(\mathbf{T}^c_{(B)})_{ab}N^b_{(9)}+\text{H.c.} \tag{48}$$

The only solution is

$$\mathbf{T}^{\,0}_{B}=\frac{1}{\sqrt{6}}(\mathbf{0}_{8\times1},\mathbf{1}_{8\times8}), \tag{49}$$

$$\mathbf{T}^{\,a}_{B}=\left(\frac{1}{\sqrt{6}}\mathbf{T}^{\dagger a}_{(1/8)},\frac{1}{2}\mathbf{D}^a_{(8)}-\frac{1}{2}\mathbf{F}^a_{(8)}\right). \tag{50}$$

Since we find that this is the only case which violates the $U_A(1)$ symmetry, we use the subscript B. The coefficients $\mathbf{C}^{abc}_{(8/9)}$ can be similarly obtained as in Eq. (23), and we have the following relation:

$$\mathbf{F}^{\,a\dagger}_{(8)}\mathbf{T}^b_B-\mathbf{T}^b_B\mathbf{F}^a_{(9)}+d_{abc}\mathbf{T}^c_B=0. \tag{51}$$

4. Chiral mixing interaction $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$

For completeness' sake we also show the $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ chiral mixing interaction. The $[(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})]$ decuplet baryon field can only mix with $[(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})]$ (mir) to compose a chiral singlet, and we find the following form of Lagrangian:

$$\bar{N}^{\,a}_{(10)}M^{c}N^{b}_{(18m)}\mathbf{C}^{abc}_{(10/18)}+\text{H.c.} \tag{52}$$

and its mirror part can be chiral invariant. Again we turn to the basis

$$g_{(10/18)}\bar{N}^{a}_{(10)}(\sigma^{c}+i\gamma_5\pi^{c})(\mathbf{T}^{c}_{(10/18)})_{ab}N^{b}_{(18m)}+\text{H.c.} \tag{53}$$

and the only solution is

$$\mathbf{T}^{\,0}_{(10/18)}=\frac{1}{\sqrt{6}}(\mathbf{0}_{10\times8},\mathbf{1}_{10\times10}), \tag{54}$$

$$\mathbf{T}^{\,a}_{(10/18)}=\left(-\frac{1}{\sqrt{3}}\mathbf{T}^{\dagger a}_{(8/10)},\frac{1}{3}\mathbf{F}^{a}_{(10)}\right). \tag{55}$$

The coefficients $\mathbf{C}^{abc}_{(8/9)}$ can be similarly obtained like Eq. (23). and we have the following relation:

$$-\mathbf{F}^{a\dagger}_{(10)}\mathbf{T}^{b}_{(10/18)}+\mathbf{T}^{b}_{(10/18)}\mathbf{F}^{a}_{(18)}+d_{abc}\mathbf{T}^{c}_{(10/18)}=0. \tag{56}$$

C. Brief summary of interactions

Altogether we have the following form of chiral invariant Lagrangian:

$$\begin{aligned}\mathcal{L}=&(\bar{N}_{(8m)}\quad\bar{N}_{(9m)}\quad\bar{N}_{(18m)}\quad\bar{N}_{(10m)})\left[(\sigma^{a}+i\gamma_5\pi^{a})\begin{pmatrix}\mathbf{0}_{8\times8}&\mathbf{0}_{8\times9}&\mathbf{0}_{8\times18}&\mathbf{0}_{8\times10}\\ \mathbf{0}_{9\times8}&g_{(9)}\mathbf{D}^{a}_{(9)}&g_{(9/18)}\mathbf{T}^{a}_{(9/18)}&\mathbf{0}_{9\times10}\\ \mathbf{0}_{18\times8}&g^{*}_{(9/18)}\mathbf{T}^{\dagger a}_{(9/18)}&g_{(18/18)}\mathbf{D}^{a}_{(18)}&\mathbf{0}_{18\times10}\\ \mathbf{0}_{10\times8}&\mathbf{0}_{10\times9}&\mathbf{0}_{10\times18}&\mathbf{0}_{10\times10}\end{pmatrix}\right.\\&\left.+(\sigma^{a}-i\gamma_5\pi^{a})\begin{pmatrix}\mathbf{0}_{8\times8}&g_{(8/9)}\mathbf{T}^{a}_{(8/9)}&g_{(8/18)}\mathbf{T}^{a}_{(8/18)}&\mathbf{0}_{8\times10}\\ g^{*}_{(8/9)}\mathbf{T}^{\dagger a}_{(8/9)}&\mathbf{0}_{9\times9}&\mathbf{0}_{9\times18}&\mathbf{0}_{9\times10}\\ g^{*}_{(8/18)}\mathbf{T}^{\dagger a}_{(8/18)}&\mathbf{0}_{18\times9}&\mathbf{0}_{18\times18}&g^{*}_{(10/18)}\mathbf{T}^{\dagger a}_{(10/18)}\\ \mathbf{0}_{10\times8}&\mathbf{0}_{10\times9}&g_{(10/18)}\mathbf{T}^{a}_{(10/18)}&\mathbf{0}_{10\times10}\end{pmatrix}\right]\begin{pmatrix}N_{(8m)}\\ N_{(9m)}\\ N_{(18)}\\ N_{(10m)}\end{pmatrix},\end{aligned} \tag{57}$$

and its mirror part is also chiral invariant,

$$\begin{aligned}\mathcal{L}_{(m)}=&(\bar{N}_{(8m)}\quad\bar{N}_{(9m)}\quad\bar{N}_{(18m)}\quad\bar{N}_{(10m)})\left[(\sigma^{a}-i\gamma_5\pi^{a})\begin{pmatrix}\mathbf{0}_{8\times8}&\mathbf{0}_{8\times9}&\mathbf{0}_{8\times18}&\mathbf{0}_{8\times10}\\ \mathbf{0}_{9\times8}&g'_{(9)}\mathbf{D}^{a}_{(9)}&g'_{(9/18)}\mathbf{T}^{a}_{(9/18)}&\mathbf{0}_{9\times10}\\ \mathbf{0}_{18\times8}&g'^{*}_{(9/18)}\mathbf{T}^{\dagger a}_{(9/18)}&g'_{(18/18)}\mathbf{D}^{a}_{(18)}&\mathbf{0}_{18\times10}\\ \mathbf{0}_{10\times8}&\mathbf{0}_{10\times9}&\mathbf{0}_{10\times18}&\mathbf{0}_{10\times10}\end{pmatrix}\right.\\&\left.+(\sigma^{a}+i\gamma_5\pi^{a})\begin{pmatrix}\mathbf{0}_{8\times8}&g'_{(8/9)}\mathbf{T}^{a}_{(8/9)}&g'_{(8/18)}\mathbf{T}^{a}_{(8/18)}&\mathbf{0}_{8\times10}\\ g'^{*}_{(8/9)}\mathbf{T}^{\dagger a}_{(8/9)}&\mathbf{0}_{9\times9}&\mathbf{0}_{9\times18}&\mathbf{0}_{9\times10}\\ g'^{*}_{(8/18)}\mathbf{T}^{\dagger a}_{(8/18)}&\mathbf{0}_{18\times9}&\mathbf{0}_{18\times18}&g'^{*}_{(10/18)}\mathbf{T}^{\dagger a}_{(10/18)}\\ \mathbf{0}_{10\times8}&\mathbf{0}_{10\times9}&g'_{(10/18)}\mathbf{T}^{a}_{(10/18)}&\mathbf{0}_{10\times10}\end{pmatrix}\right]\begin{pmatrix}N_{(8)}\\ N_{(9)}\\ N_{(18m)}\\ N_{(10)}\end{pmatrix}.\end{aligned}$$

Besides these, there is another single piece of Lagrangian which is also chiral invariant,

$$\mathcal{L}_{(B)}=g_{(B)}\bar{N}_{(8m)}(\sigma^{a}-i\gamma_5\pi^{a})\mathbf{T}^{a}_{(B)}N_{(9m)}+\text{H.c.},$$

together with its mirror part

$$\mathcal{L}_{(Bm)}=g'_{(B)}\bar{N}_{(8m)}(\sigma^{a}+i\gamma_5\pi^{a})\mathbf{T}^{a}_{(B)}N_{(9)}+\text{H.c.}$$

At the same time, we have also proven that this is the only possible case. Moreover, we can easily verify that this Lagrangian is also invariant under $U_A(1)$ chiral transformation, except $\mathcal{L}_{(B)}$ and $\mathcal{L}_{(Bm)}$. All this information is listed in Table I. Besides these Lagrangians, we still have the naive combinations $m_{(8)}\bar{N}_{(8m)}\gamma_5N_{(8)}$, $m_{(9)}\bar{N}_{(9m)}\gamma_5N_{(9)}$, $m_{(18)}\bar{N}_{(18m)}\gamma_5N_{(18)}$, and $m_{(10)}\bar{N}_{(10m)}\gamma_5N_{(10)}$. There are no meson fields, but these Lagrangians are still chiral $SU_L(3)\times SU_R(3)$ invariant and chiral $U(1)_A$ invariant.

TABLE I. Allowed chiral invariant terms with one meson field. The $\surd$ denotes that the symmetries are conserved, while x denotes they are not.

$(SU_A(3), U_A(1))$	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$ [mir]	$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$ [mir]	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$	$(\mathbf{1},\mathbf{10})\oplus(\mathbf{10},\mathbf{1})$ [mir]
$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$ [mir]	N/A	$(\surd,\surd)$	$(\surd,\surd)$	N/A
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$ [mir]	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$	N/A
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$
$(\mathbf{1},\overline{\mathbf{10}})\oplus(\overline{\mathbf{10}},\mathbf{1})$ [mir]	N/A	N/A	$(\surd,\surd)$	N/A
$(SU_A(3), U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$ [mir]	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	N/A	$(\surd,\surd)$	$(\surd,\surd)$	N/A
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$	N/A
$(\bar{\mathbf{3}},\bar{\mathbf{6}})\oplus(\bar{\mathbf{6}},\bar{\mathbf{3}})$ [mir]	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$	$(\surd,\surd)$
$(\overline{\mathbf{10}},\mathbf{1})\oplus(\mathbf{1},\overline{\mathbf{10}})$	N/A	N/A	$(\surd,\surd)$	N/A
$(SU_A(3), U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$ [mir]		
$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$	N/A	$(\surd, x)$		
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$ [mir]	$(\surd, x)$	N/A		

TABLE II. Allowed chirally invariant terms without meson field (the so-called mirror-mass terms). The $\surd$ denotes that the symmetries are conserved, while x denotes they are not.

$(SU_A(3), U_A(1))$	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$	$(\mathbf{3},\mathbf{6})\oplus(\mathbf{6},\mathbf{3})$ [mir]	$(\mathbf{10},\mathbf{1})\oplus(\mathbf{1},\mathbf{10})$
$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$ [mir]	$(\surd,\surd)$	N/A	N/A	N/A
$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$ [mir]	N/A	$(\surd,\surd)$	N/A	N/A
$(\bar{\mathbf{6}},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\bar{\mathbf{6}})$	N/A	N/A	$(\surd,\surd)$	N/A
$(\mathbf{1},\overline{\mathbf{10}})\oplus(\overline{\mathbf{10}},\mathbf{1})$ [mir]	N/A	N/A	N/A	$(\surd,\surd)$

This information is listed in Table II. These results stand in marked contrast to the two-flavor case [4,13], where the $SU_L(2)\times SU_R(2)$ symmetric interactions have both a $U_A(1)$ symmetry-conserving and a $U_A(1)$ symmetry-breaking version. Thus, the three-flavor chiral symmetry is more restrictive than the two-flavor one.

IV. CHIRAL MIXING

In this section we establish the phenomenologically preferable mixing pattern(s) and then we use the allowed chiral interactions to reproduce some of them. First we summarize the salient features of chiral mixing and axial couplings from Ref. [5].

There are three admissible scenarios (i.e. choices of pairs of chiral multiplets admixed to the $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$ one that lead to real mixing angles) when fitting the $g_A^{(0)}$ and $g_A^{(3)}$ that yield the values of F and D. Similarly, when we fit $g_A^{(3)}$ and $g_A^{(8)}$, or equivalently F and D, we predict the values for $g_A^{(0)}$ and $g_A^{(3)}$.

This is due to the fact that all three-quark baryon fields satisfy the relation $g_A^{(0)} = 3F - D = \sqrt{3}g_A^{(8)}$. Manifestly, in this way one cannot satisfy both $g_{A\,\text{expt.}}^{(0)} = 0.33 \pm 0.08$ and $g_{A\,\text{expt.}}^{(8)} = 0.34 \pm 0.07$. Thus we are left with two possible scenarios:

(1) Fit $g_A^{(0)}$ and $g_A^{(3)}$ and predict F and D. In Ref. [5] we found that there are three possible mixing patterns. Now the chiral selection rules from Sec. III allow only two of them: the case III-I mixing (see Table III), $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$; and the case IV-I mixing, $[(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})]$-$[(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})]$. However, the latter mixing violates $U_A(1)$ symmetry.

TABLE III. The Abelian and the non-Abelian axial charges and the non-Abelian chiral multiplets of $J^P = \frac{1}{2}$, Lorentz representation $(\frac{1}{2},0)$ nucleon, and Δ fields; see Refs. [4,10,11,13].

Case	Field	$g_A^{(0)}$	$g_A^{(3)}$	$\sqrt{3}g_A^{(8)}$	F	D	$SU_L(3)\times SU_R(3)$
I	$N_- = N_1 - N_2$	-1	$+1$	-1	0	$+1$	$(\mathbf{3},\bar{\mathbf{3}})\oplus(\bar{\mathbf{3}},\mathbf{3})$
II	$N_+ = N_1 + N_2$	$+3$	$+1$	$+3$	$+1$	0	$(\mathbf{8},\mathbf{1})\oplus(\mathbf{1},\mathbf{8})$
III	$N'_-(N^{(m)}_-)$	$+1$	-1	$+1$	0	-1	$(\bar{\mathbf{3}},\mathbf{3})\oplus(\mathbf{3},\bar{\mathbf{3}})$
IV	$N'_+(N^{(m)}_+)$	-3	-1	-3	-1	0	$(\mathbf{1},\mathbf{8})\oplus(\mathbf{8},\mathbf{1})$
0	$\partial_\mu N^\mu$	$+1$	$+\frac{5}{3}$	$+1$	$+\frac{2}{3}$	$+1$	$(\mathbf{6},\mathbf{3})\oplus(\mathbf{3},\mathbf{6})$

(2) Fit $g_A^{(3)}$ and $g_A^{(8)}$ and predict $g_A^{(0)}$. Since we have $g_A^{(0)} = \sqrt{3} g_A^{(8)}$ for all the three-quark nucleon fields [12], the results here can be obtained by simple refitting of the previous case. Fitting (F, D) has not been a problem, so we leave this exercise out of this paper because it generally overpredicts the $g_A^{(0)}$ by a factor of roughly $\sqrt{3} = 1.73$.

Thus we determine the mixing angles in Sec. IV A, which we then translate into statements about the admixed fields' masses in Sec. IV B. We note here that the relation $g_A^{(8)} = \frac{1}{\sqrt{3}}(3F - D)$ is a general $SU(3)$ result valid for octet fields, whereas $g_A^{(0)} = 3F - D$ is a result that depends on our specific choice of three-quark interpolating fields being admixed to the $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$ one. The latter relation changes when one considers "exotic" interpolating fields, such as certain five-quark (pentaquark) ones, for example, and that allows a simultaneous fit of $g_A^{(0)}$, $g_A^{(3)}$, and $g_A^{(8)}$, which topic is beyond the scope of this paper.

A. Phenomenology of the axial coupling constants

A basic feature of the linear chiral realization is that the axial couplings are determined by the chiral representations. For the nucleon (proton and neutron), the three-quark chiral representations of $SU_L(3) \times SU_R(3)$, $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$, $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$, and $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$ provide the nucleon isovector axial coupling $g_A^{(3)} = 1$, 1, and $5/3$ respectively. Therefore, the mixing of chiral $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$, $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$, and $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$ nucleons leads to the axial coupling

$$\begin{aligned} 1.267 &= g^{(3)}_{A(1/2,0)} \cos^2\theta + g^{(3)}_{A(1,1/2)} \sin^2\theta \\ &= g^{(3)}_{A(1/2,0)} \cos^2\theta + \frac{5}{3}\sin^2\theta, \end{aligned} \tag{58}$$

where $g^{(3)}_{A((1/2),0)}$ represents the coupling of either $(\mathbf{8}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8})$ or $(\mathbf{3}, \bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}}, \mathbf{3})$, and $g^{(3)}_{A(1,(1/2))}$ represents the coupling of $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$. The coupling $g^{(3)}_{A(1,(1/2))}$ is needed because only the coupling of $(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})$ is larger than the experimental value 1.267. We list the results of the mixing angles for all the four cases in Table IV. Three-quark nucleon interpolating fields in QCD have well-defined $U_A(1)$ chiral transformation properties (see Table III) that can be used to predict the flavor-singlet axial coupling $g^{(0)}_{A\,\mathrm{mix.}}$ and the F and D values

$$\begin{aligned} g^{(0)}_{A\,\mathrm{mix.}} &= g^{(0)}_{A((1/2),0)} \cos^2\theta + g^{(0)}_{A(1,(1/2))} \sin^2\theta \\ &= g^{(0)}_{A((1/2),0)} \cos^2\theta + \sin^2\theta, \end{aligned} \tag{59}$$

$$\begin{aligned} F &= F_{((1/2),0)} \cos^2\theta + F^{(1)}_{(1,(1/2))} \sin^2\theta, \\ &= F_{((1/2),0)} \cos^2\theta + \frac{2}{3}\sin^2\theta, \end{aligned} \tag{60}$$

$$\begin{aligned} D &= D_{((1/2),0)} \cos^2\theta + D_{(1,(1/2))} \sin^2\theta \\ &= D_{((1/2),0)} \cos^2\theta + \sin^2\theta. \end{aligned} \tag{61}$$

The mixing angle θ is extracted from Eq. (58), where we used the bare F and D values for different chiral multiplets as listed in Table III. Because of the different (bare) non-Abelian $g_A^{(3)}$ and Abelian $g_A^{(0)}$ axial couplings (see Table III), the mixing formulas Eq. (59) give substantially different predictions from one case to another; see Table IV. We can see in Table IV that the two best candidates are cases I and IV, with $g_A^{(0)} = -0.2$ and $g_A^{(0)} = 0.4$, respectively, the latter being within the error bars of the measured value $g^{(0)}_{A\,\mathrm{expt.}} = 0.33 \pm 0.08$ [2,28]. Selection rules from Sec. III allow the case III and the case IV. And so the case IV is the best candidate so long as we consider just the mixing of two nucleon fields [4].

Manifestly, a linear superposition of any three fields (except for the mixtures of cases II, III, and IV above, which yield complex mixing angles) gives a perfect fit to the central values of the experimental axial couplings $g^{(0)}_{A\,\mathrm{expt.}} = 0.33 \pm 0.08$ and $g^{(3)}_{A\,\mathrm{expt.}} = 1.267$ and predicts the F and D values, or vice versa: one may fit $g_A^{(3)}$ and $g_A^{(8)}$, (or equivalently F and D) and thus predict $g_A^{(0)}$. This has been done in Ref. [5], where there were three allowed cases: I-II, I-III, and I-IV. The selection rules from Sec. III indicate that only two of them are possible in the one-meson approximation: (1) the case I-III and (2) the case I-IV. In the former case the $U_A(1)$ symmetry is conserved, whereas in the latter the $U_A(1)$ is violated.

Such a three-field admixture introduces two new free parameters, besides the already introduced mixing angles, e.g. θ_3 and $\theta_1 (= 0)$ (which we may set to vanish in the

TABLE IV. The values of the baryon isoscalar axial coupling constant predicted from the naive mixing and $g^{(3)}_{A\,\mathrm{expt.}} = 1.267$; compare with $g^{(0)}_{A\mathrm{expt.}} = 0.33 \pm 0.03 \pm 0.05$, $F = 0.459 \pm 0.008$, and $D = 0.798 \pm 0.008$, leading to $F/D = 0.571 \pm 0.005$; Ref. [1].

Case	$g^{(3)}_{A\,\mathrm{expt.}}$	θ_i	$g^{(0)}_{A\,\mathrm{mix.}}$	$\sqrt{3} g^{(8)}_{A\,\mathrm{mix.}}$	F	D	F/D
I	1.267	39.3°	−0.20	−0.20	0.267	1	0.267
II	1.267	39.3°	2.20	2.20	0.866	0.401	2.16
III	1.267	67.2°	1.00	1.00	0.567	0.700	0.81
IV	1.267	67.2°	0.40	0.40	0.417	0.850	0.491

present approximation). For the case I-III (we shall call it here case III-I for reasons soon to be clarified) we have the relative/mutual mixing angle $\theta_{31} = \varphi$, as the two nucleon fields III and I mix due to the off-diagonal interaction Eq. (43). Thus we find two equations with two unknowns of the general form

$$\frac{5}{3}\sin^2\theta + \cos^2\theta(g_A^{(3)}(\mathrm{III})\cos^2\varphi + g_A^{(3)}(\mathrm{I})\sin^2\varphi) = 1.267, \tag{62}$$

$$\sin^2\theta + \cos^2\theta(g_A^{(0)}(\mathrm{III})\cos^2\varphi + g_A^{(0)}(\mathrm{I})\sin^2\varphi) = 0.33 \pm 0.08. \tag{63}$$

The solutions to these equations (the values of the mixing angles θ, φ) provide, at the same time, input for the prediction of F and D,

$$\cos^2\theta(F(\mathrm{III})\cos^2\varphi + F(\mathrm{I})\sin^2\varphi) + \frac{2}{3}\sin^2\theta = F, \tag{64}$$

$$\cos^2\theta(D(\mathrm{III})\cos^2\varphi + D(\mathrm{I})\sin^2\varphi) + \sin^2\theta = D. \tag{65}$$

The values of the mixing angles (θ, φ) obtained from this straightforward fit to the baryon axial coupling constants are shown in Table V. We also show the result of the case I-IV as well as IV-I in this table. Besides these cases, the cases I-II and II-I can also be used to produce the experimental $g_A^{(0)}$ and $g_A^{(3)}$, which are however not allowed from Sec. III.

B. Baryon masses

The next step is to try and reproduce this phenomenological mixing starting from a model interaction, rather than *per fiat*. As the first step in that direction we must look for a dynamical source of mixing. One such mechanism is the simplest chirally symmetric *nonderivative* one-(σ, π)-meson interaction Lagrangian, which induces baryon masses via its σ-meson coupling. Chiral symmetry is spontaneously broken through the "condensation" of the sigma field $\sigma \to \sigma_0 = \langle\sigma\rangle_0 = f_\pi$, which leads to the dynamical generation of baryon masses, as can be seen from the linearized chiral invariant interaction Lagrangians, Eqs. (19) and (28).

In this section, we study the masses of the octet baryons. There are altogether six types of octet baryon fields: N_+ $[N_{(8)}]$, N_- [contained in $N_{(9)}$] and N_μ [contained in $N_{(18)}$], as well as their mirror fields N'_+ $[N_{(8m)}]$, N'_- [contained in $N_{(9m)}$], N'_μ [contained in $N_{(18m)}$]. The nucleon mass matrix is already in a simple block-diagonal form when the nucleon fields form the following mass matrix:

$$M = \frac{1}{\sqrt{6}}\bar{N}\begin{pmatrix} 0 & g_{(8/9)} & g_{(8/18)} & m_{(8)}\gamma_5 & g_B & 0 \\ g^*_{(8/9)} & g_{(9/9)} & g_{(9/18)} & g^*_B & m_{(9)}\gamma_5 & 0 \\ g^*_{(8/18)} & g^*_{(9/18)} & g_{(18/18)} & 0 & 0 & m_{(18)}\gamma_5 \\ m_{(8)}\gamma_5 & g'_B & 0 & 0 & g'_{(8/9)} & g'_{(8/18)} \\ g'^*_B & m_{(9)}\gamma_5 & 0 & g'^*_{(8/9)} & g'_{(9/9)} & g'_{(9/18)} \\ 0 & 0 & m_{(18)}\gamma_5 & g'^*_{(8/18)} & g'^*_{(9/18)} & g'_{(18/18)} \end{pmatrix} N, \tag{66}$$

where

$$N = (N'_+, N'_-, N_\mu, N_+, N_-, N'_\mu)^T. \tag{67}$$

Since there are three nucleon fields as well as their mirror fields, there can be a nonzero phase angle. However, for simplicity, we assume all the axial couplings are real.

C. Masses due to $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$-$[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$ mixing

We use the results of Sec. III: the chirally invariant diagonal, Eqs. (19) and (28), and off-diagonal, Eq. (33), meson-baryon-baryon interactions involving

$$\begin{aligned} &(B_1, \Lambda) \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})\ [\mathrm{mir}], \\ &(B_2, \Delta) \in (\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6}), \\ &(\sigma, \pi) \in (\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}}). \end{aligned} \tag{68}$$

Here all baryons have spin $1/2$, while the isospin of B_1 and B_2 is $1/2$ and that of Δ is $3/2$. The Δ field is then represented by an isovector-Dirac spinor field Δ^i, $(i = 1, 2, 3)$.

In writing down the Lagrangians, Eqs. (19), (28), and (33), we have implicitly assumed that the parities of B_1, B_2, Λ, and Δ are the same. In principle, they are arbitrary, except for the ground state nucleon, which must be even. For instance, if B_2 has odd parity, the first term in the interaction Lagrangian, Eq. (33), must include another γ_5 matrix [30]. Here we assume the ground state nucleon is contained in either $[(\mathbf{6}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{6})]$ or $[(\bar{\mathbf{3}}, \mathbf{3}) \oplus (\mathbf{3}, \bar{\mathbf{3}})]$, and so at least one of B_1 and B_2 has even parity. Next we consider all possible cases for the parities of B_2, Λ, and Δ. The results are similar to the two-flavor ones shown in Refs. [4,13] (because we assumed good $SU(3)$ symmetry here).

Having established the mixing interaction, Eq. (33), as well as the diagonal terms, Eqs. (19) and (28), we calculate the masses of the baryon states, as functions of the pion decay constant/chiral order parameter and the coupling constants $g_1 \sim g_{(9)}$, $g_2 \sim g_{(18)}$, and $g_3 \sim g_{(9/18)}$,

TABLE V. The values of the mixing angles obtained from the simple fit to the baryon axial coupling constants and the predicted values of axial F and D couplings. The experimental values are $F = 0.459 \pm 0.008$ and $D = 0.798 \pm 0.008$, leading to $F/D = 0.575 \pm 0.005$ and $g_A^{(8)} = 0.33 \pm 0.01$; Ref. [29]. The most recent analysis of experimental values leads to $F = 0.477 \pm 0.001$ and $D = 0.835 \pm 0.001$ and $g_A^{(8)} = 0.344 \pm 0.001$ in Ref. [1]. Note that these values are more than 2σ away from the old ones, and that the new F,D add up to $F + D = 1.312 \neq 1.269 \pm 0.002$. Also $g_{A\,\mathrm{expt.}}^{(0)} = 0.33 \pm 0.08$.

Case	$g_{A\,\mathrm{expt.}}^{(3)}$	$g_A^{(0)}$	$g_A^{(8)}$	θ	φ	F	D	F/D
I-III	1.267	0.33 ± 0.08	0.19 ± 0.05	$50.7° \pm 1.8°$	$23.9° \pm 2.9°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
III-I	1.267	0.33 ± 0.08	0.19 ± 0.05	$50.7° \pm 1.8°$	$66.1° \pm 2.9°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
I-IV	1.267	0.33 ± 0.08	0.19 ± 0.05	$63.2° \pm 4.0°$	$54° \pm 23°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04
IV-I	1.267	0.33 ± 0.08	0.19 ± 0.05	$63.2° \pm 4.0°$	$36° \pm 23°$	0.399 ± 0.02	0.868 ∓ 0.02	0.460 ± 0.04

$$\begin{aligned}\mathcal{L}_{(9)} &= -g_1(\bar{B}_1\sigma B_1 - 2\bar{\Lambda}\sigma\Lambda) + \cdots,\\ \mathcal{L}_{(18)} &= -g_2(\bar{B}_2\sigma B_2 - 2\bar{\Delta}^i\sigma\Delta^i) + \cdots,\\ \mathcal{L}_{(9/18)} &= -g_3(\bar{B}_1\sigma B_2) + \cdots.\end{aligned} \tag{69}$$

Altogether we have

$$\begin{aligned}\mathcal{L} &= -f_\pi(\bar{B}_1, \bar{B}_2)\begin{pmatrix} g_1 & g_3 \\ g_3 & g_2\end{pmatrix}\begin{pmatrix} B_1 \\ B_2\end{pmatrix} + 2g_1 f_\pi\bar{\Lambda}\Lambda\\ &\quad + 2g_2 f_\pi\bar{\Delta}^i\Delta^i.\end{aligned} \tag{70}$$

We diagonalize the mass matrix and express the mixing angle in terms of diagonalized masses

$$\begin{aligned}N(N^*) &= \cos\theta B_1 + \sin\theta B_2,\\ N^*(N) &= -\sin\theta B_1 + \cos\theta B_2.\end{aligned} \tag{71}$$

We find the following double-angle formulas for the mixing angles $\theta_{1,\ldots,8}$ between B_1 and B_2 in the eight different parities scenarios:

$$\begin{aligned}\tan2\theta_1 &= \frac{\sqrt{-(2N+\Delta)(2N^*+\Delta)}}{(N+N^*+\Delta)}\\ &= -\frac{\sqrt{-(2N+\Lambda)(2N^*+\Lambda)}}{(N+N^*+\Lambda)},\end{aligned} \tag{72}$$

$$\begin{aligned}\tan2\theta_2 &= \frac{\sqrt{(2N+\Delta)(2N^*-\Delta)}}{(N-N^*+\Delta)}\\ &= -\frac{\sqrt{(2N+\Lambda)(2N^*-\Lambda)}}{(N-N^*+\Lambda)},\end{aligned} \tag{73}$$

$$\begin{aligned}\tan2\theta_3 &= \frac{\sqrt{-(2N+\Delta)(2N^*+\Delta)}}{(N+N^*+\Delta)}\\ &= -\frac{\sqrt{-(2N-\Lambda)(2N^*-\Lambda)}}{(N+N^*-\Lambda)},\end{aligned} \tag{74}$$

$$\begin{aligned}\tan2\theta_4 &= \frac{\sqrt{(2N+\Delta)(2N^*-\Delta)}}{(N-N^*+\Delta)}\\ &= \frac{\sqrt{(2N-\Lambda)(2N^*+\Lambda)}}{(-N+N^*+\Lambda)},\end{aligned} \tag{75}$$

$$\begin{aligned}\tan2\theta_5 &= \frac{\sqrt{-(2N-\Delta)(2N^*-\Delta)}}{(N+N^*-\Delta)}\\ &= -\frac{\sqrt{-(2N+\Lambda)(2N^*+\Lambda)}}{(N+N^*+\Lambda)},\end{aligned} \tag{76}$$

$$\begin{aligned}\tan2\theta_6 &= \frac{\sqrt{(2N-\Delta)(2N^*+\Delta)}}{(N-N^*-\Delta)}\\ &= -\frac{\sqrt{(2N+\Lambda)(2N^*-\Lambda)}}{(N-N^*+\Lambda)},\end{aligned} \tag{77}$$

$$\begin{aligned}\tan2\theta_7 &= \frac{\sqrt{-(2N-\Delta)(2N^*-\Delta)}}{(N+N^*-\Delta)}\\ &= -\frac{\sqrt{-(2N-\Lambda)(2N^*-\Lambda)}}{(N+N^*-\Lambda)},\end{aligned} \tag{78}$$

$$\begin{aligned}\tan2\theta_8 &= \frac{\sqrt{(2N-\Delta)(2N^*+\Delta)}}{(N-N^*-\Delta)}\\ &= \frac{\sqrt{(2N-\Lambda)(2N^*+\Lambda)}}{(N-N^*-\Lambda)},\end{aligned} \tag{79}$$

where N, N^*, Λ, and Δ represent the masses of the corresponding particles. The four angles correspond to the eight possible parities: θ_1, $(N^{*+}, \Lambda^+, \Delta^+)$; θ_2, $(N^{*-}, \Lambda^+, \Delta^+)$; θ_3, $(N^{*+}, \Lambda^-, \Delta^+)$; θ_4, $(N^{*-}, \Lambda^-, \Delta^+)$; θ_5, $(N^{*+}, \Lambda^+, \Delta^-)$; θ_6, $(N^{*-}, \Lambda^+, \Delta^-)$; θ_7, $(N^{*+}, \Lambda^-, \Delta^-)$; and θ_8, $(N^{*-}, \Lambda^-, \Delta^-)$, where $\pm$ indicate the parity of the state. Note that the angles θ_1, θ_3, and θ_5 are necessarily imaginary so long as the Δ, Λ, and N^* masses are physical (positive), and that the reality of the mixing angle(s) imposes stringent limits on the Δ, N^* resonance masses in other cases, as well.

In the present study we have three model parameters g_1, g_2, and g_3, which can be determined by different set of inputs. We can use two baryon masses and the mixing angle as inputs and predict the third baryon mass (inverse prediction). We use the Eqs. (71)–(79) for the (double) mixing angles $\theta_{1,\ldots,8}$ together with the two observed nucleon masses and the mixing angle $\theta = 67.2°$ as shown in Table IV to predict the Δ masses shown in the Table VI.

We see that only the (N^{*-}, Δ^{+}) parity combination leads to a realistic prediction of the baryon masses. Otherwise, at least one of the predicted baryon masses is off by a factor of order two. Indeed, the case $(N^{*P}, \Lambda^{P'}, \Delta^{P''}) = (-, -, +)$ predicts the (odd-parity) $SU(3)$ flavor-singlet Λ at 1140 MeV, somewhat below the measured value (1405 MeV) and $\Delta(2330)$, the nearest known candidate state being the (Particle Data Group [PDG] four star in Ref. [31]) $P_{31}(1910)$ resonance. It is curious that the flavor-singlet $\Lambda(1140)$ state lies (considerably) below the flavor-octet state $N^{*}(1535)$ even in the good flavor $SU(3)$ symmetry limit; the predicted mass difference might/ought to be improved by introducing explicit $SU(3)$ symmetry breaking strange-up/down quark mass difference.

D. Masses due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing

To improve our analysis, we may add a third chiral multiplet nucleon field. As in the previous Sec. III, we consider baryon fields

$$\begin{aligned}(B_1, \Lambda_1) &\in (\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})\ [\text{mir}],\\ (B_2, \Delta) &\in (\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6}),\\ (B_3, \Lambda_2) &\in (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3}).\end{aligned} \tag{80}$$

As discussed above, the case III-I allows one to reproduce the experimental couplings $g_A^{(0)}$ and $g_A^{(3)}$. To study this mixing, we need to use the previous Lagrangian, Eq. (69), as well as the new ones

$$\begin{aligned}\mathcal{L}'_{(9)} &= -g_4(\bar{B}_3\sigma B_3 - 2\bar{\Lambda}_1\sigma\Lambda_1) + \cdots,\\ \mathcal{L}_{(9/9)} &= -g_5 f_\pi \bar{B}_1 B_3 - g_5 f_\pi \bar{\Lambda}_1\Lambda_2 + \cdots\end{aligned} \tag{81}$$

that follow from Eq. (28), where the third nucleon field B_3 is a mirror image of B_1. We note that B_1 and B_3 couple with each other through the naive combinations $m_{(9)}\bar{N}_{(9m)}\gamma_5 N_{(9)}$. Chiral symmetry is spontaneously broken through the condensation of the sigma field $\sigma \to \sigma_0 = \langle\sigma\rangle_0 = f_\pi$, which leads to the dynamical generation of baryon masses,

$$\begin{aligned}\mathcal{L} = &-f_\pi(\bar{B}_1, \bar{B}_3, \bar{B}_2)\begin{pmatrix} g_1 & g_5 & g_3\\ g_5 & g_4 & 0\\ g_3 & 0 & g_2\end{pmatrix}\begin{pmatrix} B_1\\ B_3\\ B_2\end{pmatrix}\\ &- f_\pi(\bar{\Lambda}_1, \bar{\Lambda}_2)\begin{pmatrix} -2g_1 & g_5\\ g_5 & -2g_4\end{pmatrix}\begin{pmatrix}\Lambda_1\\ \Lambda_2\end{pmatrix} + 2g_2 f_\pi \bar{\Delta}^i\Delta^i.\end{aligned} \tag{82}$$

To solve this system in its full generality seems both too complicated and not very useful. However, since g_6 of $g_6\bar{B}_3 B_2$ vanishes, we only need five conditions to solve this system. Therefore, we just use the three nucleon candidates $N(940)$, $N(1440)$, and $N^{*}(1535)$ as well as the two mixing angles $\theta° = 63.2°$ and $\phi = 36°$. Finally we find that there are two possibilities as shown in Table VII.

Once again, the odd-parity Δ option appears as the better one. Now, the first flavor-singlet Λ lies at 1370 MeV, substantially closer to 1405 MeV than before. A second flavor-singlet Λ lies at 1850 MeV, very close to the (three star PDG, Ref. [31]) $P_{01}(1810)$ resonance. This is our best candidate in the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing scenario.

E. Masses due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ mixing

We can also study the baryon masses due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ mixing

$$B_1 \in (\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})\ [\text{mir}], \quad (B_2, \Delta) \in (\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6}). \tag{83}$$

Having established the mixing interaction Eq. (38), as well as the diagonal terms Eq. (19), we calculate the masses of the baryon states, as functions of the pion decay constant/chiral order parameter and the coupling constants $g_2 \sim g_{(18)}$ and $g_3 \sim g_{(8/18)}$,

$$\begin{aligned}\mathcal{L}_{(18)} &= -g_2(\bar{B}_2\sigma B_2 - 2\bar{\Delta}^i\sigma\Delta^i) + \cdots,\\ \mathcal{L}_{(8/18)} &= -g_3(\bar{B}_1\sigma B_2) + \cdots.\end{aligned} \tag{84}$$

TABLE VI. The values of the Δ baryon masses predicted from the isovector axial coupling $g_{A\,\text{mix.}}^{(1)} = g_{A\,\text{expt.}}^{(1)} = 1.267$ and $g_{A\,\text{mix.}}^{(0)} = 0.4$vs. $g_{A\,\text{expt}}^{(0)} = 0.33 \pm 0.08$.

$(N^{*P}, \Lambda^{P'}, \Delta^{P''})$	(N, N^{*})	Λ (MeV)	$\Lambda_{\text{expt.}}$ (MeV)	Δ (MeV)	$\Delta_{\text{expt.}}$ (MeV)
$(-,+,+)$	N(940), R(1535)	2330		2330	1910
$(-,-,+)$	N(940), R(1535)	1140	1405	2330	1910
$(-,+,-)$	N(940), R(1535)	2330		1140	
$(+,-,-)$	N(940), R(1440)	2030, 2730		2030, 2730	
$(-,-,-)$	N(940), R(1535)	1140	1405	1140	

TABLE VII. The values of the Δ and Λ baryon masses predicted from the isovector axial coupling $g^{(1)}_{A\,\mathrm{mix.}} = g^{(1)}_{A\,\mathrm{expt.}} = 1.267$ and $g^{(0)}_{A\,\mathrm{mix.}} = 0.33 \pm 0.08$ due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing.

No.	g_1	g_2	g_3	g_4	g_5	Λ_1^P (MeV)	Λ_2^P (MeV)	Δ^P (MeV)
1	−4.7	8.4	−3.4	2.9	9.8	1370^-	1850^+	2170^-
2	−7.2	4.6	7.9	9.1	−4.2	1940^+	2430^-	1200^-

Note that $g_1 \sim g_{(8)}$ is zero now. We diagonalize the mass matrix and express the mixing angle in terms of diagonalized masses. We find the following double-angle formulas for the mixing angles $\theta_{1,\ldots,4}$ between B_1 and B_2 in the four different parities scenarios:

$$\tan 2\theta_1 = -2i\frac{\sqrt{NN^*}}{N^* + N}, \qquad \Delta = -2(N^* + N), \tag{85}$$

$$\tan 2\theta_2 = \frac{2\sqrt{NN^*}}{N^* - N}, \qquad \Delta = 2(N^* - N), \tag{86}$$

$$\tan 2\theta_3 = -2i\frac{\sqrt{NN^*}}{N^* + N}, \qquad \Delta = 2(N^* + N), \tag{87}$$

$$\tan 2\theta_4 = \frac{2\sqrt{NN^*}}{N^* - N}, \qquad \Delta = -2(N^* - N), \tag{88}$$

where N, N^*, and Δ represent the masses of the corresponding particles. The four angles correspond to the four possible parities: θ_1, (N^{*+}, Δ^+); θ_2, (N^{*-}, Δ^+); θ_3, (N^{*+}, Δ^-); and θ_4, (N^{*-}, Δ^-), where $\pm$ indicate the parity of the state. Note that only θ_2 leads to a physical result. We can use the mixing angle $\theta = 67.2°$ and the nucleon mass 940 MeV to predict the excited nucleon mass and Δ mass; see Table VIII. This gives predictions of no practical value. To get a practically useful result, we need to add one of the two-meson interaction Lagrangians from Sec. III A 3, and thus a nonzero g_1 term,

$$\mathcal{L}_{(8)} = -\frac{g_1}{f_\pi}\bar{B}_1\sigma^2 B_1 + \cdots,$$

and we have four new different parities scenarios,

$$\tan 2\theta_1 = \frac{\sqrt{-(2N + \Delta)(2N^* + \Delta)}}{(N + N^* + \Delta)}, \tag{89}$$

$$\tan 2\theta_2 = \frac{\sqrt{(2N + \Delta)(2N^* - \Delta)}}{(N - N^* + \Delta)}, \tag{90}$$

$$\tan 2\theta_3 = \frac{\sqrt{-(2N - \Delta)(2N^* - \Delta)}}{(N + N^* - \Delta)}, \tag{91}$$

$$\tan 2\theta_4 = \frac{\sqrt{(2N - \Delta)(2N^* + \Delta)}}{(N - N^* - \Delta)}. \tag{92}$$

Note that only θ_1 is imaginary for positive baryon masses, i.e. unphysical. We can use the mixing angle $\theta = 67.2°$ and the two nucleon masses to predict the Δ mass; see Table IX. The nearest known candidate for the Δ(2330) state is the (four star PDG, Ref. [31]) $P_{31}(1910)$ resonance.

F. Masses due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing

To improve our analysis, we can add a third field, and altogether we consider

$$B_1 \in (\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})\ [\mathrm{mir}], \qquad (B_2, \Delta) \in (\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6}), \tag{93}$$

$$(B_3, \Lambda) \in (\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3}). \tag{94}$$

As discussed above, the case IV-I is possible to produce the experimental couplings $g_A^{(0)}$ and $g_A^{(3)}$, although this is $U_A(1)$ violated. To study this mixing, we need to use the previous Lagrangian, Eq. (84), as well as the new ones

$$\begin{aligned}\mathcal{L}'_{(9)} &= -g_4(\bar{B}_3\sigma B_3 - 2\bar{\Lambda}\sigma\Lambda) + \cdots,\\ \mathcal{L}_{(B)} &= -g_5\bar{B}_1\sigma B_3 + \cdots,\end{aligned} \tag{95}$$

that follow from Eqs. (28) and (48). Chiral symmetry is spontaneously broken through the condensation of the sigma field $\sigma \to \sigma_0 = \langle\sigma\rangle_0 = f_\pi$, which leads to the dynamical generation of baryon masses,

$$\begin{aligned}\mathcal{L} &= -f_\pi(\bar{B}_1, \bar{B}_3, \bar{B}_2)\begin{pmatrix} g_1 & g_5 & g_3 \\ g_5 & g_4 & 0 \\ g_3 & 0 & g_2 \end{pmatrix}\begin{pmatrix} B_1 \\ B_3 \\ B_2 \end{pmatrix}\\ &\quad + 2g_4 f_\pi\bar{\Lambda}\Lambda + 2g_2 f_\pi\bar{\Delta}^i\Delta^i.\end{aligned} \tag{96}$$

TABLE VIII. The values of the Δ baryon masses predicted from the isovector axial coupling $g^{(1)}_{A\,\mathrm{mix.}} = g^{(1)}_{A\,\mathrm{expt.}} = 1.267$ and $g^{(0)}_{A\,\mathrm{mix.}} = 0.4$vs. $g^{(0)}_{A\,\mathrm{expt.}} = 0.33 \pm 0.08$ due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ mixing without additional two-meson interactions.

$(N^{*P}, \Delta^{P'})$	N	N^*	$N^*_{\mathrm{expt.}}$ (MeV)	Δ (MeV)	$\Delta_{\mathrm{expt.}}$ (MeV)
(−, +)	N(940)	5320		8760	

TABLE IX. The values of the Δ baryon masses predicted from the isovector axial coupling $g^{(1)}_{A\,\mathrm{mix.}} = g^{(1)}_{A\,\mathrm{expt}} = 1.267$ and $g^{(0)}_{A\,\mathrm{mix.}} = 0.4$vs. $g^{(0)}_{A\,\mathrm{expt.}} = 0.33 \pm 0.08$ due to $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ mixing with additional two-meson interactions.

$(N^{*P}, \Delta^{P'})$	(N, N^*)	Δ (MeV)	$\Delta_{\mathrm{expt.}}$ (MeV)
$(-,+)$	N(940), R(1535)	2330	1910
$(+,-)$	N(940), R(1440)	2030, 2730	
$(-,-)$	N(940), R(1535)	1140	

Since g_6 of $g_6 \bar{B}_3 B_2$ vanishes, we only need five conditions to solve this system. Therefore, we may use the three lowest-lying nucleon states $N(940)$, $N(1440)$, and $N^*(1535)$ as well as the two mixing angles $\theta^\circ = 50.7^\circ$ and $\phi = 66.1^\circ$. Finally we find that there are two real possibilities as shown in Table X. Once again, the two odd-parity Δ options appear as the best ones. First, even-parity flavor-singlet $\Lambda(1580)$, lies very close to the (three star PDG, Ref. [31]) $P_{01}(1600)$ resonance. Second, the odd-parity flavor-singlet Λ lies at 1850 MeV, also very close to the (three star PDG, Ref. [31]) $S_{01}(1800)$ resonance. These are our best candidates in the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$ mixing scenario, that shows that this option is open.

G. Baryon masses and chiral restoration

Note that, starting from the above mass formulas one may study the behavior of baryon masses in the chiral restoration limit, i.e. as $f_\pi \to 0$. We do not wish to go into this subject in any depth here, except to point out several more-or-less immediate consequences of our results.

First we note that in the two-flavor case one often finds nucleon parity doublets in the chiral restoration limit $f_\pi \to 0$ [4]. That, however, is generally a consequence of the assumptions made about the number and kind of chiral multiplets that are being mixed. If one assumes, as in our studies above, that more than two multiplets are mixed, then, of course, there will be no parity doublets, but triplets, or generally as many states as there are admixed multiplets. Moreover, if there are more than two degenerate states, such as in our studies above, then at least two will have the same parity, i.e. the concept of "parity doublets" ceases to be meaningful and "parity multiplets" ought to be introduced. Finally, if two different flavor $SU(3)$ multiplets form one chiral multiplet, such as the **8** and **10** in the $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$, then the two-flavor $SU(3)$ multiplets may form a mass-degenerate "parity doublet" in the chiral restoration limit, even though most of the states in such doublets do not have the same flavor quantum numbers.

Various conjectures have been made about the potential relation between the observed parity doublets high in the baryon spectrum and chiral symmetry restoration, especially the restoration of the (otherwise explicitly broken) $U_A(1)$ symmetry (see Ref. [32] and references therein). Our results above viz. that there are two basic allowed scenarios that differ in the $U_A(1)$ (non)symmetry of their interactions, show immediately that the $U_A(1)$ symmetry need not play a role in the baryon spectra. In this regard we agree with the conclusions of Refs. [32,33], which used only a two-flavor model, however. Such conclusions were also previously reached in the two-flavor case in Ref. [13] and in Ref. [34], only in the more restricted case of just one $SU(2)$ parity doublet and without mirror fields. The first, limited, attempts at the three-flavor case were made in Refs. [17,18].

V. SUMMARY AND OUTLOOK

We have used the results of our previous paper [5] to construct the $SU_L(3) \times SU_R(3)$ chiral invariant interactions based on the phenomenological facts regarding the baryon axial currents, of the chiral $[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$ multiplet mixing with other nonexotic baryon field multiplets, such as the $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ and $[(\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{8})]$.

The existence of these multiplets is not limited to three-quark interpolators: they are present in the $SU(3)_L \times SU(3)_R$ Clebsch-Gordan series for the five-quark interpolating fields, as well as the seven-quark ones, etc. Indeed, these are the only nonexotic chiral multiplets, as they consist of only nonexotic flavor $SU(3)$ multiplets. The "ordinary" (vector) $SU(3)$ multiplet content of a chiral multiplet is determined by the Clebsch-Gordan series for the tensor product of the right- and left-$SU(3)$ multiplets: thus $\mathbf{1} \oplus \mathbf{8} \in (\mathbf{3},\bar{\mathbf{3}})$; $\mathbf{8} \in (\mathbf{8},\mathbf{1})$; $\mathbf{8} \oplus \mathbf{10} \in (\mathbf{6},\mathbf{3})$. Introducing multiple fields with identical chiral contents would lead to double counting, however. That is to say that the effects of multiquark fields are implicitly accounted

TABLE X. The values of the Δ and Λ baryon masses predicted from the isovector axial coupling $g^{(1)}_{A\,\mathrm{mix.}} = g^{(1)}_{A\,\mathrm{expt.}} = 1.267$ and $g^{(0)}_{A\,\mathrm{mix.}} = 0.33 \pm 0.08$ and the mass fit to $N(940)$, $N(1440)$ and $N^*(1535)$.

No.	g_1	g_2	g_3	g_4	g_5	Λ^P (MeV)	Δ^P (MeV)
1	4.6	8.0	−1.8	−6.1	9.7	1580^+	2070^-
2	−8.4	4.3	7.1	10.6	−2.4	2750^-	1124^-
3	−1.3	10.2	2.1	−2.5	9.8	640^+	2660^-
4	−8.7	8.1	7.3	7.1	2.9	1850^-	2110^-

for, unless these fields differ from the ones we assumed in some respect other than the non-Abelian chiral multiplet. Introduction of exotic chiral multiplets, on the other hand, would lead to exotic flavor $SU(3)$ multiplets in the spectrum, which are absent experimentally, however. Thus, we may conclude that these three chiral multiplets, together with their mirror images, are the *only* ones consistent with the present experimental knowledge, and that no additional chiral mixing is phenomenologically allowed, without further explanation.

The results of the three-field (''two-angle'') mixing are curious insofar as all phenomenologically permissible combinations of interpolating fields lead to the same F, D values, that are in reasonable agreement with experiment. This (unexpected) equivalence of results is a consequence of the relation $g_A^{(0)} = 3F - D$ between the flavor-singlet axial coupling $g_A^{(0)}$ and the (previously unrelated) flavor-octet F and D values. That relation is a benchmark feature of the three-quark interpolating fields and any (potential) departures from it may be attributed to interpolating fields with a number of quarks that is higher than three.

We constructed all $SU_L(3) \times SU_R(3)$ chirally symmetric baryon-one-meson interactions that mix the three basic baryon chiral multiplets (and their mirror images). All of these interactions, with only one exception, obey the $U_A(1)$ symmetry as well. We used these interactions to relate the mixing angles to the masses of physical (''mixed'') baryons. Then we tried to reproduce the phenomenological mixing angles based on observed baryon spectra. Once the number of admixed fields exceeds three there is too much freedom, i.e. too many mixing angles, in the most general form of such a mixing procedure to be constrained by only three measured numbers. That assumption can be relaxed, if/when more detailed studies become necessary if/when new observables are measured in the future.

For the purpose of simplification we used the two lowest-lying nucleon states and then fit the phenomenological values of the mixing angles and thus predicted (at least) one high-lying resonance, which we then searched for in the PDG tables; Ref. [31]. This has led us to (at least) two allowed scenarios. In this way we have made the first tentative assignments of observed baryon states to chiral multiplets. As explained above, this procedure does not necessarily lead to unique results, however. The two basic allowed scenarios differ primarily in the number of predicted flavor-singlet Λ hyperons and in the $U_A(1)$ (non) symmetry of their interactions. At this moment in time we have no reason to prefer one solution to another, other than aesthetic ones, such as the $U_A(1)$ symmetry breaking.

Manifestly, the good $U_A(1)$ symmetry limit is sufficient to reproduce the nucleon axial couplings and the low-lying spectrum, as shown in the first scenario ($[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\bar{\mathbf{3}},\mathbf{3}) \oplus (\mathbf{3},\bar{\mathbf{3}})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing), but it is not necessary, as shown in the second scenario ($[(\mathbf{6},\mathbf{3}) \oplus (\mathbf{3},\mathbf{6})]$-$[(\mathbf{1},\mathbf{8}) \oplus (\mathbf{8},\mathbf{1})]$-$[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ mixing). This result stands in contrast to the two-flavor case [4,13], where all $SU_L(2) \times SU_R(2)$ symmetric interactions have both a $U_A(1)$ symmetry-conserving and a $U_A(1)$ symmetry-breaking version. Thus, the three-flavor chiral symmetry is more restrictive and consequently more instructive than the two-flavor one.

One of our conclusions follows as a simple corollary of this result: the mass degeneracy of opposite-parity baryon resonances is not necessarily a consequence of the explicit $U_A(1)$ symmetry restoration in agreement with the conclusions drawn from the two-flavor model calculations; Refs. [32,33]. Moreover, the parity doubling need be neither one of, nor the only, consequence(s) of the spontaneous $SU_L(3) \times SU_R(3)$ symmetry restoration.

This result also shows that the $U_A(1)$ anomaly in QCD may still, but need not, be the underlying source of the spin problem [2], as was once widely thought [18]. In all likelihood it provides only a relatively small part of the solution, the largest part coming from the chiral structure of the nucleon.

The main line of applications of these results lies in the nonzero density/temperature physics: all previous attempts (see Refs. [19,21]) included only the $[(\mathbf{3},\bar{\mathbf{3}}) \oplus (\bar{\mathbf{3}},\mathbf{3})]$ baryon chiral multiplet, which naturally led to axial couplings that differ from the measured ones. Another step, left for the future, is to include the explicit chiral symmetry breaking.

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