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Gauge- and frame-independent decomposition of nucleon spin

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In a recent paper, we have shown that the way of gauge-invariant decomposition of the nucleon spin is not necessarily unique, but there still exists a preferable decomposition from the observational viewpoint. What was not complete in this argument is a fully satisfactory answer to the following questions. Does the proposed gauge-invariant decomposition, especially the decomposition of the gluon total angular momentum into its spin and orbital parts, correspond to observables which can be extracted from high-energy deep-inelastic-scattering measurements ? Is this decomposition not only gauge invariant but also Lorentz frame independent, so that it is legitimately thought to reflect an intrinsic property of the nucleon? We show that we can answer both of these questions affirmatively by making full use of a gauge-invariant decomposition of the covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame.

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I. INTRODUCTION

The so-called ‘‘nucleon spin puzzle’’ is still one of the most fundamental problems in hadron physics [1,2]. In the past few years, there have been several remarkable progresses from the observational point of view. First, a lot of experimental evidence has been accumulated, which indicates that the gluon polarization inside the nucleon is likely to be small [3–6]. At the least, it seems now widely accepted that the $U_A(1)$-anomaly motivated explanation of the nucleon spin puzzle is disfavored. Second, the quark spin fraction or the net longitudinal quark polarization Δq has been fairly precisely determined through high-statistics measurements of the deuteron spin structure function by COMPASS [7,8] and the HEREMES group [9]. According to these analyses, the portion of the nucleon spin coming from the intrinsic quark spin is around $1/3$. These observations necessarily attract a great deal of interest in the role of orbital angular momenta of the quark and gluon field inside the nucleon.

When one talks about the spin contents of the nucleon, however, one cannot be unconcerned with the unsettled theoretical issues concerning the decomposition of the nucleon spin. An especially difficult problem here is the decomposition of the gluon total angular-momentum into its intrinsic spin and orbital parts. Most people believe that the polarized gluon distribution function is an observable quantity from high-energy deep-inelastic-scattering (DIS) measurements [10,11]. On the other hand, it is often claimed that there is no gauge-invariant decomposition of the gluon total angular momentum into its spin and orbital parts [12,13]. Undoubtedly, this latter statement is closely connected with another observation that there is no gauge-invariant local operator corresponding to the 1st moment of the polarized gluon distribution in the standard framework of operator-product expansion. Since the gauge principle is one of the most important principle of physics, which demands that only gauge-invariant quantities are measurable, how to reconcile these two conflicting observations is a fundamentally important problem in the physics of nucleon spin.

As the first step of the program, which aims at clearing up the state of confusion, we have recently investigated the relationship between the known decompositions of the nucleon spin [14]. We showed that the gauge-invariant decomposition advocated by Chen *et al.* [15,16] can be viewed as a nontrivial extension of the gauge-variant decomposition given by Jaffe and Manohar [12], so as to meet the gauge-invariance requirement of each term of the decomposition. However, we have also pointed out that there is another gauge-invariant decomposition of the nucleon spin, which is closer to the Ji decomposition, while allowing the decomposition of the gluon total angular momentum into the spin and orbital parts. After clarifying the reason why the gauge-invariant decomposition of the nucleon spin is not unique, we emphasized the possible superiority of our decomposition to that of Chen *et al.* on the ground of observability. To be more concrete, we developed an argument in favor of Ji's proposal to obtain a full decomposition of the nucleon spin [17]. It supports the widely accepted experimental project, in which one first determines the total angular momentum of quarks and gluons through generalized-parton-distribution (GPD) analyses and then extracts the orbital angular-momentum contributions of quarks and gluons by subtracting the intrinsic spin parts of quarks and gluons, which can be determined through polarized DIS measurements. Unfortunately, our argument lacks a finishing touch in the respect that we did not give a rigorous proof that the quark and gluon intrinsic spin contributions in our gauge-invariant decomposition in fact coincides with the quark and gluon polarizations extracted from the polarized DIS analyses. Another question, which is not unrelated to the above problem, is as follows. Since our gauge-invariant

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decomposition as well as that of Chen *et al.* are given in a specific Lorentz frame, we could not give a definite answer to the question whether these decompositions have a frame-independent meaning or not. The purpose of the present paper is to solve these remaining problems. We will show that these questions can be solved simultaneously, by making full use of a gauge-invariant decomposition of covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame.

The plan of the paper is as follows. In Sec. II, we show that we can make a gauge-invariant decomposition of the covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame, even without fixing gauge explicitly. Next, in Sec. III, the nucleon forward matrix element of the Pauli-Lubansky vector expressed in terms of the covariant angular-momentum tensor and the nucleon momentum is utilized to obtain a gauge- and frame-independent decomposition of the nucleon spin. In Sec. IV, we clarify the relation between our decomposition and the high-energy DIS observables. The summary and conclusion of our analyses are then given in Sec. V.

II. GAUGE-INVARIANT DECOMPOSITION OF COVARIANT ANGULAR-MOMENTUM TENSOR OF QCD

Following Jaffe and Manohar [12], we start with a Belinfante symmetrized expression for the QCD energy-momentum tensor given by

$$T^{\mu\nu} = T_q^{\mu\nu} + T_g^{\mu\nu}, \tag{1}$$

where

$$T_q^{\mu\nu} = \tfrac{1}{2}\bar{\psi}(\gamma^\mu iD^\nu + \gamma^\nu iD^\mu)\psi, \tag{2}$$

$$T_g^{\mu\nu} = 2\,\mathrm{Tr}(F^{\mu\alpha}F_\alpha{}^\nu - \tfrac{1}{4}g^{\mu\nu}F^2). \tag{3}$$

Here, $T^{\mu\nu}$ is conserved, $\partial_\mu T^{\mu\nu} = 0$, symmetric, $T^{\mu\nu} = T^{\nu\mu}$, and gauge invariant. The QCD angular momentum tensor $M^{\mu\nu\lambda}$ is a rank-3 tensor constructed from $T^{\mu\nu}$ as

$$M^{\mu\nu\lambda} \equiv x^\nu T^{\mu\lambda} - x^\lambda T^{\mu\nu}. \tag{4}$$

$M^{\mu\nu\lambda}$ is conserved, $\partial_\mu M^{\mu\nu\lambda} = 0$, and gauge-invariant, if $T^{\mu\nu}$ is symmetric and conserved. Another noteworthy property of $M^{\mu\nu\lambda}$, which was emphasized by Jaffe and Manohar, is that it has no totally antisymmetric part, which means that it satisfies the identity

$$\epsilon_{\alpha\mu\nu\lambda}M^{\mu\nu\lambda} = 0, \tag{5}$$

or equivalently

$$M^{\mu\nu\lambda} + M^{\lambda\mu\nu} + M^{\nu\lambda\mu} = 0. \tag{6}$$

As shown in [12], by using the identity

$$\begin{aligned}&\bar{\psi}(x^\nu\gamma^\lambda - x^\lambda\gamma^\nu)iD^\mu\psi - \bar{\psi}\gamma^\mu(x^\nu iD^\lambda - x^\lambda iD^\nu)\psi \\ &= \epsilon^{\mu\nu\lambda\beta}\bar{\psi}\gamma_\beta\gamma_5\psi - \tfrac{1}{2}\partial_\alpha[(x^\nu\epsilon^{\mu\lambda\alpha\beta} - x^\lambda\epsilon^{\mu\nu\alpha\beta}) \\ &\quad\times\bar{\psi}\gamma_\beta\gamma_5\psi],\end{aligned} \tag{7}$$

the quark part of $M^{\mu\nu\lambda}$ can gauge invariantly be decomposed in the following way:

$$M_q^{\mu\nu\lambda} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\beta}\bar{\psi}\gamma_\beta\gamma_5\psi + \bar{\psi}\gamma^\mu(x^\nu iD^\lambda - x^\lambda iD^\nu)\psi, \tag{8}$$

up to a surface term. In remarkable contrast, it is a widespread belief that the gluon part of $M^{\mu\nu\lambda}$ cannot be gauge-invariantly decomposed into the intrinsic spin and orbital angular-momentum contributions [12,13]. The gauge-invariant version of the decomposition of $M^{\mu\nu\lambda}$ given in the paper by Jaffe and Manohar is therefore given as

$$M^{\mu\nu\lambda} = M_q^{\mu\nu\lambda} + M_g^{\mu\nu\lambda} + \text{total divergence}, \tag{9}$$

with

$$M_q^{\mu\nu\lambda} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\beta}\bar{\psi}\gamma_\beta\gamma_5\psi + \bar{\psi}\gamma^\mu(x^\nu iD^\lambda - x^\lambda iD^\nu)\psi, \tag{10}$$

$$\begin{aligned}M_g^{\mu\nu\lambda} &= 2\,\mathrm{Tr}[x^\nu F^{\mu\alpha}F_\alpha{}^\lambda - x^\lambda F^{\mu\alpha}F_\alpha{}^\nu] \\ &\quad - \tfrac{1}{2}\mathrm{Tr}F^2[x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}].\end{aligned} \tag{11}$$

Note that this is essentially the covariant version of the Ji decomposition [13]. It should also be noted that the 2nd term of $M_g^{\mu\nu\lambda}$ contributes only to Lorentz boosts, so that it has nothing to do with nucleon spin decomposition.

Somewhat surprisingly, however, basically by following the idea proposal by Chen *et al.* [15,16], we can make a gauge-invariant decomposition of $M_g^{\mu\nu\lambda}$, at least formally. The idea is to decompose the gluon field into two parts as

$$A^\mu = A^\mu_{\text{phys}} + A^\mu_{\text{pure}}, \tag{12}$$

with A^μ_{pure} a pure-gauge term transforming in the same way as the full A^μ does, and always giving null field strength, and A^μ_{phys} a physical part of A^μ transforming in the same manner as $F^{\mu\nu}$ does, i.e., covariantly. That is, the two important properties of this decomposition is the condition for the pure-gauge part of the field,

$$F^{\mu\nu}_{\text{pure}} \equiv \partial^\mu A^\nu_{\text{pure}} - \partial^\nu A^\mu_{\text{pure}} - ig[A^\mu_{\text{pure}}, A^\nu_{\text{pure}}] = 0, \tag{13}$$

and the gauge transformation properties of the two parts:

$$A^\lambda_{\text{phys}}(x) \to U(x)A^\lambda_{\text{phys}}(x)U^{-1}(x), \tag{14}$$

$$A^\lambda_{\text{pure}}(x) \to U(x)\bigg(A^\lambda_{\text{pure}}(x) + \frac{i}{g}\partial^\lambda\bigg)U^{-1}(x). \tag{15}$$

As a matter of course, these conditions are not enough to uniquely fix gauge. To uniquely fix gauge, Chen *et al.* proposed to impose some additional gauge-fixing condition, which is a generalization of the Coulomb gauge condition in the case of QED. (The detail of the gauge-fixing problem is discussed also in recent research [18,19].) Alternatively, one can take the light-cone gauge with some appropriate boundary condition for the gauge field. In either case, these extra gauge-fixing procedures necessarily break the Lorentz symmetry. Fortunately, we find it possible to accomplish a gauge-invariant decomposition of a covariant rank-3 tensor $M^{\mu\nu\lambda}$ based on the above conditions (13)–(15) only, while postponing a concrete gauge-fixing procedure until the later stage. The usefulness of such covariant formulation should become apparent if one tries to compare the relation between the nucleon spin decomposition in different gauges and in different Lorentz frames.

Now, we explain the derivation of a gauge-invariant decomposition of $M^{\mu\nu\lambda}$ in some detail, since this decomposition plays a central role in our following discussion. First, by using the identity

$$F^{\alpha\lambda} \equiv \partial^\alpha A^\lambda - \partial^\lambda A^\alpha - ig[A^\alpha, A^\lambda] = D^\alpha A^\lambda - \partial^\lambda A^\alpha, \tag{16}$$

with $D^\alpha \equiv \partial^\alpha - ig[A^\alpha, \cdot]$ being the covariant derivative for the adjoint representation of color $SU(3)$, one can easily prove the identity

$$\begin{aligned} &x^\nu F^{\mu\alpha} F_\alpha{}^\lambda - x^\lambda F^{\mu\alpha} F_\alpha{}^\nu \\ &\quad = F^{\mu\alpha}(x^\nu D_\alpha A^\lambda - x^\lambda D_\alpha A^\nu) - F^{\mu\alpha}(x^\nu \partial^\lambda - x^\lambda \partial^\nu) A_\alpha. \end{aligned} \tag{17}$$

This gives

$$\begin{aligned} &x^\nu F^{\mu\alpha} F_\alpha{}^\lambda - x^\lambda F^{\mu\alpha} F_\alpha{}^\nu \\ &= F^{\mu\alpha}(x^\nu D_\alpha A^\lambda_{\text{phys}} - x^\lambda D_\alpha A^\nu_{\text{phys}}) - F^{\mu\alpha}(x^\nu \partial^\lambda \\ &\quad - x^\lambda \partial^\nu) A^{\text{phys}}_\alpha + F^{\mu\alpha}(x^\nu D_\alpha A^\lambda_{\text{pure}} - x^\lambda D_\alpha A^\nu_{\text{pure}}) \\ &\quad - F^{\mu\alpha}(x^\nu \partial^\lambda - x^\lambda \partial^\nu) A^{\text{pure}}_\alpha. \end{aligned} \tag{18}$$

The sum of the 3rd and 4th terms can be transformed in the following way:

$$\begin{aligned} &F^{\mu\alpha}[(x^\nu D_\alpha A^\lambda_{\text{pure}} - x^\lambda D_\alpha A^\nu_{\text{pure}}) - (x^\nu \partial^\lambda - x^\lambda \partial^\nu) A^{\text{pure}}_\alpha] \\ &= F^{\mu\alpha}[x^\nu (D_\alpha A^\lambda_{\text{pure}} - \partial^\lambda A^{\text{pure}}_\alpha) \\ &\quad - x^\lambda (D_\alpha A^\nu_{\text{pure}} - \partial^\nu A^{\text{pure}}_\alpha)] \\ &= F^{\mu\alpha}\{x^\nu (\partial_\alpha A^\lambda_{\text{pure}} - \partial^\lambda A^{\text{pure}}_\alpha - ig[A^{\text{pure}}_\alpha, A^\lambda_{\text{pure}}] \\ &\quad - ig[A^{\text{phys}}_\alpha, A^\lambda_{\text{pure}}]) - x^\lambda (\partial_\alpha A^\nu_{\text{pure}} - \partial^\nu A^{\text{pure}}_\alpha \\ &\quad - ig[A^{\text{pure}}_\alpha, A^\nu_{\text{pure}}] - ig[A^{\text{phys}}_\alpha, A^\nu_{\text{pure}}])\} \\ &= -igF^{\mu\alpha}(x^\nu [A^{\text{phys}}_\alpha, A^\lambda_{\text{pure}}] - x^\lambda [A^{\text{phys}}_\alpha, A^\nu_{\text{pure}}]). \end{aligned} \tag{19}$$

Here, we have used the pure-gauge condition (13) for the pure-gauge part of A^μ. Adding up the 2nd term of (18) to the above sum, we obtain

$$\begin{aligned} &- F^{\mu\alpha}(x^\nu \partial^\lambda - x^\lambda \partial^\nu) A^{\text{phys}}_\alpha - igF^{\mu\alpha}(x^\nu [A^{\text{phys}}_\alpha, A^\lambda_{\text{pure}}] \\ &\quad - x^\lambda [A^{\text{phys}}_\alpha, A^\nu_{\text{pure}}]) = -F^{\mu\alpha}\{x^\nu (\partial^\lambda - ig[A^\lambda_{\text{pure}}, A^{\text{phys}}_\alpha]) \\ &\quad - x^\lambda (\partial^\nu - ig[A^\nu_{\text{pure}}, A^{\text{phys}}_\alpha])\} \\ &= F^{\mu\alpha}(x^\nu D^\lambda_{\text{pure}} A^{\text{phys}}_\alpha - x^\lambda D^\nu_{\text{pure}} A^{\text{phys}}_\alpha). \end{aligned} \tag{20}$$

Here, we have introduced the *pure-gauge covariant derivative* by

$$D^\lambda_{\text{pure}} \equiv \partial^\lambda - ig[A^\lambda_{\text{pure}}, \cdot]. \tag{21}$$

As a consequence of the manipulation above, we obtain a fairly simple relation:

$$\begin{aligned} &x^\nu F^{\mu\alpha} F_\alpha{}^\lambda - x^\lambda F^{\mu\alpha} F_\alpha{}^\nu \\ &\quad = F^{\mu\alpha}(x^\nu D_\alpha A^\lambda_{\text{phys}} - x^\lambda D_\alpha A^\nu_{\text{phys}}) \\ &\qquad - F^{\mu\alpha}(x^\nu D^\lambda_{\text{pure}} A^{\text{phys}}_\alpha - x^\lambda D^\nu_{\text{pure}} A^{\text{phys}}_\alpha). \end{aligned} \tag{22}$$

Now, making use of the relation $D_\alpha F^{\alpha\mu} = \partial_\alpha F^{\alpha\mu} - ig[A_\alpha, F^{\alpha\mu}]$, it is straightforward to prove the identity:

$$\begin{aligned} &\partial_\alpha \text{Tr}(F^{\alpha\mu} x^\nu A^\lambda - F^{\alpha\mu} x^\lambda A^\nu) \\ &= \text{Tr}\{(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda - x^\lambda A^\nu) - F^{\mu\alpha}(x^\nu D_\alpha A^\lambda - x^\lambda D_\alpha A^\nu) \\ &\quad + F^{\mu\lambda} A^\nu - F^{\mu\nu} A^\lambda\}. \end{aligned} \tag{23}$$

It is also obvious from the above derivation that a similar identity holds even though we replace the fields A^λ and A^ν above by their physical parts, i.e., A^λ_{phys} and A^ν_{phys}:

$$\begin{aligned} &\partial_\alpha \text{Tr}(F^{\alpha\mu} x^\nu A^\lambda_{\text{phys}} - F^{\alpha\mu} x^\lambda A^\nu_{\text{phys}}) \\ &\quad = \text{Tr}\{(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{\text{phys}} - x^\lambda A^\nu_{\text{phys}}) \\ &\qquad - F^{\mu\alpha}(x^\nu D_\alpha A^\lambda_{\text{phys}} - x^\lambda D_\alpha A^\nu_{\text{phys}}) \\ &\qquad + F^{\mu\lambda} A^\nu_{\text{phys}} - F^{\mu\nu} A^\lambda_{\text{phys}}\}. \end{aligned} \tag{24}$$

Combining (22) and (24), we thus find the relation

$$\begin{aligned} &\text{Tr}(x^\nu F^{\mu\alpha} F_\alpha{}^\lambda - x^\lambda F^{\mu\alpha} F_\alpha{}^\nu) \\ &\quad + \partial_\alpha \text{Tr}(F^{\alpha\mu} x^\nu A^\lambda_{\text{phys}} - F^{\alpha\mu} x^\lambda A^\nu_{\text{phys}}) \\ &= \text{Tr}\{(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{\text{phys}} - x^\lambda A^\nu_{\text{phys}}) - F^{\mu\alpha}(x^\nu D^\lambda_{\text{pure}} \\ &\quad - x^\lambda D^\nu_{\text{pure}}) A^{\text{phys}}_\alpha + F^{\mu\lambda} A^\nu_{\text{phys}} - F^{\mu\nu} A^\nu_{\text{phys}}\}. \end{aligned} \tag{25}$$

After all these steps, we eventually arrive at the following decomposition for the QCD angular-momentum tensor [we call it decomposition (I)]:

$$\begin{aligned} M^{\mu\nu\lambda} &= M^{\mu\nu\lambda}_{q-\text{spin}} + M^{\mu\nu\lambda}_{q-\text{OAM}} + M^{\mu\nu\lambda}_{g-\text{spin}} + M^{\mu\nu\lambda}_{g-\text{OAM}} \\ &\qquad + M^{\mu\nu\lambda}_{\text{boost}} + \text{total divergence}, \end{aligned} \tag{26}$$

where

$$M^{\mu\nu\lambda}_{q-\text{spin}} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi, \tag{27}$$

$$M^{\mu\nu\lambda}_{q-\text{OAM}} = \bar{\psi} \gamma^\mu (x^\nu i D^\lambda - x^\lambda i D^\nu) \psi, \tag{28}$$

$$M^{\mu\nu\lambda}_{g-\rm spin} = 2\,{\rm Tr}[F^{\mu\lambda}A^{\nu}_{\rm phys} - F^{\mu\nu}A^{\lambda}_{\rm phys}], \tag{29}$$

$$M^{\mu\nu\lambda}_{g-\rm OAM} = -2\,{\rm Tr}[F^{\mu\alpha}(x^\nu D^\lambda_{\rm pure} - x^\lambda D^\nu_{\rm pure})A^{\rm phys}_\alpha] + 2\,{\rm Tr}[(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{\rm phys} - x^\lambda A^\nu_{\rm phys})], \tag{30}$$

$$M^{\mu\nu\lambda}_{\rm boost} = -\tfrac{1}{2}{\rm Tr}F^2(x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}). \tag{31}$$

In the above decomposition, $M^{\mu\nu\lambda}_{q-\rm spin}$ and $M^{\mu\nu\lambda}_{q-\rm OAM}$, respectively, correspond to the spin and orbital angular-momentum (OAM) parts of quarks, while $M^{\mu\nu\lambda}_{g-\rm spin}$ and $M^{\mu\nu\lambda}_{g-\rm OAM}$ to the spin and orbital angular-momentum parts of gluons. (At the quantum level, there is some delicacy in the identification of the term $M^{\mu\nu\lambda}_{q-\rm spin}$ with the intrinsic quark spin part. This will be discussed in the next section.) We have already pointed out that the term $M^{\mu\nu\lambda}_{\rm boost}$ contributes only to the Lorentz boots. An important feature of the above decomposition (26) of $M^{\mu\nu\lambda}$ is that each piece is separately gauge invariant. Since this is already obvious for the quark part, let us confirm it below for the less trivial gluon part.

The gauge invariance of the $M^{\mu\nu\lambda}_{g-\rm spin}$ and the 2nd term of $M^{\mu\nu\lambda}_{g-\rm OAM}$ can easily be convinced if one remembers the covariant transformation property (14) of the physical part of A^μ as well as the covariant transformation property of the field strength tensor $F^{\mu\nu}$. Less trivial is the 1st term of the gluon orbital part $M^{\mu\nu\lambda}_{g-\rm OAM}$. We first notice that, under a gauge transformation, $D^\lambda_{\rm pure}A^{\rm phys}_\alpha$ transform as

$$\begin{aligned} D^\lambda_{\rm pure}A^{\rm phys}_\alpha &\equiv \partial^\lambda A^{\rm phys}_\alpha + ig[A^\lambda_{\rm pure}, A^{\rm phys}_\alpha] \to \partial^\lambda(UA^{\rm phys}_\alpha U^{-1}) \\ &\quad - ig\Big[U\Big(A^\lambda_{\rm pure} + \frac{i}{g}\partial^\lambda\Big)U^{-1}, UA^{\rm phys}_\alpha U^{-1}\Big] \\ &= U(\partial^\lambda A^{\rm phys}_\alpha - ig[A^\lambda_{\rm pure}, A^{\rm phys}_\alpha])U^{-1} \\ &= UD^\lambda_{\rm pure}A^{\rm phys}_\alpha U^{-1}. \end{aligned} \tag{32}$$

This means that $D^\lambda_{\rm pure}A^{\rm phys}_\alpha$ transforms covariantly under a gauge transformation. The gauge invariance of the 1st term of $M^{\mu\nu\lambda}_{g-\rm OAM}$ should be almost obvious from this fact. Altogether, this confirms the fact that each term of decomposition (I) is in fact separately gauge invariant.

Note that the gluon orbital angular-momentum contribution $M^{\mu\nu\lambda}_{g-\rm OAM}$ consists of two terms. Using the QCD equation of motion

$$(D^\mu F_{\mu\nu})^a = -g\bar{\psi}\gamma_\nu T^a\psi, \tag{33}$$

the 1st term of $M^{\mu\nu\lambda}_{g-\rm OAM}$ can also be expressed in the form,

$$2\,{\rm Tr}\{(D_\alpha F^{\alpha\mu})(x^\nu A^\lambda_{\rm phys} - x^\lambda A^\nu_{\rm phys})\} = -g\bar{\psi}\gamma^\mu(x^\nu A^\lambda_{\rm phys} - x^\lambda A^\nu_{\rm phys})\psi. \tag{34}$$

Undoubtedly, this term is a covariant generalization of the "potential angular-momentum" *a la* Konopinski [20] as pointed out in our previous paper [14]. Since this term is *solely* gauge invariant, one has a freedom to combine it with another gauge-invariant term, for example, with the quark orbital angular-momentum term of decomposition (I). This leads to another gauge-invariant decomposition of $M^{\mu\nu\lambda}$ given as [this will be called decomposition (II)]

$$M'^{\mu\nu\lambda} = M'^{\mu\nu\lambda}_{q-\rm spin} + M'^{\mu\nu\lambda}_{q-\rm OAM} + M'^{\mu\nu\lambda}_{g-\rm spin} + M'^{\mu\nu\lambda}_{g-\rm OAM} + M'^{\mu\nu\lambda}_{\rm boost} + \text{total divergence}, \tag{35}$$

where

$$M'^{\mu\nu\lambda}_{q-\rm spin} = \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi, \tag{36}$$

$$M'^{\mu\nu\lambda}_{q-\rm OAM} = \bar{\psi}\gamma^\mu(x^\nu iD^\lambda_{\rm pure} - x^\lambda iD^\nu_{\rm pure})\psi, \tag{37}$$

$$M'^{\mu\nu\lambda}_{g-\rm spin} = 2\,{\rm Tr}[F^{\mu\lambda}A^\nu_{\rm phys} - F^{\mu\nu}A^\lambda_{\rm phys}], \tag{38}$$

$$M'^{\mu\nu\lambda}_{g-\rm OAM} = -2\,{\rm Tr}[F^{\mu\alpha}(x^\nu D^\lambda_{\rm pure} - x^\lambda D^\nu_{\rm pure})A^{\rm phys}_\alpha], \tag{39}$$

$$M'^{\mu\nu\lambda}_{\rm boost} = -\tfrac{1}{2}{\rm Tr}F^2(x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}). \tag{40}$$

Noteworthy here is the fact that the intrinsic spin parts are just the same in decompositions (I) and (II) for both of quarks and gluons, i.e.,

$$M'^{\mu\nu\lambda}_{q-\rm spin} = M^{\mu\nu\lambda}_{q-\rm spin}, \tag{41}$$

$$M'^{\mu\nu\lambda}_{g-\rm spin} = M^{\mu\nu\lambda}_{g-\rm spin}, \tag{42}$$

whereas the orbital parts are critically different for both of quarks and gluons, i.e.,

$$M'^{\mu\nu\lambda}_{q-\rm OAM} \neq M^{\mu\nu\lambda}_{q-\rm OAM}, \tag{43}$$

$$M'^{\mu\nu\lambda}_{g-\rm OAM} \neq M^{\mu\nu\lambda}_{g-\rm OAM}, \tag{44}$$

although it holds that the sum of the quark and gluon orbital angular momenta precisely coincides in the two decompositions, i.e.,

$$M'^{\mu\nu\lambda}_{q-\rm OAM} + M'^{\mu\nu\lambda}_{g-\rm OAM} = M^{\mu\nu\lambda}_{q-\rm OAM} + M^{\mu\nu\lambda}_{g-\rm OAM}. \tag{45}$$

One might think that decomposition (II) can be thought of as a covariant generalization of the gauge-invariant decomposition of Chen *et al.* [15,16]. Actually, the gauge is not definitely fixed yet in our treatment. We still have complete freedom to choose any desired gauge compatible with the decomposition of the gluon field into its physical and pure-gauge parts. By choosing a "generalized Coulomb gauge" advocated by Chen *et al.* in a suitable Lorentz frame, the above decomposition would in fact reduce to that of Chen *et al.* On the other hand, if one takes the light-cone gauge with some residual gauge degrees of freedom, decomposition (II) reproduces the gauge-invariant decomposition of the nucleon spin proposed by Bashinsky and

Jaffe [21], which was proposed on the basis of the light-cone-gauge formulation of parton distribution functions. (For confirmation of this statement above, see the discussion in Sec. IV.) On the other hand, we already know that the Chen decomposition reduces to the Jaffe-Manohar decomposition after a particular gauge fixing. Then, the above argument altogether indicates that the known three decompositions, i.e., those of Jaffe and Manohar, of Bashinsky and Jaffe, and of Chen *et al.* are all contained in our decomposition (II) so that they are gauge equivalent. In other words, they are the same decomposition from the physical viewpoint.

We have pointed out that, in decompositions (I) and (II) of the angular-momentum tensor, the difference exists only in the orbital parts. Here, let us look into a simpler quark part more closely. What appears in our decomposition (I) is a covariant generalization of the so-called "dynamical" or "mechanical" orbital angular momentum of quarks. On the other hand, what appears in decomposition (II) is a nontrivial gauge-invariant extension of the "canonical" orbital angular momentum. This difference is of crucial physical significance, since, as emphasized in our previous paper [14], the dynamical orbital angular momentum is a measurable quantity, whereas the canonical one is not. In fact, the common knowledge of standard electrodynamics tells us that the momentum appearing in the equation of motion with the Lorentz force is the so-called dynamical momentum $\boldsymbol{\Pi} = \boldsymbol{p} - q\boldsymbol{A}$ with the full gauge field, not the canonical momentum $\boldsymbol{p}$ or its nontrivial extension $\boldsymbol{p} - q\boldsymbol{A}_{\rm pure}$. To convince it, let us consider the motion of a charged particle with mass m and a charge e ($e < 0$ for the electron) under the influence of static electric and magnetic field given as [22]

$$\boldsymbol{E} = -\nabla\phi, \qquad \boldsymbol{B} = \nabla \times \boldsymbol{A}. \tag{46}$$

The Hamiltonian, which describes the motion of the charged particle, is given by

$$H = \frac{\boldsymbol{\Pi}^2}{2m} + e\phi, \tag{47}$$

with

$$\boldsymbol{\Pi} \equiv \boldsymbol{p} - e\boldsymbol{A}. \tag{48}$$

The equation of motion for this charged particle becomes

$$m\frac{d^2\boldsymbol{x}}{dt^2} = \frac{d\boldsymbol{\Pi}}{dt} = e\left[\boldsymbol{E} + \frac{1}{2}\left(\frac{d\boldsymbol{x}}{dt} \times \boldsymbol{B} - \boldsymbol{B} \times \frac{d\boldsymbol{x}}{dt}\right)\right]. \tag{49}$$

This equation of motion dictates that the momentum accompanying the mass flow of a charged particle is the dynamical momentum $\boldsymbol{\Pi} = \boldsymbol{p} - e\boldsymbol{A}$ containing the full gauge field $\boldsymbol{A}$, not the canonical momentum $\boldsymbol{p}$ or its nontrivial extension $\boldsymbol{p} - e\boldsymbol{A}_{\rm pure}$. Similarly, the angular momentum accompanying the mass flow of a charge particle is the dynamical orbital angular momentum $\boldsymbol{x} \times \boldsymbol{\Pi} = \boldsymbol{x} \times (\boldsymbol{p} - e\boldsymbol{A})$, not $\boldsymbol{x} \times \boldsymbol{p}$ or $\boldsymbol{x} \times (\boldsymbol{p} - e\boldsymbol{A}_{\rm pure})$.

In the subsequent sections, we try to make the above statement on the observability of our decomposition more concrete first for the quark part. The analysis is then extended to the gluon part to accomplish a complete decomposition of the nucleon spin.

III. FRAME INDEPENDENCE OF OUR NUCLEON SPIN DECOMPOSITION

Our discussion in this section is based on our recommendable decomposition (I) of the QCD angular-momentum tensor $M^{\mu\nu\lambda}$ given in (26)–(31). The nucleon spin sum rule is obtained by evaluating the forward matrix element of the tensor M^{012} in the equal-time quantization, or that of the tensor M^{+12} in the light-cone quantization. This gives the normalization condition

$$\langle P, s| M^{012} |P, s\rangle / \langle P, s|P, s\rangle = \tfrac{1}{2}, \tag{50}$$

in the equal-time quantization, or

$$\langle P, s| M^{+12} |P, s\rangle / \langle P, s|P, s\rangle = \tfrac{1}{2}, \tag{51}$$

in the light-cone quantization. Here, $|P, s\rangle$ stands for a plane-wave nucleon state with momentum P_μ and spin s_μ. An alternative method to obtain the nucleon spin sum rule is to evaluate the forward matrix element of the helicity operator [23]

$$W^\mu s_\mu = \boldsymbol{J} \cdot \hat{\boldsymbol{P}} = \frac{\boldsymbol{J} \cdot \boldsymbol{P}}{|\boldsymbol{P}|}, \tag{52}$$

where

$$W^\mu = -\frac{1}{2\sqrt{P^2}}\epsilon^{\mu\alpha\beta\gamma} J_{\alpha\beta} P_\gamma, \tag{53}$$

with $J^{\alpha\beta} = M^{0\alpha\beta}$, is the Pauli-Lubansky vector [24], while P_μ and s_μ are the momentum and the spin vector of the nucleon satisfying the relations :

$$P^2 = M^2, \qquad s^2 = -1, \qquad P \cdot s = 0. \tag{54}$$

The normalization condition in this case is

$$\langle P, s| W^\mu s_\mu |P, s\rangle / \langle P, s|P, s\rangle = \tfrac{1}{2}. \tag{55}$$

In either case, for the spin decomposition of the nucleon, we need to know the forward matrix element of each term of the right-hand side of (26). We first consider the forward matrix element of $M^{\mu\nu\lambda}_{q-\rm spin}$. Although we have naively called this term the intrinsic quark spin contribution to $M^{\mu\nu\lambda}$, there is some delicacy. As first recognized by Jaffe and Manohar [12], and later elaborated in [25,26], $\bar{\psi}\gamma_\sigma\gamma_5\psi = A^{(0)}_\sigma$ is the flavor-singlet axial current and it enters $M^{\mu\nu\lambda}$ in the form $\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}A^{(0)}_\sigma$. However, Jaffe and Manohar also noticed the fact that $M^{\mu\nu\lambda}$ should have no totally antisymmetric part. This observation, combined with the fact that the total derivative term has no forward matrix element, leads to the conclusion that the forward matrix element of $M^{\mu\nu\lambda}$ cannot have a term proportional to $\epsilon^{\mu\nu\lambda\sigma}$. This means that the term of this form coming from $M^{\mu\nu\lambda}_{q-\mathrm{spin}} = \frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}A^{(0)}_\sigma$ must exactly be canceled by a similar term coming from the "orbital piece" of $M^{\mu\nu\lambda}$. First, we shall verify this fact explicitly for the quark part of $M^{\mu\nu\lambda}$. Later, we will show that a similar situation occurs also for the gluon part. In general, the forward matrix element of $M^{\mu\nu\lambda}_{q-\mathrm{spin}}$ is specified by the flavor-singlet axial charge $a^{(0)}_q$ as

$$\langle P, s|M^{\mu\nu\lambda}_{q-\mathrm{spin}}(0)|P, s\rangle = M a^{(0)}_q \epsilon^{\mu\nu\lambda\sigma} s_\sigma. \tag{56}$$

It is a widely known fact that, at the quantum level, an ambiguity arises, due to the $U_A(1)$ anomaly of QCD, concerning the relation between the flavor-singlet axial charge and the net quark polarization Δq (or the net contribution of the intrinsic quark spin to the nucleon spin). In the most popular factorization (or renormalization) scheme, i.e., in the $\overline{\mathrm{MS}}$ scheme, $a^{(0)}_q$ can just be identified with Δq. On the other hand, there is another class of renormalization scheme called the Adler-Bardeen schemes, in which $a^{(0)}_q$ is given by $a^{(0)}_q = \Delta q - 2n_f(\alpha_s/4\pi)\Delta g$ with Δg the net gluon polarization, and n_f the number of quark flavors. An advantage of the Adler-Bardeen scheme is that Δq is completely scale independent. Nonetheless, there is no compelling reason to stick to this scheme. Without any loss of generality, we can choose the $\overline{\mathrm{MS}}$ scheme, in which the forward matrix element of $M^{\mu\nu\lambda}_{q-\mathrm{spin}}$ gives the net quark spin contribution to the nucleon spin through the previously-mentioned sum rule.

Next, we investigate the forward matrix element of the quark orbital angular-momentum part $M^{\mu\nu\lambda}_{q-\mathrm{OAM}}$. This part of the current takes a general form of

$$M^{\mu\nu\lambda}(x) = x^\nu O^{\mu\lambda}(x) - x^\lambda O^{\mu\nu}(x), \tag{57}$$

so that the evaluation of its forward matrix element needs some care. The method is well known and is given by the following limiting procedure [12]:

$$\begin{aligned}\langle P, s|M^{\mu\nu\lambda}(0)|P, s\rangle &= \lim_{\Delta\to 0} i\frac{\partial}{\partial\Delta_\nu}\left\langle P+\frac{\Delta}{2}, s\,\middle|\, O^{\mu\lambda}(0)\,\middle|\, P-\frac{\Delta}{2}, s\right\rangle \\ &\quad - (\nu \leftrightarrow \lambda).\end{aligned} \tag{58}$$

(More sound formulation of this limiting procedure with use of wave packets instead of plane waves was later elaborated in [25,26].) To make use of the above formula, we first note that $M^{\mu\nu\lambda}_{q-\mathrm{OAM}}$ can be expressed as

$$M^{\mu\nu\lambda}_{q-\mathrm{OAM}} = x^\nu O^{\mu\lambda}_2 - x^\lambda O^{\mu\nu}_2, \tag{59}$$

with

$$O^{\mu\nu}_2 = \bar\psi\gamma^\mu iD^\nu\psi. \tag{60}$$

It is important to recognize that this rank-2 tensor $O^{\mu\nu}_2$ entering $M^{\mu\nu\lambda}_{q-\mathrm{OAM}}$ is different from the quark part of the QCD energy-momentum tensor

$$T^{\mu\nu}_q = \tfrac{1}{2}\bar\psi\gamma^{\{\mu}iD^{\nu\}}\psi, \tag{61}$$

by the effect of symmetrization. (Here we use the notation $a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$ and $a^{[\mu}b^{\nu]} = a^\mu b^\nu - a^\nu b^\mu$.) Then, while the nonforward matrix element of $T^{\mu\nu}_q$ is characterized by three form factors as

$$\begin{aligned}&\left\langle P+\frac{\Delta}{2}, s\,\middle|\, T^{\mu\nu}_q(0)\,\middle|\, P-\frac{\Delta}{2}, s\right\rangle \\ &= A_q(\Delta^2)P^\mu P^\nu + \frac{B_q(\Delta^2)}{2M}P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma \\ &\quad + C_q(\Delta^2)M^2 g^{\mu\nu} + O(\Delta^2),\end{aligned} \tag{62}$$

the nonforward matrix element of $O^{\mu\nu}_2$ can contain extra terms which are antisymmetric in μ and ν as

$$\begin{aligned}&\left\langle P+\frac{\Delta}{2}, s\,\middle|\, O^{\mu\nu}_2(0)\,\middle|\, P-\frac{\Delta}{2}, s\right\rangle \\ &= A_q(\Delta^2)P^\mu P^\nu + \frac{B_q(\Delta^2)}{2M}P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma \\ &\quad + \frac{\tilde B_q(\Delta^2)}{2M}P^{[\mu}\epsilon^{\nu]\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma \\ &\quad + MD_q(\Delta^2)\epsilon^{\mu\nu\alpha\beta}s_\alpha i\Delta_\beta + C_q(\Delta^2)M^2 g^{\mu\nu} + O(\Delta^2).\end{aligned} \tag{63}$$

(The above parametrizations of the nucleon matrix elements of rank-2 tensors were criticized in the paper by Bakker, Leader, and Trueman [25]. They argue that, if $T^{\mu\nu}$ transforms as a second-rank tensor, its nonforward matrix elements do not transform covariantly. Only by first factoring out the wave functions, i.e., the Dirac spinors in the case of nucleon matrix elements, the relevant function sandwiched by the initial and final wave functions transform covariantly. Nevertheless, they themselves confirmed that, despite this problem of the parametrization of the nucleon nonforward matrix elements, the treatment of Jaffe and Manohar give just the correct answer at least for the longitudinal spin sum rule of the nucleon, which is of our current interest. For the sake of simplicity, we therefore follow the treatment of Jaffe and Manohar at the cost of complete stringency.)

Now, a key observation of our nucleon spin decomposition is as follows. As shown by Shore and White [26], the two rank-2 tesnsors $T^{\mu\nu}_q$ and $O^{\mu\nu}_2$ are not completely independent. They are related through the following identity:

$$\begin{aligned}x^\nu T^{\mu\lambda}_q - x^\lambda T^{\mu\nu}_q &= x^\nu O^{\mu\lambda}_2 - x^\lambda O^{\mu\nu}_2 + \tfrac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar\psi\gamma_\sigma\gamma_5\psi \\ &\quad + \text{total divergence}.\end{aligned} \tag{64}$$

By evaluating the forward matrix element of this identity, one can prove that all the form factors, appearing in (62) and (63), are not independent but obey the following relation:

$$\tilde{B}_q(0) = 0, \qquad 2D_q(0) = a_q^{(0)}. \tag{65}$$

As a consequence, we find that the forward matrix element of $M^{\mu\nu\lambda}_{q-\mathrm{OAM}}$ is given by

$$\langle P, s|M^{\mu\nu\lambda}_{q-\mathrm{OAM}}(0)|P, s\rangle = \frac{B_q(0)}{2M} P^{\{\mu}\epsilon^{\lambda\}\nu\alpha\beta}P_\alpha s_\beta - (\nu \leftrightarrow \lambda) - M a_q^{(0)}\epsilon^{\mu\nu\lambda\sigma}s_\sigma. \tag{66}$$

As emphasized in [12] and explicitly shown in [26], the axial-charge term, which is totally antisymmetric in the indices μ, ν, λ, cancels in the forward matrix elements of $M^{\mu\nu\lambda}_{q-\mathrm{spin}}$ plus $M^{\mu\nu\lambda}_{q-\mathrm{OAM}}$ to give

$$\langle P, s|M^{\mu\nu\lambda}_{q-\mathrm{spin}}(0) + M^{\mu\nu\lambda}_{q-\mathrm{OAM}}(0)|P, s\rangle = \frac{B_q(0)}{2M} P^{\{\mu}\epsilon^{\nu\}\alpha\beta}P_\alpha s_\beta - (\nu \leftrightarrow \lambda). \tag{67}$$

It can be shown that $B_q(0)$ just coincide with the total angular momentum J_q carried by the quark fields,

$$B_q(0) = J_q. \tag{68}$$

Now we turn to the discussion of much more difficult gluon part. Despite a lot of efforts, whether the total gluon angular momentum J_g can be gauge-invariantly decomposed into the spin and orbital parts is still a controversial problem. That it is possible at the formal level has been shown in a series of paper by Chen *et al.* [15,16] and has been confirmed in our recent paper [14]. However, these decompositions were achieved in a particular Lorentz frame. What we are looking for here is a Lorentz covariant formulation. An advantage of Lorentz covariant formulation is that we can make clear the relation between the nucleon spin decompositions obtained in different Lorentz frames. Furthermore, as we shall see shortly, it also turns out to reveal an important physics, which was masked in a noncovariant formulation. We first look into the forward matrix element of our gluon-spin operator

$$M^{\mu\nu\lambda}_{g-spin} = 2\,\mathrm{Tr}[F^{\mu\lambda}A^\nu_{\mathrm{phys}} - F^{\mu\nu}A^\lambda_{\mathrm{phys}}] = 2\,\mathrm{Tr}[F^{\mu\lambda}A^\nu_{\mathrm{phys}} + F^{\nu\mu}A^\lambda_{\mathrm{phys}}]. \tag{69}$$

We first emphasize that this operator is gauge invariant, so that it is delicately different from the gauge-variant current

$$M^{\mu\nu\lambda}_{(g)}(\mathrm{spin}) \equiv 2\,\mathrm{Tr}[F^{\mu\nu}A^\nu + F^{\nu\mu}A^\lambda], \tag{70}$$

which was naively identified with the gluon-spin operator in the paper by Jaffe and Manohar [12]. In the same paper, however, they pointed out a very interesting fact. According to them, the analogy with the quark part would have led us to expect $M^{\mu\nu\lambda}_{(g)}(\mathrm{spin})$ to be

$$\epsilon^{\mu\nu\lambda\sigma}K_\sigma = 2\,\mathrm{Tr}[F^{\nu\lambda}A^\mu + A^\nu F^{\lambda\mu} + A^\lambda F^{\mu\nu}] + 2ig\,\mathrm{Tr}A^\mu[A^\nu, A^\lambda], \tag{71}$$

which is totally antisymmetric in the three indices μ, ν, λ. Here

$$k_\mu \equiv \frac{\alpha_S}{2\pi}K_\mu = \frac{\alpha_S}{2\pi}\epsilon_{\mu\nu\alpha\beta}\,\mathrm{Tr}A^\nu\left[F^{\alpha\beta} - \frac{2}{3}A^\alpha A^\beta\right] \tag{72}$$

is the gauge-variant Chern-Simons current, whose divergence is related to the well-known topological charge density of QCD as

$$\partial^\mu k_\mu = \frac{\alpha_S}{2\pi}\,\mathrm{Tr}F^{\mu\nu}\tilde{F}_{\mu\nu}. \tag{73}$$

Owing to the symmetry difference, $\epsilon^{\mu\nu\lambda\sigma}K_\sigma$ and $K^{\mu\nu\lambda}_{(g)}(\mathrm{spin})$ are not in the same representation of the Lorentz group [12]. The former belongs to $(\frac{1}{2},\frac{1}{2})$, while the latter contains $(\frac{1}{2},\frac{3}{2}) \oplus (\frac{3}{2},\frac{1}{2})$ in addition to $(\frac{1}{2},\frac{1}{2})$. Historically, several authors advocated to use the forward matrix element of the topological current to define the gluon axial charge $a_g^{(0)}(0)$ or the gluon polarization Δg [27–29]. (See also reviews [30,31].) However, some authors soon recognized that the gauge-variant nature of the topological current k_μ prevents this attempt [26,32,33]. The argument goes as follows. The nonforward matrix element of the topological current k^μ is characterized by two form factors as

$$\left\langle P + \frac{\Delta}{2}, s \,\middle|\, k^\mu \,\middle|\, P - \frac{\Delta}{2}, s\right\rangle = 2Ms^\mu a_g^{(0)}(\Delta^2) + \Delta^\mu(\Delta\cdot s)p_g(\Delta^2) + O(\Delta^2). \tag{74}$$

Naively thinking, the 2nd term of the above equation would vanish in the forward limit $\Delta^\mu \to 0$, so that one might expect that

$$\langle P, s|k^\mu|P, s\rangle = 2Ms^\mu a_g^{(0)}(0) \tag{75}$$

with the identification $a_g^{(0)}(0) = \frac{\alpha_S}{4\pi}\Delta g$. However, it was soon recognized that the gauge-variant current k^μ couples to an unphysical Goldstone mode and the form factor $p_g(\Delta^2)$ has a massless pole [32,33]. The structure of this pole depends on the adopted gauge. It turns out that the forward matrix element of the topological current is singular in general gauges. Although the matrix element is finite in the generalized axial gauges, $n \cdot A = 0$, its value still depends on the ways of taking the forward limit $\Delta \to 0$ so that it is indefinite.

Now, we go back to our gauge-invariant operator $M^{\mu\nu\lambda}_{g-\mathrm{spin}}$. It is instructive to rewrite $M^{\mu\nu\lambda}_{g-\mathrm{spin}}$ in the form that contains the topological current in itself as

$$M^{\mu\nu\lambda}_{g-\text{spin}} = \epsilon^{\mu\nu\lambda\sigma} K_\sigma - 2\,\text{Tr}\{(F^{\lambda\nu} + ig[A^\lambda, A^\nu])A^\mu\} - 2\,\text{Tr}\{F^{\mu\lambda} A^\nu_{\text{pure}} + F^{\nu\mu} A^\lambda_{\text{pure}}\}. \quad (76)$$

One might think that this manipulation is a little artificial. Note, however, that it resembles the operation in the quark part, in which the totally antisymmetric part $\frac{1}{2}\epsilon^{\mu\nu\lambda\sigma}\bar{\psi}\gamma_\sigma\gamma_5\psi$ is separated from the total quark contribution $M^{\mu\nu\lambda}_q$. An important difference with the quark case is that each term of (76) is not separately gauge invariant. Nonetheless, the left-hand side of (76) is gauge invariant by construction, so that it is logically obvious that the gauge dependencies of the three terms in the right-hand side should exactly be canceled. The argument above then indicates that the nonforward matrix element of $M^{\mu\nu\lambda}_{g-\text{spin}}$ can be specified by gauge-independent three form factors as

$$\left\langle P + \frac{\Delta}{2}, s \,\middle|\, M^{\mu\nu\lambda}_{g-\text{spin}}(0) \,\middle|\, P - \frac{\Delta}{2}, s \right\rangle = 2M\left(\frac{\alpha_S}{4\pi}\right)^{-1} a^{(0)}_g(\Delta^2)\epsilon^{\mu\nu\lambda\sigma}s_\sigma + v_g(\Delta^2)\epsilon^{\mu\nu\lambda\sigma}\Delta_\sigma(\Delta\cdot s) + w_g(\Delta^2)\Delta^\mu(\Delta^\lambda s^\nu - \Delta^\nu s^\lambda) + O(\Delta^2). \quad (77)$$

Now, an important difference with the past argument is that, since $M^{\mu\nu\lambda}_{g-\text{spin}}$ is manifestly gauge invariant, there should be no massless pole in either of the form factors $v_g(\Delta^2)$ and $w_g(\Delta^2)$. This means that the terms containing $v_g(\Delta^2)$ and $w_g(\Delta^2)$ vanish in the forward limit and the forward matrix element of $M^{\mu\nu\lambda}_{g-\text{spin}}$ is unambiguously given by

$$\langle P, s|M^{\mu\nu\lambda}_{g-\text{spin}}(0)|P, s\rangle = 2M\left(\frac{\alpha_s}{4\pi}\right)^{-1} a^{(0)}_g(0)\epsilon^{\mu\nu\lambda\sigma}s_\sigma = 2M\Delta g\epsilon^{\mu\nu\lambda\sigma}s_\sigma. \quad (78)$$

In short, although our gluon-spin operator is not necessarily totally antisymmetric in the indices μ, ν, and λ, only the totally antisymmetric part survives in its forward matrix element. Although this seems somewhat mysterious, it certainly is a consequence of the logical reasoning explained above.

Our remaining task now is to evaluate the forward matrix element of $M^{\mu\nu\lambda}_{g-\text{OAM}}$. We first remember the fact that $M^{\mu\nu\lambda}_{g-\text{OAM}}$ can be expressed in the form

$$M^{\mu\nu\lambda}_{g-\text{OAM}} = x^\nu O^{\mu\lambda}_5 - x^\lambda O^{\mu\nu}_5, \quad (79)$$

with

$$O^{\mu\nu}_5 = -2\,\text{Tr}[F^{\mu\alpha} D^\nu_{\text{pure}} A^{\text{phys}}_\alpha] + 2\,\text{Tr}[(D_\alpha F^{\alpha\mu})A^\nu_{\text{phys}}]. \quad (80)$$

This should be compared with the net gluon contribution to $M^{\mu\nu\lambda}$, which can be expressed as

$$M^{\mu\nu\lambda}_g = x^\nu T^{\mu\lambda}_g - x^\lambda T^{\mu\nu}_g, \quad (81)$$

where $T^{\mu\nu}_g$ is the gluon contribution to the symmetric QCD energy-momentum tensor given by (3). There is a simple relation between $M^{\mu\nu\lambda}_g$ and $M^{\mu\nu\lambda}_{g-\text{OAM}}$, however. That is, as is clear from (25), aside from the boost term, $M^{\mu\nu\lambda}_g$ is different from the sum of $M^{\mu\nu\lambda}_{g-\text{spin}}$ and $M^{\mu\nu\lambda}_{g-\text{OAM}}$ only by a total divergence as

$$M^{\mu\nu\lambda}_g - \text{boost} = M^{\mu\nu\lambda}_{g-\text{spin}} + M^{\mu\nu\lambda}_{g-\text{OAM}} + \text{total divergence}. \quad (82)$$

Note that this is a *key relation* in our gauge-invariant decomposition of the gluon total angular momentum into its spin and orbital parts.

Now, we can proceed just in the same way as in the quark part. The nonforward matrix element of $T^{\mu\nu}_g(0)$ and $O^{\mu\nu}_5(0)$ are parametrized as

$$\left\langle P + \frac{\Delta}{2}, s \,\middle|\, T^{\mu\nu}_g(0) \,\middle|\, P - \frac{\Delta}{2}, s \right\rangle = A_g(\Delta^2)P^\mu P^\nu + \frac{B_g(\Delta^2)}{2M} P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma + C_g(\Delta^2)M^2 g^{\mu\nu} + O(\Delta^2), \quad (83)$$

and

$$\left\langle P + \frac{\Delta}{2}, s \,\middle|\, O^{\mu\nu}_5(0) \,\middle|\, P - \frac{\Delta}{2}, s \right\rangle = A_g(\Delta^2)P^\mu P^\nu + \frac{B_g(\Delta^2)}{2M} P^{\{\mu}\epsilon^{\nu\}\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma + \frac{\tilde{B}_g(\Delta^2)}{2M} P^{[\mu}\epsilon^{\nu]\alpha\beta\sigma}s_\alpha P_\beta i\Delta_\sigma + MD_g(\Delta^2)\epsilon^{\mu\nu\lambda\sigma}i\Delta_\lambda s_\sigma + C_g(\Delta^2)M^2 g^{\mu\nu} + O(\Delta^2). \quad (84)$$

By using the limiting procedure (58), we thus have in the forward limit

$$\langle P, s|M^{\mu\nu\lambda}_g(0)|P, s\rangle = \frac{B_g(0)}{2M} P^{\{\mu}\epsilon^{\lambda\}\nu\alpha\beta}s_\alpha P_\beta - (\nu \leftrightarrow \lambda), \quad (85)$$

and

$$\langle P, s|M^{\mu\nu\lambda}_{g-\text{OAM}}(0)|P, s\rangle = \frac{B_g(0)}{2M} P^{\{\mu}\epsilon^{\lambda\}\nu\alpha\beta}s_\alpha P_\beta - (\nu\leftrightarrow\lambda) + \frac{\tilde{B}_g(0)}{2M} P^{[\mu}\epsilon^{\lambda]\nu\alpha\beta}s_\alpha P_\beta - (\nu\leftrightarrow\lambda) - 2MD_g(0)\epsilon^{\mu\nu\lambda\sigma}s_\sigma, \quad (86)$$

while we recall that

$$\langle P, s|M^{\mu\nu\lambda}_{g-\text{spin}}(0)|P, s\rangle = 2M\epsilon^{\mu\nu\lambda\sigma}s_\sigma\Delta g. \quad (87)$$

Then, in consideration of the fact that the total divergence term does not contribute to the forward matrix element, the relation (82) together with (85)–(87), demands that

$$D_g(0) = \Delta g, \qquad \tilde{B}_g(0) = 0. \tag{88}$$

We are then led to the desired result

$$\langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{spin}}(0) | P, s\rangle = 2M\Delta g \epsilon^{\mu\nu\lambda\sigma} s_\sigma, \tag{89}$$

$$\langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{OAM}}(0) | P, s\rangle = \frac{B_g(0)}{2M} P^{\{\mu} \epsilon^{\lambda\}\nu\alpha\beta} s_\alpha P_\beta - (\nu \leftrightarrow \lambda) - 2M\Delta g \epsilon^{\mu\nu\lambda\sigma} s_\sigma, \tag{90}$$

which gives a gauge-invariant decomposition of J_g into the spin and orbital parts. Again, the totally antisymmetric terms in the indices μ, ν, λ cancel in the forward matrix element of the sum of $M^{\mu\nu\lambda}_{g-\mathrm{spin}}$ and $M^{\mu\nu\lambda}_{g-\mathrm{OAM}}$ to give

$$\langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{spin}}(0) + M^{\mu\nu\lambda}_{g-\mathrm{OAM}}(0) | P, s\rangle = \frac{B_g(0)}{2M} P^{\{\mu} \epsilon^{\nu\}\alpha\beta} P_\alpha s_\beta - (\nu \leftrightarrow \lambda). \tag{91}$$

Let us summarize at this point what we have found. We found that

$$\langle P, s | M^{\mu\nu\lambda} | P, s\rangle = \langle P, s | M^{\mu\nu\lambda}_{q-\mathrm{spin}} | P, s\rangle + \langle P, s | M^{\mu\nu\lambda}_{q-\mathrm{OAM}} | P, s\rangle + \langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{spin}} | P, s\rangle + \langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{OAM}} | P, s\rangle + \text{boost}. \tag{92}$$

with

$$\langle P, s | M^{\mu\nu\lambda}_{q-\mathrm{spin}}(0) | P, s\rangle = M\Delta q \epsilon^{\mu\nu\lambda\sigma} s_\sigma, \tag{93}$$

$$\langle P, s | M^{\mu\nu\lambda}_{q-\mathrm{OAM}}(0) | P, s\rangle = \tfrac{B_q(0)}{2M} P^{\{\mu} \epsilon^{\lambda\}\nu\alpha\beta} s_\alpha P_\beta - (\nu \leftrightarrow \lambda) - M\Delta q \epsilon^{\mu\nu\lambda\sigma} s_\sigma, \tag{94}$$

$$\langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{spin}}(0) | P, s\rangle = 2M\Delta g \epsilon^{\mu\nu\lambda\sigma} s_\sigma, \tag{95}$$

$$\langle P, s | M^{\mu\nu\lambda}_{g-\mathrm{OAM}}(0) | P, s\rangle = \frac{B_g(0)}{2M} P^{\{\mu} \epsilon^{\lambda\}\nu\alpha\beta} s_\alpha P_\beta - (\nu \leftrightarrow \lambda) - 2M\Delta g \epsilon^{\mu\nu\lambda\sigma} s_\sigma. \tag{96}$$

We emphasize again that this is a completely gauge-invariant decomposition. Inserting the above decomposition into the equation $\langle P, s | W^\mu s_\mu | P, s\rangle / \langle P, s | P, s\rangle = 1/2$ [23], one gets

$$\tfrac{1}{2} = S_q + L_q + S_g + L_g = J_q + J_g, \tag{97}$$

with

$$S_q = \tfrac{1}{2}\Delta q, \tag{98}$$

$$L_q = B_q(0) - \tfrac{1}{2}\Delta q, \tag{99}$$

$$S_g = \Delta g, \tag{100}$$

$$L_g = B_g(0) - \Delta g. \tag{101}$$

This means that the individual contributions to the spin of the nucleon is invariant under the wide class of a Lorentz transformation that preserves the helicity of the nucleon. In this sense, we are now able to say that our decomposition of the nucleon spin is not only gauge-invariant but also basically Lorentz frame independent. A remaining important question is therefore as follows. Can we give any convincing argument to show the observability of the above decomposition? A central task here is to verify whether the above gluon-spin term S_g can in fact be identified with the 1st moment of the polarized gluon distribution determined by high-energy polarized DIS analyses. We try to answer this question in the next section.

IV. OBSERVABILITY OF OUR NUCLEON SPIN DECOMPOSITION

It is a widely-known fact that the quark and gluon total angular momenta, i.e., J_q and J_g, can in principle be extracted from GPD analyses [13–17]. Let us first confirm that our decomposition is compatible with this common wisdom. Here, we closely follow the analysis by Shore and White [26]. We start with the standard definition of unpolarized GPDs for quark and gluons given as

$$f_q(x,\xi,t) = \int \frac{dz^-}{2\pi} e^{i(x+(\xi/2))P^+ z^-} \times \left\langle P + \frac{1}{2}\Delta \right| \bar{\psi}(0)\gamma^+ \mathcal{L}_g(0,z^-)\psi(z^-) \left| P - \frac{1}{2}\Delta \right\rangle,$$
$$xP^+ f_g(x,\xi,t) = \int \frac{dz^-}{2\pi} e^{i(x+(\xi/2))P^+ z^-} \times \left\langle P + \frac{1}{2}\Delta \right| 2\mathrm{Tr}[F^{+\alpha}(0)\mathcal{L}_g(0,z^-)F^+_\alpha(z^-) \left| P - \frac{1}{2}\Delta \right\rangle, \tag{102}$$

where $t = \Delta^2$, while $\mathcal{L}_g(a,b) = Pe^{-ig\int_b^a A\cdot ds}$ is the standard gauge link. It is an easy exercise to derive the following 2nd moment sum rules for $f_q(x,\xi,t)$ and $f_g(x,\xi,t)$:

$$\int_{-1}^{1} x f_q(x,\xi,t) dx = \left\langle P + \frac{\Delta}{2} \right| \bar{\psi}(0)\gamma^+ D^+ \psi(0) \left| \times P - \frac{\Delta}{2} \right\rangle / (P^+)^2, \tag{103}$$

$$\int_{-1}^{1} x f_g(x,\xi,t)dx = \left\langle P + \frac{\Delta}{2} \left| 2\,\mathrm{Tr}[F^{+\alpha}(0)F^{+}_{\alpha}(0)] \right| \times P - \frac{\Delta}{2} \right\rangle / (P^+)^2. \tag{104}$$

The operators appearing in the right-hand side of (103) and (104) are, respectively, the $++$ component of the quark and gluon parts of the QCD energy-momentum tensor. Especially simple here is the forward limit $t \to 0$, $\xi \to 0$. In this limit, $f_q(x,\xi,t)$ and $f_g(x,\xi,t)$ reduce to the standard parton distribution functions of quarks and gluons, i.e., $f_q(x)$ and $f_g(x)$. Then, remembering that the nonforward nucleon matrix elements of T_q^{++} and T_g^{++} are parametrized as

$$\left\langle P + \frac{\Delta}{2}, s \left| T^{++}_{q/g}(0) \right| P - \frac{\Delta}{2}, s \right\rangle = A_{q/g}(\Delta^2) P^+ P^+ + \frac{B_{q/g}(\Delta^2)}{M} P^+ \epsilon^{+\alpha\beta\sigma} s_\alpha P_\beta i \Delta_\sigma + C_{q/g}(\Delta^2) M^2 g^{++} + O(\Delta^2), \tag{105}$$

we can easily get the following sum rules:

$$\int_{-1}^{1} x f_q(x) dx = A_q(0), \tag{106}$$

$$\int_{-1}^{1} x f_g(x) dx = A_g(0). \tag{107}$$

These quantities are nothing but the momentum fractions $\langle x \rangle^q$ and $\langle x \rangle^g$ carried by the quark and gluon fields in the nucleon. The famous momentum sum rule of QCD

$$\int_{-1}^{1} x[f_q(x) + f_g(x)]dx = \langle x \rangle^q + \langle x \rangle^g = 1, \tag{108}$$

then follows from the equation

$$\langle P, s | T_q^{++}(0) + T_g^{++}(0) | P, s \rangle / (P^+)^2 = 1. \tag{109}$$

On the other hand, by differentiating the relations (103) and (104) before taking the forward limit, we obtain the identities

$$- iP^+ \frac{\partial}{\partial \Delta_\sigma} \int_{-1}^{1} x f_q(x, 0, \Delta) dx |_{\Delta=0} = \frac{B_q(0)}{M} \epsilon^{+\sigma\alpha\beta} s_\alpha P_\beta, \tag{110}$$

$$- iP^+ \frac{\partial}{\partial \Delta_\sigma} \int_{-1}^{1} x f_g(x, 0, \Delta) dx |_{\Delta=0} = \frac{B_g(0)}{M} \epsilon^{+\sigma\alpha\beta} s_\alpha P_\beta. \tag{111}$$

Here the quantities $B_q(0)$ and $B_g(0)$ are the forward limits of the form factors appearing in the nonforward nucleon matrix element of quark and gluon parts of the QCD energy-momentum tensor. The fact that they are just proportional to the total angular momenta of quark and gluon such that [see (97)–(101)]

$$J_q = \tfrac{1}{2} B_q(0), \tag{112}$$

$$J_g = \tfrac{1}{2} B_g(0), \tag{113}$$

is the famous Ji sum rule [13–17]. To avoid confusion, we recall here that the above form factors $B_{q/g}(\Delta^2)$ are related to more familiar form factors $A_{20}^{q/g}(\Delta^2)$ and $B_{20}^{q/g}(\Delta^2)$ through the relation $B_{q/g}(\Delta^2) = A_{20}^{q/g}(\Delta^2) + B_{20}^{q/g}(\Delta^2)$. Here, $A_{20}^{q/g}(\Delta^2)$ and $B_{20}^{q/g}(\Delta^2)$ are, respectively, the 2nd moments of the unpolarized GPDs $H^{q/g}(x,\xi,\Delta^2)$ and $E^{q/g}(x,\xi,\Delta^2)$ with $\xi = 0$, so that

$$B_{q/g}(\Delta^2) = A_{20}^{q/g}(\Delta^2) + B_{20}^{q/g}(\Delta^2) = \int_{-1}^{1} x[H^{q/g}(x,0,\Delta^2) + E^{q/g}(x,0,\Delta^2)]dx. \tag{114}$$

The GPDs are measurable quantities so that J_q and J_g can, in principle, be determined empirically. Once J_q and J_g are known, it is clear from our general formula for the nucleon spin decomposition that the orbital angular momenta L_q and L_g of the quarks and gluons can be extracted just by subtracting the intrinsic spin parts of the quarks and gluons, i.e., $\frac{1}{2}\Delta q$ and Δg. A remaining critical question is then as follows. Can the intrinsic quark and gluon-spin parts defined in our gauge-invariant decomposition of the nucleon spin be identified with the corresponding quantities as measured by the high-energy DIS measurements? This is a fairly delicate question especially for the gluon polarization Δg. However, the importance of this question should not be dismissed. In fact, only in the case in which we could affirmatively answer this question, would we attain a sound theoretical basis for a completely meaningful gauge-invariant decomposition of the nucleon spin.

To answer the raised question, it is useful to remember the investigation by Bashinsky and Jaffe [21], which can be thought of as a nontrivial generalization of the light-cone-gauge formulation of parton distribution functions. The reason why we pay special attention to the formulation of Bashinky and Jaffe is twofold. The first reason is, of course, that their light-cone-gauge formulation of the parton distribution functions and the corresponding 1st moments just fits our program, which aims at finding the relation between the gluon-spin term in our decomposition and high-energy deep-inelastic-scattering observables. Another important reason, although not unrelated to the first, is that we want to show explicitly the fact that the numerical value of the gluon-spin term in the Bashinsky-Jaffe decomposition just coincides with that of the gluon-spin term of our more general decomposition. (To avoid confusion, however, we emphasize once again that the orbital angular-momentum parts of quark and gluons in the Bashinsky-Jaffe decomposition are never related to the corresponding terms in our recommendable decomposition

(I) by any gauge transformation. See the discussion later for more detail.)

Starting with the standard light-cone-gauge formulation of parton distribution functions, Bashinsky and Jaffe invented a method of constructing gauge-invariant quark and gluon distributions describing abstract QCD observables and applied this formalism for analyzing angular-momentum contents of the nucleon. In addition to the known quark and gluon polarized distribution functions, they gave a definition of gauge-invariant distributions for quark and gluon orbital angular momentum. According to their notation, these distribution functions for the quark and gluon spin and orbital angular momenta are given by

$$f_{\Delta q}(x_{Bj}) = \frac{1}{2\pi\sqrt{2}} \int d\xi^- e^{ix_{Bj}P^+\xi^-} \langle P| \psi_+^\dagger(0)\gamma^5\psi_+(\xi^-)P\rangle, \tag{115}$$

$$f_{L_q}(x_{Bj}) = \frac{\int d\xi^- e^{ix_{Bj}P^+\xi^-}\langle P| \int d^2x^\perp \psi_+^\dagger(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)\psi_+(x^\perp + \xi^-)|P\rangle}{2\pi\sqrt{2}(\int d^2x^\perp)}, \tag{116}$$

$$f_{\Delta g}(x_{Bj}) = \frac{1}{4\pi}\int d\xi^- e^{ix_{Bj}P^+\xi^-}\langle P|F^{+\lambda}(0)\epsilon^{+-}{}_\lambda{}^\chi A_\chi(\xi^-)|P\rangle, \tag{117}$$

$$f_{L_g}(x_{Bj}) = \frac{i\int d\xi^- e^{ix_{Bj}P^+\xi^-}\langle P| \int d^2x^\perp F^{+\lambda}(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)A_\lambda(x^\perp + \xi^-)|P\rangle}{4\pi(\int d^2x^\perp)}. \tag{118}$$

Here, $\psi_+ \equiv \frac{1}{2}\gamma^-\gamma^+\psi$, and

$$\mathcal{D}_i = \partial_i - ig\mathcal{A}_i \tag{119}$$

denotes the residual gauge covariant derivative corresponding to the residual gauge degrees of freedom remaining after taking the light-cone gauge $A^+ = 0$. The 1st moments of these distribution functions becomes

$$\Delta q = \frac{1}{\sqrt{2}P^+}\langle P|\psi_+^\dagger(0)\gamma^5\psi_+(0)|P\rangle, \tag{120}$$

$$L_q = \frac{1}{\sqrt{2}P^+(\int d^2x^\perp)}\left\langle P\left|\int d^2x^\perp\psi_+^\dagger(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)\psi_+(x^\perp)\right|P\right\rangle, \tag{121}$$

$$\Delta g = \frac{1}{2P^+}\langle P|F^{+\lambda}(0)\epsilon^{+-}{}_\lambda{}^\chi A_\chi(0)|P\rangle, \tag{122}$$

$$L_g = \frac{1}{2P^+(\int d^2x^\perp)}\left\langle P\left|\int d^2x^\perp F^{+\lambda}(x^\perp)(x^1 i\mathcal{D}_2 - x^2 i\mathcal{D}_1)A_\lambda(x^\perp)\right|P\right\rangle. \tag{123}$$

One might notice here the resemblance of this decomposition to our decomposition (II). To see it more closely, we take the nucleon matrix element of $M'^{\mu\nu\lambda}$ in (35) with $\mu = +$, $\nu = 1$, $\lambda = 2$:

$$\begin{aligned}\langle P,s|M'^{+12}(0)|P,s\rangle &= \langle P,s|M'^{+12}_{q-\text{spin}}(0)|P,s\rangle + \langle P,s|M'^{+12}_{q-\text{OAM}}(0)|P,s\rangle \\ &\quad + \langle P,s|M'^{+12}_{g-\text{spin}}(0)|P,s\rangle + \langle P,s|M'^{+12}_{g-\text{OAM}}(0)|P,s\rangle,\end{aligned} \tag{124}$$

where

$$M'^{+12}_{q-\text{spin}} = \frac{1}{2}\bar{\psi}\gamma_3\gamma_5\psi = \psi_+^\dagger\gamma_5\psi_+, \tag{125}$$

$$\begin{aligned}M'^{+12}_{q-\text{spin}} &= \bar{\psi}\gamma^+(x^1 iD^2_{\text{pure}} - x^2 iD^1_{\text{pure}})\psi \\ &= 2\psi_+^\dagger(x^1 iD^2_{\text{pure}} - x^2 iD^1_{\text{pure}})\psi_+,\end{aligned} \tag{126}$$

$$\begin{aligned}M'^{+12}_{g-\text{spin}} &= 2\,\text{Tr}[F^{+2}A^1_{\text{phys}} - F^{+1}A^2_{\text{phys}}] \\ &= 2\,\text{Tr}[F^{+\lambda}\epsilon^{+-}{}_\lambda{}^\chi A^{\text{phys}}_\chi],\end{aligned} \tag{127}$$

$$M'^{+12}_{g-\text{OAM}} = -2\,\text{Tr}[F^{+\lambda}(x^1 D^2_{\text{pure}} - x^2 D^1_{\text{pure}})A^{\text{phys}}_\lambda]. \tag{128}$$

Here, we have omitted the boosts and total derivative terms, which are irrelevant in our discussion here.

The above perfect correspondence indicates the following. The residual gauge covariant derivative $\mathcal{D}_i = \partial_i - ig\mathcal{A}_i$ appearing in the orbital parts of the Bashinsky-Jaffe decomposition is critically different from the standard covariant derivative containing the full gauge field. The field $\mathcal{A}_i$ contained in $\mathcal{D}_i$ would rather correspond to the pure-gauge part A_i^{pure} in our general framework. (This fact will soon be confirmed in more explicit form.) This means that the quark and gluon orbital angular momenta appearing in the Bashinsky-Jaffe decomposition are basically the canonical ones not the dynamical ones. In fact, we have already pointed out in Sec. II that the Bashinsky-Jaffe decomposition and the Chen *et al.* decomposition fall into the same category in the sense that they are both nontrivial gauge-invariant extensions of the gauge-variant Jaffe-Manohar decomposition. As repeatedly emphasized, this is not our recommendable decomposition, since no practical experimental process is known for measuring the above distribution functions for the quark and gluon orbital angular momenta and the corresponding 1st moments.

Despite this fact, one should clearly recognize the fact that the quark and gluon-spin terms in decomposition (II) are exactly the same as those of our recommendable decomposition (I), i.e.,

$$M'^{\mu\nu\lambda}_{q-\mathrm{spin}} = M^{\mu\nu\lambda}_{q-\mathrm{spin}}, \tag{129}$$

$$M'^{\mu\nu\lambda}_{q-\mathrm{spin}} = M^{\mu\nu\lambda}_{q-\mathrm{spin}}. \tag{130}$$

We therefore concentrate on the relationship between the quark and gluon-spin terms in the Bashinsky-Jaffe decomposition and those of our gauge-invariant decomposition (I). There is no problem with the quark spin part. In fact, this term is trivially gauge-invariant in itself and it has been long known that it can be measured through polarized DIS measurements. The quark spin term in our decomposition precisely coincides with this measurable quantity.

The gluon-spin part is a little more delicate, however. In fact, it is often claimed that there is no gauge-invariant decomposition of gluon total angular-momentum into its spin and orbital parts. Since the fundamental gauge principle dictates that observables must be gauge-invariant, one might suspect whether Δg is really an observable quantity or not. To clear up these unsettled issues, we first recall that, in our gauge-invariant decomposition of the covariant angular-momentum tensor, we do not actually need to fix gauge explicitly. Only conditions necessary in our decomposition is that A^μ_{pure} in $A^\mu = A^\mu_{\mathrm{phys}} + A^\mu_{\mathrm{pure}}$ satisfies the pure-gauge requirement, $F^{\mu\nu}_{\mathrm{pure}} \equiv \partial^\mu A^\nu_{\mathrm{pure}} - \partial^\nu A^\mu_{\mathrm{pure}} - ig[A^\mu_{\mathrm{pure}}, A^\nu_{\mathrm{pure}}]$ and the appropriate gauge transformation properties (14) and (15) of A^μ_{phys} and A^μ_{pure}. (The fact is that A^μ_{phys} basically contains only the gauge-independent and physics-containing part common to all gauges that resides on the physical plane [19].)

Now, assume that we impose the light-cone gauge condition $A^+ = 0$, while leaving the freedom of residual gauge transformation retaining $A^+ = 0$. Comparing (123) and (128), it must be clear by now that the gluon-spin terms in the Bashinsky-Jaffe decomposition can be thought of as the "light-cone-gauge fixed form" of our more general expression. However, careful readers might notice a delicate difference between the two expressions (123) and (128). In the gluon-spin term in our decomposition, what enters is A^{phys}_χ, i.e., the physical part of A_χ, whereas the full gauge field A_χ enters in the Δg term of the Bashinsky-Jaffe decomposition. As such, the fully gauge-invariant nature of the Δg term in the Bashinsky-Jaffe decomposition is not so obvious, which is a source of confusion. Now we will show that the full gauge field A_χ in this Δg term can be replaced by its physical part A^{phys}_χ without any approximation. (Although not so clearly written, this fact was already recognized in the paper by Bashinsky and Jaffe [21].)

The proof goes as follows. Following Bashinsky and Jaffe [21], we introduce the Fourier decomposition of $A_\lambda(\xi) \equiv A^{\mathrm{LC}}_\lambda(\xi)$ as

$$A_\lambda(\xi) = \int \frac{dk^+}{2\pi} e^{-ik^+\xi^-} \tilde{A}_\lambda(k^+, \tilde{\xi}), \tag{131}$$

where

$$\tilde{\xi} = (\xi^+, \xi^1, \xi^2) = (\xi^+, \xi^\perp). \tag{132}$$

There still remains a residual gauge symmetry. In fact, the condition $A^+ = 0$ is preserved by a gauge transformation, the parameters of which do not depend on the coordinate ξ^-. Under such a gauge transformation, $\tilde{A}_\lambda(k^+, \tilde{\xi})$ transforms as

$$\tilde{A}_\lambda(k^+, \tilde{\xi}) \to U(\tilde{\xi})\left(\tilde{A}_\lambda(k^+, \tilde{\xi}) + \frac{2\pi i \delta(k^+)}{g}\partial_\lambda\right)U^{-1}(\tilde{\xi}). \tag{133}$$

Here the inhomogeneous term appears only at $k^+ = 0$. This motivates them to split the fields $\tilde{A}_\lambda(k^+, \tilde{\xi})$ into two parts as

$$\tilde{A}_\lambda(k^+, \tilde{\xi}) = 2\pi\delta(k^+)\mathcal{A}_\lambda(\tilde{\xi}) + \tilde{G}_\lambda(k^+, \tilde{\xi}), \tag{134}$$

which, respectively, transform as

$$\tilde{G}_\lambda(k^+, \tilde{\xi}) \to U(\tilde{\xi})\tilde{G}_\lambda(k^+, \tilde{\xi})U^{-1}(\tilde{\xi}), \tag{135}$$

$$\mathcal{A}_\lambda(\tilde{\xi}) \to U(\tilde{\xi})\left(\mathcal{A}_\lambda(\tilde{\xi}) + \frac{i}{g}\partial_\lambda\right)U^{-1}(\tilde{\xi}), \tag{136}$$

under the residual gauge transformation that does not depend on ξ^-. The decomposition is unique if one requires the boundary condition $\tilde{G}_\lambda(k^+, \tilde{\xi})|_{k^+=0} \equiv 0$ [21]. In the coordinate space, this corresponds to the decomposition

$$A_\lambda(\xi) = A^{\mathrm{phys}}_\lambda(\xi) + A^{\mathrm{pure}}_\lambda(\xi), \tag{137}$$

with

$$A^{\mathrm{phys}}_\lambda(\xi) \equiv \int \frac{dk^+}{2\pi} e^{-ik^+\xi^-} \tilde{G}_\lambda(k^+, \tilde{\xi}), \tag{138}$$

$$A^{\mathrm{pure}}_\lambda(\xi) \equiv \int \frac{dk^+}{2\pi} e^{-ik^+\xi^-} 2\pi\delta(k^+)\mathcal{A}_\lambda(\tilde{\xi}) = \mathcal{A}_\lambda(\tilde{\xi}) = \mathcal{A}_\lambda(\xi^+, \xi^\perp). \tag{139}$$

A noteworthy fact here is that the pure-gauge part of $A_\lambda(\xi)$ does not depend on the coordinate ξ^-. By making use of it, one can easily convince that these two parts transform in the following way:

$$A_\lambda^{\text{phys}}(\xi) \to U(\tilde{\xi}) A_\lambda^{\text{phys}}(\xi) U^{-1}(\tilde{\xi}), \tag{140}$$

$$A_\lambda^{\text{pure}}(\xi) \to U(\tilde{\xi}) \Big(A_\lambda^{\text{pure}}(\xi) + \frac{i}{g} \partial_\lambda \Big) U^{-1}(\tilde{\xi}), \tag{141}$$

under the residual gauge transformation. This transformation rules just confirm our previous statement on the correspondence

$$\mathcal{A}_\lambda \leftrightarrow A_\lambda^{\text{pure}},$$
$$\mathcal{D}_i = \partial_i - g\mathcal{A}_i \leftrightarrow D_i^{\text{pure}} = \partial_i - gA_i^{\text{pure}}.$$

More precisely, $\mathcal{A}_\lambda$ can be thought of as a special case of our more general quantity A_λ^{pure} after choosing the light-cone gauge. This implies that the gluon-spin term in our general (gauge-invariant) decomposition in fact reduces to the corresponding piece of the Bashinsky-Jaffe decomposition given in the light-cone gauge.

Now we return to the expression for the polarized gluon distribution function $f_{\Delta g}(x)$;

$$f_{\Delta g}(x) = \frac{\frac{1}{4\pi}\int d^2\xi^\perp \int d\xi^- \int d\eta^- e^{-ixP^+\eta^-} \langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi A_\chi(\xi+\eta^-)|P\rangle}{\int d^2\xi^\perp \int d\xi^-}, \tag{142}$$

with

$$\xi = (\xi^+, \xi^-, \xi^\perp), \qquad \tilde{\xi} = (\xi^+, \xi^\perp). \tag{143}$$

Noting the fact that $A_\chi^{\text{pure}}(\xi) = \mathcal{A}_\chi(\tilde{\xi})$ does not depend on ξ^-, the contribution of the pure-gauge part is given by

$$f_{\Delta g}(x) = \frac{\frac{\delta(x)}{2P^+}\int d^2\xi^\perp \int d\xi^- \langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi \tilde{A}_\chi(\tilde{\xi})|P\rangle}{\int d^2\xi^\perp \int d\xi^-}. \tag{144}$$

Using the relation $F^{+\lambda}(\xi) = \frac{\partial}{\partial\xi^-}A^\lambda(\xi)$ that holds in the light-cone gauge, we therefore find that

$$\begin{aligned}\int d\xi^- \langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}(\tilde{\xi})|P\rangle &= \int d\xi^- \langle P|\frac{\partial}{\partial\xi^-}A^\lambda(\xi)\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}(\tilde{\xi})|P\rangle = \langle P|[A^\lambda(\xi^- = +\infty) \\ &\quad - A^\lambda(\xi^- = -\infty)]\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}_\chi(\tilde{\xi})|P\rangle - \int d\xi^- \Big\langle P \Big| A^\lambda(\xi)\epsilon^{+-}{}_\lambda{}^\chi \frac{\partial}{\partial\xi^-}\mathcal{A}_\chi(\tilde{\xi}) \Big| P \Big\rangle \\ &= \langle P|[A^\lambda(\xi^- = +\infty) - A^\lambda(\xi^- = -\infty)]\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}_\chi(\tilde{\xi})|P\rangle,\end{aligned} \tag{145}$$

since $\frac{\partial}{\partial\xi^-}\mathcal{A}_\chi(\tilde{\xi}) = 0$. In the light-cone gauge, the above surface term does not vanish, because either $A^\lambda(\xi^- = +\infty)$ or $A^\lambda(\xi^- = -\infty)$ or both remains finite. Nonetheless, as pointed out in [21], the surface term does not contribute to the polarized gluon distribution $f_{\Delta G}(x)$, since

$$\begin{aligned}\int_{-\infty}^{+\infty} d\xi^- \langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}_\chi(\tilde{\xi})P\rangle / \int_{-\infty}^{+\infty} d\xi^- &= \langle P|[A^\lambda(\xi^- = +\infty) - A^\lambda(\xi^- = -\infty)]\epsilon^{+-}{}_\lambda{}^\chi \mathcal{A}_\chi(\tilde{\xi})|P\rangle / \int_{-\infty}^{+\infty} d\xi^- \\ &= 0.\end{aligned} \tag{146}$$

On the other hand, the contribution of the physical part of $\mathcal{A}_\chi$ is given as

$$f_{\Delta g}(x) = \frac{\frac{1}{4\pi}\int d^2\xi^\perp \int d\xi^- \int d\eta^- e^{ixP^+\eta^2} \langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi A_\chi^{\text{phys}}(\xi+\eta^-)|P\rangle}{\int d^2\xi^\perp \int d\xi^-}. \tag{147}$$

Using the translational invariance

$$\begin{aligned}\langle P|F^{+\lambda}(\xi)\epsilon^{+-}{}_\lambda{}^\chi A_\chi^{\text{phys}}(\xi+\eta^-)|P\rangle \\ = \langle P|F^{+\lambda}(0)\epsilon^{+-}{}_\lambda{}^\chi A_\chi^{\text{phys}}(\eta^-)|P\rangle,\end{aligned} \tag{148}$$

we therefore obtain

$$f_{\Delta g}^{\text{phys}}(x) = \frac{1}{4\pi} \times \int d\eta^- e^{ixP^+\eta^-} \langle P|F^{+\lambda}(0)\epsilon^{+-}{}_\lambda{}^\chi A_\chi^{\text{phys}}(\eta^-)|P\rangle. \tag{149}$$

The corresponding 1st moment becomes

$$\Delta g = \int_{-1}^{1} f_{\Delta G}^{\rm phys}(x) dx = \frac{1}{2P^+} \langle P | F^{+\lambda}(0) \epsilon^{+-}{}_{\lambda}{}^{\chi} A_{\chi}^{\rm phys}(0) | P \rangle. \quad (150)$$

Note that this precisely takes the same form as our Δg term

$$\Delta g = \langle P, s | M_{g-{\rm spin}}^{+12}(0) P, s \rangle / 2P^+ = \frac{1}{2P^+} \langle P | F^{+\lambda}(0) \epsilon^{+-}{}_{\lambda}{}^{\chi} A_{\chi}^{\rm phys}(0) | P \rangle. \quad (151)$$

Needless to say, $A_{\chi}^{\rm phys}$ in (150) should be interpreted as a gauge fixed form of more general $A_{\chi}^{\rm phys}$ in (151) after taking the light-cone gauge. With this understanding, it is clear now that the numerical value of the gluon-spin term in the Bashinsky-Jaffe decomposition precisely coincides with that of the gluon-spin term in our more general decomposition. (This is just what is meant by the gauge invariance.) To put it in another way, the gluon-spin part in our gauge-invariant decomposition precisely reduces to the 1st moment of the polarized gluon distribution accessed by high-energy DIS measurements. It is widely recognized that there is no gauge-invariant local operator corresponding to the 1st moment of the polarized gluon distribution in the standard operator-product expansion. However, it should be clear by now that there is no conflict between this general statement and our finding above. The decomposition of the gauge field A_μ into its physical and pure-gauge parts is generally a nonlocal operation so that $A_{\chi}^{\rm phys}$ is not a local operator. (This is true not only for the light-cone gauge but also for the generalized Coulomb gauge advocated by Chen *et al.*) Now we can definitely say that the gluon-spin contribution to the nucleon spin measured by high-energy DIS measurements just coincide with the quantity appearing in our general decomposition of the nucleon spin discussed in the previous sections, so that it can be given a manifestly gauge-invariant and practically frame-independent meaning.

At this point, it may be useful to summarize some of the important lessons that we have learned from the present investigation. First, as repeatedly emphasized, the way of gauge-invariant decomposition of the nucleon spin is not necessarily unique. We showed that there are basically two independent decompositions of the nucleon spin, i.e., decomposition (I) specified by (26) and decomposition (II) specified by (35). Decomposition (II) contains three known decompositions of the nucleon spin, i.e., those of Jaffe and Monahar, of Bashinsky and Jaffe, and of Chen *et al*. We can say that all these decompositions are physically equivalent in the sense that they are all obtained from more general decomposition (II) by means of suitable gauge fixing. On the other hand, the physical content of decomposition (I) is critically different from decomposition (II). Decomposition (I) contains the famous Ji decomposition, although the former allows the decomposition of the total gluon angular momentum into its intrinsic spin and orbital parts, which was given up in the latter. For pedagogical reason, we think it useful to summarize this state of affairs in conceptual figures as illustrated in Fig. 1.

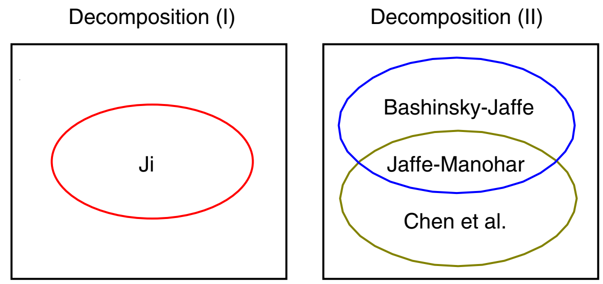

FIG. 1 (color online). Schematic picture of two independent gauge-invariant decompositions of nucleon spin and the relation with the known decompositions.

The superiority of decomposition (I) over decomposition (II) is that both of the quark and gluon orbital angular momenta can be related to concrete high-energy observables. In fact, after confirmation of the frame independence of our nucleon spin decomposition, we can now work in an arbitrary Lorentz frame. Then, the following identity must hold for the quark orbital angular momentum in decomposition (I):

$$L_q = \langle p \uparrow | M_{q-{\rm OAM}}^{012} | p \uparrow \rangle = \frac{1}{2} \int_{-1}^{1} x [H^q(x,0,0) + E^q(x,0,0)] dx - \frac{1}{2} \int_{-1}^{1} \Delta q(x) dx, \quad (152)$$

where

$$M_{q-{\rm OAM}}^{012} = \bar{\psi} \left(\boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} \right)^3 \psi. \quad (153)$$

(Note that the parton distribution functions and the polarized PDF appearing in the right-hand side of (152) are the Lorentz-frame-independent quantities.) This identity means that the quark orbital angular momentum L_q in decomposition (I) precisely coincides with the difference of the 2nd moment of the unpolarized GPD $H^q(x,0,0) + E^q(x,0,0)$ and the 1st moment of the longitudinally polarized quark distribution $\Delta q(x)$, which are both observables. Furthermore, this L_q is given as a proton matrix element of the dynamical orbital angular momentum of quarks, i.e., $\boldsymbol{x} \times \frac{1}{i} \boldsymbol{D} = \boldsymbol{x} \times \frac{1}{i} (\nabla - ig \boldsymbol{A})$ not the canonical orbital angular momentum $\boldsymbol{x} \times \frac{1}{i} \nabla$ or its gauge-invariant extension $\boldsymbol{x} \times \frac{1}{i} \boldsymbol{D}_{\rm pure} = \boldsymbol{x} \times \frac{1}{i} (\nabla - ig \boldsymbol{A}_{\rm pure})$ [17].

Similarly, for the gluon orbital angular momentum L_g in decomposition (I), the following identity must hold

$$L_g = \langle p \uparrow | M^{012}_{g-\mathrm{OAM}} | p \uparrow \rangle = \frac{1}{2} \int_{-1}^{1} x [H^g(x,0,0) + E^g(x,0,0)] dx - \int_{-1}^{1} \Delta g(x) dx, \tag{154}$$

where

$$M^{012}_{g-\mathrm{OAM}} = 2\mathrm{Tr}[E^j (\boldsymbol{x} \times \boldsymbol{D}_{\mathrm{pure}})^3 A^{\mathrm{phys}}_j] + 2\mathrm{Tr}[\rho (\boldsymbol{x} \times \boldsymbol{A}_{\mathrm{phys}})^3]. \tag{155}$$

One confirms that the gluon orbital angular momentum in decomposition (I) just coincides with the difference of the 2nd moment of the gluon GPD $H^g(x,0,0) + E^g(x,0,0)$ and the 1st moment of the longitudinally polarized gluon distribution $\Delta g(x)$. What is noteworthy here is that the relevant gluon orbital angular-momentum operator entering in this identity consists of two terms. The 1st piece is a gauge-invariant extension of the canonical orbital angular momentum of gluons. (It is physically equivalent to the usual canonical orbital angular momentum appearing, for instance, in the Jaffe-Manohar decomposition.) The 2nd piece is nothing but the potential angular-momentum term discussed in some detail in our previous paper [14]. In view of the analogous situation for the quark part, it would be legitimate now to call the sum of these two pieces, i.e., the whole part of $M^{012}_{g-\mathrm{OAM}}$ in (155), the dynamical orbital angular momentum of the gluon field.

Before ending this section, we think it instructive to call attention to some other recent investigations related to the nucleon spin decomposition. As emphasized above, the quark orbital angular momentum extracted from the combined analysis of the unpolarized GPDs and the longitudinally polarized quark distribution functions is the dynamical orbital angular momentum not the canonical one or its nontrivial gauge-invariant extension. At least until now, we have had no means to extract the canonical orbital angular momentum purely experimentally, which also means that the difference between the dynamical and canonical orbital angular momenta is not a direct experimental observable. Nevertheless, it is not impossible to estimate the size of this difference within the framework of a certain model. In fact, Burkardt and BC estimated the difference between the orbital angular momentum obtained from the Jaffe-Manohar decomposition and that obtained from the Ji decomposition within two simple toy models, and emphasize the possible importance of the vector potential in the definition of the orbital angular momentum [34]. The difference between the above two orbital angular momenta is nothing but the *potential angular momentum* in our terminology.

Also noteworthy is recent phenomenological investigations on the role of orbital angular momenta in the nucleon spin. In a recent paper, we have pointed out possible existence of a significant discrepancy between the lattice QCD predictions [35,36] for $L^u - L^d$ (the difference of the orbital angular momenta carried by up- and down-quarks in the proton) and the prediction of a typical low energy model of the nucleon, for example, the refined cloudy-bag model [37]. It is an open question whether this discrepancy can be resolved by the strongly scale-dependent nature of the quantity $L^u - L^d$ especially in the low Q^2 domain as claimed in [38], or whether the discrepancy has a root (at least partially) in the existence of two kinds of quark orbital angular momenta as indicated in [39,40]. (See also [41,42] for details.)

V. SUMMARY AND CONCLUSION

When discussing the spin structure of the nucleon, color gauge invariance has often been a cause of controversy. For instance, it is known that the polarized gluon distribution in the nucleon can be defined in terms of a nucleon matrix element of the gauge-invariant correlation function. On the other hand, one is also aware of the fact that there is no gauge-invariant local operator corresponding to the 1st moment of the polarized gluon distribution in the standard operator-product expansion. Undoubtedly, this seemingly conflicting observation has a common root as the familiar statement that there is no gauge-invariant decomposition of the gluon total angular momentum into its spin and orbital parts. Inspired by the recent proposal by Chen *et al.*, we find it possible to make a gauge-invariant decomposition of the covariant angular-momentum tensor of QCD in an arbitrary Lorentz frame. Based on this fact, we could show that our decomposition of the nucleon spin is not only gauge-invariant but also practically frame independent. We have also succeeded in establishing that each piece of our nucleon spin decomposition just corresponds to the observable extracted through combined analyses of the GPD measurements and the polarized DIS measurements, thereby supporting the standardly accepted experimental program aiming at complete decomposition of the nucleon [43–45]. In particular, the gluon-spin part of our decomposition precisely coincides with the 1st moment of the polarized gluon distribution function. In our theoretical framework, this gluon-spin part of the decomposition is given as a nucleon matrix element of the gauge-invariant operator. However, since this operator is generally nonlocal, there is no conflict with the knowledge of the standard operator-product expansion.

From a practical viewpoint, the more important lesson to be learned from our present theoretical analysis would be the *physical insight* into the measurable quark and gluon orbital angular momenta appearing in our recommendable decomposition (I). We have confirmed that the quark orbital angular momentum, which can be extracted as the difference of the 2nd moment of the unpolarized quark GPD and the 1st moment of the longitudinally polarized quark distribution, is the dynamical quark orbital angular momentum $\langle \boldsymbol{x} \times (\boldsymbol{p} - g\boldsymbol{A}) \rangle$ not the canonical one $\langle \boldsymbol{x} \times \boldsymbol{p} \rangle$. Similarly, the gluon orbital angular momentum extracted as the difference of the 2nd moment of the unpolarized gluon GPD and the 1st moment of the longitudinally polarized gluon distribution is not the canonical orbital angular momentum but the dynamical orbital angular momentum containing the potential angular momentum term in our terminology. Even though no experimental process to directly access to the canonical orbital angular momenta is known at present, one should clearly keep in mind the existence of two kinds of orbital angular momenta for both of quarks and gluons.

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