

Moduli stabilization and supersymmetry breaking in heterotic orbifold string modelsBen Dundee,^{1,3,*} Stuart Raby,^{1,3,†} and Alexander Westphal^{2,3,‡}¹*Department of Physics, The Ohio State University, 191 West Woodruff Avenue, Columbus, Ohio 43210, USA*²*Department of Physics, Stanford University, 382 Via Pueblo Mall, Stanford, California 94305, USA*³*Kavli Institute for Theoretical Physics, Santa Barbara, California 93106, USA*

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In this paper, we discuss the issues of supersymmetry breaking and moduli stabilization within the context of $E_8 \otimes E_8$ heterotic orbifold constructions and, in particular, we focus on the class of “mini-landscape” models. In the supersymmetric limit, these models admit an effective low-energy field theory with a spectrum of states and dimensionless gauge and Yukawa couplings very much like that of the minimal supersymmetric standard model. These theories contain a non-Abelian hidden gauge sector which generates a nonperturbative superpotential leading to supersymmetry breaking and moduli stabilization. We demonstrate this effect in a simple model which contains many of the features of the more general construction. In addition, we argue that once supersymmetry is broken in a restricted sector of the theory, then all moduli are stabilized by supergravity effects. Finally, we obtain the low-energy superparticle spectrum resulting from this simple model.

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I. INTRODUCTION

String theory, as a candidate theory of all fundamental interactions including gravity, is obliged to contain patterns consistent with observation. This includes the standard model gauge group and particle content, as well as an extremely small cosmological constant. If one also assumes that nature contains a low-energy supersymmetry, one would want to find patterns which are *qualitatively* close to the minimal supersymmetric standard model (MSSM). An even more ambitious goal would be to find a theory consistent with the standard model spectrum of masses and a prediction for sparticle masses which can be tested at the LHC. Some progress has been made to this end starting from different directions [1–19], i.e., free fermionic, orbifold, or smooth Calabi-Yau constructions of the heterotic string, intersecting D -brane constructions in type II string, and M or F theory constructions. Much of this progress has benefited from the requirement of an intermediate grand unified gauge symmetry which naturally delivers the standard model particle spectrum.

In this paper we focus on the “mini-landscape” of heterotic orbifold constructions [4–6,9,10], which give several models which pass a significant number of phenomenological hurdles.¹ These models have been analyzed in the supersymmetric limit. They contain an MSSM spectrum with three families of quarks and leptons, one or more pairs of Higgs doublets and an exact R parity. In the orbifold limit, they also contain a small number of vectorlike exotics and extra $U(1)$ gauge interactions felt by standard model particles. These theories also contain a

large number of standard model singlet fields, some of which are moduli, i.e., blow up modes of the orbifold fixed points. The superpotential for these orbifold theories can be calculated order by order in powers of products of superfields. This is a laborious task which is simplified by assuming that any term allowed by string selection rules appears with an order one coefficient in the superpotential. With this caveat it was shown that all vectorlike exotics and additional $U(1)$ gauge bosons acquire mass at scales of order the string scale at supersymmetric minima satisfying $F_I = D_a = 0$ for all chiral fields labeled by the index I and all gauge groups labeled by the index a . In addition, the value of the gauge couplings at the string scale and the effective Yukawa couplings are determined by the presumed values of the vacuum expectation values (VEVs) for moduli including the dilaton, S , the bulk volume and complex structure moduli, T_i , $i = 1, 2, 3$, and U and the standard model (SM) singlet fields containing the blow up moduli [22,23]. Finally, the theories also contain a hidden sector non-Abelian gauge group with QCD-like chiral matter. The problem which has yet to be addressed is the mechanism of moduli stabilization and supersymmetry breaking in the mini-landscape models.²

In this paper we focus on the problem of moduli stabilization and supersymmetry (SUSY) breaking in the context of heterotic orbifold models. In Sec. II we summarize the general structure of the Kähler and superpotential in heterotic orbifold models. The models have a perturbative superpotential satisfying modular invariance constraints, an anomalous $U(1)_A$ gauge symmetry with a dynamically generated Fayet-Iliopoulos D -term and a hidden

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QCD-like non-Abelian gauge sector generating a nonperturbative superpotential. In Sec. III we consider a simple model with a dilaton, S , one volume modulus, T , and three standard model singlets. The model has only one gaugino condensate, as is the case for the “benchmark models” of the mini-landscape [9]. We obtain a “hybrid Kachru, Kallosh, Linde, and Trivedi (KKLT)” kind of superpotential that behaves like a single condensate for the dilaton S , but as a racetrack for the T and, by extension, also for the U moduli; and an additional matter F -term, driven by the cancellation of an anomalous $U(1)_A$ D -term, is the seed for successful up lifting. Previous analyses in the literature have also used an anomalous $U(1)_A$ D -term in coordination with other perturbative or nonperturbative terms in the superpotential to accomplish SUSY breaking and up lifting [30–40]. We save a brief comparison of our work with some of these former analyses for Sec. III. In Sec. IV we discuss the other moduli and their stabilization. We conclude that a single gaugino condensate is sufficient to break supersymmetry, stabilize all the moduli and generate a de Sitter vacuum. Finally, in Sec. V we evaluate the SUSY particle spectrum relevant for the LHC. The main results from this analysis are listed in Tables VII and VIII.

II. GENERAL STRUCTURE

In this section we consider the supergravity limit of heterotic orbifold models. However, we focus on the mini-landscape models for definiteness. We discuss the general structure of the Kähler potential, \mathcal{K} , the superpotential, \mathcal{W} , and gauge kinetic function, f_a for generic heterotic orbifold models. The mini-landscape models are defined in terms of a \mathbb{Z}_6 -II orbifold of the six internal dimensions of the ten dimensional heterotic string. The orbifold is described by a three dimensional “twist” vector ν , which acts on the compact directions. We define the compact directions in terms of complex coordinates:

$$\begin{aligned} Z_1 &\equiv X_4 + iX_5, & Z_2 &\equiv X_6 + iX_7, \\ Z_3 &\equiv X_8 + iX_9. \end{aligned} \quad (1)$$

The twist is defined by the action $Z_i \rightarrow e^{2\pi i \nu_i} Z_i$ for $i = 1, 2, 3$, and for \mathbb{Z}_6 -II we have $\nu = \frac{1}{6}(1, 2, -3)$ or a $(60^\circ, 120^\circ, 180^\circ)$ rotation about the first, second, and third torus, respectively. This defines the first twisted sector. The second and fourth twisted sectors are defined by twist vectors 2ν and 4ν , respectively. Note, the third torus is unaffected by this twist. In addition, for the third twisted sector, generated by the twist vector 3ν , the second torus is unaffected. Finally, the fifth twisted sector, given by 5ν contains the CP conjugate states from the first twisted sector. Twisted sectors with unrotated tori contain $N = 2$ supersymmetric spectra. This has consequences for the nonperturbative superpotential discussed in Sec. II C. Finally, these models have three bulk volume moduli, T_i , $i = 1, 2, 3$, and one bulk complex structure modulus, U , for the third torus.

A. Anomalous $U(1)_A$ and Fayet-Illiopoulos D -term

The orbifold limit of the heterotic string has one anomalous $U(1)_A$ symmetry. The dilaton superfield S , in fact, transforms nontrivially under this symmetry. Let V_A, \tilde{V}_A be the gauge superfields with gauge covariant field strengths, $W_A^\alpha, \tilde{W}_A^\alpha$, of gauge groups, $U(1)_A, \tilde{\mathcal{G}}_A$, respectively. The Lagrangian in the global limit is given in terms of a Kähler potential [41–45]

$$\begin{aligned} \mathcal{K} = & -\log(S + \bar{S} - \delta_{\text{GS}} V_A) + \sum_a (\bar{Q}_a e^{V_a + 2q_a V_A} Q_a \\ & + \tilde{Q}_a e^{-V_a + 2\tilde{q}_a V_A} \tilde{Q}_a) \end{aligned} \quad (2)$$

and a gauge kinetic superpotential

$$\mathcal{W} = \frac{1}{2} \left[\frac{S}{4} \left(\sum_a k_a \text{Tr} W_a^\alpha W_{\alpha a} + k_A \text{Tr} W_A^\alpha W_{\alpha A} \right) + \text{H.c.} \right]. \quad (3)$$

Note q_a, \tilde{q}_a are the $U(1)_A$ charges of the “quark,” Q_a , and “antiquark,” \tilde{Q}_a , supermultiplets transforming under $\tilde{\mathcal{G}}_A$.

Under a $U(1)_A$ supergauge transformation with parameter Λ , one has

$$\delta_A V_A = -i(\Lambda - \bar{\Lambda})/2, \quad \delta_A S = -i \frac{\delta_{\text{GS}}}{2} \Lambda, \quad (4)$$

and

$$\delta_A \Phi = iq_\Phi \Lambda \Phi \quad (5)$$

for any charged multiplet Φ . The combination

$$S + \bar{S} - \delta_{\text{GS}} V_A \quad (6)$$

is $U(1)_A$ invariant. δ_{GS} is the Green-Schwarz coefficient given by

$$\delta_{\text{GS}} = 4 \frac{\text{Tr} Q_A}{192\pi^2} = \frac{(q_a + \tilde{q}_a) N_{f_a}}{4\pi^2} \quad (7)$$

where the middle term is for the $U(1)_A$ -gravity anomaly and the last term is for the $U(1)_A \times (\tilde{\mathcal{G}}_A)^2$ mixed anomaly.

The existence of an anomalous $U(1)_A$ has several interesting consequences. Because of the form of the Kähler potential [Eq. (2)] we obtain a Fayet-Illiopoulos D -term given by

$$\xi_A = \frac{\delta_{\text{GS}}}{2(S + \bar{S})} = -\frac{1}{2} \delta_{\text{GS}} \partial_S \mathcal{K} \quad (8)$$

with the D -term contribution to the scalar potential given by

$$V_D = \frac{1}{S + \bar{S}} \left(\sum_a X_a^A \partial_a \mathcal{K} \phi^a + \xi_A \right)^2 \quad (9)$$

where X_a^A are Killing vectors for $U(1)_A$. In addition, clearly the perturbative part of the superpotential must be $U(1)_A$ invariant. But moreover, it constrains the nonperturbative superpotential as well. In particular, if the dilaton appears

in the exponent, the product $e^{q\Phi^S\Phi^{\delta_{GS}/2}}$ is, and must also be, $U(1)_A$ invariant.

B. Target space modular invariance

In this section, we wish to present the modular dependence of the gauge kinetic function, the Kähler potential, and of the superpotential in as general a form as possible. Most studies in the past have worked with a universal T modulus, and neglected the effects of the U moduli altogether. Such a treatment is warranted, for example, in the \mathbb{Z}_3 orbifolds where there are no U moduli. If we want to work in the limit of a stringy orbifold grand unified theory (GUT) [46], which requires one of the T moduli to be much larger than the others, or in the \mathbb{Z}_6 -II orbifolds, however, it is impossible to treat all of the T and U moduli on the same footing.

Consider the $SL(2, \mathbb{Z})$ modular transformations of T and U given by [47–58]³

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbb{Z}, \quad (10)$$

and

$$\log(T + \bar{T}) \rightarrow \log\left(\frac{T + \bar{T}}{(icT + d)(-ic\bar{T} + d)}\right). \quad (11)$$

The Kähler potential for moduli to zeroth order is given by

$$\begin{aligned} \mathcal{K} &= - \sum_{i=1}^{h_{(1,1)}} \log(T^i + \bar{T}^i) - \sum_{j=1}^{h_{(2,1)}} \log(U^j + \bar{U}^j) \\ &= - \sum_{i=1}^3 \log(T^i + \bar{T}^i) - \log(U + \bar{U}) \end{aligned} \quad (12)$$

where the last term applies to the mini-landscape models, since in this case $h_{(1,1)} = 3$, $h_{(2,1)} = 1$. Under the modular group, the Kähler potential transforms as

$$\mathcal{K} \rightarrow \mathcal{K} + \sum_{i=1}^{h_{(1,1)}} \log|ic_i T^i + d_i|^2 + \sum_{j=1}^{h_{(2,1)}} \log|ic_j U^j + d_j|^2. \quad (13)$$

The scalar potential V is necessarily modular invariant. We have

$$V = e^{\mathcal{G}}(\mathcal{G}_I \mathcal{G}^{I\bar{J}} \mathcal{G}_{\bar{J}} - 3) \quad (14)$$

where $\mathcal{G} = \mathcal{K} + \log|\mathcal{W}|^2$. Hence for the scalar potential to be invariant under the modular transformations, the superpotential must also transform as follows:

$$\mathcal{W} \rightarrow \prod_{i=1}^{h_{(1,1)}} \prod_{j=1}^{h_{(2,1)}} (ic_i T^i + d_i)^{-1} (ic_j U^j + d_j)^{-1} \mathcal{W}, \quad (15)$$

$$\bar{\mathcal{W}} \rightarrow \prod_{i=1}^{h_{(1,1)}} \prod_{j=1}^{h_{(2,1)}} (-ic_i \bar{T}^i + d_i)^{-1} (-ic_j \bar{U}^j + d_j)^{-1} \bar{\mathcal{W}}.$$

This can be guaranteed by appropriate powers of the Dedekind η function multiplying terms in the superpotential.⁴ This is due to the fact that under a modular transformation, we have

$$\eta(T) \rightarrow (icT + d)^{1/2} \eta(T), \quad (16)$$

up to a phase, where

$$\eta(T) = \exp(-\pi T/12) \prod_{n=1}^{\infty} (1 - e^{-2\pi n T}). \quad (17)$$

The transformation of both the matter fields and the superpotential under the modular group fixes the modular dependence of the interactions. A field in the superpotential transforms as

$$\Phi_I \rightarrow \Phi_I \prod_{i=1}^{h_{(1,1)}} \prod_{j=1}^{h_{(2,1)}} (ic_i T^i + d_i)^{-n_i^I} (ic_j U^j + d_j)^{-\ell_j^I}. \quad (18)$$

The modular weights n_i^I and ℓ_j^I [60,61] depend on the localization of the matter fields on the orbifold. For states I in the i th untwisted sector, i.e., those states with internal momentum in the i th torus, we have $n_i^I = \ell_i^I = 1$, otherwise the weights are 0. For twisted sector states, we first define $\tilde{\eta}(k)$, which is related to the twisted sector $k(= 1, \dots, N - 1)$ and the orbifold twist vector v by

$$\eta_i(k) \equiv kv_i \text{ mod } 1. \quad (19)$$

Further, we require

$$\sum_i \eta_i(k) \equiv 1. \quad (20)$$

Then the modular weight of a state in the k th twisted sector is given by

$$\begin{aligned} n_i^I &\equiv (1 - \eta^i(k)) + N^i - \bar{N}^i \quad \text{for } \eta_i(k) \neq 0 \\ n_i^I &\equiv N^i - \bar{N}^i \quad \text{for } \eta_i(k) = 0. \end{aligned} \quad (21)$$

The $N^i(\bar{N}^i)$ are integer oscillator numbers for left-moving oscillators $\tilde{\alpha}^i(\bar{\alpha}^{\bar{i}})$, respectively. Similarly,

$$\begin{aligned} \ell_j^I &\equiv (1 - \eta^j(k)) - N^j + \bar{N}^j \quad \text{for } \eta_j(k) \neq 0 \\ \ell_j^I &\equiv -N^j + \bar{N}^j \quad \text{for } \eta_j(k) = 0. \end{aligned} \quad (22)$$

In general, one can compute the superpotential to arbitrary order in powers of superfields by a straightforward application of the string selection rules [62–65].

⁴These terms arise as a consequence of world sheet instantons in a string calculation. In fact, world sheet instantons typically result in more general modular functions [52–58].

³For an excellent review with many references, see [59].

One assumes that any term not forbidden by the string selection rules appears with order one coefficient. In practice, even this becomes intractable quickly, and we must cut off the procedure at some low, finite order. More detailed calculations of individual terms give coefficients dependent on volume moduli due to string world sheet instantons. In general the moduli dependence can be obtained using the constraint of target space modular invariance. Consider a superpotential term for the mini-landscape models, with three T moduli and one U modulus, of the form

$$\mathcal{W}_3 = w_{IJK} \Phi_I \Phi_J \Phi_K. \quad (23)$$

We assume that the fields $\Phi_{I,J,K}$ transform with modular weights $n_{I,J,K}^i$ and $\ell_{I,J,K}^3$ under T_i , $i = 1, 2, 3$, and U , respectively. Using the (net) transformation property of the superpotential, and the transformation property of $\eta(T)$ under the modular group, we have (for nonuniversal moduli)

$$w_{IJK} \sim h_{IJK} \prod_{i=1}^3 \eta(T_i)^{\gamma_{T_i}} \eta(U)^{\gamma_U}$$

where $\gamma_{T_i} = -2(1 - n_i^1 - n_i^2 - n_i^3)$, $\gamma_U = -2(1 - \ell_U^3 - \ell_U^2 - \ell_U^1)$.⁵ This is easily generalized for higher order interaction terms in the superpotential. We see that the modular dependence of the superpotential is rarely symmetric under interchange of the T_i or U_i . Note, when minimizing the scalar potential we shall use the approximation $\eta(T)^{\gamma_T} \approx e^{-bT}$ with $b = \pi\gamma_T/12$. (Recall, at large T , we have $\log(\eta(T)) \approx -\pi T/12$.) This approximation misses the physics near the self-dual point in the potential, nevertheless, it is typically a good approximation.

As a final note, Wilson lines break the $SL(2, \mathbb{Z})$ modular group down to a subgroup [66] (see, Appendix A). This has the effect of an additional differentiation of the moduli as they appear in the superpotential. In particular, factors of $\eta(T_i)$ are replaced by factors of $\eta(NT_i)$ or $\eta(T_i/N)$ for Wilson lines in \mathbb{Z}_N . In summary, the different modular dependence of twisted sector fields and the presence of Wilson lines leads quite generally to anisotropic orbifolds [67].

C. Gauge kinetic function and sigma model anomaly

To one loop, the string-derived gauge kinetic function is given by [61,68–72]

$$f_a(S, T) = k_a S + \frac{1}{8\pi^2} \sum_{i=1}^{h_{(1,1)}} (\alpha_a^i - k_a \delta_\sigma^i) \log(\eta(T^i))^2 + \frac{1}{8\pi^2} \sum_{j=1}^{h_{(2,1)}} (\alpha_a^j - k_a \delta_\sigma^j) \log(\eta(U^j))^2 \quad (24)$$

⁵Note, the constants γ_{T_i} , γ_U can quite generally have either sign, depending upon the modular weights of the fields at the particular vertex.

where k_a is the Kač-Moody level of the group, which we will normally take to be 1. The constants α_a^i are model dependent, and are defined as

$$\alpha_a^i \equiv \ell(\text{adj}) - \sum_{\text{rep}_I} \ell_a(\text{rep}_I) (1 + 2n_i^I).$$

$\ell(\text{adj})$ and $\ell_a(\text{rep}_I)$ are the Dynkin indices of the adjoint representation and of the matter representation I of the group G_a , respectively [73], and n_i^I are modular weights.⁶ The δ_σ^i terms are necessary to cancel an anomaly in the underlying σ model, which induces a transformation in the dilaton field under the modular group:

$$S \rightarrow S + \frac{1}{8\pi^2} \sum_{i=1}^{h_{(1,1)}} \delta_\sigma^i \log(ic_i T_i + d_i) + \frac{1}{8\pi^2} \sum_{j=1}^{h_{(2,1)}} \delta_\sigma^j \log(ic_j U^j + d_j). \quad (25)$$

It is important to note that the factor

$$(\alpha_a^i - k_a \delta_\sigma^i) \equiv \frac{b_a^{(N=2)}(i)}{|D|/|D_i|} \quad (26)$$

where $b_a^{(N=2)}(i)$ is the beta function coefficient for the i th torus. It is nonzero if and only if the k th twisted sector has an effective $N = 2$ supersymmetry. Moreover this occurs only when, in the k th twisted sector, the i th torus is not rotated. The factors $|D|$, $|D_i|$ are the degree of the twist group D and the little group D_i , which does not rotate the i th torus. For example, for the mini-landscape models with $D = \mathbb{Z}_6$ -II we have $|D| = 6$ and $|D_2| = 2$, $|D_3| = 3$ since the little group keeping the second (third) torus fixed is $\mathbb{Z}_2(\mathbb{Z}_3)$. The first torus is rotated in all twisted sectors. Hence, the gauge kinetic function for the mini-landscape models is only a function of T_2 and T_3 .

Taking into account the sigma model anomalies, the heterotic string Kähler potential has the following form, where we have included the loop corrections to the dilaton [68,70]

$$\mathcal{K} = -\log\left(S + \bar{S} + \frac{1}{8\pi^2} \sum_{i=1}^{h_{(1,1)}} \delta_\sigma^i \log(T^i + \bar{T}^i) + \frac{1}{8\pi^2} \sum_{j=1}^{h_{(2,1)}} \delta_\sigma^j \log(U^j + \bar{U}^j)\right) - \sum_{i=1}^{h_{(1,1)}} \log(T^i + \bar{T}^i) - \sum_{j=1}^{h_{(2,1)}} \log(U^j + \bar{U}^j). \quad (27)$$

The first line of Eq. (27) is modular invariant by itself, and one can redefine the dilaton, Y , such that

⁶If T_a^r are the generators of the group G_a in the representation r , then we have $\text{Tr}(T_a^r T_b^r) = \ell_a(\text{rep}_r) \delta_{ab}$.

$$\begin{aligned}
 Y \equiv S + \bar{S} + \frac{1}{8\pi^2} \sum_{i=1}^{h_{(1,1)}} \delta_{\sigma}^i \log(T^i + \bar{T}^i) \\
 + \frac{1}{8\pi^2} \sum_{j=1}^{h_{(1,2)}} \delta_{\sigma}^j \log(U^j + \bar{U}^j), \quad (28)
 \end{aligned}$$

where Y is invariant under the modular transformations.

D. Nonperturbative superpotential

In all mini-landscape models [24], and most orbifold heterotic string constructions, there exists a hidden sector with non-Abelian gauge interactions and vectorlike matter carrying hidden sector charge. In the benchmark models [9] the hidden sector gauge group is $SU(4)$ with chiral matter in the $4 + \bar{4}$ representation.

In this section, let us consider a generic hidden sector with gauge group $SU(N_1) \otimes SU(N_2) \otimes U(1)_A$, where ‘‘A’’ stands for anomalous. There are N_{f_1} and N_{f_2} flavors of quarks Q_1 and Q_2 in the fundamental representation (along with antiquarks \bar{Q}_1 and \bar{Q}_2 , in the antifundamental representations), as well as two singlet fields, called ϕ and χ . The charge assignments are listed in Table I. We assume the existence of two moduli, S and T , which enter the nonperturbative superpotential through the gauge kinetic function, namely $f = f(S, T)$. The model also allows for T dependence in the Yukawa sector.

Nonperturbative effects generate a potential for the S and T moduli. Gaugino condensation will generate a scale Λ_{SQCD} , which is determined purely by the symmetries of the low-energy theory:

$$\Lambda_a(S, T) = e^{-(8\pi^2/\beta_a)f_a(S, T)}, \quad (29)$$

where $\beta_a = 3N_a - N_{f_a}$ is the one-loop beta function coefficient of the theory. At tree level $f_a(S, T) = S$, however, we include the possibility of threshold corrections which introduce a dependence on the T modulus [68,70]. We also find that $U(1)_A$ and modular invariance together dictate a very specific form for the nonperturbative superpotential.

In the mini-landscape analysis the effective mass terms for the vectorlike exotics were evaluated. They were given as a polynomial in products of chiral MSSM singlet fields (chiral moduli). It was shown that all vectorlike exotics obtain mass⁷ when the chiral moduli obtain VEVs at supersymmetric points in moduli space. In our example let us, for simplicity, take couplings between the quarks and the field ϕ to be diagonal in flavor space. Mass terms of the form

$$\mathbb{M}_1(\phi, T)Q_1\bar{Q}_1 + \mathbb{M}_2(\phi, T)Q_2\bar{Q}_2 \quad (30)$$

are dynamically generated when ϕ receives a nonzero VEV, which we will discuss below. A key assumption is

⁷In fact, one of the $SU(4)$ quark- antiquark pairs remained massless in the two benchmark models.

TABLE I. Charge assignments for the fields in a generic hidden sector. Flavor indices are suppressed.

	ϕ	χ	Q_1	Q_2	\bar{Q}_1	\bar{Q}_2
$U(1)_A$	-1	q_χ	q_1	q_2	\bar{q}_1	\bar{q}_2
$SU(N_1)$	1	1	\square	1	$\bar{\square}$	1
$SU(N_2)$	1	1	1	\square	1	$\bar{\square}$

that those mass terms are larger than the scale of gaugino condensation, so that the quarks and antiquarks may be consistently integrated out. If this can be accomplished, then one can work in the pure gauge limit [74].⁸

Before we integrate out the meson fields, the nonperturbative superpotential (plus quark masses) for $N_{f_a} < N_a$ is of the form [75]

$$\begin{aligned}
 \mathcal{W}_{\text{NP}} = \sum_{a=1,2} \left[\mathbb{M}_a(\phi, T)Q_a\bar{Q}_a + (N_a - N_{f_a}) \right. \\
 \left. \times \left(\frac{\Lambda_a^{3N_a - N_{f_a}}}{\det Q_a \bar{Q}_a} \right)^{1/(N_a - N_{f_a})} \right], \quad (31)
 \end{aligned}$$

with $\mathbb{M}_a(\phi, T) = c_a e^{-b_a T} \phi^{q_a + \bar{q}_a}$ where c_a is a constant. Note, given the charges for the fields in Table I and using Eqs. (4), (7), and (29), one sees that \mathcal{W}_{NP} is $U(1)_A$ invariant. The Kähler potential for the hidden sector is assumed to be of the form

$$\begin{aligned}
 \mathcal{K} = -\log(S + \bar{S}) - 3\log(T + \bar{T}) + \alpha_\phi \bar{\phi} e^{-2V_A} \phi \\
 + \alpha_\chi \bar{\chi} e^{2q_\chi V_A} \chi + \sum_{a=1,2} \alpha_a (\bar{Q}_a e^{V_a + 2q_a V_A} Q_a \\
 + \bar{\bar{Q}}_a e^{-V_a + 2\bar{q}_a V_A} \bar{Q}_a). \quad (32)
 \end{aligned}$$

The quantities α_ϕ , α_χ , α_i are generally functions of the modulus T , where the precise functional dependence is fixed by the modular weights of the fields (see Sec. II B). V_i and V_A denote the vector superfields associated with the gauge groups $\mathcal{G}_i = SU(N_i)$ and $U(1)_A$.

The determinant of the quark mass matrix is given by

$$\det \mathbb{M}_a(\phi, T) = (c_a e^{-b_a T} \phi^{q_a + \bar{q}_a})^{N_{f_a}}. \quad (33)$$

We have taken the couplings between ϕ and the quarks to have exponential dependence on the T modulus, an ansatz which is justified by modular invariance (see Sec. II B). Inserting the meson equations of motion and Eq. (33) into Eq. (31), we have

$$\begin{aligned}
 \mathcal{W}_{\text{NP}} = \sum_{a=1,2} [N_a (c_a e^{-b_a T} \phi^{q_a + \bar{q}_a})^{N_{f_a}/N_a} \\
 \times [\Lambda_a(S, T)]^{(3N_a - N_{f_a})/N_a}].
 \end{aligned}$$

⁸There is a check on the consistency of this approach: at the end of the day, after calculating the VEVs of the scalars, we can verify that the mass terms for the quarks are indeed of the correct magnitude.

Note that the transformation of the superpotential under the modular group in Eq. (15) also requires that the (non-perturbative) superpotential obey

$$\mathcal{W}_{\text{NP}} \rightarrow \prod_{i=1}^{h_{(1,1)}} \prod_{j=1}^{h_{(2,1)}} (ic_i T^i + d_i)^{-1} (ic_j U^j + d_j)^{-1} \mathcal{W}_{\text{NP}}. \quad (34)$$

Because the nonperturbative Lagrangian must be invariant under all of the symmetries of the underlying string theory, it must be that [71,76–80]

$$\begin{aligned} \mathcal{W}_{\text{NP}} \equiv & A \times e^{-aS} \prod_{i=1}^{h_{(1,1)}} \prod_{j=1}^{h_{(2,1)}} (\eta(T^i))^{-2+(3/(4\pi^2\beta))\delta_i^\sigma} \\ & \times (\eta(U^j))^{-2+(3/(4\pi^2\beta))\delta_j^\sigma} \end{aligned} \quad (35)$$

where $a \equiv \frac{24\pi^2}{\beta}$ and $\beta = 3\ell(\text{adj}) - \sum_I \ell(\text{rep}_I)$ is the one-loop beta function coefficient, and A is generally a function of the chiral matter fields appearing in \mathbb{M} . This, coupled with the one-loop gauge kinetic function in Eq. (24), gives the heterotic generalization of the racetrack superpotential.

In the following section (III), we construct a simple model using the qualitative features outlined in this section. This model is novel because it requires only one non-Abelian gauge group to stabilize moduli and give a de Sitter vacuum. We have also constructed two condensate models, however, the literature already contains several examples of the ‘‘racetrack’’ in regards to stabilization of S and T moduli. Moreover in the mini-landscape models, whose features we are seeking to reproduce, there are many examples of hidden sectors containing a single non-Abelian gauge group [24], while there are no examples with multiple hidden sectors.

III. MODULI STABILIZATION AND SUPERSYMMETRY BREAKING IN THE BULK

In this section we construct a simple, generic heterotic orbifold model which captures many of the features discussed in Sec. II. In particular, it is a single gaugino condensate model with the following fields: dilaton (S), modulus (T), and MSSM singlets (ϕ_1, ϕ_2, χ). The model has one anomalous $U(1)_A$ with the singlet charges given by ($q_{\phi_1} = -2, q_{\phi_2} = -9, q_\chi = 20$). The Kähler and superpotential are given by⁹

$$\begin{aligned} \mathcal{K} = & -\log[S + \bar{S}] - 3\log[T + \bar{T}] \\ & + \bar{\phi}_1 \phi_1 + \bar{\phi}_2 \phi_2 + \bar{\chi} \chi \end{aligned} \quad (36)$$

⁹The coefficient A [Eq. (37)] is an implicit function of all other nonvanishing chiral singlet VEVs which would be necessary to satisfy the modular invariance constraints, i.e., $A = A(\langle\phi_I\rangle)$. If one rescales the $U(1)_A$ charges, $q_{\phi_i}, q_\chi \rightarrow q_{\phi_i}/r, q_\chi/r$, then the $U(1)_A$ constraint is satisfied with $r = 15p$ (assuming no additional singlets in A). Otherwise we may let r and p be independent. This rescaling does not affect our analysis, since the vacuum value of the ϕ_i, χ term in the superpotential vanishes.

$$\mathcal{W} = e^{-bT} (w_0 + \chi(\phi_1^{10} + \lambda\phi_1\phi_2^2)) + A\phi_2^p e^{-aS-b_2T}. \quad (37)$$

In addition, there is an anomalous $U(1)_A$ D -term given by

$$D_A = 20\bar{\chi}\chi - 2\bar{\phi}_1\phi_1 - 9\bar{\phi}_2\phi_2 - \frac{1}{2}\delta_{\text{GS}}\partial_S\mathcal{K} \quad (38)$$

with $\delta_{\text{GS}} = \frac{(q+\bar{q})N_f}{4\pi^2} = N_f/(4\pi^2)$.

In the absence of the nonperturbative term (with coefficient A) the theory has a supersymmetric minimum with $\langle\chi\rangle = \langle\phi_1\rangle = 0$ and $\langle\phi_2\rangle \neq 0$ and arbitrary. This property mirrors the situation in the mini-landscape models where supersymmetric vacua have been found in the limit that all nonperturbative effects are neglected. We have also added a constant $w_0 = w_0(\langle\phi_I\rangle)$ which is expected to be generated (in the mini-landscape models) at high order in the product of chiral moduli due to the explicit breaking of an accidental R symmetry which exists at lower orders [81].¹⁰ The T dependence in the superpotential is designed to take into account, in a qualitative way, the modular invariance constraints of Sec. II B. We have included only one T modulus, assuming that the others can be stabilized near the self-dual point [76,82]. Moreover, as argued earlier, the T_i and U moduli enter the superpotential in different ways (see Sec. II B). This leads to modular invariant solutions which are typically anisotropic [67].¹¹

Note that the structure, $\mathcal{W} \sim w_0 e^{-bT} + \phi_2 e^{-aS-b_2T}$ gives us the crucial progress¹²:

- (i) a hybrid KKLT kind of superpotential that behaves like a single condensate for the dilaton S , but as a racetrack for the T and, by extension, also for the U moduli; and
- (ii) an additional matter F_{ϕ_2} term driven by the cancellation of the anomalous $U(1)_A$ D -term seeds SUSY breaking with successful up lifting.

The constant b is fixed by modular invariance constraints. In general the two terms in the perturbative superpotential would have different T dependence. We have found solutions for this case as well. This is possible since the VEV of the χ term in the superpotential vanishes. The second term (proportional to A) represents the nonperturbative contribution of one gaugino condensate. The constants $a = 24\pi^2/\beta$, b_2 , and p depend on the size of the gauge group, the number of flavors, and the coefficient of the one-loop beta function for the effective $N = 2$ supersymmetry of the torus T . For the mini-landscape models, this would be either T_2 or T_3 . Finally, the coefficient of the exponential factor of the dilaton S is taken to be $A\phi_2^p$.

¹⁰The fields entering w_0 have string scale mass.

¹¹Note, we have chosen to keep the form of the Kähler potential for this single T modulus with the factor of 3, so as to maintain the approximate no-scale behavior.

¹²Note, the constants b, b_2 can have either sign. For the case with $b, b_2 > 0$ the superpotential for T is racetracklike. However for $b, b_2 < 0$ the scalar potential for T diverges as T goes to zero or infinity and compactification is guaranteed [76,82].

TABLE II. Input values for the superpotential parameters for three different cases. Case 2 has a vanishing one-loop correction for ϕ_2 .

Case	b	b_2	λ	R	p	r	A	w_0
1	$\pi/50$	$3\pi/2$	33	10	$2/5$	$15p$	160	8×10^{-15}
2	$8/125$	$3\pi/2$	0	5	$2/5$	$15p$	30	42×10^{-16}
3	$1/16$	$29\pi/20$	38	10	$2/5$	$15p$	90	6×10^{-15}
4	$-\pi/120$	$-\pi/40$	40	64	$2/3$	1	$1/10$	-5×10^{-15}
5	$-\pi/250$	$-\pi/100$	25	16	1	$10/3$	$7/5$	-7×10^{-15}

This represents the effective hidden sector quark mass term, which in this case is proportional to a power of the chiral singlet ϕ_2 . In a more general case, it would be a polynomial in powers of chiral moduli.¹³ The exponent p depends in general on the size of the gauge group, the number of flavors and the power that the field ϕ_2 appears in the effective quark mass term.

We have performed a numerical evaluation of the scalar potential with the following input parameters. We take hidden sector gauge group $SU(N)$ with $N = 5$, $N_f = 3$, and $a = 8\pi^2/N$.¹⁴ For the other input values we have considered five different possibilities given in Table II.¹⁵ We find that supersymmetry breaking, moduli stabilization, and up lifting is a direct consequence of adding the nonperturbative superpotential term.

In our analysis we use the scalar potential V given by

$$V = e^K \left(\sum_{i=1}^5 \sum_{j=1}^5 [F_{\Phi_i} \bar{F}_{\Phi_j} \mathcal{K}_{i,j}^{-1} - 3|W|^2] \right) + \frac{D_A^2}{(S + \bar{S})} + \Delta V_{\text{CW}}[\Phi_i, \bar{\Phi}_i] \quad (39)$$

where $\Phi_{i,j} = \{S, T, \chi, \phi_1, \phi_2\}$ and $F_{\Phi_i} \equiv \partial_{\Phi_i} \mathcal{W} + (\partial_{\Phi_i} \mathcal{K}) \mathcal{W}$. The first two terms are the tree level supergravity potential. The last term is a one-loop correction which affects the vacuum energy and D -term contribution.

The one-loop Coleman-Weinberg (CW) potential is in general given by

$$\Delta V_{\text{CW}} = \frac{1}{32\pi^2} \text{Str}(M^2) \Lambda^2 + \frac{1}{64\pi^2} \text{Str} \left(M^4 \log \left[\frac{M^2}{\Lambda^2} \right] \right) \quad (40)$$

¹³Holomorphic gauge invariant monomials span the moduli space of supersymmetric vacua. One such monomial is necessary to cancel the Fayet-Illiopoulos D -term (see, Appendix B).

¹⁴We have also found solutions for the case with $N = 4$, $N_f = 7$ which is closer to the mini-landscape benchmark models. Note, when $N_f > N$ we may still use the same formalism, since we assume that all the Q, \bar{Q} s get mass much above the effective QCD scale.

¹⁵Note the parameter relation $r = 15p$ in Table II is derived using $U(1)_A$ invariance and the assumption that no other fields with nonvanishing $U(1)_A$ charge enter into the effective mass matrix for hidden sector quarks. We have also allowed for two cases where this relation is not satisfied.

with the mass matrix M given by $M = M(\Phi_i)$ and Λ is the relevant cut off in the problem. We take $\Lambda = M_S \sim 10^{17}$ GeV.

We have not evaluated the full one-loop correction. Instead we use the approximate formula

$$\Delta V_{\text{CW}}[\phi_2, \bar{\phi}_2] = \frac{\lambda^2 F_2^2 |\phi_2|^2}{8\pi^2} (\log[R(\lambda|\phi_2|^2)^2] + 3/2) + \mathcal{O}(\Lambda^2) \quad (41)$$

where $F_2 = \langle F_{\phi_2} \rangle$ is obtained self-consistently and all dimensional quantities are expressed in Planck units. This one-loop expression results from the χ, ϕ_1 contributions to the Coleman-Weinberg formula. The term quadratic in the cutoff is naturally proportional to the number of chiral multiplets in the theory and could be expected to contribute a small amount to the vacuum energy, of order a few percent times $m_{3/2}^2 M_{\text{pl}}^2$. We will discuss this contribution later, after finding the minima of the potential. Finally, note that the parameters λ, R in Table II might both be expected to be significantly greater than one when written in Planck units. This is because the scale of the effective higher dimensional operator with coefficient λ in Eq. (37) is most likely set by some value between M_{pl} and M_{string} and the cutoff scale for the one-loop calculation (which determines the constant R) is the string scale and not M_{pl} .

In all cases we find a metastable minimum with all (except for two massless modes) fields massive of $\mathcal{O}(\text{TeV})$ or larger. Supersymmetry is broken at the minimum with values given in Table III. Note $\text{Re}S \sim 2.2$ and $\text{Re}T$ ranges between 1.1 and 1.6. The moduli χ, ϕ_1 are stabilized at their global minima $\phi_1 = \chi = 0$ with $F_\chi = F_{\phi_1} = 0$ in all cases. The modulus $\sigma = \text{Im}S$ is stabilized at $\sigma \approx 1$ in the racetrack cases 1, 2, and 3. This value enforces a relative negative sign between the two terms dependent on $\text{Re}T$. We plot the scalar potential V in the $\text{Re}T$ direction for case 2 ($b, b_2 > 0$) [Fig. 1(a)] and for case 4 ($b, b_2 < 0$) [Fig. 1(b)]. Note the potential as a function of $\text{Re}S$ is qualitatively the same for both cases (Fig. 2).

At the metastable minimum of the scalar potential we find a vacuum energy which is slightly negative, i.e., of order $(-0.03 \text{ to } -0.01) \times 3m_{3/2}^2 M_{\text{pl}}^2$ (see Table III). Note, however, one-loop radiative corrections to the vacuum energy are of order $(N_T m_{3/2}^2 M_S^2 / 16\pi^2)$, where N_T is

TABLE III. The values for field VEVs and soft SUSY breaking parameters at the minimum of the scalar potential. Note $F_\Phi \equiv \partial_\Phi \mathcal{W} + (\partial_\Phi \mathcal{K}) \mathcal{W}$.

	Case 1	Case 2	Case 3	Case 4	Case 5
$\langle s \rangle$	2.2	2.2	2.1	2.1	2.2
$\langle t \rangle$	1.2	1.1	1.6	1.1	1.1
$\langle \sigma \rangle$	1.0	1.0	1.0	0.0	0.0
$\langle \phi_2 \rangle$	0.08	0.08	0.08	0.03	0.06
F_S	2.8×10^{-16}	1.3×10^{-16}	2.7×10^{-16}	1.1×10^{-16}	8.0×10^{-17}
F_T	-8.7×10^{-15}	-5.1×10^{-15}	-5.0×10^{-15}	6.7×10^{-15}	9.1×10^{-15}
F_{ϕ_2}	-9.2×10^{-17}	-4.5×10^{-17}	-8.9×10^{-17}	1.3×10^{-15}	1.3×10^{-15}
D_A	4.4×10^{-31}	1.0×10^{-32}	5.9×10^{-31}	-3.8×10^{-31}	-4.8×10^{-32}
$D_A/m_{3/2}^2$	0.6	0.03	2.7	-0.7	-0.05
$V_0/(3m_{3/2}^2)$	-0.02	-0.01	-0.02	-0.03	-0.02
$m_{3/2}$	2.2 TeV	1.4 TeV	1.1 TeV	1.8 TeV	2.4 TeV

the total number of chiral multiplets [83] and we have assumed a cutoff at the string scale M_S . With typical values $N_T \sim \mathcal{O}(300)$ and $M_S/M_{\text{Pl}} \sim 0.1$, this can easily lift the vacuum energy the rest of the way to give a small positive effective cosmological constant which is thus a metastable local de Sitter minimum. Note that the constants λ , R have also been used to adjust the value of the cosmological constant as well as, and more importantly for LHC phenomenology, the value of D_A (see Fig. 3).

The two massless fields can be seen as the result of two $U(1)$ symmetries; the first is a $U(1)_R$ symmetry and the second is associated with the anomalous $U(1)_A$. The $U(1)_R$ is likely generic (but approximate), since even the ‘‘constant’’ superpotential term needed to obtain a small cosmological constant necessarily comes with $\eta(T)$ moduli dependence. Since we have approximated $\eta(T) \sim \exp(-\pi T/12)$ by the first term in the series expansion [Eq. (17)], the symmetry is exact. However higher order terms in the expansion necessarily break the $U(1)_R$ symmetry. The $U(1)_A$ symmetry is gauged.

One can express the fields S , T , and ϕ_2 in the following basis¹⁶:

$$S \equiv s + i\sigma, \quad T \equiv t + i\tau, \quad \phi_2 \equiv \varphi_2 e^{i\theta_2}. \quad (42)$$

The transformation properties of the fields σ , τ , and θ_2 under the two $U(1)$ ’s are given by

$$\begin{aligned} U(1)_R: & \begin{cases} \tau \rightarrow \tau + c \\ \sigma \rightarrow \sigma + \frac{-b_2+b}{a}c \end{cases}, \\ U(1)_A: & \begin{cases} \theta \rightarrow \theta - \frac{2}{r}c' \\ \sigma \rightarrow \sigma - \frac{9p}{a'r}c' \end{cases}, \end{aligned} \quad (43)$$

where c , c' are arbitrary constants and for the definition of r , see Footnote 9. The corresponding Nambu-Goldstone (NG) bosons are given by

$$\begin{aligned} \chi_{\text{NG}}^1 &= \frac{a}{-b_2+b} \sigma + \tau, \\ \chi_{\text{NG}}^2 &= \tilde{N} \left(-\sigma + \frac{-b_2+b}{a} \tau \right) + \frac{1}{p} \theta_2, \end{aligned} \quad (44)$$

where \tilde{N} is a normalization factor. One can then calculate the mass matrix in the $\sigma - \tau - \theta_2$ basis and find two zero eigenvalues (as expected) and one nonzero eigenvalue. The two NG modes, in all cases, can be shown to be linear combinations of the two eigenvectors of the two massless states. The $U(1)_A$ NG boson is eaten by the $U(1)_A$ gauge boson, while the $U(1)_R$ pseudo-NG boson remains as an ‘‘invisible axion’’ [84]. The $U(1)_R$ symmetry is nonperturbatively broken (by world sheet instantons) at a scale of order

$$\langle e^{\mathcal{K}/2} \mathcal{W} e^{-\pi T} \rangle \approx m_{3/2} \langle e^{-\pi T} \rangle \sim 0.02 m_{3/2} \quad (45)$$

in Planck units, resulting in an ‘‘axion’’ mass of order 10 GeV and decay constant of order M_{Pl} .¹⁷

Before discussing the rest of the moduli, in a more complete string model, and how they would be stabilized or the LHC phenomenology of the mini-version of the mini-landscape models, it is worth comparing our analysis with some previous discussions in the literature.

In a series of two papers by Dvali and Pomarol [31,32], the authors consider an anomalous $U(1)$ with two charged singlet fields. The D -term is given by¹⁸

$$D_A = q|\phi_+|^2 - |q_-|^2 + \xi. \quad (46)$$

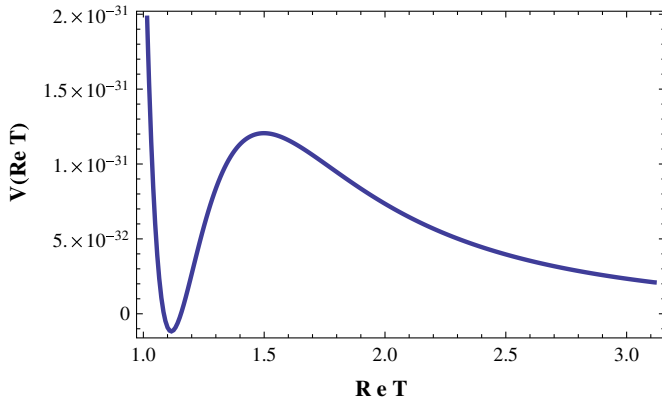
The gauge invariant superpotential is

$$\mathcal{W} = m\phi_+\phi_-, \quad (47)$$

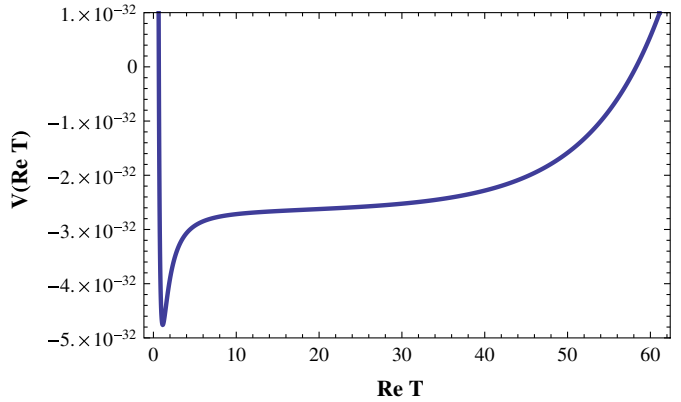
¹⁶The fields χ and ϕ_1 cannot be expressed in polar coordinates as they receive zero VEV, and cannot be canonically normalized in this basis.

¹⁷In addition, the heterotic orbifold models might very well have the standard invisible axion [85].

¹⁸We refer to the anomalous $U(1)$ as $U(1)_A$ and not $U(1)_X$, as in the papers referenced below.



(a) The scalar potential in Case 2 for $\text{Re } T$, with $b_i > 0$.



(b) The scalar potential in Case 4 for $\text{Re } T$, with $b_i < 0$.

FIG. 1 (color online). As $\text{Re } T \rightarrow \infty$, the potential for $b_i > 0$ mimics a racetrack, which can be seen from Eq. (37), for example. In the case where $b_i < 0$, however, the potential exhibits a different asymptotic behavior. As $\text{Re } T \rightarrow \infty$ the potential diverges, which means that theory is forced to be compactified [76,82].

where m has some charge under $U(1)_A$. They suggest a few different ways to generate m . The first is with some high power of 1 of the ϕ fields:

$$\mathcal{W} \sim \phi_-^q \phi_+ \Rightarrow m \equiv \langle \phi_- \rangle^{q-1}. \quad (48)$$

The second is by giving the ϕ a coupling to some quarks from a SUSY QCD (SQCD) theory that becomes strongly coupled. The scale, Λ_{SQCD} then serves as the mass term in the superpotential. They do not, however, consider dilaton dependence, and their D -term is static, not dynamic. They also work in the global SUSY limit, so they do not consider up lifting.

In a paper by Binetruy and Dudas [30], the authors assume that S can be stabilized at some finite value S_0 , possibly through some extra S dependent term in the superpotential and they assume that $F_S(S_0) = 0$. In their setup, they have an anomalous $U(1)$, some charged singlets, and some hidden sector SQCD with matter. The singlets couple to matter, and SQCD becomes strongly coupled,

generating a scale, just as in our analysis. Since they are working in the global SUSY limit, they are not concerned with up lifting.

Lalak [33] considers several types of models with an anomalous $U(1)$, some charged singlets, and some coupling to the dilaton S . In the last section, he considers superpotentials with an exponential dependence on S . He then assumes that S_0 is a (globally) supersymmetric minimum of the potential. Also, working in global SUSY, he does not address up lifting.

In a paper by Dudas and Mambrini [36], the authors consider one modulus, one singlet field, and an $SU(N)$ with one flavor of quarks. The $SU(N)$ becomes strongly coupled, and the superpotential and Kähler potential look like:

$$\mathcal{W} = w_0 + (c/X^2)e^{-aT} + m\phi^q X \quad (49)$$

$$\mathcal{K} = -3 \log(T + \bar{T} - |X|^2 - |\phi|^2), \quad (50)$$

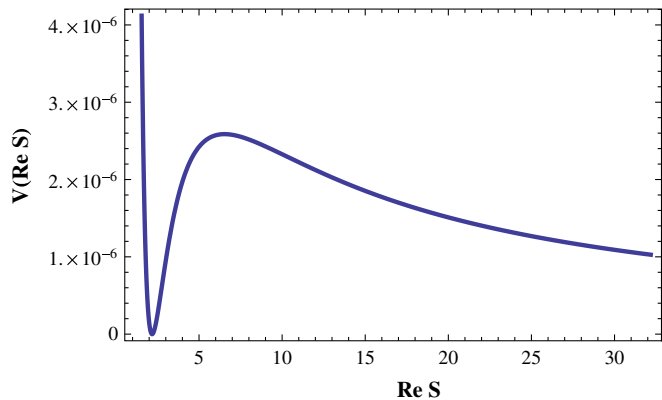


FIG. 2 (color online). The scalar potential in the $\text{Re } S$ direction for case 2.

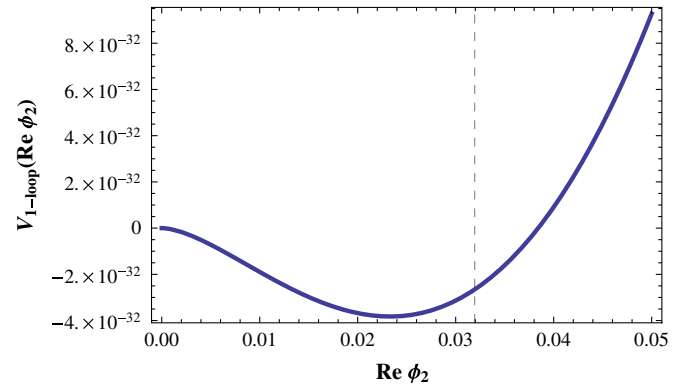


FIG. 3 (color online). The one-loop Coleman-Weinberg potential (case 4) for ϕ_2 . The dashed line represents the VEV of ϕ_2 in the minimum of the *full* potential.

where X is the meson field and ϕ is the singlet. Note, the modulus appearing in the exponent is T , not S . They find that the only consistent minimum with approximately zero cosmological constant requires $m_{3/2} \sim \xi$. So either the gravitino mass is of order the GUT scale or for the gravitino mass of order a TeV, the meson charge must satisfy $q \sim 10^{-8}$.

In a paper by Dudas *et al.* [38], the authors consider a single modulus and two singlet fields:

$$D_A = |\phi_+|^2 - |\phi_-|^2 + \xi, \quad (51)$$

$$\mathcal{W} = w_0 + m\phi_+\phi_- + a\phi_-^q e^{-bT}. \quad (52)$$

They do not discuss the origin of the constant w_0 . They suggest that m might come from nonperturbative effects. Note the latter is crucial, since m affects the up lifting of the scalar potential. They are also interested in large volume compactifications, as $t \equiv \text{Re}T \approx 60$. Given their SUSY breaking scheme, they go on to look at the low-energy spectrum. However, they neglect the D -term contributions to the soft masses, claiming that there are only two possibilities for the low-energy physics:

- (i) Because $\xi > 0$, some SM quarks and leptons carry positive $U(1)_A$ charges. This leads to scalar masses (for them) of around 100 TeV, and may give an unstable low-energy spectrum.
- (ii) All SM quarks and leptons are neutral under $U(1)_X$. This implies that there should be more matter that is charged under the MSSM and $U(1)_A$.

It seems that they have missed an important possibility, namely, that matter in the MSSM appears with $U(1)_A$ charges of both signs. This actually seems to be the generic case, at least in the mini-landscape models.

The last paper we consider, by Gallego and Serone [39], contains an analysis which is possibly most similar to that in this paper. There are however two major differences. If one neglects all nonperturbative dependence on the dilaton and Kähler moduli, then their superpotential is of the form $\mathcal{W} \supset \phi^q \chi$ and the D-term is given by $D_A = q|\chi|^2 - |\phi|^2 + \xi$. Hence the model does not have a supersymmetric minimum in the global limit, due to a conflict between $F_\chi = 0$ and $D_A = 0$. However in our model [Eq. (37)] there is a supersymmetric solution when nonperturbative effects are ignored. Finally, the authors were not able to find a supersymmetry breaking solution, like ours, with just one hidden non-Abelian gauge sector.

As an aside, we note that Casas *et al.* [79] study a similar problem of moduli stabilization and SUSY breaking, but without the anomalous $U(1)$. However, their model is very different from ours, but they do include the one-loop Coleman-Weinberg corrections.

IV. MODULI STABILIZATION CONTINUED—THE TWISTED SECTOR AND BLOW UP MODULI

In our discussion above we considered a simple model which is representative of heterotic orbifold models. Our simple model had only a few moduli, i.e., the dilaton, S , a volume modulus, T , and three chiral singlet “moduli,” χ , ϕ_1 , ϕ_2 . Any heterotic orbifold construction, on the other hand, will have several volume and complex structure moduli and, of order 50 to 100 chiral singlet moduli. The superpotential for the chiral singlet moduli is obtained as a polynomial product of holomorphic gauge invariant monomials which typically contain hundreds of terms at each order (with the number of terms increasing with the order). In the mini-landscape analysis, supersymmetric vacua satisfying $F = D = 0$ constraints to sixth order in chiral singlet moduli could be found. Although there are many flat directions in moduli space, the anomalous D -term fixes at least one holomorphic gauge invariant monomial to have a large value. Our simple model expressed this fact with the chiral singlets χ , ϕ_1 , ϕ_2 , where the VEVs were fixed by the global SUSY minimum with $\langle \phi_2 \rangle$ fixed by the $U(1)_A$ D -term.

In addition to the non-Abelian hidden gauge sector considered in the simple model, a generic orbifold vacuum also has additional $U(1)$ gauge interactions and vectorlike exotics which obtain mass proportional to chiral singlet VEVs. Some of these singlets are assumed to get large VEVs (of order the string scale). These are the ones giving mass to the extra $U(1)$ gauge sector and vectorlike exotics. These same VEVs generate nontrivial Yukawa couplings for quarks and leptons. Moreover, there are chiral singlets which get zero VEVs, such as χ and ϕ_1 . For example, in the mini-landscape benchmark model 1, the electroweak Higgs μ -term is zero in the supersymmetric limit. The question arises as to what happens to all these VEVs once supersymmetry is broken.

We now sketch the fact that the supersymmetry breaking discussed above, ensuing from F -terms, $F_S, F_T, F_{\phi_2} \neq 0$ and driven by the nonperturbative superpotential, inevitably leads to a stabilization of the many singlet moduli of the heterotic orbifold vacuum. We shall consider here 3 classes of heterotic MSSM singlets.

A. Singlets with polynomial Yukawa couplings

Let us first consider singlets having polynomial Yukawa couplings in the superpotential, which in case of a coupling arising among purely untwisted sector fields $\phi_i^{(U)}$ are perturbatively generated, and in the other case involving at least one twisted sector field $\phi_i^{(T)}$ are nonperturbatively generated (see, Sec. II B). The latter case is actually the most common situation. Restricting again for reasons of simplicity to the case of a single scalar field of the type under consideration, we can describe the two cases as follows:

(i)

$$\begin{aligned}\mathcal{K} &= -3 \log(T + \bar{T} - \bar{\phi}^{(U)} \phi^{(U)}), \\ \mathcal{W} &\supset \lambda \cdot (\phi^{(U)})^N, \quad N \geq 3.\end{aligned}$$

Note that the untwisted sector scalar fields $\phi^{(U)}$, being inherited from the bulk **248** in 10D, appear this way in the Kähler potential.

(ii)

$$\begin{aligned}\mathcal{K} &= -3 \log(T + \bar{T}) + c \bar{\phi}^{(T)} \phi^{(T)}, \\ \mathcal{W} &\supset e^{-bT} (\phi^{(T)})^N, \quad N \geq 3.\end{aligned}$$

Here the exponential dependence on T arises from the η function, which a nonperturbatively generated Yukawa coupling must have for reasons of modular invariance (see, Sec. **II B**).

(iii)

$$\begin{aligned}\mathcal{K} &= -3 \log(T + \bar{T} - \bar{\phi}^{(U)} \phi^{(U)}) + c \bar{\phi}^{(T)} \phi^{(T)} \\ \mathcal{W} &\supset \lambda e^{-bT} (\phi^{(T)})^N + \tilde{\lambda} e^{-\tilde{b}T} (\phi^{(T)})^{\tilde{N}} (\phi^{(U)})^M \\ &\text{with } M, N, \tilde{N} \geq 2.\end{aligned}$$

Here, too, the exponential dependence on T from the η function dependence of a nonperturbatively generated Yukawa coupling.

The calculation in case (i) simplifies by the fact that there K fulfills an extended no-scale relation

$$\begin{aligned}\mathcal{K}_i \mathcal{K}^{\bar{i}j} \mathcal{K}_{\bar{j}} &= 3 \quad \forall i, j = T, \phi^{(U)} \\ \mathcal{K}^i &= \mathcal{K}^{\bar{i}j} \mathcal{K}_{\bar{j}} = -\mathcal{V} \cdot \delta_T^i, \\ \mathcal{V} &\equiv (T + \bar{T} - \bar{\phi}^{(U)} \phi^{(U)})\end{aligned}\quad (53)$$

which implies for the F -term scalar potential a result of

$$\begin{aligned}V_F &= e \mathcal{K} \left[\mathcal{K}^{\phi^{(U)} \bar{\phi}^{(U)}} (|\partial_{\phi^{(U)}} \mathcal{W}|^2 \right. \\ &\quad \left. + (\partial_{\phi^{(U)}} \mathcal{W} \cdot \overline{\mathcal{K}_{\phi^{(U)}} \mathcal{W}} + \text{c.c.})) \right. \\ &\quad \left. + \frac{\mathcal{V}}{3} (T + \bar{T}) |\partial_T \mathcal{W}|^2 + (\mathcal{V} \partial_T \mathcal{W} + \text{c.c.}) \right].\end{aligned}\quad (54)$$

It is clear then that one solution to $\partial_{\phi^{(U)}} V_F = 0$ is given by

$$\partial_{\phi^{(U)}} \mathcal{W} = \partial_{\phi^{(U)}} \mathcal{V} = 0 \Rightarrow \langle \phi^{(U)} \rangle = 0 \quad (55)$$

because $\partial_{\phi^{(U)}} \partial_T \mathcal{W} \equiv 0 \quad \forall \phi^{(U)}$. This implies that those untwisted sector singlets that were stabilized at the origin in global supersymmetry by a purely untwisted sector Yukawa coupling remain so even in supergravity.

For the twisted sector case (ii) we find the scalar potential to be

$$\begin{aligned}V_F &= e \mathcal{K} [\mathcal{K}^{\phi^{(T)} \bar{\phi}^{(T)}} |D_{\phi^{(T)}} \mathcal{W}|^2 + \mathcal{K}^{T\bar{T}} (|\partial_T \mathcal{W}|^2 \\ &\quad + \underbrace{\partial_T \mathcal{W} \mathcal{K}_{\bar{T}} \mathcal{W}}_{\sim F_T} + \text{c.c.})] \sim e^{-2bT} (\bar{\phi}^{(T)} \phi^{(T)})^{N-1} \\ &\quad - F_T (T + \bar{T}) e^{-bT} (\phi^{(T)})^N + \text{c.c.}\end{aligned}\quad (56)$$

which gives two solutions to $\partial_{\phi^{(T)}} V_F = 0$ as

$$\begin{aligned}\langle \phi^{(T)} \rangle = 0 \quad \forall \quad \langle \phi^{(T)} \rangle &\sim \left(\frac{F_T (T + \bar{T})}{e^{-bT}} \right)^{1/(N-2)} \\ &\sim \left(\frac{m_{3/2}}{e^{-bT}} \right)^{1/(N-2)}.\end{aligned}\quad (57)$$

This implies that the $\phi^{(T)}$ get stabilized either at the origin, or at nonzero but small VEVs $\ll 1$. Their value in the latter case approaches $\phi^{(T)} \sim M_{\text{GUT}}$ for nonperturbative Yukawa couplings of order $N \geq 5$ and $m_{3/2} \sim \text{TeV}$ (which can be interesting for phenomenological reasons involving heavy vectorlike non-MSSM matter).

Finally, we note that case (iii) reduces to case (ii). To see this, note, that the structure of \mathcal{K} and \mathcal{W} given in case (iii) does not change the arguments given for case (i) which implies that in case (iii) we still find $\langle \phi^{(U)} \rangle = 0$. This, however, immediately gives us

$$\mathcal{W}|_{\langle \phi^{(U)} \rangle = 0} \supset \lambda e^{-bT} (\phi^{(T)})^N \quad (58)$$

which is case (ii).

B. Singlet directions which are F and D flat in global supersymmetry

There are many directions in singlet field space in our heterotic constructions which are F and D flat in global supersymmetry. Let us denote these fields by $\phi_i^{(f)}$, and the remaining set of nonflat directions in field space by χ_i . D flatness entails that the D -terms do not depend on the $\phi_i^{(f)}$. F flatness implies that $F_{\phi_i^{(f)}} = \partial_{\phi_i^{(f)}} \mathcal{W}(\phi_i^{(f)}, \chi_i) = \text{const}$ for all values of $\langle \phi_i^{(f)} \rangle$. Generically this implies that $\langle \chi_i \rangle = 0$.

Simplifying to the case of a single χ , this leads to a consideration of 2 cases

(i)

$$\begin{aligned}F_{\phi_i^{(f)}} &= 0 \quad \forall \phi_i^{(f)} \\ \Rightarrow \mathcal{W} &\supset e^{-bT} \chi^{\dagger}(\phi_i) \quad \forall \quad \mathcal{W} \supset e^{-bT} \chi^p \dagger(\phi_i), \\ p &\geq 2\end{aligned}\quad (59)$$

(ii)

$$\begin{aligned}F_{\phi_i^{(f)}} &= \text{const} \neq 0 \\ \forall \phi_i^{(f)} &\Rightarrow \mathcal{W} \supset \lambda e^{-bT} f(\bar{\phi}_j) \phi_i^{(f)}\end{aligned}\quad (60)$$

where the $\tilde{\phi}_j$ VEVs are assumed fixed by other terms in the superpotential and f is an arbitrary function of its argument.

We consider first case (i). At the supersymmetric minimum satisfying $\partial_\chi \mathcal{W} = \partial_{\phi_i} \mathcal{W} = 0$, we have $\langle \chi \rangle = 0$ with $\langle \phi_i \rangle$ arbitrary [subject, for the first case only, to the condition $f(\phi_i) = 0$]. In this example we have $\chi \in \{\chi_i\}$ and $\phi_i \in \{\phi_i^{(f)}\}$. Note the fields $\phi_i^{(f)}$ effectively do not appear in the superpotential at its minimum.

We now argue that the fields $\phi_i^{(f)}$ are stabilized by the corrections from supergravity in the F -term scalar potential. Namely, consider for sake of simplicity the case of a single such field $\phi^{(f)}$ and χ with

$$\begin{aligned} \mathcal{K} &= -3 \log(T + \bar{T}) + c \bar{\phi}^{(f)} \phi^{(f)} + c' \bar{\chi} \chi \\ \partial_\chi \mathcal{W} = \partial_{\phi^{(f)}} \mathcal{W} &\equiv 0 \quad \text{for } \langle \chi \rangle = 0. \end{aligned} \quad (61)$$

We get the F -term scalar potential in supergravity [for the twisted sector case (ii)] we find the scalar potential to be

$$\begin{aligned} V_F &= e \mathcal{K} (\mathcal{K}^{\phi^{(f)} \bar{\phi}^{(f)}} |D_{\phi^{(f)}} \mathcal{W}|^2 + \mathcal{K}^{\chi \bar{\chi}} |D_\chi \mathcal{W}|^2 + \mathcal{K}^{T \bar{T}} |D_T \mathcal{W}|^2 - 3 |\mathcal{W}|^2) = e \mathcal{K} (c \bar{\phi}^{(f)} \phi^{(f)} - \kappa) \cdot |\mathcal{W}|^2 \\ &\approx |\mathcal{W}|^2 \cdot \left[-c(\kappa - 1) \bar{\phi}^{(f)} \phi^{(f)} - \frac{c^2(\kappa - 2)}{2} (\bar{\phi}^{(f)} \phi^{(f)})^2 - \frac{c^3(\kappa - 3)}{6} (\bar{\phi}^{(f)} \phi^{(f)})^3 + \frac{c^4(4 - \kappa)}{24} (\bar{\phi}^{(f)} \phi^{(f)})^4 + \dots \right]. \end{aligned} \quad (62)$$

Note, we maintain $\langle \chi \rangle = 0$, $\mathcal{W} \neq 0$ is due to other sectors of the theory and $\kappa = (3 - \mathcal{K}^{T \bar{T}} |D_T \mathcal{W}|^2 / |\mathcal{W}|^2) \leq 3$ is a positive semidefinite number of order 3. This scalar potential is unbounded from above at large-field values, $\phi^{(f)}$, thus driving the VEV to a large-field value. To this order in V_F we find

$$\langle \phi^{(f)} \rangle \sim \frac{1}{\sqrt{c}}. \quad (63)$$

This implies that supergravity effects will serve to stabilize all the globally supersymmetric and F - and D -flat singlet fields generically at large values of $\mathcal{O}(1)$. Note, that the nonperturbative effects coming from gaugino condensation in the hidden sector will add dependence of \mathcal{W} on $\phi^{(f)}$ beyond the global mini-landscape analysis. This may render κ a weak function of ϕ^f such that we may for some of the globally supersymmetric and F - and D -flat fields $\phi^{(f)}$ have $\kappa < 1$ at small $\phi^{(f)}$ while $1 < \kappa < 3$ at larger values of $\phi^{(f)}$. In this situation the involved $\phi^{(f)}$ -type singlets will acquire vacua at both $\langle \phi^{(f)} \rangle = 0$ and $\langle \phi^{(f)} \rangle \sim 1/\sqrt{c}$. The χ -like fields will have their VEVs near the origin, i.e., they may be shifted from the origin by small SUSY breaking effects.

Let us now turn to case (ii) of F -flat but *nonsupersymmetric* singlet directions and look for vacua stabilizing $\phi^{(f)} \ll 1$ using again

$$\mathcal{K} = -3 \log(T + \bar{T}) + \bar{\phi}^{(f)} \phi^{(f)} + \bar{\chi} \chi. \quad (64)$$

The scalar potential is

$$\begin{aligned} V_F &= e \mathcal{K} [\underbrace{\mathcal{K}^{T \bar{T}} \langle D_T \mathcal{W} \rangle}_{=F_T} \overline{\partial_T \mathcal{W}} + \text{c.c.} \\ &+ \mathcal{K}^{\phi^{(f)} \bar{\phi}^{(f)}} |D_{\phi^{(f)}} \mathcal{W}|^2] \mathcal{K}^{T \bar{T}} F_T \cdot b \lambda e^{-bT} f(\chi) \phi^{(f)} \\ &+ \text{c.c.} + \mathcal{K}^{\phi^{(f)} \bar{\phi}^{(f)}} [\lambda e^{-bT} f(\chi) (1 + \bar{\phi}^{(f)} \phi^{(f)}) \\ &+ \bar{\phi}^{(f)} \langle \mathcal{W} \rangle]^2. \end{aligned} \quad (65)$$

In the desired regime of $\phi^{(f)} \ll 1$ this gives us two subcases:

ia

$$\mathcal{K}^{\phi^{(f)} \bar{\phi}^{(f)}} F_{\phi^{(f)}} \ll \mathcal{K}^{T \bar{T}} F_T$$

iib

$$\mathcal{K}^{\phi^{(f)} \bar{\phi}^{(f)}} F_{\phi^{(f)}} \gg \mathcal{K}^{T \bar{T}} F_T.$$

In case (ia), $\phi^{(f)} \ll 1$ implies that $F_{\phi^{(f)}} \equiv \lambda e^{-bT} f(\langle \chi \rangle) \ll \langle \mathcal{W} \rangle$ and thus $\partial_{\phi^{(f)}} V_F = 0$ gives us

$$\langle \phi^{(f)} \rangle \sim \frac{\langle F_{\phi^{(f)}} \rangle}{\langle \mathcal{W} \rangle} \ll 1 \quad (66)$$

which is thus a self-consistent vacuum.

In the opposite situation we get $F_{\phi^{(f)}} \equiv \lambda e^{-bT} f(\langle \chi \rangle) \gg \langle \mathcal{W} \rangle, \langle F_T \rangle$. Using again $\phi^{(f)} \ll 1$ this leads to

$$\langle \phi^{(f)} \rangle \sim \frac{\langle F_T \rangle}{\langle F_{\phi^{(f)}} \rangle} \ll 1. \quad (67)$$

Thus, even the F flat but nonsupersymmetric singlet directions of case (ii) get stabilized by supersymmetry breaking effects from the bulk moduli stabilization at generically small but nonzero VEVs.

This property, of all F - and D -flat singlet fields generically acquiring nonzero VEVs from supersymmetry breaking in the bulk moduli stabilizing sector through supergravity, dynamically ensures the decoupling of all vectorlike non-MSSM matter at low energies as checked in global supersymmetry for the mini-landscape setup.

Note, that the overall vacuum structure of the F -flat singlet fields implicates a choice of initial conditions. The amount of non-MSSM vectorlike extra matter in the mini-landscape constructions which decouples from low

energies depends on the choice of the globally F -flat singlets $\phi_i^{(f)}$ placed at their nonzero VEV vacuum instead of their zero VEV vacuum. Thus, the choice of initial conditions in the vacuum distribution among the set of globally F -flat singlet fields characterizes how close to the MSSM one can get when starting from one of the mini-landscape models.

Assuming now that one finds successful eternal inflation occurring somewhere in the mini-landscape, this choice of initial conditions turns into a question of cosmological dynamics. In this situation, all possible initial conditions of the set of globally F -flat singlets were potentially realized in a larger multiverse. The choice of initial conditions on the singlets in the globally F -flat sector would then be amenable to anthropic arguments and might be eventually determined by selection effects.

V. SUSY SPECTRUM

Now that we understand how SUSY is broken, we can calculate the spectrum of soft masses. The messenger of SUSY breaking is mostly gravity, however, there are other contributions from gauge and anomaly mediation.

A. Contributions to the soft terms

At tree level, the general soft terms for gravity mediation are given in Refs. [86–90]. The models described in this paper contain an additional contribution from the F term of a scalar field ϕ_2 . Following Refs. [86,87,90], we define

$$F^I \equiv e^{\mathcal{K}/2} \mathcal{K}^{I\bar{J}} (\bar{\mathcal{W}}_{\bar{J}} + \bar{\mathcal{W}} \mathcal{K}_{\bar{J}}). \quad (68)$$

1. Supergravity effects

Gaugino masses

The tree level gaugino masses are given by

$$M_a^{(0)} = \frac{g_a^2}{2} F^n \partial_n f_a(S) = \frac{g_a^2}{2} F^S. \quad (69)$$

At tree level, the gauge kinetic function in heterotic string theory is linear in the dilaton superfield S , and only dependent on the T modulus at one loop. It is important to note the enhancement of F^S relative to F_S : naively, one might guess that loop corrections to the gaugino masses might be important, however,

$$F^S \gg \frac{F^T}{16\pi^2}, \quad (70)$$

thus loop corrections will be neglected.

A Terms

At tree level, the A terms are given by

$$A_{IJK}^{(0)} = F^n \partial_n \mathcal{K} + F^n \partial_n \log \frac{\mathcal{W}_{IJK}}{\kappa_I \kappa_J \kappa_K}, \quad (71)$$

where

$$\mathcal{W}_{IJK} \equiv \frac{\partial^3 \mathcal{W}}{\partial \Phi^I \partial \Phi^J \partial \Phi^K} \quad (72)$$

and \mathcal{K} is the Kähler potential. Neglecting U dependence, we have

$$\mathcal{K} \supset \Phi_I \bar{\Phi}^{\bar{I}} \prod_i (T_i + \bar{T}_i)^{-n_i} \Rightarrow \kappa_I \equiv \prod_i (T_i + \bar{T}_i)^{-n_i}. \quad (73)$$

The κ_I are the Kähler metrics for the chiral multiplets, Φ_I , where as the A terms are expressed in terms of canonically normalized fields. As before, the modular weights of the matter field are given by n_I^i .

In general, there are also tree level contributions to A terms proportional to

$$- \frac{F_{\phi_2}}{\langle \phi_2 \rangle} \frac{\partial \log \mathcal{W}_{IJK}}{\partial \log \phi_2}. \quad (74)$$

These terms may be dominant, but unfortunately they are highly model dependent. They may give a significant contribution to A_b and A_τ , but in fact we find that the details of the low-energy spectrum are not significantly effected.

Scalar masses

The tree level scalar masses are given by

$$(M_I^{(0)})^2 = m_{3/2}^2 - F^n \bar{F}^{\bar{m}} \partial_n \partial_{\bar{m}} \log \kappa_I + g_G^2 f q_A^I \langle D_A \rangle \kappa_I, \quad (75)$$

where $g_G^2 = 1/\text{Re}S_0$ and we have implicitly assumed that the Kähler metric is diagonal in the matter fields. The factor f rescales the $U(1)_A$ charges q_A from the mini-landscape benchmark model 1 [9], so they are consistent with the charges q'_A in our mini-version of the mini-landscape model. We have $q'_A = q_A f = q_A \frac{48\pi^2}{\text{Tr}Q} \delta_{\text{GS}}$ with $\delta_{\text{GS}} = \frac{N_f}{4\pi^2}$ [Eq. (7)] and $\text{Tr}Q = \frac{296}{3}$ [Eq. (E.5) in Ref. [9]] such that $\frac{\text{Tr}(q)}{4\pi^2} = \delta_{\text{GS}}$.

Again neglecting U dependence, the Kähler metric for the matter fields depends only on the T moduli, and we find

$$(M_I^{(0)})^2 = m_{3/2}^2 - \sum_i \frac{n_I^i |F^{T_i}|^2}{(T_i + \bar{T}_i)^2} + g_G^2 f q_A^I \langle D_A \rangle / (2\text{Re}T_0)^{n_i}. \quad (76)$$

μ and $B\mu$ terms

The μ term can come from two different sources:

$$\begin{aligned} \mathcal{K} &\supset Z(T_i + \bar{T}_i, U_j + \bar{U}_j, \dots) \mathbf{H}^u \mathbf{H}^d, \\ \mathcal{W} &\supset \tilde{\mu}(s_I, T_i, U_j, \dots) \mathbf{H}^u \mathbf{H}^d. \end{aligned} \quad (77)$$

In the orbifold models, Kähler corrections have not been computed, so the function Z is *a priori* unknown. Such a term could contribute to the Giudice-Masiero mechanism [91]. When both $\tilde{\mu}$ and Z vanish, the supergravity contribution to the $\mu/B\mu$ terms vanish. On the other hand, in the class of models which we consider, we know that vacuum configurations exist such that $\tilde{\mu} = 0$ to a very high order in singlet fields. Moreover $\tilde{\mu} \propto \langle \mathcal{W} \rangle$ which vanishes in the supersymmetric limit, but obtains a value w_0 at higher

order in powers of chiral singlets. If μ is generated in this way, there is also likely to be a Peccei-Quinn axion [92,93]. Finally, supergravity effects will also generate a $B\mu$ term.

Loop corrections

Finally, one can consider loop corrections to the tree level expressions in [86,87,90]. This was done in Refs. [94,95], where the complete structure of the soft terms (at one loop) for a generic (heterotic) string model were computed in the effective supergravity limit. We have applied the results of [94,95] to our models and find, at most, around a 10% correction to the tree level results of [86,87,90].¹⁹

2. Gauge mediation

The mini-landscape models generically contain vectorlike exotics in the spectrum. Moreover it was shown that such states were necessary for gauge coupling unification [96]. The vectorlike exotics obtain mass in the supersymmetric limit by coupling to scalar moduli, thus they may couple to the SUSY breaking field ϕ_2 . We will consider the following light exotics to have couplings linear in the field ϕ_2 :

$$n_3 \times (\mathbf{3}, 1)_{1/3} + n_2 \times (1, \mathbf{2})_0 + n_1 \times (1, 1)_{-1} + \text{H.c.} \quad (78)$$

where the constants n_i denote the multiplicity of states and (see, Table 7 of Reference [96])

$$n_3 \leq 4 \quad \text{and} \quad n_2 \leq 3 \quad \text{and} \quad n_1 \leq 7. \quad (79)$$

The gauge mediated contributions split the gaugino masses by an amount proportional to the gauge coupling:

$$M_3^{(1)}|_{\text{gmsb}} = n_3 \frac{g_3^2}{16\pi^2} \frac{F\phi_2}{\langle\phi_2\rangle}, \quad (80)$$

$$M_2^{(1)}|_{\text{gmsb}} = n_2 \frac{g_2^2}{16\pi^2} \frac{F\phi_2}{\langle\phi_2\rangle}, \quad (81)$$

$$M_1^{(1)}|_{\text{gmsb}} = \frac{n_3 + 3n_1}{10} \frac{g_1^2}{16\pi^2} \frac{F\phi_2}{\langle\phi_2\rangle}. \quad (82)$$

It is interesting to note that this becomes more important as $\langle\phi_2\rangle$ decreases/ $F\phi_2$ increases, or if there are a large number of exotics present.

The scalar masses in gauge mediation come in at two loops, and receive corrections proportional to

$$(M_I)^2|_{\text{gmsb}} \sim \left(\frac{1}{16\pi^2}\right)^2 \left(\frac{F\phi_2}{\phi_2}\right)^2. \quad (83)$$

Unlike in the case of the gaugino masses, however, the tree level scalar masses are set by the gravitino mass. Typically,

¹⁹In estimating this result, we have assumed that the mass terms of the Pauli-Villars fields do not depend on the SUSY breaking singlet field ϕ_2 , and that the modular weights of the Pauli-Villars fields obey specific properties.

$$16\pi^2 m_{3/2} \gg \frac{F\phi_2}{\phi_2}, \quad (84)$$

and the gauge mediation contribution gives about a 10% correction to the scalar masses, in our case. We will neglect their contributions in the calculation of the soft masses below.

B. Calculation of the soft terms—relevant details from the mini-landscape

Given the relative sizes of the F terms in the SUSY breaking sectors described in this paper, it is very difficult to make model-independent statements. This stems from the fact that F^T plays a dominant role in the SUSY breaking. Because the Kähler metrics for the matter fields have generally different dependences on the T modulus, the dependence of the soft terms on F^T is typically nonuniversal. Moreover, the couplings of the SUSY breaking singlet field ϕ_2 will necessarily depend on the details of a specific model. Thus, in order to make *any* statements about the phenomenology of these models, we will have to make some assumptions. With the general features of the mini-landscape models in mind, we will make the following assumptions:

- (1) SUSY breaking is dominated by $F_{\phi_2} \neq 0$, $F_{T_3} \neq 0$, $F_S \neq 0$. All other F terms, including those due to the other T and U moduli, are subdominant;
- (2) the massless spectrum below M_S contains some vectorlike exotics;
- (3) the untwisted sector contains the following Higgs and (3rd generation) matter multiplets: \mathbf{H}_u , \mathbf{H}_d , \mathbf{Q}_3 , \mathbf{U}_3^c , \mathbf{E}_3^c ;
- (4) the first two families have the same modular weights, see Table IV;
- (5) the SUSY breaking field, ϕ_2 , lives in the untwisted, or second or fourth twisted sector, with a modular weight given by $n^3 = 0$; and
- (6) we neglect possible ϕ_2 dependence of the effective Yukawa terms.

Let us examine these assumptions in some more detail.

TABLE IV. Modular weights of the MSSM states in the mini-landscape benchmark model 1A. For the first two generations, the $U(1)_A$ charges differ depending on whether the particle is in the $\mathbf{10}$ or $\bar{\mathbf{5}}$ of $SU(5)$. See [9] for details.

MSSM particle	Modular Weight \vec{n}	$U(1)_A$ charge
\mathbf{Q}_3	(0, 1, 0)	4/3
\mathbf{U}_3^c	(1, 0, 0)	2/3
\mathbf{D}_3^c	($\frac{1}{3}, \frac{2}{3}, 0$)	8/9
\mathbf{L}_3	($\frac{2}{3}, \frac{1}{3}, 0$)	4/9
\mathbf{E}_3^c	(1, 0, 0)	2/3
First two gen.	($\frac{5}{6}, \frac{2}{3}, \frac{1}{2}$)	7/18 ($\mathbf{10}$) -5/18 ($\bar{\mathbf{5}}$)
\mathbf{H}_u	(0, 0, 1)	-2
\mathbf{H}_d	(0, 0, 1)	+2

In general, gauge coupling unification in the mini-landscape models seems to require the existence of light vectorlike exotics [96], whose masses can be as small as $\mathcal{O}(10^9 \text{ GeV})$. We further assume that these exotics couple to the SUSY breaking field ϕ_2 , giving a gauge mediated contribution to the gaugino masses above. We will make this contribution to the soft terms explicit in what follows. In assumption 2 we have specialized to the case where only ‘‘brane-localized’’ exotics are present in the model. These are states which come from the first and third twisted sectors of the model, and we refer the reader to [9,96] for more details.

The top quarks and the up Higgses live in the bulk and the string selection rules allow for the following coupling in the superpotential:

$$\mathcal{W} \supset c \mathbf{Q}_3 \mathbf{H}_u \mathbf{U}_3^c. \quad (85)$$

The coupling c is a pure number of $\mathcal{O}(1)$, and is free of any dependence on the moduli. The down and lepton Yukawas are a bit more involved, as they arise at a higher order in the stringy superpotential. We will take them to be of the following form:

$$\begin{aligned} \mathcal{W} \supset & \eta(T_1)^{p_1} \eta(T_2)^{p_2} \eta(T_3)^{p_3} (f_1 \langle \langle \mathbf{s}_l^5 \rangle \rangle \mathbf{Q}_3 \mathbf{H}_d \mathbf{D}_3^c \\ & + f_2 \langle \langle \mathbf{s}_l^5 \rangle \rangle \mathbf{L}_3 \mathbf{H}_d \mathbf{E}_3^c). \end{aligned} \quad (86)$$

The \mathbf{s}_l are other singlet fields in the model (excluding the SUSY breaking singlet field, ϕ_2 , as per our assumptions), and the numbers p_1 , p_2 , and p_3 are calculable in principle, given knowledge of the modular weights of the \mathbf{s}_l . As one might expect, the expressions for the A terms explicitly depend on the value of p_3 in such a way that changing its value may result in a significant change in A_b and A_τ at the string scale. The impact on the weak scale observables is much less severe, however, giving a correction of a few percent to the gaugino masses, and leaving the squark and slepton masses virtually unchanged. Motivated by the modular weight assignments in Table IV, we will choose $p_3 = 0$. Note this choice gives us universal A terms for the third generation.

One of the nice features of the mini-landscape models is the incorporation of a discrete (D_4) symmetry between the first two families in the low-energy effective field theory. Because of this symmetry, we expect the modular weights of these matter states to be the same [97], see Table IV. This will turn out to be very beneficial in alleviating the flavor problems that are generic in gravity mediated models of SUSY breaking: the scalar masses (at tree level) are given by a universal contribution (the gravitino mass squared) plus a contribution proportional to the modular weight. If the modular weights are the same between the first two generations, then the leading order prediction is for degenerate squark and slepton masses in the two light generations. Other contributions to the scalar masses come from gauge mediation and anomaly mediation, which do not

introduce any new flavor problems into the low-energy physics.

C. Hierarchy of F terms

Note, in Sec. III, we find (roughly)

$$F_T \gg F_S \gtrsim F_{\phi_2}, \quad (87)$$

for cases 1, 2, and 3; and

$$F_T \gtrsim F_{\phi_2} \gg F_S, \quad (88)$$

for cases 4 and 5, where

$$F_I \equiv \mathcal{W}_I + \mathcal{W} \mathcal{K}_I. \quad (89)$$

When one includes the relevant factors of the Kähler metric, we have (Table V)

$$F^T > F^S \gg F^{\phi_2} \quad (90)$$

for cases 1, 2, and 3; and

$$F^T \gg F^S \sim F^{\phi_2} \quad (91)$$

for cases 4 and 5. F^S is enhanced by a factor of $\mathcal{K}^{S5} \sim (2+2)^2$, while F^{ϕ_2} is decreased by a factor of $\mathcal{K}^{\phi_2 \bar{\phi}_2} \sim (2)^{-1/2}$.²⁰ This means that although the singlet field ϕ_2 was a dominant source of SUSY breaking, it is the least important when computing the soft terms, given the one condensate hidden sector of the known mini-landscape models studied in Sec. III.²¹ Taking the details of the mini-landscape models into account, the soft terms at the string scale are given in Table VI.

In the five chosen cases, 2, 3, and 4 have a gravitino mass less than 2 TeV. The value of the gravitino mass can be adjusted by varying w_0 . For cases, 1, 3 (4) the Higgs up (down) mass squared is negative. This is a direct result of the sign of D_A and the $U(1)_A$ charge of the Higgses [see Table IV for the $U(1)_A$ charges of all the MSSM states].²² Note, the first and second generation squarks and sleptons are lighter than the third generation states at the string

²⁰This is due to the assumed modular weight of the field ϕ_2 (assumption 5 in Sec. VB).

²¹In racetrack models F_S is suppressed by more than an order of magnitude. In these cases F_{ϕ_2} is dominant [39].

²²Note, it is well known that the D -term VEV in supergravity is of order $\langle F^i \rangle^2$ [89,98]. It is given by the relation

$$\langle D_A \rangle = 2M_A^{-2} \langle F^i \rangle \langle F_j^* \rangle \langle \partial_i \partial^j D_A \rangle. \quad (92)$$

Thus the D -term contribution to the vacuum energy is negligible, but its contribution to scalar masses can be significant. Since $|F^S|^2 < |F^T|^2$, F^T is dominant in the above relation. However, the Kähler metric of ϕ_2 which spontaneously breaks $U(1)_A$, in our case, does not include T , i.e., $\langle (\partial_T \partial^T D_A) \rangle = 0$. Hence $\langle D_A \rangle$ is suppressed compared with $|F^T|^2/M_{\text{Pl}}^2$, i.e., $\langle D_A \rangle : |F^T|^2/M_{\text{Pl}}^2 = |F^S|^2 : |F^T|^2$ where we used $\langle (\partial_S \partial^S D_A) \rangle = (M_A/M_{\text{Pl}})^2$, because of the S -dependent Fayet-Iliopoulos (FI) term [99]. However, it should be clear that we have also used the freedom available in the Coleman-Weinberg one-loop correction to further adjust the value of the D -term.

TABLE V. The hierarchy of F terms in the five examples of the single-condensate model we studied. Note that F^Φ is defined in Eq. (68). All of the F -terms contribute to the soft masses, as they are all within an order of magnitude.

	Case 1	Case 2	Case 3	Case 4	Case 5
F^S	6.6×10^{-16}	3.7×10^{-16}	4.2×10^{-16}	2.7×10^{-16}	2.1×10^{-16}
F^T	-2.2×10^{-15}	-1.2×10^{-15}	-1.4×10^{-15}	1.6×10^{-15}	2.2×10^{-15}
F^{ϕ_2}	-1.1×10^{-17}	-6.5×10^{-18}	-7.7×10^{-18}	1.9×10^{-16}	1.8×10^{-16}

TABLE VI. Boundary conditions at the string scale. n_3, n_2, n_1 refer to possible intermediate mass vectorlike exotics which couple to the SUSY breaking field ϕ_2 ; see Eq. (78).

Parameter	All Masses in GeV									
	Case 1		Case 2		Case 3		Case 4		Case 5	
$m_{3/2}$	2159		1350		1133		1808		2375	
m_{H_u}	478 <i>i</i>		168		372 <i>i</i>		688		384	
m_{H_d}	679		216		495		476 <i>i</i>		251	
M_1	$362 - 0.3n_1 - 0.1n_3$		$206 - 0.2n_1 - 0.1n_3$		$243 - 0.2n_1 - 0.1n_3$		$158 + 13n_1 + 4n_3$		$118 + 7n_1 + 2n_3$	
M_2	$362 - 1n_2$		$206 + 1n_2$		$243 - 1n_2$		$158 + 45n_2$		$118 + 23n_2$	
M_3	$362 - 1n_3$		$206 + 1n_3$		$243 - 1n_3$		$158 + 45n_3$		$118 + 23n_3$	
A_t	3901		2466		1974		-3690		-4798	
A_b	3901		2466		1974		-3690		-4798	
A_τ	3901		2466		1974		-3690		-4798	
	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3
$m_{\tilde{q}}$	1580	2288	966	1355	895	1446	1262	1657	1691	2361
$m_{\tilde{u}^c}$	1580	2225	966	1353	895	1299	1262	1734	1691	2368
$m_{\tilde{d}^c}$	1521	2246	964	1354	757	1350	1330	1709	1697	2366
$m_{\tilde{\ell}}$	1580	2203	964	1352	757	1246	1330	1759	1697	2370
$m_{\tilde{e}^c}$	1580	2225	966	1353	895	1299	1262	1734	1691	2368

scale. This is a consequence of the significant T modulus contribution to the first and second generation squark and slepton masses, due to their modular weights, Table IV. Finally we have included the possible gauge mediated SUSY breaking contribution to the gaugino masses, Table VI. This contribution is only significant for cases 4 and 5, due to the larger value of F_{ϕ_2} in these cases.

D. Weak scale observables

We do not intend this work to be a comprehensive study of the parameter space of these models, so we will limit our weak scale analysis to the five cases studied in the single-condensate model presented in this paper. The points are chosen subject to the following constraints:

- (i) $m_{h^0}|_{\text{LEP}} \approx 114.4 \text{ GeV}$,
- (ii) successful electroweak symmetry breaking,
- (iii) $m_{\tilde{\chi}^\pm} \gtrsim 94 \text{ GeV}$, and
- (iv) the low-energy spectrum is free of tachyons.

Note that we take $\text{sgn}(\mu) > 0$ and vary $\tan\beta$, and the number, n_i , of ‘‘messenger’’ exotics. We stay in the region

of small to moderate $\tan\beta$ as the mini-landscape models do not tend to predict unification of the third family Yukawas. This can be seen from Eqn. (85) and (86), for example.

Using SOFTSUSY (v3.1) [100], we performed the renormalization group equation running from the string scale to the weak scale. We use the current value of the top quark mass [101]

$$m_{\text{top}}|_{\text{world avg}} = 173.1 \text{ GeV} \quad (93)$$

and the strong coupling constant at M_Z [102]

$$\alpha_s(M_Z) = 0.1176. \quad (94)$$

The μ parameter is obtained under the requirement of radiative electroweak symmetry breaking, and is of order of the gravitino mass, as expected. This implies a fine-tuning of order

$$\frac{M_Z^2}{m_{3/2}^2} \sim \mathcal{O}(10^{-2}) \text{ to } \mathcal{O}(10^{-4}). \quad (95)$$

The results obtained from SOFTSUSY are presented in Table VII. In this analysis, we have not included any possible gauge mediated SUSY breaking contributions.

This assumes that all the vectorlike exotics have mass at the string scale. In case 2 and 3 we have the smallest gravitino masses, so the lightest SUSY partners. $\tan\beta = 25$ in order for the light Higgs mass to be above the LEP bound. Note we assume a ± 2 GeV theoretical uncertainty in the Higgs mass. In all 5 cases the Higgs mass is between the LEP bound and 121 GeV. All other Higgs masses are of order of the gravitino mass. In all 5 cases the gluino mass is less than 1 TeV and of order 600 GeV or less in cases 2, ..., 5. Thus the gluino is very observable at the LHC. In all cases, the lightest MSSM particle is the lightest neutralino. The next-to-lightest neutralino and the lightest

chargino are approximately degenerate with mass of order twice the lightest neutralino mass. In cases 2, 3, and 4 the lightest stop has mass less than 1 TeV. In cases 2 and 4, the lightest stop is also the lightest squark. Thus in these cases the gluino will predominantly decay into a top-antitop pair with missing energy (and possibly two energetic leptons). In case 3, the lightest down squarks of the first two families are lighter than the lightest stop. In these cases gluinos will decay significantly into two light quark jets plus missing energy (and possibly two energetic leptons).

In all cases the lightest MSSM particle is mostly ($\geq 99\%$) bino (see, Table VIII). We note that this is

TABLE VII. Weak scale observables, with no contribution from gauge mediation: $n_3 = n_2 = n_1 = 0$; see Eq. (78). We have listed the mass eigenstates of the squarks and sleptons. Note that for light generations, $m_{\tilde{u}_1} \approx m_{\tilde{u}_L}$, etc. The last two rows give the lightest massive modulus (m_{LMM}) [mostly Kähler modulus ($\text{Re}T$)] and the *next to* lightest massive modulus (m_{nLMM}) [mostly the dilaton ($\text{Re}S$)]. All other moduli have mass ≥ 100 TeV.

		All Masses in GeV (defined at $M_W \approx 80$ GeV, unless otherwise noted.)									
	Observable	Case 1	Case 2	Case 3	Case 4	Case 5					
Inputs	$m_{3/2}$	2159	1350	1133	1808	2375					
	$\tan\beta$	10	25	25	10	4					
	$\text{sgn}(\mu)$	+	+	+	+	+					
	n_1, n_2, n_3	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0	0, 0, 0				
EWSB	$\mu(M_{\text{SUSY}})$	2221	1317	1342	1848	2636					
	m_{h^0}	115.8	113.3	113.5	121.4	116.7					
	m_{H^0}	2299	1161	1368	1731	2717					
	m_{A^0}	2303	1173	1376	1728	2715					
	m_{H^\pm}	2305	1176	1379	1730	2716					
$M_a(M_{\text{SUSY}})$	M_1	151	83	100	68	53					
	M_2	277	155	185	128	100					
	M_3	773	457	538	370	279					
\tilde{g}	$m_{\tilde{g}}$	914	545	630	456	365					
Neut./Charg.	$m_{\tilde{\chi}_1^0}$	150	83	99	68	52					
	$m_{\tilde{\chi}_2^0}$	293	164	194	136	104					
	$m_{\tilde{\chi}_3^0}$	-2204	-1306	-1334	-1835	-2616					
	$m_{\tilde{\chi}_4^0}$	2206	1307	1335	1836	2617					
	$m_{\tilde{\chi}_1^\pm}$	293	164	194	136	104					
	$m_{\tilde{\chi}_2^\pm}$	2214	1313	1341	1839	2622					
Squarks/Sleptons		Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3	Gen. 1, 2	Gen. 3
	$m_{\tilde{u}_1}$	1712	1542	1040	921	1013	987	1283	717	1677	1107
	$m_{\tilde{u}_2}$	1704	2042	1038	1164	1006	1336	1289	1260	1683	1860
	$m_{\tilde{d}_1}$	1714	2037	1043	1150	1016	1316	1285	1223	1678	1838
	$m_{\tilde{d}_2}$	1651	2321	1036	1341	888	1379	1351	1702	1688	2364
	$m_{\tilde{e}_1}$	1532	2192	970	1227	769	1182	1334	1696	1694	2356
	$m_{\tilde{e}_2}$	1586	2206	968	1305	901	1228	1256	1750	1687	2370
	$m_{\tilde{\nu}}$	1530	2196	966	1296	764	1202	1331	1746	1692	2366
Other Obs.	$\delta\rho$	8.5×10^{-6}	3.0×10^{-5}	2.3×10^{-5}	2.1×10^{-5}	7.2×10^{-6}					
	$\delta(g-2)_\mu$	6.0×10^{-11}	3.9×10^{-10}	5.5×10^{-10}	7.0×10^{-11}	1.2×10^{-11}					
	$\text{BR}(b \rightarrow s\gamma)$	3.7×10^{-4}	3.9×10^{-4}	3.9×10^{-4}	3.6×10^{-4}	3.7×10^{-4}					
	$\text{BR}(B_s \rightarrow \mu^+ \mu^-)$	3.1×10^{-9}	2.7×10^{-9}	2.9×10^{-9}	3.1×10^{-9}	3.1×10^{-9}					
	m_{LMM}	272	175	138	531	487					
	m_{nLMM}	41 659	25 694	22 745	27 231	36 795					

TABLE VIII. Scan over $\tan\beta$ and $\text{sgn}(\mu)$.

Case		1		2		3		4		5	
		$\mu < 0$	$\mu > 0$	$\mu < 0$	$\mu > 0$	$\mu < 0$	$\mu > 0$	$\mu < 0$	$\mu > 0$	$\mu < 0$	$\mu > 0$
$\tan\beta$	lo	5	6	8	12	9	11	5	4	5	3
	hi	38	50	36	50	39	50	32	48	39	50
m_{h^0}	lo	113.5	113.2	112.4	112.4	112.4	112.4	112.4	116.2	113.5	112.5
	hi	117.4	117.2	113.7	113.4	113.6	113.7	120.5	121.8	120.8	121.9
Neut. comp.		bino	bino	bino	bino	bino	bino	bino	bino	bino	bino
		$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$	$\geq 99\%$
$m_{\tilde{\chi}_1^0}$ (GeV)	lo	149.1	148.5	82.5	82.0	98.6	98.2	68.9	67.3	53.6	51.5
	hi	151.5	149.9	84.0	82.9	99.8	98.7	69.6	70.3	55.1	55.9
$m_{\tilde{\chi}_1^\pm}$ (GeV)	lo	290.4	291.3	162.3	162.3	193.5	193.6	139.4	134.3	110.3	103.3
	hi	298.8	293.7	167.1	163.7	197.7	194.5	141.0	141.6	113.8	114.6

generically true in the models, even when there are contributions from gauge mediation. The gauge mediated contributions in Eq. (78) do not appreciably change the composition of the lightest supersymmetric particle (LSP), which one can check with the solutions in Table 7 of Ref. [96].

We have evaluated other low-energy observables using MICROMEAS [103]. As expected, the bino LSP overcloses the Universe, giving $\Omega_{\text{DM}} \gg \Omega_{\text{DM}}^{\text{OBS}} \approx 0.2$. The calculated values for the following observables are given in the last few rows of Table VII. Corrections to the ρ parameter are very small. Corrections to $(g-2)_\mu$ are significant in cases 2 and 3 which is not surprising since these are the two cases with the lightest sleptons for the first two families. We also display the results for $\text{BR}(b \rightarrow s\gamma)$ and $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$. The result for $\text{BR}(b \rightarrow s\gamma)$ is within the 2σ experimental bound (see, [104] and references therein). Given the small chargino masses and the large values of μ and the squark and CP odd Higgs masses, we obtain a branching ratio $\text{BR}(B_s \rightarrow \mu^+ \mu^-)$ consistent with the standard model.

We are not overly concerned about the fact that binos seem to overclose the Universe. In some of the heterotic orbifold models the Higgs μ term vanishes in the supersymmetric limit. Hence there is a Peccei-Quinn symmetry. Supersymmetry breaking effects are expected to shift the moduli VEVs and generate a nonvanishing μ term; spontaneously breaking the Peccei-Quinn symmetry and producing the standard invisible axion. In fact, it has been shown that Peccei-Quinn axions may be obtained in heterotic orbifold constructions [85]. In such cases it is possible that the bino decays to an axino + photon leaving an axino dark matter candidate [105–107].

However another, perhaps more important, cosmological effect must be considered. All 5 cases have a gravitino with mass less than 3 TeV. Thus there is most likely a gravitino problem. In addition the lightest moduli mass is of order (Table VII) several 100s GeV. Thus there is also a

cosmological moduli problem. But there is hope. The next lightest massive modulus (nLMM) has, in all cases, a mass above 20 TeV. A detailed cosmological analysis is beyond the scope of this paper. However, it is possible that when cosmological temperatures are of order m_{nLMM} , the Universe becomes nLMM dominated. By the time the nLMM decays all matter is diluted and then the Universe reheats to temperatures above the scale of big bang nucleosynthesis (for example, see [108]). Thus it is possible that the nLMM solves both the gravitino and light moduli problems. Of course, then the issue of obtaining the correct baryon asymmetry of the Universe and the dark matter abundance must be addressed. Both can in principle be obtained via nonthermal processes at low temperature.

In Table VIII we analyze the dependence of our results on the value of $\tan\beta$ and $\text{sgn}(\mu)$ with all other input parameters fixed. We find that only the value of the light Higgs mass is sensitive to varying $\tan\beta$. Note the lowest value of $\tan\beta$ is obtained by the Higgs mass bound, while the largest value of the light Higgs mass is obtained with the largest value of $\tan\beta$ (for both signs of μ). Additionally, at large $\tan\beta$ for $\mu < 0$ the Higgs potential becomes unbounded from below. For $\mu > 0$ we limited the analysis to $\tan\beta \leq 50$. The light Higgs mass does not go above 122 GeV for $\tan\beta \leq 50$.

VI. CONCLUSIONS

As a candidate theory of all fundamental interactions, string theory should admit at least one example of a four-dimensional vacuum which contains particle physics and early universe cosmology consistent with the two standard models. In this context, the recently found mini-landscape of heterotic orbifold constructions [4–6,9,10] provides us with very promising four-dimensional perturbative heterotic string vacua. Their low-energy effective field theory was shown to resemble that of the MSSM, assuming nonzero VEVs for certain blow up moduli fields which

parametrize resolutions of the orbifold fixed points along F - and D -flat directions in global supersymmetry.

In this paper we have dealt with the task of embedding the globally supersymmetric constructions of the heterotic mini-landscape into supergravity and then stabilizing the moduli of these compactifications, including their orbifold fixed point blow up moduli. The blow up moduli appear as chiral superfields contained in the twisted sectors of the orbifolded heterotic string theory. They are singlets under all standard model gauge groups, but are charged under several unwanted $U(1)$ gauge symmetries, including the universal anomalous $U(1)_A$ gauge symmetry of the heterotic string. Note, moduli stabilization of string compactifications is a crucial precondition for comparing to low-energy data, as well as for analyzing any early universe cosmology, such as inflation, in a given construction.

Section II served the purpose of reviewing the ingredients and structure of the heterotic 4D $N = 1$ supergravity inherited from orbifold compactifications of the 10D perturbative $E_8 \otimes E_8$ heterotic string theory. The general structure of these compactifications results in:

- (i) a standard no-scale Kähler potential for the bulk volume and complex structure moduli, as well as the dilaton, together with
- (ii) gaugino condensation in the unbroken subgroup of the hidden E_8 , and
- (iii) the fact that the nonperturbative (in the world sheet instanton sense) Yukawa couplings among the twisted sector singlet fields contain terms explicitly breaking the low-energy $U(1)_R$ symmetry.

We have shown in Sec. III that these three general ingredients, present in all of the mini-landscape constructions, effectively realize a KKLT-like setup for moduli stabilization. Here, the existence of terms explicitly breaking the low-energy $U(1)_R$ symmetry at high order in the twisted sector singlet fields is the source of the effective small term w_0 in the superpotential, which behaves like a constant with respect to the heterotic dilaton [81]. Utilizing this, the presence of just a single condensing gauge group in the hidden sector (in contrast to the racetrack setups in the heterotic literature) suffices to stabilize the bulk volume T (and, by extension, also the bulk complex structure moduli U), as well as the dilaton S at values $\langle \text{Re}T \rangle \sim 1.1 - 1.6$ and $\langle \text{Re}S \rangle \sim 2$. These are the values suitable for perturbative gauge coupling unification into $SU(5)$ - and $SO(10)$ -type GUTs distributed among the orbifold fixed points. Note, we have shown this explicitly for the case one T modulus and a dilaton, however, we believe that all bulk moduli will be stabilized near their self-dual points [76,82].

At the same time, the near cancellation of the D -term of the universal anomalous $U(1)_A$ symmetry stabilizes non-zero VEVs for certain gauge invariant combinations of twisted sector singlet fields charged under the $U(1)_A$. This feature in turn drives nonvanishing F -terms for some of the twisted sector singlet fields. Thus, together

with the F -terms of the bulk volume moduli inherited from modular invariance, it is sufficient to uplift the anti-de Sitter vacuum to near-vanishing cosmological constant.

The structure of the superpotential discussed in this paper, $\mathcal{W} \sim w_0 e^{-bT} + \phi_2 e^{-aS - b_2 T}$, behaves like a hybrid KKLT with a single condensate for the dilaton S , but as a racetrack for the T and, by extension, also for U moduli. An additional matter F_{ϕ_2} term driven by the cancellation of the anomalous $U(1)_A$ D -term seeds successful up lifting.

We note the fact that the effective constant term in the superpotential, w_0 , does not arise from a flux superpotential akin to the type IIB case. This leaves open (for the time being) the question of how to eventually fine-tune the vacuum energy to the 10^{-120} cancellation necessary.

Section IV then serves to demonstrate how the success of stabilizing the bulk moduli and breaking supersymmetry in the F -term sector, driven by the $U(1)_A$ D -term cancellation, transmits itself to the chiral singlet fields from the untwisted and twisted sectors of the orbifold compactification which contain, among others, the blow up moduli associated with the orbifold fixed points. The effects from the bulk moduli stabilization and supersymmetry breaking, transmitted through supergravity, generically suffice to stabilize all of the twisted sector singlet fields at nonzero VEVs. This property was assumed in the original mini-landscape construction in order to decouple the non-MSSM vectorlike exotic matter, and our arguments provide the first step towards a self-consistent justification for these assumptions.

In Sec. V we estimate the structure of the soft terms from the moduli sector supersymmetry breaking at the high scale. We find that the contributions from high-scale gauge mediation are subdominant (although not parametrically suppressed) compared to the gravity mediated contributions. Upon the renormalization group equation running the high-scale soft terms to the weak scale using SOFTSUSY, we obtain several benchmark patterns of sparticle and Higgs masses (see Table VII). The low-energy spectrum features an allowed window of $\tan\beta$ values for $m_{3/2} < 5$ TeV. It generically contains a light chargino/ neutralino spectrum and heavy squarks and sleptons. The lightest MSSM partner, in the 5 benchmark cases studied, is given by a bino ($> 99\%$) with mass ≥ 52 GeV. If this were the LSP, it would yield a dark matter abundance which over closes the Universe, however, the mini-landscape models offer some possible resolutions. One possibility is that the bino decays into an axino, the partner of the invisible axion responsible for canceling the θ angle of QCD, which is present in many of the mini-landscape setups [85]. We have also considered an alternative possibility that the late decay of the next-to-lightest massive modulus might ameliorate or solve the cosmological gravitino and moduli problem. This would then dilute the above mentioned cosmological abundance of binos. Of course, the nonthermal production of dark matter and a baryon asymmetry must then be addressed. Note, however,

the resolution of these cosmological questions are beyond the scope of the present paper.

Summarizing, we have given a mechanism for moduli stabilization and supersymmetry breaking for the perturbative heterotic orbifold compactifications. It relies on the same variety and number of effective ingredients as the KKLT construction of type IIB flux vacua and thus represents a significant reduction in necessary complexity, compared to the multicondensate racetrack setups utilized so far. When applied to a simplified analog of the mini-landscape heterotic orbifold compactifications, which give the MSSM at low energies, it leads to fully stabilized 4D heterotic vacua with broken supersymmetry and a small positive cosmological constant. Moreover, most of the low-energy spectrum could be visible at the LHC.

We leave some important questions, like the problem of the full fine-tuning of the vacuum energy to near vanishing, or the existence of an inflationary cosmology within these stabilized mini-landscape constructions for future work. Further study is also warranted with respect to potential cosmological moduli and gravitino problems that may be associated with sub-100 TeV moduli and gravitino mass values (see, e.g., [109]). Finally, the numerical evaluation of any particular mini-landscape vacuum requires analyzing the supergravity limit with three bulk moduli, T , one bulk complex structure modulus, U , and of order 50 blow up moduli. A detailed analysis of this more realistic situation would require a much better handle on the moduli space of heterotic orbifold models than is presently available.

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APPENDIX A: A DIFFERENT RACETRACK

The form of the gaugino condensate, given in Eq. (35), ensures that the nonperturbative part of the superpotential

is invariant under the modular group $SL(2, \mathbb{Z})$. In deriving the form of \mathcal{W}_{NP} , however, we have neglected the fact that the presence of discrete Wilson lines often break the modular group $SL(2, \mathbb{Z})$ to one of its subgroups. It has been noted [66] that turning on one or more Wilson lines breaks the modular group $SL(2, \mathbb{Z})$ down to one of its subgroups. Next we define the subgroup $\Gamma_0(p) \subset SL(2, \mathbb{Z})$. The subgroup is defined as the set of 2×2 matrices such that²³

$$\mathcal{M} \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad (\text{A1})$$

$$ad - cb = 1, \quad (\text{A2})$$

$$a, b, c, d \in \mathbb{Z}, \quad (\text{A3})$$

$$c \equiv 0 \pmod{p}, \quad p \in \mathbb{P}, \quad (\text{A4})$$

where \mathbb{P} is the set of prime integers. Under this subgroup, then, the invariant function is a linear combination of Dedekind η functions:

$$f_p(\tau) = \frac{1}{p} \sum_{\lambda=0}^{p-1} \eta\left(\frac{\tau + \lambda}{p}\right). \quad (\text{A5})$$

APPENDIX B: THE ROLE OF HOLOMORPHIC MONOMIALS

Supersymmetry can be broken by either F -terms or D -terms. In a generic supersymmetric gauge theory, $D = 0$ is satisfied only along special directions in moduli space. These directions are described by holomorphic, gauge invariant monomials (HIMs) [111–113]. The moduli space of a general heterotic string model is significantly more complex than that of our simple models. Not only are there many more fields in the picture, there are also many more gauge groups.

Consider a theory with gauge symmetry $U(1)^\rho \otimes U(1)_A$, where A stands for anomalous. The $D = 0$ constraints are

$$D_{a \neq A} \sim \sum_i q_i^a |\phi_i|^2 = 0. \quad (\text{B1})$$

A generic HIM can be written in terms of fields ϕ_i with charges q_i^j

$$\mathcal{H}[\phi_i] = \prod_i \phi_i^{n_i}, \quad n_i > 0, \quad (\text{B2})$$

²³A detailed mathematical treatment of the modular functions can be found in Ref. [110].

such that

$$\sum_i n_i q_i^j = 0, \quad \forall j \neq A. \quad (\text{B3})$$

The requirement that $n_i > 0$ is a reflection of the holomorphicity of \mathcal{H} , while the requirement that the sum over n_i (weighted by the charges) vanishes is a reflection of the gauge invariance. The general HIM in Eq. (B2) relates the VEVs of the fields ϕ as follows:

$$\frac{|\phi_1|}{\sqrt{n_1}} = \frac{|\phi_2|}{\sqrt{n_2}} = \dots. \quad (\text{B4})$$

Given this relationship, one can show that Eqs. (B1) can be satisfied. Notice that no scale is introduced in Eq. (B4): the HIMs (in general) only constrain the relative magnitudes of the ϕ VEVs, and give no information about their phases or their absolute magnitudes.

The procedure for dealing with an anomalous $U(1)_A$ works the same way. Instead of Eq. (B1), one has

$$D_A \sim \sum_i q_i^A |\phi_i|^2 + \xi = 0, \quad (\text{B5})$$

and we will assume that $\xi > 0$. In this case, one needs to find a monomial which is holomorphic and gauge invariant under all of the ρ $U(1)$ factors, but which carries a net negative charge under the anomalous $U(1)_A$ [111–113]. The situation is different than the case with nonanomalous symmetries, as a mass scale is introduced into the problem.

In a heterotic string orbifold, the FI term is generated by the mixed gauge-gravitational anomaly, and is canceled by the Green-Schwarz mechanism, which forces singlets to get VEVs of order of the FI scale (typically $\sim M_S$). Usually, several singlets participate in this cancellation, all receiving VEVs of the same order. In the mini-landscape models [9], supersymmetric vacua were obtained, prior to the consideration of any nonperturbative effects. A holomorphic gauge invariant monomial was found which is invariant under all other $U(1)$ s but with net charge under $U(1)_A$ opposite to that of the FI term. This composite field necessarily gets a nonzero VEV to cancel the FI term. Our field ϕ_2 in the simple model gives mass to the vectorlike exotics of the hidden sector and thus it also appears in the non-perturbative superpotential. In a more general heterotic model, ϕ_2 would be replaced by an HIM which also cancels the FI term.

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