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Generalized gaugino condensation in super Yang-Mills theories: Discrete R symmetries and vacua

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One can define generalized models of gaugino condensation as theories that dynamically break a discrete R symmetry but do not break supersymmetry. We consider general examples consisting of gauge and matter fields and the minimal number of gauge-singlet fields to avoid flat directions in the potential. We explore which R symmetries can arise and their spontaneous breaking. In general, we find that the discrete symmetry is $\mathbb{Z}_{2b_0R}$, and the number of supersymmetric vacua is b_0, where b_0 is the coefficient of the one-loop beta function. Results are presented for various groups, including $SU(N_c)$, $SO(N_c)$, $Sp(2N_c)$, and G_2, for various numbers of flavors, N_f, by several methods. This analysis can also apply to the other exceptional groups and, thus, all simple Lie groups. We also comment on model-building applications where a discrete R symmetry, broken by the singlet vacuum expectation values, can account for μ-type terms and allow a realistic Higgs spectrum naturally.

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I. INTRODUCTION

Gaugino condensation [1] is frequently discussed in considering problems of supersymmetry dynamics and model building. There are several distinguishing features of this nonperturbative effect: (i) it does not break supersymmetry, (ii) it breaks a discrete R symmetry, and (iii) it generates a scale dynamically (used, for instance, in the "retrofitting" procedure of [2]). In [3], a large class of theories with gauge, matter, and gauge-singlet fields with these features was introduced. This is a generalization of gaugino condensation, possessing the above properties. In particular, [3] explored the significant model-building consequences of the R symmetry being broken by an order parameter with a mass dimension of less than three. Generalized gaugino condensation also has some similarities to a confining phase superpotential [4] and related techniques [5] (especially when we consider integrating out heavy flavors). Discrete symmetries have also been studied extensively elsewhere, especially in the context of supersymmetric model building and forbidding baryon- or lepton-number–violating (e.g., proton decay) operators [6].

In this work, we explore theories with generalized gaugino condensation in more detail and for a wide range of gauge groups. This extends and generalizes the supersymmetric $SU(N_c)$ theory used in [3], which was also considered earlier by Yanagida in [7]. It is interesting to consider this class of theories in more detail for a range of gauge groups and understand their dynamics. We find, quite generally, that the discrete symmetry and number of supersymmetric vacua are counted by the one-loop beta-function coefficient, b_0. The discrete R symmetry is found by considering the R charge of an instanton in the theory with a continuous R symmetry at the classical level. We find that requiring that the instanton be uncharged under a discrete subgroup gives $\mathbb{Z}_{2b_0R}$. This calculation method is even more general than finding the number of vacua, since the instanton calculation is done without assuming the group or even the representations of the matter content.

In all of the theories we study, the R symmetry is broken to a $\mathbb{Z}_2$ by gaugino condensation and the singlet vacuum expectation values (VEVs), leading to a discrete set of supersymmetric vacua. The dynamics responsible for breaking the symmetry depend on the coupling regime. In some regions, the vacua can be calculated, in general, by utilizing a generic effective superpotential motivated by (and matching) several known examples, or by integrating out heavy quarks. In other regions of a particular theory, the specific low-energy dynamics are important. We find the number of vacua to be b_0, matching what one expects from the breaking of the R symmetry. These results are shown explicitly in several examples with different gauge groups, and it seems possible that this holds, in at least some regions, for any simple Lie group.

The paper is organized as follows: in the next section, we derive general results for the discrete R symmetry and number of vacua. In Sec. III, these results are derived in different regions of the parameter space for $SU(N_c)$ models and for a wide range of N_f, the number of flavors. Depending on the value of N_f, a few different methods are used. Section IV shows that these results hold for $SO(N_c)$, $Sp(2N_c)$, and G_2 models. We also comment on the difficulties with the exceptional groups and why it is possible these results may apply here as well. This would, then, encompass all the simple Lie groups. This work also extends the model building introduced in [3], which we comment on [including using a recent model similar to the Next-to-Minimal Supersymmetric Standard Model

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(NMSSM)] in Sec. V. A brief discussion and concluding remarks follow that.

II. GENERAL RESULTS

Many of the topics and techniques used in this work are well-summarized in the review by Peskin [8]. Let us first define some relevant quantities and conventions before the general calculations.

N_c will denote the number of colors [i.e., as in $SU(N_c)$], although some care must be taken with factors of 2 for the symplectic group. The quark superfields are Q_i and $\bar{Q}_i$, which will be in the fundamental or antifundamental representation of the group, as appropriate for the given gauge group [in the vector representation in the case of $SO(N_c)$]. However, for the general calculation of the discrete R symmetry, the quarks may be in any representation. We define the (gauge-invariant) "meson" superfield $M_{ij} = Q_i\bar{Q}_j$, where i and j run from 1 to N_f. For our purposes, it suffices to take derivatives of the superpotential W with respect to M, but one can also work directly with the Q's. The gauge singlets will be denoted by S_{ij}, where i and j again run from 1 to N_f (up to a 2 for Sp); there are N_f^2 singlets, corresponding to each of the possible flavor combinations.

Although we will use gauge singlets throughout this work, one could use some other representation of the gauge group. We rely only on a cubic self-coupling for these fields and no dimensionful couplings in the superpotential, so fields in the adjoint could also work, for instance. In a practical sense, choosing a different representation may be of use in model building or further constrained by other requirements.

The gauginos are represented by λ (in general, spinor indices will be dropped). We will also assume flavor-symmetric solutions and single-coupling constants (rather than a matrix in flavor space).

Our normalization for the group-theory constants $C(r_i)\delta_{ab} = \mathrm{Tr}\{t_a t_b\}_{r_i}$, where r_i denotes the representation, is such that the fundamental representation has $C = 1/2$. Then, for instance, the adjoint of $SU(N_c)$ has $C = N_c$.

A. The discrete *R* symmetry

To find the (nonanomalous) discrete R symmetry, we start with a continuous $U(1)_R$. The instanton breaks this symmetry, and we look at what discrete subgroup can remain (i.e., the instanton is not charged under the subgroup).

First, let us define the R charges of the superfields, where our superpotential will have interactions of the form $SQ\bar{Q} + S^3$. We will start by defining the gaugino transformation parameter, β:

$$\lambda \rightarrow \beta\lambda. \tag{1}$$

The requirement, familiar in the case of a continuous R symmetry, that W (and W_α^2) must have R charge 2 becomes a transformation by an overall factor of β^2. In other words, in this notation, the actual R charge is given by powers of β. Gaugino condensation, $\langle\lambda\lambda\rangle$, and a cubic self-interaction of the singlets (in W) both have the same total R charge. The S's must transform as follows:

$$S_{ij} \rightarrow \beta^{2/3} S_{ij}. \tag{2}$$

Finally, the quark superfields will be coupled to the singlets as $QS\bar{Q}$, which also must transform with a factor of β^2. Q_i and $\bar{Q}_i$ have the same R charge, and the fermionic component differs by a power of β from this (due to the charge of the Grassmann θ coordinate, which transforms like λ)[1]:

$$Q_i \rightarrow \beta^{2/3} Q_i, \tag{3}$$

$$\psi_Q \rightarrow \beta^{-1/3}\psi_Q. \tag{4}$$

The interaction terms mentioned above will be made more explicit in the later sections.

To determine the R charge of the instanton, we only need to know how many fermions are involved. The number of zero modes for a fermion in the representation r_i appearing in the instanton is given by twice the group theoretical coefficient defined previously:

$$n_i = 2C(r_i). \tag{5}$$

The calculation is now very simple: we have the gauginos in the adjoint representation (denoted A) and quarks in the fundamental or antifundamental representation[2] (denoted as just r_i). The total R charge (power of β) is, then,

$$2C(A) - \frac{1}{3}\sum_i 2C(r_i) = \frac{2}{3}\Big[3C_2(A) - \sum_i C(r_i)\Big], \tag{6}$$

where we used the fact that, in the adjoint, $C(A) = C_2(A)$, where $C_2(A)$ is the quadratic Casimir operator of the adjoint representation. We recognize this as being proportional to the coefficient of the one-loop beta function

$$b_0 = 3C_2(A) - \sum_i C(r_i). \tag{7}$$

Therefore, the discrete subgroup of the $U(1)_R$ that is left over by the instanton is $\mathbb{Z}_{2b_0R}$. Since this will ultimately be

[1]We note that these R charges are the same as the usual nonanomalous R charges for $SU(N_c)$, $SO(N_c)$, and $Sp(2N_c)$ when taking into account the discrete arithmetic of the group we find below.

[2]Actually, nothing here depends on the representation of the quarks; the calculation is general. In the following sections, however, we will have the quarks in the fundamental or antifundamental representations.

broken down to just a $\mathbb{Z}_2$ by gaugino condensation and the singlet VEV, we expect to see b_0 vacuum states, which we will show in the next section.

B. The number of vacua

We will calculate the number of vacua by considering a general superpotential, including an effective superpotential term. This analysis is only for when such a term exists, but we will see how to extend this result quite generally.

The interaction terms in the general superpotential, from including the singlets, S_{ij}, are

$$W_S = yS_{ij}M_{ij} + \frac{\gamma}{3}\mathrm{Tr}S^3, \tag{8}$$

with y and γ as coupling constants (for simplicity, we do not write them as more general matrices).

Let us consider an effective superpotential term of a generic form, which can incorporate the known effective superpotential term, when it exists, from $SU(N_c)$, $SO(N_c)$, $Sp(2N_c)$, and G_2 supersymmetric gauge theories. The ingredients are the energy scale of the theory, Λ, which has a power determined by the beta function; the meson superfield, which we take to just be some power of the matrix elements [this is easy to see in, e.g., flavor-symmetric solutions of $SU(N_c)$]; and the fact that the total mass dimension must be 3:

$$W_{\mathrm{eff}} = C\left(\frac{\Lambda^{b_0}}{M_{ij}^a}\right)^{1/b}, \tag{9}$$

where C is a normalization constant. We have the condition that

$$(b_0 - 2a)/b = 3, \tag{10}$$

since M_{ij} has a mass of dimension 2.

Taking a derivative with respect to M_{ij},

$$\frac{\partial W}{\partial M_{ij}} = 0 = -C\frac{a}{b}\left(\frac{\Lambda^{b_0}}{M_{ij}^a}\right)^{(1-b)/b}\frac{\Lambda^{b_0}}{M_{ij}^{a+1}} + yS_{ij}, \tag{11}$$

$$0 = -C\frac{a}{b}\frac{\Lambda^{b_0/b}}{M_{ij}^{(a/b)+1}} + yS_{ij}, \tag{12}$$

$$\Rightarrow M_{ij} = \left(\frac{C\frac{a}{b}\Lambda^{b_0/b}}{yS_{ij}}\right)^{1/[(a/b)+1]}. \tag{13}$$

We will assume that, at the minimum, we have solutions of the form $M_{ij} = v^2\delta_{ij}$ and $S_{ij} = s\delta_{ij}$. Then, the final equation above is an equation for v^2 in terms of s, while a derivative with respect to S_{ij} is

$$\frac{\partial W}{\partial S_{ij}} = 0 = yM_{ij} + \gamma(S^2)_{ij}. \tag{14}$$

Plugging in M_{ij} in terms of S_{ij} from above and evaluating everything at the minimum (in terms of v and s),

$$0 = y\left(\frac{C\frac{a}{b}\Lambda^{b_0/b}}{ys}\right)^{1/[(a/b)+1]} + \gamma s^2, \tag{15}$$

$$0 = y\left(\frac{Ca}{yb}\Lambda^{b_0/b}\right)^{1/[(a/b)+1]}s^{\{-1/[(a/b)+1]\}-2} + \gamma, \tag{16}$$

$$-\frac{\gamma}{y}\left(\frac{Ca}{yb}\Lambda^{b_0/b}\right)^{-1/[(a/b)+1]} = s^{-[2(a/b)+3]/[(a/b)+1]}, \tag{17}$$

$$s = \left(\frac{-\gamma}{y}\right)^{-[(a/b)+1]/[2(a/b)+3]}\left(\frac{Ca}{yb}\Lambda^{b_0/b}\right)^{1/[2(a/b)+3]}. \tag{18}$$

Using the mass-dimension constraint to write $(2a/b) + 3 = b_0/b$,

$$s = \left[\frac{-y^a}{\gamma^{a+b}}\left(C\frac{a}{b}\right)^b\right]^{1/b_0}\Lambda. \tag{19}$$

Therefore, there are potentially b_0 solutions and supersymmetric vacua.

The above analysis seems to be limited to the region of a theory with an effective superpotential, like $SU(N_c)$ with $N_f < N_c$, which we will explore in more detail below. However, we shall see several ways in which the same result for the number of supersymmetric vacua, b_0, holds in regions where there is not an effective superpotential. For instance, one can work in the limit of very heavy quarks and integrate them out. We will show this explicitly for $SU(N_c)$, and this is again a very general procedure (although we will not do this for a general theory).

III. $SU(N_c)$ MODELS

A. $N_f < N_c$

Working with $SU(N_c)$ supersymmetric gauge theory, with $N_f < N_c$ flavors and $b_0 = 3N_c - N_f$, there is an effective superpotential [9] given by

$$W_{\mathrm{eff}} = (N_c - N_f)\left(\frac{\Lambda^{b_0}}{\det M}\right)^{1/(N_c-N_f)}. \tag{20}$$

Adding in the N_f^2 gauge-singlet superfields S_{ij}, the superpotential is now

$$W = yS_{ff'}M_{ff'} + \frac{\gamma}{3}\mathrm{Tr}S^3 + W_{\mathrm{eff}}. \tag{21}$$

For simplicity, the singlet-quark couplings are all the same here, but the features below are stable with small changes to the couplings. One could also do a field redefinition. This is the theory considered in [3], and earlier in [7]. We also take yS such that the quarks have a mass less than Λ. We look for flavor-symmetric solutions with all the Q's having the same VEV, v, and $M_{ff'} = v^2\delta_{ff'}$. Similarly, $S_{ff'} = s\delta_{ff'}$. Then, $\det M = (v^2)^{N_f}$ at this point. Taking a derivative here,

$$\frac{\partial W_{\mathrm{eff}}}{\partial M_{ff'}} = -\Lambda^{b_0/(N_c-N_f)}(\det M)^{-[1/(N_c-N_f)]-1}\frac{\partial \det M}{\partial M_{ff'}}, \tag{22}$$

$$= -\Lambda^{b_0/(N_c-N_f)} v^{-2N_f(N_c-N_f+1)/(N_c-N_f)} (v^2)^{N_f-2}, \quad (23)$$

$$\frac{\partial W_{\text{eff}}}{\partial M_{ff'}} = -\Lambda^{b_0/(N_c-N_f)} v^{(-2N_c)/(N_c-N_f)} \delta_{ff'}, \quad (24)$$

where we have used $\partial\, \det A/\partial A_{ij} = (A^{-1})_{ji} \det A$. Then, we have

$$\frac{\partial W}{\partial M_{ff'}} = ys\delta_{ff'} - \Lambda^{b_0/(N_c-N_f)} v^{(-2N_c)/(N_c-N_f)} \delta_{ff'}, \quad (25)$$

and setting this equal to zero and solving for v^2 (explicitly putting in the phase),

$$v^2 = \Lambda^{b_0/N_c} \left(\frac{e^{2\pi i k}}{ys}\right)^{(N_c-N_f)/N_c}. \quad (26)$$

Taking a derivative of the superpotential with respect to S_{ij} and setting this equal to zero,[3]

$$\frac{\partial W}{\partial S_{ff'}} = 0 = yM_{ff'} + (S^2)_{f'f}. \quad (27)$$

Working at the flavor-symmetric minimum, plugging in $M_{ff'} = v^2 \delta_{ff'}$ from above, and using $S_{ff'} = s\delta_{ff'}$,

$$0 = y\Lambda^{b_0/N_c} \left(\frac{e^{2\pi i k}}{ys}\right)^{(N_c-N_f)/N_c} + \gamma s^2, \quad (28)$$

$$0 = \frac{y^{N_f/N_c}}{\gamma} \Lambda^{b_0/N_c} e^{2\pi i k(N_c-N_f)/N_c} s^{-(3N_c-N_f)/N_c} + 1, \quad (29)$$

$$\Rightarrow s = \left(\frac{y^{N_f} e^{2\pi i k(N_c-N_f)}}{(-\gamma)^{N_c}}\right)^{1/(3N_c-N_f)} \Lambda. \quad (30)$$

Since N_c and N_f are integers, this implies $3N_c - N_f = b_0$ solutions. This matches the general calculation in the previous section.

We note that when $\gamma \ll y$, s is very large, and, thus, the quarks are heavy while the singlets are lighter. In the opposite limit, all the fields are much lighter. We can also rewrite v^2 just in terms of the constants of the theory:

$$v^2 = \left(\frac{-\gamma e^{4\pi i k}}{y^3}\right)^{(N_c-N_f)/(3N_c-N_f)} \Lambda^2. \quad (31)$$

B. $N_f \geq N_c$

There are (at least) two ways we can proceed to analyze the case $N_f \geq N_c$: we can use the electric-magnetic duality or make all the flavors heavy and integrate them out. Let us start with the latter.

The concept of "holomorphic decoupling" (see, for instance, the review [8]) allows one to get the superpotential of the theory from a known one of a theory with more flavors. By making these extra flavors heavy and integrating them out, one should properly recover the behavior of the theory with fewer flavors. This is, then, also a constraint on the theory with more flavors, as it needs to properly describe theories with fewer flavors in the decoupling limit. In the models we are considering here, the singlet interactions always provide a mass term for the quarks. If these masses are made heavy by taking the singlet VEVs to be large (compared to the dynamical scale of the theory), the theory should have a superpotential analogous to the case studied previously, with $N_f < N_c$.

For these values of N_f, there are now also "baryons":

$$B_{i_1 i_2 \cdots i_{N_f-N_c}} = \epsilon_{i_1 i_2 \cdots i_{N_f-N_c} j_1 j_2 \cdots j_{N_c}} \epsilon_{k_1 k_2 \cdots k_{N_c}} \times Q_{j_1 k_1} Q_{j_2 k_2} \cdots Q_{j_{N_c} k_{N_c}}, \quad (32)$$

where the j's are flavor indices and the k's are color indices. There is a similar definition for "antibaryons," $\bar{B}$, where the Q's are $\bar{Q}$'s. The baryons give new flat directions, and we will add additional singlets to lift these, as well.

Additionally, when $N_f = N_c$, the classical constraint of $\det M = B\bar{B}$ is modified to be $\det M - B\bar{B} = \Lambda^{b_0}$ by nonperturbative effects [10]. There is no superpotential generated. This constraint can be implemented through a Lagrange multiplier field.

We take all the fields but the singlets to be very heavy (i.e., γ is small, so the quarks get a heavier mass than the singlets from the singlet VEV) and assume that $\bar{B} = B = 0$ at the minimum. Integrating out all the heavy degrees of freedom at the scale s (the singlet VEV), the effective superpotential is

$$W_{\text{eff}} = \langle \lambda\lambda \rangle = s^3 e^{-\{(8\pi^2)/[N_c g^2(s)]\}}, \quad (33)$$

where the denominator of the exponential has an N_c because the beta-function coefficient is now for a pure gauge theory, $3N_c$, and the 3 cancels due to cubing the scale. We have

$$\frac{8\pi^2}{g^2(s)} = \frac{8\pi^2}{g^2(\mu)} + b_0 \ln\left(\frac{s}{\mu}\right), \quad (34)$$

and b_0 is the coefficient of the beta function of the theory with the massive quarks [$= 3N_c - N_f$ for $SU(N_c)$]. Substituting this in and rewriting,

$$W_{\text{eff}} = e^{-\{(8\pi^2)/[N_c g^2(\mu)]\}} \mu^{b_0/N_c} s^{3-[(b_0)/(N_c)]}. \quad (35)$$

Again, after including an additional s^3 interaction term, minimizing W easily yields $b_0 (= 3N_c - N_f)$ solutions. This is a very general procedure and can be used in the other theories we consider as a way of extending the calculation of the number of vacua beyond the region of an effective superpotential.

To enforce that the baryons are zero at the minimum, we can use additional singlets, χ and $\tilde{\chi}$, one for each of the B's

[3] Also using $\partial \text{Tr} S^3/\partial S_{ff'} = 3(S^2)_{f'f}$.

and $\bar{B}$'s. The additional terms in the superpotential are, then, proportional to χB and $\tilde{\chi}\bar{B}$ (with all indices suppressed). The partial derivatives, with respect to the new singlets and baryons, enforce that both are at zero. These couplings also need to be large enough to prevent a runaway in this direction. In general, the R symmetry restricts any further terms with the new singlets, but in some cases (due to the specific R charges of the theory), it may be necessary to impose some other symmetry, as well.

Now, let us see this using the electric-magnetic duality [11]. The duality takes a theory at strong coupling to one at weak coupling, and vice versa. So, here, we are not in the same coupling region as above, but we can consider our original theory at strong coupling for a range of flavors of light quarks (relative to Λ) and study its analog at weak coupling through the duality. Again, there are a few ways to proceed here, but we will use the duality to make a direct connection with the calculation for $N_f < N_c$. We will do this by showing that the effective superpotential can be extended to larger values of N_f by including the singlets. (See also the detailed work of [12] for extending the effective superpotential for larger number of flavors in Su and Sp theories; in Dijkgraaf-Vafa theory [13], there have also been related studies [14] and the review [15]).

The effective superpotential we studied above is not valid for $N_f \geq N_c$, and the theory can, instead, be studied in its dual "magnetic" description [11] (besides [8], another good review is [16]). Below is basically a summary of some material in, e.g., [16]; this is a well-known way to extend the previous results to larger N_f.

The magnetic gauge group is $SU(N_f - N_c)$ (matching the number of indices of the baryon operators), with N_f flavors of quarks, q_i and $\bar{q}_i$, and N_{ij}, a gauge-invariant field. The superpotential for the dual theory is

$$W = \frac{1}{\mu} qM\bar{q}. \tag{36}$$

The scale μ relates the M of the magnetic theory, M_m (which we will not use explicitly), with the M of the electric theory, $M_{ij} = Q_i\bar{Q}_j$: $M = \mu M_m$. The scale of the magnetic theory is related to the electric theory by

$$\Lambda^{b_0}\Lambda_m^{b_{m0}} = (-1)^{N_f - N_c}\mu^{N_f}, \tag{37}$$

where the m subscript denotes the magnetic theory, and $b_{m0} = 3(N_f - N_c) - N_f$ is b_0 for the magnetic theory.

In this dual picture, let us consider arbitrary values of $\langle M\rangle$, so the magnetic quarks are massive, with a mass $\langle M\rangle/\mu$. Now, the low-energy theory has no matter besides the singlets, and the new scale of the theory is

$$\Lambda_{Lm}^{b_{Lm0}} = \frac{\det M}{\mu^{N_f}}\Lambda_m^{b_{m0}}, \tag{38}$$

where the low-energy theory has the beta-function coefficient $b_{Lm0} = 3(N_f - N_c)$.

Gaugino condensation again leads to an effective superpotential:

$$W_{\text{eff}} = (N_f - N_c)\Lambda_{Lm}^3 = (N_c - N_f)\left(\frac{\Lambda^{b_0}}{\det M}\right)^{1/(N_c - N_f)}. \tag{39}$$

This is exactly the effective superpotential for $SU(N_c)$ with $N_f < N_c$ flavors of quarks that we analyzed earlier, continued to this value of N_f.

IV. $SO(N_c)$, $Sp(2N_c)$, AND G_2 MODELS

Supersymmetric $SO(N_c)$ theories exhibit a very rich set of phenomena [17]. In particular, aside from some special cases, there is a dynamically generated effective superpotential, which fits into the general form of Eq. (9). This is generated by gaugino condensation, as well, when $N_f < N_c - 4$ (and in some branch of the theory when $N_c - 4 \leq N_f < N_c - 2$) and $N_c \geq 4$, where N_f is the number of flavors of quarks in the vector representation:

$$W_{\text{eff}} = A\left(\frac{\Lambda^{b_0}}{\det M}\right)^{1/(N_c - N_f - 2)}, \tag{40}$$

where $M_{ij} = Q_iQ_j$, $b_0 = 3(N_c - 2) - N_f$, and A is a normalization constant. The anomaly coefficients are $2(N_c - 2)$ for the adjoint and 2 for the fundamental representations.

Since the effective superpotential for the $SO(N_c)$ theory is of the same form as in the generic calculations of Sec. II, we will again have a $\mathbb{Z}_{2b_0R}$ discrete R symmetry and b_0 supersymmetric vacua. For larger N_f, we can again integrate out all the quarks (made heavy by the singlet VEVs) or use a magnetic duality to a $SO(N_f - N_c + 4)$ [17], similar to the $SU(N_c)$ calculations previously.

However, there are also several special cases for the $SO(N_c)$ theories. When $N_f = N_c - 4$, the theory is broken to $SO(4) = SU(2) \times SU(2)$, and so there are two gaugino condensates. Only when the condensates have the same relative sign does the theory have the effective superpotential above. On the other branch of the theory, there is no dynamically generated superpotential; there is a moduli space, which includes confinement without chiral symmetry breaking [17]. When $N_f = N_c - 3$, there is also a branch of the theory that includes the effective superpotential. Additionally, when $N_c = 3, 4$, there are other considerations [17].

$Sp(2N_c)$ theories also fit easily into the general framework of Sec. II and are a bit simpler. Here, we have $2N_f$ flavors of quarks in the fundamental representation. The anomaly coefficients are $4(N_c + 2)$ for the adjoint and 2 for the fundamental representations. For $N_f \leq N_c$, there is a dynamically generated superpotential (from gaugino condensation or through instantons) [18]:

$$W_{\text{eff}} = A\left(\frac{\Lambda^{b_0}}{\text{Pf}M}\right)^{1/(N_c+1-N_f)}, \tag{41}$$

where $b_0 = 3(2N_c + 2) - 2N_f$, $M_{ij} = Q_iQ_j$ is an antisymmetric tensor (e.g., the Q's are combined with the antisymmetric tensor that the group preserves), and A is a normalization constant. Again, for larger N_f, the theory can be analyzed by integrating out heavy quarks or using a duality to a $Sp[2(N_f - N_c - 2)]$ theory. There are no baryons for the $Sp(2N_c)$ theories, as they break up into mesons by virtue of the ϵ tensor being expressible in terms of the antisymmetric tensor preserved by $Sp(2N_c)$.

The exceptional group G_2 also fits into this analysis quite easily [19,20]. The beta-function coefficient is $b_0 = 12 - N_f$, and there are N_f flavors of quarks in the fundamental **7** representation. There are several gauge-invariant fields; M denotes the dimension-two composite superfield. The effective superpotential is

$$W_{\text{eff}} = A\left(\frac{\Lambda^{b_0}}{\det M}\right)^{1/(4-N_f)}, \tag{42}$$

which again matches our general form. This superpotential is generated by gluino condensation for $N_f \leq 2$ and by instantons for $N_f = 3$. For larger N_f, there is again a quantum modified moduli space and, then, a dual picture for $N_f \geq 6$.

The exceptional groups present some difficulties in attempting to extend this analysis, which is already apparent in the G_2 theory [20]. The more complicated group structure gives rise to many gauge-invariant composite fields, and so the effective potential form cannot be completely fixed from general considerations. Even so, the same arguments (such as the R symmetry and flavor symmetry) that give rise to the effective superpotential of, e.g., $SU(N_c)$ are very general and could possibly give the effective superpotential for at least some region of the theories with other exceptional groups [20]. One way to do this is to consider subgroups, which are reached by VEVs of the different gauge-invariant composite fields. For instance, the **27** of E_6 can break the group to $SO(10)$ with a singlet, fundamental, and spinor representation (**1** + **10** + **16**). This can have a generated superpotential, and turning on the VEVs will break this into smaller and smaller subgroups. So, it seems quite possible that these results could hold, with some restrictions, for theories with any simple Lie group.

V. APPLICATIONS TO MODEL BUILDING

We will now briefly discuss our results in the context of building supersymmetric models, in a similar spirit to [3]. As in [3], we have presented a mechanism for incorporating a discrete R symmetry and retrofitting (generating) mass scales of a model by using the VEV of a singlet field. The overall goal of this process is to make a model more "natural": a mass hierarchy from marginal or irrelevant couplings, rather than imposed by hand. The R symmetry can also be used to forbid unwanted operators.

First, let us consider how to build general models. As a general method, take the superpotential of the model to be modified and replace masses by an interaction with a singlet field. One may need more than one singlet in order to generate different scales or use different coupling constants at the expense of some tuning or imposing a hierarchy. Additionally, one can choose the group, number of colors, and number of flavors for the gauge theory the singlet(s) are coupled to in order to produce the discrete R symmetry desired. The R charges of the model are now (at least partially) fixed due to the singlet interactions. This can forbid unwanted operators.

As an example, we consider a generalization of the Next-to-Minimal Supersymmetric Standard Model (NMSSM)—the Minimal Supersymmetric Standard Model (MSSM), plus a singlet—as in the recent work [21] (see also, the earlier work [22]). The superpotential of the Higgs and singlet superfields is

$$W = (\mu + \lambda S)H_uH_d + \tfrac{1}{2}\mu_s S^2, \tag{43}$$

where the cubic and linear terms for S are assumed to be negligible or set to zero. This does not solve the μ problem and, in fact, adds another, μ_s. However, this model closely resembles the MSSM phenomenologically and, without tuning the scalar potential, can have the lightest neutral Higgs mass above current bounds and light top squarks.

At first glance, this model seems rather unnatural: there are two free-mass parameters, and some unwanted terms in the superpotential are simply set to zero. However, it is quite simple in our framework to alleviate these problems. We can use just a single additional singlet, $\tilde{S}$, coupled to this model to make it more natural and only require slight tuning of coupling constants to get any desired hierarchy between μ and μ_s:

$$W = \left(\frac{\alpha\tilde{S}^2}{M_p} + \lambda S\right)H_uH_d + \beta\frac{1}{2}\frac{\tilde{S}^2}{M_p}S^2. \tag{44}$$

In order to have all these terms have R charge 2, the R charge of the Higgs and S must be the same, and S has twice the R charge of $\tilde{S}$ (all mod $2b_0$). Then, cubic and linear terms of S are forbidden. In [21], μ and μ_s are free parameters taken at or below the TeV scale; $\mu = 500$ GeV and $\mu_s = 2$ TeV are used in the plots. Here, one would require $\beta \approx 4\alpha$ to generate these relative scales. This is now the same solution to the μ problem considered in [3,7].

VI. DISCUSSION AND CONCLUSION

We have constructed models with a discrete R symmetry that is respected by the instanton, but gaugino condensation and the singlet VEV ultimately break it down to just a $\mathbb{Z}_{2R}$. In [3], it was argued that this is not really an R

symmetry; combined with a 2π rotation from the Lorentz group, this is just a non-R $\mathbb{Z}_2$. However, as discussed in [3], models with a discrete R symmetry (larger than $\mathbb{Z}_{2R}$) can be very important in models with low-energy supersymmetry.

Because of the gaugino condensation and singlet VEV, we showed that there are b_0 vacuum states generally, and then in detail for the $SU(N_c)$ theories. This is basically a generalization of the discrete $\mathbb{Z}_{2N_c}$ symmetry and resulting states in $SU(N_c)$, with $N_f < N_c$ flavor of quarks and gaugino condensation. We have analyzed general theories with singlet interactions to construct the discrete R symmetry. The discrete R symmetry was found by considering an instanton and finding what discrete subgroup of the $U(1)_R$ it respects: $\mathbb{Z}_{2b_0R}$.

We have analyzed many of the more common vectorlike supersymmetric gauge theories with an effective superpotential generated by gaugino condensation or instanton effects. We studied these with some simplifications of the couplings. Although these different theories share many common features, in different regions of couplings, there are different behaviors. The common form of the effective superpotential for these gauge theories motivated a general expression, which we used to show that we expect b_0 vacuum states. For the known examples, the form of the effective superpotential follows from renormalization, gauge symmetry, and the R symmetry. These are quite general arguments for any gauge group, but the exact form of the effective superpotential is not completely fixed for the exceptional groups. These groups have several gauge-invariant composite fields, and so there is still ambiguity in the form of the effective superpotential. On the other hand, our analysis of the discrete R symmetry, its breaking, and the number of vacua calculated through integrating out heavy quarks, still applies. While a more detailed study may be possible, our general results apply here, as well, and, thus, to all simple Lie groups.

There are still several open questions to pursue. We did not explicitly analyze the theories in the so-called "conformal window," where the couplings and number of flavors would place the theory in a conformal regime. Although the previous analysis of integrating out heavy quarks (where γ is very small) still works, it would be interesting to understand the dynamics of the theory in this regime. How do the singlet interactions change the theory in this region? Are there general statements to be made here, as well? We have also restricted our analysis to certain regions of the parameter space (largely ignoring baryons), and have also limited cases of integrating out heavy-matter fields. Perhaps this can be made more precise, or maybe there are interesting special cases to be found. Theories like the $SO(N_c)$ case also have more involved dynamics depending on the number of flavors. Again, maybe there is more to be said here, as well. However, even at this point, the picture we have presented is quite general and prevalent in supersymmetric Yang-Mills theories.

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[1] T. R. Taylor, G. Veneziano, and S. Yankielowicz, Nucl. Phys. **B218**, 493 (1983); G. Veneziano and S. Yankielowicz, Phys. Lett. **113B**, 231 (1982); Hans Peter Nilles, *ibid.* **112**, 455 (1982).

[2] Michael Dine, Jonathan L. Feng, and Eva Silverstein, Phys. Rev. D **74**, 095012 (2006).

[3] Michael Dine and John Kehayias, Phys. Rev. D **82**, 055014 (2010).

[4] Kenneth A. Intriligator and N. Seiberg, Nucl. Phys. **B431**, 551 (1994).

[5] Kenneth A. Intriligator, R. G. Leigh, and N. Seiberg, Phys. Rev. D **50**, 1092 (1994); Kenneth A. Intriligator, Phys. Lett. B **336**, 409 (1994).

[6] Luis E. Ibanez and Graham G. Ross, Phys. Lett. B **260**, 291 (1991); Nucl. Phys. **B368**, 3 (1992); Kiichi Kurosawa, Nobuhito Maru, and T. Yanagida, Phys. Lett. B **512**, 203 (2001); Herbi K. Dreiner, Christoph Luhn, and Marc Thormeier, Phys. Rev. D **73**, 075007 (2006).

[7] T. Yanagida, Phys. Lett. B **400**, 109 (1997).

[8] Michael Edward Peskin, arXiv:hep-th/9702094.

[9] Ian Affleck, Michael Dine, and Nathan Seiberg, Phys. Rev. Lett. **51**, 1026 (1983); Nucl. Phys. **B241**, 493 (1984).

[10] Nathan Seiberg, Phys. Rev. D **49**, 6857 (1994).

[11] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).

[12] Philip C. Argyres and Mohammad Edalati, J. High Energy Phys. 01 (2006) 012; 02 (2006) 071; 07 (2006) 021.

[13] Robbert Dijkgraaf and Cumrun Vafa, arXiv:hep-th/0208048.

[14] Riccardo Argurio, Vanicson L. Campos, Gabriele Ferretti, and Rainer Heise, Phys. Rev. D **67**, 065005 (2003); Yves Demasure and Romuald A. Janik, Phys. Lett. B **553**, 105 (2003).

[15] Romuald A. Janik, Acta Phys. Polon. B **34**, 4879 (2003).

[16] Kenneth A. Intriligator and N. Seiberg, Nucl. Phys. B, Proc. Suppl. **45**, 1 (1996).
[17] Kenneth A. Intriligator and N. Seiberg, Nucl. Phys. **B444**, 125 (1995).
[18] Kenneth A. Intriligator and P. Pouliot, Phys. Lett. B **353**, 471 (1995).
[19] Igor Pesando, Mod. Phys. Lett. A **10**, 1871 (1995).
[20] Steven B. Giddings and John M. Pierre, Phys. Rev. D **52**, 6065 (1995).
[21] Antonio Delgado, Christopher Kolda, J. Pocahontas Olson, and Alejandro de la Puente, Phys. Rev. Lett. **105**, 091802 (2010).
[22] Michael Dine, Nathan Seiberg, and Scott Thomas, Phys. Rev. D **76**, 095004 (2007).