

Tempered two-Higgs-doublet model

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We discuss the phenomenological consequences of requiring the cancellation of quadratic divergences up to the leading two-loop order within the two-Higgs-doublet model. Taking into account existing experimental constraints, allowed regions in the parameter space, permitting the cancellation, are determined. A degeneracy between masses of scalar bosons is observed for $\tan\beta \gtrsim 40$. The possibility for CP violation in the scalar potential is discussed and regions of $\tan\beta$ - M_{H^\pm} with a substantial amount of CP violation are determined. In order to provide a source for dark matter in a minimal manner, a scalar gauge singlet is introduced and discussed. The model allows to ameliorate the little hierarchy problem by lifting the minimal scalar Higgs-boson mass and by suppressing the quadratic corrections to scalar masses. The cutoff originating from the naturality arguments is therefore lifted from ~ 0.6 TeV in the standard model to $\gtrsim 2.5$ TeV in two-Higgs-doublet model depending on the mass of the lightest scalar.

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I. INTRODUCTION

The goal of this work is to extend the standard model (SM) such that there would be no quadratic divergences to scalar masses up to the leading order at the two-loop level of the perturbation expansion. The quadratic divergences were first studied within the SM by Veltman [1], who showed that applying dimensional reduction [2] one gets the following quadratically divergent one-loop correction to the Higgs-boson (h) mass:

$$\delta^{(\text{SM})}m_h^2 = \frac{\Lambda^2}{\pi^2 v^2} \left[\frac{3}{2}m_t^2 - \frac{1}{8}(6m_W^2 + 3m_Z^2) - \frac{3}{8}m_h^2 \right], \quad (1)$$

where Λ is a UV cutoff and $v \simeq 246$ GeV denotes the vacuum expectation value of the scalar doublet. The issue of quadratic divergences was then investigated further adopting other regularization schemes (e.g. point splitting [3]) and also in [4] without reference to any regularization scheme.

Since precision measurements require a light Higgs boson, the correction (1) exceeds the mass itself even for small values of Λ , e.g. for $m_h = 130$ GeV we obtain $\delta^{(\text{SM})}m_h^2 \simeq m_h^2$ already for $\Lambda \simeq 580$ GeV. On the other hand, if we assume that the scale of new physics is widely separated from the electroweak scale, then constraints that emerge from analysis of operators of dimension 6 require $\Lambda \gtrsim$ a few TeV. The lesson from this observation is that whatever is beyond the SM physics, some amount of fine tuning is necessary; either we tune to lift the cutoff above $\Lambda \simeq 580$ GeV, or we tune when precision observables

measured at LEP are fitted.¹ Tuning both in corrections to the Higgs mass and in LEP physics is, of course, also a viable alternative which we are going to explore below. So, we will look for new physics in the TeV range which will allow to lift the cutoff implied by quadratic corrections to m_h^2 to the multi-TeV range *and* which will be consistent with all the experimental constraints—both require some amount of tuning. Within the SM the requirement $\delta^{(\text{SM})}m_h^2 = 0$ implies $m_h \simeq 310$ GeV. However, as is very well known, the present data favor a light SM Higgs boson—according to the PDG [5], after including all the available experimental data and taking into account theoretical uncertainties, the 99% C.L. upper limit for the Higgs mass reads: $m_h \leq 194$ GeV. Therefore, within the SM the one-loop condition $\delta^{(\text{SM})}m_h^2 = 0$ requires an unrealistic value of the Higgs-boson mass.

Examining closer the experimental constraints one finds also the following tension which emerges in the process of fitting all the available data to the SM (see [6] for a recent review). Hadronic asymmetry measurements ($A_{\text{FB}}^c, A_{\text{FB}}^b, Q_{\text{FB}}$) favor a heavy Higgs boson, with $m_h \sim 500$ GeV, while leptonic asymmetries ($A_{\text{LR}}, A_{\text{FB}}^l$) together with nonasymmetry precision measurements (m_W, Γ_Z, \dots) favor a Higgs mass smaller by one order of magnitude. If ($A_{\text{FB}}^c, A_{\text{FB}}^b, Q_{\text{FB}}$) are omitted from the fit one obtains $m_h \sim 50$ GeV with an upper limit $m_h < 105$ GeV at the 95% C.L. [6]. Moreover there is the LEP lower limit on the Higgs mass, $m_h > 114.4$ GeV [7]. The fit which combines all the data is therefore of low quality. That observation suggests a modification of the SM which would allow for a

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¹In terms of the effective Lagrangian approach that implies coefficients of dimension-6 operators $c_i \ll 1$.

heavy Higgs boson with a mass at least above the LEP limit. For that the SM prediction for the oblique parameters S and T must be modified by the extension of the SM that we are seeking.

Here we are going to construct a model which would both soften the little hierarchy problem by suppressing $\delta^{(\text{SM})}m_h^2$ and which would allow to lift the central value for the Higgs mass up to a value which is well above the LEP limit (presumably it would imply a better fit of the precision observables). We would like to point out also that increasing the Higgs-boson mass would ameliorate the little hierarchy problem even if $\delta^{(\text{SM})}m_h^2$ was not suppressed (since then larger cutoff would lead to the correction of the order of the mass itself).

There are only two ways to suppress $\delta^{(\text{SM})}m_h^2/m_h^2$: one can either modify the SM such that (i) larger SM-like Higgs-boson mass is allowed; or (ii) extra radiative corrections to $\delta^{(\text{SM})}m_h^2$ emerge that partially cancel (1). The best-studied example of the second approach is provided by supersymmetric theories for which $\delta m_h^2 \ll m_h^2$ up to the grand unified theory scale, however the suppression of $\delta^{(\text{SM})}m_h^2$ could also be achieved through very modest means, e.g. by introducing just extra real scalar singlets to the SM [8] (although more tuning than in the supersymmetric case is necessary). The first strategy was followed in [9] within the so-called inert doublet model² (IDM). There, a second Higgs doublet was introduced and an exact \mathbb{Z}_2 symmetry was imposed to provide a dark-matter candidate. As shown in [9] a large (400–600 GeV) SM-like Higgs-boson mass was allowed by the addition of an extra Higgs doublet (the inert doublet). It was demonstrated that the extra contributions to the oblique parameters originating from the inert doublet (with physical fields H^\pm , A and S) can cancel large effects of a heavy SM Higgs, such that $m_h \sim 400\text{--}600$ GeV is allowed. Here we propose a model which does both, i.e. suppression of $\delta^{(\text{SM})}m_h^2$ by contributions from some extra states (which implies reduced $\delta^{(\text{SM})}m_h^2/m_h^2$) and which modifies the results of the global fit such that much heavier SM Higgs boson is allowed (that also helps to decrease $\delta^{(\text{SM})}m_h^2/m_h^2$ and in addition it could eliminate the tension caused by the high LEP lower bound in the presence of a low central value from precision tests). A different approach to this problem has been proposed in [11].

Another well-known problem of the SM is the strength of CP violation which is too weak to make the electroweak baryogenesis viable [12]. It is also worth noting a too slow phase transition (within the SM) which is another difficulty for realistic baryogenesis [12].

In light of the above remarks it seems very natural to consider simple extensions of the SM scalar sector, as for

instance the two-Higgs-doublet model (2HDM). Our intention is to bring the reader's attention to a region of parameter space that not only is consistent with standard theoretical requirements (positivity and unitarity) and satisfies all the relevant experimental constraints, but also offers a simple pragmatic option to reduce the size of the quadratic corrections to scalar two-point Green's functions (so in other words to scalar masses). It has been noticed a long time ago [13] that within the 2HDM one can cancel quadratically divergent corrections to two-point Green's functions for scalar particles. Some phenomenological consequences of the cancellation were discussed already in [14]. It is well known [15] that within 2HDM extensions the oblique parameters S , T and U can be modified such that SM contributions growing with m_h ($\propto \ln m_h$) could be canceled by other terms (originating from extra scalars present in the 2HDM), so that the lightest Higgs boson could be relatively heavy, see [9]. Within the 2HDM the electroweak phase transition could also be made fast enough [16] to make electroweak baryogenesis viable. The 2HDM provides also new sources of CP violation in interactions of neutral scalars. Therefore, here we will discuss 2HDMs which do not suffer from quadratic divergences in scalar two-point Green's functions, the tempered two-Higgs-doublet model, seeking a model which also allows for CP violation in the scalar potential. Since we have argued above that the heavy SM-like Higgs boson would be more consistent with experimental data, *we will investigate how much CP violation in 2HDMs is allowed after lifting the Higgs mass well above the LEP lower limit and by imposing the conditions needed to cancel quadratic divergences (so as to ameliorate the little hierarchy problem)*. We will not address here the issue of the electroweak phase transition.

In a recent publication [17], motivated by similar arguments, we have considered a version of the IDM with CP violation introduced by replacing the SM-like Higgs doublet by a pair of doublets. There, a candidate for dark matter (DM) was provided by the lightest neutral component of the inert doublet (as in the original IDM). In the model considered here, CP violation again originates from the 2HDM, however in order to accommodate a DM candidate *in a minimal manner* (instead of introducing the inert doublet as in [17]) we extend the model by a real singlet.³ In fact, it is intriguing to note that the singlet is even more inert than the original inert doublet since it interacts only with the Higgs doublets and with right-handed neutrinos, having no gauge interactions.

The paper is organized as follows. In Sec. II we investigate theoretical and phenomenological consequences of the cancellation conditions within the general 2HDM. In

²The model was introduced in [10] in the context of dark matter.

³Although our basic motivations is different, this possibility is similar to the idea proposed in [18] for DM.

order to accommodate a DM candidate we introduce an extra real scalar gauge singlet, that is discussed in Sec. III. Section IV contains our summary.

II. NONINERT TWO-HIGGS-DOUBLET MODEL

A very appealing possibility would be to combine the IDM with the idea of canceling the one-loop quadratic divergences. However, as we have shown in [19], that is impossible because of the vacuum stability conditions in the IDM are inconsistent with the requirement of cancellation of quadratic divergences. Since our intention is to build a 2HDM, which has no quadratic divergences at least at the one-loop level therefore, in the following we will consider a general (noninert) 2HDM hoping for both a successful implementation of the cancellation condition and for a new source of CP violation. The price to pay will be the loss of a DM candidate. We return to that issue in Sec. III.

In order to accommodate CP violation we consider here a noninert 2HDM with softly broken \mathbb{Z}_2 symmetry which acts as $\Phi_1 \rightarrow -\Phi_1$ and $u_R \rightarrow -u_R$ (all other fields are neutral). The scalar potential then reads

$$\begin{aligned} V(\phi_1, \phi_2) = & -\frac{1}{2}[m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 \\ & + [m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}] + \frac{1}{2}\lambda_1(\phi_1^\dagger \phi_1)^2 \\ & + \frac{1}{2}\lambda_2(\phi_2^\dagger \phi_2)^2 + \lambda_3(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\ & + \lambda_4(\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2}[\lambda_5(\phi_1^\dagger \phi_2)^2 + \text{H.c.}]. \end{aligned} \quad (2)$$

The minimization conditions at $\langle \phi_1^0 \rangle = v_1/\sqrt{2}$ and $\langle \phi_2^0 \rangle = v_2/\sqrt{2}$ can be formulated as follows:

$$\begin{aligned} m_{11}^2 &= v_1^2 \lambda_1 + v_2^2 (\lambda_{345} - 2\nu), \\ m_{22}^2 &= v_2^2 \lambda_2 + v_1^2 (\lambda_{345} - 2\nu), \end{aligned} \quad (3)$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \text{Re}\lambda_5$ and $\nu \equiv \text{Re}m_{12}^2/(2v_1 v_2)$.

We assume that ϕ_1 and ϕ_2 couple to down- and up-type quarks, respectively, (the so-called 2HDM II).

A. One-loop quadratic divergences

The cancellation of one-loop quadratic divergences for the scalar two-point Green's functions at zero external momenta (G_i , $i = 1, 2$) implies [13] in the case of 2HDM type II:

$$\begin{aligned} G_1 &\equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4\right) \\ &- 3\frac{m_b^2}{c_\beta^2} = 0, \end{aligned} \quad (4)$$

$$\begin{aligned} G_2 &\equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2}\left(\frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4\right) \\ &- 3\frac{m_t^2}{s_\beta^2} = 0, \end{aligned} \quad (5)$$

where $v^2 \equiv v_1^2 + v_2^2$, $\tan\beta \equiv v_2/v_1$ and we adopt the notation: $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$. We note that when $\tan\beta$ is large, the two quark contributions can be comparable. In the type II model the mixed, $\phi_1 - \phi_2$, Green's function is not quadratically divergent.

In the general CP -violating case, the quartic couplings λ_i can be expressed in terms of the mass parameters and elements of the rotation matrix needed for diagonalization of the scalar masses (see, for example, Eqs. (3.1)–(3.5) of [20]):

$$\begin{aligned} \lambda_1 &= \frac{1}{c_\beta^2 v^2} [c_1^2 c_2^2 M_1^2 + (c_1 s_2 s_3 + s_1 c_3)^2 M_2^2 \\ &+ (c_1 s_2 c_3 - s_1 s_3)^2 M_3^2 - s_\beta^2 \mu^2], \end{aligned} \quad (6)$$

$$\begin{aligned} \lambda_2 &= \frac{1}{s_\beta^2 v^2} [s_1^2 c_2^2 M_1^2 + (c_1 c_3 - s_1 s_2 s_3)^2 M_2^2 \\ &+ (c_1 s_3 + s_1 s_2 c_3)^2 M_3^2 - c_\beta^2 \mu^2], \end{aligned} \quad (7)$$

$$\begin{aligned} \lambda_3 &= \frac{1}{c_\beta s_\beta v^2} \{c_1 s_1 [c_2^2 M_1^2 + (s_2^2 s_3^2 - c_3^2) M_2^2 \\ &+ (s_2^2 c_3^2 - s_3^2) M_3^2] + s_2 c_3 s_3 (c_1^2 - s_1^2) (M_3^2 - M_2^2)\} \\ &+ \frac{1}{v^2} [2M_{H^\pm}^2 - \mu^2], \end{aligned} \quad (8)$$

$$\lambda_4 = \frac{1}{v^2} [s_2^2 M_1^2 + c_2^2 s_3^2 M_2^2 + c_2^2 c_3^2 M_3^2 + \mu^2 - 2M_{H^\pm}^2], \quad (9)$$

$$\text{Re } \lambda_5 = \frac{1}{v^2} [-s_2^2 M_1^2 - c_2^2 s_3^2 M_2^2 - c_2^2 c_3^2 M_3^2 + \mu^2], \quad (10)$$

$$\begin{aligned} \text{Im } \lambda_5 &= \frac{-1}{c_\beta s_\beta v^2} \{c_\beta [c_1 c_2 s_2 M_1^2 - c_2 s_3 (c_1 s_2 s_3 + s_1 c_3) M_2^2 \\ &+ c_2 c_3 (s_1 s_3 - c_1 s_2 c_3) M_3^2] \\ &+ s_\beta [s_1 c_2 s_2 M_1^2 + c_2 s_3 (c_1 c_3 - s_1 s_2 s_3) M_2^2 \\ &- c_2 c_3 (c_1 s_3 + s_1 s_2 c_3) M_3^2]\}, \end{aligned} \quad (11)$$

where $\mu \equiv v^2 \nu$ while $c_i = \cos\alpha_i$ and $s_i = \sin\alpha_i$ refer to the neutral-Higgs-sector rotation matrix R , the latter parametrized in terms of the angles α_1 , α_2 and α_3 according to the convention of [21].

It will be useful to adopt the following relation (emerging from the diagonalization of the neutral Higgs mass matrix [22]) between M_1^2 , M_2^2 and M_3^2 :

$$M_3^2 = \frac{M_1^2 R_{13}(-R_{11} + R_{12} \tan\beta) + M_2^2 R_{23}(-R_{21} + R_{22} \tan\beta)}{R_{33}(R_{31} - R_{32} \tan\beta)}. \quad (12)$$

Substitution of (12) into (6)–(11) allows to express the quartic couplings through the mixing angles together with M_1^2 , M_2^2 , $M_{H^\pm}^2$ and μ^2 (eliminating M_3^2). Then, inserting the appropriate quartic couplings into the conditions for cancellation of the quadratic divergences (4) and (5) we obtain two linear equations for M_1^2 and M_2^2 with coefficients depending on the mixing angles α_i (as well as on $M_{H^\pm}^2$ and μ^2). Therefore, for a given choice of α_i 's, the squared neutral-Higgs masses M_1^2 , M_2^2 and M_3^2 can be determined from the cancellation conditions (4) and (5) in terms of $\tan\beta$, μ^2 and $M_{H^\pm}^2$.

Scalar masses resulting from a scan over α_i , M_{H^\pm} and $\tan\beta$ are shown in Fig. 1 for $\mu = 200$ GeV and

$\mu = 500$ GeV. The charged Higgs-boson mass was varied between 300 GeV and 700 GeV. Since only large $\tan\beta$ will turn out to be allowed we have chosen to display plots with $40 \leq \tan\beta \leq 50$ in order to illustrate a specific property of the scalar spectrum that is visible at large $\tan\beta$. Under the scan, the M_i^2 were calculated along the lines described above. The only extra constraints (the cancellation of quadratic divergences was, of course, guaranteed implicitly by the construction) imposed were $M_i^2 > 0$ and $M_1 \leq M_2 \leq M_3$. A striking degeneracy of the neutral-Higgs masses is observed for the case of large $\tan\beta$. This degeneracy can be understood by expanding M_i^2 for large $\tan\beta$. The cancellation conditions, Eqs. (4) and (5) can then be expressed as follows:

$$Y_{11}M_1^2 + Y_{12}M_2^2 - Y_{13}(4m_b^2 + \mu^2) = \mathcal{O}\left(\frac{1}{\tan\beta}\right), \quad (13)$$

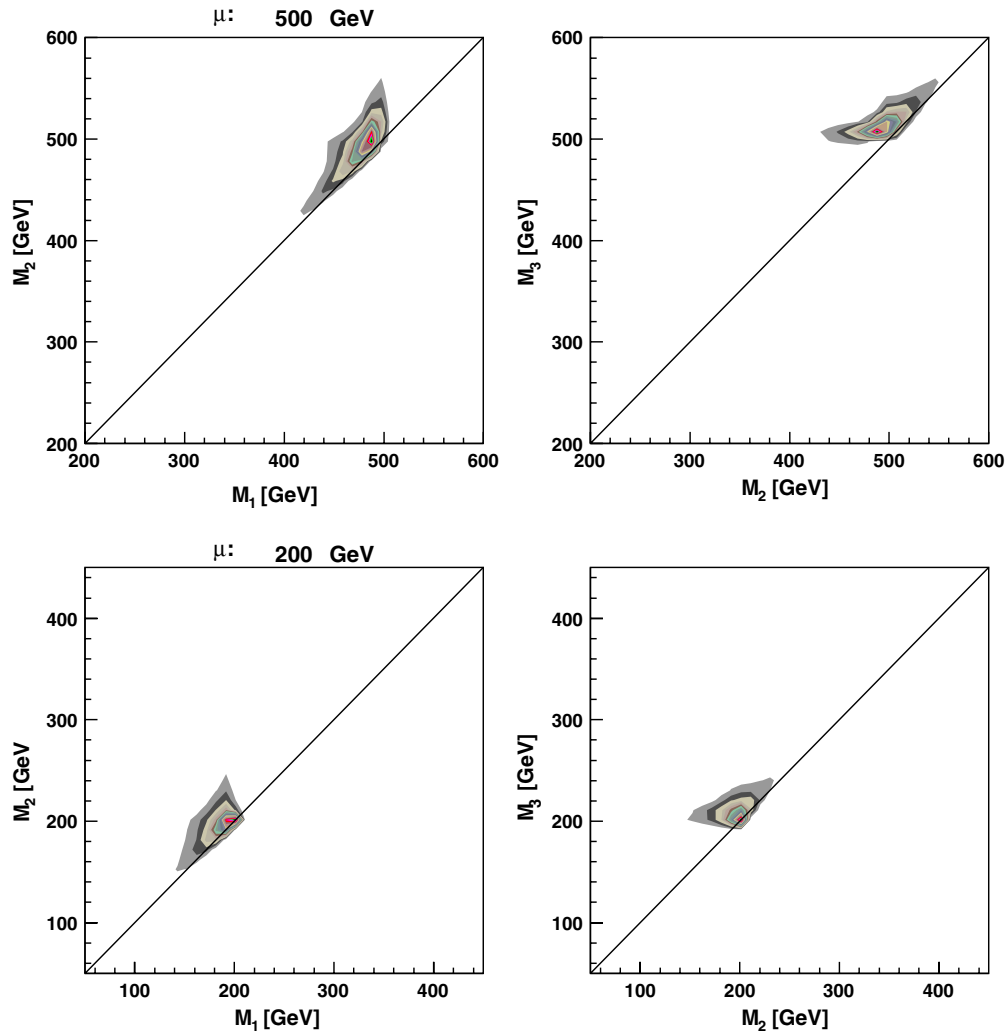


FIG. 1 (color online). Distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right), resulting from a scan over the full range of α_i , $\tan\beta \in (40, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 500$ GeV (top) and $\mu = 200$ GeV (bottom). No constraints are imposed other than the cancellation of quadratic divergences (4) and (5), $M_i^2 > 0$ and $M_1 < M_2 < M_3$. The color (grey-scale) coding indicates increasing density (while scanning over the parameter space) of allowed points as one moves inward from the boundary.

$$2R_{12}R_{22}\tan\beta(M_1^2 - M_2^2) - R_{33}[-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2] = \mathcal{O}\left(\frac{1}{\tan\beta}\right), \quad (14)$$

where⁴

$$\begin{aligned} Y_{11} &\equiv -R_{12}R_{13}R_{31}^2 + R_{11}^2R_{32}R_{33}, \\ Y_{12} &\equiv -R_{22}R_{23}R_{31}^2 + R_{21}^2R_{32}R_{33}, \\ Y_{13} &\equiv R_{32}R_{33}, \\ \bar{m}^2 &\equiv \frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2. \end{aligned} \quad (15)$$

First, it is useful to notice that (14) implies

$$M_1^2 - M_2^2 \sim \frac{1}{\tan\beta} \frac{R_{33}}{R_{12}R_{22}} \times [-4\bar{m}^2 - 2M_{H^\pm}^2 + 12m_t^2 + \mu^2]. \quad (16)$$

Therefore, for $\tan\beta \gg 1$, we expect to have $M_1^2 \simeq M_2^2$ unless $|R_{12}R_{22}| \ll 1$ or $-2M_{H^\pm}^2 + \mu^2$ is very large.⁵ Second, solving (13) and (14) one finds that to leading order (for large $\tan\beta$) $M_1^2 = M_2^2 = \mu^2 + 4m_b^2$.

On the other hand, expanding (12) for $\tan\beta \gg 1$ one obtains

$$M_3^2 = -\frac{M_1^2R_{12}R_{13} + M_2^2R_{22}R_{23}}{R_{32}R_{33}} + \mathcal{O}\left(\frac{1}{\tan\beta}\right). \quad (17)$$

Therefore (invoking unitarity of R) it is seen that the degeneracy $M_1 = M_2$ implies that also $M_1 = M_2 = M_3$. Finally we can conclude that for large $\tan\beta$ one obtains $M_1 \simeq M_2 \simeq M_3 \simeq \mu^2 + 4m_b^2$, this explains the approximate degeneracy observed in Fig. 1.⁶

B. Two-loop leading quadratic divergences

The generic form of the quadratically divergent contributions to scalar two-point Green's functions at zero external momenta reads [4]

$$\delta G_i = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda) \left[\ln\left(\frac{\Lambda}{\bar{\mu}}\right) \right]^n + \dots, \quad (18)$$

where n corresponds to $(n+1)$ -loop contribution, λ stands for relevant coupling constants, $\bar{\mu}$ is the renormalization scale and $f_n^{(i)}(\lambda)$ is a calculable (order by order) function (polynomial) of the couplings. It should be noticed that at

⁴Note that $Y_{11} + Y_{12} = Y_{13}$.

⁵Note that cancellations between the $M_{H^\pm}^2$ and μ^2 terms are possible. This is why the degeneracy survives even for μ as large as $\mu = 500$ GeV, see Fig. 1.

⁶The reader should be warned that the above expansions are justified if the coefficients of subleading terms $\propto 1/\tan\beta$ are not enhanced by special values of the mixing angles (that would correspond to CP conservation in the scalar sector). Since here we are interested in the case of CP violation, we will not elaborate on those CP conserving limits.

the $(n+1)$ -loop level there exist also subleading contributions that contain terms $\propto \Lambda^2(\ln\Lambda)^m$ with $m < n$, so for instance at the two-loop level the leading contribution is $\propto \Lambda^2 \ln\Lambda$ while there are also subleading terms $\propto \Lambda^2$. The coefficients of the leading terms, $f_n^{(i)}(\lambda)$, can be determined recursively adopting a nice algorithm noticed by Einhorn and Jones [4]:

$$(n+1)f_{n+1}^{(i)} = \bar{\mu} \frac{\partial}{\partial \bar{\mu}} f_n^{(i)} = \sum_I \beta_{\lambda_I} \frac{\partial}{\partial \lambda_I} f_n^{(i)}, \quad (19)$$

where the sum runs over coupling constants that contribute to the coefficient $f_n^{(i)}$. Hereafter we will limit ourselves to the leading two-loop contributions. Therefore, to calculate $f_1^{(i)}$, only the one-loop coefficient $f_0^{(i)}$ and one-loop beta functions are needed. As beta functions for the 2HDM are known [23] the cancellation condition for quadratic divergences up to the leading two-loop order can easily be determined:

$$G_1 + \delta G_1 = 0 \quad \text{and} \quad G_2 + \delta G_2 = 0 \quad (20)$$

with

$$\begin{aligned} \delta G_1 &= \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_1} + 4\beta_{\lambda_3} + 2\beta_{\lambda_4}] \\ &\quad \times \ln\left(\frac{\Lambda}{\bar{\mu}}\right) \\ \delta G_2 &= \frac{v^2}{8} [9g_2\beta_{g_2} + 3g_1\beta_{g_1} + 6\beta_{\lambda_2} + 4\beta_{\lambda_3} \\ &\quad + 2\beta_{\lambda_4} - 24g_t\beta_{g_t}] \ln\left(\frac{\Lambda}{\bar{\mu}}\right). \end{aligned} \quad (21)$$

In what follows, adopting (6)–(11) we will be solving the conditions (20) for the scalar masses M_i^2 for a given set of α_i 's, $\tan\beta$, μ^2 and $M_{H^\pm}^2$. For the renormalization scale we will adopt v , so $\bar{\mu} = v$.⁷ Then those masses together with the corresponding coupling constants, will be adopted to find predictions of the model for various observables which can be confronted with experiments.

C. Positivity and unitarity constraints

The requirements of positivity for the 2HDM model potential are well known [10]:

$$\lambda_{1,2} > 0, \quad (23)$$

$$\lambda_3 > -\sqrt{\lambda_1\lambda_2}, \quad \lambda_L \equiv \lambda_3 + \lambda_4 - |\lambda_5| > -\sqrt{\lambda_1\lambda_2}. \quad (24)$$

One could also require the above conditions to be satisfied up the unification scale $\Lambda \sim 10^{15}$ GeV. That approach,

⁷Since we are using tree-level relations between quartic couplings and scalar masses, the renormalization scale should be of the order of the masses themselves, that is why we adopt here $\bar{\mu} = v$. For a more exhaustive discussion of the renormalization-scale dependence, see the first paper of [24].

resulting in much stronger constraints in terms of allowed scalar masses and $\tan\beta$ was followed in Ref. [25]. Here, as we consider UV completion appearing at the scale of a few TeV we do not follow that line of reasoning.

D. Experimental constraints

We impose the following experimental constraints:

- (i) The oblique parameters T and S
- (ii) $B_0 - \bar{B}_0$ mixing
- (iii) $B \rightarrow X_s \gamma$
- (iv) $B \rightarrow \tau \bar{\nu}_\tau X$
- (v) $B \rightarrow D \tau \bar{\nu}_\tau$
- (vi) LEP2 Higgs-boson nondiscovery
- (vii) R_b
- (viii) The muon anomalous magnetic moment
- (ix) Electron electric dipole moment

For more details concerning the implementation of the experimental constraints, see Refs. [17,20,26]. Subject to all these constraints, we find allowed solutions of (20). The recent paper [27] also contains an exhaustive analysis of experimental constraints on the 2HDM type II. The lower limit on the charged Higgs-boson mass adopted here, $M_{H^\pm} \geq 300$ GeV (basically determined by the $b \rightarrow s \gamma$

constraint) agrees roughly with the 95% C.L. lower limit, 316 GeV, obtained in [27] irrespective of $\tan\beta$.

E. Allowed regions

Imposing the above conditions we find allowed regions in the $\tan\beta - M_{H^\pm}$ plane as illustrated by the red domains in the $\tan\beta - M_{H^\pm}$ plane, see Figs. 2 and 3 for fixed values of μ . The allowed regions were obtained scanning over the mixing angles α_i and solving the two-loop cancellation conditions (20). Imposing also unitarity in the Higgs-Higgs-scattering sector [28–30] (yellow regions), the allowed regions are only slightly reduced. Requiring that also experimental constraints listed in Sec. IID are satisfied one obtains the green regions shown in the figure.

For parameters that are consistent with unitarity, positivity, experimental constraints and the two-loop cancellation conditions (20), we show in Figs. 4 and 5 scalar masses resulting from a scan over α_i , M_{H^\pm} and $\tan\beta$. Those plots could be compared with Fig. 1. One should however remember that in the two-loop case also unitarity, positivity and experimental constraints are taken into account. Note that in Figs. 2 and 3, consistent solutions are obtained only for $\tan\beta \gtrsim 15$ while at the one-loop level, also a small low- $\tan\beta$ region was allowed after imposing all

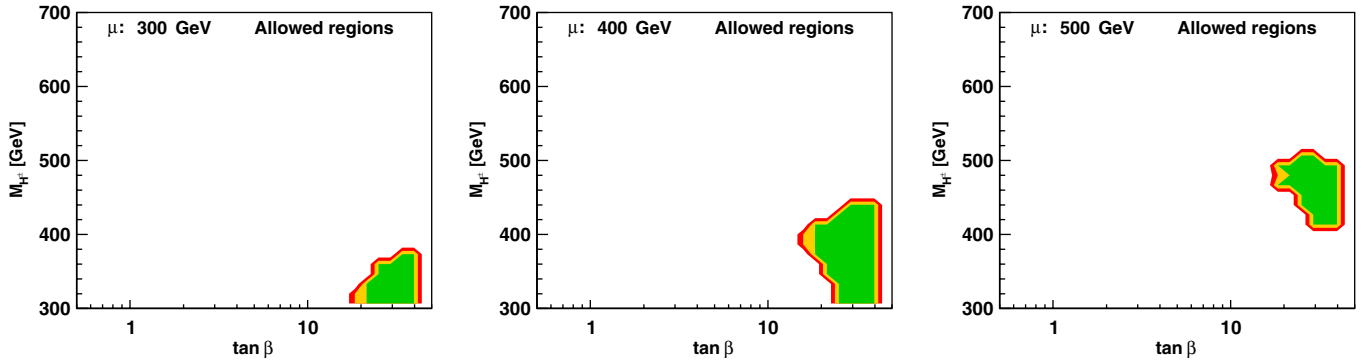


FIG. 2 (color online). Two-loop allowed regions in the $\tan\beta - M_{H^\pm}$ plane, for $\Lambda = 2.5$ TeV, for $\mu = 300, 400, 500$ GeV (as indicated). Red (outer grey): positivity is satisfied; yellow (light grey): positivity and unitarity both satisfied; green (inner grey): also experimental constraints satisfied at the 95% C.L., as specified in the text.

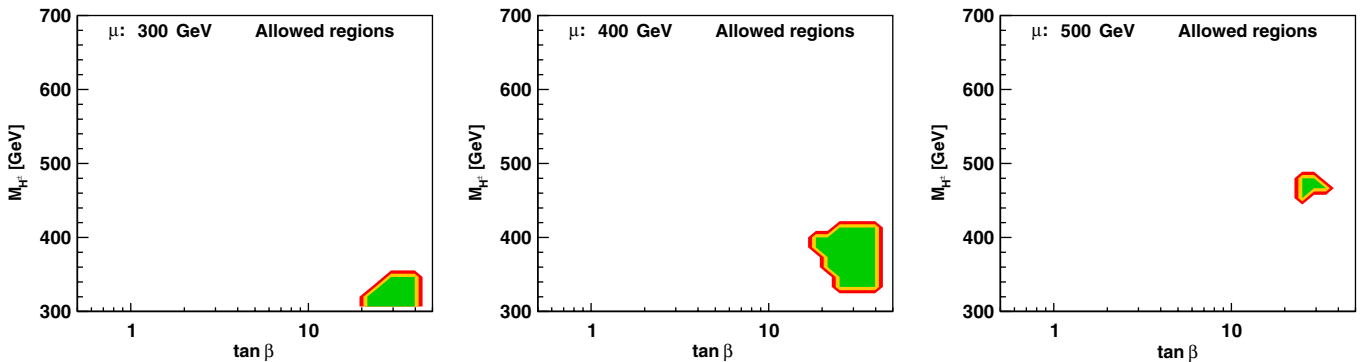


FIG. 3 (color online). Similar to Fig. 2 for $\Lambda = 6.5$ TeV.

the constraints. That small low- $\tan\beta$ region is disallowed after the two-loop corrections are imposed, see [19] for the one-loop result. As we have noticed for the one-loop spectrum, large $\tan\beta$ implies similar scalar masses. This is indeed what is being observed in Figs. 4 and 5 also for the two-loop case. The allowed solutions “peak” around $M_{H^\pm} \sim \mu$ with $20 \lesssim \tan\beta \lesssim 50$. For $\mu = 200$

and 600 GeV there are hardly any solutions for $\Lambda = 2.5$ TeV and no solutions were found for $\Lambda = 6.5$ TeV.

F. CP violation

Here we are going to discuss the possibility of having CP violation in the scalar potential (2), subject to the two-loop cancellation of quadratic divergences (20). In order to

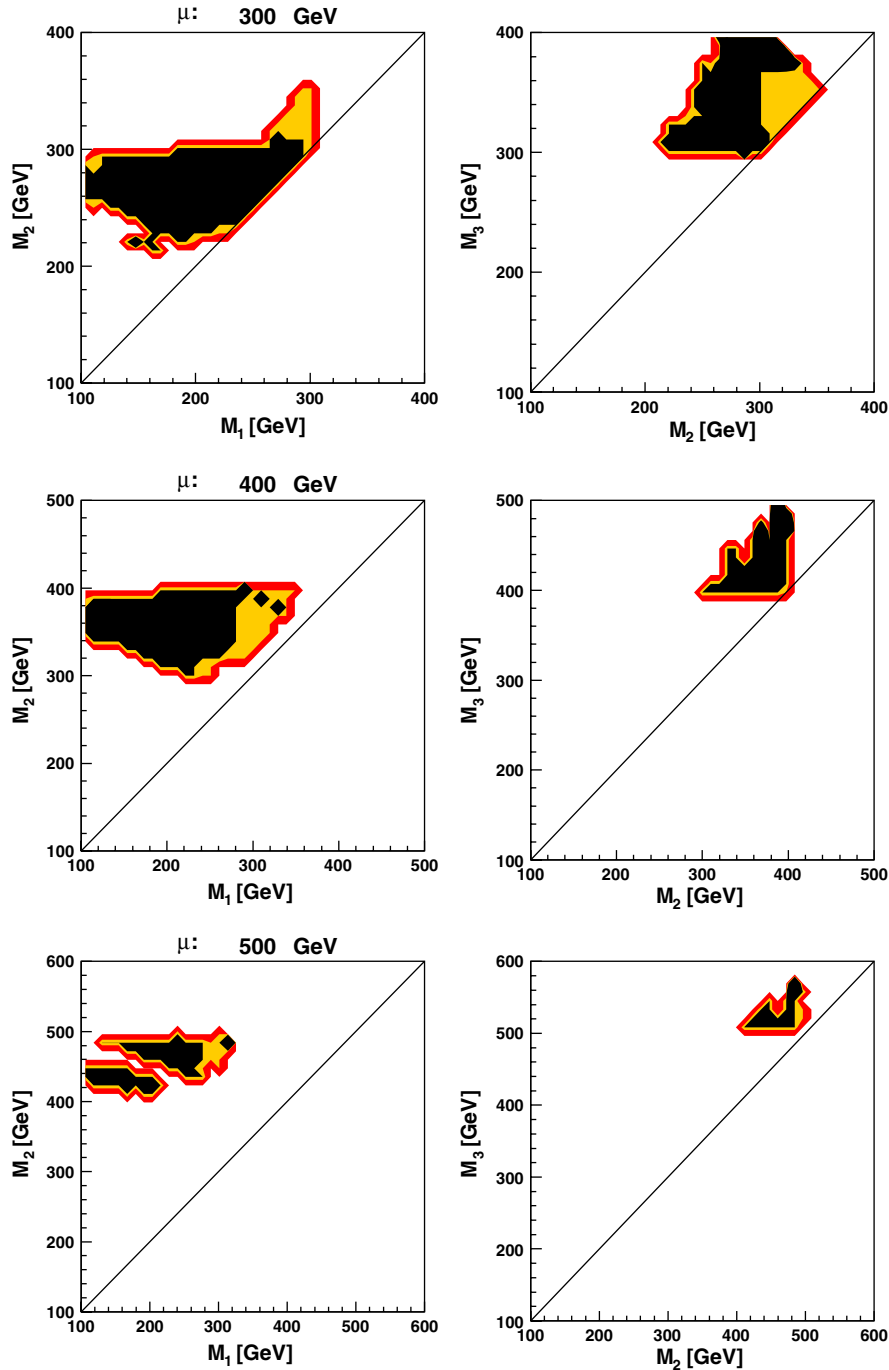
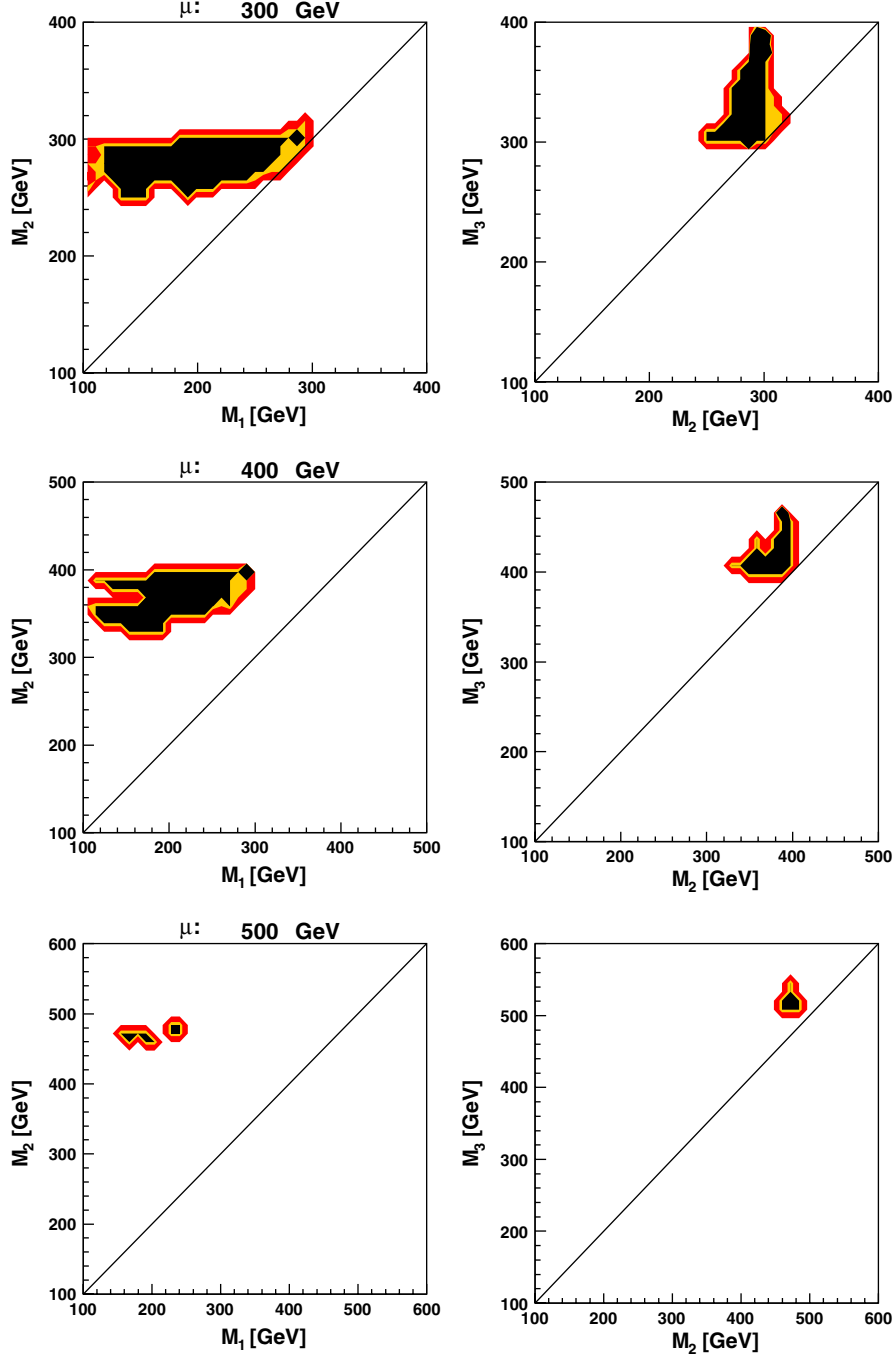


FIG. 4 (color online). Two-loop distributions of allowed masses M_2 vs M_1 (left panels) and M_3 vs M_2 (right) for $\Lambda = 2.5$ TeV, resulting from a scan over the full range of α_i , $\tan\beta \in (0.5, 50)$ and $M_{H^\pm} \in (300, 700)$ GeV, for $\mu = 300, 400, 500$ GeV. Red (outer grey): Positivity is satisfied; yellow (light grey): positivity and unitarity both satisfied; black: also experimental constraints satisfied at the 95% C.L., as specified in the text.


 FIG. 5 (color online). Similar to Fig. 4 for $\Lambda = 6.5$ TeV and $\mu = 300, 400, 500$ GeV.

parametrize the magnitude of CP violation we adopt the rephasing invariants introduced by Lavoura and Silva [31] (see also [32]). We shall here use the basis-invariant formulation of these invariants J_1, J_2 and J_3 as proposed by Gunion and Haber [33]. As is proven there (theorem #4) the Higgs sector is CP conserving if and only if all J_i are real. In the basis adopted here the invariants read [17]:

$$\text{Im} J_1 = -\frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) \text{Im} \lambda_5, \quad (25)$$

$$\begin{aligned} \text{Im} J_2 = & -\frac{v_1^2 v_2^2}{v^8} [((\lambda_1 - \lambda_3 - \lambda_4)^2 - |\lambda_5|^2) v_1^4 \\ & + 2(\lambda_1 - \lambda_2) \text{Re} \lambda_5 v_1^2 v_2^2 - ((\lambda_2 - \lambda_3 - \lambda_4)^2 \\ & - |\lambda_5|^2) v_2^4] \text{Im} \lambda_5, \end{aligned} \quad (26)$$

$$\text{Im} J_3 = \frac{v_1^2 v_2^2}{v^4} (\lambda_1 - \lambda_2) (\lambda_1 + \lambda_2 + 2\lambda_4) \text{Im} \lambda_5. \quad (27)$$

It is seen that there is no CP violation when $\text{Im}\lambda_5 = 0$, see [17] for more details.

As we have noted earlier, $\tan\beta$ above ~ 40 implies approximate degeneracy of scalar masses. That could be catastrophic for CP violation since it is well known that the exact degeneracy $M_1 = M_2 = M_3$ results in vanishing invariants $\text{Im}J_i$ and no CP violation (exact degeneracy implies $\text{Im}\lambda_5 = 0$). Using the one-loop conditions (4) and (5) one immediately finds that $\lambda_1 - \lambda_2 = 4(m_b^2/c_\beta^2 - m_t^2/s_\beta^2)/v^2$, which implies

$$\begin{aligned} \text{Im}J_1 &= 4 \text{Im}\lambda_5 \frac{c_\beta^2 m_t^2 - s_\beta^2 m_b^2}{v^2} \\ &= -4 \text{Im}\lambda_5 \left(\frac{m_b}{v}\right)^2 + \mathcal{O}\left(\frac{\text{Im}\lambda_5}{\tan\beta^2}\right). \end{aligned} \quad (28)$$

In fact the above result shows even more than we have anticipated. If $\tan\beta$ is large then $\text{Im}J_1$ is suppressed not only by $\text{Im}\lambda_5 \simeq 0$ (as caused by $M_1 \simeq M_2 \simeq M_3$) but also by the factor (m_b^2/v^2) , as implied by the cancellation conditions (4) and (5). The same suppression factor appears for $\text{Im}J_3$. The case of $\text{Im}J_2$ is more involved, however when m_b^2/v^2 is neglected all the invariants (25)–(27) have the same simple asymptotic behavior

$$\text{Im}J_i \sim \frac{\text{Im}\lambda_5}{\tan^2\beta} \quad (29)$$

for large $\tan\beta$. It is also worth noticing that $\tan\beta = m_t/m_b (\simeq 38)$ implies $\lambda_1 = \lambda_2$, which in turn leads to exact vanishing of $\text{Im}J_1$ and $\text{Im}J_3$. Qualitatively those conclusions survive at the two-loop level. For a quantitative illustration we plot in Figs. 6 and 7 maximal values of the invariants in the $\tan\beta$ – M_{H^\pm} plane with all the necessary constraints imposed, seeking regions which still allow for substantial CP violation. At high values of $\tan\beta$ these invariants are of the order of 10^{-3} , in qualitative agreement with the discussion above. Note that the SM corresponding invariant $\text{Im}Q = (V_{ud}V_{cb}V_{ub}^*V_{cd}^*) \simeq 2 \times 10^{-5} \sin\delta_{\text{KM}}$ [12], for V_{ij} and δ_{KM} being elements of the CKM matrix and CP -violating phase, respectively. Therefore the model considered here offers at least 2 orders of magnitude enhancement comparing to the SM.

G. Stability and the determination of the cutoff

It should be emphasized here that the conditions (20) eliminate the quadratic divergences only up to the leading two-loop corrections. Even though the subleading two-loop and higher effects are suppressed by powers of coupling constants and powers of $1/(16\pi^2)$, nevertheless since the $\ln\Lambda$ term is growing, there exists always Λ large enough, that the hierarchy problem reappears: loop corrections to masses are again of the order of the masses themselves. In fact that observation allows to determine

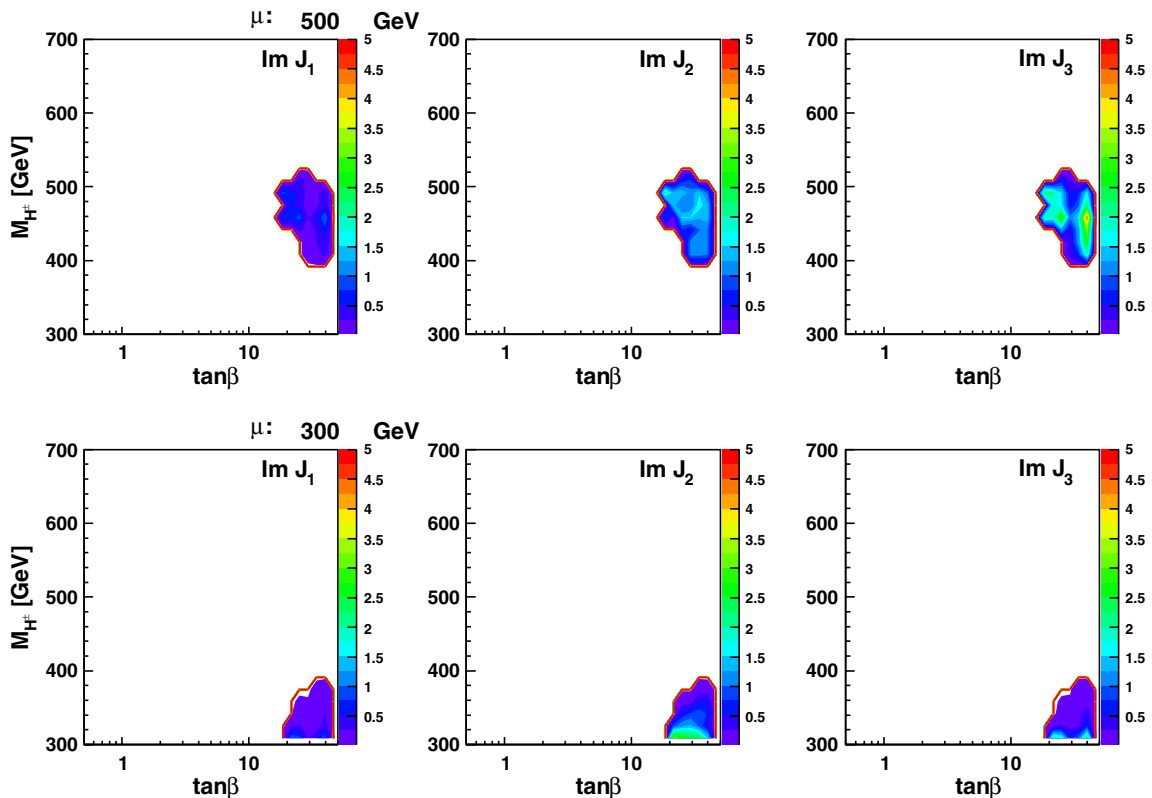
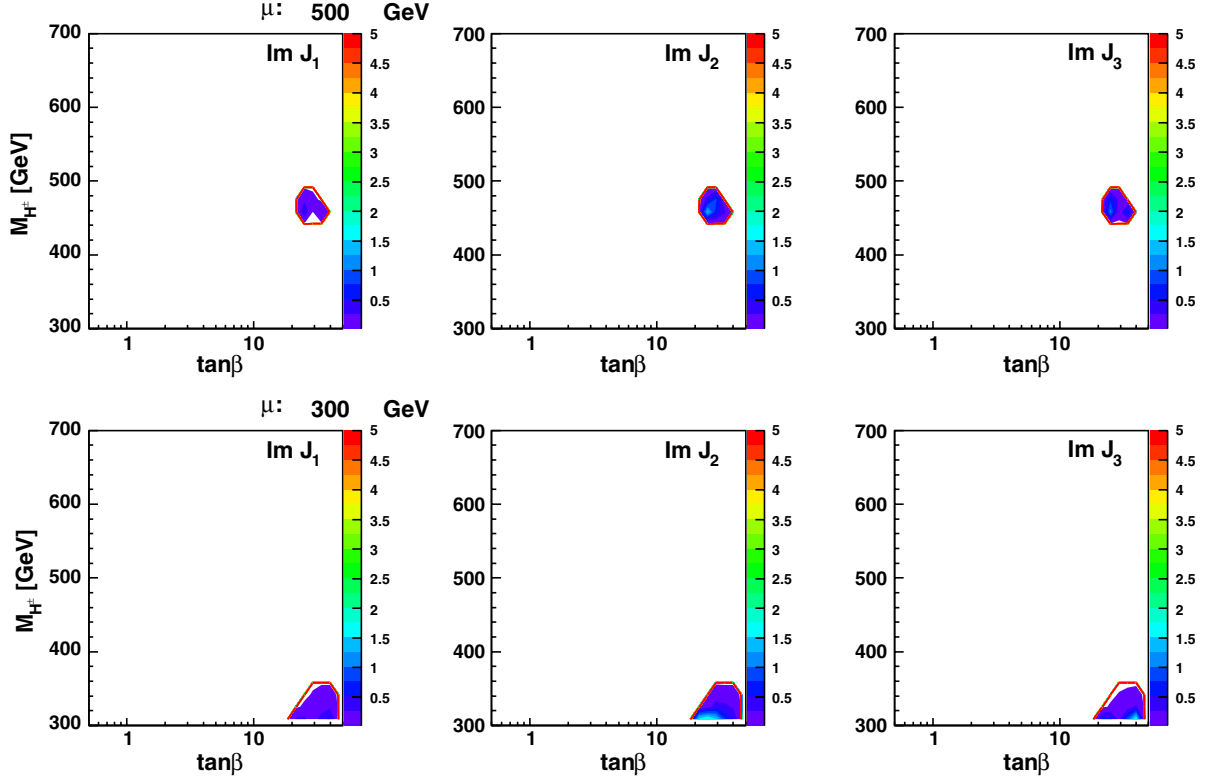


FIG. 6 (color online). Imaginary parts of the rephasing invariants $|\text{Im}J_i|$ at the two-loop level for $\Lambda = 2.5$ TeV, for $\mu = 500$ GeV (top) and $\mu = 300$ GeV (bottom). The color (grey-scale) coding in units 10^{-3} is given along the right vertical axis.

FIG. 7 (color online). Similar to Fig. 6 for $\Lambda = 6.5$ TeV.

the value of the cutoff up to which higher order corrections do not reintroduce the hierarchy problem (see [24] for the analogous strategy within the SM). In general, quadratic corrections to scalar masses have the form of [(18)]

$$\delta M_i^2 = \Lambda^2 \sum_{n=0} f_n^{(i)}(\lambda) \left[\ln\left(\frac{\Lambda}{v}\right) \right]^n + \dots, \quad (30)$$

where v is chosen as a renormalization scale. The following naïve estimation of $f_n^{(i)}$ is sufficient:

$$f_n^{(i)} \sim \left(\frac{4\pi}{16\pi^2} \right)^{n+1} = \left(\frac{1}{4\pi} \right)^{n+1}, \quad (31)$$

where the relevant coupling constants were conservatively assumed to be of the order of 4π .⁸ Here we choose as the cutoff the maximal value of Λ such that the higher order corrections do not exceed the mass of the lightest scalar, M_1 :

$$\Lambda \lesssim 4\pi M_1. \quad (32)$$

Then, e.g. for $M_1 = 200$ (500) GeV the cutoff is at least at $\Lambda \sim 2.5$ (6.3) TeV. Of course, larger M_1 would imply higher Λ .

Having the cutoff determined, we should address the issue of higher-loop corrections to Eqs. (20) that ensure

⁸This estimate agrees qualitatively with the two-loop result obtained for f_1 in the SM, see Eq. (21) in [24].

vanishing of the quadratic corrections up to leading two-loop effects. As is seen from (30), the generic form of the condition for vanishing quadratic divergence is the following:

$$\frac{\lambda}{(4\pi)^2} + \frac{\lambda^2}{(4\pi)^4} \ln\left(\frac{\Lambda}{v}\right) + \frac{\lambda^2}{(4\pi)^4} + \frac{\lambda^3}{(4\pi)^6} \ln^2\left(\frac{\Lambda}{v}\right) + \dots = 0, \quad (33)$$

where λ stands for a typical coupling constant. The last two terms shown above (subleading two- and leading three-loop effects) have been neglected in the present analysis. It is then easy to see that even for the cutoff as large as $\Lambda = 6.5$ TeV using a very conservative (large) value for the typical coupling, $\lambda = 4\pi$, the precision of the adopted approximation is of the order of 12%. Note that whenever $\lambda < 4\pi$ or $\Lambda < 6.5$ TeV, the adopted approximations work better.

III. 2 DOUBLET + 1 SINGLET HIGGS MODEL: THE CASE FOR DARK MATTER

In this scenario we combine CP violation present in the noninert 2HDM (allowing for softly broken \mathbb{Z}_2 symmetry) with a real scalar φ which is a gauge singlet. The singlet provides a natural DM candidate (see [8,34,35]). In this case the scalar potential is the following:

$$\begin{aligned}
V(\phi_1, \phi_2, \varphi) = & -\frac{1}{2}m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 \\
& + [m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.}] + \frac{1}{2}\lambda_1 (\phi_1^\dagger \phi_1)^2 \\
& + \frac{1}{2}\lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
& + \lambda_4 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \frac{1}{2}[\lambda_5 (\phi_1^\dagger \phi_2)^2 + \text{H.c.}] \\
& + \mu_\varphi^2 \varphi^2 + \frac{1}{24}\lambda_\varphi \varphi^4 \\
& + \varphi^2 (\eta_1 \phi_1^\dagger \phi_1 + \eta_2 \phi_2^\dagger \phi_2). \tag{34}
\end{aligned}$$

Note that the term $\propto \varphi^2 \phi_1^\dagger \phi_2$ is forbidden as it breaks the \mathbb{Z}_2 symmetry in a hard way. Since φ is supposed to be the DM candidate, in order to ensure its stability we have imposed an extra discrete symmetry \mathbb{Z}'_2 such that $\varphi \rightarrow -\varphi$ while other fields are neutral. The symmetry excludes terms odd in φ . The potential should be arranged such that the symmetry remains unbroken, so that $\langle \varphi \rangle = 0$. For that it is sufficient to require

$$\mu_\varphi^2 > 0 \quad \& \quad \lambda_\varphi, \eta_1, \eta_2 > 0. \tag{35}$$

Then it is easy to see that if the standard 2HDM stability conditions (23) and (24) are fulfilled then the potential (34) is also positive definite. For the mass of the singlet we obtain: $m_\varphi^2 = 2\mu_\varphi^2 + \eta_1 v_1^2 + \eta_2 v_2^2$.

Since φ is a gauge singlet, therefore in the presence of right-handed neutrinos which are also gauge singlets the following Yukawa interaction is allowed [8]:

$$\mathcal{L}_Y = -\varphi \overline{(\nu_R)^c} Y_\varphi \nu_R + \text{H.c.} \tag{36}$$

Note that for a number of right-handed neutrino flavours greater than 1, the Yukawa matrix Y_φ is in general (depending on the quantum numbers of ν_R under \mathbb{Z}'_2 , see [8]) nonvanishing.

In this model the conditions for cancellation of quadratic divergences are slightly modified:

$$\begin{aligned}
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left(\frac{1}{2}\eta_1 + \frac{3}{2}\lambda_1 + \lambda_3 + \frac{1}{2}\lambda_4 \right) - 3\frac{m_b^2}{c_\beta^2} &= 0, \\
\frac{3}{2}m_W^2 + \frac{3}{4}m_Z^2 + \frac{v^2}{2} \left(\frac{1}{2}\eta_2 + \frac{3}{2}\lambda_2 + \lambda_3 + \frac{1}{2}\lambda_4 \right) - 3\frac{m_t^2}{s_\beta^2} &= 0, \\
\frac{\lambda_\varphi}{2} + 4(\eta_1 + \eta_2) - 8 \text{Tr}\{Y_\varphi Y_\varphi^\dagger\} &= 0. \tag{37}
\end{aligned}$$

The last condition above guarantees vanishing quadratic divergence in corrections to the φ mass. Since for the positivity we assumed $\lambda_\varphi, \eta_1, \eta_2 > 0$, it is clear from the above equation that the presence of the Yukawa coupling Y_φ is mandatory to extend the condition for cancellation of quadratic divergences to the singlet field as well. It should also be mentioned that the presence of the singlet does influence the two-loop corrections to the

quadratic divergences, those effects are neglected as being small ($\propto \eta_i$).

As we have already mentioned the extra singlet φ provides a candidate for the DM. To estimate its present abundance we consider the dominant annihilation channels for φ . The Lagrangian describing relevant cubic and quartic scalar interactions reads

$$\mathcal{L} = -\varphi^2 (\kappa_i v H_i + \lambda_{ij} H_i H_j + \lambda_\pm H^+ H^-), \tag{38}$$

where

$$\kappa_i = \eta_1 R_{i1} c_\beta + \eta_2 R_{i2} s_\beta, \tag{39}$$

$$\lambda_{ij} = \frac{1}{2} [\eta_1 (R_{i1} R_{j1} + s_\beta^2 R_{i3} R_{j3}) + \eta_2 (R_{i2} R_{j2} + c_\beta^2 R_{i3} R_{j3})], \tag{40}$$

$$\lambda_\pm = \eta_1 s_\beta^2 + \eta_2 c_\beta^2. \tag{41}$$

A detailed study of the DM within this model [with extended conditions for the cancellation of quadratic divergences (37)] will be presented elsewhere [36]. However here we would like to show that it is indeed natural to expect the right DM abundance in the presence of the singlet. For an illustration we will assume that the DM annihilation cross section is of the order of the contributions from the lightest neutral Higgs boson H_1 of mass M_1 .

For an estimate of the DM abundance, we will consider two $\varphi\varphi$ annihilation mechanisms. First we assume that $\varphi\varphi$ annihilate to $\gamma\gamma, q\bar{q}, l^+l^-, W^+W^-$ and ZZ through s -channel H_1 exchange. Then, following [37] we obtain⁹ in the nonrelativistic approximation the following result for the thermally averaged annihilation cross section

$$\langle \sigma v \rangle_1 = \frac{4\kappa_1^2 v^2}{(4m_\varphi^2 - M_1^2)^2 + M_1^2 \Gamma_{H_1}^2} \left[\frac{\Gamma_{H_1}(2m_\varphi)}{2m_\varphi} \right], \tag{42}$$

where $\Gamma_{H_1}(2m_\varphi)$ stands for the decay width of H_1 calculated at $M_1 = 2m_\varphi$ (in the following numerical calculations we will use the SM width for the estimate). Now we have to add the contribution from the $H_1 H_1$ final state. There are two contributions: due to s -channel Higgs exchange, and due to the four-point coupling. We find in the nonrelativistic approximation

$$\begin{aligned}
\langle \sigma v \rangle_2 = & \frac{1}{32\pi} \frac{1}{m_\varphi^2} \left(1 - \frac{M_1^2}{m_\varphi^2} \right)^{1/2} \theta(m_\varphi - M_1) \\
& \times \left| \lambda_{11} + \frac{\kappa_1 \tilde{\lambda}_{111} v^2}{4m_\varphi^2 - M_1^2 + iM_1 \Gamma_{H_1}} \right|^2, \tag{43}
\end{aligned}$$

⁹This cross section is also to be found in the literature [35], however our result is smaller by a factor of 2. We have included both the combinatoric factor $1/2$ in $\varphi\varphi H_1$ vertices and statistical factors (both in the initial and in the final states) in $\langle \sigma v \rangle$.

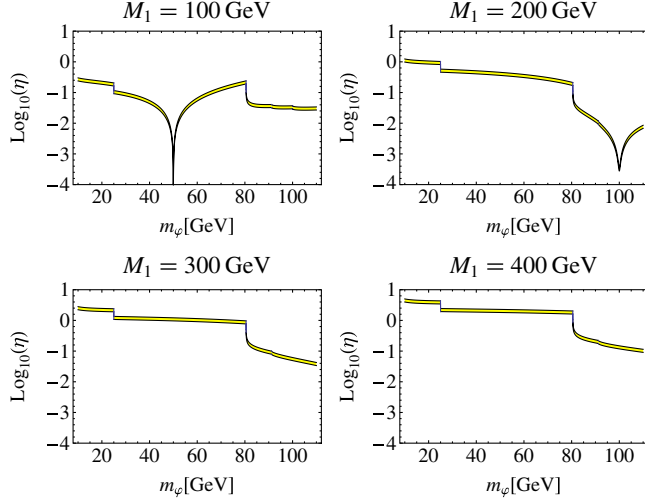


FIG. 8 (color online). Inert-scalar coupling η (vs m_φ) required by the observed DM abundance $\Omega_{\text{DM}}h^2 = 0.106 \pm 0.008$ [39] within a 3- σ band. As indicated above each panel, the lightest Higgs-boson mass ranges from $M_1 = 100$ to 400 GeV.

where the quartic $\varphi^2 H_1 H_1$ coupling λ_{11} is defined by Eq. (40) and the trilinear $H_1 H_1 H_1$ coupling normalized to v is denoted by $\tilde{\lambda}_{111}$ [38]. For an order-of-magnitude estimate of the DM abundance, we will use here $\tilde{\lambda}_{111} = 3M_1^2/v^2$ (this choice, together with (44), reproduces results which would be obtained for the SM Higgs doublet ϕ_{SM} coupled to the singlet through the term $\eta\varphi^2|\phi_{\text{SM}}|^2$) and parameterize κ_1 and λ_{11} through one variable η as follows:

$$2\lambda_{11} = \kappa_1 \equiv \eta. \quad (44)$$

Then, following [37] for cold relics one has to solve the following equation to determine the freeze-out temperature from $x_f = m_\varphi/T_f$:

$$x_f = \ln \left[0.038 \frac{m_{\text{Pl}} m_\varphi}{(g_\star x_f)^{1/2}} \langle \sigma v \rangle \right], \quad (45)$$

where $\langle \sigma v \rangle \equiv \langle \sigma v \rangle_1 + \langle \sigma v \rangle_2$ and g_\star counts relativistic degrees of freedom at annihilation and m_{Pl} denotes the Planck mass. It turns out that in the range of parameters we are interested in, $x_f \sim \mathcal{O}(25)$, so that this is indeed the case of cold dark matter, it also implies that $g_\star \approx 10$ –100. Then the present density of φ 's is given by

$$\Omega_\varphi h^2 = 1.07 \times 10^9 \frac{x_f}{g_\star^{1/2} m_{\text{Pl}} \langle \sigma v \rangle}. \quad (46)$$

In Fig. 8 we show the 3- σ allowed band in $\log(\eta)$ vs m_φ , as constrained by the observed DM abundance $\Omega_{\text{DM}}h^2 = 0.106 \pm 0.008$ [39]. For $m_h = 100$ and 200 GeV we

observe consequences of resonant behavior at $m_\varphi = m_h/2$. The thresholds seen at $m_\varphi \approx 25$ and 80 GeV are caused by the rapid change in g_\star as a function of temperature and by the opening of the W^+W^- channel for the decay of a Higgs boson of mass $m_h = 2m_W$, respectively.

One can conclude that the singlet could indeed provide a realistic candidate for DM: for any m_φ between ~ 1 GeV and ~ 500 GeV there exists an allowed value η for which $\Omega_\varphi h^2$ agrees with the experimental data. Note that if we had found only solutions with $\eta \gtrsim 1$ and light φ ($m_\varphi \lesssim v$) then this scenario would be jeopardized since the minimization condition requires $m_\varphi^2 > \eta_1 v_1^2 + \eta_2 v_2^2$ (as $\mu_\varphi^2 > 0$). As seen from Fig. 8, this is not the case. Most of the allowed region corresponds indeed to $\eta \lesssim 10^{-1}$ if the singlet mass is not too low.

IV. SUMMARY

The goal of this work was to build a minimal realistic model which would allow for softening the little hierarchy problem through suppression of the quadratic divergences in scalar boson mass corrections and through lifting the mass of the lightest Higgs boson. That could be accomplished within two-Higgs-doublet models. Phenomenological consequences of requiring no quadratic divergences in corrections to scalar masses within the 2HDM were discussed. The 2HDM type II was analyzed taking into account existing experimental constraints. Allowed regions in the parameter space were determined. An interesting scalar mass degeneracy was observed for $\tan\beta \gtrsim 40$. The issue of possible CP violation in the scalar potential was addressed and regions of $\tan\beta$ – M_{H^\pm} with substantial strength of CP violation were identified. In order to accommodate a possibility for dark matter a scalar gauge singlet was added to the model. Requirements necessary for correct present abundance of dark matter were estimated.

The model we considered here allows to soften the little hierarchy problem by lifting the minimal scalar Higgs-boson mass and by suppressing the one-loop quadratic corrections to scalar masses. The cutoff implied by the naturalness arguments is lifted from ~ 600 GeV in the SM up to at least $\gtrsim 2.5$ TeV, depending on the mass of the lightest scalar.

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- [1] M. J. G. Veltman, *Acta Phys. Pol. B* **12**, 437 (1981).
- [2] W. Siegel, *Phys. Lett.* **84B**, 193 (1979); D.M. Capper, D.R.T. Jones, and P. van Nieuwenhuizen, *Nucl. Phys.* **B167**, 479 (1980).
- [3] P. Osland and T. T. Wu, *Z. Phys. C* **55**, 569 (1992); **55**, 585 (1992).
- [4] M. B. Einhorn and D. R. T. Jones, *Phys. Rev. D* **46**, 5206 (1992).
- [5] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
- [6] M. S. Chanowitz, [arXiv:0806.0890](https://arxiv.org/abs/0806.0890).
- [7] R. Barate *et al.* (ALEPH Collaboration, DELPHI Collaboration, L3 Collaboration, OPAL Collaboration, and The LEP Working Group for Higgs Boson Searches), *Phys. Lett. B* **565**, 61 (2003).
- [8] B. Grzadkowski and J. Wudka, *Phys. Rev. Lett.* **103**, 091802 (2009).
- [9] R. Barbieri, L. J. Hall, and V. S. Rychkov, *Phys. Rev. D* **74**, 015007 (2006).
- [10] N.G. Deshpande and E. Ma, *Phys. Rev. D* **18**, 2574 (1978).
- [11] M. Aoki, S. Kanemura, and O. Seto, *Phys. Rev. Lett.* **102**, 051805 (2009); *Phys. Rev. D* **80**, 033007 (2009).
- [12] W. Bernreuther, *Lect. Notes Phys.* **591**, 237 (2002).
- [13] C. Newton and T. T. Wu, *Z. Phys. C* **62**, 253 (1994).
- [14] E. Ma, *Int. J. Mod. Phys. A* **16**, 3099 (2001).
- [15] J. F. Gunion, H. E. Haber, G. L. Kane, and S. Dawson, *The Higgs Hunter's Guide* (Addison-Wesley, Reading, MA, 1990).
- [16] L. Fromme, S. J. Huber, and M. Seniuch, *J. High Energy Phys.* **11** (2006) 038.
- [17] B. Grzadkowski, O. M. Ogreid, and P. Osland, *Phys. Rev. D* **80**, 055013 (2009).
- [18] M. Kadastik, K. Kannike, and M. Raidal, *Phys. Rev. D* **81**, 015002 (2010).
- [19] B. Grzadkowski and P. Osland, [arXiv:0910.4068](https://arxiv.org/abs/0910.4068).
- [20] A. W. El Kaffas, P. Osland, and O. M. Ogreid, *Nonlinear Phenom. Complex Syst.* **10**, 347 (2007).
- [21] E. Accomando *et al.*, [arXiv:hep-ph/0608079](https://arxiv.org/abs/hep-ph/0608079).
- [22] W. Khater and P. Osland, *Nucl. Phys.* **B661**, 209 (2003).
- [23] D. Kominis and R. S. Chivukula, *Phys. Lett. B* **304**, 152 (1993).
- [24] C. F. Kolda and H. Murayama, *J. High Energy Phys.* **07** (2000) 035; J. A. Casas, J. R. Espinosa, and I. Hidalgo, *J. High Energy Phys.* **11** (2004) 057.
- [25] P. M. Ferreira and D. R. T. Jones, *J. High Energy Phys.* **08** (2009) 069.
- [26] A. W. El Kaffas, P. Osland, and O. M. Ogreid, *Phys. Rev. D* **76**, 095001 (2007).
- [27] O. Deschamps, S. Descotes-Genon, S. Monteil, V. Niess, S. T'Jampens, and V. Tisserand, *Phys. Rev. D* **82**, 073012 (2010).
- [28] S. Kanemura, T. Kubota, and E. Takasugi, *Phys. Lett. B* **313**, 155 (1993).
- [29] A. G. Akeroyd, A. Arhrib, and E. M. Naimi, *Phys. Lett. B* **490**, 119 (2000); A. Arhrib, [arXiv:hep-ph/0012353](https://arxiv.org/abs/hep-ph/0012353).
- [30] I. F. Ginzburg and I. P. Ivanov, [arXiv:hep-ph/0312374](https://arxiv.org/abs/hep-ph/0312374); *Phys. Rev. D* **72**, 115010 (2005).
- [31] L. Lavoura and J. P. Silva, *Phys. Rev. D* **50**, 4619 (1994).
- [32] G. C. Branco, M. N. Rebelo, and J. I. Silva-Marcos, *Phys. Lett. B* **614**, 187 (2005).
- [33] J. F. Gunion and H. E. Haber, *Phys. Rev. D* **72**, 095002 (2005).
- [34] V. Silveira and A. Zee, *Phys. Lett. B* **161**, 136 (1985); J. McDonald, *Phys. Rev. D* **50**, 3637 (1994); M. C. Bento, O. Bertolami, and R. Rosenfeld, *Phys. Lett. B* **518**, 276 (2001); H. Davoudiasl, R. Kitano, T. Li, and H. Murayama, *Phys. Lett. B* **609**, 117 (2005); J. J. van der Bij, *Phys. Lett. B* **636**, 56 (2006).
- [35] C. P. Burgess, M. Pospelov, and T. ter Veldhuis, *Nucl. Phys.* **B619**, 709 (2001).
- [36] B. Grzadkowski and P. Osland (work in progress).
- [37] E. W. Kolb and M. S. Turner, *The Early Universe* (Addison-Wesley, Reading, MA, 1990).
- [38] P. Osland, P. N. Pandita, and L. Selbuz, *Phys. Rev. D* **78**, 015003 (2008).
- [39] C. Amsler *et al.* (Particle Data Group), *Phys. Lett. B* **667**, 1 (2008).