Constraints on interacting scalars in two-time field theory and no scale models in one-time field theory

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In this paper I determine the general form of the physical and mathematical restrictions that arise on the interactions of gravity and scalar fields in the 2T field theory setting, in $d + 2$ dimensions, as well as in the emerging shadows in d dimensions. These constraints on scalar fields follow from an underlying $Sp(2, R)$ gauge symmetry in phase space. Determining these general constraints provides a basis for the construction of 2T supergravity, as well as physical applications in 1T-field theory, that are discussed briefly here and more detail elsewhere. In particular, no scale models that lead to a vanishing cosmological constant at the classical level emerge naturally in this setting.

I. PROLOGUE: THE ROLE OF THE $Sp(2, R)$ GAUGE SYMMETRY

The progress of two-time physics (2T physics), from classical mechanics to the standard model, supersymmetric 2T field theory, 2T general relativity, and cosmology, is summarized in [1]. More recently, 2T field theory computational techniques for 2T physics that are directly in $4 + 2$ dimensions have emerged [2,3] and mathematical fields that adopted the notions of 2T physics have advanced [4].

2T physics is the fundamental solution to the question of how to make a theory with two timelike dimensions causal, ghost-free, and unitary directly in d space and 2-time dimensions. Field theories based on the principles of 2T physics in $d + 2$ dimensions, including the standard model and general relativity, automatically have the right mix of gauge symmetries and constraints to make them compatible with their conventional formulation in field theory in 1T physics in $3 + 1$ dimensions. Conventional classical or quantum mechanics as well as 1T field theory emerge as "shadows" [1] of 2T-physics systems in one less time and one less space dimensions. Beyond this compatibility with 1T physics, 2T physics predicts systematically additional physical information, in the form of hidden symmetries and dualities among the shadows, that is systematically missed information in the conventional 1T formulation. One of these shadows, called the "conformal shadow," is the one most familiar to particle physicists. In this shadow conformal symmetry SO(4,2) is a 4-dimensional nonlinear realization of the linear Lorentz symmetry in $4 + 2$ dimensions. The new aspects of 2T physics follow from demanding a fundamental $Sp(2, R)$ gauge symmetry in phase space, which in turn requires a primary ambient spacetime, with no more and no less than two timelike dimensions, in which conventional spacetime is embedded.

The role of the fundamental $Sp(2, R)$ gauge symmetry in 2T physics is similar to the world sheet gauge symmetry in string theory. In the case of $Sp(2, R)$ there are three gauge generators, $Q_{ij}(X, P)$, $i, j = 1, 2$ arranged into a 2×2 symmetric matrix, Q_{11}, Q_{22}, $Q_{12} = Q_{21}$, constructed from phase space degrees of freedom on the worldline $X^M(\tau), P_M(\tau)$ with a target spacetime in $d + 2$ dimensions labeled by M. Comparing to string theory, the $Q_{ij}(X, P)$ are analogous to the Virasoro operators constructed from string coordinates $x^\mu(\tau, \sigma)$ and its derivatives (or canonical oscillators, i.e. phase space). In both cases the particle or string may interact with an infinite set of background fields (electromagnetism, gravity, etc.) that appear in the expressions of the $Sp(2, R)$ or Virasoro generators. All such $Q_{ij}(X, P)$ in 2T physics have been classified up to canonical transformations[1] [5].

Demanding $Sp(2, R)$ gauge symmetry on the worldline is similar to demanding local conformal symmetry on the world sheet. To preserve the $Sp(2, R)$ symmetry, the background fields that appear in $Q_{ij}(X, P)$ must satisfy certain equations that follow from requiring closure of the $Q_{ij}(X, P)$ as the $Sp(2, R)$ Lie algebra under Poisson brackets. The resulting field equations are analogous to the equations for background fields in string theory that follow from demanding exact world sheet conformal symmetry.

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[1]The simplest example of the Q_{ij} is $Q_{11} = X \cdot X$, $Q_{12} = X \cdot P$ $Q_{22} = P \cdot P$ where the dot product involves the flat background metric η_{MN} in $d + 2$ dimensions. If η_{MN} had been a Euclidean metric or a 1T Minkowski metric, the three equations $Q_{ij} = 0$ yield only trivial solutions. Nontrivial as well as ghost-free physics occurs only when two timelike dimensions are admitted. The solutions are called shadows when a gauge is chosen to embed $3 + 1$ phase space in $4 + 2$ phase space (see Fig. 2 in [1]). Because many different embeddings exist such that "time" and "Hamiltonian" are identified differently as part of $4 + 2$ phase space, the 1T physics is different in different $3 + 1$ shadows, as seen by observers restricted to those shadows. Nevertheless many relations do exist among the shadows. This is the new information predicted by 2T physics that is systematically missed in 1T physics. For each choice of $Q_{ij}(X, P)$ including background fields [5], there is a corresponding set of shadows analogous to Fig. 1 in [1]. Hence the systematically missed hidden information in 1T physics is vast.

However, in the case of $Sp(2, R)$ these are "kinematic equations," that guarantee the absence of ghosts without determining the dynamics of the background fields. To determine the dynamics, consistently with the kinematics imposed by $Sp(2, R)$, a unique action principle that has its own gauge symmetries is constructed, as illustrated in [6,7]. In this paper I will discuss the extent to which this approach permits generalizations while still imposing constraints on the interactions of scalar fields in 2T field theory.

Since the $Q_{ij}(X, P)$, like the Virasoro operators, are gauge generators, the gauge invariant space (at the particle level) is defined as the subspace of phase space that satisfies $Q_{ij}(X, P) = 0$. As can be verified through the example in footnote 1, there is nontrivial physics content in the equations $Q_{ij}(X, P) = 0$ provided there are two time-like dimensions in target space X^M [1]. Furthermore, the available gauge symmetry is just right to eliminate the ghosts of two timelike dimensions. Therefore, unitarity at the quantum mechanics level (wave functions $\leftrightarrow$ fields) is achieved only for a spacetime with 2 times, no less and no more.

Unitarity is also insured for spinning particles on a $d + 2$ dimensional target space (analogs of Neveu-Schwarz degrees of freedom in string theory) or supersymmetric systems (analogs of Green-Schwarz degrees of freedom in string theory) by enlarging the gauge symmetry to $OSp(n|2)$ [8,9] or with appropriate supercoset symmetries [10–15], respectively. The latter leads to novel twistor and supertwistor representations of various systems [13], including the 2T-physics version of the twistor superstring in $4 + 2$ dimensions.

2T field theory is constructed with similar techniques to string field theory. The basic equations for free scalar fields are the physical state conditions that correspond to gauge invariance under $Sp(2, R)$ $Q_{ij}(X, \partial)\Phi(X) = 0$ (with an appropriate ordering of X and $P = -i\partial/\partial X$). This is analogous to the Virasoro constraints applied on string fields.[2]

A systematic approach to include interactions in 2T field theory is to construct the Bechhi-Rouet-Stora-Tyutin (BRST) operator for $Sp(2, R)$, use it as the kinetic operator in a BRST field theory, and then include interactions [16]. The inclusion of interactions modifies the BRST operator [16], just as it does in string field theory.

A complementary approach to 2T field theory is a procedure analogous to the construction of the string effective action as a field theory. It consists of a field theory action in $d + 2$ dimensions that reproduces the same equations for the background fields that were obtained by demanding the $Sp(2, R)$ gauge symmetry on the worldline in the presence of backgrounds [6,7]. In this paper I will discuss, in this approach, the generalizations of the gravity action in [6,7] when many scalar fields are present.

One can verify that the two approaches, BRST or effective action, lead to the same "kinematical" and "dynamical" equations after redundant fields are integrated out in the BRST field theory [16].[3] In contrast to the kinematical equations that are algebraic or at most first order in derivatives [see, e.g., Q_{11}, Q_{12} in footnote 2, or see Eqs. (2.3)], the dynamical equations are those that include Klein-Gordon or Dirac-like differential operators (such as $Q_{22}\Phi = \partial \cdot \partial\Phi + \cdots \sim 0$) whose details include also interactions. Kinematical and dynamical equations for all spinning fields follow from either one of the two complementary approaches as shown in [5,9].

These were the 2T-physics principles that led to 2T field theory for all spinning fields. They were used to construct the standard model [18], supersymmetric generalizations [19,20], and general relativity [6,7] as unitary field theories in $4 + 2$ dimensions. The action principle provides all the necessary ingredients to perform computations directly in $d + 2$ dimensions.

Recent examples of the computation of n-point Green's functions in $4 + 2$ dimensions that obey all the "kinematic equations" of 2T physics was given by

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[2]For example, for a scalar field in a flat background in $d + 2$ dimensions (see footnote [1]), these equations are $Q_{11}\Phi = X^2\Phi = 0$, $Q_{12}\Phi = (X \cdot \partial + \partial \cdot X)\Phi = 0$, and $Q_{22}\Phi = \partial \cdot \partial\Phi = 0$. The first two equations are called "kinematic" and the last "dynamic." The solution of the first equation is $\Phi(X) = \delta(X^2)\phi(X)$ indicating that $\phi(X) \rightarrow \phi(X) + X^2\Lambda(X)$ is a gauge symmetry. This also illustrates the origin of the delta function $\delta(W)$ that occurs in the actions (2.6) and (2.12), since in flat space $W = X^2$ as seen in Eq. (2.2). Inserting this solution in the second equation, and using the homogeneity of the delta function, one obtains $\delta(X^2)(2X \cdot \partial + d - 2)\phi(X) = 0$ indicating that $\phi(X)$ must be homogeneous of degree $(2 - d)/2$. Combining these properties one learns that $\phi(X)$ depends really on d coordinates rather than $d + 2$ coordinates. The various shadows of the surviving degrees of freedom in Φ correspond to how the phase space in d dimensions is embedded in the phase space in $d + 2$ dimensions. The final (shadow) form of the last dynamical equation depends on this embedding. The conformal shadow is one of those examples.

[3]Our BRST approach of [16] is directly applicable with all background fields in the $Q_{ij}(X, P)$ as in [5]. This was adopted recently by mathematicians in [4] (with a limited subset of background fields) to develop a mathematical topic called "tractors." Although not discussed in [4], I suggest that this mathematical topic can be further generalized by using the BRST operators for $OSp(n|2)$ that includes spin degrees of freedom, as in [9] or supersymmetric degrees of freedom as in [10–15]. Deeper physical and mathematical results would be obtained by deriving the kinematic constraints from an improved action principle (as in [17]) or with a delta function [as in Eq. (2.6) and [6]] as opposed to applying them as external constraints as done in [4].

Weinberg [3].[4] Independent computations in [2] provide some examples of shadows in the context of Green functions, which are consistent with the shadows discussed before in classical or quantum mechanics (see, e.g., [1]) as well as directly in field theory [21,22]. I expect that such notions and techniques that are inherently in $d + 2$ dimensions are bound to open new avenues of computation as well as provide a deeper view of space and time.

II. 2T-GRAVITY ACTION AND PHYSICAL CONSIDERATIONS

The 2T-physics description of a particle moving in a gravitational background at the classical mechanics level is given by the worldline action [1]

$$S = \int d\tau \left(\dot{X}^M P_M - \frac{1}{2} A^{ij} Q_{ij}(X, P) \right), \qquad (2.1)$$

with the following realization of the $\text{Sp}(2, R)$ generators [5,6]:

$$Q_{11} = W(X), \quad Q_{12} = V^M(X)P_M, \quad Q_{22} = G^{MN}(X)P_M P_N. \qquad (2.2)$$

For comparison see the flat space version in footnote [1] and the more general cases in [5]. Closure of the $\text{Sp}(2, R)$ algebra under Poisson brackets is obtained, e.g., $\{Q_{11}, Q_{12}\} = 2Q_{11}$ etc., when the background fields $W(X)$, $V^M(X)$, $G^{MN}(X)$ satisfy the following kinematical

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[4]In the initial version of Ref. [3] Weinberg was not aware that the constraints he used to derive his 6-dimensional Green's functions were identical to the "kinematic equations" of 2T-physics field theory in $4 + 2$ dimensions. The constraints for tensors or spinors were previously derived from an underlying principle based on $\text{Sp}(2, R)$ gauge symmetry [5], and related extended gauge symmetries with spin [8,9], and these appeared in realistic 2T-physics models in flat spacetime, including the standard model in $4 + 2$ dimensions [18]. Furthermore, the reduction to $3 + 1$ dimensions of the Green's functions discussed in [3] parallels the corresponding reduction of the fields discussed in [21,22] just for the conformal shadow. This link to conformal symmetry, that led to the 6-dimensional equations with some educated guesswork, goes back to Dirac [23–31]. However, until the emergence of 2T physics it was not realized that (i) there is a fundamental $\text{Sp}(2, R)$ phase space gauge principle behind these equations, or their generalizations to all tensors or spinors, which is universal for all physics (as reviewed in [1]), and (ii) that these 6-dimensional equations can systematically be connected to many 1T-physics shadows in 4 dimensions, not only the conformal shadow. In fact, historically both of these points developed without any knowledge of Dirac's work. I suggest that Weinberg's 6-dimensional Green's functions go well beyond conformal Green's functions in 4 dimensions. They can also be reduced to other shadows just like in [2,21,22]. This technique should produce Green's functions for more complicated field theories that correspond to the shadows described earlier in 2T physics [1]. This is an extension to Green's functions of a similar remark often mentioned in my 2T-physics papers as being a computational technique that may produce nonperturbative information in 1T field theory.

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equations, which amount to homothety conditions on the geometry described by the metric G^{MN}: [5,6]

$$V^M = \frac{1}{2} G^{MN} \partial_N W, \quad V^M \partial_M W = 2W, \quad \mathcal{L}_V G^{MN} = -2G^{MN}. \qquad (2.3)$$

Here $\mathcal{L}_V G^{MN}$ is the Lie derivative of the metric,

$$\mathcal{L}_V G^{MN} \equiv -\nabla^M V^N - \nabla^N V^M$$
$$= V^K \partial_K G^{MN} - \partial_K V^M G^{KN} - \partial_K V^N G^{MK}. \qquad (2.4)$$

These coupled nonlinear geometrical equations imply uniquely that G_{MN} can be written as [6]

$$G_{MN} = \nabla_M V_N = \frac{1}{2}(\partial_M \partial_N W + \Gamma^P_{MN} \partial_P W). \qquad (2.5)$$

The kinematic equations (2.3), (2.4), and (2.5) that follow from $\text{Sp}(2, R)$ are the crucial constraints that remove ghosts from the metric degrees of freedom $G^{MN}(X)$ in a 2T spacetime. They can be solved exactly [6,7], showing that what remains undetermined in $G_{MN}(X)$ in $d + 2$ dimensions are only the degrees of freedom and gauge symmetries of a conventional shadow metric $g_{\mu\nu}(x)$ in d dimensions, plus gauge degrees of freedom and prolongations of the shadow that are immaterial to the 1T physical phenomena in the shadow [6,7]. The dynamics of the surviving shadow $g_{\mu\nu}(x)$ in d dimensions (plus matter degrees of freedom) is determined by the dynamical equations in $d + 2$ dimensions that follow from a 2T field theory action principle as discussed below.

As outlined in the previous section, the homothety conditions (2.3), (2.4), and (2.5) on the background fields are analogous to those obtained in string theory by demanding conformal symmetry on the world sheet. So, with a similar approach to string theory, one can construct an effective action principle that yields the same set of equations (2.3), (2.4), and (2.5) by using the variational principle. I proposed such an action in [6] by including one additional field $\Omega(X)$ that I called the dilaton,[5]

$$S = \gamma \int d^{d+2} X \sqrt{G}[\delta(W)\{a_d \Omega^2 R(G) + \frac{1}{2}\partial\Omega \cdot \partial\Omega$$
$$- V(\Omega)\} + \delta'(W)\{a_d \Omega^2 (4 - \nabla^2 W) + a_d \partial W \cdot \partial\Omega^2\}]. \qquad (2.6)$$

Here the constant $a_d = \frac{(d-2)}{8(d-1)}$ for $d + 2$ dimensions, as well as the form of the action, are uniquely determined by the requirement that the variation of this action reproduces the kinematic equations (2.3), (2.4), and (2.5) [6]. The potential energy is also fixed up to the overall λ which is a dimensionless coupling constant $V(\Omega) = \lambda \frac{d-2}{2d} \Omega^{2d/(d-2)}$.

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[5]The field Ω here is rescaled compared to Ref. [6], namely, $\Omega_{\text{old}} = \sqrt{a}\Omega$.

Note the unusual but crucial delta function $\delta(W(X))$ and its derivative. In this action $W(X)$ is treated as a field that is varied just as the other fields $\Omega(X)$, $G^{MN}(X)$ are varied. The variation of the action with respect to each field takes the following form:

$$\delta S = \int d^{d+2}X\sqrt{G}\{\delta G^{MN}[\delta(W)A^G_{MN} + \delta'(W)B^G_{MN}$$
$$+ \delta''(W)C^G_{MN}] + \delta\Omega[\delta(W)A^\Omega + \delta'(W)B^\Omega$$
$$+ \delta''(W)C^\Omega] + \delta W[\delta'(W)A^W + \delta''(W)B^W$$
$$+ \delta'''(W)C^W]\}, \qquad (2.7)$$

where the coefficients of the delta functions $(A^G_{MN}, B^G_{MN}, C^G_{MN})$, $(A^\Omega, B^\Omega, C^\Omega)$, and (A^W, B^W, C^W) are explicitly given in [6]. All of these coefficients must vanish in order to minimize the action since δG^{MN}, $\delta\Omega$, δW are arbitrary while the delta function and its derivatives are linearly independent distributions. The equations derived from the coefficients of $\delta'(W)$, $\delta''(W)$, $\delta'''(W)$, are the "kinematic equations" while those derived from the coefficients of $\delta(W)$ are the "dynamical equations" for each field. These kinematic equations do not contain interactions [see however new generalization in Eq. (3.10)] and are in precise agreement[6] with Eqs. (2.3), (2.4), and (2.5) derived from the Sp(2, R) algebra of the $Q_{ij}(X, P)$.

The action (2.6) has its own new gauge symmetries. It was shown in [6,7] that by fixing some of these gauge symmetries, and solving the kinematic (not the dynamic) equations for all the fields, W, Ω, G^{MN} the physics of the 2T action may be reduced to 1T physics in various shadows. The conformal shadow, which is the most familiar to particle physicists, is then described by an effective action in d dimensions which takes the conventional form of a conformally coupled scalar $\phi(x)$ [shadow of $\Omega(X)$]) interacting with the gravitational field $g_{\mu\nu}(x)$ [shadow of $G_{MN}(X)$] as follows:

$$S(g, \phi) = \int d^dx\sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + a_dR\phi^2 - V(\phi)\right). \qquad (2.8)$$

For the special value of a_d given above, it is well known that this action has a local scaling Weyl symmetry $S(\tilde{g}, \tilde{\phi}) = S(g, \phi)$ under the gauge transformation

$$\tilde{g}_{\mu\nu}(x) = e^{2\lambda(x)}g_{\mu\nu}(x), \qquad \tilde{\phi}(x) = e^{-((d-2)/2)\lambda(x)}\phi(x). \qquad (2.9)$$

Tracing back the origin of this symmetry, it was discovered that it emerges as a remnant of the general coordinate transformations in the extra dimensions, while not being a Weyl symmetry of the original action [7]. Hence, this is

an accidental symmetry of the conformal shadow. With this symmetry, the role of the shadow $\phi(x)$ is that of a "compensator" as known in conventional 1T conformal symmetry.[7] It can be gauge fixed to a constant $\phi(x) \rightarrow \phi_0$ if so desired, thus obtaining the 1T physics of pure Einstein gravity with an undetermined cosmological constant

$$S(g, \phi_0) = \frac{1}{2\kappa_d^2}\int d^dx\sqrt{-g}[R(g) - \Lambda_d], \qquad (2.10)$$

where the Newton and cosmological constants in d dimensions are determined by ϕ_0, λ

$$\frac{1}{2\kappa_d^2} = a_d\phi_0^2, \qquad \frac{\Lambda_d}{2\kappa_d^2} = V(\phi_0) = \lambda\frac{d-2}{2d}\phi_0^{2d/(d-2)}. \qquad (2.11)$$

Now we come to one of the main points of the present paper concerning the interactions of matter with the gravitational field in the 2T formulation. The Sp(2, R)-consistent rules for matter fields of Klein-Gordon, Dirac, and Yang-Mills types in $d + 2$ dimensions were given in [6]. For scalar fields the consistent rules turn out to be more general as compared to [6] as follows.

The Sp(2, R)-consistent action is unique when there is only one scalar field Ω as in (2.6). When there are more scalar fields $\Phi_m(X)$, $m = 1, 2, 3\ldots$, (all arranged into real fields), I will show in the next section that the interactions include a function $U(\Phi)$, a potential energy $V(\Phi)$, and a sigma-model–type metric $g_{mn}(\Phi)$, with some constraints on them, as follows:

$$S = \gamma\int d^{d+2}X\sqrt{G}\{\delta(W)[a_dU(\Phi)R(G)$$
$$- \tfrac{1}{2}g_{mn}(\Phi)G^{MN}\partial_M\Phi^m\partial_N\Phi^n - V(\Phi)] + \delta'(W)$$
$$\times [a_dU(\Phi)(4 - \nabla^2W) + a_d\partial W \cdot \partial U(\Phi)]\}, \qquad (2.12)$$

where $a_d = \frac{d-2}{8(d-1)}$. For a single field, U, V, g are unique as given in Eq. (2.6), $U = \Omega^2$, $g_{\Omega\Omega} = -1$, and $V = \lambda\frac{d-2}{2d}\Omega^{2d/(d-2)}$.

Some general physics requirement are as follows. (i) First, the metric in field space, $g_{mn}(\Phi)$, must not have any zero eigenvalues for generic values of the fields Φ_m; this insures that all scalar fields have a kinetic term. (ii) Second, each positive eigenvalue of g_{mn} corresponds to a physical scalar field while each negative eigenvalue corresponds to a negative norm ghost. At least one negative eigenvalue occurs as seen in the example of the single field $U = \Omega^2$. For each additional negative eigenvalue the full theory must have some extra gauge symmetries beyond the

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[6]See Sec. IV-A in [7] for a more detailed discussion of the relation of these equations to Sp(2, R).

[7]I emphasize that the shadow $\phi(x)$ may be fixed to a constant, but the original field $\Omega(X)$ is not a constant, it still depends on the extra dimensions beyond x^μ in a specific way.

accidental Weyl symmetry of Eq. (2.9) to remove negative norm scalars. (iii) Third, $U(\Phi)$ should satisfy certain positivity conditions that insure that gravity is an attractive (not repulsive) force, at least in the patch of spacetime that makes up our Universe, as will be discussed below.

To illustrate these points it is useful to review the example of U, g_{mn} that was given in [6]

$$U(\Phi) = \Omega^2 - \sum_{i=1}^{N} S_i^2, \qquad (2.13)$$

$$g_{mn}(\Phi) = -\frac{1}{2} \frac{\partial^2 U}{\partial \Phi^m \partial \Phi^n} = \text{diag}(-1, +1, \ldots, 1), \quad (2.14)$$

where $\Phi_0 \equiv \Omega$, $\Phi_i \equiv S_i$ In this example the metric g_{mn} and the function $U(\Phi)$ have $SO(N, 1)$ symmetry. This symmetry as well as the form of $U(\Phi)$ emerges uniquely in 2T supergravity if one requires that each scalar has the standard canonical normalization of its kinetic term (i.e. constant metric g_{mn}). To understand the physical effects of the negative eigenvalue consider the conformal shadow analogous to Eq. (2.8) in which all shadow scalars are conformally coupled to gravity with the special value of a_d and the quadratic $U(\Phi)$ of Eq. (2.13). This special coupling of scalars to R insures that in the conformal shadow there is an overall local Weyl symmetry that rescales all shadow scalars equally [as in (2.9)]. This Weyl symmetry is essential both to generate the Newton constant and to remove one negative norm ghost as explained in [6,7,32]. Indeed, the field Ω has an extra minus sign in the kinetic term as compared to the fields S_i. This sign of Ω is the sign required in the conformal shadow of Eq. (2.8) in order to obtain a positive Newton constant upon the gauge fixing of the Weyl symmetry, as in Eq. (2.10). However, this sign is the wrong sign for the kinetic energy, and it makes the shadow ϕ of Ω a negative norm ghost field. This is no problem since the Weyl gauge symmetry removes this ghost from the spectrum when the gauge is fixed to obtain the Einstein action of Eq. (2.10). The additional scalars $S_i(X)$ must have the positive eigenvalue of g_{mn} in the kinetic term so that their shadows $s_i(x)$ are physical, positive norm, scalars. For this reason, in the expression of $U(\Phi) = \Omega^2 - \sum S_i^2$ there must be a relative minus sign between Ω and the other scalars S_i. Now, one notices that the full $U(\Phi)$ rather than only Ω^2 plays the role of an effective Newton constant $16\pi G = (a_d U(\Phi))^{-1} = (a_d \Omega^2 - a_d \sum S_i^2)^{-1}$. Hence, one must consider some positivity requirements that limits the fields $\Phi_m(X)$ to the region $U(\Phi) > 0$ in field space in order to have a positive G that results in an attractive force for gravity.

What could go wrong if the dynamics of the scalar fields, including various choices for the potential energy[8] $V(\Phi)$,

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[8]The most general $V(\Phi)$ compatible with (2.13) and (2.14) is homogeneous and may be written as $V(\Phi) = \Omega^{2d/(d-2)} v(\frac{S_i}{\Omega})$, where $v(x_i)$ is any function of its arguments [6,7].

permit field configurations in which the effective Newton constant $16\pi G = (a_d U(\Phi))^{-1}$ switches sign? A conservative approach to prevent physical disasters is to demand positivity; in particular, in the quadratic example in Eq. (2.13) one may require a field space that satisfies $\Omega^2 > \sum_{i=1}^{N} S_i^2$. However, this is an artificial condition that may be violated by the dynamics of the coupled field equations. It is more interesting to investigate the physics of what happens if the dynamics lets $U(\Phi)$ evolve to the configuration $U(\Phi) = 0$ and even switch sign. In regions of spacetime where $U(\Phi)$ is negative there would be effectively antigravity (repulsive forces) rather than gravity (attractive forces).

In the conformal shadow familiar to particle physicists antigravity has not been observed, so what are the observable effects if the positivity condition is not obeyed by the dynamics? This question was investigated in a simplified and exactly solvable cosmological model [32], where it was shown that $U(\Phi) = 0$ corresponds to the big bang, while the region $U(\Phi) < 0$ is a pre–big-bang region that is not accessible in our own Universe, thus avoiding phenomenological problems. Interesting cosmological questions arise for realistic and complete models, such as whether our spacetime region of the Universe is compatible with the existence of various other antigravity regions of spacetime in the early or later epochs during the evolution of the Universe? Answering such questions has a bearing in a fundamental theory on which forms of $U(\Phi)$ are consistent with the physics we observe.

Having explained the nature of the physical issues that arise through the quadratic example $U(\Phi) = [\Omega^2 - \sum S_i^2]$, I will next prove that the form of the action in Eq. (2.12) is required generally for consistency with the $Sp(2, R)$ homothety constraints (2.3), (2.4), and (2.5) on the geometry $G_{MN}(X)$ in $d + 2$ dimensions. Afterwards I will discuss more general forms of $U(\Phi)$ and the additional gauge symmetries required when the corresponding metric $g_{mn}(\Phi)$ in field space has more than one negative eigenvalue.

At this point it is important to note that the emergent shadow field theory that follows from (2.12) is a special one among all the possible 1T field theories containing scalar fields coupled to gravity. For example, when $U(\Phi) = [\Omega^2 - \sum S_i^2]$, the emergent shadow [6,7] similar to (2.10) including the shadows of the Ω, S_i, requires that all scalar fields [such as Higgs, grand unified generalizations, scalar superpartners in supersymmetry (SUSY) theories, inflaton, etc.] must couple to R quadratically with the same coefficient a_d up to $\pm$ signs (see, e.g., [6,7]). This is allowed but not motivated by a principle in a generic 1T field theory. In this paper we learned that there is some freedom in the choice of $U(\Phi)$, $g_{mn}(\Phi)$, but this freedom is further reduced when additional requirements, such as symmetries in supergravity, are considered, as will be explained in Sec. IV. After expressing the conformal

shadow (familiar to particle physicists) in the Einstein frame[9] the tight relations among interactions produced by the functions $U(\Phi)$, $g_{mn}(\Phi)$ could be physically distinguishable from a comparable generic 1T field theory that may not motivate the same type of restrictions. At the present time *elementary* scalar fields in particle physics (such as the Higgs particle) have not been constrained by experiment. So an immediate test of certain patterns of scalar couplings in 1T field theory, as motivated by 2T field theory, is not available, but this could change in the future.

III. 2T-GRAVITY ACTION AND CONSISTENCY WITH Sp(2, *R*)

To justify that the proposed action (2.12) is the most general form for a 2T field theory of interacting scalars and gravity, I now compare the Sp(2, *R*) equations in Eq. (2.3), (2.4), and (2.5) to the kinematic equations that result from the variation of the action. The variation δS has the form of Eq. (2.7) except that instead of the single scalar field Ω there are now many fields Φ^m. The kinematic equations are those proportional to $\delta'(W)$, $\delta''(W)$, $\delta'''(W)$, so I concentrate on those terms symbolized in Eq. (2.7) by the letters $(B^G_{MN}, C^G_{MN}; B^{\Phi_m}, C^{\Phi_m}; A^W, B^W, C^W)$, all of which must vanish to extremize the action.

Using steps similar to Refs. [6,7] I derive these coefficients. First, note that the coefficients C^G_{MN}, C^{Φ_m}, C^W all have the same common factor $(G^{MN}\partial_M W \partial_N W - 4W)$. The only way that all C coefficients vanish is for the field $W(X)$ to satisfy the equation

$$G^{MN}\partial_M W \partial_N W = 4W. \tag{3.1}$$

This matches precisely the first two equations in Eq. (2.3) that follow from the Sp(2, *R*) worldline gauge symmetry, once the vector V^M is identified as

$$V^M = \tfrac{1}{2}G^{MN}\partial_N W. \tag{3.2}$$

To arrive at this result it is important to emphasize that the same $U(\Phi)$ must appear in all three terms in the action (2.12) where they are indicated. One could have started with three different functions $U_1(\Phi)$, $U_2(\Phi)$, $U_3(\Phi)$, in those three terms and then find out that they must be the same $U(\Phi)$ because otherwise there would be additional terms to cancel, proportional to $\partial_M W \partial_N W$ or $\partial_M W \partial_N U_i$ in B^G_{MN} and C^G_{MN}, that would be inconsistent with the Sp(2, *R*) constraints (2.3), (2.4), and (2.5). So, $U_1 = U_2 = U_3 = U$.

---

[9]The Einstein frame is obtained by a Weyl transformation in the conformal shadow (recall this is really a general coordinate transformation in the extra dimensions [7]). See, e.g., [32] for the quadratic example of Eq. (2.13), in which this Weyl gauge amounts to eliminating the dilaton ϕ in favor of the physical fields s_i, such as $\phi = \pm[1/2\kappa_d^2 + \sum s_i^2]^{1/2}$, where $(2\kappa_d^2)^{-1}$ is the Newton constant and ϕ, s_i are the conformal shadows of the original fields Ω, S_i.

is another consequence of demanding consistency with the Sp(2, *R*) constraints (2.3), (2.4), and (2.5).

Next examine the coefficients B^G_{MN}, B^{Φ_m}, B^W, A^W that are given by [6]

$$0 = B^G_{MN} = a_d U(\Phi)[G_{MN}(-6 + \nabla^2 W + \partial W \cdot \partial \ln U(\Phi))$$
$$- \nabla_M \partial_N W], \tag{3.3}$$

$$0 = B^{\Phi_m} = g_{mn}(\Phi)\partial W \cdot \partial \Phi^n + 2a_d \frac{\partial U}{\partial \Phi^m}(6 - \nabla^2 W), \tag{3.4}$$

$$0 = B^W = a_d[U(\Phi)(16 - 2\nabla^2 W) - 2\partial W \cdot \partial U(\Phi)], \tag{3.5}$$

$$0 = A^W = a_d U(\Phi)R(G) - 2a_d \nabla^2 U(\Phi)$$
$$- \tfrac{1}{2}g_{mn}(\Phi)\partial\Phi^m \cdot \partial\Phi^n - V(\Phi), \tag{3.6}$$

where the dot in $(A \cdot B)$ means contraction with G^{MN} and ∇^2 is the Laplacian constructed with the metric G^{MN}. By contracting Eq. (3.3) with G^{MN} and using $G^{MN}G_{MN} = d + 2$, one can derive an equation for $\nabla^2 W$

$$(d + 2)(-6 + \partial W \cdot \partial \ln U(\Phi)) + (d + 1)\nabla^2 W = 0. \tag{3.7}$$

Combining this equation with the $B^W = 0$ equation (3.5), the two unknown quantities $\nabla^2 W$ and $\partial W \cdot \partial \ln U(\Phi)$ are uniquely determined as

$$\nabla^2 W = 2(d + 2), \quad \partial W \cdot \partial \ln U(\Phi) = -2(d - 2). \tag{3.8}$$

Plugging this result back into Eq. (3.3) one finds

$$G_{MN} = \nabla_M V_N = \tfrac{1}{2}\nabla_M \partial_N W = \tfrac{1}{2}(\partial_M \partial_N W + \Gamma^P_{MN}\partial_P W). \tag{3.9}$$

This is precisely equivalent to the homothety condition on the geometry required by the Sp(2, *R*) constraints (2.3), (2.4), and (2.5). There remains dealing with the kinematic equations $B^{\Phi_m} = A^W = 0$ of Eqs. (3.4), (3.5), and (3.6).

With the results (3.1), (3.2), (3.3), (3.4), (3.5), (3.6), (3.7), (3.8), and (3.9) that follow from the action, so far it is evident that the 2T field theory action principle proposed in Eq. (2.12) is the most general one compatible with the underlying Sp(2, *R*) constraints (2.3), (2.4), and (2.5) on the geometry, as well as general coordinate invariance, without higher order derivatives of the spacetime metric G_{MN}. This unique action has also led to the unique kinematic conditions on $W(X)$ and $U(\Phi)$ shown in Eq. (3.8). The later condition on $U(\Phi)$ may be written as a homothety condition $V^M \partial_M U(\Phi(X)) = -(d - 2)U(\Phi(X))$, where the left side is the Lie derivative for a scalar field using the vector V^M in Eq. (3.2).

Next, solve Eq. (3.4), $B^{\Phi_m} = 0$, after substituting $(6 - \nabla^2 W) = -2(d - 1)$ as follows:

$$\partial W \cdot \partial \Phi^n = \frac{1}{2}(d - 2)g^{nm}(\Phi)\frac{\partial U}{\partial \Phi^m}. \tag{3.10}$$

This kinematic constraint is also a generalized homothety condition on the fields Φ^n which follow from Sp(2, *R*) in

the presence of interactions provided by $g^{nm} \frac{\partial U}{\partial \Phi^m}$. In the simple quadratic example of Eqs. (2.13) and (2.14), the homothety condition (3.10) for the scalars Φ^m becomes simply, $V^M \partial_M \Phi^m = -\frac{1}{2}(d-2)\Phi^m$, which is the Sp(2, R) kinematic condition familiar from previous studies of 2T physics either in the BRST approach [16] or the flat space-time 2T field theory approach [18]. In flat spacetime, with $V^M = X^M$ (see footnote [2]) this equation simply means that $\Phi(X)$ is homogeneous $\Phi(tX) = t^{-(1/2)(d-2)}\Phi(X)$. So, for more general $U(\Phi)$, $g_{mn}(\Phi)$, the kinematic equation (3.10) should be understood as the generalized homothety or "homogeneity" condition, including interactions. Solving this equation in a convenient choice of spacetime coordinates in curved space (see examples in [6,7]) reduces the field dependence on spacetime X^M by one coordinate. Since Eq. (3.10) is a first order differential equation in a single coordinate, it can always be solved exactly for any interaction $g^{nm} \frac{\partial U}{\partial \Phi^m}$.

Next, multiply both sides of Eq. (3.10) by $\frac{\partial \ln U}{\partial \Phi^n}$. After summing over n, the left-hand side becomes $\partial W \cdot \partial \ln U(\Phi)$; and using its derived value in Eq. (3.8), the right-hand side of (3.10) yields

$$g^{nm}(\Phi)\frac{\partial U}{\partial \Phi^m}\frac{\partial U}{\partial \Phi^n} = -4U(\Phi). \qquad (3.11)$$

It is interesting to note the similarity of this equation to Eq. (3.1), although one is in field space Φ^m while the other is in position space X^M. In a similar way one can also obtain the following equation from (3.4) by multiplying both sides of (3.10) with $\partial W \cdot \partial \Phi^m$:

$$g_{mn}(\Phi)\partial W \cdot \partial \Phi^m \partial W \cdot \partial \Phi^n = -(d-2)^2 U(\Phi). \quad (3.12)$$

It should be noted that Eqs. (3.11) and (3.12) are regarded as conditions on the metric $g_{mn}(\Phi)$ in field space, which restrict the types of possible interactions of the scalars for a given $U(\Phi)$.

An example of a metric $g_{mn}(\Phi)$ and a $U(\Phi)$ that satisfy all of these constraints, Eqs. (3.10), (3.11), and (3.12), is the quadratic example given in Eqs. (2.13) and (2.14). In this example the metric is constant and both the kinetic term and the $U(\Phi)R$ terms in the action have SO(N, 1) symmetry. Furthermore, the homothety condition (3.10) takes the simple form $\partial W \cdot \partial \Phi^m = -(d-2)\Phi^m$, which is the curved space generalization of the Sp(2, R) constraint given in footnote [2], and is easily solved [6,7].

Another example of $g_{mn}(\Phi)$ and $U(\Phi)$ that satisfy the constraints in Eqs. (3.10), (3.11), and (3.12) is

$$U(\Phi) = \Omega^2, \qquad g^{mn} = \begin{pmatrix} -1 & -\frac{S^j}{\Omega} \\ -\frac{S^i}{\Omega} & \left(g^{ij} - \frac{S^i S^j}{\Omega^2}\right) \end{pmatrix},$$
$$g_{mn} = \begin{pmatrix} \left(-1 + \frac{S^k g_{kl} S^l}{\Omega^2}\right) & -\frac{g_{jk} S^k}{\Omega} \\ -\frac{g_{il} S^l}{\Omega} & g_{ij} \end{pmatrix}, \qquad (3.13)$$

where the $N \times N$ submetric $g_{ij}(\Omega, S)$ is an arbitrary metric in field space. In this second example $U(\Phi) = \Omega^2$ is positive definite, while the Sp(2, R) homothety constraint (3.10) takes the form $\partial W \cdot \partial \Phi^m = -(d-2)\Phi^m$ for $\Phi^m = (\Omega, S^i)$, which is the same as the other example.

Finally, there is the equation $A^W = 0$ in (3.6). Actually, this is not a kinematic equation, but rather it is a dynamical equation since second order spacetime derivatives appear. To analyze this equation we need to take into account the dynamical equations $A^G_{MN} = 0$ and $A^{\Phi_m} = 0$ for all the fields Φ^m and G_{MN}. It turns out that $A^W = 0$ is automatically satisfied provided the dynamical equations $A^G_{MN} = 0$ and $A^{\Phi_m} = 0$ are satisfied (see [6,7]), so this is not an additional constraint to contend with.

IV. MORE SYMMETRY AND CONSTRAINTS ON SCALARS

More constraints on the scalar couplings $U(\Phi)$, $V(\Phi)$, $g_{mn}(\Phi)$ can arise because of stronger symmetries. Specific examples of this occur with 2T supersymmetry [19,20] which restricts the form of $V(\Phi)$, and 2T supergravity [33,34] which restricts the form of $g_{mn}(\Phi)$ to be a function constructed from $U(\Phi)$. Without going into the details of the 2T supersymmetry, it is possible to understand the effects of supersymmetry on the scalars in 2T supergravity by considering the conformal shadows of the scalars that are expected to appear in conventional 1T SUSY and 1T supergravity theories.

For example, for conventional $\mathcal{N} = 1$ SUSY in 4-dimensions, the main effect on $V(\Phi)$ is that it must be constructed from complex fields (chiral multiplets) in the form of D-terms and F-terms with a holomorphic super-potential $f(\Phi)$, in a well-known form that we do not need to elaborate on here (for reviews see [35,36]).

More constraints are found in supergravity. For example, for conventional $\mathcal{N} = 1$ supergravity in 4 dimensions, there is a Kähler potential coupled to R that also determines the metric in field space that occurs in the kinetic term of complex chiral multiplets (see, e.g., formulas 31.6.57 to 31.6.61 in [36]).

From such 1T shadows of 2T supergravity, with various numbers of supercharges $\mathcal{N}$, it is straightforward to deduce the corresponding constraints on $g_{mn}(\Phi)$, $U(\Phi)$ in 2T supersymmetric field theory beyond the constraints already described in the previous section. We will not be specific here for various $\mathcal{N}$, but only indicate that one typical constraint is that $g_{mn}(\Phi)$ is constructed from $U(\Phi)$ as a second derivative in field space, such as

$$g_{mn}(\Phi) = -\frac{1}{2}\frac{\partial^2 U(\Phi)}{\partial \Phi^m \partial \Phi^n}. \qquad (4.1)$$

Actually, the constraint in $\mathcal{N} = 1$ supergravity is even stronger in terms of complex fields that yield a Kähler metric in field space

$$g_{m\bar{n}}(\Phi, \bar{\Phi}) = -\frac{\partial^2 U(\Phi, \bar{\Phi})}{\partial \Phi^m \partial \bar{\Phi}^{\bar{n}}}. \qquad (4.2)$$

Rather than the specific form (complex or real), the derivative form is what we wish to pursue here to make the following observations. For this reason we will stick to real fields and the metric in (4.1) to maintain a consistent notation with the previous sections (the result will be similar for complex fields).

The derivative form of the metric permits a different approach to solving the homothety constraint (3.4) on the scalars. Inserting Eq. (4.1) into (3.4) and using $(6 - \nabla^2 W) = -2(d-1)$ as before, $B^{\Phi^m} = 0$ takes the form

$$-\frac{1}{2}\frac{\partial^2 U(\Phi)}{\partial \Phi^m \partial \Phi^n}\partial W \cdot \partial \Phi^n - \frac{1}{2}(d-2)\frac{\partial U}{\partial \Phi^m} = 0. \quad (4.3)$$

Rather than solving this in the form of (3.10), which still holds, the chain rule leads to a simpler result, namely,

$$\partial W \cdot \partial\frac{\partial U}{\partial \Phi^m} + (d-2)\frac{\partial U}{\partial \Phi^m} = 0. \qquad (4.4)$$

This homothety constraint is a linear equation in $\frac{\partial U}{\partial \Phi^m}$ and, other than being in curved space, it looks the same as the $\mathrm{Sp}(2, R)$ kinematic constraint for the scalar field in footnote [2]). Combined with the result in (3.8), $\partial W \cdot \partial \ln U(\Phi) = -2(d-2)$, this implies that

$$\partial W \cdot \partial \Phi^m = -(d-2)\Phi^m, \qquad (4.5)$$

and that $U(\Phi)$ and $V(\Phi)$ must be homogeneous of degree 2 and $2d/(d-2)$, respectively, in field space

$$U(t\Phi) = t^2 U(\Phi), \qquad V(t\Phi) = t^{2d/(d-2)}V(\Phi). \quad (4.6)$$

Hence, when the metric is constructed as a second derivative of $U(\Phi)$, as in 2T supergravity, an additional consequence is that $U(\Phi)$ and $V(\Phi)$ are homogeneous as indicated. Furthermore, $U(\Phi)$ must also satisfy the nonlinear equation that follows from Eq. (3.11):

$$\frac{\partial U}{\partial \Phi^m}\left(\frac{\partial \Phi \otimes \partial \Phi}{\partial^2 U}\right)^{mn}\frac{\partial U}{\partial \Phi^n} = 2U(\Phi). \qquad (4.7)$$

These are a lot of constraints on $U(\Phi)$, as well as $g_{mn}(\Phi)$, that eliminate previously possible solutions. Nevertheless, there still remains some freedom. Note that a homogeneous $U(\Phi)$ of degree 2 does not necessarily mean quadratic, since ratios of fields may also occur in $U(\Phi)$. Some additional symmetry conditions, such as global or local gauge symmetries that must be respected in the full theory can narrow down the possibilities. For example, asking for a global $SO(N, 1)$ symmetry in the kinetic term completely nails down both $U(\Phi)$ and $g_{mn}(\Phi)$ to have the quadratic form in Eq. (2.13) and (2.14). Alternatively, asking for no particular symmetry of $U(\Phi)$ but only asking for a canonical normalization of the kinetic term with a constant metric g_{mn} also nails down $U(\Phi)$ to be the same quadratic of Eq. (2.13).

As an example, applying this quadratic case for complex scalars Φ^m, $\bar{\Phi}^m$, to construct a 2T $\mathcal{N} = 1$ supergravity

theory in $4 + 2$ dimensions [37] yields the following potential energy $V(\Phi, \bar{\Phi})$ when U is given by $U(\Phi, \bar{\Phi}) = (\Phi^0 \bar{\Phi}^0 - \sum \Phi^i \bar{\Phi}^i)$:

$$V(\Phi, \bar{\Phi}) = \frac{\partial f(\Phi)}{\partial \Phi^m}\frac{\partial \bar{f}(\bar{\Phi})}{\partial \bar{\Phi}^n}g^{mn}, \qquad f(t\Phi) = t^3 f(\Phi). \quad (4.8)$$

Here g^{mn} is the constant $SU(N, 1)$ metric[10] in Eq. (2.14), and $f(\Phi)$ is the analytic superpotential which must be homogeneous of degree 3. Again this does not necessarily mean that $f(\Phi)$ is cubic, since ratios of fields may also occur (in nonrenormalizable effective theories). Although this looks like a simple F-term in the potential, I emphasize that this is the full form after all the supergravity machinery, including the Kähler potential is taken into account as explained in [37]. The simplicity occurs because of the special form of the quadratic function $U(\Phi, \bar{\Phi})$. Note that due to the indefinite metric g_{mn}, this V is not a positive definite potential energy. Having a negative contribution to the potential energy, despite the exact local supersymmetry, is typical in supergravity. An additional positive definite D-term is added to this potential energy in the standard form when Yang-Mills gauge fields are coupled to supergravity (see, e.g., [35,36]).

I have shown that in 2T field theory there are a variety of constraints on scalar fields. The first and foremost are the $\mathrm{Sp}(2, R)$ constraints on the overall theory which results in kinematic equations on all the fields of all spins. These kinematic equations are related to the gauge symmetries that remove ghosts and make the theory unitary directly in $d + 2$ dimensions. Their effect on scalar fields is twofold. First, the scalar field must obey a kinematic equation, whose most general form, including interactions, is given in Eq. (3.10). The solutions of the kinematic equations correspond to the shadows in 1T field theory and help interpret the predictions of 2T physics in the language of 1T physics. Second, the possible interactions of scalars among themselves $V(\Phi)$ and with gravity described by $U(\Phi)$, $g_{mn}(\Phi)$ must obey certain conditions, including especially (3.11) and (3.12) which are regarded as restrictions on the metric in field space. Supersymmetry puts further severe constraints on $V(\Phi)$, and g_{mn}, $U(\Phi)$ in the form (4.6) and (4.7), while gauge symmetries and global symmetries of the overall theory narrow down the possible interactions.

The patterns of scalar field interactions predicted by 2T physics for the 1T conformal shadow (familiar setting in particle physics) can also be introduced in 1T field theory by hand, but are not necessarily motivated by a similar principle.

---

[10]When both the kinetic terms and U are all rewritten in terms of real fields, the symmetry of both U and the kinetic terms is actually $SO(2N, 2)$, with two negative eigenvalues of the metric when rewritten in a real basis. But, because of the complex nature of the superpotential, what appears in V is the metric for the subgroup $SU(N, 1)$.

V. EPILOGUE: NO SCALE MODELS AND THE COSMOLOGICAL CONSTANT

In a separate paper I will discuss 2T supergravity. The present study of scalars was motivated by trying to solve some puzzles for how to supersymmetrize 2T gravity. The essential issue was that the accidental Weyl symmetry (2.9) in the conformal shadow was crucial to remove the ghost [the negative eigenvalue in the metric $g_{mn}(\Phi)$] associated with the field Ω. Recall that the conformal shadow of Ω played the role of a conformal compensator familiar in 1T field theory. To supersymmetrize, this ghost compensator could be made a member of a chiral multiplet, and hence there would be an additional ghost due to the complex field nature of the compensator chiral multiplet. Not only that, there would also be fermionic partners of these ghosts. I was puzzled for a long time how these additional bosonic and fermionic ghosts could be removed consistently with supersymmetry, and was wondering if supersymmetrizing 2T gravity would require a different approach than chiral multiplets?[11]

Just one brief remark[12] on SU(2, 2|1) was sufficient to show me the way and solve all the puzzles as follows. In 2T SUSY field theory with $\mathcal{N}$ supersymmetries in $4 + 2$ dimensions there is a global SU(2, 2|$\mathcal{N}$) symmetry [19,20]. The SU(2,2) part of it is the linearly realized SO(4,2) of the $4 + 2$ dimensions. To construct $\mathcal{N} = 1$ supergravity, just as SO(4,2) is turned into a local symmetry, the full SU(2, 2|1) must also become local. If such a 2T supergravity theory exists in $4 + 2$ dimensions it must be that the conformal shadow in $3 + 1$ dimensions is also locally symmetric under SU(2, 2|1). Indeed, Ferrara pointed out that such a formulation of 1T Poincaré supergravity was considered some time ago in a conformal formalism [39,40]. The SU(2, 2|1) has all the local symmetries to remove all the ghosts associated with the compensator. Specifically, the local superconformal S supersymmetry removes the fermionic member of the compensator chiral multiplet, the gauged U(1) R symmetry (with a nonpropagating auxiliary vector field) removes the phase of the remaining complex boson, and finally the local Weyl symmetry fixes the compensator to the Newton constant as in Eq. (2.10). From SU(2, 2|1) there remains only the local Lorentz and the local Q supersymmetry, which are the evident local symmetries of Poincaré supergravity. Hence, the road to 2T supergravity is clear.

The details of the 2T supergravity will be given elsewhere [37], but here I outline the resulting conformal shadow with particular emphasis on the scalars. The shadow of the $\mathcal{N} = 1$ Poincaré supergravity in $3 + 1$

dimensions contains the following supermultiplets: the graviton supermultiplet $(e_\mu^a, \psi_\mu^\alpha, b_\mu, z)$ where $b_\mu, z (\equiv s + ip)$ are auxiliary fields [39], plus the chiral supermultiplets (ϕ^m, ψ^m, F^m) labeled by $m = 0, 1, \ldots, N$, where F^m are auxiliary fields. The bosonic part of the Lagrangian that can be compared to conventional supergravity is[13]

$$
\frac{1}{e} \mathcal{L}_{\text{bose}} = \left\{ U(\phi, \bar{\phi}) \left(\frac{1}{6} R(g) - z\bar{z} + g^{\mu\nu} b_\mu b_\nu \right) \right.
$$
$$
+ \frac{\partial^2 U}{\partial \phi^m \partial \bar{\phi}^n} (g^{\mu\nu} \partial_\mu \phi^m \partial_\nu \bar{\phi}^n - F^m \bar{F}^n)
$$
$$
+ \left[-(zF^m + ib^\mu \partial_\mu \phi^m) \frac{\partial U}{\partial \phi^m} + \frac{\partial f}{\partial \phi^m} F^m \right.
$$
$$
\left. \left. + 3\bar{z} f(\phi) \right] + \text{c.c.} \right\},
$$

where $f(\phi)$ is the superpotential. The multiplet labeled by $m = 0$ contains the field $\phi^0(x)$ that plays the role of the complex compensator so it describes a supermultiplet of negative norm ghosts. There is just the required amount of gauge symmetry to remove them from the physical spectrum, and generate from them the Newton constant, as described above. In particular the U(1) gauge field that removes the phase of $\phi^0(x)$ is the auxiliary field b_μ. To exhibit the U(1) gauge symmetry associated with b_μ I define the following covariant derivative with a nonlinear action of the U(1) transformation (it becomes linear only for quadratic U):

$$
D_\mu \phi^m = \partial_\mu \phi^m + ib_\mu \frac{\partial U}{\partial \bar{\phi}^n} \left(\frac{\partial \bar{\phi} \otimes \partial \phi}{\partial^2 U} \right)^{nm}, \qquad (5.1)
$$

where the last factor is the inverse of the Kähler metric. Then this Lagrangian takes the U(1) gauge invariant form

$$
\frac{1}{e} \mathcal{L}_{\text{bose}} = \left\{ \frac{1}{6} U(\phi, \bar{\phi}) R(g) + \frac{\partial^2 U}{\partial \phi^m \partial \bar{\phi}^n} g^{\mu\nu} D_\mu \phi^m D_\nu \bar{\phi}^n \right.
$$
$$
+ \left(\frac{\partial \bar{\phi} \otimes \partial \phi}{\partial^2 U} \right)^{nm} \frac{\partial \bar{f}}{\partial \bar{\phi}^n} \frac{\partial f}{\partial \phi^m} + \frac{\partial^2 U}{\partial \phi^m \partial \bar{\phi}^n}
$$
$$
\times \left(F + \bar{z}\phi + \left(\frac{\partial \phi \otimes \partial \bar{\phi}}{\partial^2 U} \right) \frac{\partial \bar{f}}{\partial \bar{\phi}} \right)^m
$$
$$
\times \left(\bar{F} + z\bar{\phi} + \left(\frac{\partial \bar{\phi} \otimes \partial \phi}{\partial^2 U} \right) \frac{\partial f}{\partial \phi} \right)^n - \bar{z} \left(\phi^m \frac{\partial f}{\partial \phi^m} \right.
$$
$$
\left. \left. - 3f(\phi) \right) - z \left(\bar{\phi}^m \frac{\partial \bar{f}}{\partial \bar{\phi}^m} - 3f(\bar{\phi}) \right) \right\}.
$$

---

[11]For example, the linear multiplet [38] does not require the complexification, but after all it is equivalent to the chiral multiplet.

[12]I thank S. Ferrara for clarifying some aspects of gauge symmetries in supergravity [39–41].

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[13]See, e.g., Eq. (31.6.57) in [36], where the Newton constant terms are dropped, and the auxiliary fields b_μ, s, p are renormalized by a convenient numerical factor of $2/3$. Note also that the factor of $1/6$ in front of R comes from $2a_d = \frac{1}{6}$ for $d = 4$ (coming from $d + 2 = 6$). Here we have $2a_d$ instead of a_d because of the complex basis for the fields. Similarly, the factor of 3 in front of $3\bar{z}f(\phi)$ more generally is given as $\frac{d+2}{2}$.

This shows that gauge invariance is possible only when $U(\phi, \bar{\phi})$ satisfies the nonlinear condition in Eq. (4.7). The gauge invariance is required in order to remove the negative norm ghost.

The last two lines in this form vanish after integrating out the auxiliary fields F^m and z. In any case the last line does not even appear because 2T supegravity already requires in Eq. (4.8) that $f(\phi)$ had to be homogeneous of degree 3; hence the auxiliary field z dropped out automatically anyway. Hence, the bosonic Lagrangian simplifies greatly to the form[14]

$$\frac{1}{e} \mathcal{L}_{\text{bose}} = \frac{1}{6} U(\phi, \bar{\phi}) R(g) + \frac{\partial^2 U}{\partial \phi^m \partial \bar{\phi}^n} g^{\mu\nu} D_\mu \phi^m D_\nu \bar{\phi}^n$$
$$+ \left(\frac{\partial \bar{\phi} \otimes \partial \phi}{\partial^2 U} \right)^{nm} \frac{\partial \bar{f}}{\partial \bar{\phi}^n} \frac{\partial f}{\partial \phi^m}. \tag{5.2}$$

The potential energy is then

$$V(\phi, \bar{\phi}) = - \left(\frac{\partial \bar{\phi} \otimes \partial \phi}{\partial^2 U} \right)^{nm} \frac{\partial \bar{f}}{\partial \bar{\phi}^n} \frac{\partial f}{\partial \phi^m}, \tag{5.3}$$

as given in Eq. (4.8). Note that this is homogeneous of degree 4, $V(t\phi, t\bar{\phi}) = t^4 U(\phi, \bar{\phi})$. Recall that here we also require that $U(\phi, \bar{\phi})$ is homogeneous of degree 2, and that it must satisfy the complex version of the nonlinear condition (4.7)

$$U(t\phi, t\bar{\phi}) = t^2 U(\phi, \bar{\phi}),$$
$$\frac{\partial U}{\partial \phi^m} \left(\frac{\partial \phi \otimes \partial \bar{\phi}}{\partial^2 U} \right)^{mn} \frac{\partial U}{\partial \bar{\phi}^n} = U(\phi, \bar{\phi}). \tag{5.4}$$

Only the $U(\phi, \bar{\phi})$ that can solve these equations (with nonzero eigenvalues in the Kähler metric $g_{mn} =$

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[14]This form is invariant under the following infinitesimal Weyl transformation that generalizes Eq. (2.9): $\delta_\lambda g_{\mu\nu} = 2\lambda(x) g_{\mu\nu}$ and $\delta_\lambda \phi^m = -\frac{d-2}{2} \lambda(x) g^{mn} \frac{\partial U}{\partial \bar{\phi}^n}$, and $\delta_\lambda b_\mu = 0$, where $g^{mn} \equiv \left(\frac{\partial \phi \otimes \partial \bar{\phi}}{\partial^2 U} \right)^{mn}$. Note that the Weyl transformation of ϕ^m is derived from the Sp(2, R) condition on the parent field Φ^m in $d + 2$ dimensions in Eq. (3.10). As explained in [7], this implies that the Weyl trasformation for all the fields in the shadow amounts to a reparametrization of the coordinates in the extra dimensions. There is no Weyl symmetry in the higher dimensional theory. Note also that this Weyl symmetry holds even for the more general case of $g_{mn}(\phi, \bar{\phi})$ that satisfies the complex version of Eq. (4.7), $\frac{\partial U}{\partial \phi^m} g^{mn} \frac{\partial U}{\partial \bar{\phi}^n} = -U$, even when g_{mn} is not constructed from derivatives of U (i.e., not taking into account the local supersymmetry conditions in supergravity). For any such $g_{mn}(\phi, \bar{\phi})$, after integrating out the gauge field b_μ, the kinetic term for the scalars takes the following form: $g_{mn} D\phi^m \cdot D\bar{\phi}^n = g^{\mu\nu}(g_{mn} \partial_\mu \phi^m \partial_\nu \bar{\phi}^n - J_\mu J_\nu)$, where $J_\mu = \frac{1}{2}(i\partial_\mu \phi^m \frac{\partial \ln U}{\partial \phi^m} - i\partial_\mu \bar{\phi}^m \frac{\partial \ln U}{\partial \bar{\phi}^m})$.

$-\frac{\partial^2 U}{\partial \phi^m \partial \bar{\phi}^n}$) are admitted in the $3 + 1$ dimensional conformal shadow of 2T supergravity in $4 + 2$ dimensions.

A $U(\phi, \bar{\phi})$ that satisfies Eq. (5.4), together with an analytic homogeneous superpotential that satisfies $f(t\phi) = t^3 f(\phi)$, determines fully the scalar field interactions in 2T supergravity and its conformal shadow given in Eq. (5.2). *This is the restriction on scalars in 1T physics that arises from 2T supergravity.*

An example of a $U(\phi, \bar{\phi})$ that satisfies these equations is the complex version of the quadratic case in Eq. (2.13) that I discussed several times in this paper. In a complex basis it has the form

$$\text{example}: U(\phi, \bar{\phi}) = \phi^0 \bar{\phi}^0 - \sum_{i=1}^{N} \phi^i \bar{\phi}^i = -g_{mn} \phi^m \bar{\phi}^n. \tag{5.5}$$

This constant metric $g_{mn} = \text{diag}(-1, +1, \ldots, +1)$ leads to canonically normalized complex scalars, with an automatic linearly realized SU(N, 1) global symmetry in the kinetic and R-terms of the action (5.2). This symmetry may be broken by the choice of the superpotential $f(\phi)$. In this example, the action (5.2) requires that all scalars must be conformally coupled to gravity. Note that not only the compensator, but all scalars are conformally coupled. This is possible in 1T field theory, but it is not motivated by a principle, like it is in 2T physics.

It is worth mentioning that the work on 2T supergravity has led in a natural way to a class of no scale models of 1T supergravity that are a good starting point for understanding the basic problem of the smallness of the cosmological constant [42]. In particular, the quadratic U of Eq. (5.5) immediately produces an attractive no scale model with the potential energy V given above in Eq. (4.8), as detailed in [37]. It is a coincidence that right after resolving the 2T supergravity puzzles, and having constructed the quadratic model, a brief discussion with C. Kounnas whom I ran into unexpectedly has attracted my attention to the no scale ideas [42–46] for which the 2T supergravity path is quite natural. Specifically, a no scale model is obtained from the above 2T supergravity approach simply by taking the following basis for the fields $\phi^\pm = (\phi^0 \pm \phi^N)/\sqrt{2}$, and then writing the potential (4.8) in the form:

$$V(\phi, \bar{\phi}) = -\frac{\partial f}{\partial \phi^0} \frac{\partial \bar{f}}{\partial \bar{\phi}^0} + \frac{\partial f}{\partial \phi^N} \frac{\partial \bar{f}}{\partial \bar{\phi}^N} + \sum_{i=1}^{N-1} \frac{\partial f}{\partial \phi^i} \frac{\partial \bar{f}}{\partial \bar{\phi}^i},$$
$$= -\frac{\partial f}{\partial \phi^+} \frac{\partial \bar{f}}{\partial \bar{\phi}^-} - \frac{\partial f}{\partial \phi^-} \frac{\partial \bar{f}}{\partial \bar{\phi}^+} + \sum_{i=1}^{N-1} \frac{\partial f}{\partial \phi^i} \frac{\partial \bar{f}}{\partial \bar{\phi}^i}. \tag{5.6}$$

If one takes a superpotential $f(\phi)$ that depends only ϕ^+ and ϕ^i, i.e., $\frac{\partial f}{\partial \phi^-} = 0$, then in the remaining $V(\phi, \bar{\phi})$

each term is strictly positive for all values of the fields. For the scalars, the minimum of the potential $V(\phi, \bar{\phi})$ can only occur when each term vanishes at field values that satisfy $\frac{\partial f}{\partial \phi^i} = 0$ for $i = 1, \ldots, N-1$, while $\frac{\partial f}{\partial \phi^+} =$ anything (since $\frac{\partial f}{\partial \phi^-} = 0$). Therefore the absolute minimum of the potential is necessarily at zero $V_{\min} = 0$, yielding automatically a vanishing cosmological constant even after a spontaneous breakdown of symmetries that cause phase transitions in the history of the Universe (such as electroweak, SUSY, grand unification, inflation, etc.).[15]

The field ϕ^+ can be gauge fixed conveniently as in footnote [15] by using the Weyl gauge symmetry in Eq. (2.9), thus generating the Newton constant $(2\kappa^2)^{-1}$. In this specific gauge the quadratic example can be compared to the no scale model discussed in [43]. The homogeneous superpotential of Eq. (4.8) may be written as $f(\phi) = (\phi^+)^3 \rho(\phi^i / \phi^+)$, with an arbitrary $\rho(z^i)$. This $\rho(z^i)$ may be chosen to fit particle physics phenomenology, including SUSY breaking, which may be clarified in experiments at the LHC if the SUSY scale is within its reach.[16] The remaining field ϕ^- is unfixed at the minimum of the potential at the classical level (a flat direction, hence no scale). Quantum corrections can stabilize the remaining ϕ^- field. There exist schemes [46] that may explain the smallness of the observed cosmological constant after the quantum corrections.

One lesson of the $\mathcal{N} = 1$ supergravity example above is that $g_{mn}(\Phi)$ can have more than one negative eigenvalues (see footnote [10]). To kill the corresponding additional ghosts, there must be Yang-Mills type gauge symmetries, such as the U(1) R symmetry in the SU(2, 2|1), or its generalizations for higher $\mathcal{N}$. Furthermore higher $\mathcal{N}$ supergravity admits gauge symmetries with nonpropagat-

ing auxiliary vector fields. Such gauge symmetries, combined with the Weyl symmetry, are then used to remove all the negative norm ghost scalar fields, leaving behind the familiar scalar fields in 1T supergravity theories that are described as moduli in coset spaces of certain noncompact U-duality groups. The benefit of keeping the negative norm scalars in the initial formulation of the 2T supergravity theory is to make evident hidden symmetries and then using the gauge symmetries in the most convenient way (see, for example, [32]) to analyze the physics in the 1T shadows.

The 2T approach has been indicating in many settings, including gravity and supergravity in this paper, that there is an ambient $d + 2$ dimensional spacetime in which the fundamental form of the theory resides. The shadows in d dimensions distort the fundamental form of the equations, just like an observer's choice of coordinates in general relativity inserts a distortion. Unlike general relativity, in 2T physics this distortion leads to different choices of time in 1T physics, and hence to different 1T-physics interpretations. The conventional formulation of 1T physics is just one of the shadows, namely, the conformal shadow familiar in particle physics. This familiarity is the reason to concentrate mainly on the conformal shadow in many of the discussions because this helps to digest the physical meaning of 2T physics at least in one familiar setting. However, the benefits of the 2T formulation will be mainly in exploring the other shadows and in using the duality relationships among the shadows to develop useful computational techniques as well as new insights about the meaning of space and time, as discussed partially in [2,21,22]. In this regard, Weinberg's recent results for Green's functions [3], which amount to Green's functions in flat $4 + 2$ dimensional 2T field theory including the standard model [18], is one of the explicit examples that can be explored by using the reduction techniques to various shadows as suggested in footnote [4].

On the fundamental theory side, 2T physics for strings and branes and 2T superfield theory in higher dimensions is still underdeveloped (for their status see [1]). I note that SUSY Yang-Mills theory has already been constructed in $10 + 2$ dimensions as will be presented in the near future [47]. Further exploration of the fundamentals in 2T physics along these lines should lead next to supergravity in $10 + 2$ and $11 + 2$ dimensions, thus providing a 2T version of M theory.

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[15]This discussion is before one goes to the Einstein frame. The Einstein frame is most easily reached by making convenient Weyl and U(1) gauge choices for the conformal shadow of Φ^+, namely, Im $(\phi^+) = 0$ and Re $\phi^+ = (\phi^- + \bar{\phi}^-)^{-1}(\frac{1}{2\kappa^2} + \sum \phi_i \bar{\phi}_i)$ for the quadratic example. After a similar gauge choice for any U, and using the homogeneity of the potential V, the effective potential in the Einstein frame is written as $V(\phi, \bar{\phi}) = (\text{Re } \phi_+)^4 V(\frac{\phi}{\phi^+}, \frac{\bar{\phi}}{\bar{\phi}^+}) = V_{\text{eff}}(\phi^+, \phi_i, \bar{\phi}^+, \bar{\phi}_i)$. Hence, they are proportional to each other $V_{\text{eff}} \sim V$ with an overall positive coefficient $(\text{Re } \phi^+)^4$. Therefore, the discussion of the minimum of the potential is the same in the Einstein frame. Furthermore, including the D-terms do not change this discussion because D-terms are strictly positive and they must vanish separately at the minimum of the potential.

[16]An interesting modification of the quadratic form is $U(\phi, \bar{\phi}) = \alpha \phi^+ \bar{\phi}^+ - \phi^+ \bar{\phi}^- - \phi^- \bar{\phi}^+ + \sum \phi_i \bar{\phi}_i$ with any constant α. This gives a nondiagonal but constant g_{mn} and a potential $V(\phi, \bar{\phi}) = -\frac{\partial f}{\partial \phi^+}\frac{\partial \bar{f}}{\partial \bar{\phi}^-} - \frac{\partial f}{\partial \phi^-}\frac{\partial \bar{f}}{\partial \bar{\phi}^+} - \alpha \frac{\partial f}{\partial \phi^-}\frac{\partial \bar{f}}{\partial \bar{\phi}^-} + \sum_{i=1}^{N-1}\frac{\partial f}{\partial \phi_i}\frac{\partial \bar{f}}{\partial \bar{\phi}_i}$. When $\frac{\partial f}{\partial \phi^-} = 0$ this still reduces to a no scale model with the strictly positive V, but a different U for any α. The parameter α can be used for phenomenological purposes as discussed elsewhere.

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