PHYSICAL REVIEW D **82,** 123533 (2010)

Evolution of the Universe to the present inert phase

I. F. Ginzburg and K. A. Kanishev
Sobolev Institute of Mathematics and Novosibirsk State University, Novosibirsk, Russia

M. Krawczyk and D. Sokołowska
Institute of Theoretical Physics, University of Warsaw, Warsaw, Poland
(Received 9 November 2010; published 30 December 2010)

We assume that the current state of the Universe can be described by the inert doublet model, containing two scalar doublets, one of which is responsible for electroweak-symmetry breaking and masses of particles and the second one having no couplings to fermions and being responsible for dark matter. We consider possible evolutions of the Universe to this state during cooling down of the Universe after inflation. We found that in the past the Universe could pass through phase states having no dark matter candidate. In the evolution via such states, in addition to a possible electroweak-symmetry breaking phase transition (second order), the Universe sustained one first-order phase transition or two phase transitions of the second order.

DOI: 10.1103/PhysRevD.82.123533 PACS numbers: 95.35.+d, 14.80.Da, 14.80.Fd, 98.80.−k

I. INTRODUCTION

According to the standard cosmological model, about 25% of the Universe is made of dark matter (DM). Different candidates for DM particles are now discussed in the literature. One of the widely discussed models is the inert doublet model (IDM) [1]—a Z_2 symmetric two-Higgs doublet model (2HDM) with a suitable set of parameters. The model contains one ‘‘standard’’ scalar (Higgs) doublet ϕ_S, responsible for electroweak-symmetry breaking and the masses of fermions and gauge bosons as in the standard model (SM), and one scalar doublet ϕ_D, which does not receive vacuum expectation value (VEV) and does not couple to fermions.[1]

In this model 4 degrees of freedom of the Higgs doublet ϕ_S are as in the SM: Three Goldstone modes become longitudinal components of the electroweak (EW) gauge bosons, and one mode becomes the Higgs boson (here denoted as h_S). All the components of the scalar doublet ϕ_D are realized as the massive scalar D particles: two charged $D^\pm$ and two neutral ones D_H and D_A. By construction, they possess a new conserved multiplicative quantum number, and therefore the lightest particle among them can be considered as a candidate for a DM particle.

Assuming, as usual, that DM particles are neutral, we consider such a variant of the IDM, in which masses of D particles are

$$M_{D^\pm}, M_{D_A} \geq M_{D_H} \quad \text{or} \quad M_{D^\pm}, M_{D_H} \geq M_{D_A}. \qquad (1)$$

Possible masses of these D particles are constrained by the accelerator and astrophysical data (see, e.g., [2,3]).

In this paper we assume that the current state of the Universe is described by the IDM. We discuss possible variants of the history of the phase states of the Universe during its cooling down after inflation. In some respects, this analysis can be considered as a particular case of analysis [4,5]. We use below some notations from Refs. [4–7].

[1] Our notations are similar to those in the general 2HDM with the change $\phi_1 \to \phi_S$, $\phi_2 \to \phi_D$.

II. THE LAGRANGIAN

In this paper we consider an electroweak-symmetry breaking (EWSB) via the Brout-Englert-Higgs-Kibble mechanism, as described by the Lagrangian

$$\mathcal{L} = \mathcal{L}_{gf}^{SM} + \mathcal{L}_H + \mathcal{L}_Y(\psi_f, \phi_S), \quad \mathcal{L}_H = T - V. \qquad (2)$$

Here, $\mathcal{L}_{gf}^{SM}$ describes the $SU(2)\times U(1)$ standard model interaction of gauge bosons and fermions, which is independent on the realization of the Brout-Englert-Higgs-Kibble mechanism. In the considered case the Higgs scalar Lagrangian $\mathcal{L}_H$ contains the standard kinetic term T and the potential V with two scalar doublets ϕ_S and ϕ_D. The $\mathcal{L}_Y$ describes the Yukawa interaction of fermions ψ_f with only one scalar doublet ϕ_S, having the same form as in the SM with the change $\phi \to \phi_S$.

Potential.—The potential must be Z_2 symmetric in order to describe the IDM. Without loss of generality it can be written in the following form:

$$V = -\frac{1}{2}[m_{11}^2(\phi_S^\dagger\phi_S) + m_{22}^2(\phi_D^\dagger\phi_D)] + \frac{\lambda_1}{2}(\phi_S^\dagger\phi_S)^2 + \frac{\lambda_2}{2}(\phi_D^\dagger\phi_D)^2 + \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D) + \lambda_4(\phi_S^\dagger\phi_D) \times(\phi_D^\dagger\phi_S) + \frac{\lambda_5}{2}[(\phi_S^\dagger\phi_D)^2 + (\phi_D^\dagger\phi_S)^2], \qquad (3a)$$

with all parameters real and with an additional condition[2]

[2] In the general Z_2 symmetric potential, the last term has the form $[\tilde\lambda_5(\phi_S^\dagger\phi_D)^2 + \tilde\lambda_5^*(\phi_D^\dagger\phi_S)^2]$. The physical content of theory cannot be changed by the global phase rotation $\phi_a \to \phi_a e^{i\alpha_a}$ ($a = S, D$). Starting with an arbitrary complex $\tilde\lambda_5 = |\tilde\lambda_5|e^{i\rho}$, we select $\alpha_S - \alpha_D = \rho/2 + \pi/2$ to get (3) with negative $\lambda_5 = -|\tilde\lambda_5|$.

$$\lambda_5 < 0. \tag{3b}$$

The IDM is realized in some regions of parameter space of this potential. To study thermal evolution, we will consider also other possible vacuum states of such a potential, realized at other values of the parameters.

To make some equations shorter, we use the following abbreviations:

$$\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5, \qquad R = \frac{\lambda_{345}}{\sqrt{\lambda_1 \lambda_2}}. \tag{4}$$

Discrete symmetries.—The potential (3) is invariant under two discrete symmetry transformations of a Z_2 type:

$$S\colon \phi_S \xrightarrow{S} -\phi_S, \qquad \phi_D \xrightarrow{S} \phi_D, \quad \mathrm{SM} \xrightarrow{S} \mathrm{SM}, \tag{5}$$

$$D\colon \phi_S \xrightarrow{D} \phi_S, \qquad \phi_D \xrightarrow{D} -\phi_D, \qquad \mathrm{SM} \xrightarrow{D} \mathrm{SM}, \tag{6}$$

where SM denotes the SM fermions and gauge bosons.

We call these transformations S transformation and D transformation, respectively. In the case when vacuum has vanishing vacuum expectation values, $\langle \phi_S \rangle = \langle \phi_D \rangle = 0$, the mentioned above invariance of V results in the D-parity and S-parity conservation in the processes involving only scalars (or scalars and gauge bosons). The Yukawa term violates S symmetry even if $\langle \phi_S \rangle = \langle \phi_D \rangle = 0$, while it respects D symmetry in any order of perturbation theory.

Positivity constraints.—To have a stable vacuum, the potential must be positive at large quasiclassical values of fields $|\phi_i|$ (positivity constraints), for an arbitrary direction in the (ϕ_S, ϕ_D) plane. These conditions limit possible values of λ_i (see, e.g., [8]). In terms of parameters (4), positivity constraints, which are needed in our analysis, can be written as

$$\lambda_1 > 0, \qquad \lambda_2 > 0, \qquad R + 1 > 0. \tag{7}$$

III. THERMAL EVOLUTION

The main goal of this paper is to consider an evolution of the Universe during its cooling down to the present inert phase. For this purpose we consider thermal evolution of the Lagrangian, following the approach presented in Refs. [5,9].

Potential.—Since the Hubble constant is small, we assume a statistical equilibrium at every temperature T. In this approximation, at the finite temperature, the ground state of system is given by a minimum of the Gibbs potential

$$V_G = \mathrm{Tr}(V e^{-\hat{H}/T}) / \mathrm{Tr}(e^{-\hat{H}/T}). \tag{8}$$

In the first nontrivial approximation and high enough temperature, the obtained Gibbs potential has the same form as the basic potential $V(3)$, i.e., as the potential at zero temperature. The coefficients $\lambda' s$ of the quartic terms in the potential V_G and V coincide, while the mass terms vary with temperature T, as follows:

$$\begin{aligned} &m_{11}^2(T) = m_{11}^2 - c_1 T^2, \qquad m_{22}^2(T) = m_{22}^2 - c_2 T^2, \\ &c_1 = \frac{3\lambda_1 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32} + \frac{g_t^2 + g_b^2}{8}, \\ &c_2 = \frac{3\lambda_2 + 2\lambda_3 + \lambda_4}{12} + \frac{3g^2 + g'^2}{32}. \end{aligned} \tag{9}$$

Here g and g' are the EW gauge couplings; $g_t \approx 1$ and $g_b \approx 0.03$ are values of the SM Yukawa couplings for t and b quarks, respectively.

Generally, each of coefficients c_1 and c_2 can be either positive or negative. However, in virtue of positivity conditions (7) their sum is positive:

$$c_2 + c_1 > 0, \tag{10}$$

even neglecting (positive) contributions from gauge bosons W/Z and fermions.

We will show later on that for a realization of the present inert vacuum with a neutral dark matter particle one needs $\lambda_4 + \lambda_5 < 0$ (24). Therefore, at $R > 0$ we have $\lambda_3 > 0$. Taking into account that $\lambda_5 < 0$ (3b), we obtain that $c_1 > 0$, $c_2 > 0$. At $R < 0$ there are no constraints on signs of $c_{1,2}$. So we have

$$R > 0\colon\ c_1 > 0,\, c_2 > 0; \qquad R < 0\colon \text{ arbitrary signs of } c_{1,2}. \tag{11}$$

Yukawa interaction.—The form of Yukawa interaction and values of Yukawa couplings do not vary during thermal evolution.

IV. EXTREMA OF THE POTENTIAL

Following Ref. [4] we first consider extrema of the potential (3) at arbitrary values of parameters. The extrema conditions

$$\left.\frac{\partial V}{\partial \phi_i}\right|_{\phi_i = \langle \phi_i \rangle} = 0, \qquad \left.\frac{\partial V}{\partial \phi_i^\dagger}\right|_{\phi_i = \langle \phi_i \rangle} = 0 \qquad (i = S, D) \tag{12}$$

define the extremum values $\langle \phi_S \rangle$ and $\langle \phi_D \rangle$ of the fields ϕ_S and ϕ_D, respectively. The extremum with the lowest energy (the global minimum of the potential) realizes the vacuum state of the system. Other extrema are saddle points, maxima or local minima of the potential.

The most general solution of (12) can be written in the following form:

$$\begin{aligned} &\langle \phi_S \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_S \end{pmatrix}, \\ &\langle \phi_D \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} u \\ v_D \end{pmatrix} \\ &(v^2 = v_S^2 + |v_D^2| + u^2), \end{aligned} \tag{13}$$

since for each electroweak-symmetry violating extremum with $\langle\phi_S\rangle \neq 0$, one can choose the z axis in the weak isospin space so that

$$\langle\phi_S\rangle \sim \begin{pmatrix} 0 \\ v_S \end{pmatrix},$$

with real positive v_S (choosing a "neutral direction" in the weak isospin space).

Neutral extrema.—The solutions of (12) with $u = 0$ are called neutral extrema, as they respect $U(1)$ symmetry of electromagnetism. For these extrema the conditions (12) can be written as a system of two degenerate cubic equations:

$$\begin{aligned} &v_S(-m_{11}^2 + \lambda_1 v_S^2 + \lambda_{345} v_D^2) = 0, \qquad v_S^2 \geq 0, \\ &v_D(-m_{22}^2 + \lambda_2 v_D^2 + \lambda_{345} v_S^2) = 0, \qquad v_D^2 \geq 0. \end{aligned} \tag{14}$$

This system has four solutions. One solution defines electroweak symmetric extremum *EWs* and there are three solutions electroweak-symmetry violating (*EWv*) extrema: inert extremum I_1, inertlike extremum I_2 and mixed extremum M. Below we list their VEVs and extrema energies $\mathcal{E}_a$:

$$\boldsymbol{EWs}\colon v_D = 0, \qquad v_S = 0, \qquad \mathcal{E}_{EWs} = 0; \tag{15}$$

$$\boldsymbol{I_1}\colon v_D = 0, \qquad v_S^2 = \frac{m_{11}^2}{\lambda_1}, \qquad \mathcal{E}_{I_1} = -\frac{m_{11}^4}{8\lambda_1}; \tag{16}$$

$$\boldsymbol{I_2}\colon v_S = 0, \qquad v_D^2 = \frac{m_{22}^2}{\lambda_2}, \qquad \mathcal{E}_{I_2} = -\frac{m_{22}^4}{8\lambda_2}; \tag{17}$$

$$\boldsymbol{M}\colon \begin{cases} v_S^2 = \frac{m_{11}^2\lambda_2 - \lambda_{345} m_{22}^2}{\lambda_1\lambda_2 - \lambda_{345}^2}, \\ v_D^2 = \frac{m_{22}^2\lambda_1 - \lambda_{345} m_{11}^2}{\lambda_1\lambda_2 - \lambda_{345}^2}; \\ \mathcal{E}_M = -\frac{m_{11}^4\lambda_2 - 2\lambda_{345} m_{11}^2 m_{22}^2 + m_{22}^4\lambda_1}{8(\lambda_1\lambda_2 - \lambda_{345}^2)}. \end{cases} \tag{18}$$

Some of Eqs. (16)–(18) can give also negative values of v_S^2 or v_D^2, in contradiction with the basic condition for the extremum (14). In such a case the extremum, described by the corresponding equations, is absent.

The energy differences between $I_{1,2}$ and M extrema are as follows:

$$\begin{aligned} \mathcal{E}_{I_1} - \mathcal{E}_M &= \frac{(m_{11}^2\lambda_{345} - m_{22}^2\lambda_1)^2}{8\lambda_1^2\lambda_2(1 - R^2)}; \\ \mathcal{E}_{I_2} - \mathcal{E}_M &= \frac{(m_{22}^2\lambda_{345} - m_{11}^2\lambda_2)^2}{8\lambda_1\lambda_2^2(1 - R^2)}. \end{aligned} \tag{19}$$

Charge-breaking extremum.—For $u \neq 0$ the extremum violates not only EW symmetry but also the $U(1)$ electromagnetic symmetry, leading to the electric charge nonconservation. According to general analysis in Refs. [4,7,10–12], this extremum can realize vacuum state only if

$$\lambda_4 + \lambda_5 > 0. \tag{20}$$

We will see later on that if this condition is fulfilled the DM particle cannot be neutral, which contradicts (1).

V. VACUUM STATES

Below, we describe briefly the properties of neutral extrema, listed in the previous section, provided in each case that it realizes a true vacuum.

A. Electroweak symmetric vacuum *EWs*

The electroweak symmetric extremum with $\langle\phi_S\rangle = \langle\phi_D\rangle = 0$ exists for all values of parameters of the potential (3). It respects the D and S symmetries of the potential. This extremum is a minimum, realizing vacuum state, at

$$m_{11}^2 < 0, \qquad m_{22}^2 < 0. \tag{21}$$

In this case, gauge bosons and fermions are massless, while scalar doublets ϕ_S and ϕ_D have masses equal to $|m_{11}|/\sqrt{2}$ and $|m_{22}|/\sqrt{2}$, respectively.

B. Inert vacuum I_1

In the case when I_1 extremum realizes vacuum, the inert doublet model describes reality. The standard field decomposition near the I_1 vacuum has the form

$$\phi_S = \begin{pmatrix} G^+ \\ \frac{v + h_S + iG}{\sqrt{2}} \end{pmatrix}, \qquad \phi_D = \begin{pmatrix} D^+ \\ \frac{D_H + iD_A}{\sqrt{2}} \end{pmatrix}, \tag{22}$$

where $G^\pm$ and G are the Goldstone modes, while h_S and $D = D_H, D_A, D^\pm$ are the scalar particles. Here the Higgs particle h_S interacts with the fermions and gauge bosons just as the Higgs boson in the SM. According to our basic assumption on the Yukawa interaction (2), the D particles do not interact with fermions. Neither are there interactions of D particles with gauge bosons V_i of the type $D_j V_1 V_2$.

Symmetry properties.—The inert vacuum state violates the S symmetry (5). However, this state is invariant under the D transformation (6) just as the whole Lagrangian (2). Therefore the D parity is conserved, and due to this fact the lightest D particle is stable, being a good DM candidate.

Allowed region of parameters.—For the inert extremum to exist it is necessary that $m_{11}^2 > 0$ (16). In accordance with (16) and (17), the extremum I_1 can be a vacuum only if $m_{11}^2/\sqrt{\lambda_1} > m_{22}^2/\sqrt{\lambda_2}$. An additional condition arises from a comparison of I_1 and M extrema. In virtue of (19), at $1 - R^2 < 0$ the extremum M can exist but its energy is larger than the energy of the I_1 extremum—so that the extremum I_1 realizes vacuum. At $1 - R^2 > 0$ the inert extremum still can be the vacuum in the case when the mixed extremum does not exist, i.e., if at least one of quantities v_S^2 and v_D^2 defined by Eqs. (18) is negative.

Note that, due to the positivity constraint $1 + R > 0$ (7) in the case when $1 - R^2 < 0$, we have $R > 1$. For the opposite case, i.e., with $1 - R^2 > 0$, the quantity R can be either positive or negative.

Particle properties.—The quadratic part of the potential written in terms of physical fields h_S, D_H, D_A, and $D^\pm$ (22) gives the following masses of scalars:

$$\begin{aligned} M_{h_S}^2 &= \lambda_1 v^2 = m_{11}^2, \\ M_{D^\pm}^2 &= \frac{\lambda_3 v^2 - m_{22}^2}{2}, \\ M_{D_A}^2 &= M_{D^\pm}^2 + \frac{\lambda_4 - \lambda_5}{2} v^2, \\ M_{D_H}^2 &= M_{D^\pm}^2 + \frac{\lambda_4 + \lambda_5}{2} v^2. \end{aligned} \tag{23}$$

The requirement (1) that the lightest D particle is a neutral one results in the condition

$$\lambda_4 + \lambda_5 < 0. \tag{24}$$

Since $\lambda_5 < 0$ (3b), the "scalar" D_H is lighter than "pseudoscalar" D_A.

As in the standard 2HDM, scalars D_H and D_A have opposite P parities, but, since they do not couple to fermions, there is no way to assign to them a definite value of P parity.[3] However, their relative parity does matter, and, for example, vertex ZD_HD_A is allowed while vertices ZD_HD_H and ZD_AD_A are forbidden.

C. Inertlike vacuum I_2

If we compare (16) and (17), the inertlike vacuum I_2 looks "mirror-symmetric" to the inert vacuum I_1. The interactions among scalars and between scalars and gauge bosons in both cases are identical in form with the change $\phi_S \leftrightarrow \phi_D$. The only difference between I_2 and I_1 is given by the Yukawa interaction.

The main formulas for this state are similar to those for the vacuum I_1 with obvious replacements. The corresponding field decomposition is given by

$$\phi_S = \begin{pmatrix} S^+ \\ \frac{S_H + iS_A}{\sqrt{2}} \end{pmatrix}, \qquad \phi_D = \begin{pmatrix} G^+ \\ \frac{v + h_D + iG}{\sqrt{2}} \end{pmatrix}, \tag{25}$$

with one Higgs particle h_D and four S particles: S_H, S_A, and $S^\pm$.

Symmetry properties.—The inertlike vacuum I_2 violates D symmetry (6). This state as well as the Higgs potential is invariant under the S transformation (5). However, in contrast to the D parity in the inert vacuum, S parity is not conserved by the whole Lagrangian because of the form of Yukawa interaction.

Allowed regions of parameters.—The inertlike extremum exists for $m_{22}^2 > 0$. In order to have an inertlike vacuum it is necessary that $m_{11}^2/\sqrt{\lambda_1} < m_{22}^2/\sqrt{\lambda_2}$. For $1 - R^2 < 0$ there are no additional demands. If $1 - R^2 > 0$, the inertlike extremum I_2 can be a vacuum only if at least one of the quantities v_S^2, v_D^2, defined by Eqs. (18), is negative. All these conditions are similar to those for the inert vacuum I_1.

Particle properties.—The masses of the Higgs boson h_D and S scalars are given by [cf. (23)]

$$\begin{aligned} M_{h_D}^2 &= \lambda_2 v^2 = m_{22}^2, \\ M_{S^\pm}^2 &= \frac{\lambda_3 v^2 - m_{11}^2}{2}, \\ M_{S_A}^2 &= M_{S^\pm}^2 + \frac{\lambda_4 - \lambda_5}{2} v^2, \\ M_{S_H}^2 &= M_{S^\pm}^2 + \frac{\lambda_4 + \lambda_5}{2} v^2. \end{aligned} \tag{26}$$

The Higgs boson h_D couples to gauge bosons just as the Higgs boson of the SM; however, it does not couple to fermions at the tree level. The S scalars do interact with fermions. All fermions, by construction interacting only with ϕ_S with vanishing VEV $\langle\phi_S\rangle = 0$, are massless. (Small mass can appear only as a loop effect.)

Here there are no candidates for dark matter particles.

D. Mixed vacuum M

The mixed vacuum M[4] violates both D and S symmetries. In this vacuum we have massive fermions and no candidates for a DM particle, like in the SM. The decomposition around the mixed vacuum looks as follows:

$$\phi_S = \begin{pmatrix} \rho_S^+ \\ \frac{v_S + \rho_S + i\chi_S}{\sqrt{2}} \end{pmatrix}, \qquad \phi_D = \begin{pmatrix} \rho_D^+ \\ \frac{v_D + \rho_D + i\chi_D}{\sqrt{2}} \end{pmatrix}, \tag{27}$$

where the ρ_S^+ and ρ_D^+ lead to two orthogonal combinations G^+ and H^+, while ρ_S and ρ_D (χ_S and χ_D) lead to two orthogonal combinations h and H (G and A), respectively. There are five Higgs bosons—two charged $H^\pm$ and three neutral ones: the CP-even h and H and CP-odd A.

Allowed regions of parameters.—In accordance with (18) and (19) the mixed extremum is the global minimum of potential, i.e., the vacuum, if and only if the following conditions hold: $1 - R^2 > 0$ and $v_S^2 > 0$, $v_D^2 > 0$. For VEVs squared given by Eqs. (18) the latter conditions can be transformed to the relations between mass parameters m_{11}^2 and m_{22}^2:

$$\begin{aligned} &\text{at } 1 > R > 0\text{: } 0 < R\frac{m_{11}^2}{\sqrt{\lambda_1}} < \frac{m_{22}^2}{\sqrt{\lambda_2}} < \frac{m_{11}^2}{R\sqrt{\lambda_1}}; \\ &\text{at } 0 > R > -1\text{: } \frac{m_{22}^2}{\sqrt{\lambda_2}} > R\frac{m_{11}^2}{\sqrt{\lambda_1}}, \frac{m_{22}^2}{\sqrt{\lambda_2}} > \frac{m_{11}^2}{R\sqrt{\lambda_1}}. \end{aligned} \tag{28}$$

[3]Note that the rephasing transformation $\phi_1 \to \phi_1$, $\phi_2 \to i\phi_2$, changing sign of λ_5, results in change $D_H \leftrightarrow D_A$ in the I_1 state.

[4]Sometimes called a normal extremum N; see, e.g., [12].

Particle properties.—Masses of scalars are as follows (see, e.g., [4,6]):

$$M_{H^\pm}^2 = -\frac{\lambda_4 + \lambda_5}{2} v^2, \qquad M_A^2 = -v^2 \lambda_5 \qquad (v^2 = v_S^2 + v_D^2). \tag{29}$$

The neutral CP-even mass matrix is equal to

$$\mathcal{M} = \begin{pmatrix} \lambda_1 v_S^2 & \lambda_{345} v_S v_D \\ \lambda_{345} v_S v_D & \lambda_2 v_D^2 \end{pmatrix}. \tag{30}$$

Note that the extremum M can be minimum only if both diagonal elements of the mass matrix and its determinant are positive, i.e., $\lambda_1 \lambda_2 v_S^2 v_D^2 (1 - R^2) > 0$, in agreement with the above-mentioned conditions. It means also that in the case where the mixed extremum is a minimum, it is the global minimum = the vacuum.

The mass matrix (30) gives masses of the neutral CP-even Higgs bosons:

$$M_{h,H}^2 = \frac{\lambda_1 v_S^2 + \lambda_2 v_D^2 \pm \sqrt{(\lambda_1 v_S^2 + \lambda_2 v_D^2)^2 - 4 \det \mathcal{M}}}{2}, \tag{31}$$

with sign + for the H and sign − for h.

Couplings of the physical Higgs bosons to fermions and gauge bosons have the standard form as for the 2HDM with the model I Yukawa interaction.

VI. EVOLUTION OF PHASE STATES OF THE UNIVERSE

In this section we consider a possible phase history of the Universe, leading to the inert vacuum I_1 today, by using the thermal evolution described in Sec. III.

To summarize properties of different vacua and to classify all possible ways of evolution of the Universe, we will use phase diagrams in the $(\mu_1(T), \mu_2(T))$ plane, where

$$\mu_1(T) = m_{11}^2(T)/\sqrt{\lambda_1}, \qquad \mu_2(T) = m_{22}^2(T)/\sqrt{\lambda_2}. \tag{32}$$

Let us recall (Sec. III) that in our approximation during cooling down of the Universe parameters λ_i are fixed, while mass parameters m_{ii}^2 vary. These variations result in a modification of the vacuum state and a possible change of its nature. Possible types of evolution depend on the value of parameter R (4) and are depicted in Figs. 1–3. The possible current states of the Universe are represented in these figures by small black dots $P = (\mu_1, \mu_2)$.[5] Since currently we are in the inert phase, we have $\mu_1 > 0$ for each possible today point P (see Sec. V B). The parameter μ_2 can be both positive (points $P1$ and $P3$) and negative (points $P2$, $P4$, and $P5$).

[5]We distinguish present-day values of parameters $\mu_i \equiv \mu_i(0)$ and their values $\mu_i(T)$ at some temperature T.

TABLE I. Possible values of parameters and correspondent symmetry of initial Universe.

R	$\tilde{c}$	Initial state of Universe
>0	>0	*EWs*
<0	>0	*EWs*
	<0	*EWv*

In accordance with (9), a particular evolution leading to a given physical vacuum state P is represented by a ray that ends at the point P. Arrows on these rays are directed towards a growth of time (decreasing of temperature). The direction of the ray is determined by parameters [cf. (9)]

$$\tilde{c}_1 = c_1/\sqrt{\lambda_1}, \qquad \tilde{c}_2 = c_2/\sqrt{\lambda_2}, \qquad \tilde{c} = \tilde{c}_2/\tilde{c}_1. \tag{33}$$

According to (9), for $\tilde{c} > 0$ in the initial state of the Universe ($T \to \infty$), $m_{11}^2 < 0$ and $m_{22}^2 < 0$. For this range of parameters the initial phase state of the Universe is electroweak symmetric. If $\tilde{c} < 0$ in the initial state of the Universe, either m_{11}^2 or m_{22}^2 is positive. For these parameters the initial state of the Universe is electroweak-symmetry violating[6]—*EWv*. The correspondence between signs of R and $\tilde{c}$ is given by Eq. (11). The list of opportunities is presented in Table I.

For different possible positions of today's point P, we consider typical evolutions for various possible values of parameter $\tilde{c}$. In the figures below, all representative rays are shown; they are labeled by two numbers, with the first one corresponding to the label of the final point P.

A. The case $R > 1$

The phase diagram for this case is presented in Fig. 1. It contains one quadrant with *EWs* phase and two sectors, describing the I_1 and I_2 phases. These sectors are separated by the phase transition line $\mu_1 = \mu_2$ (thick black line). Two typical positions of today's state are represented by points $P1$ ($\mu_2 > 0$) and $P2$ ($\mu_2 < 0$). Since according to (11) both $\tilde{c}_1$ and $\tilde{c}_2 > 0$ ($\tilde{c} > 0$), all possible phase evolutions are represented by rays 11 and 12 for today's point $P1$ and by a ray 21 which leads to today's point $P2$.

Ray 11: $\tilde{\boldsymbol{c}} > \boldsymbol{\mu_2/\mu_1} > \mathbf{0}$.—The Universe started from the *EWs* state and after the second-order EWSB transition at $m_{11}^2(T) = 0$, i.e., at the temperature

$$T_{EWs,1} = \sqrt{m_{11}^2/c_1} = \sqrt{\mu_1/\tilde{c}_1}, \tag{34}$$

has entered the present inert phase I_1.

[6]Such an opportunity was discussed by a number of authors—see, e.g., [13]. Certainly, it is not ruled out, but it contradicts a key idea of the modern approach—the state at very high temperatures has high symmetry, which is broken at cooling down of the Universe. In this sense this opportunity is unnatural.

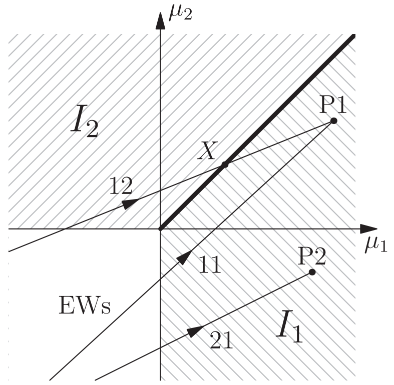

FIG. 1. Phase diagram for the $R > 1$ case.

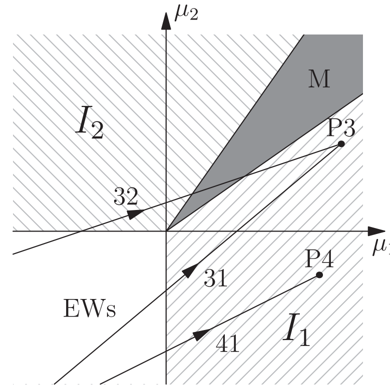

FIG. 2. Phase diagram for the $1 > R > 0$ case.

Ray 12: $\mathbf{0 < \tilde{c} < \mu_2/\mu_1}$.—The Universe started from the *EWs* state. Then it went through the EWSB second-order phase transition into the inertlike phase I_2 at $m_{22}^2(T) = 0$, i.e., at the temperature equal to

$$T_{EWs,2} = \sqrt{m_{22}^2/c_2} = \sqrt{\mu_2/\tilde{c}_2}. \tag{35}$$

The next transition is the phase transition from the inertlike phase I_2 into today's inert phase I_1 at the point X, where $\mu_2(T) = \mu_1(T)$, i.e., at the temperature

$$T_{2,1} = \sqrt{\frac{\mu_1 - \mu_2}{\tilde{c}_1 - \tilde{c}_2}}. \tag{36}$$

That is the first-order phase transition with the latent heat given by

$$\begin{aligned} Q_{I_2 \to I_1} &= T\frac{\partial \mathcal{E}_{I_2}}{\partial T} - T\frac{\partial \mathcal{E}_{I_1}}{\partial T}\bigg|_{\mu_2(T) \to \mu_1(T)} \\ &= (\mu_2 \tilde{c}_1 - \mu_1 \tilde{c}_2) T_{2,1}^2/4. \end{aligned} \tag{37}$$

Ray 21: $\boldsymbol{\mu_2 < 0}$.—The Universe started from the *EWs* state and after the second-order EWSB transition at the temperature (34) has entered today's I_1 phase.

B. The case $1 > R > 0$

The phase diagram for this case is presented in Fig. 2. As compared to the previous case (Fig. 1), in the upper right quadrant the new sector with the mixed phase M appears, described in accordance with (28) by the equation

$$0 < R\mu_1 < \mu_2 < \mu_1/R. \tag{38}$$

As before, since $R > 0$, we have $\tilde{c} > 0$.

Since currently we are in the inert vacuum, possible today's states are of the type of points $P3$ and $P4$, for which

$$\mu_2 < R\mu_1. \tag{39}$$

All possible phase evolutions are represented in Fig. 2 by three rays, with rays 31 and 32 having today's end point $P3$ while ray 41 is pointing at $P4$.

For rays 31 and 41 phase evolutions are as for rays 11 and 21, respectively. A new situation appears for ray 32.

Ray 32: $\mathbf{0 < \tilde{c} < \mu_2/\mu_1}$.—The Universe started from the *EWs* state. Then at the temperature given by (35) it went through the EWSB second-order phase transition into the inertlike phase I_2. At the subsequent cooling down, the Universe goes through the mixed phase M into the present inert phase I_1. The second-order phase transitions $I_2 \to M$ and $M \to I_1$ happened at the following temperatures:

$$\begin{aligned} T_{\text{phtr}}\colon T_{2,M} &= \sqrt{(\mu_1 - R\mu_2)/(\tilde{c}_1 - R\tilde{c}_2)}, \\ T_{M,1} &= \sqrt{(R\mu_1 - \mu_2)/(R\tilde{c}_1 - \tilde{c}_2)}. \end{aligned} \tag{40}$$

In accordance with equations in Sec. V, at the transition point $I_2 \to M$ masses of S_H and h vanish, while at the transition point $M \to I_1$ masses of h and D_H become 0. At a small distance from the transition point with the temperature T_{phtr}, these masses grow as a function of the temperature T as $M_a^2 = A_a|T^2 - T_{\text{phtr}}^2|$, with different coefficients A_a.

C. The case $0 > R > -1$

The phase diagram is presented in Fig. 3. In this case, as follows from (28), the mixed phase M is realized in a wider region than for the case $0 < R < 1$ (Fig. 2), even beyond an upper right quadrant of this plane,[7] namely,

$$\mu_2 > \mu_1/R, \qquad \mu_2 > \mu_1 R. \tag{41}$$

[7] This case was overlooked in the literature; see, e.g., [1] and also [2]. We thank G. Gil and B. Gorczyca for discussion on this point.

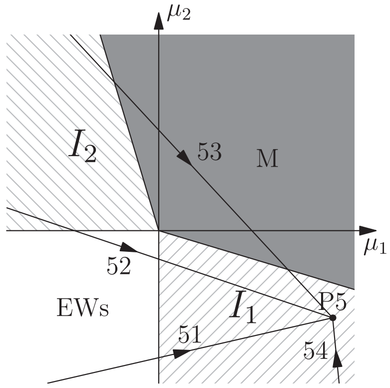

FIG. 3. Phase diagram for the $0 > R > -1$ case.

Since currently we are in the inert vacuum ($\mu_1 > 0$), for today's point $P5$ we have

$$\mu_2 < R\mu_1 \qquad (\mu_2 < 0). \tag{42}$$

New opportunities appear due to a larger freedom for temperature coefficients c_i, as in accordance with (11) in this case $\tilde{c}$ can be negative.

All possible phase evolutions leading to the point $P5$ are represented in Fig. 3 by four rays: 51, 52, 53, and 54.

Ray 51 describes similar evolution as rays 21 and 41. New are rays 52, 53, and 54 with the common feature $\tilde{c} < 0$, resulting in a lack of the electroweak symmetry in the very early stages of the Universe (see Table I).

Ray 52: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} > \mu_2/\mu_1$.—Here a high-temperature state of the Universe is the inertlike vacuum I_2. With cooling down the Universe went through electroweak symmetric phase *EWs* into the present I_1 phase. The second-order phase transitions $I_2 \rightarrow EWs$ and $EWs \rightarrow I_1$ happened, respectively, at the temperatures

$$T_{2,EWs} = \sqrt{\mu_2/\tilde{c}_2}, \qquad T_{EWs,1} = \sqrt{\mu_1/\tilde{c}_1}. \tag{43}$$

Ray 53: $\tilde{c}_1 > 0$, $\tilde{c}_2 < 0$, $\tilde{c} < \mu_2/\mu_1$.—Here a high-temperature state of the Universe is an inertlike vacuum I_2. With cooling down the Universe passed through the mixed phase M into the present I_1 phase. The phase transitions $I_2 \rightarrow M$ and $M \rightarrow I_1$ are of the second order; they happened at the temperatures given by Eqs. (40).

Ray 54: $\tilde{c}_1 < 0$, $\tilde{c}_2 > 0$.—For this ray the Universe stays in the inert vacuum I_1 during the whole evolution.

D. Summary of possible evolutions

We find that in the considered approximation the thermal evolution of the Universe to the current inert phase can be studied effectively in the (μ_1, μ_2) plane, at fixed values of quartic parameters λ_i. Different types of such evolution, represented as the directed rays, depend crucially on two parameters: R (4), describing the allowed-for various vacua regions of the (μ_1, μ_2) plane, and $\tilde{c}$ (33), determining the direction of rays. The first one depends only on ratios between the coefficients of the quartic part of potential λ_i. The second one depends both on mentioned parameters λ_i, which are unknown up to now, and on precisely known gauge and Yukawa couplings.

One can distinguish the following types of evolution to the current inert phase:

(I) The Universe evolves from the initial electroweak symmetric state.

(a) The simplest evolution to the inert phase is realized through a single EWSB phase transition of the second-order type (rays 11, 21, 31, 41, and 51). Dark matter appears at this single transition simultaneously with EWSB.

(b) After the first EWSB phase transition, the Universe passes into the inertlike phase I_2. Then it passes into the inert phase I_1 either directly (the first-order phase transition, ray 12) or through the mixed phase M (two second-order phase transitions, ray 32). In both cases dark matter appears only after the last phase transition to the inert phase.

(II) The Universe evolves from the initial state having no electroweak symmetry.

(a) The initial phase is the inert one I_1. The evolution contains no phase transitions. Dark matter existed always (ray 54).

(b) The initial phase is the inertlike one I_2. It contains no dark matter. Evolution to the current inert phase I_1 undergoes through two second-order phase transitions:

(i) either via the mixed phase M (ray 53)

(ii) or via the *EWs* phase, i.e., with a *temporary* appearance of electroweak symmetry (ray 52).

In both cases dark matter appears only after the last phase transition to the inert phase I_1.

Each of these evolutions can be realized in a wide range of parameters.

VII. RESULTS AND DISCUSSION

Main results.—The most important observation we made in this paper is as follows: If the current state of the Universe is described by the IDM, then during the thermal evolution the Universe could pass through various intermediate phases, different from the inert one. These possible intermediate phases contained no dark matter, which appeared only at the relatively late stage of cooling down of the Universe.

A complete set of possible ways of evolution of the Universe, including both EW symmetric (*EWs*) and EW nonsymmetric (EW violating, *EWv*) initial states, can be summarized as follows (symbol I or II over arrows corresponds to the type of phase transition):

$$EWs \xrightarrow{\mathrm{II}} \begin{cases} I_1 & \text{(rays 11, 21, 31, 41, 51)} \\ I_2 & \begin{cases} \xrightarrow{\mathrm{II}} M \xrightarrow{\mathrm{II}} I_1 & \text{(ray 32)} \\ \xrightarrow{\mathrm{I}} I_1 & \text{(ray 12),} \end{cases} \end{cases} \tag{44}$$

$$EWv: \quad \begin{matrix} I_2 \xrightarrow{\mathrm{II}} & \begin{cases} EWs \xrightarrow{\mathrm{II}} I_1 & \text{(ray 52)} \\ M \xrightarrow{\mathrm{II}} I_1 & \text{(ray 53)} \end{cases} \\ I_1 \to I_1 & \qquad\qquad \text{(ray 54).} \end{matrix}$$

We see that a simple EWSB with a direct transition to the inert phase, as well as sequences of two or three transitions from *EWs* to the inert phase, is possible. We found also that the current inert state of the Universe can be obtained both from the initial high-temperature state with EW symmetry and from the initial state without this symmetry.

As far as the charged DM is concerned, still in principle allowed by the data, if heavy enough, it seems to be excluded in the IDM (see the appendix).

To find what scenario of evolution is realized in nature, one should measure all parameters of potential. The program to measure these parameters at the LHC and the International Linear Collider is under preparation.

Outlook.

A. Extra phase transitions at lower than EWSB temperature (and especially first-order phase transition at the evolution represented by ray 12) can influence baryogenesis even stronger than transformation of the standard second-order EWSB transition into the first-order one due to term $\phi^3 T$ [14]. Moreover, in contrast to the standard picture, the considered scenarios allow for the phase transition to the current inert phase at a relatively low temperature, giving a new starting point for calculation of today's abundance of the neutral DM components of the Universe and other phenomena.

B. In this paper we calculated thermal evolution of the Universe in the very high-temperature approximation, i.e., for $T^2 \gg |m_{ii}^2|$. The most interesting effects are expected at lower temperatures, where more precise calculations are necessary. The simplest expected modifications of the presented description are as follows:

(1) Appearance of cubic terms like $\phi^3 T$ [14].—These terms are important near the phase transition point, as they can transform some second-order phase transition into the first-order transition.

(2) The parameters become dependent on temperature in a more complicated way than that given by (9). Therefore, the rays depicting thermal evolutions in Figs. 1–3 can become nonstraight. The bending of these rays can be different in different points of our plots and at different λ_i. It can give a possible spectrum of phase evolutions even reacher than discussed above.

However, we expect that the general picture will not change too much.

Possible extensions of the model.—The model we focused on in this paper contains two doublets. One can consider a similar model with three doublets (for the particular case of a three-Higgs doublet model), as is discussed in Ref. [15]. It contains two standard Higgs doublets ϕ_{S1} and ϕ_{S2}, coupled to fermions, and one Higgs doublet ϕ_D having no coupling to fermions. The potential is invariant under S and D transformations:

$$\begin{aligned} &\{\phi_{S1}, \phi_{S2}\} \xrightarrow{S} -\{\phi_{S1}, \phi_{S2}\}, \qquad \phi_D \xrightarrow{S} \phi_D, \\ &\{\phi_{S1}, \phi_{S2}\} \xrightarrow{D} \{\phi_{S1}, \phi_{S2}\}, \qquad \phi_D \xrightarrow{D} -\phi_D, \\ &\qquad \mathrm{SM} \xrightarrow{S} \mathrm{SM}, \qquad \mathrm{SM} \xrightarrow{D} \mathrm{SM}. \end{aligned} \tag{45}$$

This model can incorporate all phenomena which appear in the standard 2HDM at the same time it contains DM particles as in the IDM. Thermal evolution of its parameters gives a very diverse phase diagram, which contains in addition to the phases discussed above other phases, discussed in Refs. [4,5,9], and their mixtures. However, even for this model our main conclusion about possible transformation of the Universe through the phase without DM holds.

Reacher variants of both S and D sectors can be considered similarly. For example, one widely discussed model of this type (see, e.g., [16]) contains the same S sector as in our paper, but the D sector consists of one doublet and one singlet scalar, noninteracting with fermions. This model has phenomenology similar to that discussed above, but one hopes to derive more or less natural values of couplings starting from $SO(10)$ universality at the grand unified theory scale. The biography of the Universe in this model can be studied as above; obviously, it will give a more diverse phase story.

ACKNOWLEDGMENTS

We are thankful to I. Ivanov, M. Dubinin, and R. Nevzorov for useful discussions. M. K. and D. S. thank Grzegorz Gil and Bogumiła Gorczyca for important clarification, as well as Piotr Chankowski for useful comments. This work was partly supported by Polish Ministry of Science and Higher Education Grant No. N202 230337. The work of M. K. and D. S. was supported in addition by EU Marie Curie Research Training Network HEPTOOLS, under Contract No. MRTN-CT-2006-035505, FLAVIAnet Contract No. MRTN-CT-2006-035482. The work of I. G. and K. K. was also supported by Grants No. RFBR 08-02-00334-a and No. NSh-3810.2010.2 and Program of Department of Physical Sciences RAS "Experimental and theoretical studies of fundamental interactions related to LHC."

APPENDIX: IF DM IS CHARGED

Model-independent analysis shows that the case with a charged DM particle is not ruled out absolutely, but charged DM particles must be heavier than $100q$ TeV, where q is electric charge of the DM particle in units of electron charge [17]. For the IDM it means $M_{H^\pm} > 100$ TeV. Such a heavy mass seems to be unnatural in the modern particle physics with natural energy scale $\lesssim 1$ TeV. Obviously, such an opportunity cannot be tested at colliders in the estimable future.

The case with a charged DM particle can be realized in the IDM only if $\lambda_4 + \lambda_5 > 0$ (23). Since perturbativity constraints $|\lambda_i| \lesssim 8\pi$ must hold (see, for example, [6]), a very large $D^\pm$ mass can arise only from a very large negative m_{22}^2 (23). The position on the (μ_1, μ_2) plane of the actual state of the Universe, for the anticipated in the SM value of the Higgs mass $M_{h_S} \lesssim 200$ GeV and $M_{D^\pm} \gtrsim 100$ TeV, corresponds to $\mu_1 > 0$, $\mu_2 < 0$, with a large ratio of their absolute values $\gtrsim 10^5$–10^6, if $\lambda_1/\lambda_2 \sim 1$.

According to Refs. [4,7,10], if $\lambda_4 + \lambda_5 > 0$, then the mixed phase M cannot exist [see, e.g., (29)] while the charge-breaking phase can. The charge-breaking vacuum can be realized if in addition $|R_3| < 1$, where $R_3 = \lambda_3/\sqrt{\lambda_1\lambda_2}$. Simple analysis shows that the phase diagrams for this case are similar to those in Figs. 2 and 3, with the replacement $R \to R_3$. Rays similar to rays 41, 51, 52, and 54 give nothing new in comparison with the cases discussed in Sec. III.

The really new opportunity could appear for the ray similar to ray 53 in Fig. 3 (with rays going through the charge-breaking vacuum). However, this opportunity is ruled out. Indeed, to realize it one needs $c_2 < 0$ and $|c_2|/c_1 > |m_{22}^2|/m_{11}^2 \gtrsim (10^5$–$10^6)$. The latter inequality contradicts the $c_1 > -c_2$ relation (10) based on the positivity condition. It means that in our simple model Universe evolution to the current inert phase cannot pass through the charge-breaking phase.

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