Constituent quark masses obtained from hadron masses with contributions of Fermi-Breit and Glozman-Riska hyperfine interactions

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We use the color-spin and flavor-spin interaction Hamiltonians with SU(3) flavor symmetry breaking to obtain meson and baryon mass formulas. Adjusting these masses with experimental masses we determine the constituent quark masses. We discuss the constituent quark masses obtained from meson and baryon mass fits. The results for constituent quark masses are very similar in the case of two different phenomenological models: Fermi-Breit and Glozman-Riska hyperfine interactions.

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I. INTRODUCTION

The quark model predicts that the masses of the mesons and baryons are given by the sum of the constituent masses of the quarks and the hyperfine splitting [1–3]. As described in [1], the term with hyperfine splitting emanates from a spin-spin interaction produced by one gluon exchange, which also depends on the masses of the quarks (for mesons this term contains a mass sum of a quark pair and for baryons it contains a sum over all the possible quark pairs).

In Ref. [4] the baryons were studied in a quark model with color hyperfine interactions. Some parameters of meson and baryon spectroscopy, including some predictions of the quark masses, can be found in [5]. In Ref. [6], from the meson and baryon relations, the authors fitted both the mass differences and mass ratios with a single set of quark masses. In [7] estimations of masses were made via magnetic moments of baryons and via the vector meson masses. In Refs. [8,9] it was shown that, in the constituent quark model, the Feynman-Hellmann theorem and semiempirical mass formulas can be applied to give useful information about the masses of mesons and baryons.

In this paper we use a schematic study (two-quark interaction) to calculate meson and baryon theoretical masses from which the constituent quark masses can be obtained. In Sec. II we give the complete method of how to calculate theoretical masses; in Sec. III we explain the fitting procedure and our main results; discussion and conclusions are given in Sec. IV.

II. CONTRIBUTION OF THE HYPERFINE INTERACTIONS

Using the colored version of the Fermi-Breit hyperfine interaction (FB HFI) [10–13] and the Glozman-Riska hyperfine interaction (GR HFI) [14–16], we present estimations of the theoretical meson and baryon masses. A strong FB HFI Hamiltonian [13] has the following form:

$$H_{\rm FB} = C\sum_{i>j}\frac{\vec{\sigma}_i\vec{\sigma}_j}{m_i m_j}(\lambda_i^C\lambda_j^C), \qquad (1)$$

and it has explicit color and spin exchange dependence and implicit (by way of quark masses) flavor dependence. Here σ_i are the Pauli spin matrices, λ_i^C are the color Gell-Mann matrices, and C is a constant. The FB contribution to hadron masses is given by $m_{\nu,\rm FB} = \langle\nu|\langle\chi|H_{\rm FB}|\chi\rangle|\nu\rangle$, where χ denotes the spin wave function and ν is the flavor wave function. The strong GR Hamiltonian [14] is

$$H_{\rm GR} = -C_\chi\sum_{i<j}(-1)^{\alpha_{ij}}(\lambda_i^F\lambda_j^F)\left(\frac{\vec{\sigma}_i\vec{\sigma}_j}{m_i m_j}\right),$$
$$(-1)^{\alpha_{ij}} = \left\{\begin{matrix} -1, & q\bar{q} \\ +1, & qq \text{ or } \bar{q}\,\bar{q}\end{matrix}\right\}, \qquad (2)$$

where λ_i^F are Gell-Mann matrices for flavor SU(3), σ_i are the Pauli spin matrices, and C_χ is a constant. We employ this schematic flavor-spin interaction between quarks and antiquarks which leads to a Glozman-Riska HFI contribution to hadron masses: $m_{\nu,\rm GR} = \langle\nu|\langle\chi|H_{\rm GR}|\chi\rangle|\nu\rangle$, where m_i are the constituent quark effective masses: $m_{\rm u} = m_d \neq m_{\rm s}$ and ν—flavor wave functions.

For mesons, there are two flavor SU(3) multiplets according to product: $3\otimes\bar{3} = 1 + 8$, i.e. one singlet and one octet. For baryons, according to $3\otimes 3\otimes 3 = 1 + 8_{\rm MS} + 8_{\rm MA} + 10$, there is one singlet, two octets [one mixed symmetric (MS) and one mixed antisymmetric (MA)] and one decuplet. Young diagrams for these $SU(3)_{\rm F}$ multiplets, as well as the weight diagrams, can be found in textbooks (see e.g. [17,18]). Regarding baryons, in the case of octet members (as they have the MS and MA part of the flavor wave function), the contribution of HFI is

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calculated as the sum of the symmetric and the antisymmetric part, so we start from the following expression: $|\nu\uparrow\rangle = \frac{1}{\sqrt{2}}|\nu_{\mathrm{MS}} + \nu_{\mathrm{MA}}\rangle$.

III. FITTING PROCEDURE

For our calculations we used multidimensional least-square fit of masses, using subroutine "lfit" from Numerical Recipes in FORTRAN [19], modified according to the instructions in the last paragraph of Sec. 15.4 of [19]. We fitted simultaneously all parameters, m_u, m_s, m_c, and the constant C, in such a way that we minimized χ^2 between the measured and the theoretical masses. Equations for meson and baryon theoretical masses (see Sec. IV) are at first linearized by expansion in Taylor's series up to the first order, and in that way we obtained the corresponding system of linear equations for differences between experimental and theoretical masses in which the unknown variables are corrections of the parameters. These corrections, as well as their corresponding uncertainties, are determined using linear least-square method, and then the parameters are accounted for the values of these obtained corrections. With these new (corrected) parameters the previous procedure is repeated until the fit converges, that is while the χ^2 value between the experimental and theoretical masses decreases. In that way, after several iterations, we obtain final values for the parameters and their uncertainties. The uncertainties are estimated during the fitting procedure as square roots of the corresponding diagonal elements of covariance matrix, according to Eq. (15.4.15) of Ref. [19].

For every analyzed system of equations a fast convergence is obtained, even in the case when starting values of parameters differ very much from their final values, which favors the correctness of our theoretical model as well as our fitting method.

IV. RESULTS AND DISCUSSION

We calculated FB and GR contributions to meson and baryon masses. The mass formulas of mesons and baryons with FB HFI included are given in Eqs. (3) and (4), while their mass formulas with GR HFI included can be found in [15] [Eqs. (6)–(10) for mesons and Eqs. (11)–(13) for baryons].

Here we study the hadron masses using FB and GR HFI in schematic approximation (two-particle interaction). The masses of constituent quarks $m_u (= m_d)$, m_s, m_c, and the constants C^m and C^b are calculated from the least-square fit of the theoretical equations for meson and baryon masses with HFI included. The corresponding experimental masses are taken from the Particle Data Group site [20].

We give the equations for meson masses with FB interaction included for the following mesons: light pseudoscalar mesons π, K, light vector mesons ρ, K^*, ω, φ, charmed mesons D, D^*, strange charmed mesons D_S, D_S^*, and double charmed mesons η_c, J/ψ [Eq. (3)]. In these equations, the constant for mesons is denoted by C^m. We did not calculate the η and η' contribution because of their mixing and they cannot be described within such a model. Because of that mixing (due to the same quantum numbers), their flavor wave functions are given only in a first approximation [21] and therefore calculations are not sufficiently precise. The mixing of the states also changes the properties and shifts masses from the theoretical predictions.

$$\begin{aligned}
&m_\pi = 2m_u - \frac{3C^m}{m_u^2}; \qquad m_K = m_u + m_s - \frac{3C^m}{m_u m_s}\\
&m_\rho = 2m_u + \frac{C^m}{m_u^2}; \qquad m_{K*} = m_u + m_s + \frac{C^m}{m_u m_s}\\
&m_\omega = 2m_u + \frac{C^m}{m_u^2}; \qquad m_\varphi = 2m_s + \frac{C^m}{m_s^2}\\
&m_D = m_u + m_c - \frac{3C^m}{m_u m_c}; \qquad m_{D*} = m_u + m_c + \frac{C^m}{m_u m_c}\\
&m_{Ds} = m_s + m_c - \frac{3C^m}{m_s m_c}; \qquad m_{Ds*} = m_s + m_c + \frac{C^m}{m_s m_c}\\
&m_{\eta c} = 2m_c - \frac{3C^m}{m_c^2}; \qquad m_{J/\psi} = 2m_c + \frac{C^m}{m_c^2}. \qquad (3)
\end{aligned}$$

We also present the theoretical mass equations with FB HFI included for the following baryons: light baryon octet N, Σ, Ξ, Λ, light baryon decuplet Δ, Σ^*, Ξ^*, Ω, and heavy baryons Σ_c, Ξ_{cc}, Λ_c, Σ_c^*, Ω_c [Eq. (4)]. The constant for baryons is denoted by C^b.

TABLE I. Constituent quark masses obtained from the meson fits. For each combination of mesons, the results are obtained from their masses with FB HFI included (upper rows) and with GR HFI (lower rows). Note that constant C^m differs for the two HFIs.

Fit No.	Mesons	Quark masses (MeV) $m_u = m_d$	m_s	m_c	C^m ($\times 10^7$ MeV^3)	χ^2 ($\times 10^6$)
1	π, K, ρ, K^*, ω, φ, D	318.12 ± 0.11	477.29 ± 0.21	1591.42 ± 0.21	1.5977 ± 0.0017	0.095 336
	D^*, D_S, D_S^*, η_c, J/ψ	304.55 ± 0.11	534.33 ± 0.19	1576.94 ± 0.21	2.1331 ± 0.2346	0.304 832
2	π, K, ρ, K^*, ω, φ	308.05 ± 0.12	484.91 ± 0.24	⋯	1.5043 ± 0.0017	0.000 411
		292.35 ± 0.12	552.80 ± 0.22	⋯	2.0284 ± 0.0023	0.143 289
3	D, D^*, D_S, D_S^*	454.75 ± 0.47	547.48 ± 0.49	1524.36 ± 0.35	2.6936 ± 0.0095	0.003 475
	η_c, J/ψ	453.68 ± 0.44	546.82 ± 0.49	1524.94 ± 0.35	4.0280 ± 0.0138	0.003 480

$$
\begin{aligned}
m_N &= 3m_u - 3C^b \frac{1}{2m_u^2} \\
m_\Sigma &= 2m_u + m_s + 2C^b \frac{1}{m_u^2}\left(\frac{1}{4} - \frac{m_u}{m_s}\right) \\
m_\Xi &= m_u + 2m_s + 2C^b \frac{1}{m_s^2}\left(\frac{1}{4} - \frac{m_s}{m_u}\right) \\
m_\Lambda &= 2m_u + m_s - 3C^b \frac{1}{2m_u^2} \\
m_\Delta &= 3m_u + 3C^b \frac{1}{2m_u^2} \\
m_{\Sigma*} &= 2m_u + m_s + C^b \frac{1}{m_u^2}\left(\frac{1}{2} + \frac{m_u}{m_s}\right) \\
m_{\Xi*} &= m_u + 2m_s + C^b \frac{1}{m_s^2}\left(\frac{1}{2} + \frac{m_s}{m_u}\right) \qquad (4) \\
m_\Omega &= 3m_s + 3C^b \frac{1}{2m_s^2} \\
m_{\Sigma c} &= 2m_u + m_c + 2C^b \frac{1}{m_u^2}\left(\frac{1}{4} - \frac{m_u}{m_c}\right) \\
m_{\Xi cc} &= m_u + 2m_c + 2C^b \frac{1}{m_c^2}\left(\frac{1}{4} - \frac{m_c}{m_u}\right) \\
m_{\Lambda c} &= 2m_u + m_c - 3C^b \frac{1}{2m_u^2} \\
m_{\Sigma * c} &= 2m_u + m_c + C^b \frac{1}{m_u^2}\left(\frac{1}{2} + \frac{m_u}{m_c}\right) \\
m_{\Omega c} &= 2m_s + m_c + C^b \frac{1}{m_s^2}\left(\frac{1}{2} + \frac{m_s}{m_c}\right).
\end{aligned}
$$

For each set of equations, the minimized χ^2 values for masses are calculated by the formula $\chi^2 = \sum_{i=1}^{N}[(T_i - E_i)^2/\sigma_i^2]$, where T_i is the model prediction for the hadron mass, E_i the experimental hadron mass, and σ_i the uncertainty of the mass.

The values of constituent quark masses and their uncertainties (in MeV) are given in Tables I and II. It is noticeable that the fits are satisfactory for both HFIs and in the cases of all mesons, light mesons, and light baryons. They show that the constituent mass is only slightly modified by the dynamics of confinement. The constituent masses differ for the quarks in mesons and those in baryons. This is not unexpected since they are confined in quite different systems. Also, as noted in [8], there are no theoretical reasons why the masses determined from the baryon fits should coincide exactly with those determined from the meson fits because these quark masses are constitutive masses, i.e. effective ones. Roncaglia *et al.* [8,9] also showed that constraints (inequalities) for mass differences are stronger for baryons than for mesons, and therefore the requirement for the same set of quark masses for both baryons and mesons would result in significantly poorer fits [8].

In the case of heavy mesons, the fit resulted in somewhat increased values for u and s quarks, and in the case of heavy baryons fit even did not converge. Therefore, our results suggest that both of the analyzed HFIs can be used in a first approximation for modeling the term with hyperfine splitting in the quark model only for light mesons and light baryons. That is, both HFIs well describe the systems which contain quarks from $SU(3)_F$, i.e. light quarks: u, d, s. In order to verify this conclusion, we also show the results for the case when hadrons with two c quarks are excluded (see Table III): meson fit without η_c and J/ψ, and baryon fit without Ξ_{cc}. From these two fits we can calculate mass differences $m_s - m_u$ and $m_c - m_s$ which may be then compared with the corresponding inequalities given by Roncaglia *et al.* [8,9], obtained from the Feynman-Hellmann theorem. We can see from Table III that in this case the constituent quark masses obtained for FB HFI satisfy all of the inequalities given in [8,9], contrary to Tables I and II, where FB HFI breaks some of these inequalities. Also, from Table III one can see that GR HFI violates one or more of the inequalities, but to a lesser extent than in Tables I and II. Even more, in this case the obtained constituent quark masses are more realistic than those from Tables I and II. From each set of equations (except for heavy baryons) we calculated masses of constituent quarks and constants C^m and C^b, and we got slightly different values, but all of them are from expected mass ranges of constituent quarks. As one can see from Table IV, our predictions for constituent quark masses are in the range of masses obtained using different phenomenological models. When comparing these two HFIs, it is interesting to note that the values of obtained quark masses are similar although one interaction is color-spin, and the other one is flavor-spin. As seen from Tables I and II, two different HFIs result with masses of constituent quarks which are more similar than those obtained by the same HFIs applied to different groups of hadrons (e.g. light

TABLE II. The same as Table I, but for baryon fits. Note that for heavy baryons fit did not converge and constitutive quark masses could not be obtained.

Fit. No.	Baryons	Quark masses (MeV) $m_u = m_d$	m_s	m_c	C^b ($\times 10^7$ MeV^3)	χ^2 ($\times 10^6$)
1	N, Σ, Ξ, Λ, Δ, Σ^*, Ξ^*	381.53 ± 0.09	537.26 ± 0.18	1368.15 ± 0.27	1.3454 ± 0.0021	0.153 491
	Ω, Σ_c, Ξ_{cc}, Λ_c, Σ_c^*, Ω_c	541.04 ± 0.18	655.00 ± 0.22	1358.62 ± 0.25	2.4878 ± 0.0047	0.125 597
2	N, Σ, Ξ, Λ, Δ, Σ^*, Ξ^*	362.52 ± 0.10	538.68 ± 0.20	$\cdots$	1.2789 ± 0.0020	0.001 165
	Ω	500.42 ± 0.20	621.20 ± 0.25	$\cdots$	1.6951 ± 0.0043	0.002 719

TABLE III. Constituent quark masses obtained from hadron fits when hadrons with two c-quarks are excluded. The upper rows correspond to FB HFI, and the lower rows to GR HFI.

	Hadrons	Quark masses (MeV) $m_u = m_d$	m_s	m_c	C ($\times 10^7$ MeV^3)	χ^2 ($\times 10^6$)
Mesons	π, K, ρ, K^*, ω, φ, D	314.75 ± 0.12	466.80 ± 0.21	1627.31 ± 0.25	1.5546 ± 0.0017	0.033 123
	D^*, D_S, D_S^*, Λ_c, Σ_c^*, Ω_c	300.88 ± 0.11	526.65 ± 0.19	1605.62 ± 0.26	2.0660 ± 0.0023	0.263 221
Baryons	N, Σ, Ξ, Λ, Δ, Σ^*, Ξ^*	365.69 ± 0.09	530.08 ± 0.18	1700.17 ± 0.37	1.2513 ± 0.0019	0.015 451
	Ω, Σ_c, Λ_c, Σ_c^*, Ω_c	506.26 ± 0.19	623.40 ± 0.22	1629.69 ± 0.35	1.8163 ± 0.0041	0.032 378

TABLE IV. Predicted masses (in MeV) of constituent quarks in different phenomenological models.

Mass	Ref. [5]	Ref. [8]	Ref. [1]	Ref. [22]	Ref. [6]	Ref. [7]	Ref. [15]
$m_u = m_d$	220	300	310	290	360	337.5	311
m_s	419	475	483	460	540	486	487
m_c	1628	1640	$\cdots$	1650	1710	1550	1592

mesons, heavy mesons, light baryons, ...). For example, results for $m_u (= m_d)$ and m_s for FB HFI applied on light and heavy mesons are more different than results for $m_u (= m_d)$ and m_s for FB and GR HFIs applied on light mesons.

When comparing FB, the color-spin interaction, with GR HFI, which is a flavor-spin interaction, it is interesting that we obtained similar results. We can conclude that both HFIs describe masses of light mesons and baryons very well. It shows that both HFIs well describe systems which contain quarks from $SU(3)_F$. In the case of heavy baryons, the fits for both HFIs did not converge, which might indicate that the quark model which includes these two HFIs is not a satisfactory approximation for heavy baryons. It might also indicate that FB and GR HFI are not the complete effective two-quark interactions and therefore they cannot be successfully applied to quark systems out of the SU(3) group. If we do not take into account hadrons with two c quarks, the FB HFI becomes a good approximation, even for hadrons having one heavy c quark. We also show that the constituent quark masses are very sensitive to the system in which they are confined, and their values differ less or more in different systems (for example, heavy mesons and light mesons). We obtained that the least-square fit is more accurate for FB than for GR interaction in the case of light mesons, but in the case of heavy mesons and light baryons accuracy of the fit is similar.

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