Patterns of dynamical gauge symmetry breaking

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We construct and analyze theories with a gauge symmetry in the ultraviolet of the form $G \otimes G_b$, in which the vectorial, asymptotically free G_b gauge interaction becomes strongly coupled at a scale where the G interaction is weakly coupled and produces bilinear fermion condensates that dynamically break the G symmetry. Comparisons are given between Higgs and dynamical symmetry-breaking mechanisms for various models.

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I. INTRODUCTION

An outstanding question at present concerns the origin of electroweak symmetry breaking (EWSB), in which the electroweak gauge symmetry of the standard model (SM), based on the gauge group $G_{\rm EW} = {\rm SU}(2)_L \otimes {\rm U}(1)_Y$, where ${\rm SU}(2)_L$ and ${\rm U}(1)_Y$ are the factor groups for weak isospin and hypercharge, is broken to the electromagnetic ${\rm U}(1)_{\rm em}$ subgroup. The standard model hypothesizes that this symmetry breaking is due to the vacuum expectation value (VEV) of a fundamental Higgs field that transforms as $T = 1/2$ and $Y = 1$. Similarly, the minimal supersymmetric standard model attributes electroweak symmetry breaking to nonzero VEVs of the (scalar components of) two Higgs chiral superfields with $T = 1/2$ and $Y = \pm 1$. A rather different approach is taken by technicolor (TC) theories. In these, the vectorial, asymptotically free technicolor gauge interaction becomes strongly coupled at the TeV scale, producing condensates of technifermions that break $G_{\rm EW}$ to ${\rm U}(1)_{\rm em}$. Other possibilities have also been studied, such as electroweak symmetry breaking due to boundary conditions on gauge fields in higher dimensions. Experiments at the Large Hadron Collider (LHC) are currently underway to answer the question of the origin of electroweak symmetry breaking.

In general, a comparative study of Higgs-type and dynamical approaches to the breaking of gauge symmetries gives insights into both of these approaches. In this paper we shall carry out such a study. We shall consider a class of gauge theories with a direct-product gauge symmetry of the Lagrangian, of the form

$$G_{\rm UV} = G \otimes G_b, \tag{1.1}$$

such that as the theory evolves from some high energy scale to lower energies, the G_b interaction becomes strongly coupled at a scale Λ_b, where the G interaction is weakly coupled, and produces bilinear fermion condensates that transform as nonsinglets under G and hence dynamically break the G symmetry to a subgroup $H \subset G$, i.e.,

$$G \to H \quad \text{induced by } G_b. \tag{1.2}$$

(The subscript b on G_b and Λ_b refers to their roles in the breaking of G.) The condition that the G interaction is weakly coupled at the scale Λ_b is similar to the fact that the electroweak interaction is weakly coupled at the scale $2^{-1/4} G_F^{-1/2} \simeq 250$ GeV where it is broken. However, our study is not an attempt to construct a semirealistic theory of dynamical EWSB, but instead focuses on gaining insights into the differences between Higgs-type and dynamical symmetry breaking through comparative analyses of various models.

In order for the dynamical symmetry in Eq. (1.2) to occur, the following conditions are necessary and are therefore assumed here: (i) the G_b gauge interaction is asymptotically free, so that the running coupling $\alpha_b(\mu) = g_b(\mu)^2/(4\pi)$ increases as the reference energy scale μ decreases; (ii) G_b, considered by itself, is a vectorial gauge symmetry, so that it does not self-break when it forms condensates, but instead remains exact; and (iii) the content of fermions that are nonsinglets under G_b is sufficiently small so that as the G_b interaction evolves from the ultraviolet to lower energy scales, $\alpha_b(\mu)$ increases sufficiently to exceed the critical value for the formation of the requisite G-breaking fermion condensates rather than evolving in a chirally symmetric manner. We consider several types of symmetries G, both vectorial and chiral, and of both direct-product and (semi)simple type. Although G_b, considered by itself (with the G interaction turned off), is vectorial, the full gauge symmetry $G_{\rm UV}$ is chiral in all of the cases that we consider. The $G_{\rm UV}$ symmetry thus requires that the fermions that are nonsinglets under G_b have zero intrinsic masses.

One can generalize the analysis further to deal with gauge symmetries of the form

$$G_{\rm UV} = G \otimes \left[\prod_{i=1}^{k} G_{b_i}\right], \tag{1.3}$$

where k strongly coupled gauge interactions G_{b_i}, $1 \le i \le k$, play a role in the dynamical breaking of G. We will focus on the simplest case, $k = 1$, but will also comment on models with $k = 2$.

This paper is organized as follows. In Sec. II we review two illustrative examples of the type of induced gauge symmetry breaking that we consider. In Sec. III we carry out a comparative study of the breaking of an SU(3) gauge symmetry to SU(2) by a Higgs field in the fundamental representation and by a dynamical mechanism. We also discuss how color $SU(3)_c$ would be broken in a modified standard model with a strongly coupled $SU(2)_L$ interaction. In Sec. IV we carry out a comparative study of the breaking of an SU(3) gauge symmetry by a Higgs field in the adjoint representation and by a dynamical mechanism. This is generalized to SU(N) in Sec. V. Some further discussion and our conclusions are given in Secs. VI and VII.

II. SOME EXAMPLES OF INDUCED DYNAMICAL SYMMETRY BREAKING

A. QCD Breaking Electroweak Symmetry

As background for our work, we first briefly review two examples of induced dynamical symmetry breaking of weakly coupled gauge symmetries by strongly coupled gauge interactions. In addition to the physics that is responsible for the main electroweak symmetry breaking at the scale ~250 GeV, there is another source of EWSB, albeit at a much smaller mass scale. This is quantum chromodynamics (QCD). The color $SU(3)_c$ gauge interaction produces bilinear quark condensates at a scale $\Lambda_{QCD} \sim 250$ MeV, in the most attractive channel $3 \times \bar{3} \rightarrow 1$, of the form $\langle \bar{q}q \rangle = \langle \bar{q}_L q_R \rangle + \text{H.c.}$ Because these quark condensates transform as weak $T = 1/2$, $|Y| = 1$ quantities, they break G_{EW} to electromagnetic $U(1)_{em}$. Indeed, one could imagine a hypothetical world in which the electroweak symmetry were not broken at the normal scale, but instead remained valid all the way down to the QCD scale. In this world [assuming that the $SU(3)_c$, $SU(2)_L$, and $U(1)_Y$ running gauge couplings had approximately their usual values], QCD would be the main source of EWSB [1,2]. Such a theory would be of the form of Eqs. (1.1) and (1.2), with

$$G = G_{EW}, \qquad G_b = SU(3)_c, \qquad H = U(1)_{em}. \quad (2.1)$$

In this hypothetical world the W and Z would pick up masses given by $m_W^2 = g^2 f_\pi^2/4$ and $m_Z^2 = (g^2 + g'^2) f_\pi^2/4$, where g and g' are the $SU(2)_L$ and $U(1)_Y$ running gauge couplings at the scale Λ_{QCD}, and f_π is the pion decay constant.

B. Electroweak Symmetry Breaking by Technicolor

Technicolor models embody the idea of dynamical electroweak symmetry breaking [1] (recent reviews include [3]). In these models, the gauge symmetry that is broken is (the electroweak part of) the SM gauge group $G = G_{SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. At the scale where G_{SM} is broken, all of the three gauge interactions corresponding to its factor groups are weakly coupled. The technicolor gauge interaction is associated with the group $G_b = G_{TC}$. Typically, $G_{TC} = SU(N_{TC})$ with some value of N_{TC} such as 2, so these models can be described in the notation of Eqs. (1.1) and (1.2) by

$$G = G_{SM}, \quad G_b = SU(N_{TC}), \quad H = SU(3)_c \otimes U(1)_{em}. \quad (2.2)$$

The (vectorial, asymptotically free) technicolor gauge interaction produces condensates of technifermions $\langle \bar{F}F \rangle = \langle \bar{F}_L F_R \rangle + \text{H.c.}$ that transform as weak $T = 1/2$, $|Y| = 1$ and hence break G_{EW} to $U(1)_{em}$, as indicated in Eq. (2.2). Technicolor models are embedded in extended technicolor (ETC) in order to communicate the electroweak symmetry breaking to the quarks and leptons. These TC/ETC theories are subject to a number of constraints from induced flavor-changing neutral processes, precision electroweak data, and limits on pseudo-Nambu-Goldstone bosons (PNGBs).

Technicolor models can be classified into two generic types: (i) one-family models, in which the technifermions comprise one SM family, and (ii) one-doublet models, in which, among the technifermions, there is only a single electroweak doublet. One-family (but not one-doublet) technicolor models feature a color-octet technivector meson resonance, as well as color-nonsinglet pseudo-Nambu-Goldstone bosons. Many searches for technihadrons have been carried out [3]. Recent LHC results from the ATLAS and CMS experiments have set lower limits of order 1.5 TeV on a color-octet technivector meson [4,5].

III. BREAKING AN SU(3) GAUGE SYMMETRY TO SU(2)

In this section we shall compare Higgs and dynamical mechanisms for breaking an SU(3) gauge symmetry to SU(2). We assume that the fermion content of the theory is such that the fermions that are only nonsinglets under SU(3) form a vectorlike sector. We shall begin by considering an abstract asymptotically free SU(3) theory at a sufficiently high scale that it is weakly coupled.

A. Higgs mechanism to break SU(3) to SU(2)

The requisite breaking can be accomplished by including a Higgs field ϕ that transforms as a fundamental (triplet) representation of the SU(3) group, with a potential

$$V = \frac{\mu^2}{2} \phi^\dagger \phi + \frac{\lambda}{4} (\phi^\dagger \phi)^2, \quad (3.1)$$

where $\mu^2 < 0$ (and $\lambda > 0$ in order for V to be bounded from below). This potential is minimized for a nonzero value of the ϕ vacuum expectation value. Without loss of generality, one can use the SU(3) gauge invariance to define directions in SU(3) space so that this has the form $\langle \phi \rangle = (0, 0, 1)^T v$, and one can perform a global phase redefinition on ϕ to make v real. This breaks SU(3) to the SU(2) subgroup generated by the the SU(3) generators T_a with $a = 1, 2, 3$ (in the usual Gell-Mann ordering of these generators). Of the six real components of the ϕ field, five are Nambu-Goldstone bosons and are absorbed by the five gauge bosons in the coset space SU(3)/SU(2) to form the longitudinal polarization states of the resultant massive vector bosons. The resultant vector boson masses are $\propto g_3(v)v$, where $g_3 \equiv g_3(\mu)$ is the running SU(3) gauge coupling at the scale $\mu = v$. The sixth component of the ϕ field forms a physical Higgs boson with a mass $\sim\sqrt{\lambda}v$. This is a singlet under the residual SU(2) gauge interaction.

As noted above, we assume that this breaking occurs at a scale v that is large compared with the scale where the running SU(3) gauge coupling $\alpha_3(\mu) = g_3(\mu)^2/(4\pi)$ would have grown to O(1) and the theory would thus have become strongly coupled. This assumption is necessary for this model to fall under the class of theories that are considered in this paper. If one were to relax this assumption, the analysis would become more complicated, because one would not be able to perform a perturbative analysis of the Higgs sector. (For a recent discussion of this strongly coupled case and further references, see [6].) Below the scale v, the resultant SU(2) theory would have a fermion sector consisting of the SU(2)-nonsinglet components of the original SU(3) fermion sector, together with the SU(2) gluons, with a gauge coupling inherited from the original SU(3) theory. This SU(2) theory would then evolve further into the infrared. With a sufficiently small fermion content $\{f\}$, the SU(2) coupling would eventually increase to O(1) at a lower scale Λ_2, where the SU(2) interaction would confine and produce bilinear fermion condensates. There would thus be a spectrum of SU(2)-singlet meson and (bosonic) baryons, together with glueballs (which would mix with the mesons to produce mass eigenstates) at this lower scale Λ_2.

There are several properties of this Higgs mechanism that will be contrasted with the induced dynamical breaking mechanism to be discussed next. First, *a priori*, one has the freedom to choose the coefficient μ^2 in the Higgs potential (3.1) to be positive or negative. Since one wants to construct the Higgs mechanism to break SU(3), one chooses $\mu^2 < 0$, but this sign choice could be considered to be *ad hoc*, since one does not give any deeper explanation for this choice. Second, the Higgs mechanism predicts physical pointlike Higgs particle(s), whereas in a dynamical mechanism, although the G_b interaction leads to various G_b-singlet bound states, including some with angular momentum $J = 0$, the properties of these states are not, in general, the same as those of a Higgs particle. Third, as is well known, this potential is unstable to large loop corrections and is thus sensitive to the nature of the ultraviolet completion of the theory (i.e., has a hierarchy problem). A fourth and related point is that the Higgs sector is not asymptotically free; i.e., the beta function for the quartic coupling, $d\lambda/dt$, is positive, where $t = \ln\mu$. Because of this, if one fixes λ at the scale v, say, then one must worry about a possible Landau pole in λ that could occur at a scale $\mu \gg v$. An equivalent way to phrase this is that if one fixes λ at a high scale in the ultraviolet, then λ decreases as μ decreases and is subject to an upper bound at a much lower scale such as v [7].

B. Induced dynamical breaking of SU(3) to SU(2)

In this subsection we discuss how one can produce the breaking of the SU(3) symmetry to SU(2) in a dynamical manner. For G_b we choose the smallest non-Abelian Lie group, $SU(2)_b$, so that $G_{UV} = SU(3) \otimes SU(2)_b$, in the notation of Eq. (1.1). To the set of fermions $\{f\}$ transforming vectorially under SU(3) we add the following chiral fermions [where a and α denote SU(3) and $SU(2)_b$ gauge indices, respectively, and the numbers in parentheses denote the dimensionalities of the representations of $G_{\rm UV}$]: (i) $\zeta_L^{a\alpha}$:(3, 2); (ii) η_L^{α}:(1, 2); and (iii) $\chi_{p,R}^{a}$:(3, 1) with $p = 1, 2$. This set of fermions is similar to the set that one of us used in Ref. [6]. Since the $SU(2)_b$ gauge interaction is asymptotically free, as the reference energy scale μ decreases from large values, the running coupling $\alpha_b(\mu)$ increases. The $SU(2)_b$-nonsinglet fermions comprise four chiral Weyl fermions or, equivalently, two Dirac fermions. This is well below the estimated critical number $N_{f,\rm cr} \sim 8$ beyond which the $SU(2)_b$ theory would evolve into the infrared in a chirally symmetric manner [8]. Therefore, we can conclude that as μ decreases to the scale $\mu = \Lambda_b$ such that $\alpha_b(\mu) \sim O(1)$, the $SU(2)_b$ interaction produces bilinear fermion condensates. The most attractive channel for the fermion condensation is $2 \times 2 \to 1$. One such condensate is of the form $\langle \epsilon_{\alpha\beta}\zeta_L^{a\alpha\,T} C \zeta_L^{b\beta}\rangle$, where $\epsilon_{\alpha\beta}$ is the antisymmetric tensor density for $SU(2)_b$. This is automatically antisymmetrized in the SU(3) indices a, b and hence is proportional to

$$\langle \epsilon_{abc}\epsilon_{\alpha\beta}\zeta_L^{a\alpha\,T} C \zeta_L^{b\beta}\rangle, \tag{3.2}$$

where ϵ_{abc} is the antisymmetric tensor density for SU(3). The condensate (3.2) transforms as conjugate fundamental ($\bar{3}$) representation of SU(3) and therefore dynamically breaks SU(3) to an SU(2) subgroup. A second condensate formed by the $SU(2)_b$ interaction is

$$\langle \epsilon_{\alpha\beta}\zeta_L^{a\alpha\,T} C \eta_L^{\beta}\rangle. \tag{3.3}$$

This transforms as a fundamental representation of SU(3) and hence also breaks it to an SU(2) subgroup. One can use vacuum alignment arguments [9] to infer that these SU(2) subgroups are the same. Then, without loss of generality, one may choose the index $c = 3$ in the condensate (3.2) and $a = 3$ in the condensate (3.3). The residual SU(2) subgroup preserved by these condensates is thus the one generated by T_a, $a = 1, 2, 3$ in SU(3). The fermions $\zeta_L^{a\,\alpha}$ and η_L^{α} with $a = 1, 2, 3$, $\alpha = 1, 2$ involved in these condensates gain dynamical masses of order Λ_b and are integrated out of the low-energy effective field theory that is operative at scales $\mu < \Lambda_2$. The two copies of $\chi^a_{p,L}$ decompose as two doublets under the resultant SU(2) for $a = 1, 2$ (while the $a = 3$ components form two singlets). The fermion content of this low-energy SU(2) theory thus consists of these two doublets, together with the SU(2)-nonsinglet components of the set $\{f\}$. With an asymptotically free SU(2), the coupling $\alpha_2(\mu)$, which is inherited from the weakly coupled SU(3) theory, will increase as μ decreases below Λ_b, and if the fermion content is sufficiently small so that $\alpha_2(\mu)$ grows to O(1) at a lower scale Λ_2, the SU(2) gauge interaction will confine and form bilinear fermion condensates at this scale. Given that α_2 is small at the scale Λ_b and increases only logarithmically, it follows that $\Lambda_2 \ll \Lambda_b$.

If one were to turn off the SU(3) gauge interaction, the $SU(2)_b$ theory would have a classical U(4) or equivalently SU(4) ⊗ U(1) global chiral symmetry. The U(1) is broken by $SU(2)_b$ instantons, so that the actual nonanomalous global chiral symmetry would be the SU(4) (generated by global transformations of the $\zeta^{a\alpha}_L$ and η^α_L among each other for a fixed α). The bilinear condensates would break this to Sp(4), with the resultant appearance of five Nambu-Goldstone bosons. Turning on the SU(3) gauge interaction explicitly breaks this global symmetry, although the breaking is weak, since α_3 is small.

We contrast this dynamical breaking with the corresponding Higgs mechanism presented above. First, the dynamical mechanism is more predictive, in the sense that once one has specified the gauge interaction G_b and the fermion content, the resulting fermion condensation and symmetry breaking follow automatically; one does not have to make an *ad hoc* choice of a parameter, as one does with the choice $\mu^2 < 0$ in the Higgs potential (3.1). Second, the theory does not suffer from a hierarchy problem, i.e., is not sensitively dependent on an ultraviolet completion, in contrast to the Higgs mechanism. Third, by construction, both the SU(3) and the $SU(2)_b$ sectors are asymptotically free, again in contrast with the Higgs mechanism, in which the quartic coupling is not asymptotically free.

C. Induced breaking of $SU(3)_c$ in a modified standard model

Here we discuss another way to break an SU(3) gauge symmetry dynamically. In this case we will take the SU(3) to be the color $SU(3)_c$ group of the standard model. The point here is that with a modification of the properties of the standard model, color $SU(3)_c$ would, in fact, be dynamically broken by the $SU(2)_L$ gauge interaction. Our analysis also addresses a fundamental question in particle physics. One of the profound properties of nature is the fact that it is the chiral part of $G_{\rm SM}$ that is broken, leaving as a residual exact subgroup a symmetry that is vectorial, namely, $H = SU(3)_c \otimes U(1)_{\rm em}$. This is naturally explained in the standard model Higgs mechanism and also in technicolor theories. One is led, then, to ask how general this property is in quantum field theory. That is, can one construct a model that exhibits dynamical breaking of a vectorial gauge symmetry? Clearly, this requires more than one gauge interaction to be present, since if one has just a single vectorial gauge interaction and it becomes strongly coupled and produces condensates, then the most attractive channel is $R_i \times \bar{R}_i \to 1$ for the one or more fermion representations R_i in the theory, so it does not self-break [10,11].

Let us thus consider a theory with the same gauge group, $G_{\rm SM}$, but make two changes: (i) first, we remove the usual breaking of $SU(2)_L$ at the 250 GeV scale, and (ii) we arrange the values of the gauge couplings so that at a scale Λ_2 considerably larger than $\Lambda_{\rm QCD}$, where $SU(3)_c$ [and $U(1)_Y$] are weakly coupled, the $SU(2)_L$ interaction becomes strongly coupled, with $\alpha_2(\Lambda_2) = g(\Lambda_2)^2/(4\pi)$ of order unity. The $SU(2)_L$ sector contains $N_{\rm gen}(N_c + 1) =$ 12 chiral fermion doublets (where $N_{\rm gen}$ denotes the number of SM fermion generations), so that the $SU(2)_L$ gauge interaction is asymptotically free, with leading coefficient

$$(b_1)_{SU(2)_L} = \frac{1}{3}[22 - (N_c + 1)N_{\rm gen}]. \tag{3.4}$$

Given that there is no breaking of $G_{\rm EW}$, the fermions are massless, so they all contribute to the $SU(2)_L$ beta function. To illustrate this dynamical breaking in the simplest context, we assume $N_{\rm gen} = 1$, so that there are four chiral $SU(2)_L$ doublets, or, equivalently, $N_f = 2$ Dirac doublets. This is well within the phase in which $SU(2)_L$ confines and spontaneously breaks global chiral symmetry. The model thus contains one family of SM fermions:

$$Q^{ai}_L = \begin{pmatrix} u^a \\ d^a \end{pmatrix}_L,$$

u^a_R, d^a_R,

$$L^i_L = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L,$$

and e_R, where a and i are $SU(3)_c$ and $SU(2)_L$ gauge indices, respectively.

The most attractive channel for the strongly coupled $SU(2)_L$ interaction is $2 \times 2 \to 1$, and it produces several condensates in this channel. The first of these is of the form $\langle \epsilon_{k\ell} Q^{a,k\,T}_L C Q^{b,\ell}_L \rangle$, where $\epsilon_{k\ell}$ is the antisymmetric tensor density for $SU(2)_L$. This is automatically antisymmetric in $SU(3)_c$ indices and hence is proportional to

$$\langle \epsilon_{abc} \epsilon_{k\ell} Q^{a,k\,T}_L C Q^{b,\ell}_L \rangle = 2\langle \epsilon_{abc} u^{a\,T}_L C d^b_L \rangle. \tag{3.5}$$

This transforms as a $(3 \times 3)_{\rm as} = \bar{3}$ under $SU(3)_c$ (where the subscript "as" stands for antisymmetric) and hence breaks $SU(3)_c$ to a subgroup $SU(2)_c$. It also breaks $U(1)_Y$. As is clear from the fact that electric charge satisfies $Q = T_3 + (Y/2)$ and the fact that the condensate (3.5) is invariant under $SU(2)_L$, it also violates electric charge invariance. Without loss of generality, we choose the breaking direction of $SU(3)_c$ as the third direction, so that the u_L^a and d_L^a quarks with color indices $a = 1, 2$ occur in the condensate (3.5) and hence gain dynamical masses of order Λ_2.

The strong $SU(2)_L$ interaction would also produce the condensate

$$\langle \epsilon_{k\ell} Q_L^{a,k\,T} C L_L^\ell \rangle = \langle u_L^{a\,T} C e_L - d_L^{a\,T} C \nu_{e,L} \rangle. \tag{3.6}$$

This also breaks $SU(3)_c$ to an $SU(2)_c$ subgroup and violates hypercharge and electric charge. As in our discussion above, a vacuum alignment argument can be used to infer that the condensate (3.6) breaks $SU(3)_c$ to the same $SU(2)_c$ as the condensate (3.5), so that the color index in Eq. (3.6) has the value $a = 3$. This $SU(2)_c$ is the one generated by the color generators (T_a) with $a = 1, 2, 3$. Thus, this model is of the form in Eqs. (1.1) and (1.2) with

$$G = SU(3)_c \otimes U(1)_Y, \qquad G_b = SU(2)_L, \qquad H = SU(2)_c. \tag{3.7}$$

In addition to breaking these gauge symmetries, the condensate (3.5) breaks baryon number by $\Delta B = 2/3$, while the condensate (3.6) breaks B by $\Delta B = 1/3$ and lepton number L by $\Delta L = 1$. The quarks u_L^a, d_L^a with $a = 1, 2, 3$ and the leptons e_L, and $\nu_{e,L}$ involved in these condensates gain dynamical masses of order Λ_2. (The actual mass eigenstates involve linear combinations of these fields.) Similarly, the five gluons in the coset $SU(3)_c/SU(2)_c$ corresponding to the broken generators of $SU(3)_c$ gain dynamical masses of order

$$m_g \sim g_3(\Lambda_2)\Lambda_2 \tag{3.8}$$

and the Abelian $U(1)_Y$ gauge boson B gains a mass

$$m_B \sim g'(\Lambda_2)\Lambda_2. \tag{3.9}$$

Since by our assumptions, $SU(3)_c$ and $U(1)_Y$ are weakly coupled at this scale, the masses of these five gluons and of the one B boson are smaller than the dynamically produced fermion masses.

Of the quarks and leptons in this $N_{\rm gen} = 1$ model, all of the components of the $N_c + 1 = 4$ $SU(2)_L$ doublets are involved in the condensates (3.5) and (3.6) and gain dynamical masses of order Λ_2. These fermions are thus integrated out of the low-energy effective theory below Λ_2. The $SU(2)_c$ gauge symmetry of this low-energy effective field theory remains exact. The content of nonsinglet fermions in this low-energy theory consists of u_R^a and d_R^a with $a = 1, 2$, which form two Weyl fermions, or, equivalently, one Dirac fermion. The $SU(2)_c$ gauge coupling α_{2c} is inherited from the $SU(3)_c$ theory and is small at Λ_2, but eventually grows to O(1) at a much lower $\Lambda_{2c} \ll \Lambda_2$. At this lower scale Λ_{2c}, the $SU(2)_c$ theory confines and produces a bilinear fermion condensate,

$$\langle \epsilon_{ab} u_R^{a\,T} C d_R^b \rangle, \tag{3.10}$$

where here ϵ_{ab} is the antisymmetric tensor density of $SU(2)_c$. This $SU(2)_c$ theory has a classical U(2), or, equivalently, $SU(2) \otimes U(1)$ global chiral symmetry defined by transformations that mix up the u_R^a and d_R^a fields (for fixed a). The U(1) is broken by $SU(2)_c$ instantons, so that the actual nonanomalous global chiral symmetry is SU(2). In general, an SU(2) gauge theory with N_d massless chiral Weyl fermions transforming according to the fundamental representation (with $N_d = 2k$ even to avoid a global Witten anomaly) has an $SU(2k)$ global chiral symmetry corresponding to transformations that mix up the $2k$ chiral doublets. Formation of condensates involving these doublets breaks this global symmetry to $Sp(2k)$. Since the orders of these groups are $4k^2 - 1$ and $k(2k + 1)$, respectively, this entails the breaking of $2k^2 - k - 1$ generators of $SU(2k)$, and the resultant appearance of this number of massless Nambu-Goldstone bosons. In this $SU(2)_c$ theory, there are $N_d = 2$ chiral fermions, i.e., $k = 1$, so the SU(2) global chiral symmetry is equivalent to Sp(2), and there is no chiral symmetry breaking due to the formation of the condensate (3.10).

It is also worthwhile to comment on the situation concerning global chiral symmetry at the higher scale, above Λ_2. In the present model with its one generation of SM fermions, if one turns off the $SU(3)_c$ and $U(1)_Y$ couplings, then, at an energy above Λ_2, the $SU(2)_L$ theory has a nonanomalous global $SU(N_d)$ symmetry, where $N_d = N_c + 1 = 4$. The condensates (3.5) and (3.6) break this to Sp(4), leading to the appearance of five Nambu-Goldstone bosons. Since the NGBs couple in a derivative manner, their scattering amplitudes are suppressed by powers of center-of-mass energy $\sqrt{s}/\Lambda_2$ and hence they are progressively more weakly coupled as the energy scale decreases further below Λ_2 [13]. Turning on the $SU(3)_c$ and $U(1)_Y$ couplings explicitly breaks the global SU(4) symmetry, but also the would-be NGBs are absorbed to form the longitudinal components of the five vector bosons in the coset $SU(3)_c/SU(2)_c$. This process is reminiscent of the mechanism by which technicolor gives masses to the W and Z bosons.

Our analysis here answers the question that we posed at the beginning of this subsection concerning the breaking of a vectorial, in contrast to a chiral, gauge symmetry. Our answer is that it is certainly possible for a vectorial gauge symmetry to be broken, if this breaking is induced by another strongly coupled interaction. The reason that $SU(2)_L$ does not break $SU(3)_c$ in nature is a consequence of the fact that $SU(2)_L$ is broken well above the scale where its coupling would have become large enough to produce the condensates (3.5) and (3.6). The resultant W and Z are massive and weakly coupled and their interactions are too weak to induce such condensates. Indeed, even if the $SU(2)_L$ symmetry were not broken at this higher scale, it would be broken by the quark condensates at the QCD scale, as discussed above, before it could become strong enough to break $SU(3)_c$.

IV. INDUCED DYNAMICAL BREAKING OF A GAUGE SYMMETRY BY ADJOINT FIELDS: AN ILLUSTRATIVE MODEL WITH $G = SU(3)$

A. Higgs mechanism

We next consider induced breaking of a gauge symmetry by fields that transform as the adjoint representation of the gauge group. In this section we discuss SU(3) because of some special properties that it has, and in the next section we discuss $SU(N)$ for general $N \geq 4$. We begin by constructing a Higgs mechanism for this breaking. We assume that the theory contains a Higgs field ϕ transforming according to the adjoint (i.e., octet) representation of SU(3), with an appropriate Higgs potential. We will write the components of ϕ as ϕ^i_j, $1 \leq i, j \leq 3$; these are subject to the trace condition $\mathrm{Tr}(\phi) = \sum_{i=1}^3 \phi^i_i = 0$. In general, when using the adjoint representation of $SU(N)$, in addition to the notation ϕ^i_j with $1 \leq i,\ j \leq N$, it will also be convenient to use an equivalent notation ϕ_a, with $1 \leq a \leq N^2 - 1$, that indicates the 1–1 correspondence with the $N^2 - 1$ generators T_a of $SU(N)$. Thus the ϕ^i_j form the entries of a matrix given by $\sqrt{2}\sum_{a=1}^{N^2-1} \phi_a T_a$.

We will require that the Higgs part of the Lagrangian be invariant under the replacement $\phi \to -\phi$. It follows that the Higgs potential contains only quadratic and quartic terms in ϕ. For a general $SU(N)$ theory with a Higgs field in the adjoint representation, there are two independent quartic terms, proportional to $[\mathrm{Tr}(\phi^2)]^2$ and $\mathrm{Tr}(\phi^4)$. For the special values $N = 2$ or $N = 3$, $[\mathrm{Tr}(\phi^2)]^2 = 2\mathrm{Tr}(\phi^4)$, so there is only one independent quartic term. For the present case of SU(3), the Higgs potential may thus be written as

$$V = \frac{\mu^2}{2} \mathrm{Tr}(\phi^2) + \frac{\lambda}{4}[\mathrm{Tr}(\phi^2)]^2. \tag{4.1}$$

Here we take $\mu^2 < 0$ to get the symmetry breaking. This potential is minimized for a Higgs field VEV of the form

$$\langle\phi\rangle = T_8 v, \tag{4.2}$$

where v can be made real by a global rephasing of ϕ and T_8 is the second member of the Cartan subalgebra of SU(3),

$$T_8 = \frac{1}{2\sqrt{3}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}. \tag{4.3}$$

The VEV (4.2) breaks SU(3) according to the pattern

$$\mathrm{SU}(3) \to \mathrm{SU}(2) \otimes \mathrm{U}(1). \tag{4.4}$$

Since SU(3) has order eight, while $SU(2) \otimes U(1)$ has order four, there are four broken generators of SU(3), namely, the T_a with $a = 4, 5, 6, 7$ in the standard Gell-Mann basis. The corresponding components ϕ_a are Nambu-Goldstone bosons and are absorbed to become the longitudinal components of the massive vector bosons. Four physical Higgs fields remain, with masses $\sim\sqrt{\lambda}v$. Of these, ϕ_a, $a = 1, 2, 3$ transform as the adjoint representation of the residual SU(2) gauge interaction, and assuming that it confines, they are thus confined in SU(2)-singlet bound states. Since we have assumed that the SU(2) theory is weakly coupled at the scale $\mu = v$, and since its coupling increases only logarithmically, the SU(2) confinement scale Λ_2 is much smaller than v. In passing, we note that although a Higgs VEV of the form

$$\langle\phi\rangle = \mathrm{diag}(a, b, -(a+b)) \quad \text{with } |a| \neq |b|$$

is, *a priori*, possible, and would break SU(3) to U(1) rather than $SU(2) \otimes U(1)$, it does not minimize the Higgs potential.

B. Dynamical breaking mechanism with adjoint fields

To study the dynamical breaking of the SU(3) symmetry by an $SU(N_b)$ gauge interaction, we must choose a value of N_b and a requisite sector comprised of one or more fermion fields that transform as nonsinglets under both $G = SU(3)$ and $G_b = SU(N_b)$. For our model we choose a chiral fermion that transforms as an adjoint of SU(3) and a fundamental representation of $SU(N_b)$:

$$(\psi^i_{j,L})^\alpha\colon (8, N_b), \tag{4.5}$$

where the numbers in parentheses are the dimensions of the representation under $G_{\mathrm{UV}} = SU(3) \otimes SU(N_b)$, the indices i, j are SU(3) indices, and $\alpha = 1, \ldots, N_b$ is an $SU(N_b)$ index. Because $(\psi^i_{j,L})^\alpha$ transforms according to a self-adjoint representation of SU(3), it does not contribute any gauge anomaly to the SU(3) theory. We take $N_b = 2$, the minimal value, so $G_b = SU(2)_b$. As above, we will also use the equivalent notation $\phi_{a,L}$, $1 \leq a \leq 8$. The $(\psi^i_{j,L})^\alpha$ form the components of a matrix given by $\sqrt{2}\sum_{a=1}^8 \psi^\alpha_{a,L} T_a$.

The $SU(2)b$ gauge interaction is asymptotically free, with the leading beta function coefficient $b_1 = 14/3$ (see the Appendix for notation). The fermion $(\psi^i_{j,L})^\alpha$ amounts to 8 Weyl doublets, or, equivalently, 4 Dirac doublets, of SU(2). Since this number is well below the estimated critical value $N_{f,\mathrm{cr}} \simeq 8$ separating the (zero-temperature) phase with confinement and spontaneous chiral symmetry breaking from the chirally symmetric phase [8], we can conclude that the SU(2) interaction confines and produces bilinear condensates. These occur in the most attractive $SU(2)_b$ channel, which is $2 \times 2 \to 1$, with a condensate of the form $\langle\epsilon_{\alpha\beta}(\psi^i_{j,L})^{\alpha T} C(\psi^k_{\ell,L})^\beta\rangle$, where here $\epsilon_{\alpha\beta}$ is the antisymmetric tensor density for $SU(2)_b$. The $\epsilon_{\alpha\beta}$ contraction antisymmetrizes the bilinear fermion product, so that in the full Clebsch-Gordan decomposition,

$$8 \times 8 = 1_s + 8_s + 8_a + 10_a + \overline{10}_a + 27_s \tag{4.6}$$

(where the subscripts s and a denote symmetric and antisymmetric combinations), the above condensate must be one of the antisymmetric products, namely, 8_a or 10_a. We next use a vacuum alignment argument [9], according to which the symmetry breaking should preserve as large a subgroup symmetry as possible. Now, relative to the maximal subgroup $\mathrm{SU}(2) \otimes \mathrm{U}(1)$, an octet of SU(3) has the decomposition

$$8_{\mathrm{SU}(3)} = 3_0 + 2_1 + 2_{-1} + 1_0, \tag{4.7}$$

where the numbers on the right-hand side are the dimensionalities of the SU(2) representations and the subscripts are the hypercharges with the Gell-Mann normalization. In contrast, the decuplet has the decomposition

$$10_{\mathrm{SU}(3)} = 4_1 + 3_0 + 2_{-1} + 1_{-2}. \tag{4.8}$$

Of these, only the octet contains a piece that is a singlet under $\mathrm{SU}(2) \otimes \mathrm{U}(1)$. Using a vacuum alignment argument, we therefore can infer that the condensate transforms as the 1_0 piece of the octet of SU(3) and hence has the form

$$\langle \epsilon_{\alpha\beta}(\psi^i_{j,L})^{\alpha T} C(\psi^j_{\ell,L})^{\beta}\rangle \propto (T_8)^i_\ell \Lambda^3_b. \tag{4.9}$$

In the equivalent notation using $\psi^\alpha_{a,L}$ with $1 \le a \le 8$, the condensate has the form $\langle \epsilon_{\alpha\beta} f_{ab8}(\psi_{a,L})^{\alpha T} C(\psi_{b,L})^{\beta}\rangle$, where the f_{abc} are the structure constants of the SU(3) Lie algebra. The nonzero structure constants f_{ab8} with $a < b$ that enter here are f_{458} and f_{678}. The condensate (4.9) dynamically breaks SU(3) to $\mathrm{SU}(2) \otimes \mathrm{U}(1)$, as in Eq. (4.4). As in the Higgs case, there are four broken generators, namely, the T_a with $a = 4, 5, 6, 7$. The Nambu-Goldstone modes involving $\psi^\alpha_{a,L}$, $a = 4, 5, 6, 7$, contracted on the $\mathrm{SU}(2)_b$ indices $\alpha = 1, 2$ to form $\mathrm{SU}(2)_b$ singlets, are absorbed by the corresponding SU(3) gauge bosons, forming longitudinal polarization states and giving them masses. The remaining $\psi^\alpha_{a,L}$ fermions with $a = 1, 2, 3, 8$ are bound in $\mathrm{SU}(2)_b$-singlet states. Furthermore, of these bound states, the ones with $a = 1, 2, 3$ transform as the adjoint representation of the residual SU(2) gauge interaction, and assuming that it confines, they are confined in SU(2)-singlet bound states. This is reminiscent of the situation with the corresponding components of Higgs fields in the situation where one uses a Higgs mechanism for the breaking.

A comment is in order here concerning chiral symmetry in this model. The $\psi^\alpha_{a,L}$ fermions have zero Lagrangian masses, and hence, if one turns off the SU(3) gauge interaction completely, the theory has a (nonanomalous) SU(8) global chiral symmetry. In general, a full set of bilinear fermion condensates breaks this to Sp(8). As noted above, the breaking of $\mathrm{SU}(2k)$ to $\mathrm{Sp}(2k)$ entails $2k^2 - k - 1$ broken generators and corresponding massless Nambu-Goldstone bosons. With $k = 4$, this means 27 NGBs in the present case. When one turns on the SU(3) gauge coupling, this explicitly breaks the global SU(8) chiral symmetry, and moreover, the vacuum alignment argument suggests which condensates form, as we have discussed above.

V. INDUCED BREAKING OF AN SU(N) SYMMETRY BY ADJOINT FIELDS

A. General

In this section we carry out a comparative study of a Higgs mechanism versus dynamical breaking of an SU(N) gauge symmetry with $N \ge 4$ by fields transforming according to the adjoint representation of this group. Two general types of breaking patterns of the SU(N) symmetry will be relevant. Both of these involve breaking to a maximal subgroup of SU(N), with the same rank (dimension of the Cartan subalgebra of mutually commuting generators) as SU(N), namely, $N - 1$. However, these subgroups have different orders (numbers of generators). The first of these symmetry-breaking patterns is

$$\mathrm{SU}(N) \to \mathrm{SU}(N-1) \otimes \mathrm{U}(1). \tag{5.1}$$

The residual symmetry group has order

$$o[\mathrm{SU}(N-1) \otimes \mathrm{U}(1)] = (N-1)^2 \tag{5.2}$$

so the symmetry reduction in Eq. (5.1) involves the breaking of

$$\Delta o = 2(N-1) \tag{5.3}$$

generators of SU(N), which is the dimension of the coset

$$\mathrm{SU}(N)/[\mathrm{SU}(N-1) \otimes \mathrm{U}(1)]. \tag{5.4}$$

The second type of symmetry-breaking pattern leads to a residual symmetry involving three factor groups. To describe this, it is convenient to deal separately with the cases of even and odd N. For even $N = 2k$, a possible symmetry-breaking pattern is

$$\mathrm{SU}(N) \to \mathrm{SU}(N/2) \otimes \mathrm{SU}(N/2) \otimes \mathrm{U}(1). \tag{5.5}$$

The residual symmetry group has order

$$o[\mathrm{SU}(N/2) \otimes \mathrm{SU}(N/2) \otimes \mathrm{U}(1)] = \frac{N^2}{2} - 1 = 2k^2 - 1, \tag{5.6}$$

so that (5.5) involves the breaking of

$$\Delta o = \frac{N^2}{2} = 2k^2 \tag{5.7}$$

generators of SU(N).

For odd $N = 2k + 1$, a possible symmetry-breaking pattern is

$$\mathrm{SU}(N) \to \mathrm{SU}((N+1)/2) \otimes \mathrm{SU}((N-1)/2) \otimes \mathrm{U}(1). \tag{5.8}$$

The residual symmetry group has order

$$o[\mathrm{SU}((N+1)/2)\otimes \mathrm{SU}((N-1)/2)\otimes \mathrm{U}(1)] = \frac{N^2-1}{2} = 2k(k+1), \tag{5.9}$$

so that (5.8) involves the breaking of

$$\Delta o = \frac{N^2-1}{2} = 2k(k+1) \tag{5.10}$$

generators of SU(N). The symmetry-breaking patterns (5.5) and (5.8) can be expressed in a unified manner as

$$\mathrm{SU}(N) \to \mathrm{SU}(N-\ell)\otimes \mathrm{SU}(\ell)\otimes \mathrm{U}(1), \tag{5.11}$$

where $\ell = [N/2]_{ip}$ and $[\nu]_{ip}$ denotes the integral part of the real number ν.

As we will discuss further below in the context of dynamical symmetry breaking, a vacuum alignment argument prefers a symmetry-breaking pattern that yields the largest residual symmetry. The size of the subgroup that constitutes the residual symmetry can be characterized by its rank and order. All of the patterns above satisfy the condition that the rank of the residual symmetry group should be maximal, i.e., the same as that of SU(N), namely, $N-1$. Concerning the differences in the orders of the various possible subgroups resulting from the symmetry breaking of SU(N), we calculate, for even $N=2k$, the difference

$$o[\mathrm{SU}(N-1)\otimes \mathrm{U}(1)] - o[\mathrm{SU}(N/2)\otimes \mathrm{SU}(N/2)\otimes \mathrm{U}(1)] = \frac{(N-2)^2}{2} = 2(k-1)^2. \tag{5.12}$$

This difference is positive semidefinite, and positive-definite for $k \geq 2$, i.e., $N \geq 4$. For odd $N = 2k+1$,

$$o[\mathrm{SU}(N-1)\otimes \mathrm{U}(1)] - o[\mathrm{SU}((N+1)/2) \otimes \mathrm{SU}((N-1)/2)\otimes \mathrm{U}(1)] = \frac{(N-1)(N-3)}{2} = 2k(k-1). \tag{5.13}$$

This difference is also positive semidefinite, and positive-definite for $k \geq 2$, i.e., $N \geq 5$. Hence, as these calculations show, a vacuum alignment argument prefers the breaking pattern (5.1) for both even and odd $N \geq 4$. The special case $N=3$ has been analyzed above, and leads to breaking of the SU(3) group to SU(2) $\times$ U(1), which is also of the form (5.1) with $N=3$.

There are other symmetry-breaking patterns that could, *a priori*, occur. SU(N) could, in principle, break to a non-maximal subgroup, i.e., a subgroup with rank smaller than the rank of SU(N), namely, $N-1$. For example, in principle SU(3) could, *a priori*, break to U(1), SU(4) could break to SU(2) $\otimes$ U(1), and so forth. However, in the context of the Higgs mechanism, these symmetry-breaking patterns do not occur as minima of the Higgs potential, and in the dynamical symmetry-breaking context, they are disfavored by vacuum alignment arguments.

B. SU(N) breaking with an adjoint Higgs field

First, we discuss the mechanism for breaking an SU(N) gauge symmetry with a Higgs field Φ in the adjoint representation [14]. The components of the Higgs field are denoted Φ^i_j. We impose a $\Phi \to -\Phi$ symmetry. Then the Higgs potential has the general form

$$V = \frac{\mu^2}{2}\mathrm{Tr}(\Phi^2) + \frac{\lambda_1}{4}[\mathrm{Tr}(\Phi^2)]^2 + \frac{\lambda_2}{4}\mathrm{Tr}(\Phi^4), \tag{5.14}$$

where we take $\mu^2 < 0$ to produce the symmetry breaking. Since Φ is a Hermitian matrix, it can be diagonalized by a unitary transformation. It follows that one can write

$$\Phi^i_j = \delta^i_j \phi_j \quad (\text{no sum on } j), \tag{5.15}$$

for $1 \leq i, j \leq N$. Substituting Eq. (5.15) into Eq. (5.14) gives

$$V = \frac{\mu^2}{2}\sum_{i=j}^{N}\phi_j^2 + \frac{\lambda_1}{4}\left(\sum_{j=1}^{N}\phi_j^2\right)^2 + \frac{\lambda_2}{4}\sum_{j=1}^{N}\phi_j^4. \tag{5.16}$$

Since $\mathrm{Tr}(\Phi)=0$, the ϕ_j satisfy the condition

$$\sum_{j=1}^{N}\phi_j = 0. \tag{5.17}$$

Hence, ϕ only involves $N-1$ independent fields, and V only depends on $N-1$ of the components ϕ_j, which we take to be ϕ_j with $j = 1, \ldots, N-1$. Now

$$[\mathrm{Tr}(\Phi^2)]^2 \geq \mathrm{Tr}(\Phi^4), \tag{5.18}$$

as can be seen from the explicit expression for the difference,

$$[\mathrm{Tr}(\Phi^2)]^2 - \mathrm{Tr}(\Phi^4) = 2\left[\sum_{1\leq i<j\leq N-1}\phi_i^2\phi_j^2 + \left(\sum_{i=1}^{N-1}\phi_i^2\right) \times \left(\sum_{j=1}^{N-1}\phi_j\right)^2\right] \geq 0. \tag{5.19}$$

As noted above, if N is equal to 2 or 3, then $[\mathrm{Tr}(\Phi^2)]^2 = 2\,\mathrm{Tr}(\Phi^4)$, so that there is only one independent quartic term, and its coefficient, $(1/4)[\lambda_1 + (\lambda_2/2)]$, must be positive. For $N \geq 4$, the two quartic terms are independent, and the condition that V be bounded from below requires that $\lambda_1 > 0$.

For $\lambda_2 > 0$, it is again convenient to consider the cases of even $N = 2k$ and odd $N = 2k+1$ separately. For $\lambda_2 > 0$ and even $N = 2k$, V is minimized if the VEV of Φ has the form given by

$$\langle \phi_i \rangle = \frac{v}{\sqrt{2N}} \times \begin{cases} 1 & \text{for } 1 \leq i \leq k \\ -1 & \text{for } k+1 \leq i \leq 2k. \end{cases} \tag{5.20}$$

The normalization of v in Eq. (5.20) and in the equations below is determined by the definition $\mathrm{Tr}(\Phi^2) = (1/2)v^2$, analogous to the usual normalization condition $\mathrm{Tr}(T_a T_b) = (1/2)\delta_{ab}$ for the generators of SU(N). At this minimum, one finds

$$v^2 = \frac{-2\mu^2}{[\lambda_1 + \frac{\lambda_2}{N}]}. \tag{5.21}$$

The VEV (5.20) breaks SU($2k$) according to (5.5). The value of the potential at the minimum is

$$V_{\min} = \frac{-\mu^4}{4\Big[\lambda_1 + \frac{\lambda_2}{N}\Big]}. \tag{5.22}$$

For $\lambda_2 > 0$ and odd $N = 2k+1$ with $k \geq 2$, V is minimized if the VEV of Φ has the form

$$\langle \phi_i \rangle = v\Big[\frac{k}{2(k+1)(2k+1)}\Big]^{1/2} \times \begin{cases} 1 & \text{for } 1 \leq i \leq k+1 \\ -\frac{k+1}{k} & \text{for } k+2 \leq i \leq 2k+1. \end{cases} \tag{5.23}$$

(The special case $k = 1$, i.e., $N = 3$, was dealt with above.) The minimization condition determines v according to

$$v^2 = \frac{-2\mu^2}{\Big[\lambda_1 + \Big(\frac{N^2+3}{N(N+1)(N-1)}\Big)\lambda_2\Big]}. \tag{5.24}$$

This VEV (5.23) breaks SU($2k+1$) according to (5.8). The value of the potential at the minimum is

$$V_{\min} = \frac{-\mu^4}{4\Big[\lambda_1 + \Big(\frac{N^2+3}{N(N+1)(N-1)}\Big)\lambda_2\Big]}. \tag{5.25}$$

It is possible for λ_2 to have a restricted range of negative values [15],

$$-\Big(\frac{N(N-1)}{N^2 - 3N + 3}\Big)\lambda_1 < \lambda_2 < 0. \tag{5.26}$$

For λ_2 in this range, V is minimized if Φ has the VEV

$$\langle \phi_i \rangle = \frac{v}{\sqrt{2N(N-1)}} \times \begin{cases} 1 & \text{for } 1 \leq i \leq N-1 \\ -(N-1) & \text{for } i = N, \end{cases} \tag{5.27}$$

where

$$v^2 = \frac{-2\mu^2}{\Big[\lambda_1 + \Big(\frac{N^2-3N+3}{N(N-1)}\Big)\lambda_2\Big]}. \tag{5.28}$$

The VEV (5.27) breaks SU(N) according to Eq. (5.1). The value of V at this minimum is

$$V_{\min} = \frac{-\mu^4}{4\Big[\lambda_1 + \Big(\frac{N^2-3N+3}{N(N-1)}\Big)\lambda_2\Big]}. \tag{5.29}$$

Note that all three of the minimal values (5.22), (5.25), and (5.29) have the form

$$V_{\min} = \frac{\mu^2 v^2}{8} \tag{5.30}$$

for the respective three values of v^2. The lower limit on the allowed negative range of λ_2 in Eq. (5.26) is evident from Eq. (5.29), since in this equation $V_{\min} \to -\infty$ as λ_2 approaches this lower limit from above. The fact that $\lambda_2 = 0$ is the boundary between the two types of minima is evident from the difference between the values of the minima for even N,

$$V_{\min,\lambda_2>0} - V_{\min,\lambda_2<0} = \frac{-(N-2)^2\lambda_2\mu^4}{4\Big[\lambda_1 + \frac{\lambda_2}{N}\Big][N(N-1)\lambda_1 + (N^2-3N+3)\lambda_2]}, \tag{5.31}$$

and for odd N,

$$V_{\min,\lambda_2>0} - V_{\min,\lambda_2<0} = \frac{-N^3(N-1)(N-3)\lambda_2\mu^4}{4[N(N^2-1)\lambda_1 + (N^2+3)\lambda_2][N(N-1)\lambda_1 + (N^2-3N+3)\lambda_2]}. \tag{5.32}$$

Both of these differences are proportional to λ_2, explicitly showing the switch in global minimum as λ_2 reverses sign. The reason for the residual U(1) invariance in these symmetry-breaking patterns obtained with a Higgs field Φ transforming according to the adjoint representation of SU(N) is that since Φ can be diagonalized, as noted above, its VEV can be expressed as a linear combination of coefficients multiplied by the $N-1$ diagonal Cartan generators of SU(N). Indeed, without loss of generality, one can define axes in SU(N) space so that it points entirely along one such Cartan generator, which can be denoted as T_C. Then $\exp(i\theta T_C)$ commutes with $\langle\Phi\rangle$, yielding the U(1) invariance.

From the formulas for Δo, the number of broken generators for the various symmetry-breaking patterns, one can immediately infer the number of gauge bosons of SU(N) that become massive. Thus, for $\lambda_2 > 0$ and even $N = 2k$, the symmetry breaking (5.5) involves the breaking of $N^2/2 = 2k^2$ generators, so that of the $N^2 - 1$ (real) components of Φ, $N^2/2$ are absorbed to become the longitudinal components of the gauge bosons corresponding to these broken generators, which pick up masses $\propto gv$. The remaining $N^2/2 - 1 = 2k^2 - 1$ real components of Φ are physical Higgs bosons. For $\lambda_2 > 0$ and odd $N = 2k+1$, the symmetry breaking (5.1) involves the breaking of $(N^2-1)/2 = 2k(k+1)$ generators, so that of the

$N^2 - 1$ (real) components of Φ, $(N^2-1)/2$ are absorbed to become the longitudinal components of the gauge bosons corresponding to these broken generators. The remaining $(N^2-1)/2 = 2k(k+1)$ real components of Φ are physical Higgs bosons. For $\lambda_2 < 0$, the symmetry breaking (5.1) involves the breaking of $2(N-1)$ generators, and an equal number of Nambu-Goldstone bosons, which are absorbed to become the longitudinal components of the gauge bosons in the coset (5.4). The remaining $(N-1)^2$ real components of Φ are physical Higgs bosons.

C. Dynamical mechanism for SU(N) breaking by an adjoint field

For the analysis of dynamical symmetry breaking of SU(N) by an adjoint field, we analyze a model of the form of Eq. (1.1), in which

$$G = \mathrm{SU}(N), \qquad G_b = \mathrm{SU}(N_b). \tag{5.33}$$

For the fermions that transform under both SU(N) and G_b we use

$$(\psi^i_j)^\alpha_L \colon (N^2-1, N_b), \tag{5.34}$$

where here and below, α is the G_b gauge index. Thus, we assign each of the N^2-1 components of $(\psi^i_j)^\alpha_L$ to transform according to the fundamental representation of SU(N_b). The numbers in parentheses in Eq. (5.34) are the dimensions of the representations with respect to the factor groups in Eq. (5.33). The N_b copies of fermions in the adjoint representation of SU(N) contribute zero gauge anomaly to SU(N). As stated earlier, these and the other fermions that we include are taken to have zero Lagrangian masses since mass terms would violate the full G_{UV} symmetry, which is chiral.

The choice of the rest of the G_b-nonsinglet fermions in the model depends on the value of N_b. We first consider the possibility that $N_b = 2$. Now, N is even$\Longleftrightarrow N^2-1$ is odd. The SU$(2)_b$ theory must have an even number of chiral doublet fermions in order to avoid a global anomaly, so if N is odd, the N^2-1 $(\psi^i_j)^\alpha_L$ form an acceptable SU$(2)_b$ fermion sector by themselves, while if N is even, then we obtain an acceptable fermion sector by adding an odd number of additional SU$(2)_b$ doublets. We shall choose this odd number to be the minimal value, namely, one, with the fermion

$$\omega^\alpha_L \quad \text{included for even } N. \tag{5.35}$$

For these two cases, the SU$(2)_b$ beta function has as its leading coefficient

$$b_1 = \begin{cases} \frac{1}{3}(23 - N^2) & \text{for } N \text{ odd} \\ \frac{1}{3}(22 - N^2) & \text{for } N \text{ even.} \end{cases} \tag{5.36}$$

The requirement that the SU$(2)_b$ theory be asymptotically free is thus that $N < \sqrt{23}$ for odd N and $N < \sqrt{22}$ for even N. These amount to the possibilities $N = 3$ for odd N and $N = 2, 4$ for even N. We have already dealt with the case $N = 3$ above, so here we focus on the case $N = 4$. As discussed in the introduction, these are necessary but not sufficient conditions; we also must require that the fermion content of the SU$(2)_b$ theory be sufficiently small that as the reference energy scale μ decreases, the coupling $\alpha_b(\mu)$ will increase sufficiently so that the SU$(2)_b$ gauge interaction will produce bilinear fermion condensates instead of evolving in a chirally symmetric manner into the infrared. For SU(2), the critical number of Dirac fermions, $N_{f,\mathrm{cr}}$, below which this condensation will occur is estimated to be $N_{f,\mathrm{cr}} \simeq 8$ [8]. Because SU(2) has only (pseudo)real representations, we can rewrite the theory with a given number of chiral Weyl doublets as a theory with half this number of Dirac doublets. For $N = 4$, we would have $N^2 = 16$ chiral doublets, or eight Dirac doublets, which is marginal. Assuming that the SU$(2)_b$ sector does, indeed, produce bilinear fermion condensates, these would occur in the most attractive channel, which is $2 \times 2 \to 1$ in SU$(2)_b$. These would have either the form

$$\langle \epsilon_{\alpha\beta} [\psi^{\alpha}_{a,L}{}^T C \psi^\beta_{b,L}]_{\mathrm{as}} \rangle \tag{5.37}$$

or the form

$$\langle \epsilon_{\alpha\beta} \psi^{\alpha}_{a,L}{}^T C \omega^\beta_L \rangle, \tag{5.38}$$

where in Eq. (5.37) the symbol $[\ldots]_{\mathrm{as}}$ means an antisymmetric SU(4) combination of the two adjoint fermion fields. In both cases, the condensate thus transforms as an adjoint of SU(4). A vacuum alignment argument implies that the condensates form in such a way as to preserve the largest subgroup in SU(4). The order of the subgroup SU(3) ⊗ U(1) is 9, which is greater than the order of the subgroup SU(2) ⊗ SU(2) ⊗ U(1), which is 7. Hence, from a vacuum alignment argument, one may infer that the condensate is proportional to the SU(4) generator $T_{15} = (2\sqrt{6})^{-1}\mathrm{diag}(1,1,1,-3)$, leading to the $N = 4$ special case of the symmetry-pattern pattern (5.1).

We next consider possible values $N_b \geq 3$ for the gauge group symmetry SU(N_b) responsible for the dynamical breaking of SU(N). In this case, for the rest of the G_b-nonsinglet fermions we choose

$$\omega^\alpha_{p,L} \colon (N^2-1)(1, \bar{N}_b), \tag{5.39}$$

where the notation $\bar{N}_b$ means the conjugate fundamental representation and here the copy number takes on the values $1 \leq p \leq N^2 - 1$. This ensures that the SU(N_b) theory has zero gauge anomaly. With the fermions (5.34) and (5.39) [and with the SU(N) interaction taken as negligibly weak], the SU(N_b) theory is vectorlike. This is in accord with one of the conditions that we imposed above, which guarantees that the G_b symmetry does not self-break when it becomes strongly coupled. Expressed in manifestly vectorial form, it has N^2-1 Dirac fermions transforming according to the fundamental representation of SU(N_b).

The beta function for the $SU(N_b)$ coupling has leading coefficient

$$(b_1)_{SU(N_b)} = \frac{1}{3}[11N_b - 2(N^2 - 1)]. \quad (5.40)$$

The requirement that the $SU(N_b)$ theory be asymptotically free is thus

$$N_b > \frac{2(N^2 - 1)}{11}. \quad (5.41)$$

As noted above, this is a necessary, but not sufficient, condition for the $SU(N_b)$ theory to produce the requisite condensates. We must also require that, for a given value of N_b, the fermion content of the $SU(N_b)$ sector must be small enough so that as the theory evolves down in energy scale, it produces condensates instead of evolving into the infrared in a chirally symmetric (conformal) manner. For a vectorial asymptotically free $SU(N)$ gauge theory with N_f copies of Dirac fermions (with zero Lagrangian masses) in the fundamental representation, if N_f is smaller than a critical value, $N_{f,\mathrm{cr}}$, then as the reference scale decreases from large values, the coupling will eventually grow large enough to form condensates which generically break the global chiral symmetry. In contrast, if $N_f > N_{f,\mathrm{cr}}$, then the theory will evolve from the ultraviolet to the infared without any spontaneous chiral symmetry breaking, yielding conformal behavior. A combined analysis of the beta function and solutions of the Dyson-Schwinger equation for the fermion propagator in the approximation of one-gluon exchange yields the result [8]

$$N_{f,\mathrm{cr}} = \frac{2N_b(50N_b^2 - 33)}{5(5N_b^2 - 3)}. \quad (5.42)$$

Although the Dyson-Schwinger analysis does not directly incorporate effects of either confinement or instantons, it has been shown that these two effects affect $N_{f,\mathrm{cr}}$ in opposite ways, so that neglecting both of them can still yield a reasonably accurate result [16]. Recent lattice simulations of SU(3) gauge theory with variable numbers N_f of light fermions in the fundamental representation are [taking account of theoretical uncertainties in both Eq. (5.42) and the lattice work] broadly consistent with Eq. (5.42) [17]. The lattice study of SU(3) with fermions in the fundamental representation provides further evidence indicating that technicolor theories with a slowly running coupling associated with an approximate infrared fixed point can lead to a reduced TC contribution to the S parameter [17]. Although this lattice work does not test the prediction of $N_{f,\mathrm{cr}}$ for $N_b \neq 3$, it makes it plausible that this prediction could also be reasonably accurate. For these values of N_b, Eq. (5.42) rapidly approaches the asymptotic large-N_b form $N_{f,\mathrm{cr}} \simeq 4N_b$. We thus require that N_b be sufficiently large that the $SU(N_b)$ theory with its $N_f = N^2 - 1$ Dirac fermions will exhibit spontaneous chiral symmetry breaking and confinement instead of evolving down in energy in a chirally symmetric non-Abelian Coulomb (conformal) phase. Using the prediction of Eq. (5.42), we thus obtain the lower bound $N_{f,\mathrm{cr}} \simeq 4N_b > N^2 - 1$, i.e.,

$$N_b > \frac{(N^2 - 1)}{4}. \quad (5.43)$$

With the fermion content as specified via Eqs. (5.34) and (5.39), and in the approximation that one turns off the $SU(N)$ gauge interaction, the $SU(N_b)$ sector has a classical global symmetry of the form $U(N^2-1)_\psi \otimes U(N^2-1)_\omega$, or, equivalently, $SU(N^2-1)_\psi \otimes SU(N^2-1)_\omega \otimes U(1)_\psi \otimes U(1)_\omega$, where the subscripts indicate which fields are involved in the respective symmetry transformations. Both the $U(1)_\psi$ and $U(1)_\omega$ are broken by $SU(N_b)$ instantons, but the linear combination corresponding to the difference of the currents for the ψ and ω fields is conserved in the presence of instantons. We will denote this symmetry as $U(1)'$. The actual (nonanomalous) global symmetry of the G_b theory at the high scale is thus

$$SU(N^2-1)_\psi \otimes SU(N^2-1)_\omega \otimes U(1)'. \quad (5.44)$$

We comment on this further below.

Now we turn on the $SU(N)$ gauge interaction. This explicitly breaks the above global chiral symmetry. However, just as the breaking of chiral $SU(2)_L \otimes SU(2)_R$ symmetry in QCD by electroweak interactions is weak, so also here this breaking is weak, since α_G is small at the scale Λ_b. We can fix the initial value of $\alpha_b(\mu)$ at a high value of μ so that as μ decreases to the scale Λ_b, this coupling grows sufficiently large to produce bilinear fermion condensates. These condensates will occur in the most attractive channel, which, for the above fermion content, is $N_b \times \bar{N}_b \to 1$. In general, these condensates would be of the form $\langle \psi^{\alpha\ T}_{a,L} C\omega_{p,\alpha,L}\rangle$. A vacuum alignment argument implies that these condensates will form in a manner so as to preserve the largest residual gauge symmetry. We regard this implication as very plausible, but add the obvious caveat that one must remember the theoretical uncertainties that are present in such a strongly coupled theory. Combining this implication from the vacuum alignment argument with our discussion above, we infer that the symmetry-breaking pattern is that $SU(N)$ breaks to the maximal subgroup $SU(N-1) \otimes U(1)$ as in Eq. (5.1), so that the condensate would have the form

$$\langle \psi^{\alpha\ T}_{a,L} C\omega_{p,\alpha,L}\rangle \quad \text{with} \quad a = N^2 - 1. \quad (5.45)$$

That is, it would transform like the last of the generators in the Cartan algebra of $SU(N)$,

$$(T_{a=N^2-1})^i_j = \frac{1}{\sqrt{2N(N-1)}}\delta^i_j \times \begin{cases} 1 & \text{for } 1 \le i \le N-1 \\ -(N-1) & \text{for } i = N. \end{cases} \quad (5.46)$$

Here we emphasize an important contrast between this dynamical symmetry-breaking mechanism and the Higgs mechanism. In our introductory discussion above, we have already noted a number of the differences between the Higgs mechanism and a dynamical mechanism for breaking a gauge symmetry. Among other differences, for example, the Higgs mechanism leads to the appearance of at least one physical pointlike Higgs field, whereas a dynamical mechanism does not yield such a particle (although it may yield composite $J = 0$ bound states). Furthermore, if one uses a Higgs mechanism to break SU(N), then by appropriate choices of the parameters, one can guarantee that the minimum of the potential occurs for a Higgs VEV of the form (5.20) or (5.23), so that the symmetry breaking is of the type (5.5) or (5.8), rather than (5.1). However, in the dynamical approach to SU(N) breaking, once one specifies the gauge and fermion content, there are no free parameters, and the theory is, in principle, completely predictive. Although the dynamical symmetry-breaking mechanism involves a strongly coupled gauge sector, one can use most attractive channel criteria and vacuum alignment arguments to make a plausible inference about what form the bilinear fermion condensate will take, namely, as discussed above, the form that preserves the largest residual symmetry, $\mathrm{SU}(N-1) \otimes \mathrm{U}(1)$. These most attractive channel and vacuum alignment properties would be manifest if one were to explicitly calculate the effective potential for the composite operator represented by the condensate, along the lines of Ref. [18]. In this context, one may recall that the Higgs potential was partially motivated by the original Ginzburg-Landau free energy functional in phenomenological models of superconductivity, and retrospectively, from the perspective of the Bardeen-Cooper-Schrieffer theory and the Cooper pair condensate, one may view the Ginzburg-Landau free energy functional as an approximate way to represent the physics of this Cooper pair condensate. This is, of course, not a precise isomorphism, but only a partial correspondence. As recalled above, there are important differences between a Higgs and dynamical mechanism for breaking a gauge symmetry. To the extent that one may regard a Higgs potential as embodying some of the same physics as an effective potential for a composite operator represented by bilinear fermion condensate(s), one may observe that the pattern of symmetry breaking inferred from the dynamical approach makes definite predictions for the coefficients in the corresponding Higgs potential. First, because the Lagrangian in the dynamical model is invariant under the separate global transformations $\psi^{\alpha}_{a,L} \to -\psi^{\alpha}_{a,L}$ and $\omega^{\alpha}_{p,L} \to -\omega^{\alpha}_{p,L}$, it follows that an analogous effective potential for the condensate (5.45) should not contain odd powers of this condensate. Our dynamical model for the symmetry breaking of an SU(N) gauge theory using fermions transforming as an adjoint representation of SU(N) then predicts that in a corresponding Higgs approach, in order to obtain the same pattern of symmetry breaking, the coefficients of the Higgs potential should have the following properties: (i) $\mu^2 < 0$, for symmetry breaking; (ii) $\lambda_2 < 0$, yielding the specific symmetry-breaking pattern (5.1); and the stability properties that (iii) $\lambda_1 > 0$ and (iv) λ_2 satisfy the lower bound in Eq. (5.26).

With the symmetry-breaking pattern as given by (5.1), there are then $2(N-1)$ broken generators of SU(N), and Nambu-Goldstone modes formed from the fermion condensates are absorbed by the gauge bosons corresponding to these broken generators, forming the longitudinal components of the resultant massive vector bosons. These masses are of order $g\Lambda_b$. This is reminiscent of the process whereby Nambu-Goldstone modes in the technicolor mechanism for electroweak symmetry breaking are absorbed to give the $W^{\pm}$ and Z bosons their masses. The SU(N_b)-nonsinglet fermions involved in the condensate (5.45) gain dynamical masses of order Λ_b and are integrated out of the low-energy effective theory that is operative at scales μ below Λ_b. Since, by construction, the SU(N_b) theory confines, the spectrum of the SU(N_b) theory includes a set of SU(N_b)-singlet mesons, baryons, and glueballs that form at the scale Λ_b.

VI. REMARKS ON OTHER DIRECTIONS OF STUDY

We comment here on some other related directions of study that could be interesting to pursue. One could construct models with dynamical symmetry breaking of other gauge symmetries and compare results with those obtained via Higgs scenarios. An example of this would be models with extended electroweak gauge groups such as $G = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \otimes \mathrm{U}(1)_{B-L}$ and $G = \mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$, for which dynamical mechanisms were presented in Ref. [19]. In a more abstract direction, one could consider groups such as $G = \mathrm{SO}(N)$. One could also study the breaking of SU(N) by fields transforming according to representations other than the fundamental and adjoint, such as the rank-2 symmetric and antisymmetric tensor representations.

One could also study situations in which the G gauge interaction is not weakly coupled at the scale Λ_b where the G_b interaction becomes strongly coupled, so that there is generically a combination of self-breaking of G and induced breaking of G by G_b. Indeed, in reasonably ultraviolet-complete ETC theories, the sequential breaking of the ETC gauge symmetry down to the residual exact technicolor symmetry typically involves both self-breaking of ETC, which is a strongly coupled, chiral gauge symmetry, and induced breaking by an auxiliary gauge interaction called hypercolor in [20]. A similar statement applies to ultraviolet completions of topcolor-assisted technicolor models that include the necessary additional gauge interactions to produce the required symmetry breakings [3,21].

Although our study is primarily intended as a comparison of gauge symmetry breaking by dynamical and Higgs mechanisms in a general field theoretic context, it is appropriate to address the question of possible dynamical symmetry breaking of a grand unified symmetry. We recall that there has long been interest in grand unified theories (GUTs) which embed the three factor groups of G_{SM} in a single group, since this would unify quarks and leptons, predict the ratios of the three SM gauge couplings, and quantize electric charge [22–24]. Much work on GUTs has been done in a supersymmetric context, since supersymmetry remedies the gauge hierarchy problem of the standard model and since the minimal supersymmetric standard model naturally yields gauge coupling unification. There have also been studies of the question of whether some type of grand unification could feasibly be achieved in a theoretical context involving dynamical electroweak symmetry breaking [25]. It is natural to ask whether one could use induced dynamical breaking of a GUT gauge symmetry such as SU(5) or SO(10), which is weakly coupled at the GUT scale, M_{GUT}, using a (vectorial non-Abelian, asymptotically free) G_b gauge interaction that becomes strongly coupled at this scale. One could, of course, argue that such an approach differs from the original purpose of the grand unification, which was to obtain an ultraviolet-scale theory with only a single gauge group and gauge coupling. Indeed, such an induced GUT symmetry-breaking scenario appears problematic, since in order to produce the requisite bilinear fermion condensates, the G_b interaction would necessarily have to confine, and this would generically lead to stable G_b-singlet baryons with masses of order M_{GUT}. With plausible estimates for the relevant reaction cross sections, one finds that these G_b-singlet baryons would contribute far too much to the dark matter in the universe [26]. Interestingly, even if a dynamical approach to breaking a GUT symmetry were not excluded by its production of excessive dark matter, it would predict that a GUT group such as SU(5) would preferentially break to $SU(4) \otimes U(1)$ rather than the SM group, $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. In the conventional SU(5) GUT, the latter breaking to G_{SM} is obtained by a Higgs mechanism with a Higgs field transforming as the adjoint representation [22]. Modern GUT theories also make use of string-inspired mechanisms for the GUT gauge symmetry breaking, including higher-dimension operators and Wilson lines [24].

VII. CONCLUSIONS

In conclusion, in this paper we have constructed and analyzed theories with a gauge symmetry in the ultraviolet of the form $G \otimes G_b$, in which the vectorial, asymptotically free G_b gauge interaction becomes strongly coupled at a scale where the G interaction is weakly coupled and produces bilinear fermion condensates that dynamically break the G symmetry. We have compared the results to those obtained with a Higgs mechanism. There are many interesting contrasting properties of these two approaches to breaking a gauge symmetry. The Higgs mechanism is perturbative, and one has the freedom, by appropriate choices of parameters in the Higgs potential, to determine whether and, in general, how the symmetry breaks. In contrast, the dynamical approach is arguably more predictive, in the sense that, provided that one has chosen the gauge and field content of the G_b sector appropriately, there are no free parameters to vary; the G_b gauge interaction will confine and produce fermion condensates that break the G symmetry. Most attractive channel and vacuum alignment arguments provide a plausible guide to enable one to infer which channel(s) have fermion condensation, and what the form of this condensation is, thereby predicting the resultant pattern of symmetry breaking. In the dynamical models that we have constructed, we produce this breaking by introducing fermions that are nonsinglets under both G and G_b. In the course of our analysis, we have discussed how the gauge symmetry G can be broken not just for the case where it is chiral (as in electroweak symmetry breaking), but also for the case where it is vectorial. We have compared Higgs and dynamical mechanisms for breaking SU(3) via a Higgs field or condensate transforming according to the fundamental or adjoint representation. We have also carried such an analogous study for SU(N) with $N \geq 4$. Our present study helps to elucidate the differences between the Higgs and dynamical mechanisms for breaking a gauge symmetry. We believe such theoretical studies are useful since it is still an open question what mechanism is responsible for breaking electroweak gauge symmetry or a grand unified symmetry.

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APPENDIX

Here we define some notation used in the text. For a gauge group G_j we denote the running gauge coupling as $g_j(\mu)$, where μ is the Euclidean reference momentum, and we denote $\alpha_j(\mu) = g_j(\mu)^2/(4\pi)$. The beta function is $\beta_{G_j} = dg_j/dt$, where $dt = d\ln\mu$. We write

$$\frac{d\alpha_j}{dt} = -\frac{\alpha_j^2}{2\pi}\left[b_1 + \frac{b_2\alpha_j}{4\pi} + O(\alpha_j^3)\right], \tag{A1}$$

where the first two coefficients, b_1 and b_2, are scheme-independent. For a representation R of a Lie group G, the quadratic Casimir invariant $C_2(R)$ is defined by $\sum_{a=1}^{\mathrm{order}(G)} \sum_{j=1}^{\dim(R)} (T_a)_{ij}(T_a)_{jk} = C_2(R)\delta_{ik}$.

[1] S. Weinberg, Phys. Rev. D **19**, 1277 (1979); L. Susskind, Phys. Rev. D **20**, 2619 (1979).

[2] C. Quigg and R. Shrock, Phys. Rev. D **79**, 096002 (2009).

[3] C. T. Hill and E. H. Simmons, Phys. Rep. **381**, 235 (2003); Workshop on Dynamical Electroweak Symmetry Breaking, Southern Denmark University, Odense, 2008, http://hep.sdu.dk/dewsb; R. S. Chivukula, M. Narain, and J. Womersley (Particle Data Group), J. Phys. G **37**, 1340 (2010); F. Sannino, Acta Phys. Pol. B **40**, 3533 (2009).

[4] G. Aad *et al.* (ATLAS Collaboration), Phys. Rev. Lett. **105**, 161801 (2010); , Report No. CONF-2010-093.

[5] CMS Collaboration, arXiv:1010.0203.

[6] S. Nussinov and R. Shrock, Phys. Rev. D **82**, 034031 (2010).

[7] Indeed, if one were to envision fixing λ at an arbitrarily high scale μ, then the value of λ at a low scale would approach zero, the property known as triviality.

[8] T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. Lett. **77**, 1214 (1996); T. Appelquist, A. Ratnaweera, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. D **58**, 105017 (1998); T. Appelquist, K. Lane, and U. Mahanta, Phys. Rev. Lett. **61**, 1553 (1988).

[9] R. Dashen, Phys. Rev. D **3**, 1879 (1971); S. Weinberg, Phys. Rev. D **13**, 974 (1976); M. Peskin, Nucl. Phys. **B175**, 197 (1980); J. Preskill, Nucl. Phys. **177**, 21 (1981).

[10] In general, if one has fermions transforming according to the representations R_1 and R_2 of a gauge group, then a measure of the attractiveness of the condensation channel $R_1 \times R_2 \to R_{\rm cond}$ is $\Delta C_2 = C_2(R_1) + C_2(R_2) - C_2(R_{\rm cond})$, where $C_2(R)$ is the quadratic Casimir invariant for the representation R.

[11] This is to be contrasted with the situation in a strongly coupled chiral gauge theory, where the associated gauge symmetry does self-break [12].

[12] S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. **B169**, 373 (1980).

[13] This suppression of low-energy amplitudes involving (P)NGBs is demonstrated experimentally via the success of low-energy pion theorems, as discussed, e.g., in S. L. Adler and R. F. Dashen, *Current Algebra* (Benjamin, New York, 1968).

[14] L.-F. Li, Phys. Rev. D **9**, 1723 (1974); V. Elias, S. Eliezer, and A. R. Swift, Phys. Rev. D **12**, 3356 (1975).

[15] Although some of our discussion in this section is review, the bounded range for negative λ_2 in Eq. (5.26) and its consequences do not seem to have been noted before in the literature.

[16] S. J. Brodsky and R. Shrock, Phys. Lett. B **666**, 95 (2008).

[17] Recent lattice simulations with fermions in the fundamental representation include T. Appelquist, G. Fleming, and E. Neil, Phys. Rev. Lett. **100**, 171607 (2008); Phys. Rev. D **79**, 076010 (2009); T. Appelquist *et al.*, Phys. Rev. Lett. **104**, 071601 (2010); A. Deuzeman, M. P. Lombardo, and E. Pallante, Phys. Lett. B **670**, 41 (2008); Proc. Sci. LATTICE2008 (**2008**) 060 [arXiv:0810.1719]; Phys. Rev. D **82**, 074503 (2010); X.-Y. Jin and R. Mawhinney, Proc. Sci., LAT2008 (2008) 059 [arXiv:0812.0413]; Z. Fodor, K. Holland, J. Kuti, D. Nogradi, and C. Schroeder, Phys. Lett. B **681**, 353 (2009); Proc. Sci. LAT2009 (**2009**) 058 [arXiv:0911.2934]; arXiv:0911.2463; A. Hasenfratz, arXiv:1004.1004. Recent reviews include T. Appelquist, Prog. Theor. Phys. **180**, 72 (2009); G. Fleming, in Int. Conf. on High Energy Physics ICHEP, Paris, 2010, http://www.ichep2010.fjr.

[18] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D **10**, 2428 (1974).

[19] T. Appelquist and R. Shrock, Phys. Rev. Lett. **90**, 201801 (2003).

[20] T. Appelquist and J. Terning, Phys. Rev. D **50**, 2116 (1994); T. Appelquist and R. Shrock, Phys. Lett. B **548**, 204 (2002); T. Appelquist, M. Piai, and R. Shrock, Phys. Rev. D **69**, 015002 (2004).

[21] T. A. Ryttov and R. Shrock, Phys. Rev. D **82**, 055012 (2010).

[22] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

[23] H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).

[24] S. Raby, in (Particle Data Group), J. Phys. G **37**, 193 (2010).

[25] E. Farhi and L. Susskind, Phys. Rev. D **20**, 3404 (1979); N. D. Christensen and R. Shrock, Phys. Rev. D **72**, 035013 (2005); S. B. Gudnason, T. A. Ryttov, and F. Sannino, Phys. Rev. D **76**, 015005 (2007); N. Chen and R. Shrock, Phys. Rev. D **78**, 035002 (2008).

[26] T. A. Ryttov and R. Shrock (unpublished).