Normal tau polarization as a sensitive probe of *CP* violation in chargino decay

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CP violation in the spin-spin correlations in chargino production and subsequent two-body decay into a tau and a tau-sneutrino is studied at the ILC. From the normal polarization of the tau, an asymmetry is defined to test the *CP*-violating phase of the higgsino mass parameter μ. Asymmetries of more than $\pm 70\%$ are obtained, also in scenarios with heavy first and second generation sfermions. Bounds on the statistical significances of the *CP* asymmetries are estimated. As a result, the normal tau polarization in the chargino decay is one of the most sensitive probes to constrain or measure the phase φ_μ at the ILC, motivating further detailed experimental studies.

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I. INTRODUCTION

Within the framework of the standard electro-weak model (SM), the complex Cabbibo-Kobayashi-Maskawa (CKM) matrix is the origin for *CP* violation [1–3]. It explains all the current laboratory data, but it is not sufficient to generate the matter-antimatter asymmetry of the universe [4]. Thus, further theories have to be investigated that offer new sources of *CP* violation [5]. The minimal supersymmetric standard model (MSSM) is a promising extension of the SM [6]. To prevent the supersymmetric partners of the known particles from appearing below the LEP and Tevatron energy scale, there has to be supersymmetry (SUSY) break down, at least at the electroweak energy scale [7]. Several of the supersymmmetric parameters can be complex, including the higgsino mass parameter μ.

Concerning low energy observables, the corresponding SUSY *CP* phases lead to *T*-violating electric dipole moments (EDMs) that already would be far beyond the experimental bounds [3,8–12]. This constitutes the SUSY *CP* problem: The SUSY phases have to be considerably suppressed unless cancellations appear between different EDM contributions or the SUSY spectrum is beyond the TeV scale [13–17]. And, indeed, attempts to naturally solve the SUSY *CP* problem often still require a certain amount of tuning among the SUSY masses, phases, and parameters. Recently proposed models are *split-SUSY* [18], *inverted-hierarchy* models [19] and also *focus-point* scenarios that attempt to restore naturalness [20]. These models have also been proposed to solve the SUSY flavour problem, to ensure proton stability, and to fulfil cosmological bounds, like constraints on dark matter or primordial nucleosynthesis. It is clear that *CP*-sensitive observables outside the low energy sector have to be proposed and measured in order to tackle the SUSY *CP* problem and to reveal the underlying SUSY model.

Concerning future collider experiments at the LHC [21] and ILC [22], SUSY phases alter SUSY particle masses, cross sections, branching ratios [23,24], and longitudinal polarizations of final-state fermions [25]. Although the SUSY phases can change these *CP*-even observables by an order of magnitude or more, only *CP*-odd observables are a direct evidence of *CP* violation [26]. *CP*-odd rate asymmetries of cross sections, distributions, or partial decay widths [27], however, usually do not exceed 10%, as they require the presence of absorptive phases, unless they are resonantly enhanced [28–31]. At tree level, larger *T*-odd and *CP*-odd observables can be defined with triple or epsilon products of particle momenta and/or spins [32,33].

At the LHC, *CP*-odd triple product asymmetries have been studied in the decays of third generation squarks [34–38] and three-body decays of neutralinos, which originate from squarks [38–40]. Since triple products are not boost invariant, compared to epsilon products, some of these studies have included boost effects at the LHC [37–40]. For the ILC, triple product asymmetries have been studied in the production and decay of neutralinos [41–45] and charginos [45–47], also using transversely polarized beams [48]. The result of these studies is that the largest asymmetries of the order of 60% can be obtained if final fermion polarizations are analyzed, like the normal tau polarization in neutralino decay $\tilde{\chi}_i^0 \to \tilde{\tau}_k \tau$ [41,42].

Although the experimental reconstruction of taus is much more involved than those for electrons or muons, tau decays in principle allow for a measurement of their polarization [49,50]. The fermion polarization contains additional and unique information on the SUSY couplings [51,52], which will be lost if only electron or muon momenta are considered for measurements. Taus might also be the dominant final-state leptons. In particular, in inverted hierarchy models, sleptons of the first and second generations are heavy, such that electron and muon final states will be rare in SUSY particle decay chains. We are thus motivated to analyze the potential of *CP*-odd effects in chargino production

$$e^+ + e^- \to \tilde{\chi}_i^\pm + \tilde{\chi}_j^\mp, \qquad i,j = 1,2, \tag{1}$$

with longitudinally polarized beams and the subsequent two-body decay of one chargino into a polarized tau

$$\tilde{\chi}_i^\pm \to \tau^\pm + \tilde{\nu}_\tau^{(*)}. \tag{2}$$

For the chargino decay, we define a T-odd asymmetry from the tau polarization normal to the plane spanned by the e^- beam and the tau momentum. It is highly sensitive to the CP phase φ_μ of the higgsino mass parameter μ, which enters the chargino sector. Since φ_μ is also the most constrained SUSY CP phase from EDM searches, this asymmetry is particularly important. Besides a MSSM scenario with light sleptons, we thus also analyze the asymmetries in an inverted hierarchy scenario, which relaxes the strong EDM constraints on the SUSY phases. Scenarios with an inverted hierarchy are attractive for our process of chargino production and decay, since not only is the chargino production cross section enhanced due to missing destructive sneutrino interference, but the rate of chargino decay into taus is also amplified.

In Sec. II, we present our formalism. We briefly review chargino mixing in the complex MSSM and give the relevant parts of the interaction Lagrangian. We calculate the τ spin-density matrix, the normal τ polarization and present the corresponding analytical formulae in the spin-density matrix formalism [53]. In Sec. III, we present our numerical results. We summarize and conclude in Sec. IV.

II. FORMALISM

A. Chargino mixing and complex couplings

In the MSSM, the charged winos $\tilde{W}^\pm$ and higgsinos $\tilde{H}^\pm$ mix, after electro-weak symmetry breaking, and form the chargino mass eigenstates $\tilde{\chi}_{1,2}^\pm$. In the $(\tilde{W}, \tilde{H})$ basis, their mixing is defined by the complex chargino mass matrix [6]

$$M_{\tilde{\chi}} = \begin{pmatrix} M_2 & m_W\sqrt{2}\sin\beta \\ m_W\sqrt{2}\cos\beta & \mu \end{pmatrix}. \tag{3}$$

At tree level, the chargino system depends on the SU(2) gaugino mass parameter M_2, the higgsino mass parameter μ, and the ratio $\tan\beta = v_2/v_1$ of the vacuum expectation values of the two neutral CP-even Higgs fields. We parameterize the CP violation by the physical phase φ_μ of $\mu = |\mu|e^{i\varphi_\mu}$, taking by convention M_2 real and positive, absorbing its possible phase by redefining the fields.

By diagonalizing the chargino matrix [6]

$$U^* M_{\tilde{\chi}} V^\dagger = \mathrm{diag}(m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^\pm}), \tag{4}$$

we obtain the chargino masses $m_{\tilde{\chi}_2^\pm} \geq m_{\tilde{\chi}_1^\pm} \geq 0$, as well as their couplings. In Appendix C, we give the analytic expressions for the two independent, unitary diagonalization matrices U and V. We shall use them for a qualitative understanding of the chargino mixing in the presence of a nonvanishing CP phase $\varphi_\mu \neq 0$.

At the ILC, chargino production $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-$ proceeds at tree level via $\tilde{\nu}_e$ exchange in the t channel, and Z, γ exchange in the s channel, see the Feynman diagrams in Fig. 1. Note that the photon exchange contribution vanishes for nondiagonal chargino production $i \neq j$. The relevant terms in the MSSM Lagrangian for chargino production are [6,54,55]

$$\mathcal{L}_{\gamma\tilde{\chi}_j^+\tilde{\chi}_i^-} = -eA_\mu \overline{\tilde{\chi}_i^+}\gamma^\mu \tilde{\chi}_j^+ \delta_{ij}, \qquad e > 0, \tag{5}$$

$$\mathcal{L}_{e\tilde{\nu}_e\tilde{\chi}_i^+} = -gV_{i1}^* \overline{\tilde{\chi}_i^+}^C P_L e \tilde{\nu}_e^* + \text{H.c.}, \tag{6}$$

$$\mathcal{L}_{Z^0\tilde{\chi}_i^+\tilde{\chi}_j^-} = \frac{g}{\cos\theta_w} Z_\mu \overline{\tilde{\chi}_j^+}\gamma^\mu [O_{ji}^{\prime L} P_L + O_{ji}^{\prime R} P_R]\tilde{\chi}_i^+, \tag{7}$$

with $i, j = 1, 2$, and the projectors $P_{L,R} = (1 \mp \gamma^5)/2$. In Eq. (5) "$e$" refers to the electron coupling; in Eq. (6) it refers to the electron spinor field. Furthermore,

$$O_{ij}^{\prime L} = -V_{i1}V_{j1}^* - \frac{1}{2}V_{i2}V_{j2}^* + \delta_{ij}\sin^2\theta_w, \tag{8}$$

$$O_{ij}^{\prime R} = +U_{i1}U_{j1}^* - \frac{1}{2}U_{i2}U_{j2}^* + \delta_{ij}\sin^2\theta_w, \tag{9}$$

with the weak mixing angle θ_w and the weak coupling constant $g = e/\sin\theta_w$. For diagonal chargino production, $i = j$, the Z-chargino couplings are real, see Eqs. (8) and (9), and the production amplitude has no CP-violating terms at tree level.

For the subsequent chargino decay into the tau $\tilde{\chi}_i^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$, the contribution to the Lagrangian is [6]

$$\mathcal{L}_{\tilde{\chi}_i^+\tau\tilde{\nu}_\tau} = -g\overline{\tilde{\chi}_i^+}^C (c_{i\tau}^L P_L + c_{i\tau}^R P_R)\tau\tilde{\nu}_\tau^* + \text{H.c.}, \tag{10}$$

with the left and right τ-$\tilde{\nu}_\tau$-chargino couplings

$$c_{i\tau}^L = V_{i1}^*, \qquad c_{i\tau}^R = -Y_\tau U_{i2} \tag{11}$$

and the Yukawa coupling

$$Y_\tau = \frac{m_\tau}{\sqrt{2}m_W\cos\beta}. \tag{12}$$

In the following, we present the analytical formulae for the normal tau polarization, which will be a sensitive probe of CP violation in the chargino system.

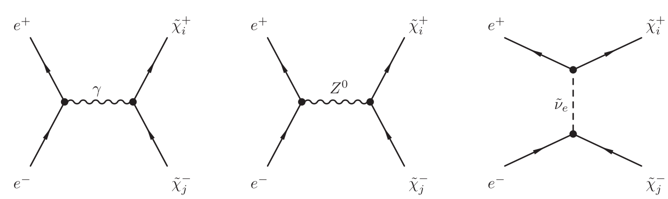

FIG. 1. Feynman diagrams for chargino production.

B. Tau spin-density matrix

The unnormalized 2×2, hermitean τ spin-density matrix for chargino production, Eq. (1), and decay, Eq. (2), reads

$$\rho^{\lambda_\tau \lambda'_\tau} \equiv \int (|\mathcal{M}|^2)^{\lambda_\tau \lambda'_\tau} d\mathcal{L}ips, \tag{13}$$

with the amplitude $\mathcal{M}$ and the Lorentz invariant phase-space element $d\mathcal{L}ips$, for details see Appendix B. The τ helicities are denoted by λ_τ and λ'_τ. In the spin-density matrix formalism [53], the amplitude squared for the full process: production and decay, is given by[1]

$$(|\mathcal{M}|^2)^{\lambda_\tau \lambda'_\tau} = |\Delta(\tilde{\chi}_i)|^2 \sum_{\lambda_i, \lambda'_i} (\rho_{\rm P})^{\lambda_i \lambda'_i} (\rho_{\rm D})^{\lambda_\tau \lambda'_\tau}_{\lambda'_i \lambda_i}, \tag{14}$$

with the chargino helicities denoted λ_i, λ'_i, and an implicit sum over the helicities of chargino $\tilde{\chi}_j$, whose decay is not further considered. The amplitude squared decomposes into the remnant of the chargino propagator

$$\Delta(\tilde{\chi}_i) = \frac{1}{p^2_{\tilde{\chi}_i} - m^2_{\tilde{\chi}_i} + i m_{\tilde{\chi}_i} \Gamma_{\tilde{\chi}_i}}, \tag{15}$$

with mass $m_{\tilde{\chi}_i}$ and width $\Gamma_{\tilde{\chi}_i}$ of the decaying chargino and the unnormalized spin-density matrices $\rho_{\rm P}$ for production (P) and $\rho_{\rm D}$ for decay (D). They can be expanded in terms of the Pauli matrices σ^a

$$(\rho_{\rm P})^{\lambda_i \lambda'_i} = 2[{\rm P}\delta^{\lambda_i \lambda'_i} + \Sigma^a_{\rm P} (\sigma^a)^{\lambda_i \lambda'_i}], \tag{16}$$

$$(\rho_{\rm D})^{\lambda_\tau \lambda'_\tau}_{\lambda'_i \lambda_i} = {\rm D}^{\lambda_\tau \lambda'_\tau} \delta_{\lambda'_i \lambda_i} + (\Sigma^a_{\rm D})^{\lambda_\tau \lambda'_\tau} (\sigma^a)_{\lambda'_i \lambda_i}, \tag{17}$$

with an implicit sum over $a = 1, 2, 3$. For chargino production, the analytical formulae for the coefficients P and $\Sigma^a_{\rm P} = \Sigma^\mu_{\rm P} s^a_{\tilde{\chi}_i,\mu}$, which depend on the chargino spin vectors $s^a_{\tilde{\chi}_i}$, are explicitly given in Ref. [54]. In that convention, a sum over the helicities λ_j, λ'_j of chargino $\tilde{\chi}^\mp_j$, whose decay we do not further consider, gives the factor of 2 in Eq. (16).

For the chargino $\tilde{\chi}^\pm_i$ decay into a tau, Eq. (2), we define a set of tau spin basis vectors s^b_τ, see Appendix A. We then expand the coefficients of the decay matrix $\rho_{\rm D}$, Eq. (17),

$${\rm D}^{\lambda_\tau \lambda'_\tau} = {\rm D}\delta^{\lambda_\tau \lambda'_\tau} + {\rm D}^b (\sigma^b)^{\lambda_\tau \lambda'_\tau}, \tag{18}$$

$$(\Sigma^a_{\rm D})^{\lambda'_\tau \lambda_\tau} = \Sigma^a_{\rm D} \delta^{\lambda_\tau \lambda'_\tau} + \Sigma^{ab}_{\rm D} (\sigma^b)^{\lambda_\tau \lambda'_\tau}, \tag{19}$$

with an implicit sum over $b = 1, 2, 3$. A calculation of the expansion coefficients yields

$$\begin{aligned} {\rm D} &= \frac{g^2}{2}(|V_{i1}|^2 + Y^2_\tau |U_{i2}|^2)(p_{\tilde{\chi}_i} \cdot p_\tau) \\ &\quad - g^2 Y_\tau \Re\mathfrak{e}\{V_{i1} U_{i2}\} m_{\tilde{\chi}_i} m_\tau, \end{aligned} \tag{20}$$

$${\rm D}^b = \mathop{}^{+}_{(-)} \frac{g^2}{2}(|V_{i1}|^2 - Y^2_\tau |U_{i2}|^2) m_\tau (p_{\tilde{\chi}_i} \cdot s^b_\tau), \tag{21}$$

$$\Sigma^a_{\rm D} = \mathop{}^{-}_{(+)} \frac{g^2}{2}(|V_{i1}|^2 - Y^2_\tau |U_{i2}|^2) m_{\tilde{\chi}_i} (p_\tau \cdot s^a_{\tilde{\chi}_i}), \tag{22}$$

$$\begin{aligned} \Sigma^{ab}_{\rm D} &= g^2 Y_\tau \Re\mathfrak{e}\{V_{i1} U_{i2}\}[(p_{\tilde{\chi}_i} \cdot p_\tau)(s^a_{\tilde{\chi}_i} \cdot s^b_\tau) \\ &\quad - (p_\tau \cdot s^a_{\tilde{\chi}_i})(p_{\tilde{\chi}_i} \cdot s^b_\tau)] \\ &\quad - \frac{g^2}{2}(|V_{i1}|^2 + Y^2_\tau |U_{i2}|^2) m_{\tilde{\chi}_i} m_\tau (s^a_{\tilde{\chi}_i} \cdot s^b_\tau) \\ &\quad - g^2 Y_\tau \Im\mathfrak{m}\{V_{i1} U_{i2}\}[p_{\tilde{\chi}_i}, p_\tau, s^a_{\tilde{\chi}_i}, s^b_\tau], \end{aligned} \tag{23}$$

with the weak coupling constant g and the Yukawa coupling Y_τ, *cf.* Eq. (12). The formulae are given for the decay of a positive chargino $\tilde{\chi}^+_i \to \tau^+ \tilde{\nu}_\tau$. The signs in parentheses in Eqs. (21) and (22) hold for the charge conjugated decay $\tilde{\chi}^-_i \to \tau^- \tilde{\nu}^*_\tau$.

The spin-spin correlation term $\Sigma^{ab}_{\rm D}$ in Eq. (23) explicitly depends on the imaginary part $\Im\mathfrak{m}\{V_{i1} U_{i2}\}$ of the chargino matrices U and V, cf. Eqs. (C1) and (C2) in Appendix C. Thus this term is manifestly CP sensitive, i.e. sensitive to the phase φ_μ of the chargino sector. The imaginary part is multiplied by the totally antisymmetric (epsilon) product

$$\mathcal{E}^{ab} \equiv [p_{\tilde{\chi}_i}, p_\tau, s^a_{\tilde{\chi}_i}, s^b_\tau] \equiv \epsilon_{\mu\nu\rho\sigma} p^\mu_{\tilde{\chi}_i} p^\nu_\tau s^{a,\rho}_{\tilde{\chi}_i} s^{b,\sigma}_\tau. \tag{24}$$

We employ the convention for the epsilon tensor $\epsilon_{0123} = 1$. Since each of the spacial components of the four-momenta p or spin vectors s changes sign under a naive time transformation $t \to -t$, the epsilon product $\mathcal{E}^{ab}$ is T odd.

Inserting the density matrices Eqs. (16) and (17) into Eq. (14), we get for the amplitude squared

$$\begin{aligned} (|\mathcal{M}|^2)^{\lambda_\tau \lambda'_\tau} &= 4|\Delta(\tilde{\chi}_i)|^2[({\rm PD} + \Sigma^a_{\rm P} \Sigma^a_{\rm D})\delta^{\lambda_\tau \lambda'_\tau} \\ &\quad + ({\rm PD}^b + \Sigma^a_{\rm P} \Sigma^{ab}_{\rm D})(\sigma^b)^{\lambda_\tau \lambda'_\tau}], \end{aligned} \tag{25}$$

with an implicit sum over $a, b = 1, 2, 3$.

Note that the terms proportional to m_τ in Eqs. (20), (21), and (23) are negligible at high particle energies $E \gg m_\tau$, in particular ${\rm D}^b$ can be neglected in Eq. (25).

C. Normal tau polarization and CP-sensitive asymmetry

The τ polarization for the overall event sample is given by the expectation value of the Pauli matrices $\boldsymbol{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ [56]

$$\boldsymbol{\mathcal{P}} = \frac{{\rm Tr}\{\rho \boldsymbol{\sigma}\}}{{\rm Tr}\{\rho\}}, \tag{26}$$

with the τ spin-density matrix ρ, as given in Eq. (13). In our convention for the polarization vector $\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2,$

[1] In the following, in order to avoid cluttering the notation, we drop the $\pm$ superscripts on the chargino in formulae: $\tilde{\chi}_i$.

$\mathcal{P}_3$), the components $\mathcal{P}_1$ and $\mathcal{P}_3$ are the transverse and longitudinal polarizations in the plane spanned by $\mathbf{p}_\tau$ and $\mathbf{p}_{e^-}$, respectively, and $\mathcal{P}_2$ is the polarization normal to that plane. See our definition of the tau spin basis vectors s_τ^b in Appendix A.

The normal τ polarization is equivalently defined as

$$\mathcal{P}_2 \equiv \frac{N(\uparrow) - N(\downarrow)}{N(\uparrow) + N(\downarrow)}, \tag{27}$$

with the number of events N with the τ spin up ($\uparrow$) or down ($\downarrow$) with respect to the quantization axis $\mathbf{p}_\tau \times \mathbf{p}_{e^-}$, cf. Eq. (A12) in Appendix A. The normal τ polarization can thus also be regarded as an asymmetry

$$\mathcal{A}_\tau \equiv \mathcal{P}_2 = \frac{\sigma(\mathcal{T} > 0) - \sigma(\mathcal{T} < 0)}{\sigma(\mathcal{T} > 0) + \sigma(\mathcal{T} < 0)} \tag{28}$$

of the triple product

$$\mathcal{T} = \boldsymbol{\xi}_\tau \cdot (\mathbf{p}_\tau \times \mathbf{p}_{e^-}), \tag{29}$$

where $\boldsymbol{\xi}_\tau$ is the direction of the τ spin vector for each event. The triple product $\mathcal{T}$ is included in the spin-spin correlation term $\Sigma_{\rm P}^a \Sigma_{\rm D}^{ab}$ of the amplitude squared $|\mathcal{M}|^2$, Eq. (25). The asymmetry thus probes the term which contains the epsilon product $\mathcal{E}^{ab}$, Eq. (24).

Since, under naive time reversal $t \to -t$, the triple product $\mathcal{T}$ changes sign, the tau polarization $\mathcal{P}_2$, Eq. (28), is T odd. Because of CPT invariance [57], T-odd observables are related to CP-odd observables. In particular, the tau polarization $\mathcal{P}_2$ is also CP odd if contributions from absorptive phases, e.g. from intermediate s-state resonances or final-state interactions, can be neglected.[2]

Inserting now the explicit form of the density matrix ρ, Eq. (13), into Eq. (26), together with Eq. (25), we obtain the T-odd asymmetry

$$\mathcal{A}_\tau = \mathcal{P}_2 = \frac{\int \Sigma_{\rm P}^a \Sigma_{\rm D}^{ab=2} d\mathcal{L}ips}{\int {\rm PD} d\mathcal{L}ips}, \tag{31}$$

where we have used the narrow width approximation for the propagators in the phase-space element $d\mathcal{L}ips$, see Eq. (B4). Note that in the denominator of $\mathcal{A}_\tau$, Eq. (31), all spin correlation terms vanish $\int \Sigma_{\rm P}^a \Sigma_{\rm D}^a d\mathcal{L}ips = 0$ when integrated over phase space. In the numerator only the spin-spin correlation term $\Sigma_{\rm P}^a \Sigma_{\rm D}^{ab=2}$ contributes, since only $\Sigma_{\rm D}^{ab}$, Eq. (23), contains the T-odd epsilon[3] product $\mathcal{E}^{ab}$, see Eq. (24).

D. Parameter dependence of the CP-sensitive asymmetry

To qualitatively understand the dependence of the asymmetry $\mathcal{A}_\tau$, Eq. (31), on the parameters of the chargino sector, we study in some detail its dependence on the τ-$\tilde{\nu}_\tau$-chargino couplings $c_{i\tau}^L$ and $c_{i\tau}^R$, cf. Eq. (10). From the explicit form of the decay terms D, Eq. (20), and $\Sigma_{\rm D}^{ab}$, Eqs. (23) and (24), we find that the asymmetry

$$\mathcal{A}_\tau = \eta_i \frac{\int \Sigma_{\rm P}^a \mathcal{E}^{ab=2} d\mathcal{L}ips}{\int {\rm P}(p_{\tilde{\chi}_i} \cdot p_\tau) d\mathcal{L}ips}, \tag{32}$$

with $(p_{\tilde{\chi}_i} \cdot p_\tau) = (m_{\tilde{\chi}_i}^2 - m_{\tilde{\nu}_\tau}^2)/2$, is proportional to the decay coupling factor

$$\eta_i = \frac{\Im\mathfrak{m}\{c_{i\tau}^L c_{i\tau}^{R*}\}}{\frac{1}{2}(|c_{i\tau}^L|^2 + |c_{i\tau}^R|^2)} = \frac{Y_\tau \Im\mathfrak{m}\{V_{i1} U_{i2}\}}{\frac{1}{2}(|V_{i1}|^2 + Y_\tau^2 |U_{i2}|^2)}, \tag{33}$$

with $\eta_i \in [-1, 1]$.

Using the explicit forms of the matrix elements U and V, see Eqs. (C1) and (C2), we can rewrite the factor η_i for $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$ or $\tilde{\chi}_2^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$ decay, respectively,

$$\eta_1 = \frac{-Y_\tau \sin(\theta_1)\cos(\theta_2)}{\frac{1}{2}[\cos^2(\theta_2) + Y_\tau^2 \sin^2(\theta_1)]} \sin(\gamma_1 + \phi_1), \tag{34}$$

$$\eta_2 = \frac{Y_\tau \cos(\theta_1)\sin(\theta_2)}{\frac{1}{2}[\sin^2(\theta_2) + Y_\tau^2 \cos^2(\theta_1)]} \sin(\gamma_2 + \phi_2), \tag{35}$$

with the angles $\theta_{1,2}$, which describe chargino mixing and $\gamma_{1,2}$ and $\phi_{1,2}$, which describe their CP properties, cf. Eqs. (C3) and (C4) in Appendix C.

Since η_i is proportional to the Yukawa coupling Y_τ, Eq. (12), the asymmetry will be enhanced for increasing $\tan\beta$. Then the phase dependence of the asymmetries will be $\mathcal{A}_\tau \propto \eta_i \propto \sin(\gamma_i + \phi_i) \approx \sin(\varphi_\mu)$, since we find ϕ_1, $\gamma_2 \to \varphi_\mu$, and ϕ_2, $\gamma_1 \to 0$ for $\tan\beta \gg 1$. Note that the asymmetry will be additionally suppressed if $\tan\beta$ is small, since that not only results in $Y_\tau \ll 1$ but also leads to ϕ_1, $\gamma_1 \to 0$, and $\phi_2 \to -\varphi_\mu$, $\gamma_2 \to \varphi_\mu$, which means that the phase dependent part of η_i vanishes $\sin(\gamma_i + \phi_i) \to 0$.

Second, we expect maximal asymmetries for equal left and right chargino couplings $|c_{i\tau}^L| \approx |c_{i\tau}^R|$, where the coupling factor can be maximal $\eta_i = \pm 1$, see Eq. (33). Concerning the mixing of the charginos, parametrized by the angles $\theta_{1,2}$, we expect maximal asymmetries in a mixed

[2]Note that in principle one can define a true CP asymmetry from the T-odd asymmetry by adding the corresponding asymmetry for the charge conjugated process [35,58]

$$\mathcal{A}_\tau^{CP} = \frac{1}{2}(\mathcal{A}_\tau + \bar{\mathcal{A}}_\tau). \tag{30}$$

Here, we denote with $\bar{\mathcal{A}}_\tau$ the tau polarization asymmetry for the charge conjugated process $\tilde{\chi}_i^- \to \tau^- \tilde{\nu}_\tau^*$. For our analysis at tree level, where no absorptive phases are present, we have $\mathcal{A}_\tau^{CP} = \mathcal{A}_\tau$; thus, we will study $\mathcal{A}_\tau$ in the following.

[3]Note that for $\tilde{\chi}_i^+ \tilde{\chi}_j^-$ production, with $i \neq j$, there is, in principle, also a contribution from the CP-violating normal chargino polarization $\Sigma_{\rm P}^{a=2}$ to the asymmetry $\mathcal{A}_\tau$, which projects out the CP-even parts of $\Sigma_{\rm D}^{ab=2}$. However, $\Sigma_{\rm P}^{a=2}$ is numerically small in our scenarios with large $\tan\beta$, so that we do not discuss its impact here; see Refs. [45,47] for CP asymmetries in chargino production.

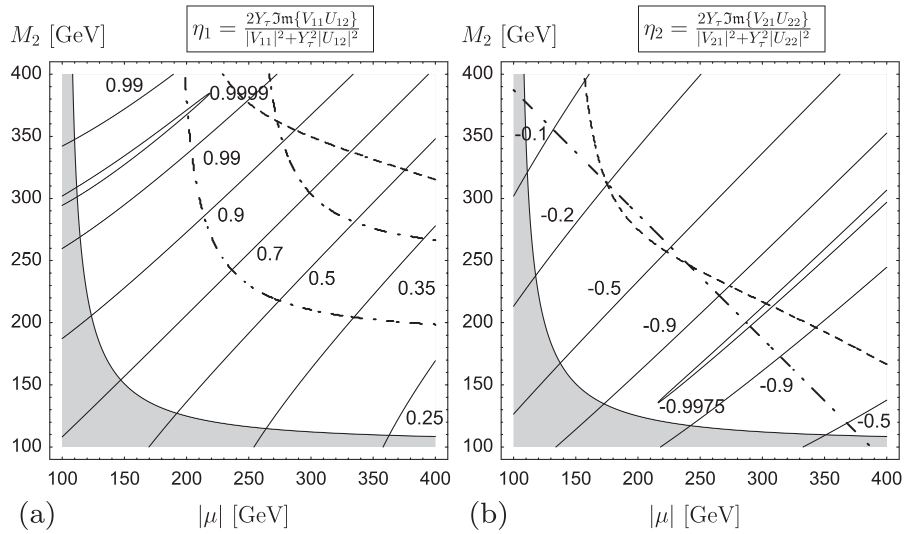

FIG. 2. Contour lines in the M_2 - $|\mu|$ plane for (a) the proportionality factor η_1, Eq. (34), of the asymmetry $\mathcal{A}_\tau$ for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$, and the scenario as defined in Table I; (b) the proportionality factor η_2, Eq. (35), of the asymmetry $\mathcal{A}_\tau$ for $e^+e^- \to \tilde{\chi}_2^\pm\tilde{\chi}_1^\mp$, $\tilde{\chi}_2^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$, and the scenario as given in Table II. In each case with a center-of-mass energy $\sqrt{s} = 500$ GeV. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV. In (a), the band between the dashed-dotted lines and in (b) the triangle below the dashed-dotted line mark the kinematically allowed regions, see Figs. 3(c) and 6(c), respectively.

gaugino-higgsino scenario $|\mu| \approx M_2$ however "corrected" by the Yukawa coupling, such that $\cos(\theta_2) \approx Y_\tau \sin(\theta_1)$ for $\tilde{\chi}_1^\pm$ decay and $\sin(\theta_2) \approx Y_\tau \cos(\theta_1)$ for $\tilde{\chi}_2^\pm$ decay, see Eqs. (34) and (35), respectively.

In Figs. 2(a) and 2(b), we show the η factors η_1 and η_2 in the M_2-μ plane for the scenarios as given in Table I and II, accordingly.

III. NUMERICAL RESULTS

We present numerical results for the CP-sensitive asymmetry $\mathcal{A}_\tau$, Eq. (31), for chargino production $e^+e^- \to \tilde{\chi}_i^\pm\tilde{\chi}_j^\mp$ and decay $\tilde{\chi}_i^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. We choose a center-of-mass energy of $\sqrt{s} = 500$ GeV and longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$. We include all spin correlations between chargino production and decay, since only they include CP-violating terms at tree level.

We study the dependence of the asymmetry on the MSSM parameters $\mu = |\mu|e^{i\varphi_\mu}$, M_2, and $\tan\beta$. For $\tilde{\chi}_1^-\tilde{\chi}_1^+$ production, we study the dependence on the beam polarizations. The feasibility of measuring the asymmetry also depends on the chargino production cross sections and decay branching ratios, which we discuss in detail. Finally, to get a lower bound on the event rates necessary to observe the asymmetry, we also give its theoretical statistical significance S_τ, Eq. (D3).

TABLE I. Scenario for chargino production $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ and decay $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. The mass parameters M_2, $|\mu|$, $M_{\tilde{E}}$, and $M_{\tilde{L}}$ are given in GeV.

$\tan\beta$	φ_μ	M_2	$\|\mu\|$	$M_{\tilde{E}} = M_{\tilde{L}}$
25	0.5π	380	240	200

Calculated mass spectrum

$\tilde{\ell}$	m [GeV]	$\tilde{\chi}$	m [GeV]
$\tilde{e}_R$, $\tilde{\mu}_R$	205	$\tilde{\chi}_1^0$	175
$\tilde{e}_L$, $\tilde{\mu}_L$	206	$\tilde{\chi}_2^0$	238
$\tilde{\nu}_e$, $\tilde{\nu}_\mu$	189	$\tilde{\chi}_3^0$	247
$\tilde{\tau}_1$	177	$\tilde{\chi}_4^0$	405
$\tilde{\tau}_2$	230	$\tilde{\chi}_1^\pm$	225
$\tilde{\nu}_\tau$	189	$\tilde{\chi}_2^\pm$	405
$BR(\tilde{\chi}_1^+ \to \tau^+\tilde{\nu}_\tau) = 30$ [%]			$\sigma_P(e^+e^- \to \chi_1^-\chi_1^+) = 417$ [fb]

TABLE II. Scenario for $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ production and decay $\tilde{\chi}_{1,2}^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. The mass parameters M_2, $|\mu|$, $M_{\tilde{E}}$, and $M_{\tilde{L}}$ are given in GeV.

$\tan\beta$	φ_μ	M_2	$|\mu|$	$M_{\tilde{E}} = M_{\tilde{L}}$
25	0.5π	250	200	150

Calculated mass spectrum

$\tilde{\ell}$	m [GeV]	$\tilde{\chi}$	m [GeV]
$\tilde{e}_R$, $\tilde{\mu}_R$	156	$\tilde{\chi}_1^0$	115
$\tilde{e}_L$, $\tilde{\mu}_L$	157	$\tilde{\chi}_2^0$	177
$\tilde{\nu}_e$, $\tilde{\nu}_\mu$	136	$\tilde{\chi}_3^0$	210
$\tilde{\tau}_1$	125	$\tilde{\chi}_4^0$	294
$\tilde{\tau}_2$	183	$\tilde{\chi}_1^\pm$	170
$\tilde{\nu}_\tau$	136	$\tilde{\chi}_2^\pm$	294

$\mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+ \tilde{\nu}_\tau) = 28$ [%] $\mathrm{BR}(\tilde{\chi}_2^+ \to \tau^+ \tilde{\nu}_\tau) = 14$ [%]

$\sigma_P(e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp) = 444$ [fb]

For the calculation of the chargino decay widths and branching ratios, we consider the two-body decays [45]

$$\tilde{\chi}_i^+ \to W^+ \tilde{\chi}_k^0, \tilde{e}_L^+ \nu_e, \tilde{\mu}_L^+ \nu_\mu, \tilde{\tau}_{1,2}^+ \nu_\tau, e^+ \tilde{\nu}_e, \mu^+ \tilde{\nu}_\mu, \tau^+ \tilde{\nu}_\tau,$$
$$\tilde{\chi}_2^+ \to \tilde{\chi}_1^+ Z^0, \tilde{\chi}_1^+ H_1^0, \tag{36}$$

for $i = 1$, 2 and $k = 1, \ldots, 4$. We neglect three-body decays, which are suppressed by phase space. In order to reduce the number of parameters, we use the grand unified theories (GUT) inspired relation $|M_1| = 5/3 M_2 \tan^2\theta_w$. This significantly constrains the neutralino sector [59]. We take stau mixing into account and set the mass of the trilinear scalar coupling parameter to $A_\tau = 250$ GeV. Since its phase does not contribute to the asymmetry, we fix it to $\varphi_{A_\tau} = 0$ as well as $\varphi_1 = 0$, which is the phase of the gaugino mass parameter M_1. We fix the soft-breaking parameters $M_{\tilde{E}} = M_{\tilde{L}}$ in the slepton sector. When varying μ and M_2 this can lead to excluded regions in the plots where the lightest stau $\tilde{\tau}_1$ is the LSP $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$, which we indicate by a dashed line, *cf.* Fig. 2. The Higgs mass parameter is fixed to $M_A = 1000$ GeV. Our results are insensitive to this choice as long as we stay in the decoupling limit.

In order to show the full phase dependence on φ_μ, we relax the constraints from the EDMs. Our purpose is to demonstrate that a nonvanishing *CP* phase in the chargino sector would lead to large asymmetries. Their measurement in chargino decays will be a sensitive probe to constrain the phase φ_μ independently from EDM measurements. There can be cancellations between different contributions to the EDMs, which can in principle be achieved by tuning parameters and *CP* phases outside the chargino sector [14–16].

On the other hand, large phases can be in agreement with the EDM bounds for scenarios with heavy first and second generation sfermion masses of the order of 10 TeV. The third generation can stay light with masses of the order of 100 GeV [19]. Such scenarios are particularly interesting also for our process. Heavy electron sneutrinos enhance chargino production cross sections, while at the same time the decay channels into taus $\tilde{\chi}_i^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$ will be dominating. We will study such a scenario at the end of the numerical section.

A. Chargino pair production $e^+ + e^- \to \tilde{\chi}_1^+ + \tilde{\chi}_1^-$ and decay $\tilde{\chi}_1^+ \to \tau^+ + \tilde{\nu}_\tau$

We first study the production of the lightest chargino pair $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$. Note that the production amplitude is purely real for equal chargino pair production $\tilde{\chi}_i^+ \tilde{\chi}_i^-$. Then the Z-chargino couplings are real, see Eqs. (8) and (9), and the t-channel sneutrino amplitude depends only on the modulus of the sneutrino couplings $|V_{i1}|^2$, see Eq. (6). Thus, at tree level, a *CP* asymmetry in general can only receive *CP*-odd contributions from the chargino decay. We center our numerical discussion around a reference scenario, see Table I, which is in some sense optimized to give large significances.

We choose beam polarizations of $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$, which enhance the γ exchange and γZ interference contributions in the production with respect to the destructive contributions from $\gamma\tilde{\nu}_e$ and $Z\tilde{\nu}_e$ interference. This favors higher production cross sections and asymmetries compared to the reversed polarizations $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (0.8|-0.6)$ since γZ interference then becomes destructive too.

1. M_2 - $|\mu|$ dependence

In Fig. 3(a), we show contour lines of the chargino pair production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-)$ in the M_2 - $|\mu|$ plane for the scenario given in Table I. For $\mu \lesssim 200$ GeV, the cross section σ_P mainly receives γ exchange and γZ interference contributions, which add

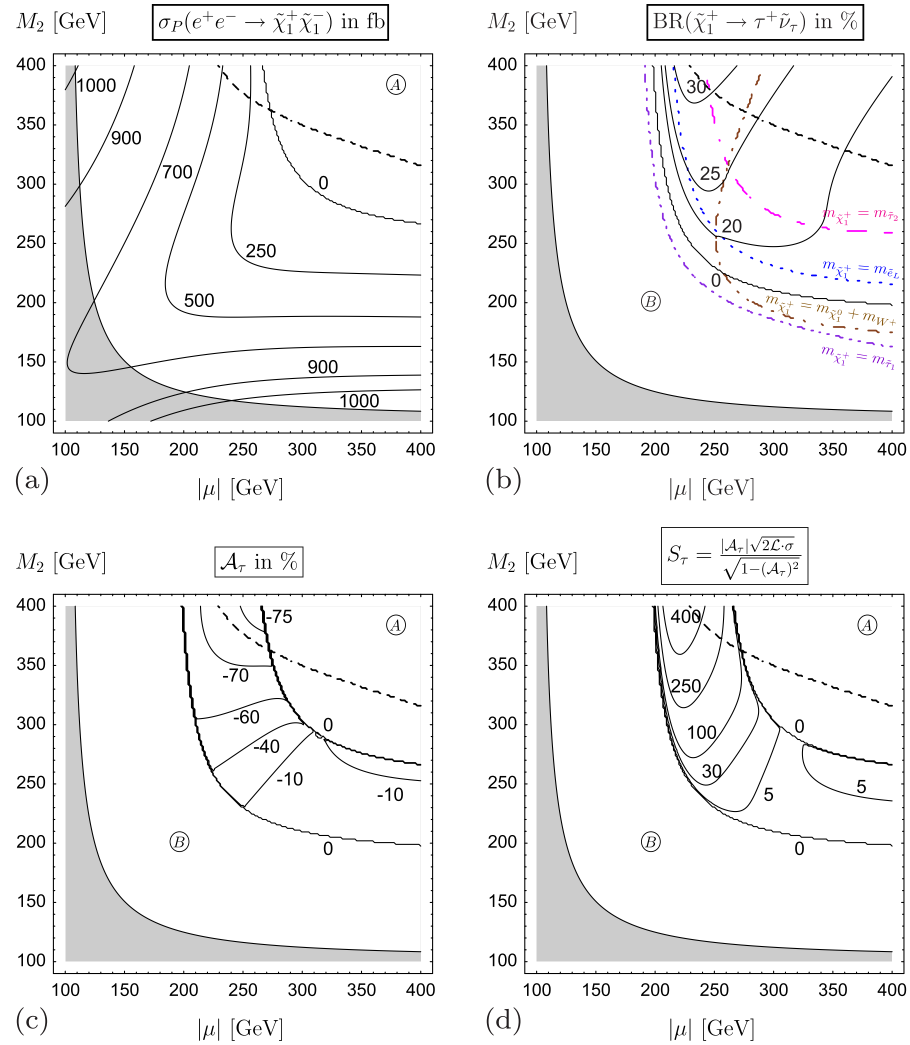

FIG. 3 (color online). Contour lines in the M_2 - $|\mu|$ plane of (a) the production cross section, (b) branching ratio, (c) CP-sensitive asymmetry of the normal tau polarization, and (d) its significance for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$, with a center-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$, and an integrated luminosity $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. The other SUSY parameters are defined in Table I. The area Ⓐ above the zero contour line of the production cross section is kinematically forbidden by $\sqrt{s} < 2m_{\tilde{\chi}_1^\pm}$ and the area Ⓑ below the zero contour line of the branching ratio by $m_{\tilde{\chi}_1^+} < m_{\tilde{\nu}_\tau}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\tau}_1} < m_{\tilde{\chi}_1^0}$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

to the contributions from pure Z and $\tilde{\nu}_e$ exchange to give more than $\sigma_P = 1000$ fb. These contributions are about twice as large as the destructive contributions from $\gamma\tilde{\nu}_e$ and $Z\tilde{\nu}_e$ interference. For $|\mu|$, $M_2 \gtrsim 200$ GeV, the cross section is reduced by the growing contributions from $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ interference. For $|\mu| \gtrsim 200$ GeV and $M_2 \lesssim 200$ GeV, the $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ interference contributions again become weaker, so that the production cross section is dominated by pure $\tilde{\nu}_e$ exchange. In Appendix E, Figs. 12(a) and 12(c), we present the different interference contributions to the production cross section σ_P. For our reference scenario, see Table I; we also show the distribution $d\sigma_P/d\theta$ in the chargino scattering angle $\theta = \sphericalangle(\mathbf{p}_{e^-}, \mathbf{p}_{\tilde{\chi}_1^-})$, see Figs. 11(a), 11(c), and 11(e).

In Fig. 3(b), we show contours for the chargino branching ratio into the tau $\mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+\tilde{\nu}_\tau)$ as a function of $|\mu|$, and M_2. We indicate the thresholds of the rivalling decay channels by lines of different shape. The branching ratios $\mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+\tilde{\nu}_\ell)$ into the light leptons $\ell = e, \mu$ are typically of the same order as that for the decay into the tau. Branching ratios into left sleptons are of the order of $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\ell}_L^+\nu_\ell) < 3\%$, $\ell = e, \mu$. The competitive chargino decays into staus can reach up to $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\tau}_1^+\nu_\tau) = 54\%$ and $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\tau}_2^+\nu_\tau) = 15\%$. Above the dashed-dotted contour in Fig. 3(b), the decay into the W boson opens, with $\mathrm{BR}(\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0) < 5\%$. The other chargino decays $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_n^0$, $n = 2, 3, 4$ are kinematically excluded, since already $m_{\tilde{\chi}_2^0} \approx m_{\tilde{\chi}_1^\pm}$.

In Fig. 3(c), we show the asymmetry $\mathcal{A}_\tau$, Eq. (31), within its kinematically allowed range in the M_2 - $|\mu|$ plane for chargino production $\sqrt{s} \geq 2m_{\tilde{\chi}_1^\pm}$ and decay $m_{\tilde{\chi}_1^\pm} \geq m_{\tilde{\nu}_\tau}$. Although the cross section $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+\tilde{\nu}_\tau)$, which enters the denominator of Eq. (31), increases with increasing M_2, the asymmetry attains its maximum of more than $\mathcal{A}_\tau = -70\%$ for large $M_2 \gtrsim 350$ GeV. The reason is that the coupling factor η_1, Eq. (33), to which the asymmetry is proportional, is here also maximal for large M_2. As discussed in Sec. II D, η_1, and thus the asymmetry, is largest for $|V_{11}| \sim |Y_\tau U_{12}|$. For $\tan\beta = 25$, we have $Y_\tau \sim 1/3$, resulting in a maximum of η_1 and the asymmetry for $|\mu| \sim M_2/2$, see Fig. 2(a), which is in good agreement with the location of the maximum of the asymmetry in Fig. 3(c). The different interference contributions to the asymmetry are presented in Figs. 12(b) and 12(d), see Appendix E. In Figs. 11(b), 11(d), and 11(f), we show the distribution $d\mathcal{A}_\tau/d\theta$ in the chargino scattering angle.

In Fig. 3(d), we show the corresponding theoretical significance S_τ, which is defined in Eq. (D3), Appendix D for $\mathcal{L} = 500$ fb^{-1}.

2. φ_μ and $\tan\beta$ dependence

In Figs. 4(a) and 4(b), we show the φ_μ dependence of $\mathcal{A}_\tau$ and S_τ, respectively, for different values of $\tan\beta$. First, the asymmetry is increasing for increasing $\tan\beta$ since $\mathcal{A}_\tau \sim Y_\tau$, as discussed in Sec. II D. Second, concerning the phase dependence of the asymmetry, we find for large $\tan\beta$ that $\mathcal{A}_\tau \sim \sin(\varphi_\mu)$, as also discussed in Sec. II D. We observe from Fig. 4(a) the almost perfect sinusoidal behavior of $\mathcal{A}_\tau$ for large $\tan\beta = 20$. For smaller values of $\tan\beta$, the sine-shape of the asymmetry gets less pronounced, such that its maxima are not necessarily obtained

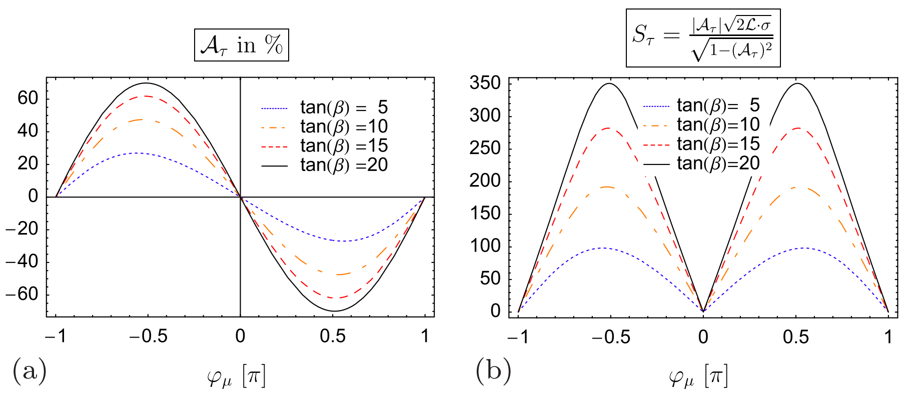

FIG. 4 (color online). Phase dependence of (a) the CP-sensitive asymmetry of the normal tau polarization and (b) its significance for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$, for various values of $\tan\beta$ with $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$ at $\sqrt{s} = 500$ GeV, and $\mathcal{L} = 500$ fb^{-1}. The other SUSY parameters are defined in Table I.

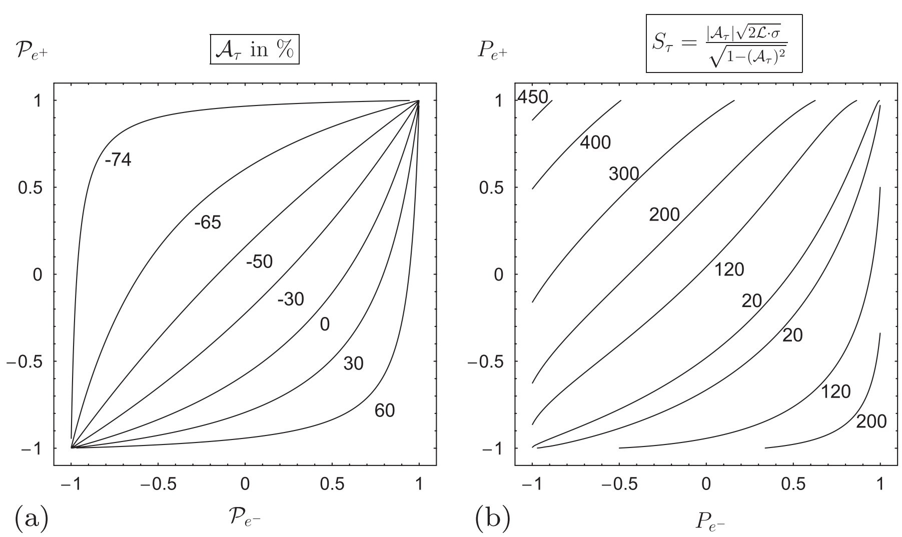

FIG. 5. Contour lines in the $\mathcal{P}_{e^+}$ - $\mathcal{P}_{e^-}$-plane of (a) the CP-sensitive asymmetry of the normal tau polarization and (b) its significance for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$, with $\sqrt{s} = 500$ GeV and $\mathcal{L} = 500$ fb^{-1} for the scenario in Table I.

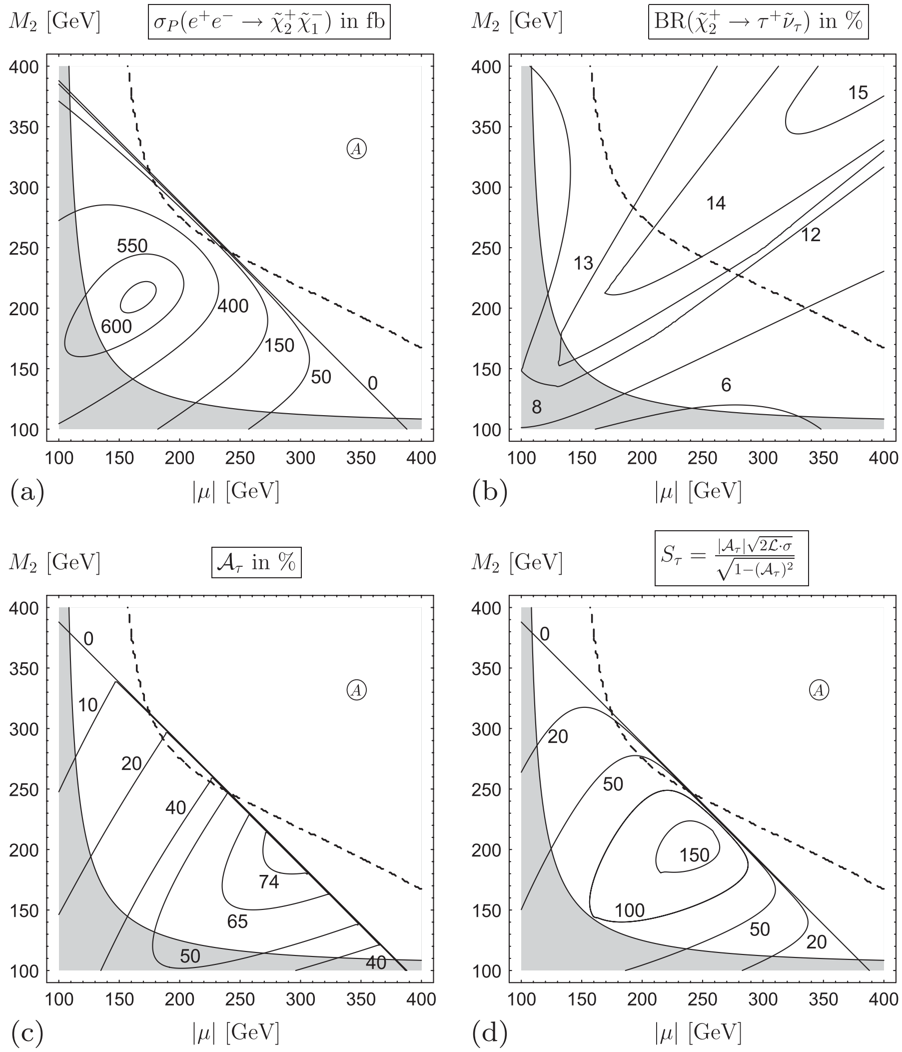

FIG. 6. Contour lines in the M_2 - $|\mu|$ plane of (a) the production cross section, (b) branching ratio, (c) CP-sensitive asymmetry of the normal tau polarization, and (d) its significance for $e^+e^- \to \tilde{\chi}_2^\pm\tilde{\chi}_1^\mp$; $\tilde{\chi}_2^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$, with a center-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$, and an integrated luminosity $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. The other SUSY parameters are defined in Table II. The area Ⓐ above the zero contour line of the cross section is kinematically forbidden by $\sqrt{s} < m_{\tilde{\chi}_2^+} + m_{\tilde{\chi}_1^-}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\tau}_1} < \tilde{\chi}_1^0$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

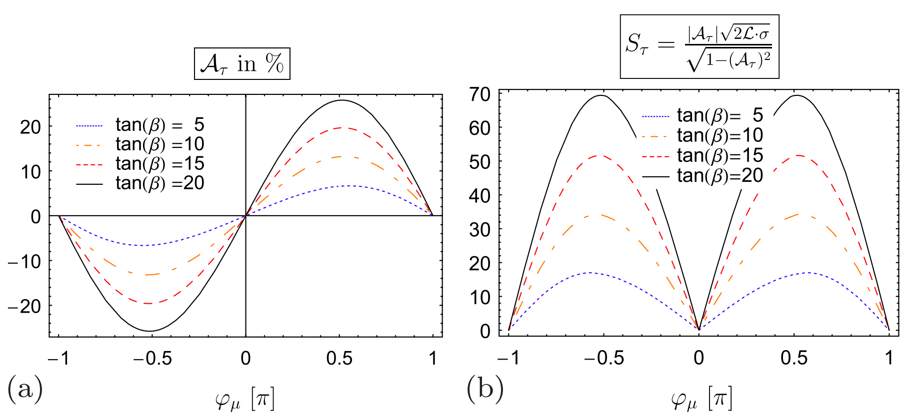

FIG. 7 (color online). Phase dependence of (a) the CP-sensitive asymmetry of the normal tau polarization and (b) its significance for $e^+e^- \to \tilde{\chi}_2^\pm\tilde{\chi}_1^\mp$; $\tilde{\chi}_2^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$ for various values of tanβ with $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$ at $\sqrt{s} = 500$ GeV and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. The other SUSY parameters are defined in Table II.

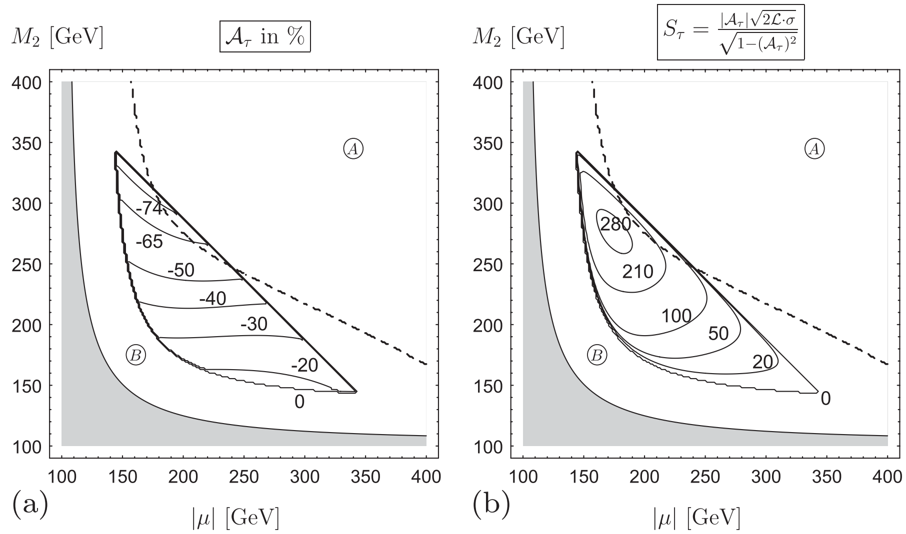

FIG. 8. Contour lines in the M_2 - $|\mu|$ plane of (a) the CP-sensitive asymmetry of the normal tau polarization and (b) its significance for $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\pm$; $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$, with a center-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$, and an integrated luminosity $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. The other SUSY parameters are defined in Table II. The area Ⓐ is kinematically forbidden by $\sqrt{s} < m_{\tilde{\chi}_1^+} + m_{\tilde{\chi}_2^-}$, and the area Ⓑ is kinematically forbidden by $m_{\tilde{\chi}_1^+} < m_{\tilde{\nu}_\tau}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\tau}_1} < \tilde{\chi}_1^0$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

for maximal CP phases $\varphi_\mu = \pm\pi/2$ but are slightly shifted away. It is remarkable that in our scenario, see Table I, the asymmetry can still be $\mathcal{A}_\tau = \pm 22\%$ even for $\varphi_\mu = \pm 0.1\pi$. Small phases φ_μ are suggested by the experimental upper bounds on the EDMs, and the asymmetry will be a sensitive probe even for small CP phases.

3. Beam polarization dependence

In Fig. 5(a), we show the beam polarization dependence of the asymmetry $\mathcal{A}_\tau$ for our benchmark scenario, see Table I. For unpolarized beams the asymmetry is $\mathcal{A}_\tau = -43\%$ and varies between $\mathcal{A}_\tau = -74\%$ for $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$ and $\mathcal{A}_\tau = +60\%$ for $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (0.8|-0.6)$. The strong dependence of the asymmetry on the beam polarizations is due to the enhancement of the chargino production channels with $\tilde{\nu}_e$ exchange for negative electron beam polarization $\mathcal{P}_{e^-} < 0$ and positive positron beam polarization $\mathcal{P}_{e^+} > 0$. For oppositely polarized beams $\mathcal{P}_{e^-} > 0$, $\mathcal{P}_{e^+} < 0$, the Z exchange contributions are enhanced, and those of $\tilde{\nu}_e$ are suppressed. Since the Z contributions enter with opposite sign, also the sign of $\mathcal{A}_\tau$ changes, see Fig. 5(a).

The corresponding theoretical statistical significance S_τ is shown in Fig. 5(b). The production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-)$ varies from 332 fb for unpolarized

TABLE III. Scenario for chargino pair production $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$ and decay $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$ with heavy first and second slepton generations. The mass parameters M_2, $|\mu|$, $M_{\tilde{E}}$, $M_{\tilde{E}_\tau}$, $M_{\tilde{L}}$, and $M_{\tilde{L}_\tau}$ are given in GeV.

$\tan\beta$	φ_μ	M_2	$\lvert\mu\rvert$	$M_{\tilde{E}} = M_{\tilde{L}}$	$M_{\tilde{E}_\tau} = M_{\tilde{L}_\tau}$
25	0.5π	380	240	15×10^3	200

Calculated mass spectrum

$\tilde{\ell}$	m [GeV]	$\tilde{\chi}$	m [GeV]
$\tilde{e}_R$, $\tilde{\mu}_R$	15×10^3	$\tilde{\chi}_1^0$	175
$\tilde{e}_L$, $\tilde{\mu}_L$	15×10^3	$\tilde{\chi}_2^0$	238
$\tilde{\nu}_e$, $\tilde{\nu}_\mu$	15×10^3	$\tilde{\chi}_3^0$	247
$\tilde{\tau}_1$	177	$\tilde{\chi}_4^0$	405
$\tilde{\tau}_2$	230	$\tilde{\chi}_1^\pm$	225
$\tilde{\nu}_\tau$	189	$\tilde{\chi}_2^\pm$	405

$\mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+\tilde{\nu}_\tau) = 49$ [%] $\qquad \sigma_P(e^+e^- \to \tilde{\chi}_1^-\tilde{\chi}_1^+) = 805$ [fb]

beams to 418 fb for $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$ and 121 fb for $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (0.8|-0.6)$. Thus, the largest values of σ_P are obtained for polarized beams, where $\tilde{\nu}_e$ exchange contributions are enhanced. The significance then reaches up to $S_\tau = 450$.

B. Production of an unequal pair of charginos $e^+ + e^- \to \tilde{\chi}_i^+ + \tilde{\chi}_j^-$ and decay $\tilde{\chi}_i^+ \to \tau^+ + \tilde{\nu}_\tau$

For $\tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$ production, the CP-sensitive asymmetry $\mathcal{A}_\tau$ in principle also receives nonvanishing CP-odd contributions from the production. However, in our benchmark scenario with large $\tan\beta = 25$, see Table II, those contributions are smaller than 1%. The dominant contributions will still be from the decay, and we discuss the asymmetries for the decay of $\tilde{\chi}_1^\pm$, and $\tilde{\chi}_2^\pm$, separately.

1. Decay of $\tilde{\chi}_2^+ \to \tau^+ + \tilde{\nu}_\tau$

In Fig. 6(a), we show the M_2 - $|\mu|$ dependence of the production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_2^+\tilde{\chi}_1^-)$, which can attain values of several hundred fb. In contrast to the production of an equal pair of charginos $e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_i^-$, the cross section receives destructive contributions from $Z\tilde{\nu}_e$ interference only. The dominant contribution is from pure $\tilde{\nu}_e$ exchange. With increasing $|\mu|$, that contribution decreases faster than the $Z\tilde{\nu}_e$ interference term, and the production cross section is reduced.

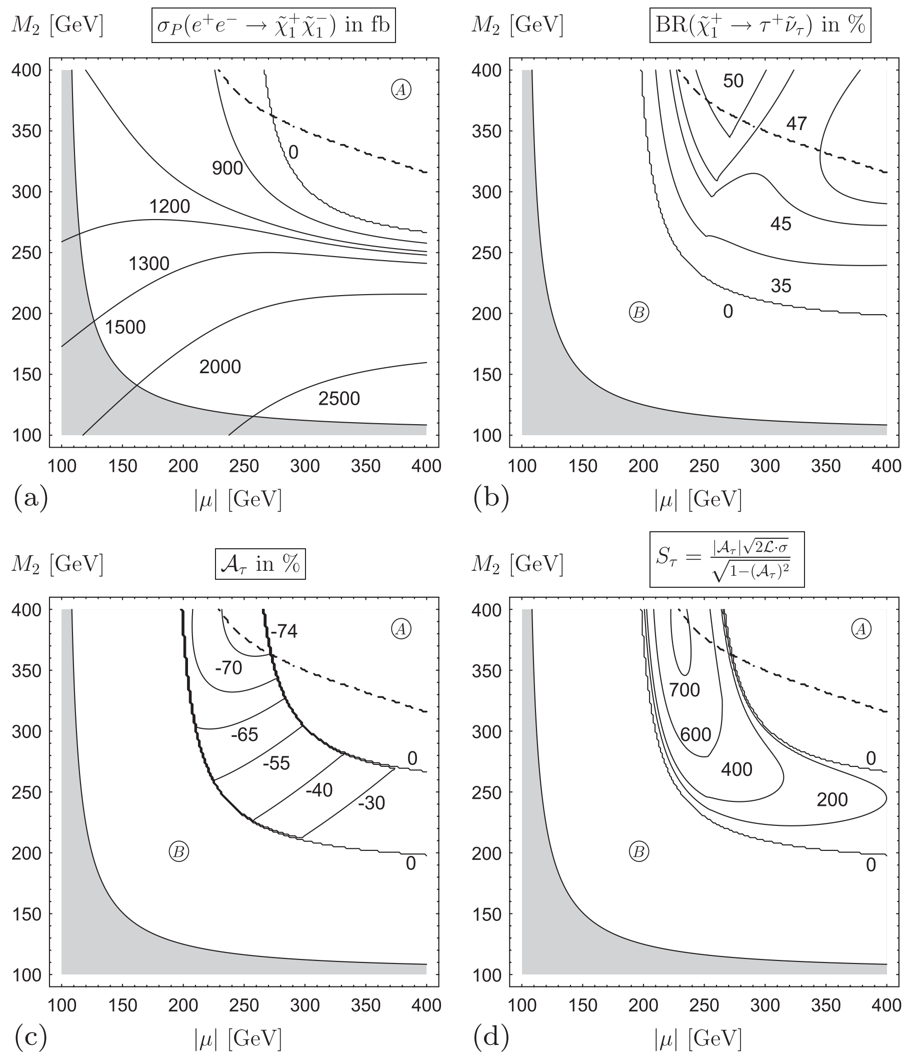

FIG. 9. Contour lines in the M_2 - $|\mu|$ plane of (a) the production cross section, (b) branching ratio, (c) CP-sensitive asymmetry of the normal tau polarization, and (d) its significance for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^\pm \to \tau^\pm\tilde{\nu}_\tau^{(*)}$ for a spectrum of heavy 1st and 2nd slepton generations as given in Table III, with a center-of-mass energy $\sqrt{s} = 500$ GeV, longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$, and an integrated luminosity $\mathcal{L} = 500\ \mathrm{fb}^{-1}$. The area Ⓐ above the zero contour line of the production cross section is kinematically forbidden by $\sqrt{s} < 2m_{\tilde{\chi}_1^\pm}$ and the area Ⓑ below the zero contour line of the branching ratio by $m_{\tilde{\chi}_1^+} < m_{\tilde{\nu}_\tau}$. Above the dashed line the lightest neutralino is no longer the LSP since $m_{\tilde{\tau}_1} < \tilde{\chi}_1^0$. In the grey-shaded area $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

In contrast to the lightest chargino $\tilde{\chi}_1^+$, the decay of the heavy chargino $\tilde{\chi}_2^+ \to \tau^+ \tilde{\nu}_\tau$ is kinematically allowed in the entire M_2 - $|\mu|$ plane, see Fig. 6(b). However, its branching ratio is small $\mathrm{BR}(\tilde{\chi}_2^+ \to \tau^+ \tilde{\nu}_\tau) < 14\%$ since all other decay channels can open, except $\mathrm{BR}(\tilde{\chi}_2^+ \to \tilde{\chi}_4^0 W^+)$, see Eq. (36). Within the parameter range of Fig. 6(b), we find $\mathrm{BR}(\tilde{\chi}_2^+ \to \ell \tilde{\nu}_\ell) < 28\%$ and $\mathrm{BR}(\tilde{\chi}_2^+ \to \tilde{\ell}_L \nu_\ell) < 15\%$ for $\ell = e, \mu$, as well as $\mathrm{BR}(\tilde{\chi}_2^+ \to \tilde{\chi}_2^0 W^+) < 41\%$. Other decays can reach up to $\mathrm{BR}(\tilde{\chi}_2^+ \to \tilde{\chi}_i^0 W^+) = 7\%$ for $i = 1, 3$, and $\mathrm{BR}(\tilde{\chi}_2^+ \to \tilde{\chi}_1^+ H_1^0) = 21\%$.

Figure 6(c) shows the asymmetry $\mathcal{A}_\tau$. It reaches more than 70% and is enhanced kinematically by the rising destructive interference $Z\tilde{\nu}_e$ in the production process for $|\mu| \gtrsim 200$ GeV. These lead to lower contributions to P, and hence to larger asymmetries, cf. Eq. (31). In addition, the coupling factor η_2, Eq. (33), is maximal for $|\mu| \approx 300$ GeV and $M_2 \approx 200$ GeV, the condition for maximal interference of the gaugino-higgsino components. See the discussion in Sec. II D and Fig. 2(b).

The corresponding significance S_τ, Eq. (D3), which is shown in Fig. 6(d), is smaller than for the production of an equal pair of charginos, as the cross section $\sigma_P(e^+e^- \to \tilde{\chi}_2^+ \tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_2^+ \to \tau^+ \tilde{\nu}_\tau)$ is smaller by a factor of about 2. However, for $\mathcal{L} = 500\ \mathrm{fb}^{-1}$, it can still attain values of $S_\tau = 150$.

The φ_μ dependence of the asymmetry and its significance is shown for various values of $\tan\beta$ in Fig. 7. Again, we can clearly observe the two striking features, $\mathcal{A}_\tau \propto Y_\tau$ and $\mathcal{A}_\tau \propto \sin(\varphi_\mu)$. These are due to the special dependence of the asymmetry on the τ - $\tilde{\nu}_\tau$-chargino couplings and can be qualitatively understood, see the discussion in Sec. II D.

2. *Decay of* $\tilde{\chi}_1^+ \to \tau^+ + \tilde{\nu}_\tau$

In Fig. 8(a), we show the $M_2 - |\mu|$ - $|\mu|$ dependence of the asymmetry. Large values, up to $\mathcal{A}_\tau = -74\%$, are obtained towards $M_2 \sim 2|\mu|$, where also η_1, Eq. (33), is maximal; compare with the asymmetry in Fig. 3(c). The corresponding branching ratio $\mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+ \tilde{\nu}_\tau)$ does not exceed 30%. The decay channels into the light leptons $\mathrm{BR}(\tilde{\chi}_1^+ \to \ell^+ \tilde{\nu}_\ell)$, $\ell = e, \mu$, and into the lightest stau $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\tau}_1^+ \nu_\tau)$ are the most competitive ones with up to 20% and 30%, respectively. Towards the production threshold, $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\ell}_L \nu_\ell)$ is of the order of 5%. Together with the production cross section $\sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_2^-)$, as shown in Fig. 6(a), the product of production and decay branching ratio $\sigma = \sigma_P \times \mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+ \tilde{\nu}_\tau)$ can be as large as 140 fb. The statistical significance, shown in Fig. 8(b), reaches $S_\tau = 200$ for $\mathcal{L} = 500\ \mathrm{fb}^{-1}$.

C. Inverted hierarchy scenario

The phase φ_μ of the higgsino mass parameter μ contributes to the EDMs of the electron [8], the neutron [9], and the mercury atom [10,11] already at the one-loop level. The dominant contribution to the electron EDM from chargino exchange, for example, is proportional to $\sin\varphi_\mu$ [8,14]. The phase φ_μ is thus strongly constrained by the experimental upper limits on the EDMs with $|\varphi_\mu| \lesssim 0.1\pi$, in general [15]. However, the bounds from the EDMs are highly model dependent. For instance, cancellations between different SUSY contributions to the EDMs can resolve the restrictions on the phases [14], although a proper fine tuning of SUSY parameters is required. On the other hand, the bounds on the phases might disappear if lepton flavour violation is included [16].

Another way to fulfil the EDM bounds is to assume sufficiently heavy sleptons and/or squarks. Since sparticle masses of the order of 10 TeV are required [14], such solutions are unnatural. Models with heavy sparticles have been discussed in the literature, like split-SUSY [18] or focus-point scenarios [20]. If only the first and second generation squarks are heavy, naturalness can be reconciled, while experimental constraints can still be

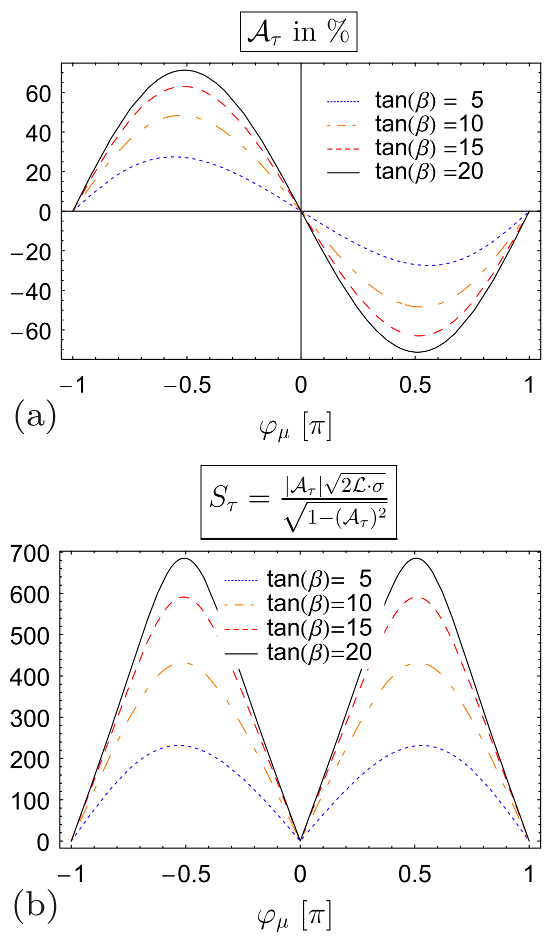

FIG. 10 (color online). Phase dependence of (a) the CP-sensitive asymmetry of the normal tau polarization and (b) its significance for $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$; $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$ for a spectrum of heavy 1st and 2nd slepton generations as given in Table III for various values of $\tan\beta$, with $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$ at $\sqrt{s} = 500$ GeV, and $\mathcal{L} = 500\ \mathrm{fb}^{-1}$.

fulfilled, see *e.g.* Ref. [19] for such an *inverted hierarchy* approach.

Heavy sfermions of the first and second generations are particularly interesting for our process of chargino production $e^+e^- \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp$ and decay into the tau $\tilde{\chi}_i^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. First, the negative contributions from sneutrino $\tilde{\nu}_e$ interference to the production cross sections are considerably reduced. Second, the chargino branching ratio into the tau is enhanced, since the chargino decay channels $\tilde{\ell}_L \nu_\ell$ and $\ell \tilde{\nu}_\ell$ are closed due to the heavy sleptons of the first and second generations $\ell = e, \mu$. In order to compare with our previous results, we take the parameters as in Table I but now choose heavy soft-breaking masses for the selectrons and smuons $M_{\tilde{L}} = M_{\tilde{E}} = 15$ TeV. See the new reference scenario and the resulting mass spectrum in Table III.

1. Parameter dependence

In Fig. 9(a), we show the M_2 - $|\mu|$dependence of the cross section for chargino production $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$ for our new reference scenario with heavy sneutrinos. Because of the heavy electron sneutrino $m_{\tilde{\nu}_e} = 15$ TeV, the negative interference contributions from $Z\tilde{\nu}_e$ and $\gamma\tilde{\nu}_e$ are thoroughly suppressed, which enhances the cross section. In the scenario with light sneutrinos, in particular, for large values of $|\mu|$, the pure $\tilde{\nu}_e$ exchange is the largest contribution to the cross section; see the discussion concerning Fig. 3(a) in Sec. III A 1. Although the constructive $\tilde{\nu}_e$ exchange contributions are also lost for heavy sneutrinos, there is still a net surplus in the production cross section, compare Fig. 3(a) and 9(a), since they are of the same order as one of the destructive channels $Z\tilde{\nu}_e$ or $\gamma\tilde{\nu}_e$.

The branching ratio $\mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+ \tilde{\nu}_\tau)$ is only reduced by the rivaling decay into the lightest stau, which is at least $\mathrm{BR}(\tilde{\chi}_1^+ \to \tilde{\tau}_1^+ \nu_\tau) = 50\%$ in Fig. 9(b). The product of production and decay $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-) \times \mathrm{BR}(\tilde{\chi}_1^+ \to \tau^+ \tilde{\nu}_\tau)$ for heavy sneutrinos is thus of the order of several hundred fb, see Table III. In contrast to the strong impact of a heavy sneutrino on the cross section, the asymmetry $\mathcal{A}_\tau$ is only slightly enhanced; compare Fig. 9(c) with Fig. 3(c). The asymmetry is mainly determined by the coupling factor η_1, see Eq. (33), which still allows for asymmetries of more than 70%. Together with the enhanced cross section, this leads to sizable significances of the order of several hundred standard deviations over statistical fluctuations, which we show in Fig. 9(d). Also the phase φ_μ and $\tan\beta$ dependence of the asymmetries are still governed by the coupling factor η_1, Eq. (33). In Fig. 10(a), we observe the same sinuslike dependence of $\mathcal{A}_\tau$, which increases with increasing $\tan\beta$, cf. Figure 4(a) and see the discussion in Sec. II D.

To summarise, the *CP*-sensitive asymmetries in the decay of a chargino into a polarized tau are a powerful tool to probe φ_μ, which might be large, in particular, in scenarios with flavour violation [16], or heavy sfermions of the first and second generations [19].

IV. SUMMARY, AND CONCLUSIONS

We have studied *CP* violation in chargino production with longitudinally polarized beams $e^+e^- \to \tilde{\chi}_i^\pm \tilde{\chi}_j^\mp$ and the subsequent two-body decay of one of the charginos into a tau and a sneutrino $\tilde{\chi}_i^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. We have given full analytical expressions for the amplitude squared, taking into account the complete spin correlations between production and decay. We have defined a *T*-odd asymmetry $\mathcal{A}_\tau$, which is the normal tau polarization, and which is sensitive to the *CP* phase φ_μ of the higgsino mass parameter μ.

In a numerical discussion, we have considered equal chargino pair production $e^+e^- \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$ and unequal chargino pair production $e^+e^- \to \tilde{\chi}_1^\pm \tilde{\chi}_2^\mp$, with the ensuing decay of either $\tilde{\chi}_1^\pm$ or $\tilde{\chi}_2^\pm$, respectively. We have studied the dependence of the asymmetries on the MSSM parameters of the chargino sector M_2, $|\mu|$, φ_μ, and $\tan\beta$. The asymmetries are considerably enhanced for large $\tan\beta$, where the tau Yukawa coupling is enhanced. The size of the asymmetries also strongly depends on the gaugino-higgsino composition of the charginos and can be maximal for equally sized left and right τ - $\tilde{\nu}_\tau$-chargino couplings.

We have found that $\mathcal{A}_\tau$ can attain values of more than 70%. The asymmetry is already present at tree level and can be sizable even for small phases of μ, as suggested by the experimental limits on EDMs. Moreover, by choosing different beam polarizations, the Z, γ, and $\tilde{\nu}_e$ contributions can be enhanced or suppressed. A proper choice of beam polarizations can thus considerably enhance both the asymmetry and the production cross sections. An analysis of statistical errors shows that the asymmetries are well accessible in future e^+e^- collider experiments in the 500 GeV range with high luminosity and longitudinally polarized beams.

Since the phase φ_μ of the higgsino mass parameter μ is the most constrained SUSY *CP* phase, as suggested by EDM bounds, a measurement of the normal tau polarization will be a powerful tool to constrain φ_μ independently from the low energy measurements. Moreover, we have shown that the asymmetry can be sizable in inverted hierarchy scenarios, with heavy sfermions of the first and second generations, where the EDM constraints on the SUSY phases are less severe.

To summarise, *T*-odd asymmetries in the decay of a chargino into a polarized tau are one of the most sensitive probes to measure or constrain φ_μ at the ILC. Since the feasibility of measuring the tau polarization can only be addressed in a detailed experimental study, we want to motivate such thorough analyses to explore the potential of measuring SUSY *CP* phases at high energy colliders.

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APPENDIX A: MOMENTA, AND SPIN VECTORS

We choose a coordinate frame for the center-of-mass system such that the momentum $\mathbf{p}_{\tilde{\chi}_j}$ of the chargino $\tilde{\chi}_j$ points in the z direction [54]. The scattering angle $\sphericalangle(\mathbf{p}_{e^-}, \mathbf{p}_{\tilde{\chi}_j})$ is denoted by θ and the azimuthal angle is set to zero. Explicitly, the momenta are then [54]

$$p_{e^-}^{\mu} = E_b(1, -\sin\theta, 0, \cos\theta), \tag{A1}$$

$$p_{\tilde{\chi}_i}^{\mu} = (E_{\tilde{\chi}_i}, 0, 0, -q), \tag{A2}$$

$$p_{e^+}^{\mu} = E_b(1, \sin\theta, 0, -\cos\theta), \tag{A3}$$

$$p_{\tilde{\chi}_j}^{\mu} = (E_{\tilde{\chi}_j}, 0, 0, q), \tag{A4}$$

with the beam energy $E_b = \sqrt{s}/2$ and [54]

$$E_{\tilde{\chi}_i} = \frac{s + m_{\tilde{\chi}_i}^2 - m_{\tilde{\chi}_j}^2}{2\sqrt{s}}, \tag{A5}$$

$$E_{\tilde{\chi}_j} = \frac{s + m_{\tilde{\chi}_j}^2 - m_{\tilde{\chi}_i}^2}{2\sqrt{s}}, \tag{A6}$$

$$q = \frac{\sqrt{\lambda(s, m_{\tilde{\chi}_i}^2, m_{\tilde{\chi}_j}^2)}}{2\sqrt{s}}, \tag{A7}$$

with

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz). \tag{A8}$$

For the chargino decay $\tilde{\chi}_i^{\pm} \to \tau^{\pm} \tilde{\nu}_{\tau}^{(*)}$, we parametrize the tau momentum in terms of the decay angle $\theta_D = \sphericalangle(\mathbf{p}_{\tau}, \mathbf{p}_{\tilde{\chi}_i})$ and its azimuth ϕ_D,

$$(p_{\tau}^{\mu})^T = \begin{pmatrix} E_{\tau} \\ -|\mathbf{p}_{\tau}| \sin\theta_D \cos\phi_D \\ |\mathbf{p}_{\tau}| \sin\theta_D \sin\phi_D \\ -|\mathbf{p}_{\tau}| \cos\theta_D \end{pmatrix}, \tag{A9}$$

$$|\mathbf{p}_{\tau}| = \frac{m_{\tilde{\chi}_i}^2 - m_{\tilde{\nu}_{\tau}}^2}{2(E_{\tilde{\chi}_i} - q\cos\theta_D)}. \tag{A10}$$

The τ spin vectors are defined as

$$s_{\tau}^{1,\mu} = \left(0, \frac{\mathbf{s}_{\tau}^2 \times \mathbf{s}_{\tau}^3}{|\mathbf{s}_{\tau}^2 \times \mathbf{s}_{\tau}^3|}\right), \tag{A11}$$

$$s_{\tau}^{2,\mu} = \left(0, \frac{\mathbf{p}_{\tau} \times \mathbf{p}_{e^-}}{|\mathbf{p}_{\tau} \times \mathbf{p}_{e^-}|}\right), \tag{A12}$$

$$s_{\tau}^{3,\mu} = \frac{1}{m_{\tau}}\left(|\mathbf{p}_{\tau}|, \frac{E_{\tau}}{|\mathbf{p}_{\tau}|}\mathbf{p}_{\tau}\right), \tag{A13}$$

with

$$s_{\tau}^a \cdot s_{\tau}^b = -\delta^{ab}, \qquad s_{\tau}^a \cdot p_{\tau} = 0. \tag{A14}$$

The chargino and tau spin vectors fulfil the completeness relation [53,54]

$$\sum_c s_{\tilde{\chi}_k}^{c,\mu} \cdot s_{\tilde{\chi}_k}^{c,\nu} = -g^{\mu\nu} + \frac{p_{\tilde{\chi}_k}^{\mu} p_{\tilde{\chi}_k}^{\nu}}{m_{\tilde{\chi}_k}^2}. \tag{A15}$$

APPENDIX B: PHASE SPACE

For chargino production $e^+e^- \to \tilde{\chi}_i^{\pm}\tilde{\chi}_j^{\mp}$ and subsequent decay of one of the charginos $\tilde{\chi}_i^{\pm} \to \tau^{\pm}\tilde{\nu}_{\tau}^{(*)}$, the Lorentz invariant phase-space element decomposes into two-body phase-space elements [60]

$$d\mathcal{L}ips(s; p_{\tilde{\chi}_j}, p_{\tau}, p_{\tilde{\nu}_{\tau}}) = \frac{1}{2\pi} d\mathcal{L}ips(s; p_{\tilde{\chi}_i}, p_{\tilde{\chi}_j}) ds_{\tilde{\chi}_i} d\mathcal{L}ips(s_{\tilde{\chi}_i}; p_{\tau}, p_{\tilde{\nu}_{\tau}}). \tag{B1}$$

The decay of the other chargino $\tilde{\chi}_j^{\mp}$ is not considered further. The constituent parts are

$$d\mathcal{L}ips(s; p_{\tilde{\chi}_i}, p_{\tilde{\chi}_j}) = \frac{q}{8\pi\sqrt{s}} \sin\theta d\theta, \tag{B2}$$

$$d\mathcal{L}ips(s_{\tilde{\chi}_i}; p_{\tau}, p_{\tilde{\nu}_{\tau}}) = \frac{1}{2(2\pi)^2} \frac{|\mathbf{p}_{\tau}|^2}{m_{\tilde{\chi}_i}^2 - m_{\tilde{\nu}_{\tau}}^2} d\Omega_D, \tag{B3}$$

with $d\Omega_D = \sin\theta_D d\theta_D d\phi_D$, and $s_{\tilde{\chi}_i} = p_{\tilde{\chi}_i}^2$.

We use the narrow width approximation for the chargino propagator $\Delta(\tilde{\chi}_i)$, cf. Equation (15),

$$\int |\Delta(\tilde{\chi}_i)|^2 ds_{\tilde{\chi}_i} = \frac{\pi}{m_{\tilde{\chi}_i}\Gamma_{\tilde{\chi}_i}}. \tag{B4}$$

This approximation should be justified for $(\Gamma_{\tilde{\chi}_i}/m_{\tilde{\chi}_i})^2 \ll 1$, which holds in our case for chargino widths $\Gamma_{\tilde{\chi}_i} \lesssim 1$ GeV and masses $m_{\tilde{\chi}_i} \approx 100$ GeV. However, the naive $\mathcal{O}(\Gamma/m)$ expectation of the error can easily receive large off-shell corrections of an order of magnitude and more, in particular, at threshold or due to interferences with other resonant or nonresonant processes. For a recent discussion of these issues, see, for example, Ref. [61].

APPENDIX C: CHARGINO DIAGONALIZATON MATRICES

The matrices U and V, which diagonalize the chargino matrix $M_{\tilde{\chi}}$, see Eq. (3), can be parametrized by [23]

$$U = \begin{pmatrix} e^{i\gamma_1} & 0 \\ 0 & e^{i\gamma_2} \end{pmatrix} \begin{pmatrix} \cos\theta_1 & e^{i\phi_1}\sin\theta_1 \\ -e^{-i\phi_1}\sin\theta_1 & \cos\theta_1 \end{pmatrix}, \quad \text{(C1)}$$

$$V = \begin{pmatrix} \cos\theta_2 & e^{-i\phi_2}\sin\theta_2 \\ -e^{i\phi_2}\sin\theta_2 & \cos\theta_2 \end{pmatrix}. \quad \text{(C2)}$$

The mixing angles $-\pi/2 \le \theta_{1,2} \le 0$ are

$$\frac{t(2\theta_1)}{2\sqrt{2}m_W} = \frac{\sqrt{M_2^2 c^2(\beta) + |\mu|^2 s^2(\beta) + M_2|\mu| s(2\beta) c(\varphi_\mu)}}{M_2^2 - |\mu|^2 - 2m_W^2 c(2\beta)}, \quad \text{(C3a)}$$

$$\frac{t(2\theta_2)}{2\sqrt{2}m_W} = \frac{\sqrt{M_2^2 s^2(\beta) + |\mu|^2 c^2(\beta) + M_2|\mu| s(2\beta) c(\varphi_\mu)}}{M_2^2 - |\mu|^2 + 2m_W^2 c(2\beta)}, \quad \text{(C3b)}$$

with the short hand notations $s(\alpha) = \sin(\alpha)$, $c(\alpha) = \cos(\alpha)$, and $t(\alpha) = \tan(\alpha)$. For a CP-violating chargino system $\varphi_\mu \neq 0$, the following CP phases enter

$$t(\phi_1) = s(\varphi_\mu)\left[c(\varphi_\mu) + \frac{M_2}{|\mu| t(\beta)}\right]^{-1}, \quad \text{(C4a)}$$

$$t(\phi_2) = -s(\varphi_\mu)\left[c(\varphi_\mu) + \frac{M_2 t(\beta)}{|\mu|}\right]^{-1}, \quad \text{(C4b)}$$

$$t(\gamma_1) = -\frac{s(\varphi_\mu)}{\left[c(\varphi_\mu) + \frac{M_2(m_{\tilde{\chi}_1}^2 - |\mu|^2)}{|\mu| m_W^2 s(2\beta)}\right]}, \quad \text{(C4c)}$$

$$t(\gamma_2) = \frac{s(\varphi_\mu)}{\left[c(\varphi_\mu) + \frac{M_2 m_W^2 s(2\beta)}{|\mu|(m_{\tilde{\chi}_2}^2 - M_2^2)}\right]}. \quad \text{(C4d)}$$

The chargino masses are

$$m_{\tilde{\chi}_{1,2}}^2 = \frac{1}{2}(M_2^2 + |\mu|^2 + 2m_W^2 \mp \kappa), \quad \text{(C5)}$$

with

$$\kappa^2 = (M_2^2 - |\mu|^2)^2 + 4m_W^4 c^2(2\beta) + 4m_W^2[M_2^2 + |\mu|^2 + 2M_2|\mu| s(2\beta) c(\varphi_\mu)]. \quad \text{(C6)}$$

APPENDIX D: THEORETICAL STATISTICAL SIGNIFICANCE

To measure a nonzero value of the asymmetry $\mathcal{A}_\tau$, Eq. (27), over statistical fluctuations, we define its theoretical statistical significance by [48]

$$S_\tau = \frac{|\mathcal{A}_\tau|}{\sigma_{\mathcal{A}}}, \quad \text{(D1)}$$

such that S_τ is the expected number of standard deviations $\sigma_{\mathcal{A}}$ to which the asymmetry $\mathcal{A}_\tau$ can be determined to differ from zero. Since the variance is given by

$$\sigma_{\mathcal{A}}^2 = \frac{1 - |\mathcal{A}_\tau|^2}{2N}, \quad \text{(D2)}$$

with the total number of events $N = \mathcal{L}\sigma$, we find

$$S_\tau = \frac{|\mathcal{A}_\tau|\sqrt{2\mathcal{L}\sigma}}{\sqrt{1 - |\mathcal{A}_\tau|^2}}. \quad \text{(D3)}$$

Note that our definition of the statistical significance S_τ is purely based on the theoretical signal rate and its asymmetry. Detector efficiencies, event reconstruction efficiencies, and contributions from CP-even backgrounds are neglected, which would reduce the significance. The definition has thus to be regarded as an absolute upper bound only. In order to give realistic values of the statistical significances to observe a CP signal, a detailed experimental study is necessary.

APPENDIX E: ANGULAR DISTRIBUTIONS, AND INTERFERENCE CONTRIBUTIONS

For the scenario as given in Table I, in Fig. 11, we show the angular distributions of the production cross section σ_P and the asymmetry $\mathcal{A}_\tau$ in the chargino scattering angle $\theta = \sphericalangle(\mathbf{p}_{e^-}, \mathbf{p}_{\tilde{\chi}_1^-})$. The distributions are defined by

$$\frac{d\sigma_P}{d\theta} = \frac{\sqrt{\lambda(s, m_{\tilde{\chi}_i}^2, m_{\tilde{\chi}_j}^2)}}{8\pi s^2} \mathrm{P}(\theta; XY)\sin\theta, \quad \text{(E1)}$$

$$\frac{d\mathcal{A}_\tau}{d\theta} = \frac{\sqrt{\lambda(s, m_{\tilde{\chi}_i}^2, m_{\tilde{\chi}_j}^2)}}{32\pi^2 s}\sin\theta \times \frac{\int \Sigma_{\mathrm{P}}^a(\theta; XY)\Sigma_{\mathrm{D}}^{ab=2}(\theta) d\mathcal{L}ips(s_{\tilde{\chi}_i}; p_\tau, p_{\tilde{\nu}})}{\int \mathrm{PD} d\mathcal{L}ips}, \quad \text{(E2)}$$

with the phase-space elements $d\mathcal{L}ips$ as defined in Appendix B. The three different channels for chargino production, see Fig. 1, are labeled by $X, Y \in \{Z, \gamma, \tilde{\nu}\}$. We show their corresponding interference contributions to the production cross section and the asymmetry in Figs. 11 and 12.

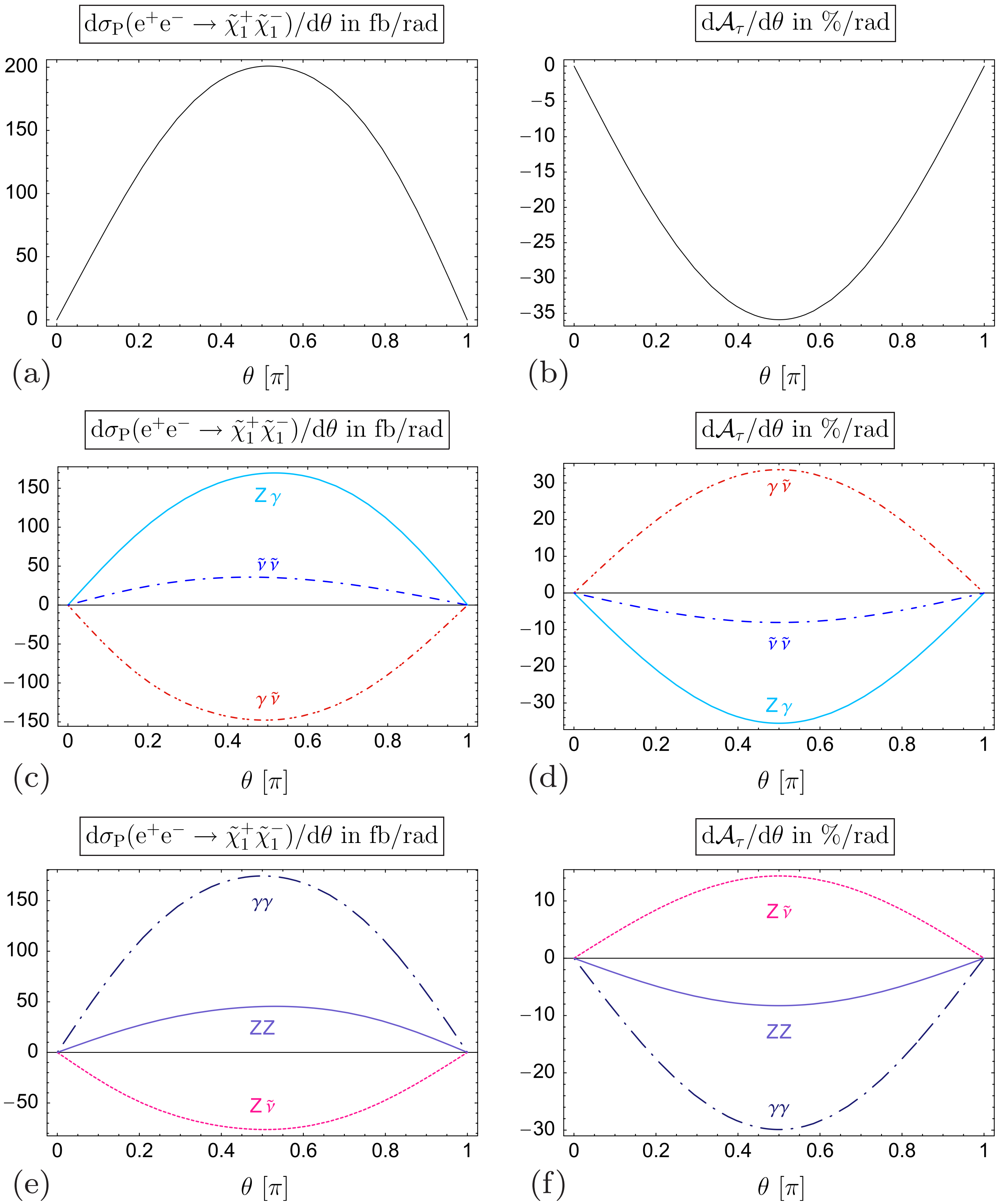

FIG. 11 (color online). Angular distributions in the chargino scattering angle $\theta = \sphericalangle(\mathbf{p}_{e^-}, \mathbf{p}_{\tilde{\chi}_1^-})$ of (a) the production cross section, see Eq. (E1); (c) & (e) the interference contributions to the production cross section; (b) the *CP*-sensitive asymmetry of the normal tau polarization, see Eq. (E2); and (d) & (f) the interference contributions to the asymmetry for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. The SUSY parameters are given in Table I, with a center-of-mass energy $\sqrt{s} = 500$ GeV and longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$.

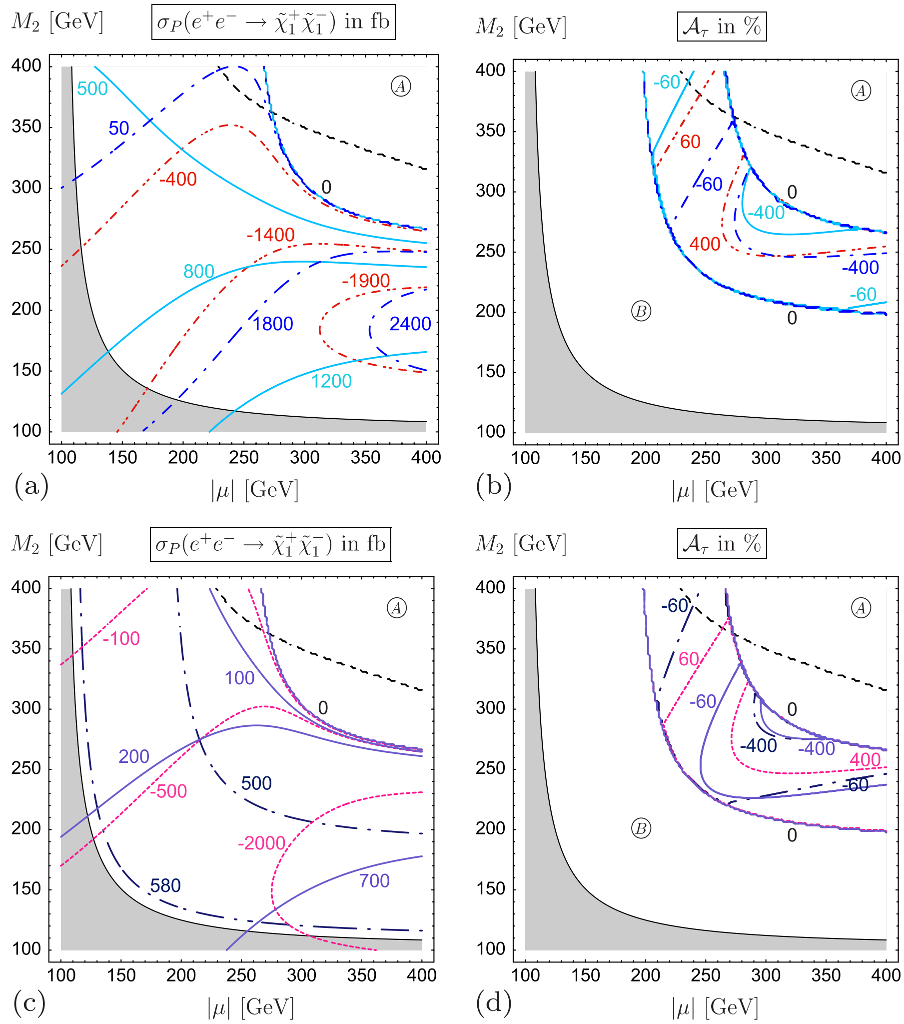

FIG. 12 (color online). Different interference contributions to (a) & (c) the chargino production cross section, and (b) & (d) to the CP-sensitive asymmetry of the normal tau polarization for $e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-$, $\tilde{\chi}_1^\pm \to \tau^\pm \tilde{\nu}_\tau^{(*)}$. The color and line shaping is chosen as in Fig. 11 and refers to the $Z\gamma$, $\tilde{\nu}\,\tilde{\nu}$, and $\gamma\tilde{\nu}$ contributions for (a) & (b) and to the $\gamma\gamma$, ZZ, and the $Z\tilde{\nu}$ contributions for (c) & (d), respectively. The underlying SUSY parameters are given in Table I, with a center-of-mass energy $\sqrt{s} = 500$ GeV and longitudinally polarized beams $(\mathcal{P}_{e^-}|\mathcal{P}_{e^+}) = (-0.8|0.6)$.

[1] N. Cabibbo and R. Gatto, Phys. Rev. Lett. **5**, 114 (1960); N. Cabibbo, Phys. Lett. **12**, 137 (1964); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] A. J. Buras, arXiv:hep-ph/0505175.

[3] C. Amsler *et al.* (Particle Data Group), Phys. Lett. B **667**, 1 (2008).

[4] F. Csikor, Z. Fodor, and J. Heitger, Phys. Rev. Lett. **82**, 21 (1999); M. B. Gavela, P. Hernandez, J. Orloff, and O. Pene, Mod. Phys. Lett. A **9**, 795 (1994); M. B. Gavela, P. Hernandez, J. Orloff, O. Pene, and C. Quimbay, Nucl. Phys. **B430**, 382 (1994); M. Beyer, *CP Violation in Particle, Nuclear and Astrophysics* (Springer, New York, 2002).

[5] A. Riotto, arXiv:hep-ph/9807454; W. Bernreuther, Lect. Notes Phys. **591**, 237 (2002); A. Masiero and O. Vives, New J. Phys. **4**, 4 (2002).

[6] H. E. Haber and G. L. Kane, Phys. Rep. **117**, 75 (1985); H. P. Nilles, Phys. Rep. **110**, 1 (1984); M. Drees, R. Godbole, and P. Roy, *Theory and Phenomenology of Sparticles: An Account of Four-Dimensional $N = 1$ Supersymmetry in High Energy Physics* (World Scientific, Singapore, 2004).

[7] M. Claudson, L. J. Hall, and I. Hinchliffe, Nucl. Phys. **B228**, 501 (1983).

[8] E. D. Commins, S. B. Ross, D. DeMille, and B. C. Regan, Phys. Rev. A **50**, 2960 (1994); B. C. Regan, E. D. Commins, C. J. Schmidt, and D. DeMille, Phys. Rev. Lett. **88**, 071805 (2002).

[9] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006); Y. Kizukuri and N. Oshimo, Phys. Rev. D **45**, 1806 (1992); **46**, 3025 (1992); J. Polchinski and M. B. Wise, Phys. Lett. B **125**, 393 (1983); S. Abel, S. Khalil, and O. Lebedev, Nucl. Phys. **B606**, 151 (2001); S. Abel and O. Lebedev, J. High Energy Phys. 01 (2006) 133; M. Pospelov and A. Ritz, Ann. Phys. (N.Y.) **318**, 119 (2005).

[10] W. C. Griffith, M. D. Swallows, T. H. Loftus, M. V. Romalis, B. R. Heckel, and E. N. Fortson, Phys. Rev. Lett. **102**, 101601 (2009); K. V. P. Latha, D. Angom, B. P. Das, and D. Mukherjee, Phys. Rev. Lett. **103**, 083001 (2009).

[11] T. Falk, K. A. Olive, M. Pospelov, and R. Roiban, Nucl. Phys. **B560**, 3 (1999).

[12] Y. K. Semertzidis *et al.* (EDM Collaboration), AIP Conf. Proc. **698**, 200 (2004).

[13] Y. K. Semertzidis, Nucl. Phys. B, Proc. Suppl. **131**, 244 (2004); J. R. Ellis, S. Ferrara, and D. V. Nanopoulos, Phys. Lett. B **114**, 231 (1982); W. Buchmuller and D. Wyler, Phys. Lett. B **121**, 321 (1983); F. del Aguila, M. B. Gavela, J. A. Grifols, and A. Mendez, Phys. Lett. B **126**, 71((1983); **129**, 473(E) (1983); D. V. Nanopoulos and M. Srednicki, Phys. Lett. B **128**, 61 (1983); M. Dugan, B. Grinstein, and L. J. Hall, Nucl. Phys. **B255**, 413 (1985); C. S. Huang and W. Liao, Phys. Rev. D **62**, 016008 (2000).

[14] T. Ibrahim and P. Nath, Phys. Rev. D **57**, 478 (1998); **58**, 019901(E) (1998); **60**, 079903(E) (1999); **60**, 119901(E) (1999); Phys. Lett. B **418**, 98 (1998); Phys. Rev. D **58**, 111301 (1998); **60**, 099902(E) (1999); **61**, 093004 (2000); M. Brhlik, G. J. Good, and G. L. Kane, Phys. Rev. D **59**, 115004 (1999); S. Yaser Ayazi and Y. Farzan, Phys. Rev. D **74**, 055008 (2006).

[15] see, e.g., A. Bartl, T. Gajdosik, W. Porod, P. Stockinger, and H. Stremnitzer, Phys. Rev. D **60**, 073003 (1999); A. Bartl, T. Gajdosik, E. Lunghi, A. Masiero, W. Porod, H. Stremnitzer, and O. Vives, Phys. Rev. D **64**, 076009 (2001); L. Mercolli and C. Smith, Nucl. Phys. B **817**, 1 (2009); V. D. Barger, T. Falk, T. Han, J. Jiang, T. Li, and T. Plehn, Phys. Rev. D **64**, 056007 (2001).

[16] A. Bartl, W. Majerotto, W. Porod, and D. Wyler, Phys. Rev. D **68**, 053005 (2003); K. A. Olive, M. Pospelov, A. Ritz, and Y. Santoso, Phys. Rev. D **72**, 075001 (2005); J. R. Ellis, J. S. Lee, and A. Pilaftsis, J. High Energy Phys. 10 (2008) 049; S. A. Abel, A. Dedes, and H. K. Dreiner, J. High Energy Phys. 05 (2000) 013.

[17] S. Y. Choi, M. Drees, and B. Gaissmaier, Phys. Rev. D **70**, 014010 (2004).

[18] N. Arkani-Hamed, S. Dimopoulos, G. F. Giudice, and A. Romanino, Nucl. Phys. **B709**, 3 (2005); G. F. Giudice and A. Romanino, Nucl. Phys. **B699**, 65 (2004); **B706**, 65 (2005); N. Arkani-Hamed and S. Dimopoulos, J. High Energy Phys. 06 (2005) 073.

[19] J. Bagger, J. L. Feng, and N. Polonsky, Nucl. Phys. **B563**, 3 (1999); S. Dimopoulos and G. F. Giudice, Phys. Lett. B **357**, 573 (1995); A. G. Cohen, D. B. Kaplan, F. Lepeintre, and A. E. Nelson, Phys. Rev. Lett. **78**, 2300 (1997); G. R. Dvali and A. Pomarol, Phys. Rev. Lett. **77**, 3728 (1996); L. Randall, arXiv:hep-ph/9706475; H. C. Cheng, J. L. Feng, and N. Polonsky, Phys. Rev. D **57**, 152 (1998); J. Hisano, K. Kurosawa, and Y. Nomura, Phys. Lett. B **445**, 316 (1999); J. A. Bagger, J. L. Feng, N. Polonsky, and R. J. Zhang, Phys. Lett. B **473**, 264 (2000); B.C. Allanach *et al.*, Nucl. Phys. B, Proc. Suppl. **135**, 107 (2004); M. S. Carena and A. Freitas, Phys. Rev. D **74**, 095004 (2006); K. Desch, J. Kalinowski, G. Moortgat-Pick, K. Rolbiecki, and W. J. Stirling, J. High Energy Phys. 12 (2006) 007; arXiv:0710.4937.

[20] J. L. Feng, K. T. Matchev, and T. Moroi, Phys. Rev. D **61**, 075005 (2000); J. L. Feng and F. Wilczek, Phys. Lett. B **631**, 170 (2005).

[21] S. Abdullin *et al.* (CMS Collaboration), J. Phys. G **28**, 469 (2002); (), Technical Design Report No. CERN-LHCC-99-15; G. Weiglein *et al.* (LHC/LC Study Group), Phys. Rep. **426**, 47 (2006).

[22] J. Brau *et al.* (ILC Collaboration), arXiv:0712.1950; J. A. Aguilar-Saavedra *et al.* (ECFA/DESY LC Physics Working Group), arXiv:hep-ph/0106315; T. Abe *et al.* (American Linear Collider Working Group), arXiv:hep-ex/0106055; K. Abe *et al.* (ACFA Linear Collider Working Group), arXiv:hep-ph/0109166; J. A. Aguilar-Saavedra *et al.*, Eur. Phys. J. C **46**, 43 (2006).

[23] A. Bartl, K. Hidaka, T. Kernreiter, and W. Porod, Phys. Lett. B **538**, 137 (2002); Phys. Rev. D **66**, 115009 (2002); A. Bartl, S. Hesselbach, K. Hidaka, T. Kernreiter, and W. Porod, Phys. Lett. B **573**, 153 (2003); Phys. Rev. D **70**, 035003 (2004).

[24] K. Rolbiecki, J. Tattersall, and G. Moortgat-Pick, arXiv:0909.3196.

[25] T. Gajdosik, R. M. Godbole, and S. Kraml, J. High Energy Phys. 09 (2004) 051.

[26] T. Han and Y. Li, Phys. Lett. B **683**, 278 (2010).

[27] S. Y. Choi and M. Drees, Phys. Lett. B **435**, 356 (1998); M. Aoki and N. Oshimo, Mod. Phys. Lett. A **13**, 3225 (1998); W. M. Yang and D. S. Du, Phys. Rev. D **65**, 115005 (2002); **67**, 055004 (2003); T. Ibrahim and P. Nath, Phys. Rev. D **71**, 055007 (2005); K. Rolbiecki and J. Kalinowski, Phys. Rev. D **76**, 115006 (2007); Acta Phys. Pol. B **38**, 3557 (2007); **39**, 1585 (2008); P. Osland and A. Vereshagin, Phys. Rev. D **76**, 036001 (2007); P. Osland, J. Kalinowski, K. Rolbiecki, and A. Vereshagin, arXiv:0709.3358; M. Frank and I. Turan, Phys. Rev. D **76**, 076008 (2007); E. Christova, H. Eberl, W. Majerotto, and S. Kraml, J. High Energy Phys. 12, (2002) 021 ; E. Christova, E. Ginina, and M. Stoilov, J. High Energy Phys. 11 (2003) 027; E. Christova, H. Eberl, E. Ginina, and W. Majerotto, J. High Energy Phys. 02 (2007) 075; Phys. Rev. D **79**, 096005 (2009); E. Ginina, E. Christova, and H. Eberl, Proc. Sci., CHARGED2008 (2008) 013 [arXiv:0812.1129]; H. Eberl, T. Gajdosik, W. Majerotto, and B. Schrausser, Phys. Lett. B **618**, 171 (2005); S. M. R. Frank, arXiv:0909.3969; H. Eberl, S. M. R. Frank, and W. Majerotto, arXiv:0912.4675; H. EberlS. M. R. Frank AIP Conf. Proc. **1200**, 518 (2010).

[28] A. Pilaftsis, Nucl. Phys. **B504**, 61 (1997); J. R. Ellis, J. S. Lee, and A. Pilaftsis, Phys. Rev. D **70**, 075010 (2004); **72**, 095006 (2005).

[29] S. Y. Choi, J. Kalinowski, Y. Liao, and P. M. Zerwas, Eur. Phys. J. C **40**, 555 (2005).

[30] E. Accomando *et al.*, arXiv:hep-ph/0608079; H. K. Dreiner, O. Kittel, and F. von der Pahlen, J. High Energy Phys. 01 (2008) 017; O. Kittel and F. von der Pahlen, J. High Energy Phys. 08 (2008) 030.

[31] M. Nagashima, K. Kiers, A. Szynkman, D. London, J. Hanchey, and K. Little, Phys. Rev. D **80**, 095012 (2009).

[32] G. Valencia, arXiv:hep-ph/9411441; G. C. Branco, L. Lavoura, and J. P. Silva, *CP Violation* (Oxford University, New York, 1999).

[33] For recent reviews see, for example, G. Moortgat-Pick, K. Rolbiecki, J. Tattersall, and P. Wienemann, arXiv:0910.1371O. Kittel, J. Phys. Conf. Ser. **171**, 012094 (2009); S. Kraml, arXiv:0710.5117; S. Hesselbach, arXiv:0709.2679.

[34] A. Bartl, E. Christova, K. Hohenwarter-Sodek, and T. Kernreiter, Phys. Rev. D **70**, 095007 (2004).
[35] A. Bartl, E. Christova, K. Hohenwarter-Sodek, and T. Kernreiter, J. High Energy Phys. 11 (2006) 076.
[36] A. Bartl, T. Kernreiter, and W. Porod, Phys. Lett. B **538**, 59 (2002); K. Kiers, A. Szynkman, and D. London, Phys. Rev. D **74**, 035004 (2006); **75**, 075009 (2007).
[37] F. Deppisch and O. Kittel, J. High Energy Phys. 09 (2009) 110.
[38] J. Ellis, F. Moortgat, G. Moortgat-Pick, J. M. Smillie, and J. Tattersall, Eur. Phys. J. C **60**, 633 (2009).
[39] P. Langacker, G. Paz, L. T. Wang, and I. Yavin, J. High Energy Phys. 07 (2007) 055.
[40] G. Moortgat-Pick, K. Rolbiecki, J. Tattersall, and P. Wienemann, J. High Energy Phys. 01 (2010) 004.
[41] A. Bartl, T. Kernreiter, and O. Kittel, Phys. Lett. B **578**, 341 (2004); O. Kittel, arXiv:hep-ph/0311169.
[42] S. Y. Choi, M. Drees, B. Gaissmaier, and J. Song, Phys. Rev. D **69**, 035008 (2004).
[43] For further studies with neutralino 2-body decays at the ILC, see A. Bartl, H. Fraas, T. Kernreiter, and O. Kittel, Eur. Phys. J. C **33**, 433 (2004); A. Bartl, H. Fraas, O. Kittel, and W. Majerotto, Phys. Rev. D **69**, 035007 (2004); Eur. Phys. J. C **36**, 233 (2004); A. Bartl, H. Fraas, T. Kernreiter, O. Kittel, and W. Majerotto, arXiv:hep-ph/0310011; J. A. Aguilar-Saavedra, Nucl. Phys. B **697**, 207 (2004); S. Y. Choi and Y. G. Kim, Phys. Rev. D **69**, 015011 (2004); A. Bartl, K. Hohenwarter-Sodek, T. Kernreiter, O. Kittel, and M. Terwort, J. High Energy Phys. 07 (2009) 054.
[44] For studies with neutralino 3-body decays at the ILC, see Y. Kizukuri and N. Oshimo, Phys. Lett. B **249**, 449 (1990); S. Y. Choi, H. S. Song, and W. Y. Song, Phys. Rev. D **61**, 075004 (2000); J. A. Aguilar-Saavedra, Phys. Lett. B **596**, 247 (2004); A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, and G. A. Moortgat-Pick, J. High Energy Phys. 08 (2004) 038; S. Y. Choi, B. C. Chung, J. Kalinowski, Y. G. Kim, and K. Rolbiecki, Eur. Phys. J. C **46**, 511 (2006).
[45] O. Kittel, Ph.D thesis, Bayerische Julius-Maximilians-Universität Würzburg, 2004, arXiv:hep-ph/0504183.
[46] For studies with chargino 3-body decays at the ILC, see Y. Kizukuri and N. Oshimo, arXiv:hep-ph/9310224; A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, T. Kernreiter, and G. Moortgat-Pick, Eur. Phys. J. C **51**, 149 (2007).
[47] For studies with chargino 2-body decays at the ILC, see S. Y. Choi, A. Djouadi, M. Guchait, J. Kalinowski, H. S. Song, and P. M. Zerwas, Eur. Phys. J. C **14**, 535 (2000); A. Bartl, H. Fraas, O. Kittel, and W. Majerotto, Phys. Lett. B **598**, 76 (2004); O. Kittel, A. Bartl, H. Fraas, and W. Majerotto, Phys. Rev. D **70**, 115005 (2004); J. A. Aguilar-Saavedra, Nucl. Phys.**B717**, 119 (2005); A. Bartl, K. Hohenwarter-Sodek, T. Kernreiter, O. Kittel, and M. Terwort, Nucl. Phys. **B802**, 77 (2008).
[48] G. A. Moortgat-Pick *et al.*, Phys. Rep. **460**, 131 (2008); A. Bartl, K. Hohenwarter-Sodek, T. Kernreiter, and H. Rud, Eur. Phys. J. C **36**, 515 (2004); A. Bartl, H. Fraas, S. Hesselbach, K. Hohenwarter-Sodek, T. Kernreiter, and G. A. Moortgat-Pick, J. High Energy Phys. 01 (2006) 170; S. Y. Choi, M. Drees, and J. Song, J. High Energy Phys. 09 (**2006**) 064; A. Bartl, H. Fraas, K. Hohenwarter-Sodek, T. Kernreiter, G. Moortgat-Pick, and A. Wagner, Phys. Lett. B **644**, 165 (2007); A. Bartl, K. Hohenwarter-Sodek, T. Kernreiter, and O. Kittel, J. High Energy Phys. 09 (2007) 079.
[49] S. A. Abel, M. Dittmar, and H. K. Dreiner, Phys. Lett. B **280**, 304 (1992); H. K. Dreiner, arXiv:hep-ph/9211203.
[50] See, e.g., S. Berge, W. Bernreuther, and J. Ziethe, Phys. Rev. Lett. **100**, 171605 (2008); S. Berge and W. Bernreuther, Phys. Lett. B **671**, 470 (2009).
[51] E. Boos, H. U. Martyn, G. A. Moortgat-Pick, M. Sachwitz, A. Sherstnev, and P. M. Zerwas, Eur. Phys. J. C **30**, 395 (2003).
[52] M. Perelstein and A. Weiler, J. High Energy Phys. 03 (2009) 141.
[53] H. E. Haber, in *Proceedings of the 21st SLAC Summer Institute on Particle Physics*, edited by L. DeProcel and C. Dunwoodie(Stanford University, Stanford, 1993), p. 231.
[54] For chargno/neutralino production in the spin-density matrix formalism, see G. A. Moortgat-Pick, H. Fraas, A. Bartl, and W. Majerotto, Eur. Phys. J. C **7**, 113 (1999); Eur. Phys. J. C **9**, 521 (1999); **9**, 549(E) (1999); G. Moortgat-Pick, *Spineffekte in Chargino-/Neutralino Produktion und Zerfall* (Shaker Verlag, Aachen, Germany, 2000).
[55] S. Y. Choi, A. Djouadi, H. K. Dreiner, J. Kalinowski, and P. M. Zerwas, Eur. Phys. J. C **7**, 123 (1999).
[56] F. M. Renard, *Basics of Electron Positron Collisions* (Editions Frontieres, Dreux, France, 1981).
[57] G. Luders, K. Dan. Vidensk. Selsk. Mat. Fys. Medd. **28N5**, 1 (1954); W. Pauli, *Niels Bohr and the Development of Physics* (Mc Graw-Hill, New York, 1955); R. Jost, Helv. Phys. Acta **30**, 409 (1957); **36**, 77 (1963); R. F. Streater and A. S. Wightman, *PCT, Spin and Statistics, and All That* (Addison-Wesley, Reading, MA, 1989).
[58] G. Valencia, Phys. Rev. D **39**, 3339 (1989).
[59] H. K. Dreiner, C. Hanhart, U. Langenfeld, and D. R. Phillips, Phys. Rev. D **68**, 055004 (2003); H. K. Dreiner, S. Heinemeyer, O. Kittel, U. Langenfeld, A. M. Weber, and G. Weiglein, Eur. Phys. J. C **62**, 547 (2009); in *Proceedings for the LCWS/ILC 2007 workshop at DESY, Hamburg, Germany*, econf C0705302:SUS06 (2007); H. K. Dreiner, O. Kittel, and U. Langenfeld, Phys. Rev. D **74**, 115010 (2006); Eur. Phys. J. C **54**, 277 (2008); in *Proceedings for the LCWS/ILC 2007 workshop at DESY, Hamburg, Germany*, econf C0705302:SUS05-POL01 (2007);H. K. Dreiner, S. Grab, D. Koschade, M. Kramer, B. O'Leary, and U. Langenfeld, Phys. Rev. D **80**, 035018 (2009); T. Behnke *et al.* (ILC Collaboration), arXiv:0712.2356; J. Brau *et al.* (ILC Collaboration), arXiv:0712.1950; G. Aarons *et al.* (ILC Collaboration), arXiv:0709.1893.
[60] E. Byckling, K. Kajantie, *Particle Kinematics* (Wiley, New York, 1973); G. Costa *et al.*, *Kinematics and Symmetries*, edited by M. Nicolić, TEPP Vol. BD. 1 (Institut National De Physique Nucléaire et de Physique des Particules, Paris, 1979).
[61] K. Hagiwara *et al.*, Phys. Rev. D **73**, 055005 (2006); D. Berdine, N. Kauer, and D. Rainwater, Phys. Rev. Lett. **99**, 111601 (2007); N. Kauer, Phys. Lett. B **649**, 413 (2007); J. High Energy Phys. 04 (2008) 055; C. F. Uhlemann and N. Kauer, Nucl. Phys. **B814**, 195 (2009); M. A. Gigg and P. Richardson, arXiv:0805.3037.