Continuum thermodynamics of the SU(N_c) gluon plasma

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We study the thermodynamics of SU(N_c) pure gauge theories for $N_c = 3$, 4 and 6. The continuum and thermodynamic limits of bulk quantities such as the pressure (p), energy density (ϵ) and the entropy density (s) are taken by using several different lattice spacings and volumes. There is no window of temperature in which a nontrivial conformal theory describes bulk thermodynamics. We extract the latent heat of the first-order deconfinement phase transitions and observe good scaling with N_c. For all quantities that we measure, strong N_c scaling holds, except, possibly, very close to the transition temperature, T_c; however we are unable to find strong evidence for scaling with the 't Hooft coupling in thermal quantities at the small values of N_c which we study.

I. INTRODUCTION

Interest in SU(N_c) gauge theories with large N_c began with the pioneering studies of [1], where it was shown that in $1 + 1$ dimensions the limit $N_c \rightarrow \infty$ when taken such that the gauge coupling, $g \rightarrow 0$, with the 't Hooft coupling $\lambda = N_c g^2$ fixed gave rise to an interesting and nontrivial but tractable theory. In this so-called 't Hooft scaling limit stable mesons exist with properties which parallel much of known hadron phenomenology. The limit is nonperturbative, and the computation of any correlation function requires the summation of an infinite number of well-characterized Feynman diagrams. Corrections to this limit appear in powers of $1/N_c$ in general, and in powers of $1/N_c^2$ for the pure gauge theory. Since then many models have used this 't Hooft limit [2], including the currently fashionable conformal cousins of QCD: the so-called $\mathcal{N} = 4$ supersymmetric theories which turn out to be tractable using the AdS/CFT correspondence. Lattice simulations of theories with small $N_c \geq 3$ can test the smoothness of approach to the 't Hooft limit.

A contemporary reason for studying the large N_c theory is in the light it could throw on the phase diagram of QCD. At any fixed N_c with two flavors of massless quarks, the theory is expected to have a second-order chiral symmetry restoring transition at finite temperature, T, which is in the O(4) universality class. If a tiny mass is given to the quarks, then the theory has a crossover at finite temperature instead of a phase transition. The limit of infinite quark mass corresponds to the pure gauge theory, which has a first-order deconfining thermal transition for all $N_c > 2$. Hence, at an intermediate quark mass there is a critical point, in the Ising universality class, which ends a line of first-order deconfining transitions [3]. We represent this information in the diagram of Fig. 1. In the regions with the chiral crossover (region A) and the large N_c deconfinement transition (region B) the phase diagrams in the plane of T and

baryon chemical potential, μ, are topologically distinct; a representative phase diagram from each of these regions is also shown. In region A one expects a first-order phase transition line dividing the chiral symmetry broken phase
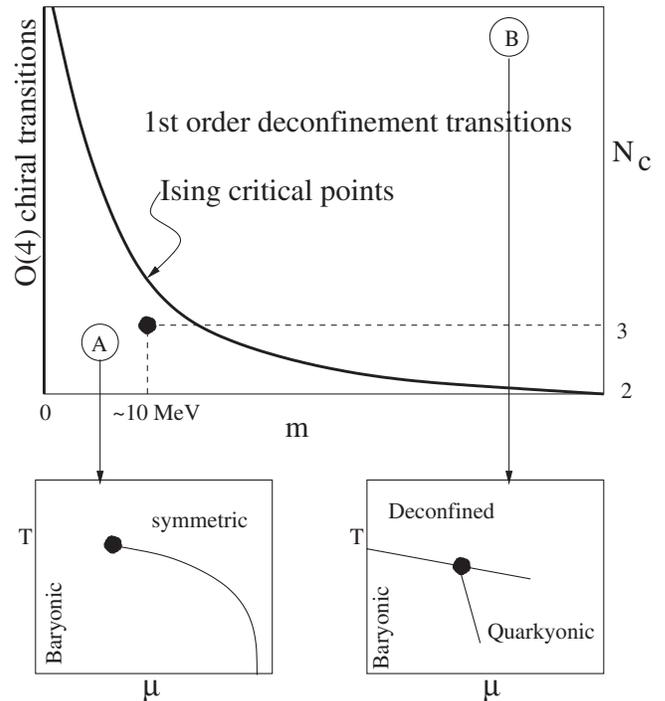
FIG. 1. The diagram on top shows the regions of first- and second-order thermal phase transitions for SU(N_c) gauge theories with two flavors of fermions as N_c and the quark mass, m, are varied. We also show the phase diagrams in two interesting regions—(A) the chiral crossover region and (B) the large N_c deconfinement region. The topology of the phase diagrams in region A is expected to involve a line of first-order transitions ending in a critical point at finite chemical potential μ. In region B large N_c arguments indicate that the phase diagram contains three lines of first-order phase transitions separating pairs of phases labeled baryonic, deconfined and quarkyonic, with a triple-point where all three phases coexist.

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from the deconfined gluoN_c plasma phase with a critical end point at finite μ. In region B one might expect a triple-point with coexistence of the three phases—gluoN_c plasma, baryonic, i.e., chiral symmetry broken and confined, and quarkyonic, i.e., chirally symmetric and confined [4]. Since the arguments for the existence of a quarkyonic phase are based on a picture of the large N_c theory which arises in the 't Hooft limit, it is interesting to explore the range of validity of such large N_c arguments.

In this paper our primary interest is in examining the thermodynamics of SU(N_c) pure gauge theory in the large volume and continuum limits. The deconfinement transition in SU(N_c) theories has been studied in [5–9]. The latent heat of the transition was studied in [6,8] on spatial volumes, V, of size $T\sqrt[3]{V} < 3$. The equation of state (EOS) has been studied with lattice spacing of $a = 1/(5T)$ up to $2T_c$ in [10] and up to $3.5T_c$ in [11]. Previous work has shown that the continuum limit of the EOS [12] is hard to extract on such coarse lattices [8,9]. In view of this, we have studied the equation of state at smaller lattice spacing in the extended temperature range $0.97T_c \leq T \leq 4T_c$ (preliminary results were presented in [3]). We extrapolate to the continuum limit using the nonperturbative beta-functions determined earlier [9]. We perform finite-size scaling studies by changing the spatial volume up to $T\sqrt[3]{V} = 4$ for $a = 1/(6T_c)$ and $T\sqrt[3]{V} \simeq 3.5$ for $a = 1/(8T_c)$. We use statistics significantly larger than used before in this context, and comparable to that used in studies of thermodynamics for SU(3) pure gauge theory.

One of the purposes of a study like this is to perform lattice simulations for small $N_c > 3$, and, from measurements of any physical quantity, find the series expansion for it around $N_c = \infty$ in powers of $1/N_c^2$. In this work we assume that the $N_c = \infty$ limit exists and that there is a series expansion around it, and ask what our data implies for the radius of convergence of this series. Technically, this also means that for each series expansion, we ask how reliably the $N_c \to \infty$ limit can be taken from measurements for a few small values of N_c.

In agreement with earlier studies, we find good evidence for scaling to $N_c \to \infty$ keeping T/T_c fixed. Such a "strong N_c scaling", previously observed for many quantities on the lattice, also gives small $1/N_c$ corrections. We also examine 't Hooft's limiting procedure. It is clear that the notion of 't Hooft scaling has to be defined carefully in any theory with a nonvanishing beta-function [2]. First, because one has to use a (scale dependent) renormalized coupling with its attendant scheme ambiguities. Second, because of this scheme dependence, $1/N_c$ corrections may be moved between the operator expectation values and the 't Hooft coupling. Even keeping these uncertainties in mind, we find that scaling at fixed λ gives large corrections (in the expansion around $N_c = \infty$) at small N_c, including the physically interesting case of $N_c = 3$.

The plan of this paper is as follows. In the next section, we summarize the formulas used for calculation of the various thermodynamic quantities. Next, in Sec. III we discuss the latent heat of the deconfinement transitions. In Sec. IV we investigate the conformal symmetry breaking measure, which is the trace of the energy-momentum tensor, $\Delta = \epsilon - 3p$. In Sec. V we discuss the pressure and the remaining bulk thermodynamic quantities. Section VI is devoted to a comparison of results from theories with different number of colors, to get an estimate of the size of the leading $O(1/N_c)$ corrections. Section VII analyzes the calculated thermodynamic quantities, to infer properties of the gluoN_c plasma. The Appendix contains a detailed discussion of the beta-functions used in this study.

II. FORMALISM AND DEFINITIONS

The thermodynamics of the SU(N_c) gauge theory can be obtained from the partition function,

$$Z(V, T) = \int \mathcal{D}U \exp(-S),$$
$$S = \beta \sum_{x, \mu, \nu < \mu} \{1 - \text{Real } P_{\mu\nu}(x)\} \tag{1}$$

calculated on a space-time lattice with N_s lattice sites in each of the spatial directions and N_t in the time direction; the lattice sites are labeled by the 4-component index x and directions by Greek indices, $1 \leq \mu, \nu \leq 4$. The bare gauge coupling, $g^2 = 2N_c/\beta$ determines the lattice spacing, a, which is implicit in the above equations. The spatial volume is $V = N_s^3 a^3$ and the temperature is $T = 1/(aN_t)$. Since we investigate finite-size effects, it is useful to introduce the aspect ratio, $\zeta = N_s/N_t = T\sqrt[3]{V}$. $P_{\mu\nu}(x)$ is the trace of the product of SU(N_c) valued link matrices U around the plaquette in the μ, ν plane starting at site x. The trace is normalized such that $P_{\mu\nu}(x) = 1$ if the link matrices are set to the identity.

The expectation value of the plaquette,

$$P = \frac{1}{6N_s^3 N_t} \sum_{x, \mu < \nu} \text{Real } P_{\mu\nu}(x), \tag{2}$$

over the ensemble at any temperature, T, is one of the primary observables on the lattice. The other is the expectation value of the Wilson line,

$$L = \frac{1}{N_s^3} \sum_{\mathbf{x}} \text{tr} \prod_{x_4=1}^{N_t} U_{\hat{t}}(x), \tag{3}$$

which is the spatial average of the product of timelike link variables wrapping around the lattice in the time direction at each spatial site, $\mathbf{x}$. $\langle L \rangle$ is the order parameter of the confinement-deconfinement transition and changes from zero to nonzero values at the (first-order) phase transition temperature T_c. In the deconfined phase $\langle L \rangle$ has N_c allowed values. Often $\langle |L| \rangle$ is examined, although it is not an order

parameter, since it has only two allowed values in the transition region: being close to vanishing in the confined phase and nonzero in the deconfined phase. These observables and the scaling of T_c to the continuum have been reported earlier [9].

The integral in (1) is performed by Monte Carlo sampling, using a combination of pseudoheatbath and over-relaxation steps, where all SU(2) subgroups of the SU(N_c) group elements are touched. For details of the algorithm used and its performance, see [9]. We studied SU(4) and SU(6) theories in the temperature range between T_c, and about $4T_c$. Since a major focus of this study is to get results in the continuum and thermodynamic limit, over the whole temperature range, we use two different lattice spacings, $a = 1/(6T)$ and $1/(8T)$, at each temperature and several different ζ [13]. The temperature scale for these theories was set in [9], where it was shown that near T_c the results are in the scaling regime. With the running of the coupling obtained in that study, we found very good agreement between the thermodynamic quantities extracted on lattices with the different a at all T. We have also made a few simulation runs for SU(3) gauge theories at large aspect ratios, to complement existing studies of latent heat in SU(3) gauge theories.

Bulk thermodynamic quantities are obtained by taking suitable derivatives of the partition function, $Z(V, T)$. In particular, the energy density and pressure are given by

$$\epsilon = \frac{T^2}{V} \frac{\partial \log Z}{\partial T}\bigg|_V, \qquad p = T \frac{\partial \log Z}{\partial V}\bigg|_T. \qquad (4)$$

The entropy density is given by the identity $s/T^3 = (\epsilon + p)/T^4$.

A quantity that is easy to calculate on the lattice is the trace of the energy-momentum tensor, $\Delta = \epsilon - 3p$. This is of some interest for models of the QCD plasma, since it is a direct measure of the conformal symmetry breaking. Using the above relations it is easy to show that

$$\frac{\Delta}{T^4} = 6N_t^4 \frac{\partial \beta}{\partial \log a} \langle \delta P \rangle, \quad \text{where}$$
$$\delta P = P(\beta, T) - \langle P(\beta, T = 0) \rangle. \qquad (5)$$

The expectation value of δP must be taken over the finite temperature ensemble. The subtraction of the plaquette expectation value at $T = 0$ must be done at the same lattice spacing as the finite temperature simulation. This serves to remove an ultraviolet divergence from the plaquette. It also makes sure that Δ/T^4 vanishes at $T = 0$, since both the pressure and the energy density vanish there. The derivative multiplying this nonperturbative factor is closely related to the beta-function of the theory. In the Appendix we have a discussion of the choices of beta-functions and their influence on Δ/T^4.

The suggestion of [14] was that since $p = T \log Z/V$, in the thermodynamic limit, the pressure can be evaluated

with respect to some reference value by integrating the plaquette expectation value—

$$\frac{p}{T^4} = \frac{1}{VT^3} \log Z = \frac{p_0}{T^4} + \frac{T}{V} \int_{\beta_0}^{\beta} d\beta' \frac{\partial \log Z}{\partial \beta}$$
$$\simeq 6 \int_{\beta_0}^{\beta} d\beta' \delta P(\beta'). \qquad (6)$$

Here p_0 is the pressure at the reference temperature $T_0 = 1/N_t a(\beta_0)$. For the gluoN_c plasma the pressure is expected to be very small below T_c, even as close to T_c as $T \sim 0.9T_c$. Conventionally one takes the reference temperature to be such a value and sets $p_0 = 0$, as in the second expression, where the derivative has also been written out explicitly. Once Δ and p are known, ϵ and s can be evaluated.

Asymptotic freedom in SU(N_c) gauge theories lead us to expect that at sufficiently high temperatures one should obtain a free gas of gluons. Then thermodynamic quantities reach their ideal gas (i.e., Stefan-Boltzmann: SB) limits. There are lattice corrections to this limit [15]. When the pressure is evaluated through the integral method one has

$$\frac{\epsilon_{SB}}{T^4} = 3\frac{p_{SB}}{T^4} = \frac{\pi^2 d_A}{15} G(N_t) \quad \text{where}$$
$$G(N_t) = 1 + \frac{8\pi^2}{21} \frac{1}{N_t^2} + \cdots \qquad (7)$$

and $d_A = N_c^2 - 1$. Since $aT = 1/N_t$, the correction terms come in powers of the lattice spacing a, and vanish in the continuum limit. The full $G(N_t)$ is also known exactly from numerical computations, and listed in [15]. When we discuss the numerical computations later we will use this full computation of $G(N_t)$ and not the series expansion above. The difference between the two is about 2% for $N_t = 6$. We draw attention to the factor of d_A: in the large N_c limit it is often replaced by N_c^2, but at the small values of N_c we used, the difference is statistically significant. We use d_A in this work, and thereby subsume some of the formally subleading $1/N_c$ corrections into this scaling.

One of the pieces of physics we are interested in is the latent heat of the transition. In a thermodynamically large volume this is defined by the formula

$$\frac{\Delta\epsilon}{T_c^4} = \lim_{\delta T \to 0}\left(\frac{\epsilon(T_c + \delta T)}{T_c^4} - \frac{\epsilon(T_c - \delta T)}{T_c^4}\right)$$
$$= \lim_{\delta T \to 0}\left(\frac{\Delta(T_c + \delta T)}{T_c^4} - \frac{\Delta(T_c - \delta T)}{T_c^4}\right) \qquad (8)$$

where the second equality follows from the fact that p is continuous across a first-order phase transition. These formulas cannot be directly used at finite volume. We describe a method for determining $\Delta\epsilon/T_c^4$ in Sec. III.

Some remarks about the computation are best placed here. It was observed earlier that at lattice spacing of $a \leq 1/(6T_c)$ the two-loop beta-function with a $1/N_t^2$ correction provides a good description of the change of measured length scales with the gauge coupling g^2. Therefore, one

should be able to perform continuum extrapolation of thermodynamic quantities for $T \gtrsim T_c$ using data on lattices with $N_t \geq 6$. This expectation should be correct except if there are large corrections in powers of a to the operators involved in defining components of the energy-momentum tensor. We show evidence later that there are no such large corrections. Another possible subtlety could reside in having to take the $V \to \infty$ (thermodynamic) limit before taking the continuum ($a \to 0$) limit. Such subtleties arise only when there are large correlation lengths, ξ. Here we have a first-order phase transition with $\xi \lesssim 1/T$ [16,17]. Hence, the full machinery of finite-size scaling need not be invoked if due caution is exercised; the extraction of the latent heat is one such case. We will mention these tests at appropriate places in the remainder of the paper.

III. THE LATENT HEAT

When two copies of a system with a first-order transition are held at temperatures $T_c \pm \delta T$, the difference in their energy densities is finite. Since the transition is of first-order, this difference remains finite even when $\delta T \to 0$, provided one has thermodynamically large systems. When V is large but not infinite, the correct limit may be obtained as long as $\delta T/T_c$ is much larger than $\mathcal{O}(\xi^3/V)$. If V is large enough, then this allows one to come close enough to T_c to have confidence in the result. For smaller ξ^3/V, this procedure breaks down. By examining the reason for the breakdown, we can develop a different procedure to estimate the latent heat.

Finite-size effects at first-order phase transitions are best understood in terms of constrained free energies, F_c. This is the free energy of a system in which $|L|$ or δP are restricted to have fixed values but other parameters are allowed to vary according to the temperature. F_c near a first-order phase transition has multiple minima; the deepest corresponds to the value the constrained variable has in

the stable phase when $V \to \infty$. As T changes, the depth of the minima change, and on crossing T_c the deepest minimum flips. However, because the finite system sees finite barriers between the minima, the system explores all the phases. Hence, the discontinuity is rounded off. As the thermodynamic limit, $V \to \infty$, is approached, the "wrong phase" minimum becomes infinitely higher and the barrier separating it from the true vacuum also becomes infinitely high. As a result, the transition sharpens and gives the correct thermodynamic limit.

The histogram of an observable obtained from its Monte Carlo history is proportional to $\exp(-VF_c/T)$. The object of a finite-size scaling study of something like the latent heat is to be able to identify the thermodynamically stable phase from histograms such as those in Fig. 2. The same figure also illustrates the problem which is usually faced in computing the latent heat in $SU(N_c)$ gauge theories. Although the histogram of $|L|$ has multiple well-identifiable maxima, the histogram of δP has a single peak. If the specific heat in each of the phases is large, then the two-peak nature of Δ/T^4 could well be hidden until extremely large volumes are reached. Finite-size scaling methods were developed in the past to extract reliably the specific heat when multiple maxima are clearly developed [18,19]. However they are not applicable here, and we need to use different techniques. We adapt one which was first applied to gauge theories in [20].

Since the phases are well resolved by $|L|$, we can try to use the following criterion. The cold confined phase could be identified by requiring that $|L| \leq L_c$, and the hot deconfined phase by $|L| \geq L_h$. The results of such a phase separation are stable as long as L_c and L_h both lie in the valley between the peaks of $|L|$. Examples of the probability density of δP obtained in the cold and hot phases so defined are shown in Fig. 3. The figures illustrate the fact that these probability densities are very stable—at each fixed volume the pure phase probability distributions of δP
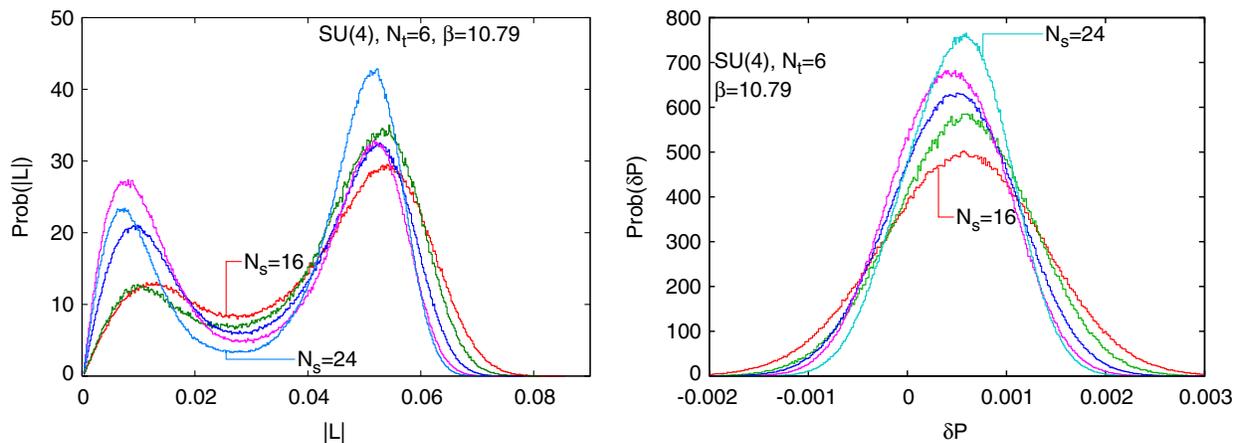
FIG. 2 (color online).   Histograms of $|L|$ and δP normalized to have unit area under the curves. A multipeak structure in the histogram corresponds to multiple (local) minima of the constrained free energy. As N_s changes between the values shown, in steps of two, the histograms move up or down monotonically in the region where they are labeled.
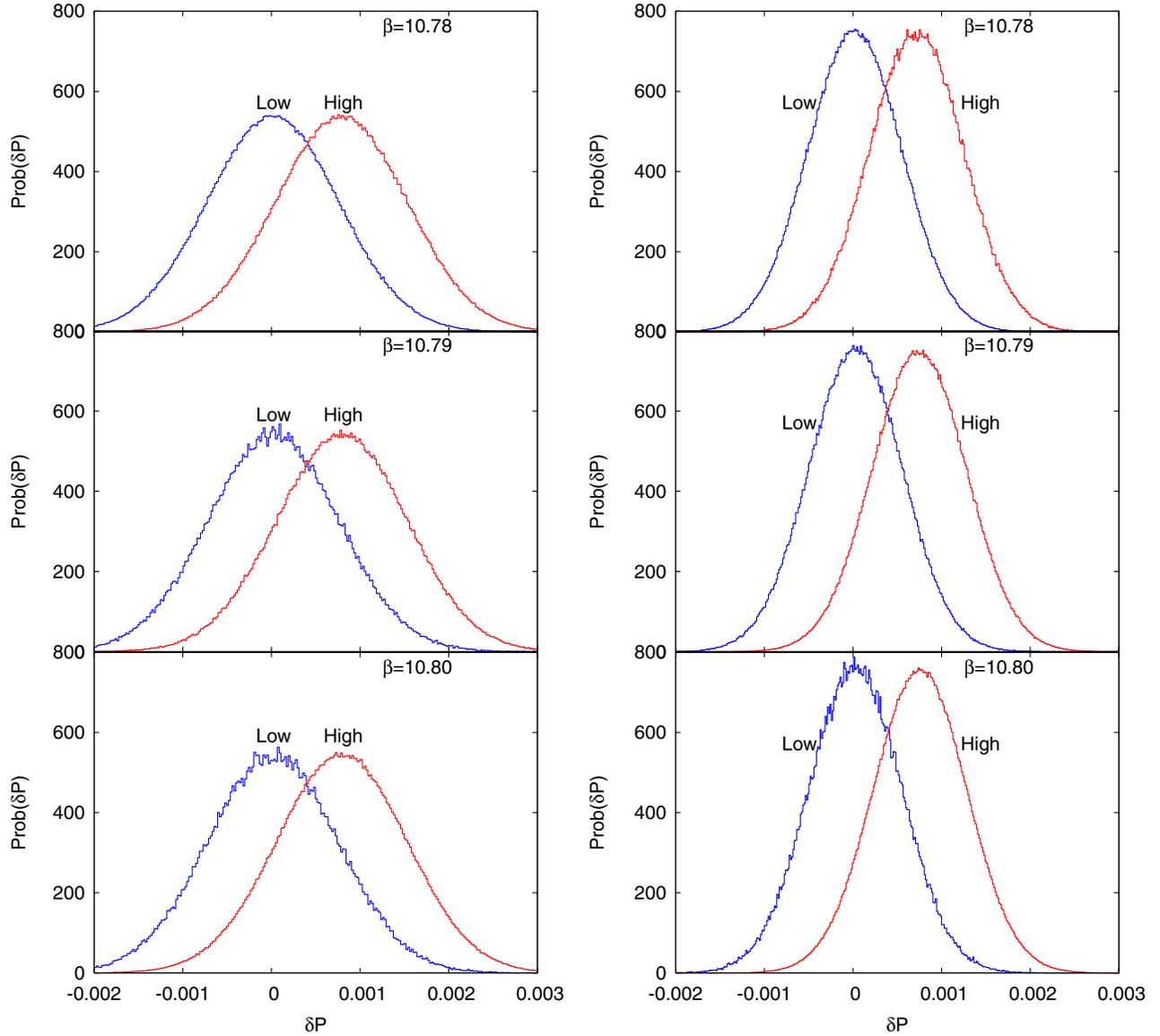
FIG. 3 (color online). Histograms of δP in the two phases of SU(4) gauge theory as defined using $L_c = 0.024$ and $L_h = 0.031$. The panels on the left are results from 6×16^3 lattices and those on the right from 6×20^3. The histograms are normalized so that the area under each is unity. The pure phase histograms at three different couplings exactly overlap on each lattice.

are identical for a range of couplings around T_c. If we had not separated out the panels for different β, the curves for the three different cases would have been indistinguishable apart from small statistical fluctuations. Furthermore, as one changes V (with fixed a) the mean value of δP in the hot and cold phases remain the same. It appears that the difference between the values of δP in the two phases is very stable under the variation of both T and V near T_c. It also turns out to be fairly stable under changes of L_c and L_h.

The agreement of the histograms of δP for the different β in the transition regime show that it is possible to reliably extract the limiting values of Δ/T^4 in each of the phases. Examination of (5) shows that knowing the difference in δP between the hot and cold phases one can extract easily

the jump in Δ/T^4 and hence the latent heat density, since the remaining factors are well-understood. In Table I we summarize our results for the latent heat for SU(N_c) gauge theories with $N_c = 3$, 4 and 6. In the entry for the latent heat, the first error is statistical while the second is a systematic error, i.e., the change in the result if $L_h - L_c$ changes by $\pm 20\%$. The results of [6] are higher, but those of [8] are consistent with ours, within the larger statistical and systematic errors of that study.

In SU(4) gauge theory we found that the extracted value of $\Delta\epsilon/T_c^4$ is fairly stable at fixed a as we change V. In fact, as is clear from Table I, when $a = 1/(6T_c)$ we found that there is no statistically significant change in the estimate of this quantity for $\zeta \geq 3$. The results for $a = 1/(8T_c)$ are consistent with this conclusion. For the SU(3) theory, on

TABLE I.   The latent heat of SU(N_c) gauge theories for $N_c = 3$, 4 and 6. The thermodynamic limit of $\Delta\epsilon/T_c^4$ is seen to be under control for $N_c = 4$ and 6, as is the continuum limit. For all N_c, the ratio $\Delta\epsilon/\Delta_{max}$ scales well. The numbers in brackets are the errors on the least significant digits, the first is statistical and the second systematic.

N_c	N_t	N_s	β	$|L|_c$	$|L|_h$	$\Delta\epsilon/T_c^4$	$\Delta\epsilon/\Delta_{max}$
		16	5.6908	0.055	0.075	2.06(1)(3)	
	4	24	5.6919	0.055	0.075	1.93(1)(3)	
3		32	5.6922	0.055	0.075	1.90(2)(2)	
		24	5.8934	0.02	0.03	1.79(2)(4)	0.65(2)
	6	32	5.8938	0.02	0.03	1.54(2)(5)	
		48	5.8940	0.022	0.032	1.44(4)(3)	
	8	32	6.0609	0.013	0.019	1.67(4)(4)	0.68(3)
		16	10.79	0.024	0.031	4.85(5)(6)	0.88(2)
	6	20	10.79	0.024	0.031	4.64(4)(5)	
4		24	10.79	0.024	0.031	4.57(4)(3)	0.85(2)
		22	11.08	0.013	0.017	4.58(5)(6)	
	8	24	11.08	0.013	0.017	4.32(6)(6)	0.82(2)
		28	11.08	0.013	0.017	4.33(8)(6))	
	6	14	24.84	0.025	0.03	12.20(10)(4)	
6		18	24.84	0.025	0.03	12.47(4)(2)	0.92(2)
	8	20	25.46	0.012	0.015	11.93(34)(5)	0.90(3)

the other hand, it seems that $\zeta \gtrsim 6$ is needed for an estimate of the latent heat with equally small systematic errors, i.e., finite volume corrections are larger for the SU(3) theory. One sees that in going from $N_c = 3$ to 4 the latent heat density scales faster than d_A. This ties in with the intuition developed from a study of correlation lengths [16,17] that the SU(3) theory is weakly first-order. One expects that correlation lengths should also become shorter with increasing N_c [8], and hence finite volume effects should be less pronounced.

Following the analysis of the Appendix, we understand that the lack of clear scaling of $\Delta\epsilon/(d_A T^4)$ to the continuum limit is not due to the use of an inappropriate beta-function. One possible explanation is that the large finite volume effects mask the approach to the continuum. If so, then one should be able to eliminate it by using another quantity with the same effect. Since Δ/T^4 has a very sharp peak as a function of T, and that measurement would also be related to the latent heat, we list the quantity $\Delta\epsilon/\Delta_{max}$ in the table. As one can see, this ratio has much better scaling properties, and the thermodynamic and continuum limits of the ratio are very well determined.

The data collected for $N_t = 8$ in Table I is fitted extremely well by the form

$$\frac{\Delta\epsilon}{d_A T_c^4} = 0.388(3) - \frac{1.61(4)}{N_c^2}, \qquad (9)$$

where the numbers in brackets are the statistical errors on the last digit of the central values. Interestingly, the fit yields $\Delta\epsilon/(d_A T_c^4) = 0.014 \pm 0.014$ at $N_c = 2$, where there is a second-order finite temperature transition, and hence $\Delta\epsilon = 0$. If one adds a term of $\mathcal{O}(1/N_c^4)$, then the fit changes marginally: the limiting value for $N_c \to \infty$ is stable at the 3σ level. The coefficient of the $\mathcal{O}(1/N_c^2)$

term changes by 16%, and the next correction term is marginal, its value being less than 10% of the total for $N_c = 3$. The extended series extrapolated to $N_c = 2$ is still consistent with vanishing latent heat of this theory. Although $N_c = 2$ must be the limit of the validity of the series expansion around $1/N_c = 0$, it seems to be well-behaved at $N_c = 3$. In agreement with this, a reliable value of the $N_c \to \infty$ limit of $\Delta\epsilon/(d_A T^4)$ can be extracted. This is an example of successful strong scaling; $\Delta\epsilon/(d_A T_c^4)$ is well described by just two terms of the series in $1/N_c^2$ even at small N_c.

The series for the fourth root of the above quantity may be of interest, since $\sqrt[4]{\Delta\epsilon}$ has mass dimension unity [7,8]. This gives

$$\left(\frac{\Delta\epsilon}{d_A T_c^4}\right)^{1/4} = 0.798(7) - \frac{1.09(9)}{N_c^2}. \qquad (10)$$

The quality of the fit, as judged by the value of χ^2/DOF, is worse, but still within the limits of acceptability. This result above seems to have improved convergence properties around $N_c = \infty$. However, on extending the fit to include the $\mathcal{O}(1/N_c^4)$ term, we find that the correction terms are unstable against changes. The coefficient of the second term reduces to half its value, and the coefficient of the third term is 6–7 times larger. The values of these terms are comparable to each other for $N_c = 3$, opening the possibility that even higher order terms, or a resummation of the whole series, need to be taken into account. From the previous analysis it seems that the series expansion for $\Delta\epsilon/(d_A T_c^4)$ comes close to performing this resummation. We shall show later that other mass dimension four quantities such as p, ϵ and s also have good strong scaling properties.

IV. CONFORMAL SYMMETRY BREAKING

As discussed in Sec. II, Δ/T^4 is easily calculated on the lattice. We have seen in the case of the latent heat, however, finite volume and cutoff effects need to be controlled. In the SU(3) gauge theory, it is known that Δ/T^4 rises rapidly near T_c and peaks at about $1.1T_c$ [14]. The rapid rise is, of course, dictated by the existence of a latent heat, but the shift in the peak away from T_c is not yet understood. We examine the N_c dependence of this peak.

In Fig. 4, we show the sensitivity of Δ/T^4 to ζ and a. Except in the immediate vicinity of T_c there seems to be little sensitivity to ζ. The cutoff dependence is also insignificant, except in the vicinity of the peak, $\Delta_{\max}$. The good agreement between data obtained with $a = 1/(6T)$ and $1/(8T)$ show that the results of the measurement with either lattice spacing can be taken to be an estimate of the continuum limit. We choose the conservative alternative of using $a = 1/(8T)$ as a determination of the continuum results. The peak of Δ/T^4 for SU(4) gauge theory is in the range $1.04 \leq T/T_c \leq 1.08$.

The systematics of Δ/T^4 for SU(6) gauge theories is also shown in the same figure. The trends are very similar to those in SU(4). Results for different ζ agree very well. The approach to the continuum limit is also very similar to that discussed for SU(4). Again, in this case, we can take

the results obtained with $a = 1/(8T)$ to be an estimate of the continuum limit. In going from $N_c = 3$ to 6, the peak in Δ/T^4 moves closer to T_c.

It is of phenomenological interest to note that Δ/T^4 is not small even at $2T_c$. In fact, as one can see in Fig. 4, one has

$$\frac{\Delta}{T^4}\bigg|_{2T_c} \simeq 0.1 d_A. \qquad (11)$$

For $N_c = 3$ this implies $\Delta^{1/4} \simeq T = 2T_c$. This is a natural scale, and therefore the theory is far from conformal.

We end this section with the investigation of an intriguing observation made in [21,22]: in the temperature range $1.1T_c \leq T \leq 4T_c$, for the SU(3) pure glue plasma, Δ/T^2 seems to be roughly constant. Phenomenological models of the gluon plasma have introduced mass scales and obtained such a behavior [23]. We investigated this modified scaling behavior at larger N_c (see Fig. 5). The dimensionless quantity $\Delta/(d_A T^2 T_c^2)$ for SU(3) gauge theory is seen to have little temperature dependence from just above T_c to about $4T_c$. Unfortunately, the data for SU(3) theory is noisy at larger T. For SU(6) the error bars are smaller and one can observe that this quantity falls with T. This implies that the temperature dependence of Δ could be slower than T^2. It would be interesting in future to expand the range of N_c and T in order to study this further.
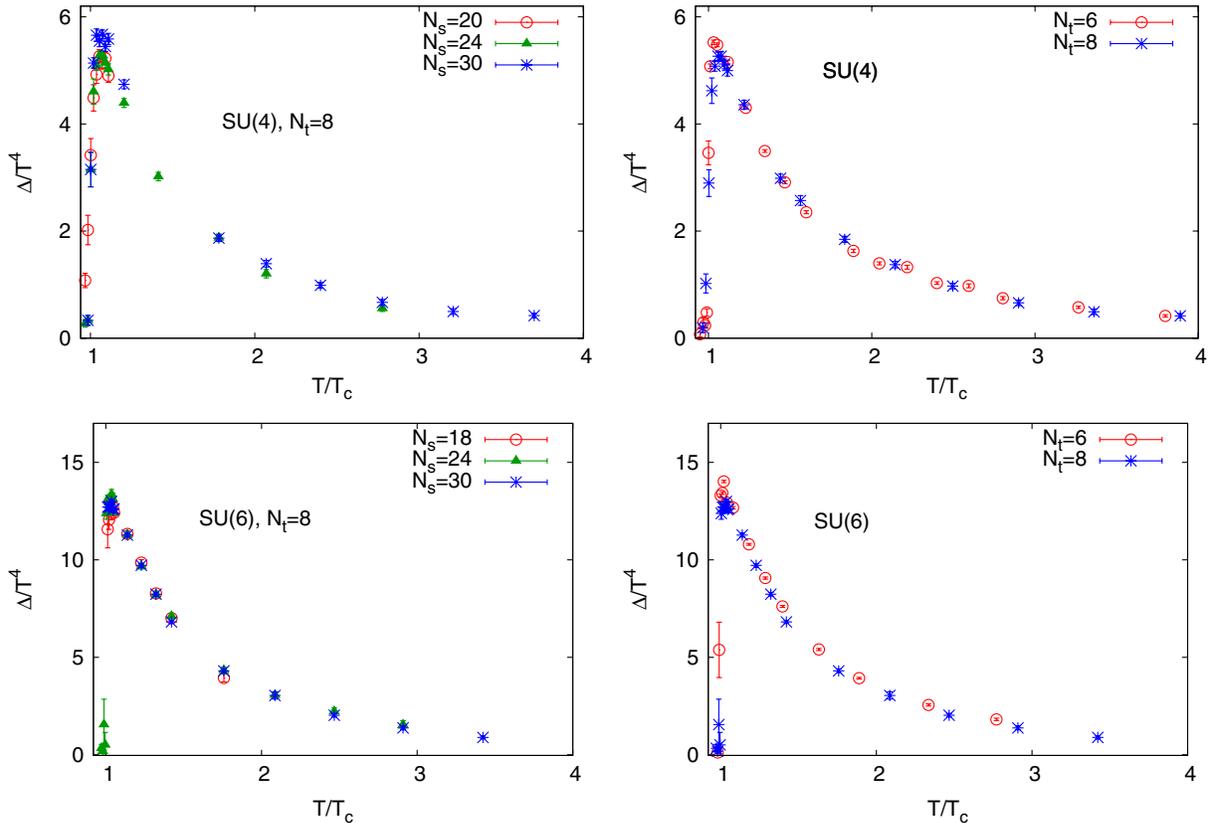
FIG. 4 (color online). Systematics of Δ/T^4 in SU(4) and SU(6) lattice gauge theories. Both finite-size and finite lattice spacing effects are small, except, possibly, very close to T_c.
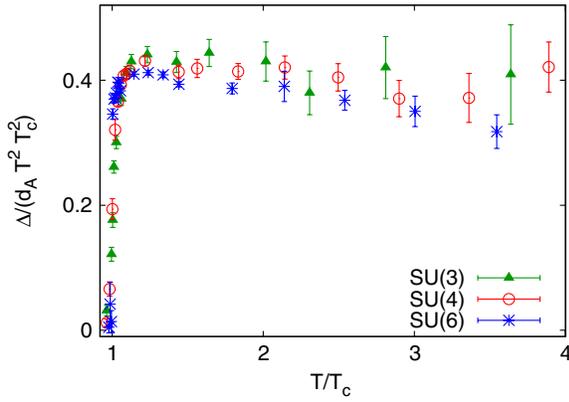
FIG. 5 (color online). Testing a different scaling [22] of the conformal symmetry breaking measure: $\Delta/(d_A T^2 T_c^2)$, where $d_A = N_c^2 - 1$, for $N_c = 3$, 4, and 6.

V. OTHER BULK THERMODYNAMIC QUANTITIES

The pressure is calculated using the method outlined in (6). The integration requires interpolation of the measured points, and there could be a systematic error arising from this. We have estimated this error by comparing linear and quadratic interpolations. We found that point by point the

error is small. Since the integration errors increase over the range of β and reach a maximum at the highest β that we use, it is sufficient to report the magnitude of that error compared to the statistical uncertainty. We can estimate the significance of the error by the t statistic

$$ t = \frac{|I_2 - I_1|}{\sqrt{\sigma^2(I_2) + \sigma^2(I_1)}} \tag{12} $$

where I_2 is the integral estimated through a quadratic interpolation, I_1 is the estimate using a linear interpolation and $\sigma^2(I_{1,2})$ is the statistical error in the estimate of $I_{1,2}$. This measure for SU(4) is 1.03 and for SU(6) it is 0.007. This source of error is therefore almost negligible. As a result, the systematic error is almost entirely due to the neglect of p_0, the pressure at the lowest temperature where the integration is started.

Figure 6 shows the cutoff and volume dependence in the calculation of pressure for the SU(4) and SU(6) theories. The results are normalized by the known (asymptotic) finite cutoff correction for an ideal gluon gas, p_{SB}, which was described earlier. Hence, the pressure, so normalized, should go to unity. We find that finite volume effects are negligible. Finite lattice spacing effects also turn out to be negligible once we normalize the pressure by p_{SB} [24].
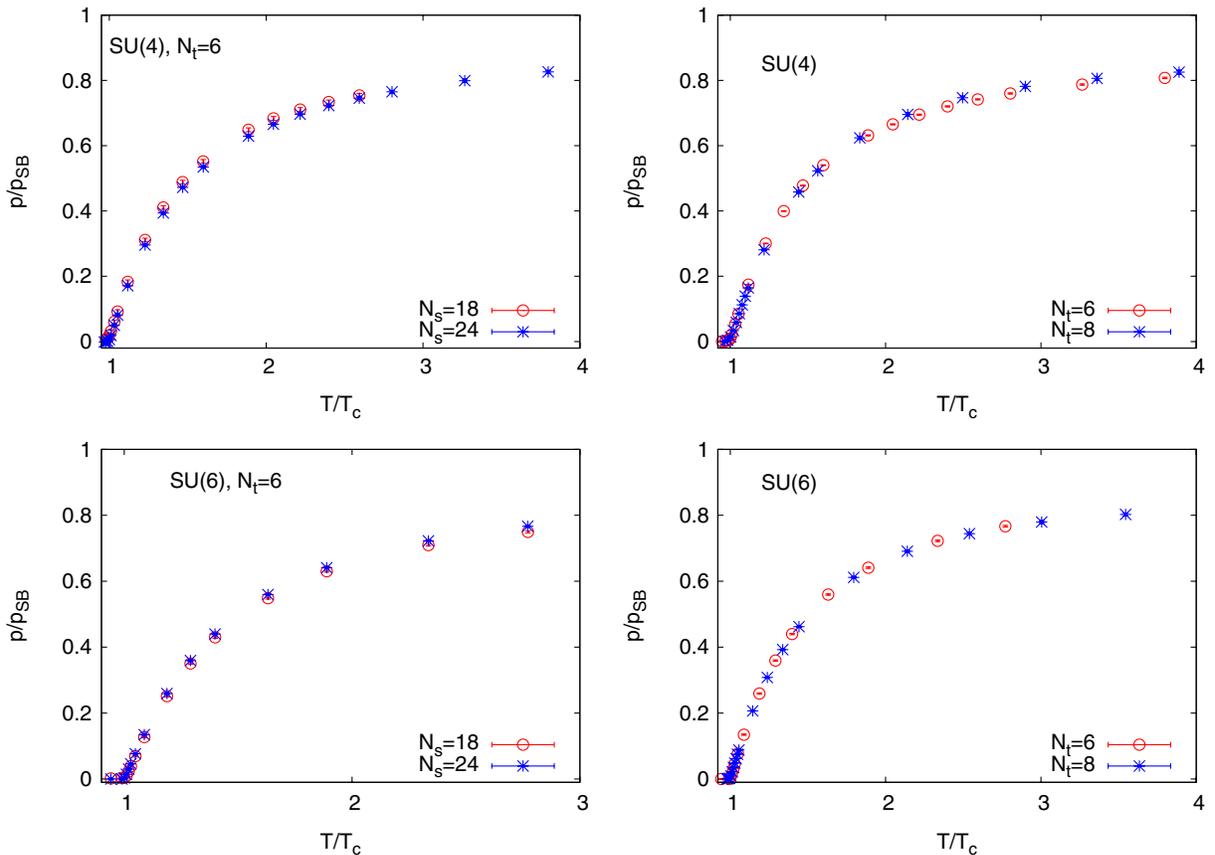
FIG. 6 (color online). Study of systematics in the calculation of pressure for SU(4) and SU(6) gauge theory. Both finite-size and finite lattice spacing effects seem to be under control.
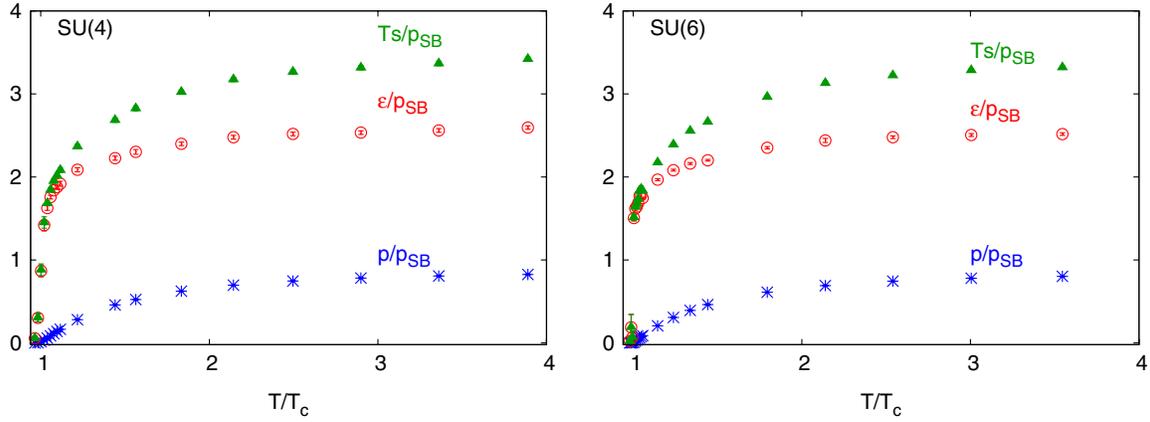
FIG. 7 (color online).   Results for thermodynamic quantities in SU(4) (left) and SU(6) (right) gauge theories. Shown are the pressure, energy density and entropy density, in units of p_{SB}. The ideal gas limits for these quantities should be 1, 3 and 4, respectively.

These results indicate that it is safe to identify the continuum limit with the $N_t = 8$ measurements. Since the volume dependence is negligible, at each T we use the largest volume on which reliable results are available as an indication of the thermodynamic limit.

At asymptotically large T the SU(N_c) gauge theory should go over to the ideal gluon gas. However, even at the highest temperatures which we have probed, i.e., $T \simeq 4T_c$, the ratio p/p_{SB} is far from unity (see Fig. 6). Given p/T^4 and Δ/T^4 we obtain also the other bulk thermodynamic quantities: ϵ/T^4 and s/T^3. These results are collected in Fig. 7. One sees clear deviations from the ideal gas limit in these two quantities as well. All three quantities also show a very slow rise throughout the measured range of T. Both ϵ and s show a rapid jump near T_c, stronger for SU(6) than for SU(4).

VI. N_c SCALING OF THERMODYNAMIC OBSERVABLES

As discussed earlier, we distinguish between strong N_c scaling and 't Hooft scaling. The first is scaling with N_c of

thermodynamic quantities at fixed T/T_c, and the second, the scaling with N_c at fixed λ. We examine scaling by combining our results for the continuum limit of bulk thermodynamic quantities for $N_c = 4$ and 6, with a reanalysis of the older $N_c = 3$ data of [14] using our techniques.

Strong N_c scaling has been observed on the lattice in many contexts [7,8,10,11]. The continuum limit of bulk thermodynamic quantities that we have extracted are also consistent with this limiting procedure, except for $N_c = 3$ near T_c. In Fig. 8 we show the energy density and pressure, each normalized by its ideal gas value, for $N_c = 3$, 4 and 6. As noted earlier, there are clear deviations from the ideal gas behavior, but the scaled quantities are almost independent of N_c. Since the ideal gas values scale with d_A, one expects that at large T all bulk thermodynamic quantities should scale accordingly.

We see that p/p_{SB} is independent of N_c over the whole temperature range above T_c within the accuracy of our measurement. Such a statement is also true for the energy density except when T is close to T_c. Close to T_c the energy
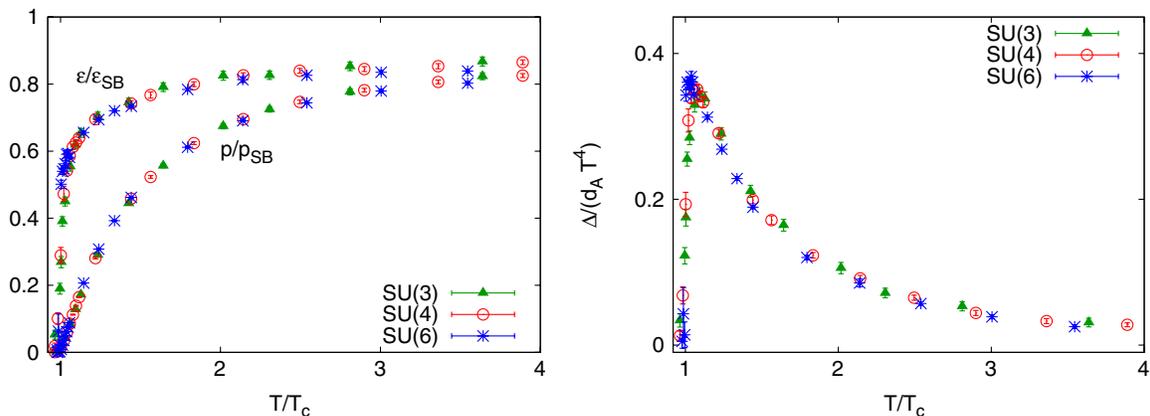
FIG. 8 (color online).   The first panel shows the energy density and pressure in SU(N_c) pure gauge theories, normalized to the corresponding ideal gas values. The second panel shows Δ/T^4, normalized by $d_A = N_c^2 - 1$, for the same theories.

density does not scale as the ideal gas, i.e., as d_A, between $N_c = 3$ and larger values of N_c. This is, of course, a consequence of the fact that the latent heat density does not scale as d_A for $N_c = 3$ (see Table I). Consequently, Δ/T^4 also fails to scale with d_A in the vicinity of T_c. In Fig. 8 we also show that the peak of $\Delta/(T^4 d_A)$ is smaller for $N_c = 3$ than for $N_c = 4$ and 6 (the latter two are almost identical in value). We also see that the peak is rounded and shifted away from T_c at $N_c = 3$. As discussed in Sec. III, this could be due to finite-size effects, since $N_c = 3$ has a weaker first-order phase transition. With this exception, strong N_c scaling seems to work very well for bulk thermodynamic quantities; the subleading corrections are too small to be seen over the statistical errors.

Next we turn to evidence for 't Hooft scaling. Since the critical point is known very precisely for several N_c [9], we test this scaling using $\lambda_c = N_c g_R^2(2\pi T_c)$ [25]. The change of λ_c with N_c is very much larger than the statistical errors, and is seen using both the nonperturbative and the two-loop beta-functions, although it is somewhat larger with the former. The change is nonmonotonic when the two-loop beta-function is used. In this context, we recall the result shown in the Appendix: the nonperturbative beta-function is preferred near T_c. Restricting ourselves to using only this leaves the three smallest values of N_c.

Even with these three, it is clear from Fig. 9 that the data do not fall on a straight line and hence a single correction term does not suffice. Using a second correction term, a description of the confinement-deconfinement phase boundary, i.e., the variation of λ_c with $1/N_c^2$, is

$$\lambda_c = \begin{cases} 9.8771(4) - \frac{14.2562(2)}{N_c^2} + \frac{54.7830(2)}{N_c^4} & \text{(non-perturbative)} \\ 9.9904(6) + \frac{1.2081(3)}{N_c^2} - \frac{23.5709(3)}{N_c^4} & \text{(two-loop)}. \end{cases}$$
(13)
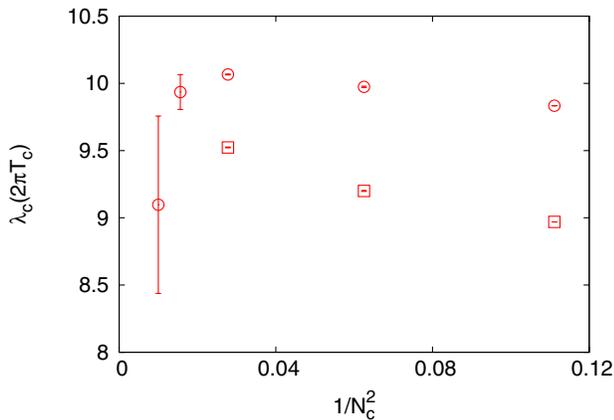
FIG. 9 (color online). The value of the 't Hooft coupling at the finite temperature transition, λ_c, for different N_c. The renormalized gauge coupling is evaluated at scale of $2\pi T_c$, so that the result is independent of the lattice spacing. The boxes denote results obtained using the nonperturbative beta-function and the circles using the two-loop beta-function in the V scheme.

Since the fit is linear in the parameters, the formal solution for χ^2 minimization can be written down along with parameter errors. At $N_c = 3$ the values of the $\mathcal{O}(1/N_c^2)$ and $\mathcal{O}(1/N_c^4)$ terms are comparable. Although they are small corrections to the leading, $\mathcal{O}(N_c^0)$ term, this behavior of the series could indicate that $N_c = 3$ lies near the radius of convergence of the series around $N_c = \infty$.

If this is so, then summing three terms of the series is not numerically accurate. However, with three pieces of data fitting a large number of terms is an ill-conditioned problem. As a result, one would do better to fit a resummation of the series, provided such a resummation has a small number of parameters. Unfortunately, there is no theory for the shape of the phase boundary. In its absence we try the usual trick of estimating a Padé resummation of the series. With three terms the best that we can try to do is to fit the lowest order expansion

$$\lambda_c(N_c) = \lambda_c(\infty) + \frac{a/N_c^2}{1 + N_*^2/N_c^2}.$$
(14)

The best fit gives $N_* \simeq 4$, roughly consistent with the series analysis. As a result, the fitted value of $\lambda_c(\infty)$ is sensitive to the form of the remaining function, and cannot be reliably extracted using data for N_c near 4. It would be useful to improve the computations with $N_c > 6$ in order to extract this quantity with better accuracy.

Next, we extend such a test to a bulk thermodynamic quantity. Our results for s/s_{SB} as functions of T/T_c and $\lambda(2\pi T)$ are shown in Fig. 10. As we show in the figure, strong scaling holds with good precision since s/s_{SB} is almost independent of N_c down to the smallest temperatures that we have studied. For scaling at fixed λ, convergence of the series is clearly bad for $\lambda \simeq 9$ or larger, because the theories for different N_c begin to drop out of the plasma phase; Figure 9 shows that theories with smaller N_c drop to the confined phase at smaller λ. At any fixed λ in this region one has to go to N_c large enough that the theories are all in the same phase before one can observe good scaling with N_c. We also find that the convergence of the series in $1/N_c^2$ is acceptable when $\lambda < 8.6$ [27]. In the range $8.6 < \lambda < 9$, the physically interesting theory with $N_c = 3$ is close to the radius of convergence of the series expansion, and the effect of the correction terms is large.

Figure 10 also displays a comparison of the scaled entropy with predictions in a $\mathcal{N} = 4$ supersymmetric Yang-Mills theory, computed [28] using the AdS/CFT correspondence

$$\frac{s}{s_{SB}} = \frac{3}{4} + \frac{45}{32}\zeta(3)\left(\frac{1}{\lambda}\right)^{3/2} + \cdots.$$
(15)

Although one does not expect this computation to be valid in the realistic nonsupersymmetric theories under investigation, it is sometimes said to agree with lattice results. Here we show that the agreement is poor, except at the highest possible temperatures. It has been argued [11] that
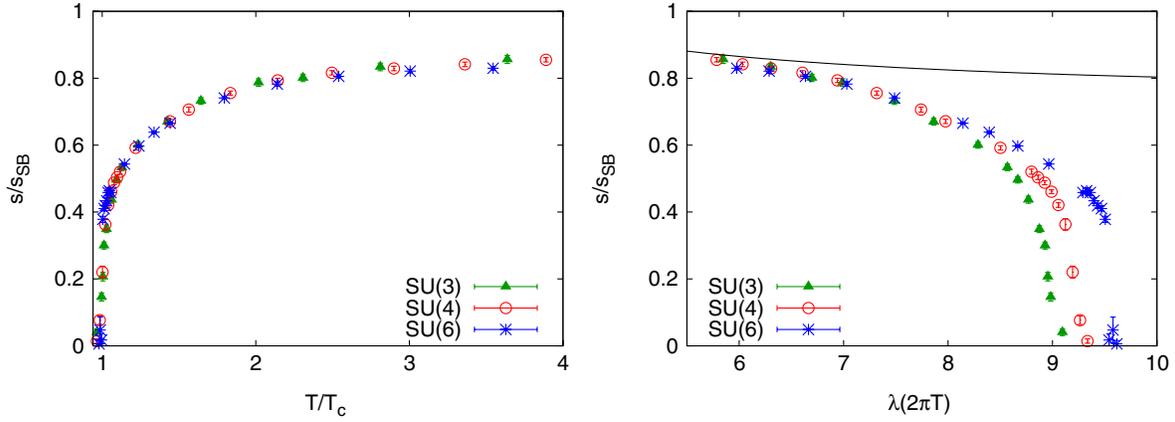
FIG. 10 (color online). The scaled entropy density, $s/s_{\rm SB}$, as a function of T/T_c and the 't Hooft coupling $\lambda = N_c g_R^2$. The gauge coupling is evaluated at the scale $2\pi T$ using the nonperturbative beta-function for the appropriate theory.

more realistic AdS-QCD models should be used for such a comparison; the analysis of such models lies beyond the scope of this paper.

VII. SUMMARY AND DISCUSSION

In this paper we have studied the thermodynamics of the gluoN_c plasma, by numerical simulations of SU(4) and SU(6) gauge theories, and comparing them with a reanalysis of existing data [14] for SU(3). Our focus is on taking the continuum and thermodynamic limits, by using multiple spatial volumes at each cutoff and by using significantly smaller cutoffs than used previously for the EOS of $N_c > 3$. As discussed in the introduction, in terms of statistics, lattice spacing and spatial volumes, this work brings the study of the thermodynamics of pure gauge SU(N_c) theories with small $N_c > 3$ at par with the state of the art for $N_c = 3$, while also throwing some new light on the SU(3) theory.

We had shown in an earlier study of the deconfinement transition [9] that observations made with finite lattice spacing could be continued to the continuum using the renormalization group equations. When measurements are made with lattice spacing $a \lesssim 1/(8T_c)$ we had found that the two-loop beta-function suffices. When the lattice spacing $a = 1/(6T_c)$ is used a nonperturbative beta-function was introduced which could be used to continue the lattice results to the continuum limit [9]. In this paper we used these earlier results to obtain the continuum thermodynamics of pure gauge SU(4) and SU(6) theories. We also made a reanalysis of the older SU(3) data using this technique (as detailed in the Appendix).

One of our important results (see Sec. III) is an extraction of the latent heat of the deconfinement transition, $\Delta\epsilon/T_c^4$, for $N_c = 3$, 4 and 6. We found that $\Delta\epsilon/(d_A T_c^4)$, where $d_A = N_c^2 - 1$, increases between $N_c = 3$ and larger N_c, indicating that the first-order transition grows stronger

with increasing N_c. Our results are compatible with [8]. One expects stronger transitions to have smaller finite volume effects, and our observations support this notion. We found some scale breaking in the measurement of $\Delta\epsilon/T_c^4$, and observed that $\Delta\epsilon/\Delta_{\rm max}$ shows better scaling properties. We also saw that in the large N_c limit one has $\Delta\epsilon/(d_A T_c^4) = 0.388 \pm 0.003$ (see Eq. (9)).

Further study of bulk thermodynamic quantities started with measurements of Δ/T^4 (details are given in Sec. IV). We found good scaling of this quantity, and a reliable continuum limit, for $T > T_c$. For SU(3) some scale breaking is observed very close to T_c where the peak of this quantity lies. The cause remains obscure, although there are some indications that lead us to conjecture that this could be due to finite volume effects. Future studies are planned to understand this remaining ambiguity. In a range of temperature up to $4T_c$ we found that $\Delta \propto T^2$ [21,22], or, possibly, slower.

The pressure, p/T^4, was obtained using the so-called integral method (see Sec. V). Finite volume and lattice spacing effects in this measurement are under good control. We extracted the energy density, ϵ/T^4, and the entropy density, s/T^3, using these two primary measurements. In the whole range of T all these quantities lie substantially below the ideal gas values (see Figs. 7 and 8). Nevertheless, p/T^4, considered as a function of T/T_c, scales very well with d_A [10,11]. Subleading corrections in N_c are hard to see at the level of accuracy we have reached (see Fig. 8). Similar scaling with d_A is also seen for ϵ/T^4 and s/T^4, except, possibly, in a small region near T_c.

In Fig. 11 we present a plot of the normalized energy density, $\epsilon/\epsilon_{\rm SB}$ against the normalized pressure, $p/p_{\rm SB}$ following [29]. The diagonal line is the line of all conformal theories, with the ideal gluon gas being one special point on it. Weak-coupling results [30] lie below this. One sees that the lattice data for $N_c = 3$, 4 and 6 lie further below. Indeed the topology of these relations is such that
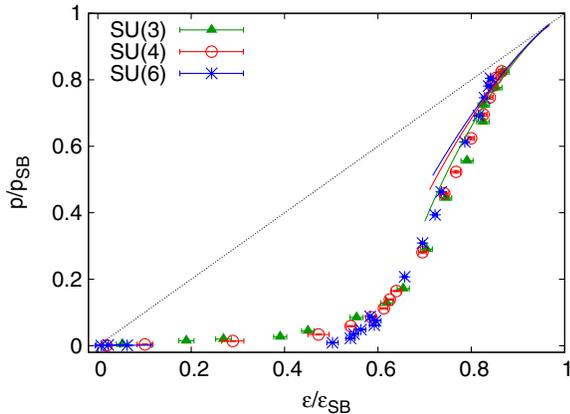
FIG. 11 (color online). Approach to conformality in $SU(N_c)$ gauge theories. The diagonal line contains results from all possible conformal theories; the upper right end is the special case of a free massless theory. Also shown are the weak-coupling results from [30] (with increasing N_c the curves move up monotonically). Since the lattice data lie closer to the weak-coupling results than to the diagonal line, there seems to be no window where a strongly-coupled conformal theory describes $SU(N_c)$ pure gauge thermodynamics.

the weak-coupling predictions are always a better approximation to the lattice data than conformal theories. This figure gives clear evidence that there is no window of temperature in which the EOS can be described by a strongly-coupled conformal theory better than by weak-coupling theory.

Yet another reason for making high-precision measurements of bulk thermodynamics for $N_c \neq 3$ is to understand the usefulness of the large N_c limit (see Sec. VI). In agreement with previous results, we find that the strong scaling limit obtained by taking fixed T and $N_c \rightarrow \infty$ works very well. Corrections in powers of $1/N_c$ are small, as a result of which the large N_c results for the entropy density, for example, can be directly applied for $N_c = 3$ with about 1% error. However, when the same data is analyzed as a function of the 't Hooft coupling, the finite N_c corrections are large for $\lambda > 8.6$. The phase boundary for the large N_c theory, expanded in powers of $1/N_c^2$, seems to have a radius of convergence smaller than $1/9$ (see Eq. (14)). Since this observation has ramifications for all models of thermal QCD which proceed from the large N_c approximation, including string-based models, we plan further measurements in the near future to explore the applicability of the 't Hooft limit.

ACKNOWLEDGMENTS

APPENDIX A: THE BETA-FUNCTION

The lattice theory is cut off at a length scale of a. When a is small various quantities have a perturbation expansion in g_R, the renormalized coupling determined at a scale κa (where κ can depend on the scheme). Then the derivative in Eq. (5) can be written as

$$\frac{\partial \beta}{\partial \ln a} = \frac{\partial \beta}{\partial g_R} \frac{\partial g_R}{\partial \ln a}, \tag{A1}$$

where the last factor is the negative of the beta-function. Because of the fact that $SU(N_c)$ gauge theories are asymptotically free, one expects that a weak-coupling determination of the beta-function should suffice when a is small enough. In [9] it was shown that the two-loop beta-function is a sufficient description of the flow of the renormalized coupling at the scale of $a \lesssim 1/(8T_c)$. Confidence in the efficacy of two-loop scaling is enhanced by the fact that the scheme dependence in the extraction of the QCD scale was small.

Although the two-loop beta-function was insufficient to describe the flow of the coupling at larger a, it was shown that for $a \simeq a_c(6)$, where $a_c(N_t) = 1/(N_t T_c)$, a simple correction of the form

$$a_c(N_t)\Lambda = R\left(\frac{1}{\beta_0 g_R^2(a_c)}\right)\left[1 + \frac{c_2}{N_t^2}\right], \qquad \text{with}$$
$$R(x) = e^{-x/2} x^{\beta_1/2\beta_0^2} \tag{A2}$$

suffices, where the two-loop beta-function is $-\beta_0 g_R^3 - \beta_1 g_R^5$ (for an alternative approach see [31]). The values of c_2 were presented in Table (4) of [9]. In this paper the integration of the beta-function is started from the scale $a_c(8)$. In the calculation of Sec. IV, since we have taken data at lattice spacings as low as $a_c(6)$, we have used such a nonperturbatively corrected beta-function in the V-scheme (this differs from the conventions of [3]). Any nonperturbative beta-function will include finite lattice spacing corrections [32] into the scaling, just as the above function does. Such corrections are nonuniversal.

A simplified version of the tests of scaling in [9,33] can be presented using the step-scaling function $\Delta\beta(\beta)$. This is the change in the bare coupling, $\Delta\beta$, required to reproduce the physics observed at a bare coupling β, when the lattice spacing is doubled. If β is chosen to be the lattice coupling where the deconfinement transition is observed for a given N_t, then $\beta - \Delta\beta$ is the lattice coupling at the deconfinement transition when N_t is changed to $N_t/2$. In Fig. 12 we show the result of using the beta-functions given above and the step-scaling function given in [14].

We also examined the sensitivity of the equation of state to the choice of the beta-function. Figure 13 displays results for Δ/T^4 obtained in SU(3) and SU(4) gauge theory, for lattices with $a = 1/(8T)$. For the SU(4) theory, differences between the E-scheme and the V-scheme are statistically insignificant at all temperatures. Similarly,
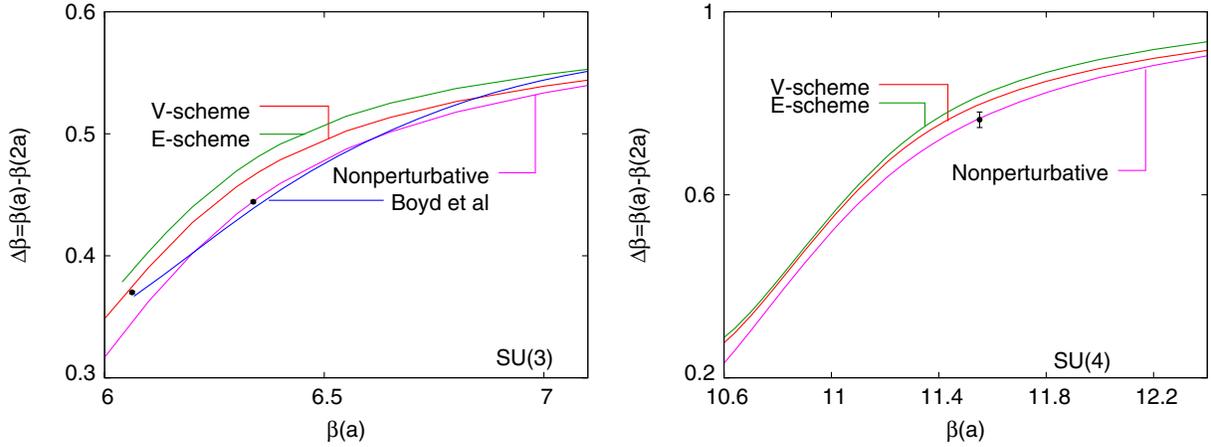
FIG. 12 (color online).   The step-scaling function in SU(3) (left) and SU(4) (right) gauge theory. The data points are for the location of the deconfinement transition. For SU(3) they are taken from [14] and for SU(4) from [9]. The nonperturbative beta-function of [14] is tuned for use with $a \lesssim a_c(4)$. We have used a nonperturbative beta-function [9,33] which can be used for $a \lesssim a_c(6)$.
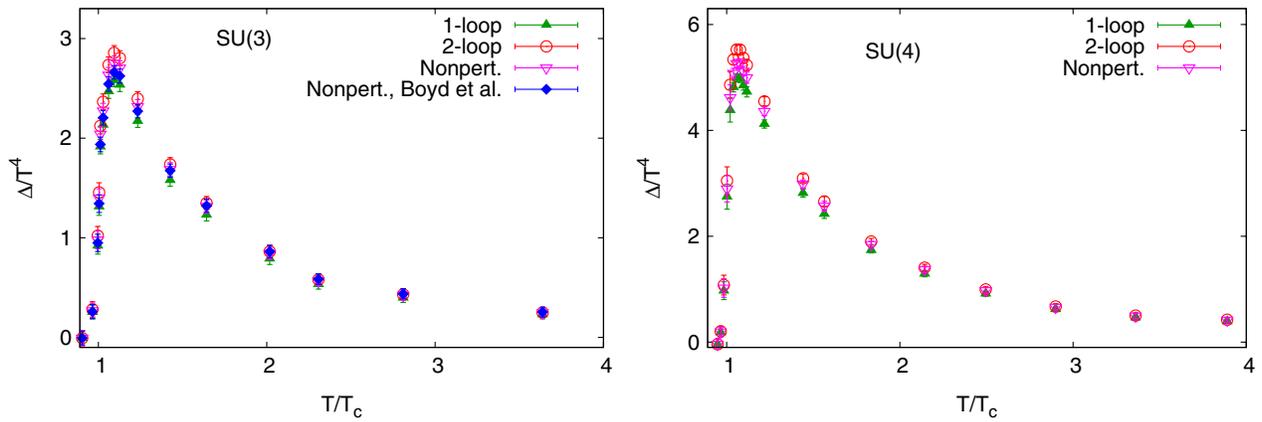
FIG. 13 (color online). Δ/T^4 for SU(3) gauge theory on $32^3 \times 8$ lattices (left) and SU(4) on $30^3 \times 8$ lattices (right). Shown are the analyses using various different beta-functions (based on the V scheme). For SU(3), where the data is taken from [14] we also show the analysis using the nonperturbative beta-function of that paper.

the difference between these and the nonperturbative beta-function of (A2) are also insignificant at these temperatures.

The results are similar for the SU(3) gauge theory, where we have reanalyzed the data of [14]. The one-loop and the two-loop beta-functions give coincident results for $T \gtrsim 1.5 T_c$. In [33] a nonperturbative beta-function of the form in (A2) was used to describe scaling of T_c. The results of using this for Δ/T^4 are shown. These three are close to each other, as is the result of [14].

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