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Space- and timelike electromagnetic kaon form factors

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A simultaneous investigation of the space- and timelike electromagnetic form factors of the charged kaon is presented within the framework of light-cone QCD, with perturbative k_T factorization including Sudakov suppression. The effects of power suppressed subleading twists and the genuine soft QCD corrections turn out to be dominant at low- and moderate-energies/momentum transfers. Our predictions agree well with the available moderate-energy experimental data, including the recent results from the CLEO measurements and certain estimates based on the phenomenological analyses of J/ψ decays.

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I. INTRODUCTION

Electromagnetic form factors are interesting physical observables in hadronic physics which directly provide insights into the hadronic constituents, charge distributions, currents, color, and flavor within the hadrons. Their precise knowledge is of fundamental importance for a realistic and accurate description of exclusive nuclear reactions that serve as ideal testing grounds for understanding the dynamics of confinement in QCD that have been grappling with physicists ever since the discovery of asymptotic freedom.

In the last few decades, there has been significant experimental efforts in extracting hadronic form factors (e.g., see [1–3] for the charged pion form factors) from various exclusive processes. However, in the case of the charged kaon form factors, their behavior was very severely constrained due to absence of reliable experimental data. Since the mid-90s, kaon photo/electroproduction experiments on reactions such as $A(\gamma, K)YB$ and $A(e, e'K)YB$ (target A, produced hyperon Y and recoil B) have invited some renewed interest in the study of kaon form factors, although the existing data is still too limited, restricted only to the very low spacelike region, as low as $-q^2 \leq 0.2\ \mathrm{GeV}^2$ [4]. In the timelike region, there are more scattered data up to several GeV^2 (albeit with very large error bars) for timelike processes such as $\gamma^* \to K^+K^-$, extracted from annihilation reactions such as $J/\psi \to e^+e^- \to h^+h^-$ $(h = \pi, K, \ldots)$ by applying suitable experimental cuts. A compilation of previously extracted kaon form factors for $q^2 = Q^2 < 10\ \mathrm{GeV}^2$ is given in [2]. Recently, high precision measurements by the CLEO Collaboration with first ever identified timelike kaons for $Q^2 > 4\ \mathrm{GeV}^2$ have yielded the following results: $|G_K(13.48\ \mathrm{GeV}^2)| = 0.063 \pm 0.004(\mathrm{stat}) \pm 0.001(\mathrm{syst})$ and $Q^2|G_K(13.48\ \mathrm{GeV}^2)| = 0.85 \pm 0.05(\mathrm{stat}) \pm 0.02(\mathrm{syst})\ \mathrm{GeV}^2$ [3]. Note that in this paper, we shall use the symbol G_K for the timelike kaon form factor to distinguish it from the spacelike counterpart F_K.

Notwithstanding the aforementioned problem of paucity of quality statistics of the existing kaon data, the purpose of this paper is an effort to make a prediction for the charged kaon form factors using the framework of perturbative factorization [5,6]. In this way, we hope to throw some light on their possible behavior, especially at the phenomenological intermediate energy region, where significant nonperturbative effects tend to spoil the asymptotic perturbative QCD (pQCD) results like the celebrated quark counting rule that predicts the scaling behavior $\{F, G\}_K(Q^2) \sim 1/Q^2$ [5,7]. Analyses of the pion form factors convincingly show that the standard pQCD with only twist-2 effects are much too small to explain the currently available experimental data at low and moderate energies. This calls for the inclusion of nonperturbative corrections from the genuine soft QCD [8–12] and the subleading twists that can give rise to unnaturally large contributions at moderate range of Q^2 values, in particular, twist-3 enhancements were seen to be quite large in the previous studies for the spacelike pion form factor [13–17], the spacelike kaon form factor [17,18], and in the studies of $B \to \pi$ transition form factors [19–22]. In fact, the analysis presented in [23] shows a possible scenario where the contributions from the twist-3 terms in the timelike region can turn out to be exceptionally large. This seemed to resolve the bulk of the existing experimental discrepancy between the space- and the timelike pion data.

In this paper, following [17,23] we extend the analysis to the space- and the timelike kaon form factors, where in addition to the twist-2 and twist-3 terms we explicitly include twist-4 corrections. Thereby, we show that the large twist-3 contributions are indeed a nontrivial aspect

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of our result in comparison with the other twist contributions, e.g., the 2-particle twist-4 contributions are explicitly shown to be about a third of the magnitude of the twist-2 terms. The paper is organized as follows: the second section briefly reviews the theoretical background, the third section deals with the details of our numerical analysis, results, and discussions of the essential features of our results, and finally, we give our conclusions. For the purpose of bookkeeping, we provide a collection of relevant mathematical formulas in the Appendix.

II. HARD AND SOFT KAON FORM FACTORS

A. Factorized pQCD

The basic definitions of the space- and timelike electromagnetic form factors are given in terms of the following local matrix elements of the electromagnetic quark currents $J_\mu^{\rm em}$:

$$\begin{aligned} e(P'+P)_\mu F_K(Q^2) &= \langle K^\pm(P')|J_\mu^{\rm em}(0)|K^\pm(P)\rangle;\\ e(P'-P)_\mu G_K(Q^2) &= \langle K^+(P')K^-(P)|J_\mu^{\rm em}(0)|0\rangle;\\ J_\mu^{\rm em} &= \sum_f e_f \bar{q}_f \gamma_\mu q_f, \end{aligned} \tag{1}$$

where e is the electronic charge and f is the flavor of the valence quark q_f with charge e_f. In terms of light cone coordinates, $P = (Q/\sqrt{2}, 0, \mathbf{0}_T)$ and $P' = (0, Q/\sqrt{2}, \mathbf{0}_T)$ are the incoming and outgoing external kaon momenta in the Breit frame. In the spacelike domain, $q^2 = (P' - P)^2 = -Q^2 \leq 0$, whereas for the timelike domain $q^2 = (P' + P)^2 = Q^2 \geq 0$. Here, Q is assumed to be much larger compared to the kaon mass m_K, so that P and P' almost lie along the light cone directions.

In our approach, the total contributions to the charged kaon form factors come from the factorizable "hard" parts $\{F, G\}_K^{\rm hard}(Q^2)$ calculable within a perturbative framework, and the nonfactorizable "soft" parts $\{F, G\}_K^{\rm soft}(Q^2)$ relying on some nonperturbative techniques. The calculation of the hard parts rests on the essential assumption that at suitable high-energy scales, the form factors are *factorizable*, i.e., separable into parts dominated by short- and long-distance dynamics. The short-distance dynamics are represented by the kernel of interactions between highly off-shell partons, above the so-called *factorization scale* μ_F. While the long-distance dynamics below the factorization scale are implicitly encoded within the kaonic wave functions/distribution amplitudes (DA) with near-on-shell partons. Note that due to the well-known *impulse* or *frozen* approximation applicable for all high-energy exclusive mechanisms, the dominant contributions come entirely from the leading Fock states, i.e., the $q\bar{q}$ valence quark configurations. The higher Fock states are neglected with contributions relatively suppressed by higher powers of $1/Q^2$. Figure 1 shows two representative Feynman diagrams (there are 4 diagrams each for the space- and timelike cases) with leading order (in the QCD coupling) hard kernels each having a single hard gluon exchange. These are convoluted with the incoming and outgoing kaon DAs to obtain the hard factorized kaon form factors. In this analysis, we calculate $\{F, G\}_K^{\rm hard}(Q^2)$ up to twist-4 accuracy in the 2-particle sector, including explicit "k_T" or transverse momentum dependence (TMD) of the constituent valence partons. The nonfactorizable soft contributions, on the other hand, can either be calculated using *Drell-Yan-West* type of overlapping wave functions ansatz [24], or from QCD sum rules incorporating the local quark-hadron duality principle [8]. Both of these approaches to parametrize the genuine soft contributions are known to give very similar results. In this work, we follow the latter approach. The above assertion can then be summarized by

$$\begin{aligned} \{F,G\}_K(Q^2) &= \{F,G\}_K^{\rm soft}(Q^2) + \{F,G\}_K^{\rm hard}(Q^2);\\ \{F,G\}_K^{\rm hard}(Q^2) &= \Delta\{F,G\}_K^{\rm twist2}(Q^2) + \Delta\{F,G\}_K^{\rm twist3}(Q^2)\\ &\quad + \Delta\{F,G\}_K^{\rm twist4}(Q^2). \end{aligned} \tag{2}$$

The principal inputs for determining the factorized kaon form factors are the collinear/light cone DAs which encode all the nonperturbative physics. They are universal in nature (frame or process independent); in a sense, once they are determined at a certain process, they could yield

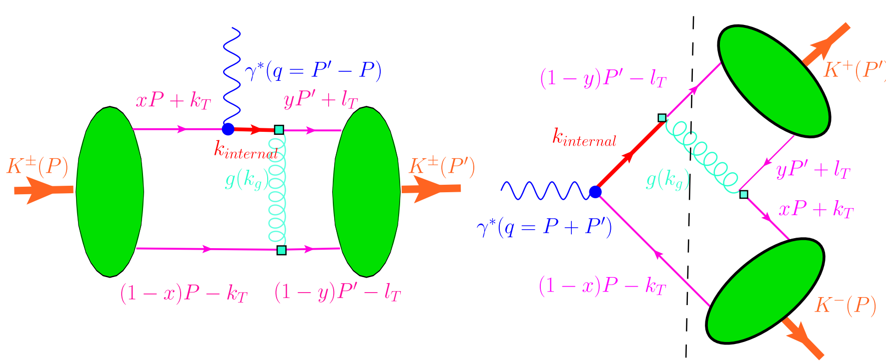

FIG. 1 (color online). Leading order Feynman diagrams in pQCD for hard contributions to the charged kaon form factors in the spacelike (left) and the timelike (right) region.

predictions for another. To next-to-leading order in conformal twist expansion there is one 2-particle twist-2 DA $\phi_{2;K}$ with an axial-vector structure, two 2-particle twist-3 DAs ($\phi^{p}_{3;K}$ with a pseudoscalar structure and $\phi^{\sigma}_{3;K}$ with a pseudotensor structure), and finally, two 2-particle twist-4 DAs ($\mathbb{A}_{4;K}$ and $\mathbb{B}_{4;K} = g_{4;K} - \phi_{2;K}$) both having pseudoscalar structures [25–27]. As an example for K^-, we display the twist-2 DA in terms of the following pseudoscalar matrix element with $\xi = 2x - 1$:

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 s(-z)|K^-(P)\rangle = iP_\mu \int_0^1 dx e^{i\xi(Pz)}\phi_{2;K}(x, \mu^2), \tag{3}$$

with the normalization condition

$$N_{2;K} = \int_0^1 \phi_{2;K}(x, \mu^2)dx = \frac{f_K}{2\sqrt{2N_c}}, \tag{4}$$

where f_K is the kaon decay constant defined in the local limit $z \to 0$ by

$$\langle 0|\bar{u}(0)\gamma_\mu\gamma_5 s(0)|K^-(P)\rangle = if_K P_\mu. \tag{5}$$

In the above equations, x is the collinear/light cone momentum fraction ($x_i = k^+/P^+$) carried by the individual valence quarks (x for the s quark and $\bar{x} = 1 - x$ for the antiquark $\bar{u}$). Note that the gauge-connection factor in the above matrix element is assumed implicitly. To the leading logarithmic accuracy, $\phi_{2;K}$ satisfies the well-known *Efremov-Radyushkin-Brodsky-Lepage evolution* equation [5,6] and can be expressed as an irreducible representation of the special collinear conformal group $\mathbb{SL}(2,\mathbb{R})$, in terms of standard *Gagenbauer polynomials* $C_n^{3/2}(\xi)$:

$$\phi_{2;K}(x, \mu^2) = \phi^{(\mathrm{as})}_{2;K}(x)\sum_{n=0}^{\infty} a_n^K(\mu_0^2)C_n^{3/2}(\xi)\left(\frac{\alpha_s(\mu_0^2)}{\alpha_s(\mu^2)}\right)^{-4\gamma_n^{(0)}/9} + \mathcal{O}(\alpha_s), \tag{6}$$

with the asymptotic twist-2 DA given by

$$\phi^{(\mathrm{as})}_{2;K}(x) = \phi_{2;K}(x, \mu^2 \to \infty) = \frac{3f_K}{\sqrt{2N_c}}x(1-x). \tag{7}$$

The standard QCD $\overline{\mathrm{MS}}$ running coupling $\alpha_s(\mu^2)$ to 2-loop accuracy is given by

$$\frac{\alpha_s(\mu^2)}{\pi} = \frac{1}{\beta_0\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})} - \frac{\beta_1\ln(\ln(\mu^2/\Lambda^2_{\mathrm{QCD}}))}{\beta_0^3\ln^2(\mu^2/\Lambda^2_{\mathrm{QCD}})} \tag{8}$$

with $\Lambda_{\mathrm{QCD}} \approx 0.2$ GeV, $\beta_0 = (11N_c - 2N_f)/12 = 9/4$, and $\beta_1 = (51N_c - 19N_f)/24 = 4$ for $N_c = N_f = 3$. The ratio of the QCD couplings represents the renormalization group evolution of the Gagenbauer moments a_n^K from the normalization scale $\mu_0 \approx 1$ GeV to the generic scale μ, with the leading order (in QCD coupling) anomalous dimensions given by

$$\gamma_n^{(0)} = \frac{4}{3}\left\{\frac{1}{4} + \sum_{k=2}^{n+1}\frac{1}{k} - \frac{1}{2(n+1)(n+2)}\right\} \geq 0. \tag{9}$$

The Gagenbauer moments represent the genuine nonperturbative inputs to the DAs and are usually determined using lattice simulations or from QCD sum rules. In this work, we use the latter inputs, since the moments for the higher twist DAs are yet to be determined precisely in lattice QCD. Note that the lower order moments in both approaches are known to be in good agreement with each other. However, dealing with such an infinite number of terms in the nonasymptotic DAs becomes a matter of technical challenge as the higher-order moments are extremely difficult to determine. Hence, for practical simplicity of calculation, one truncates the infinite series up to the first couple of terms only. Moreover, the increasing anomalous dimensions tend to suppress the higher-order terms. In this analysis, we consider the series up to the term with the second moment a_2^K. The rest of the nonasymptotic collinear DAs, i.e., the 2-particle twist-3 and twist-4 DAs, which we also consider in this work, have more elaborate expressions and are, therefore, relegated to the Appendix along with their renormalization group evolutions. A summary of the relevant DA parameters determined from QCD sum rules at the normalization scale of $\mu_0 \approx$ 1 GeV is presented in Table I.

A common feature of light cone DAs is that they are *endpoint* dominated due to large kinematic enhancements when the light cone momentum fractions tend to the endpoints (i.e., $x \to 0, 1$). One possible way to suppress such artificial enhancement is to use the Brodsky-Huang-Lepage Gaussian parametrization [29], where the intrinsic transverse momentum dependence of the valance partons within the full kaon wave function $\Psi_{t;K}$ (for each twist

TABLE I. Various input parameters for twist-2, twist-3, and twist-4 wave functions.

$K^\pm$ parameters	At $\mu_0 = 1$ GeV	Units	$K^\pm$ parameters	At $\mu_0 = 1$ GeV	Units
$m_{u,d}$	5.6 ± 1.6 [27]	MeV	a_2^K	0.25 ± 0.15 [27]	...
m_s	137 ± 27 [27]	MeV	f_K	$1.22f_\pi$, $f_\pi = 131$ [28]	MeV
$\mathcal{M}_{u,d}$	0.33	GeV	f_{3K}	0.0045 ± 0.0015 [27]	GeV^2
$\mathcal{M}_s$	0.45	GeV	ω_{3K}	-1.2 ± 0.7 [27]	...
m_K	493	MeV	$\delta_K^2 \equiv \delta^2$	0.20 ± 0.06 [27]	GeV^2
a_1^K	0.06 ± 0.03 [27]	...	ω_{4K}	0.2 ± 0.1 [27]	...

$t = 2, 3, 4$) is explicitly modeled by including an additional wave function $\Sigma_{t;K}$, i.e.,

$$\Psi_{t;K}(x, \mathbf{k}_T, \mu^2, \mathcal{M}_{\{u,d,s\}}) = A_{t;K}\phi_{t;K}(x, \mu^2)\Sigma_{t;K}(x, \mathbf{k}_T, \mathcal{M}_{\{u,d,s\}}), \quad (10)$$

where the form of $\Sigma_{t;K}$ is chosen similar to that of a harmonic oscillator wave function that can maximally suppress such endpoint effects and given by

$$\Sigma_{t;K}(x, \mathbf{k}_T, \mathcal{M}_{\{u,d,s\}}) = \frac{16\pi^2\beta_{t;K}^2}{x(1-x)}\exp\left[-\beta_{t;K}^2\left(\frac{\mathcal{M}_s^2 + \mathbf{k}_T^2}{x} + \frac{\mathcal{M}_{u,d}^2 + \mathbf{k}_T^2}{1-x}\right)\right]. \quad (11)$$

Note that the constituent quark masses $\mathcal{M}_{\{u,d,s\}}$ are introduced to parametrize the QCD vacuum effects, while the parameters $A_{t;K}$ and $\beta_{t;K}$ for the individual twists are phenomenologically extracted as described in the next section (also, see [17]). Next to obtain the full TMD-modified kaon DAs $\tilde{\mathcal{P}}_{t;K}$ in the impact parameter or b representation, we use the Brodsky-Lepage definition of the DA [5], yielding

$$\begin{aligned}\tilde{\mathcal{P}}_{t;K}(x, b, \mu^2, \mathcal{M}_{\{u,d,s\}}) &= \int_0^{1/b^2}\frac{d^2\mathbf{k}_T}{16\pi^3}\Psi_{t;M}(x, \mathbf{k}_T, \mu^2, \mathcal{M}_{\{u,d,s\}}) \\ &= A_{t;K}\phi_{t;K}(x, \mu^2)\exp\left[-\beta_{t;K}^2\left(\frac{\mathcal{M}_s^2}{x} + \frac{\mathcal{M}_{u,d}^2}{1-x}\right)\right] \\ &\quad\times\exp\left[-\frac{b^2x(1-x)}{4\beta_{t;K}^2}\right]. \quad (12)\end{aligned}$$

In Figs. 2 and 3, we show the various collinear DAs (which are endpoint enhanced) and the TMD-modified Gaussian DAs (which are endpoint suppressed), respectively. We also display the corresponding asymptotic forms of the DAs.

The inclusion of the transverse momentum dependence in the hard scattering kernel at the same time also serves as a natural regulator for possible endpoint enhancements. However, this leads to the appearance of large logarithms in the kernel due to an incomplete cancellation between soft gluon bremsstrahlung and radiative corrections that may spoil the perturbative convergence and, hence, the validity of the collinear factorization. While the large single logarithms such as $\alpha_s \ln Q^2$ can be effectively tackled using usual renormalization group techniques like ultraviolet divergences, the large double logarithms or *Sudakov* logarithms involving "k_T" dependence such as $\alpha_s \ln^2(Q^2/k_T^2)$, arising from the overlap of the leading soft and collinear kinematic regions of radiative gluons, cannot be similarly handled in ordinary fixed order perturbation theory. The alternative is to use resummation techniques to all orders in the strong coupling constant α_s, which organizes the double logarithms within exponential Sudakov factors to eventually get systematically absorbed by a redefinition of the DAs. Such Sudakov factors represent the perturbative tail of the DAs and suppress nonperturbative enhancement that arise from constituent partonic configurations, which involve large impact space separations. For a review of the Sudakov form factors and their application to exclusive physics, the reader is referred to [11,22,30,31]. There may be other radiative collinear double logarithms such as $\alpha_s \ln^2 x$, which may be resummed using *threshold resummation* [20,21] to suppress additional collinear enhancements in the kernel. The threshold resummation along with the Sudakov resummation arising from different subprocesses in pQCD factorization provides natural suppression to the endpoint and

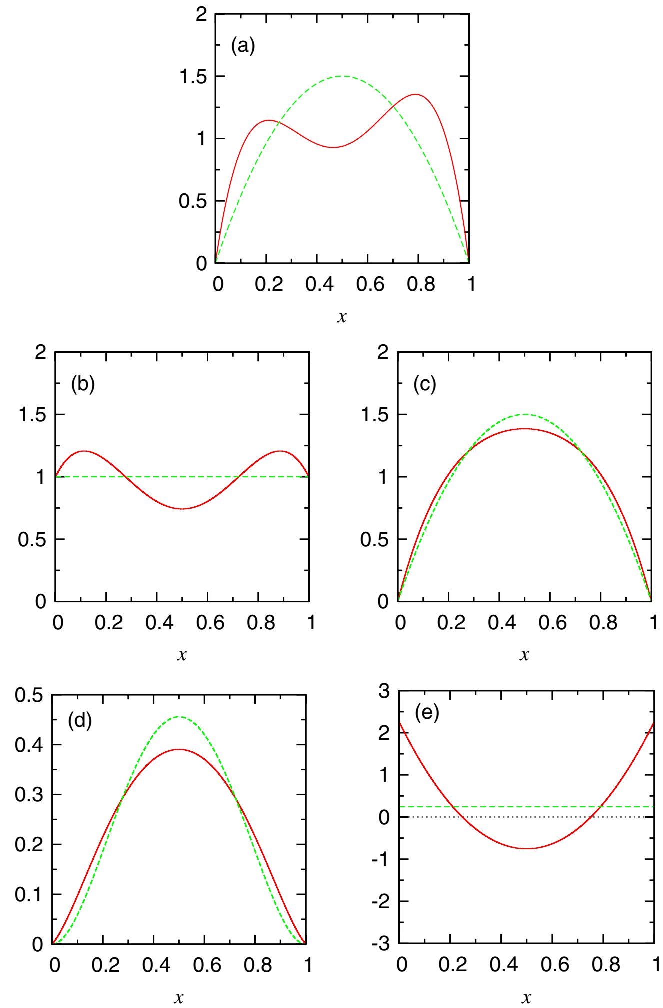

FIG. 2 (color online). The collinear twist-2 and the 2-particle twist-3 and twist-4 DAs for the kaon (modulo, the normalizations $N_{t;K}$), shown using solid (red) lines, i.e., (a) $\phi_{2;K}(x, \mu_0^2)$, (b) $\phi_{3;K}^p(x, \mu_0^2)$, (c) $\phi_{3;K}^\sigma(x, \mu_0^2)$, (d) $m_K^2\mathbb{A}_{4;K}(x, \mu_0^2)$, and (e) $m_K^2 g_{4;K}(x, \mu_0^2)$, along with their respective asymptotic DAs, shown using dashed (green) lines. The DAs are defined at the scale $\mu_0 = 1$ GeV.

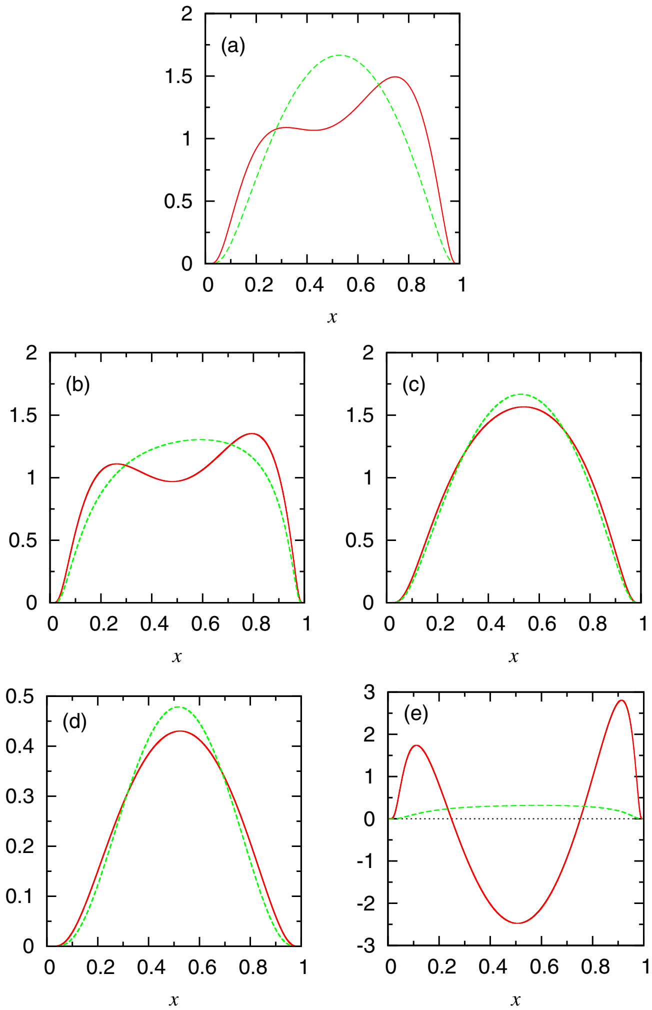

FIG. 3 (color online). The TMD-modified twist-2 and the 2-particle twist-3 and twist-4 Gaussian DAs for the kaon (modulo, the normalizations $N_{t;K}$), shown using solid (red) lines, i.e., (a) $\tilde{\mathcal{P}}_{2;K}(x,\mu_0^2)$, (b) $\tilde{\mathcal{P}}^{p}_{3;K}(x,\mu_0^2)$, (c) $\tilde{\mathcal{P}}^{\sigma}_{3;K}(x,\mu_0^2)$, (d) $m_K^2\tilde{\mathcal{P}}^{\mathbb{A}}_{4;K}(x,\mu_0^2)$, and (e) $m_K^2\tilde{\mathcal{P}}^{g}_{4;K}(x,\mu_0^2)$, along with their respective asymptotic DAs, shown using dashed (green) lines. The DAs are defined at the scale $\mu_0 = 1$ GeV.

other nonperturbative enhancements and are relevant in the range of currently probed energy/momentum transfer values. The upshot is that the hard perturbative contributions are enhanced relative to the nonperturbative contributions improving convergence significantly and making pQCD evaluation of exclusive form factors self-consistent toward lower values of Q^2, where it may not be otherwise justified.

Such techniques of systematic organization of the potentially large logarithmic contributions is a modification from the standard collinear factorization and is generally termed as the "k_T *factorization*" that has been widely applied to inclusive as well as exclusive processes [32]. However, unlike the familiar collinear factorization theorem, the k_T factorization is currently considered only at the level of a conjecture, which is yet to be proven to all orders in perturbation theory (this is a highly debatable issue and, in fact, not yet fully recognized, e.g., see [33] for a different viewpoint). To demonstrate that k_T factorization is indeed a systematic tool demands higher-order calculations, which may be very challenging. However, in this paper, we shall implicitly assume the validity of such a modified factorization without proving it and restrict ourselves at the tree level analysis of the k_T-dependent hard kernel. Moreover, in [34], the k_T factorization was proven at the level of twist-2 accuracy, while the collinear factorization was explicitly shown to be valid at the twist-3 accuracy in the case of the $\pi\gamma^* \to \gamma$ transition form factor. Our analysis, therefore, is based on the key assumption that the same formalism could be straightforwardly extended to the elastic kaon form factors.

At the leading order $\sim 1/Q^2$, the twist-2 and the 2-particle twist-4 terms contribute to the hard kernels, which have exactly the same expression given by

$$T_{\rm hard}^{(t=2,4;{\rm LO})}(x,y,Q^2,\mathbf{k}_T,\boldsymbol{l}_T,\mu^2) = \frac{\pm 16\pi\mathcal{C}_F\alpha_s(\mu^2)xQ^2}{(xQ^2\pm\mathbf{k}_T^2)(xyQ^2\pm(\mathbf{k}_T-\boldsymbol{l}_T)^2)}, \tag{13}$$

while, the $\mathcal{O}(1/Q^4)$ power suppressed 2-particle twist-3 and twist-4 hard kernels are, respectively, given by

$$T_{\rm hard}^{(t=3)}(x,y,Q^2,\mathbf{k}_T,\boldsymbol{l}_T,\mu^2) = \frac{32\pi\mathcal{C}_F\alpha_s(\mu^2)x}{(xQ^2\pm\mathbf{k}_T^2)(xyQ^2\pm(\mathbf{k}_T-\boldsymbol{l}_T)^2)};$$
$$T_{\rm hard}^{(t=4;{\rm NLO})}(x,y,Q^2,\mathbf{k}_T,\boldsymbol{l}_T,\mu^2) = \frac{48\pi\mathcal{C}_F\alpha_s(\mu^2)}{(xQ^2\pm\mathbf{k}_T^2)(xyQ^2\pm(\mathbf{k}_T-\boldsymbol{l}_T)^2)}. \tag{14}$$

In the above expressions, $\mathcal{C}_F = 4/3$, the "+" signs correspond to the spacelike case and the "−" signs correspond to the timelike case; $\mathbf{k}_T$ and $\boldsymbol{l}_T$ are, respectively, the initial and final relative transverse momenta of the valence quarks, and x and y are the corresponding light cone momentum fractions. Note that the factors in the denominators that arise from the parton propagators develop poles in the timelike region.

To obtain the hard form factors, we use the following momentum space projection operator for the DAs with the different twist structures:

$$\mathcal{M}^K_{\alpha\beta} = \frac{i}{4}\Big\{\not{P}\gamma_5\Big(\Psi_{2;K} - \frac{1}{4}m_K^2\Psi^{\mathbb{A}}_{4;K}\partial^2_{k_T}\Big) + m_K^2\gamma_5\Big(\frac{\bar{\not{P}}}{\bar{P}\cdot P}\frac{\partial}{\partial x}\Big(\int_0^x\Psi^{\mathbb{B}}_{4;K}\Big) - \Psi^{\mathbb{A}}_{4;K}\partial_{k_T}\Big) - \mu_K\gamma_5\Big(\Psi^p_{3;K} - \frac{i}{6}\sigma_{\mu\nu}n^\mu\bar{n}^\nu\frac{\partial}{\partial x}\Psi^\sigma_{3;K} + \frac{i}{6}\sigma_{\mu\nu}P^\mu\Psi^\sigma_{3;K}\partial^\nu_{k_T}\Big)\Big\}_{\alpha\beta}; \qquad \bar{P} = |P|\bar{n}, \tag{15}$$

where $\mu_K = \frac{m_K^2}{m_u+m_s}$ GeV is the "chiral-enhancement" parameter arising in the standard definition of the 2-particle twist-3 DAs (see, the Appendix), $\Psi^{\mathbb{B}}_{4;K} = \Psi^{g}_{4;K} - \Psi_{2;K}$, $\partial\!\!\!/_{k_T} \equiv \gamma^\mu \partial/\partial k_T^\mu$, $n = (1, 0, \mathbf{0}_T)$ the unit vector in the "+" direction, and $\bar{n} = (0, 1, \mathbf{0}_T)$ the unit vector in the "−" direction. Setting the renormalization/factorization scale to the magnitude of the incoming or outgoing kaon momentum, i.e., $\mu = |P| = |P'| = Q/\sqrt{2}$ and convolving the projection operators for the kaon DAs with the hard kernels using the factorization formula (symbolically, $\mathcal{M}^\dagger_{K;\mathrm{out}} \otimes T^{\mathrm{LO}}_{\mathrm{hard}} \otimes \mathcal{M}_{K;\mathrm{in}}$), we have

$$(P' \pm P)_\mu \{F, G\}_K^{\mathrm{hard}}(Q^2) = \int_0^1 dxdy \int \frac{d^2\mathbf{k}_T}{16\pi^3} \frac{d^2\boldsymbol{l}_T}{16\pi^3} \left(\frac{4\pi\alpha_s(t)\mathcal{C}_F}{3}\right) \exp[-i\mathbf{k}_T\cdot\mathbf{b}_1 - i\boldsymbol{l}_T\cdot\mathbf{b}_2] \times \mathbf{Tr}\left[\frac{\gamma^\nu \mathcal{M}^\dagger_{K;\mathrm{out}} \gamma_\nu k\!\!\!/_{\mathrm{internal}} \gamma_\mu \mathcal{M}_{K;\mathrm{in}}}{(k^2_{\mathrm{internal}} + i\epsilon)(k_g^2 + i\epsilon)} + 3\,\mathrm{diagrams}\right] \mathcal{U}_{\mathrm{RGE}}(t, \mu) S_t(x) \times \exp[-S(x, y, |\mathbf{k}_T| \sim 1/b_1, |\boldsymbol{l}_T| \sim 1/b_2, \mu)], \tag{16}$$

where k_{internal} and k_g are the internal quark and gluon momenta, respectively, as shown in Fig. 1. Also, in the above equation

$$\mathcal{U}_{\mathrm{RGE}}(t, \mu) = \exp\left[4 \int_t^\mu \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}^2))\right]; \quad \gamma_q(\alpha_s(\bar{\mu}^2)) = -\frac{\alpha_s(\bar{\mu}^2)}{\pi}, \tag{17}$$

represents the renormalization group evolution or "RGE" factor for the scattering kernel from the "upper-factorization" scale $t = \max(\sqrt{x}Q, 1/b_1, 1/b_2)$ to the renormalization scale $\mu = Q/\sqrt{2}$, and γ_q is the quark anomalous dimension. The expression for the Sudakov exponent $S(x, y, b_1, b_2, Q)$ (after absorbing the RGE factor from the kernel) is given by [31]

$$\begin{aligned} S(x, y, b_1, b_2, Q) &= s(xQ, b_1) + s((1-x)Q, b_1) + s(yQ, b_2) \\ &+ s((1-y)Q, b_2) + 2\int_{1/b_1}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}^2)) \\ &+ 2\int_{1/b_2}^t \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu}^2)), \end{aligned} \tag{18}$$

where

$$s(xQ, 1/b) \equiv s(x\mu, 1/b) = \int_{1/b}^{x\mu} \frac{d\bar{\mu}}{\bar{\mu}} \left[\ln\left(\frac{x\mu}{\bar{\mu}}\right) \mathcal{A}(\alpha_s(\bar{\mu}^2)) + \mathcal{B}(\alpha_s(\bar{\mu}^2))\right], \tag{19}$$

where the "lower-factorization" scales $1/b_1$, $1/b_2 > \Lambda_{\mathrm{QCD}}$ serve to separate the perturbative from the nonperturbative transverse distances, which are also typically the scales that provide a natural starting point of the evolution of the kaon wave functions. In the above equations, the so-called "cusp" anomalous dimensions $\mathcal{A}$ and $\mathcal{B}$, to 1-loop accuracy are given by

$$\begin{aligned} \mathcal{A}(\alpha_s(\mu^2)) &= \mathcal{C}_F \frac{\alpha_s(\mu^2)}{\pi} + \left[\left(\frac{67}{27} - \frac{\pi^2}{9}\right) N_c - \frac{10}{27} N_f + \frac{8}{3}\beta_0 \ln\left(\frac{e^{\gamma_E}}{2}\right)\right] \left(\frac{\alpha_s(\mu^2)}{\pi}\right)^2, \\ \mathcal{B}(\alpha_s(\mu^2)) &= \frac{2}{3} \frac{\alpha_s(\mu^2)}{\pi} \ln\left(\frac{e^{2\gamma_E - 1}}{2}\right). \end{aligned} \tag{20}$$

The exact form of the threshold resummation "jet" function $S_t(x)$ in Eq. (16) involves a one parameter integration, but in practice it is more convenient to take the simple parametrization proposed in [20,21]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2 + c)}{\sqrt{\pi}\Gamma(1 + c)} [x(1-x)]^c, \tag{21}$$

where the parameter $c \approx 0.3$ for light pseudoscalar mesons like the pion and the kaon.

Now we present the factorized result for the hard kaon form factors up to twist-4 corrections as follows:

$$\begin{aligned} \{F, G\}_K^{\mathrm{hard}}(Q^2) &= \delta\{F, G\}_K^{\mathrm{twist2}}(Q^2) + \delta\{F, G\}_K^{\mathrm{twist3}}(Q^2) + \delta\{F, G\}_K^{\mathrm{twist4}}(Q^2); \\ \delta\{F, G\}_K^{\mathrm{twist4}}(Q^2) &= \delta\{F, G\}_K^{\mathrm{twist4;LO}}(Q^2) + \delta\{F, G\}_K^{\mathrm{twist4;NLO}}(Q^2), \end{aligned} \tag{22}$$

where the leading twist-2 and twist-4 corrections are expressed by the following integral representations in the impact parameter space

$$\delta\{F,G\}_K^{\text{twist2}}(Q^2) + \delta\{F,G\}_K^{\text{twist4;LO}}(Q^2) = 32\pi Q^2 \mathcal{C}_F \int_0^1 x dx dy \int_0^\infty b_1 db_1 b_2 db_2 \alpha_s(t) \Big[\pm \frac{1}{2} \mathcal{P}_{2;K}(x,b_1)\mathcal{P}_{2;K}(y,b_2) \mp m_K^2 \frac{b_2^2}{8} \mathcal{P}_{2;K}(x,b_1)\mathcal{P}_{4;K}^{\mathbb{A}}(y,b_2) \mp m_K^2 \frac{b_1^2}{8} \mathcal{P}_{4;K}^{\mathbb{A}}(x,b_1)\mathcal{P}_{2;K}(y,b_2) + \mathcal{O}(m_K^4 b_1^4, m_K^4 b_2^4) \Big] H_\pm(x,y,Q,b_1,b_2) \exp[-S(x,y,b_1,b_2,Q)] S_t(x), \tag{23}$$

while, the power suppressed twist-3 and twist-4 corrections are given by

$$\delta\{F,G\}_K^{\text{twist3}}(Q^2) + \delta\{F,G\}_K^{\text{twist4;NLO}}(Q^2) = 32\pi Q^2 \mathcal{C}_F \int_0^1 dx dy \int_0^\infty b_1 db_1 b_2 db_2 \alpha_s(t) \Big[\frac{\mu_K^2}{Q^2} \Big(\bar{x} \mathcal{P}_{3;K}^p(x,b_1)\mathcal{P}_{3;K}^p(y,b_2) + \frac{(1+x)}{6} \frac{\partial}{\partial x} \mathcal{P}_{3;K}^\sigma(x,b_1)\mathcal{P}_{3;K}^p(y,b_2) + \frac{1}{2} \mathcal{P}_{3;K}^\sigma(x,b_1)\mathcal{P}_{3;K}^p(y,b_2) \Big) + \frac{3m_K^2}{2Q^2} \Big(\int_0^x d\zeta \mathcal{P}_{4;K}^{\mathbb{B}}(\zeta,b_1) \Big) \Big(\mathcal{P}_{2;K}(y,b_2) - m_K^2 \frac{b_2^2}{4} \mathcal{P}_{4;K}^{\mathbb{A}}(y,b_2) \Big) + \mathcal{O}(m_K^4 b_1^4, m_K^4 b_2^4) \Big] H_\pm(x,y,Q,b_1,b_2) \exp[-S(x,y,b_1,b_2,Q)] S_t(x), \tag{24}$$

where $\mathcal{P}_{t;K}(x,b) \equiv \tilde{\mathcal{P}}_{t;K}(x,b,1/b^2,\mathcal{M}_{\{u,d,s\}})$, and $\mathcal{P}_{4;K}^{\mathbb{B}} = \mathcal{P}_{4;K}^g - \mathcal{P}_{2;K}$. Note that the superscripts "LO" and "NLO" used in the above equation should not be confused with the usual terminologies associated with perturbative expansions in terms of α_s, but rather in the sense of operator product expansion terms. In the impact representation, the space- and timelike hard kernels (the part of the scattering kernel that is common to all the twists) could be expressed in terms of standard Bessel functions K_0, I_0, J_0, and $H_0^{(1)}$ and are given by

$$H_+(x,y,Q,b_1,b_2) = K_0(\sqrt{xy}Qb_2)[\theta(b_1-b_2)K_0(\sqrt{x}Qb_1)I_0(\sqrt{x}Qb_2) + \theta(b_2-b_1)K_0(\sqrt{x}Qb_2)I_0(\sqrt{x}Qb_1)]; \tag{25}$$

$$H_-(x,y,Q,b_1,b_2) = \Big(\frac{i\pi}{2}\Big)^2 H_0^{(1)}(\sqrt{xy}Qb_2)[\theta(b_1-b_2)H_0^{(1)}(\sqrt{x}Qb_1) \times J_0(\sqrt{x}Qb_2) + \theta(b_2-b_1)H_0^{(1)}(\sqrt{x}Qb_2)J_0(\sqrt{x}Qb_1)], \tag{26}$$

where H_+ is a real-valued function and H_- is a complex-valued function of real arguments.

Apropos of our derived formulas Eqs. (23) and (24), it is noteworthy to mention that in [35] it was suggested that the Sudakov factors must be analytically continued from the spacelike to the timelike case. This may not be generally true. The Sudakov factors in [36] (see, Sec. 3.1 of this reference) arise directly from "form factor-type" kernels, which are not universal quantities and may vary with processes. There the analytic continuation is perfectly justified. However, for an approach based on the factorization theorem, one uses "universal" Sudakov factors $S(Q)$ arising from the overlap of the soft and collinear processes below the factorization scale, as in the present context. As explained in [37], these Sudakov factors are to be considered as an integral part of the DAs and, thus, they are universal quantities as well, depending only on the magnitude of the energy scale $Q \geq 0$. Note that the Q dependence of the Sudakov factor in Eqs. (18) and (19), stems from the dependence on the collinear components of the external pion 4-momenta which are given by $P^+ = P'^- = Q/\sqrt{2}$, in the Breit frame. As such, it is important that one does not analytically continue but rather use the same Sudakov factor in both the space and timelike cases.

B. Nonfactorizable Soft QCD

In [8] it was shown that the spacelike low-energy pion data below $Q^2 \sim 10\ \text{GeV}^2$ is dominated by the soft pion form factor which accounts for more that 70% of the data. Such soft QCD contributions are nonfactorizable and are beyond the realm of ordinary perturbation theory. Since no systematic method is currently available to calculate these nonperturbative effects, one is compelled to use some model ansatz to obtain a rough estimate of their contributions, viz., in [8] the soft pion form factor in the spacelike region was calculated using the *local duality* or LD model in QCD sum rules. In our present work, we extend the same result to the spacelike kaon form factor which is then given by

$$F_K^{\text{soft}}(Q^2)|_{\text{LD}} = 1 - \frac{1 + 6s_0/Q^2}{(1+4s_0/Q^2)^{3/2}} \approx \frac{6s_0}{Q^4} + \mathcal{O}\Big(\frac{1}{Q^6}\Big); \qquad s_0 \approx 4\pi^2 f_K^2. \tag{27}$$

Now, on one hand, an *ab initio* derivation of the corresponding timelike soft form factor seems *a priori* unfeasible using QCD sum rules, since the local duality principle is strictly applicable for the spacelike region only. On the other hand, a naive analytic continuation of the spacelike formula, i.e., by a replacement of $Q^2 \to -Q^2$, leads to an undesirable pole in the denominator of the soft form factor:

$$G^{\text{soft}}_{K;\text{analytic}}(Q^2) \Rightarrow 1 - \frac{1 - 6s_0/Q^2}{(1 - 4s_0/Q^2)^{3/2}} \approx \frac{6s_0}{Q^4} + \mathcal{O}\left(\frac{1}{Q^6}\right). \tag{28}$$

Since here our primary goal is to obtain an estimate for the smooth continuum part of the kaon spectra for intermediate energies, which in reality is, however, dominated by low-energy timelike resonances that obscure the smooth continuum. With the "oversimplified" assumption that these resonance peaks behave as background "noise," superimposed on a smooth continuum spectrum, we choose the functional form of the timelike soft form factor to be the same as that of the spacelike expression, which is a smooth function for the entire range of Q^2, we consider, i.e.,

$$G^{\text{soft}}_{K}(Q^2) = 1 - \frac{1 + 6s_0/Q^2}{(1 + 4s_0/Q^2)^{3/2}} + \mathcal{O}\left(\frac{1}{Q^6}\right). \tag{29}$$

Moreover, for large enough $Q^2\sim$ above 5 GeV^2, both expressions Eqs. (27) and (28) when expanded in inverse powers of Q^2 yield the same leading term of $\mathcal{O}(\frac{1}{Q^4})$. Hence, the particular choice of the soft form factors should not matter significantly at large-Q^2 values where the perturbative predictions become more reliable and dominant.

In the present context, a vital aspect deserves some consideration. Since the inclusion of the soft form factors has been somewhat *ad hoc*, without any correspondence among the hard and the soft contributions, there could be chances of possible double counting of contributions especially at low energies. Thus, it becomes clear that we must correct the hard factorized results in the low-Q^2 region to ensure that the respective contributions lie within their domains of validity. This is achieved by enforcing the gauge invariance condition through the *vector Ward identity* $\{F, G\}_K(Q^2 = 0) = 1$, which is *a priori* not ensured in perturbative calculations. Since the soft form factors satisfies $\{F, G\}^{\text{soft}}_K(Q^2 = 0) = 1$, we must have $\{F, G\}^{\text{hard}}_K(Q^2 = 0) = 0$. But this is unfortunately not satisfied by Eqs. (23) and (24) where the contributions tend to diverge rapidly in the vicinity of $Q^2 = 0$. Therefore, the essential task is to match the large-Q^2 results of $\{F, G\}^{\text{hard}}_K(Q^2)$ with the low-Q^2 results of $\{F, G\}^{\text{soft}}_K(Q^2)$. Here we shall modify the argument given in [12] for the twist-2 case to be applicable for the twist-3 and twist-4 power corrections. The simplest way is to "power correct" for the singular $\sim 1/Q^2$ (leading twist-2 and twist-4) and $\sim 1/Q^4$ (subleading twist-3 and twist-4) behaviors, respectively, at small Q, by introducing some characteristic low-energy mass scale M_0 that may lead to the onset of the genuine nonperturbative soft dynamics. For the soft form factors modeled *via* the local duality principle, the scale $M_0^2 = 2s_0$ is a natural choice [38]. It can then be shown that for the leading twist-2 and twist-4 hard corrections, it is sufficient to make the modification [12]

$$\begin{aligned}\delta\{F, G\}^{\text{twist2}}_K(Q^2) &+ \delta\{F, G\}^{\text{twist4;LO}}_K(Q^2)\\ &\to \Delta\{F, G\}^{\text{twist2}}_K(Q^2) + \Delta\{F, G\}^{\text{twist4;LO}}_K(Q^2)\\ &= \left(\frac{Q^2}{2s_0 + Q^2}\right)^2 (\delta\{F, G\}^{\text{twist2}}_K(Q^2)\\ &+ \delta\{F, G\}^{\text{twist4;LO}}_K(Q^2)).\end{aligned} \tag{30}$$

For the case of the subleading twist-3 and twist-4 hard corrections, we perform the following replacement:

$$\begin{aligned}\delta\{F, G\}^{(t=3,4)}_K(Q^2) &= \delta\widetilde{\{F, G\}}^{(t=3,4)}_K(Q^2)\frac{M_0^4}{Q^4}\\ &\to \delta\widetilde{\{F, G\}}^{(t=3,4)}_K(Q^2)\frac{M_0^4}{M_0^4 + Q^4},\end{aligned} \tag{31}$$

where we write $\delta\{F, G\}^{(t=3,4)}_K \equiv \delta\{F, G\}^{\text{twist3}}_K + \delta\{F, G\}^{\text{twist4;NLO}}_K$ for brevity. Now, to maintain the Ward identity, we correct for the wrong $Q^2 = 0$ limit of the above expression

$$\begin{aligned}\Delta\{F, G\}^{(t=3,4)}_K(Q^2) &= -\delta\widetilde{\{F, G\}}^{(t=3,4)}_K(Q^2)\Phi_n(Q^2/M_0^2)\\ &+ \delta\widetilde{\{F, G\}}^{(t=3,4)}_K(Q^2)\frac{M_0^4}{M_0^4 + Q^4},\end{aligned} \tag{32}$$

where we introduce the smooth function $\Phi_n(z)$ (with $z = Q^2/M_0^2$) with the essential property that $\Phi_n(0) = 1$ and $z\Phi_n(z) \to 0$ as $z \to \infty$, for a suitable choice of the positive integer n, to preserve the asymptotics of $\delta\{F, G\}^{(t=3,4)}_K(Q^2)$. A natural choice for $\Phi_n(z)$ could be $\Phi_n(z) = 1/(1 + z^n)^2$, concurrent with the $\sim 1/Q^{2n}$ scaling behavior of the respective power suppressed terms. For the present purpose, it is sufficient to take $n = 2$, yielding

$$\begin{aligned}\Delta\{F, G\}^{(t=3,4)}_K(Q^2) &= \delta\widetilde{\{F, G\}}^{(t=3,4)}_K(Q^2)\frac{M_0^4}{M_0^4 + Q^4}\\ &\times\left(1 - \frac{M_0^4}{M_0^4 + Q^4}\right)\\ &= \delta\{F, G\}^{(t=3,4)}_K(Q^2)\left(\frac{Q^4}{M_0^4 + Q^4}\right)^2.\end{aligned} \tag{33}$$

In principle, this can also be achieved with larger integer values of n that would lead to $(Q^{2n}/(M_0^{2n} + Q^{2n}))^2$ in front of the hard parts. However, as $n \to \infty$, this factor becomes a step function, which is no longer smooth. Hence, a minimal value of n is preferable and we arrive at the Ward identity modified result:

$$
\begin{aligned}
&\delta\{F,G\}_K^{\rm twist3}(Q^2) + \delta\{F,G\}_K^{\rm twist4;NLO}(Q^2) \\
&\quad \to \Delta\{F,G\}_K^{\rm twist3}(Q^2) + \Delta\{F,G\}_K^{\rm twist4;NLO}(Q^2) \\
&\quad = \left(\frac{Q^4}{4s_0^2+Q^4}\right)^2 (\delta\{F,G\}_K^{\rm twist3}(Q^2) \\
&\quad + \delta\{F,G\}_K^{\rm twist4;NLO}(Q^2)).
\end{aligned} \tag{34}
$$

The prefactors only alter the low-energy behavior of the hard contributions and ensure the correct power law in maintaining a smooth matching between the large-Q^2 behavior of $\{F,G\}_K^{\rm hard}(Q^2)$ to the low-Q^2 behavior of $\{F,G\}_K^{\rm soft}(Q^2)$ (see, Fig. 4). This leads to our final expression for the total electromagnetic kaon form factors, correct up to $\mathcal{O}(\frac{1}{Q^4})$ accuracy, given by

$$
\begin{aligned}
\{F,G\}_K(Q^2) = {}& \{F,G\}_K^{\rm soft}(Q^2) + \Delta\{F,G\}_K^{\rm twist2}(Q^2) \\
& + \Delta\{F,G\}_K^{\rm twist3}(Q^2) + \Delta\{F,G\}_K^{\rm twist4}(Q^2),
\end{aligned}
$$

where

$$
\begin{aligned}
\{F,G\}_K^{\rm soft}(Q^2) &= 1 - \frac{1+6s_0/Q^2}{(1+4s_0/Q^2)^{3/2}}, \\
\Delta\{F,G\}_K^{\rm twist2}(Q^2) &= \left(\frac{Q^2}{2s_0+Q^2}\right)^2 \delta\{F,G\}_K^{\rm twist2}(Q^2), \\
\Delta\{F,G\}_K^{\rm twist3}(Q^2) &= \left(\frac{Q^4}{4s_0^2+Q^4}\right)^2 \delta\{F,G\}_K^{\rm twist3}(Q^2), \\
\Delta\{F,G\}_K^{\rm twist4}(Q^2) &= \left(\frac{Q^2}{2s_0+Q^2}\right)^2 \delta\{F,G\}_K^{\rm twist4;LO}(Q^2) \\
&\quad + \left(\frac{Q^4}{4s_0^2+Q^4}\right)^2 \delta\{F,G\}_K^{\rm twist4;NLO}(Q^2),
\end{aligned} \tag{35}
$$

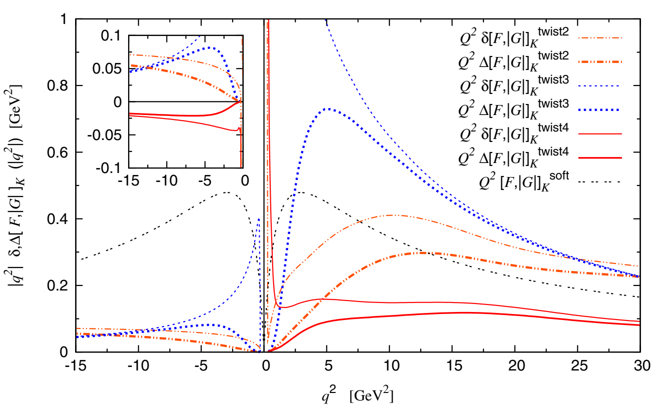

FIG. 4 (color online). Relative contributions of the soft $\{F,G\}_K^{\rm soft}$ (double-dot black lines), twist-2 $\Delta\{F,G\}_K^{\rm twist2}$ (thick double-dot dashed orange lines), twist-3 $\Delta\{F,G\}_K^{\rm twist3}$ (thick dotted blue lines), and twist-4 $\Delta\{F,G\}_K^{\rm twist4}$ (thick solid red lines) terms in Eq. (35). The same terms without the prefactor modifications are also displayed.

where the δ's are replaced by the Δ's to include the respective prefactors.

III. RESULTS AND DISCUSSION

To obtain the Gaussian parameters of the kaon wave functions, we use the following two sets of constraints valid for the individual twists ($t = 2, 3, 4$): the first set of constraints is obtained from the leptonic decay $K \to \mu + \nu_\mu$, and given by

$$
\int_0^1 dx \int \frac{d^2\mathbf{k}_T}{16\pi^3} \Psi_{t;K}(x, \mathbf{k}_T, \mu_0^2, \mathcal{M}_{\{u,d,s\}}) = N_{t;K}, \tag{36}
$$

with $N_{t;K}$ being the normalization constant for the collinear DAs; and the second follows from the phenomenological fact that the average transverse momentum of the valence partons in light mesons is about $\langle \mathbf{k}_T^2\rangle^{1/2}_{\pi,K,\eta\cdots} \approx 0.35$ GeV, i.e.,

$$
\langle \mathbf{k}_T^2\rangle_K = \frac{\int dx \int d^2\mathbf{k}_T |\mathbf{k}_T^2| |\Psi_{t;K}(x,\mathbf{k}_T,\mu_0^2,\mathcal{M}_{\{u,d,s\}})|^2}{\int dx \int d^2\mathbf{k}_T |\Psi_{t;K}(x,\mathbf{k}_T,\mu_0^2,\mathcal{M}_{\{u,d,s\}})|^2}. \tag{37}
$$

The Gaussian parameters determined in this way for $\mu_0 \approx 1$ GeV are collected in Table II. Note that due to the rather mild scale dependences of these parameters, which practically remain constant for the entire range of intermediate energies that is considered in this work, their scale variations have been kept fixed to reduce the numerical complexity. However, we do consider their variation with the changes in the collinear DA parameters, summarized in Table I, that is required for our estimation of the theoretical error. Once all the phenomenological parameters are determined, we proceed to calculate the hard contributions. For calculations, we use the full nonasymptotic collinear DAs derived from light cone QCD sum rules [25–27].

In Fig. 4, we plot the individual terms of Eq. (35), i.e., $\{F,G\}_K^{\rm soft}$, $\Delta\{F,G\}_K^{\rm twist2}$, $\Delta\{F,G\}_K^{\rm twist3}$, and $\Delta\{F,G\}_K^{\rm twist4}$, which should give an idea about the relative magnitude of each contribution for intermediate values of Q^2 up to 30 GeV^2. For comparison, we also display the results obtained without including the prefactor modifications, which do not show any appreciable difference for Q^2 values beyond ~5–10 GeV^2. As expected, the standard twist-2 contributions are much smaller compared to the soft QCD and the twist-3 power corrections at moderate energies. However, the twist-4 contributions are seen to be indeed small (about 1/3 of the magnitude of the twist-2), which are, in fact, negative in the spacelike region. In the timelike region, since all the hard contributions are complex, it only makes sense to plot the modulus of the individual twist corrections. It is notable that the general enhancement of all the timelike hard contributions relative to the spacelike ones can be attributed to the timelike parton propagators developing poles that are absent in the spacelike region. To illustrate this point, it is useful

TABLE II. The phenomenologically determined Gaussian parameters for twist-2, twist-3 and twist-4 wavefunctions. The numbers in the parentheses $(\cdots)_{\rm as}$ correspond to parameters for the asymptotic wavefunctions.

$A_{t;K}(t = 2, 3, 4)$	At $\mu_0 = 1$ GeV	Units	$(\beta_{t;K})^2(t = 2, 3, 4)$	At $\mu_0 = 1$ GeV	Units
$A_{2;K}$	$2.06(2.07)_{\rm as}$	$\cdots$	$(\beta_{2;K})^2$	$0.78(0.89)_{\rm as}$	GeV^{-2}
$A^p_{3;K}$	$2.23(2.28)_{\rm as}$	$\cdots$	$(\beta^p_{3;K})^2$	$0.71(0.79)_{\rm as}$	GeV^{-2}
$A^\sigma_{3;K}$	$2.08(2.07)_{\rm as}$	$\cdots$	$(\beta^\sigma_{3;K})^2$	$0.88(0.89)_{\rm as}$	GeV^{-2}
$A^{\mathbb{A}}_{4;K}$	$2.07(2.00)_{\rm as}$	$\cdots$	$(\beta^{\mathbb{A}}_{4;K})^2$	$0.91(0.93)_{\rm as}$	GeV^{-2}
$A^g_{4;K}$	$5.22(2.28)_{\rm as}$	$\cdots$	$(\beta^g_{4;K})^2$	$0.57(0.79)_{\rm as}$	GeV^{-2}

to plot the part of the hard kernel $H_\pm(x, y, Q, b_1, b_2)$ that is common to all the twist corrections to the hard form factors. Figure 5 shows the variation of the space- and timelike kernels $H_\pm$ (in the impact parameter space) as a function of Q^2 for some arbitrary fixed values of the parameters x, y, b_1, and b_2. It immediately becomes clear that the real-valued spacelike kernel H_+ has a rapidly decaying exponential behavior, whereas the complex-valued timelike kernel H_- has rather large amplitude oscillatory real and imaginary components which decay very gradually with increasing Q^2. In reference to Eqs. (13) and (14), we note that if $\{x, y\} \ll 1 \Rightarrow \{xQ^2, xyQ^2\} \sim \{\mathbf{k}_T^2, \boldsymbol{l}_T^2\} \ll Q^2$, and if $\{x, y\} \sim 1 \Rightarrow \{xQ^2, xyQ^2\} \sim Q^2$, so that the terms in the denominators tend to cancel each other in the timelike but not in the spacelike domain. This explains why the amplitude of the timelike oscillations in H_- grow larger and larger near the endpoints $x, y \to 0$.

The most striking feature of our results in Fig. 4 is the anomalously large twist-3 contribution in the timelike region, similar to what was seen for the pion [23], dominating all the other corrections for the entire range of low and moderate energies. This huge asymmetry between the space- and timelike twist-3 contributions comes from the additional parametric enhancement of the twist-3 DAs due to the chiral parameter μ_K which makes them particularly sensitive to the chiral scale. It is the combination of this parametric enhancement along with the occurrence of the timelike poles in the hard kernel that leads to such a characteristic anomalous twist-3 behavior which is completely missing in the twist-2 or even in the twist-4. At this point, one may also worry about possible large contributions from the 3-particle twist-3 sector (related to the 2-particle twist-3 sector through QCD equations of motion) that was not considered in this work. Here, we note that such a possibility can safely be precluded since the 3-particle twist-3 DA receives large parametric suppression from the nonperturbative parameter $f_{3K} \approx 0.0045$ GeV^2, numerically very much smaller compared to the analogous 2-particle twist-3 parameter $\mu_K \approx 1.5$ GeV, which greatly enhances the contribution from the 2-particle sector.

Further, it is important to note that (a) the "active" soft gluons that may also arise from the 3-particle twist-3 DA or higher twist DAs likewise, bring about additional power

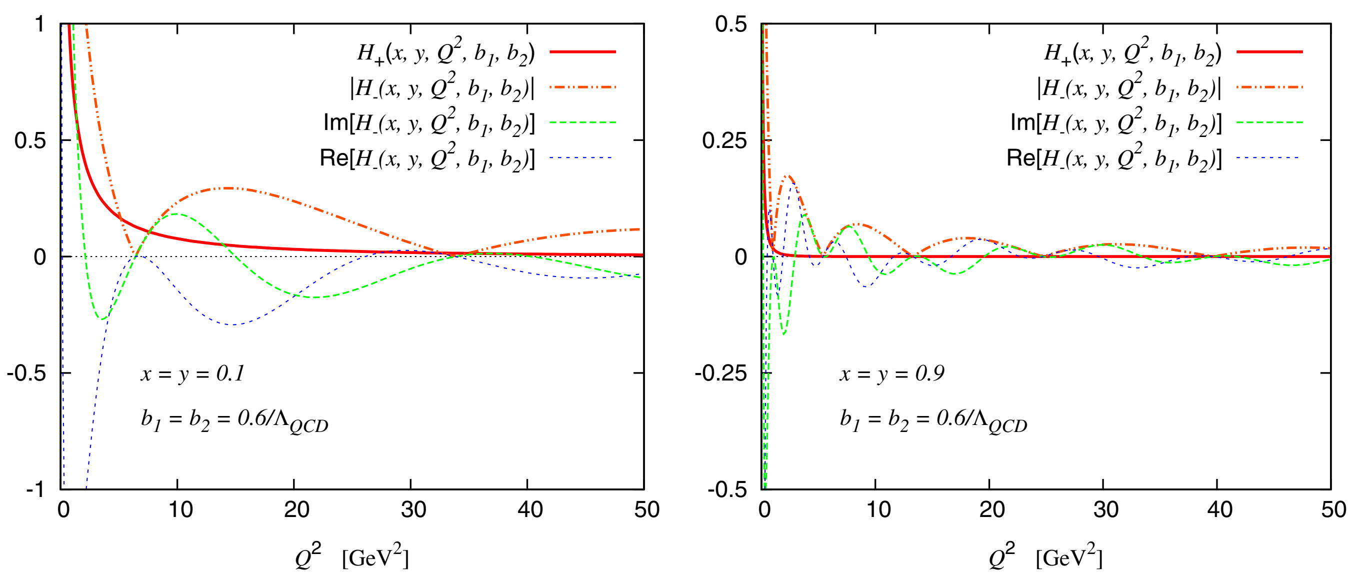

FIG. 5 (color online). The space- and timelike hard kernels $H_\pm(x, y, Q, b_1, b_2)$ in the impact space representation for two sets of choices for the collinear momentum fractions with arbitrary fixed b_1, b_2: $x = y = 0.1$ (left plot) and $x = y = 0.9$ (right plot).

corrections and, therefore, can be safely neglected at large-Q^2 values, and (b) the "long-distance" soft gluons that may be a possible source of the breakdown of k_T factorization, cannot probe the small "color-dipole" configurations of the $q\bar{q}$ hadronic bound state at high enough Q^2 (color transparency). The remaining collinear gluons are assumed to be effectively tackled within the present k_T-factorization scheme, where the inclusion of the 2-particle twist-3 corrections indeed turn out to be the most crucial aspect at the moderate-Q^2 regime. Note, however, that all such nonperturbative power corrections including the soft contributions rapidly fall off with increasing Q^2, and beyond $\sim$50–100 GeV^2 the standard twist-2 terms start dominating the asymptotic regime, yielding back numerically the *bona fide* asymptotic behavior given by the Farrar and Jackson result [7]

$$\{F, G\}_K^{\text{asy}}(Q^2) = \frac{8\pi\alpha_s(Q^2)f_K^2}{Q^2}. \qquad (38)$$

Our final prediction for the total scaled electromagnetic kaon form factors $\{F, G\}_K$ [from Eq. (35)] up to twist-4 accuracy in the range of intermediate energies/momentum transfers is presented in Fig. 6, along with the result for the soft form factors $\{F, G\}_K^{\text{soft}}$ and the standard asymptotic QCD result of Farrar and Jackson [7] for comparison. To estimate the theoretical error, we studied the variation of the wave function parameters provided in Tables I and II. In addition, we varied the chiral parameter $\mu_K = m_K^2/(m_u + m_s)$ which is often taken to be slightly lower $\sim$1.3–1.5 GeV in the literature [17,19–23,26,39,40] than its naive value about 1.7 GeV expressed in terms of the current quark masses. In this analysis, we take $\mu_K = 1.5 \pm 0.2$ GeV and include its variation in the error estimate. The shaded area, thus obtained, can be regarded as our rough estimate for the theoretical error, where the solid (red) curve corresponds to the central values of the parameters. While our result is relatively insensitive to the choice of the parameters in the spacelike region, the timelike result turns out to be very sensitive to the choice of μ_K whose variation alone amounts for more than 90% of the error bar. The error due to the rest of the model parameters is generously overestimated to include possible uncertainties due to the soft parts which we do not *a priori* take into account. Thus, we should stress that our pQCD based error estimate in the low-Q^2 region (which apparently looks small) must be considered in a very conservative sense and cannot be taken seriously below $\sim$5 GeV^2. A more rigorous error analysis is impossible at the moment due to poor quality of the experimental data.

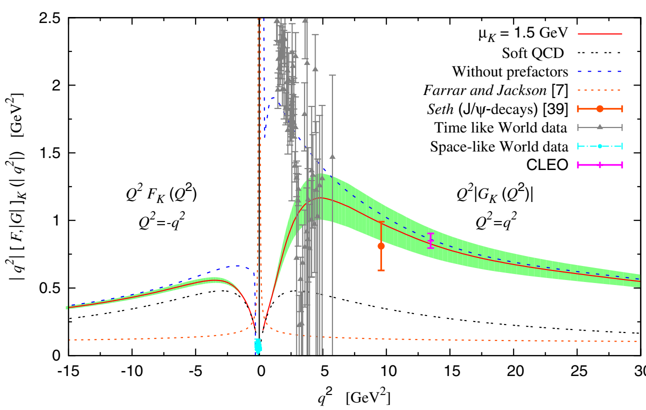

FIG. 6 (color online). The total scaled kaon form factors, denoted by the thick solid (red) lines; the soft form factors $\{F, G\}_K^{\text{soft}}$, denoted by the double-dot (black) lines; and the asymptotic QCD result [7], denoted by the short-dashed (orange) lines. The shaded area is our estimated theoretical error. The experimental data taken from [2,3] and the phenomenological result [45] is shown for comparison.

Several comments are now in order:

(i) The width of our error bar is large enough to completely subsume effects due to further inclusion of higher twists (e.g., twist-5 and twist-6), subleading Fock states and higher helicity components whose contributions should be tiny, not exceeding even 1%.

(ii) Our leading order (in α_s) scattering kernels are apparently gauge dependent arising from the contributions of the single hard gluon propagator. However, in the context of the $\pi\gamma^* \to \gamma$ transition form factor, it can be shown through a systematic order by order calculation using k_T factorization that there is indeed a cancellation of the gauge dependences between the quark-level diagrams of the hard kernel and the effective diagrams of the pion wave function [41], so that the net result turns out to be gauge invariant to all orders. It is, thus, believed that the same technique can be straightforwardly extended to other hadronic elastic and transition form factors, including the present context of the kaon form factors, at least up to the level of next-to-leading-order (in α_s) corrections.

(iii) The factorized hard form factors further suffer from renormalization/factorization scale dependent ambiguities that typically emerge from the truncation of the perturbative series and would be absent if we were able to obtain an all-order result in the QCD coupling α_s. To minimize the scale dependence in our present investigation, we adhere to a fixed prescription with the scales set to the momentum transfer Q [37,41], as mentioned previously in the context of the Sudakov factor. In this way, we hope to improve the reliability and self-consistency of the perturbative prediction and reduce the influence from higher-order corrections.

(iv) Nevertheless, a naive estimation of the twist-2 next-to-leading order (in α_s) contributions to the kaon form factors, using available next-to-leading-order radiative corrections for the pion form factor in asymptotic QCD, can be roughly expressed as [42]

$$Q^2\{F, G\}_K^{\rm NLO} \approx (0.903\ {\rm GeV}^2)\alpha_s^2(Q^2)\frac{f_K^2}{f_\pi^2}, \quad (39)$$

which yields a rather nominal contribution $\sim$20%–30% that is roughly of the same order of magnitude as the twist-4 contributions obtained in our analysis. It is to be noted that the above estimation is based on the usual collinear factorization approach [42] which do not take transverse degrees of freedom into account. The inclusion of the k_T dependence of the kernel may further reduce the magnitude of the next-to-leading order radiative corrections, as was shown in the cases of the pion [43] and the nucleon [44] form factors. It goes without saying that a full systematic next-to-leading order calculation (including the subleading twist-3) within the k_T-factorization scheme, which is missing until now, would be indispensable in resolving this issue about the definitive magnitude of the subleading corrections.

On the experimental side, as seen in Fig. 6, currently the spacelike region is completely devoid of data points at Q^2 values higher than $\sim$0.2 GeV^2. This makes it difficult, if not impossible, to compare such low-energy data with our predictions based on a pQCD approach which becomes unreliable and diverges rapidly in the vicinity of the Landau pole $\Lambda_{\rm QCD} \approx 0.2$ GeV. For the timelike region, there existed some older kaon data at relatively higher energies but with very poor statistics [2]. For such measurements the data above $Q^2 > 4.7$ GeV^2 had either upper limits or errors $\geq 50\%$. However, the recent CLEO measurements [3] at $Q^2 = 13.48$ GeV^2, apparently with a very small error bar of $\pm 15\%$, can provide first possible opportunity to critically test theoretical predictions, although they do not shed light on the variation with Q^2, which is a distinguishing feature of our result. Clearly, not only the moderate-energy timelike data seems reasonably reconciled, at higher energies both the CLEO result and the recent phenomenological prediction from J/ψ decays: $M_{J/\psi}^2|G_K(M_{J/\psi}^2 = 9.59\ {\rm GeV}^2)| = 0.81 \pm 0.18$ GeV^2 [45] lie within reasonable range of our prediction for the total timelike form factor. This is surprisingly consistent with the pion form factor results presented in [23], also obtained within the light cone k_T-factorization approach, that agreed well with most of the available moderate-energy data (with statistics far better than the kaon data), including the CLEO data and a similar phenomenological prediction [46] based on J/ψ decay analysis. Note that in the analysis [23], the central value of the twist-3 chiral parameter μ_π was also taken to be 1.5 GeV.

To this end, we consider the pion to kaon form factor ratios. In Fig. 7, using the central result for the pion form factors from [23] (with $\mu_\pi = 1.5$ GeV) we plot its variation with Q^2. The theoretical errors of the present work and [23] are added in quadrature to obtain the error band as

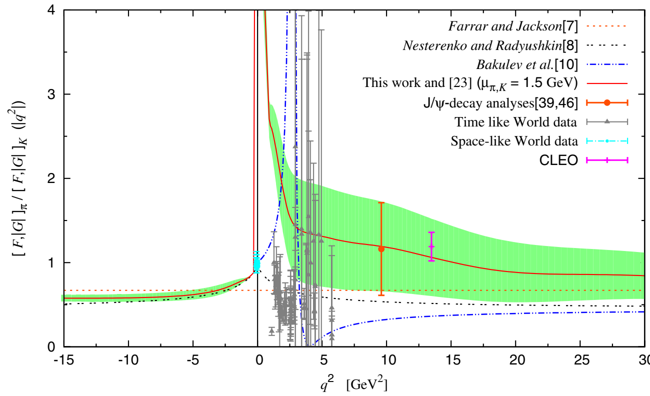

FIG. 7 (color online). Variation of the ratio of the pion and kaon timelike form factors with Q^2 in different approaches. The pion and kaon experimental data are taken from [1–4].

shown in the figure. The large error should not come as a surprise as the errors of both the pion and kaon factors are large. We now compare this result with other theoretical predictions and available experimental data. Note that the standard asymptotic pQCD result of Farrar and Jackson [7] yields a Q^2 independent ratio,

$$\left|\frac{\{F, G\}_\pi^{\rm asy}(Q^2)}{\{F, G\}_K^{\rm asy}(Q^2)}\right| = \frac{f_\pi^2}{f_K^2} = 0.67. \quad (40)$$

Clearly, the central value of our timelike ratio deviates appreciably from the asymptotic value at low and moderate Q^2, but however, it gradually approaches the asymptotic value at large Q^2, and so does the spacelike ratio. While our prediction fails to agree with the very low-energy timelike data points [2], showing the limitations of pQCD at such low-Q^2 values, the higher $Q^2 \sim 4$ GeV^2 data points can somewhat be accommodated within our error bars. At the same time, our timelike ratio at $Q^2 = 13.48$ GeV^2, i.e.,

$$\left|\frac{G_\pi(13.48\ {\rm GeV}^2)}{G_K(13.48\ {\rm GeV}^2)}\right| = 1.06 \pm 0.46, \quad (41)$$

is surprisingly close to the CLEO value: 1.19 ± 0.17 at $Q^2 = 13.48$ GeV^2 [3], and the result obtained by taking the ratio of the phenomenologically estimated timelike pion form factor [46] and the timelike kaon form factor [45], with the respective errors again added in quadrature

$$\left|\frac{G_\pi(M_{J/\psi}^2 = 9.48\ {\rm GeV}^2)}{G_K(M_{J/\psi}^2 = 9.48\ {\rm GeV}^2)}\right| = 1.16 \pm 0.55. \quad (42)$$

It is also noteworthy mentioning that the recent analyses [47,48] based on relativistic quark models up to $-q^2 \sim 10$ GeV^2, yielded the ratio of the form factors quite similar to what we obtain in the spacelike region. Finally, in Fig. 7, we compare our result, evidently working better towards large-Q^2 values, with the soft QCD results obtained from

QCD sum rules, which are instead known to yield reliable predictions at low- and moderate-Q^2 values. For example, the plot corresponding to the LD-model result [8] not only agrees well with the very low-Q^2 spacelike data (not resolved in the figure), but also with the low-energy timelike data when naively used in the timelike region. While, the analytically continued timelike LD-model result [see, Eq. (28)] [10] at low energies yields a plot very different from that of [8], but toward larger-Q^2 values both yield very similar predictions. Nevertheless, the QCD sum rules results significantly differ from the CLEO result and the one obtained from the phenomenological J/ψ decay analysis. It is to be noted that in spite of the additional inclusion of the hard contributions, our spacelike ratio of the total form factors does not differ significantly from that of [8], except at the very low $Q^2 \sim 0.2$ GeV^2 below which our result rapidly blows up.

To sum up, in this paper we tried to systematically study the higher twist effects, namely, the twist-3 and twist-4 corrections to the standard twist-2 pQCD charged kaon form factors by adopting minimal model dependence arising from the inclusion of (a) the transverse degrees of freedom in the kaon wave functions/distribution amplitudes, and (b) the nonfactorizable soft QCD corrections *via* local duality. The work presented here extends and completes the analyses of the previous work [17,23]. Assuming the validity of the k_T-factorization ansatz through the explicit transverse momentum dependence of the scattering kernel, we showed a nontrivial twist-3 contribution in the 2-particle sector which along with the large soft QCD corrections turn out to be the real hallmark of the "TMD-modified pQCD + soft QCD" approach to determine the space- and timelike kaon form factors. Other corrections, such as the 2-particle twist-4, were explicitly shows to have minor contributions only. To this end, the available moderate-energy experimental kaon data seems to be reasonably reconciled with the range of our predictions. It is also reassuring that the same approach works equally well independently for the electromagnetic pion form factors, which adds confidence to the arguments used in obtaining our results. It may, therefore, be speculated why the factorized result works so well for both the pion and kaon form factors in obtaining estimates, at least in the correct "ballpark," in spite of factors like the resonances, hadronization, and other final state interaction, naively neglected in this approach, that may render the factorized pQCD result questionable at the presently probed phenomenological region. However, to draw definite conclusion it is invaluable to have more high precision intermediate energy data, rather than to base our conclusions on such poor quality data.

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APPENDIX

2-particle collinear distribution amplitudes

The 2-particle twist-3 collinear DAs for the charged kaon (say, K^-) are defined at the scale of $\mu_0 \approx 1$ GeV, in terms of the following nonlocal matrix elements [25–27]:

$$\langle 0|\bar{u}(z) i\gamma_5 s(-z)|K^-(P)\rangle = \mu_K \int_0^1 dx e^{i\xi(Pz)} \phi^p_{3;K}(x, \mu_0^2),$$

$$\langle 0|\bar{u}(z)\sigma_{\alpha\beta}\gamma_5 s(-z)|K^-(P)\rangle = -\frac{i}{3}\mu_K\left\{1 - \left(\frac{m_u + m_s}{m_K}\right)^2\right\}(P_\alpha z_\beta - P_\beta z_\alpha) \times \int_0^1 dx e^{i\xi(Pz)} \phi^\sigma_{3;K}(x, \mu_0^2) \quad \text{(A1)}$$

with $\mu_K = m_K^2/(m_u + m_s)$ and $\xi = 2x - 1$. Note that the gauge-link factors (Wilson line) in the matrix elements are to be implicitly understood. The normalization conditions for the above twist-3 DAs are given by

$$N^{p,\sigma}_{3;K} = \int_0^1 dx \phi^{p,\sigma}_{3;K}(x, \mu^2) = \frac{f_K}{2\sqrt{2N_c}}, \quad \text{(A2)}$$

which have the following asymptotic forms:

$$\phi^{p(\mathrm{as})}_{3;K}(x) = \frac{f_K}{2\sqrt{2N_c}}, \qquad \phi^{\sigma(\mathrm{as})}_{3;K}(x) = \frac{3f_K}{\sqrt{2N_c}} x(1-x). \quad \text{(A3)}$$

The explicit formulas for the nonasymptotic 2-particle twist-3 collinear DAs, expressed as a series expansion over conformal spins at next-to-leading order, are given by [26]

$$\begin{aligned}\phi^p_{3;K}(x,\mu^2) = {}& \phi^{p(\mathrm{as})}_{3;K}(x)\{1 + (30\eta_{3K}(\mu^2) - \tfrac{5}{2}\rho_K^2(\mu^2))C_2^{1/2}(\xi) \\ & + (-3\eta_{3K}(\mu^2)\omega_{3K}(\mu^2) - \tfrac{27}{20}\rho_K^2(\mu^2) \\ & - \tfrac{81}{10}\rho_K^2(\mu^2)a_2^K(\mu^2))C_4^{1/2}(\xi)\}, \\ \phi^\sigma_{3;K}(x,\mu^2) = {}& \phi^{\sigma(\mathrm{as})}_{3;K}(x)\{1 + (5\eta_{3K}(\mu^2) - \tfrac{1}{2}\eta_{3K}(\mu^2)\omega_{3K}(\mu^2) \\ & - \tfrac{7}{20}\rho_K^2(\mu^2) - \tfrac{3}{5}\rho_K^2(\mu^2)a_2^K(\mu^2))C_2^{3/2}(\xi)\}\end{aligned} \quad \text{(A4)}$$

with

$$\eta_{3K} = \frac{f_{3K}}{f_K}\frac{1}{\mu_K}; \qquad \rho_K = \frac{m_K}{\mu_K},$$

the nonperturbative parameters f_{3K} and ω_{3K} being defined through the following matrix elements of local twist-3 operators:

$$\langle 0|\bar{u}(0)\sigma_{\mu\nu}\gamma_5 g_s G_{\alpha\beta} s(0)|K^-(P)\rangle = if_{3K}(P_\alpha P_\mu g_{\nu\beta} - P_\alpha P_\nu g_{\mu\beta} - P_\beta P_\mu g_{\nu\alpha} + P_\beta P_\nu g_{\alpha\mu}),$$
$$\langle 0|\bar{u}(0)\sigma_{\mu\lambda}\gamma_5[iD_\beta, g_s G_{\alpha\lambda}]s(0) - \tfrac{3}{7}i\partial_\beta \bar{u}(0)\sigma_{\mu\lambda}\gamma_5 g_s G_{\alpha\lambda}s(0)|K^-(P)\rangle = \tfrac{3}{14}if_{3K}P_\alpha P_\beta P_\mu \omega_{3K} + \mathcal{O}(\text{higher twist}), \quad \text{(A5)}$$

where g_s is the strong coupling and $G_{\alpha\beta}$ is the gluon field tensor. To the leading order, the scale dependence of various twist-3 parameters are given by

$$\rho_K(\mu^2) = L^{\gamma^{(0)}_{3;s\bar{u}}/\beta_0}\rho_K(\mu_0^2); \qquad \gamma^{(0)}_{3;s\bar{u}} = 1,$$
$$\eta_{3K}(\mu^2) = L^{\gamma^{(0)}_{3;\eta}/\beta_0}\eta_{3K}(\mu_0^2); \qquad \gamma^{(0)}_{3;\eta} = \tfrac{4}{3}\mathcal{C}_F + \tfrac{1}{4}\mathcal{C}_A,$$
$$\omega_{3K}(\mu^2) = L^{\gamma^{(0)}_{3;\omega}/\beta_0}\omega_{3K}(\mu_0^2); \qquad \gamma^{(0)}_{3;\omega} = -\tfrac{7}{24}\mathcal{C}_F + \tfrac{7}{12}\mathcal{C}_A,$$
$$a_1^K(\mu^2) = L^{\gamma^{(0)}_1/\beta_0}a_1^K(\mu_0^2); \qquad \gamma^{(0)}_1 = \tfrac{2}{3}\mathcal{C}_F,$$
$$a_2^K(\mu^2) = L^{\gamma^{(0)}_2/\beta_0}a_2^K(\mu_0^2); \qquad \gamma^{(0)}_2 = \tfrac{25}{24}\mathcal{C}_F, \quad \text{(A6)}$$

where $L = \alpha_s(\mu^2)/\alpha_s(\mu_0^2)$, $\mathcal{C}_F = (N_c^2 - 1)/2N_c$, and $\mathcal{C}_A = N_c$. However, the strange quark being massive, there is operator mixing of the ones in Eq. (A6) with those of twist-2 operators, so that the resulting leading order renormalization group equations give the following scale dependences:

$$f_{3K}(\mu^2) = L^{55/36\beta_0}f_{3K}(\mu_0^2) + \tfrac{2}{19}(L^{1/\beta_0} - L^{55/36\beta_0}) \times [f_K m_s](\mu_0^2) + \tfrac{6}{65}(L^{55/36\beta_0} - L^{17/9\beta_0}) \times [f_K m_s a_1^K](\mu_0^2),$$
$$[f_{3K}\omega_{3K}](\mu^2) = L^{26/9\beta_0}[f_{3K}\omega_{3K}](\mu_0^2) + \tfrac{1}{170}(L^{1/\beta_0} - L^{26/9\beta_0})[f_K m_s](\mu_0^2) + \tfrac{1}{10}(L^{17/9\beta_0} - L^{26/9\beta_0})[f_K m_s a_1^K](\mu_0^2) + \tfrac{2}{15}(L^{43/(18\beta_0)} - L^{26/9\beta_0})[f_K m_s a_2^K](\mu_0^2). \quad \text{(A7)}$$

The 2-particle twist-4 collinear DAs modify the twist-2 axial matrix element and are given by

$$\langle 0|\bar{u}(z)\gamma_\mu\gamma_5 s(-z)|K^-(P)\rangle = iP_\mu\int_0^1 dx e^{i\xi(Pz)}\Big[\phi_{2;K}(x,\mu_0^2) + \frac{1}{4}m_K^2 z^2\mathbb{A}_{4;K}(x,\mu_0^2)\Big] + \frac{i}{2}f_K m_K^2\frac{1}{Pz}z_\mu\int_0^1 dx e^{i\xi(Pz)}\mathbb{B}_{4;K}(x,\mu_0^2), \quad \text{(A8)}$$

where $\mathbb{B}_{4;K} = g_{4;K} - \phi_{2;K}$, with the normalization conditions expressed as

$$N_{4;K}^{\mathbb{A},g} = \int_0^1 dx\{\mathbb{A}, g\}_{4;K}(x,\mu^2) = \frac{f_K}{2\sqrt{2N_c}}, \quad \text{(A9)}$$

and the asymptotic forms, namely,

$$\mathbb{A}^{(\text{as})}_{4;K}(x) = \frac{15f_K}{\sqrt{2N_c}}x^2(1-x)^2, \qquad g^{(\text{as})}_{4;K}(x) = \frac{f_K}{2\sqrt{2N_c}}. \quad \text{(A10)}$$

Next we display the explicit forms of the nonasymptotic twist-4 collinear DAs at next-to-leading order in conformal spin [26]:

$$\mathbb{A}_{4;K}(x,\mu^2) = \frac{3f_K}{\mathcal{N}_{\mathbb{A}}\sqrt{2N_c}}x(1-x)\Big\{\frac{16}{15} + \frac{24}{35}a_2^K(\mu^2) + 20\eta_{3K}(\mu^2) + \frac{20}{9}\eta_{4K}(\mu^2) + \Big(-\frac{1}{15} + \frac{1}{16} - \frac{7}{27}\eta_{3K}(\mu^2)\omega_{3K}(\mu^2) - \frac{10}{27}\eta_{4K}(\mu^2)\Big)C_2^{3/2}(\xi) + \Big(-\frac{11}{210}a_2^K(\mu^2) - \frac{4}{135}\eta_{3K}(\mu^2)\omega_{3K}(\mu^2)\Big)C_4^{3/2}(\xi)\Big\} + \frac{f_K}{2\mathcal{N}_{\mathbb{A}}\sqrt{2N_c}}\Big(-\frac{18}{5}a_2^K(\mu^2) + 21\eta_{4K}(\mu^2)\omega_{4K}(\mu^2)\Big) \times\{2x^3(10 - 15x + 6x^2)\ln x + 2\bar{x}^3(10 - 15\bar{x} + 6\bar{x}^2)\ln\bar{x} + x\bar{x}(2 + 13x\bar{x})\},$$
$$g_{4;K}(x,\mu^2) = \frac{f_K}{2\sqrt{2N_c}}\Big\{1 + \Big(1 + \frac{18}{7}a_2^K(\mu^2) + 60\eta_{3K}(\mu^2) + \frac{20}{3}\eta_{4K}(\mu^2)\Big)C_2^{1/2}(\xi) + \Big(-\frac{9}{28}a_2^K(\mu^2) - 6\eta_{3K}(\mu^2)\omega_{3K}(\mu^2)\Big)C_4^{1/2}(\xi)\Big\}, \quad \text{(A11)}$$

where $\bar{x} = 1 - x$, and in the notation of [25], $\delta^2 \equiv m_K^2\eta_{4K}$ and $\epsilon \equiv 21/8\omega_{4K}$. Note that the additional factor $\mathcal{N}_{\mathbb{A}} \approx 3.5$ in the denominator of $\mathbb{A}_{4;K}$ is a contrast to the expression given in [26], which is introduced to normalize the DA. The nonperturbative parameters η_{4K} and ω_{4K} are defined through the following matrix elements of local twist-4 operators:

$$\langle 0|\bar{u}(0)\gamma_\alpha ig_s\tilde{G}_{\mu\nu}s(0)|K^-(P)\rangle = -\tfrac{1}{3}f_K m_K^2 \eta_{4K}(P_\mu g_{\nu\alpha} - P_\nu g_{\mu\alpha}),$$
$$\langle 0|\bar{u}(0)[iD_\mu, ig_s\tilde{G}_{\nu\lambda}]\gamma_\lambda s(0) - \tfrac{4}{9}i\partial_\mu \bar{u}(0)ig_s\tilde{G}_{\nu\lambda}\gamma_\lambda s(0)|K^-(P)\rangle = f_K m_K^2 \eta_{4K}\omega_{4K}(P_\mu P_\nu - \tfrac{1}{4}m_K^2 g_{\mu\nu}) + \mathcal{O}(\text{twist } 5), \tag{A12}$$

where $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}G^{\rho\sigma}$ is the dual gluon field tensor. Taking into account the mixing with the operators of lower twists, the leading order (in QCD coupling) renormalization group evolution of the twist-4 parameters are

$$\eta_{4K}(\mu^2) = L^{\gamma_{4;\eta}^{(0)}/\beta_0}\eta_{4K}(\mu_0^2) + \tfrac{1}{8}(1 - L^{\gamma_{4;\eta}^{(0)}/\beta_0}); \qquad \gamma_{4;\eta}^{(0)} = \tfrac{2}{3}\mathcal{C}_F,$$
$$[\eta_{4K}\omega_{4K}](\mu^2) = L^{\gamma_{4;\eta\omega}^{(0)}/\beta_0}[\eta_{4K}\omega_{4K}](\mu_0^2); \qquad \gamma_{4;\eta\omega}^{(0)} = \tfrac{5}{6}\mathcal{C}_A. \tag{A13}$$

Finally, we present the various Gegenbauer polynomials used in the above formulas:

$$C_0^{1/2}(\xi) = 1, \qquad C_1^{1/2}(\xi) = \xi, \qquad C_2^{1/2}(\xi) = \tfrac{1}{2}(3\xi^2 - 1), \qquad C_3^{1/2}(\xi) = \tfrac{1}{2}\xi(5\xi^2 - 3),$$
$$C_4^{1/2}(\xi) = \tfrac{1}{8}(35\xi^4 - 30\xi^2 + 3), \qquad C_0^{3/2}(\xi) = 1, \qquad C_1^{3/2}(\xi) = 3\xi,$$
$$C_2^{3/2}(\xi) = \tfrac{3}{2}(5\xi^2 - 1), \qquad C_3^{3/2}(\xi) = \tfrac{5}{2}\xi(7\xi^2 - 3). \tag{A14}$$

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