Flavor violating transitions of charged leptons from a seesaw mechanism of dimension seven

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A mechanism has been suggested recently to generate the neutrino mass out of a dimension-seven operator. This is expected to relieve the tension between the occurrence of a tiny neutrino mass and the observability of other physics effects beyond it. Such a mechanism would inevitably entail lepton flavor violating effects. We study in this work the radiative and purely leptonic transitions of the light charged leptons. In so doing we make a systematic analysis of the flavor structure by providing a convenient parametrization of the mass matrices in terms of independent physical parameters and diagonalizing them explicitly. We illustrate our numerical results by sampling over two *CP* phases and one Yukawa coupling which are the essential parameters in addition to the heavy lepton mass. We find that with the stringent constraints coming from the muon decays and the muon-electron conversion in nuclei taken into account the decays of the tau lepton are severely suppressed in the majority of parameter space. There exist, however, small regions in which some tau decays can reach a level that is about 2 orders of magnitude below their current bounds.

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I. INTRODUCTION

The tiny neutrino mass and significant lepton mixing can be incorporated in the three canonical seesaw mechanisms [1–3]. From the point of view of effective field theories they correspond to the three possible realizations at the tree level [4] of the unique dimension-five operator that induces a neutrino mass [5]. The tininess of the neutrino mass is generally attributed to the existence of very heavy new particles or very small couplings between the new particles and those that we already know of. In such a circumstance it is usually hard to detect other effects beyond the neutrino mass.

The above tension between the occurrence of a tiny neutrino mass and the testability of other physical phenomena can be alleviated by postponing the appearance of higher-dimensional operators relevant to the neutrino mass. There are two basic approaches to accomplish this. One can compose new fields so that the operators first occur at one [6], two [7,8], or even three loop order [9]. Since the loop effects provide additional suppressing factors besides a product of multiple couplings, one may gain in the couplings between the new and known particles. In the second approach, one introduces several new fields that belong to certain high dimensional representations of the gauge group. To induce an effective mass operator one has to go through several steps to connect those fields to the light lepton fields which are in low dimensional representations. In this multistep seesaw, a tiny neutrino mass can be induced without requiring all new particles to be very heavy or their couplings to light particles to be all small.

A realistic model in the second approach has been recently proposed in Ref. [10]. It introduces a vectorlike fermion triplet and a scalar four-plet so that the effective operator responsible for a neutrino mass first appears at dimension seven. The potential signatures of the new particles at the Tevatron and LHC have been studied with a special focus on the leptonic decays of the triply charged scalars. The idea of employing higher-dimensional representations has been further pursued in Ref. [11], where the neutrino mass is induced from a dimension nine operator. For a systematic effective field theory approach to neutrino mass operators of a dimension higher than five, we refer to Ref. [12].

Any mechanism for the generation of a neutrino mass and mixing is necessarily correlated with the physics of charged leptons. Before one can be sure that the physical processes involving new heavy particles at high energy colliders are relevant to neutrino physics, it is necessary to examine that the parameter regions assumed in the analysis of high energy processes are respected by precision low energy tests in the charged leptons. Particularly relevant in this respect are lepton flavor violating (LFV) decays of charged leptons and muon-electron (μe) conversion in nuclei that are severely suppressed in the standard model (SM). The experimental bounds on LFV decays of

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the muon are already very stringent [13,14], and the sensitivity to its radiative decay is expected to be upgraded by orders of magnitude in the MEG experiment within the next few years [15]. Significant progress has also been made in LFV decays of the tau lepton, thanks to the large data sample collected in recent years at the B factories [16–19]. Associated with the radiative decays of charged leptons are the precise measurements of or stringent bounds on their electromagnetic dipole moments [20–22]. For μe conversion in nuclei, the current most stringent constraints arise for titanium and gold [23,24]. PRISM/PRIME is expected to enhance their experimental sensitivity by several orders of magnitude in the future [25]. These bounds will provide strong constraints on the parameter space that will be useful in assessing the feasibility of detecting collider processes relevant to the neutrino mass generation. This motivates us to do a systematic investigation of the LFV transitions of the charged leptons in the model of high dimensional representations [10]. LFV decays have been previously studied in a similar fashion in various models of neutrino mass generation, like supersymmetric models [26], seesaw models [27–29], mirror fermions [30], little Higgs [31], and color-octet particles [32], to mention a few amongst many. Reader should consult Refs. [33,34] for a more complete list of literature on the subject. Similarly, the μe conversion in nuclei has also been widely considered in many scenarios of new physics beyond SM, such as supersymmetric models [35], seesaw models [36], littlest Higgs with T parity [37], Z' models [38], and so on. The formulas and elaborate discussion of μe conversion in nuclei have been given in Refs. [33,39,40].

In the next section we shall make a complete analysis on the flavor structure in the model of high dimensional representation. The mass matrices are parametrized in terms of physical parameters and then diagonalized approximately. The LFV decays, the contribution to dipole moments of charged leptons, and μe conversion in nuclei are then calculated in Sec. III. In Sec. IV we illustrate our numerical results by sampling over a few parameters that are potentially interesting. We discuss and conclude in the last section.

II. MODEL

To avoid the occurrence of the dimension-five neutrino mass operator at tree level, one should exclude the fields that carry the same quantum numbers as those in the three canonical seesaw models. Since the neutrinos are in the doublet representation, the easiest approach would be to arrange a Yukawa coupling that connects the neutrinos to a new scalar field and a new fermion field which differ in weak isospin by $1/2$. One way to accomplish this is to introduce a scalar multiplet with weak isospin $3/2$ and a fermion multiplet with weak isospin 1. This avoids the type 1 and type 2 seesaws automatically, while the type 3 is avoided by assigning a different hypercharge to the fermion multiplet. The is indeed the basic idea behind the model building in Ref. [10]. The new fields are denoted as

$$\Phi = \begin{pmatrix} \Phi_3 \\ \Phi_2 \\ \Phi_1 \\ \Phi_0 \end{pmatrix}(3/2, 3/2), \qquad \Sigma = \begin{pmatrix} \Sigma_2 \\ \Sigma_1 \\ \Sigma_0 \end{pmatrix}(1, 1), \tag{1}$$

where the numbers in the parentheses stand for the weak isospin I and hypercharge $Y/2$, respectively, and the subscripts to the fields indicate the electric charges in units of $|e|$. The fermion fields Σ are assumed to be vectorlike to avoid chiral anomaly. The relevant SM fields are the Higgs doublet and the lepton fields (with the subscripts L, R denoting chirality),

$$\begin{aligned} H &= \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}(1/2, 1/2), \\ F_L &= \begin{pmatrix} n_L \\ f_L \end{pmatrix}(1/2, -1/2), \qquad f_R(0, -1). \end{aligned} \tag{2}$$

A. Yukawa couplings and mass matrices

The neutrality in the hypercharge allows the following terms and their Hermitian conjugates: $F_L^* f_R H$, $F_L f_R \Phi$, $F_L \Sigma H^*$, $F_L \Sigma^* \Phi$, $\Sigma^* \Sigma$. It is possible to assign the lepton number, $L(f_R) = L(F_L) = 1$, $L(\Sigma) = -1$, $L(\Phi) = -2$. Then L is violated when Φ develops a vacuum expectation value (VEV). Now we write the terms in a form that respects $SU(2)_L$ and Lorentz symmetries. The first and last terms are trivial, $\bar{F}_L f_R H$, $\bar{\Sigma}\Sigma$, while the second one is forbidden. For the third term, H^* should be replaced by $\tilde{H} = i\sigma_2 H^*$ to preserve its identity as a doublet. To form a Lorentz scalar without complex conjugation out of two spinor fields, we can use the charge-conjugated fields. The required invariant form for the third term is, in terms of Clebsch-Gordan coefficients,

$$\begin{aligned} (\bar{F}_L^C \tilde{H} \Sigma)_0 &= \frac{1}{\sqrt{3}}(-\bar{f}_L^C H^- \Sigma_2 + \bar{n}_L^C H^{0*} \Sigma_0) \\ &\quad - \frac{1}{\sqrt{6}}(\bar{f}_L^C H^{0*} - \bar{n}_L^C H^-)\Sigma_1, \end{aligned} \tag{3}$$

where the subscript 0 on the left indicates its weak isospin of the product. We have used the notation for charge conjugation that $\psi_L^C = (\psi_L)^C$ and $\psi^C = \mathcal{C}\gamma^0\psi^*$ with $\mathcal{C} = i\gamma^0\gamma^2$.

For the fourth term in the list we note that, since the vector representation of $SU(2)$ is strictly real, $(\Sigma_0^*, \Sigma_1^*, \Sigma_2^*)$ is a vector when $(\Sigma_2, \Sigma_1, \Sigma_0)$ is. The invariant form is thus,

$$\begin{aligned} (\bar{\Sigma} F_L \Phi)_0 &= \frac{1}{2}(\bar{\Sigma}_0 n_L \Phi_0 - \bar{\Sigma}_2 f_L \Phi_3) + \frac{1}{2\sqrt{3}}(\bar{\Sigma}_2 n_L \Phi_2 \\ &\quad - \bar{\Sigma}_0 f_L \Phi_1) + \frac{1}{\sqrt{6}}(\bar{\Sigma}_1 f_L \Phi_2 - \bar{\Sigma}_1 n_L \Phi_1). \end{aligned} \tag{4}$$

Including the generation index in SM, the mass terms and Yukawa couplings are summarized as follows:

$$-\mathcal{L}_{\rm Yuk+mass} = m_\Sigma \bar{\Sigma}\Sigma + [y_{ij}\bar{F}_{Li} f_{Rj} H + x_j(\bar{F}^C_{Lj}\tilde{H}\Sigma)_0 + z_j(\bar{\Sigma}F_{Lj}\Phi)_0 + \text{H.c.}], \quad (5)$$

where y is a 3×3 complex matrix, x, z are each a three-component complex column vector, and m_Σ is real positive by definition.

When the electric neutral components of H and Φ develop a VEV,

$$\langle H^0\rangle = \frac{\rm v_2}{\sqrt{2}}, \qquad \langle \Phi_0\rangle = \frac{\rm v_4}{\sqrt{2}}, \quad (6)$$

the Yukawa terms will contribute to the masses of the neutral and singly charged fermions. For simplicity we shall assume in this work that both VEV's are real positive. The mass terms are written as

$$\mathcal{L}_{\rm mass} = \mathcal{L}_{\rm mass,2} + \mathcal{L}_{\rm mass,1} + \mathcal{L}_{\rm mass,0}, \quad (7)$$

with the number in the subscript denoting the electric charge. The field Σ_2 has only a bare mass:

$$-\mathcal{L}_{\rm mass,2} = m_\Sigma \bar{\Sigma}_2\Sigma_2, \quad (8)$$

while the fields of other charges also derive masses from Yukawa couplings so that mixing between the light and heavy particles can appear. For the singly charged fields, we have

$$-\mathcal{L}_{\rm mass,1} = \bar{\Psi}_{1L} M_1 \Psi_{1R} + \bar{\Psi}_{1R} M_1^\dagger \Psi_{1L}, \quad (9)$$

where the four-component column fields and the 4×4 mass matrix are

$$\Psi_{1R} = \begin{pmatrix} f_R \\ \Sigma^C_{1L}\end{pmatrix}, \quad \Psi_{1L} = \begin{pmatrix} f_L \\ \Sigma^C_{1R}\end{pmatrix}, \quad M_1 = \begin{pmatrix} \frac{\rm v_2}{\sqrt{2}} y & -\frac{\rm v_2}{2\sqrt{3}}x^* \\ 0 & m_\Sigma \end{pmatrix}. \quad (10)$$

Since the neutral fermions are generically Majorana, their mass terms are apparently more complicated. With the help of charge-conjugated fields, they are

$$-\mathcal{L}_{\rm mass,0} = \tfrac{1}{2}\bar{\Psi}_{0R} M_0 \Psi_{0L} + \tfrac{1}{2}\bar{\Psi}_{0L} M_0^\dagger \Psi_{0R}, \quad (11)$$

where the fields and mass matrix are

$$\Psi_{0R} = \begin{pmatrix} n^C_L \\ \Sigma^C_{0L} \\ \Sigma_{0R}\end{pmatrix}, \qquad \Psi_{0L} = \begin{pmatrix} n_L \\ \Sigma_{0L} \\ \Sigma^C_{0R}\end{pmatrix}, \quad M_0 = \begin{pmatrix} 0_{3\times 3} & \frac{\rm v_2}{\sqrt{6}}x & \frac{\rm v_4}{2\sqrt{2}}z \\ \frac{\rm v_2}{\sqrt{6}}x^T & 0 & m_\Sigma \\ \frac{\rm v_4}{2\sqrt{2}}z^T & m_\Sigma & 0 \end{pmatrix}. \quad (12)$$

These mass matrices will be analyzed and diagonalized in the later subsection.

B. Scalar potential

Before we diagonalize the mass matrices of leptons we discuss the scalar potential for completeness. Remember that the fields Φ and H have the quantum numbers $I = Y/2 = 3/2,\ 1/2$, respectively. The possible quadratic terms are, $H^\dagger H$ and $\Phi^\dagger\Phi$, while there can be no trilinear terms. For the quartic terms there are two sets of them: either Φ and $\Phi^\dagger$, and H and $H^\dagger$ come in pairs, or one $\Phi^\dagger$ is accompanied by three H. For the first set, the following ones are obvious:

$$(1a)\ (H^\dagger H)^2, \qquad (\Phi^\dagger\Phi)^2, \qquad (H^\dagger H)(\Phi^\dagger\Phi). \quad (13)$$

The pure H term is unique. This can also be understood as follows: with two identical H's of $I = 1/2$ one can only construct an $I = 1$ form that is symmetric in the two H's. (The $I = 0$ form vanishes identically.) This is also the case with two $\tilde{H}$'s. From the two $I = 1$ forms one can construct a unique $I = 0$ term (that is symmetric, though not necessarily, in the two forms). Indeed, it is easy to check that $H^\dagger\tau^a H H^\dagger \tau^a H = (H^\dagger H)^2$ where τ^a are the Pauli matrices. But this is not the case with the Φ field. With two identical Φ's of $I = 3/2$ we can construct two forms that are symmetric in them, one of $I = 3$ and the other of $I = 1$. This implies that there are two independent, invariant, pure Φ terms. Similarly with the half-H and half-Φ terms. The additional terms in the first set are thus

$$(1b)\,(\Phi^\dagger T^a_{3/2}\Phi)(\Phi^\dagger T^a_{3/2}\Phi), \quad (H^\dagger\tau^a H)(\Phi^\dagger T^a_{3/2}\Phi), \quad (14)$$

where $T^a_{3/2}$ stand for the generator matrices for $I = 3/2$.

For the second set of quartic terms involving three $\tilde{H}$'s and one Φ, we first combine three $\tilde{H}$'s into a form of $I = \frac{3}{2}$ which must be symmetric due to Bose symmetry. Then, out of this form and one Φ field, we form an $I = 0$ term:

$$(\Phi\tilde{H}\tilde{H}\tilde{H})_0 = \frac{1}{2}\Phi_0(H^{0*})^3 + \frac{\sqrt{3}}{2}\Phi_1 H^-(H^{0*})^2 + \frac{\sqrt{3}}{2}\Phi_2(H^-)^2 H^{0*} + \frac{1}{2}\Phi_3(H^-)^3. \quad (15)$$

The normalization in the above term looks a bit unusual due to the appearance of identical fields. The most general scalar potential is thus

$$\begin{aligned} V = &-\mu_H^2 H^\dagger H - \mu_\Phi^2 \Phi^\dagger\Phi + \lambda_H(H^\dagger H)^2 + \lambda_\Phi(\Phi^\dagger\Phi)^2 \\ &+ \lambda'_\Phi(\Phi^\dagger T^a_{3/2}\Phi)(\Phi^\dagger T^a_{3/2}\Phi) + \lambda H^\dagger H\Phi^\dagger\Phi \\ &+ \tfrac{1}{2}\lambda'(H^\dagger\tau^a H)(\Phi^\dagger T^a_{3/2}\Phi) + [\kappa(\Phi\tilde{H}\tilde{H}\tilde{H})_0 + \text{H.c.}], \end{aligned} \quad (16)$$

where all couplings except κ are real. We note in passing that the λ'_Φ term was missing in Ref. [10].

The VEVs of the scalar fields are determined by requiring the vanishing of the first derivatives and positive-definiteness of the matrix of the second derivatives of the potential. We can always choose, by a global $U(1)_Y$ transformation, one of the VEVs, say, v_2, to be real positive. Then one can see that $\kappa \mathrm{v}_4$ must be real, and the vanishing conditions become

$$\begin{aligned} &-\mu_H^2+\lambda_H \mathrm{v}_2^2+\tfrac{1}{2}\lambda|\mathrm{v}_4|^2+\tfrac{3}{8}\lambda'|\mathrm{v}_4|^2+\tfrac{3}{4}\mathrm{v}_2\kappa \mathrm{v}_4=0,\\ &-\tfrac{1}{2}\mu_\Phi^2|\mathrm{v}_4|^2+\tfrac{1}{2}\lambda_\Phi|\mathrm{v}_4|^4+\tfrac{9}{8}\lambda'_\Phi|\mathrm{v}_4|^4+\tfrac{1}{4}\lambda \mathrm{v}_2^2|\mathrm{v}_4|^2\\ &\quad+\tfrac{3}{16}\lambda' \mathrm{v}_2^2|\mathrm{v}_4|^2+\tfrac{1}{8}\kappa \mathrm{v}_4\mathrm{v}_2^3=0. \end{aligned} \tag{17}$$

Since $\mathrm{v}_4 \neq 0$ breaks the custodial symmetry, it is natural to assume $|\mathrm{v}_4| \ll \mathrm{v}_2$. Assuming further that the quartic couplings are perturbative, we have to good precision that

$$\mathrm{v}_2 \approx \sqrt{\frac{\mu_H^2}{\lambda_H}}, \qquad \frac{\mathrm{v}_4}{\mathrm{v}_2} \approx \frac{2\kappa^*\mu_H^2}{8\lambda_H\mu_\Phi^2-(4\lambda+3\lambda')\mu_H^2}. \tag{18}$$

Since the κ term breaks lepton number, it would be easy to attribute the tininess of v_4 to that of κ. With so many free parameters at hand it is no problem to guarantee that the above is the true vacuum. If κ vanishes, it would be necessary to fine-tune the parameters to get a tiny v_4, which we shall not pursue further.

C. Diagonalization of lepton mass matrices

We continue to assume for the sake of simplicity that both VEVs are real positive. To diagonalize the mass matrices, we first parameterize them without losing generality in terms of independent physical parameters by following the procedure advocated in Ref. [41]. We sketch below how this is done.

With three generations in SM there is always one massless neutral mode, which can be ν_1 [in normal hierarchy (NH)] or ν_3 [in inverted hierarchy (IH)] according to the oscillation data. (With $n_g \geq 3$ generations there are $n_g - 2$ massless modes while there is none with less generations.) We describe the case of NH in some detail and will record the result for IH later. By applying a judicious unitary transformation to n_{Lj}, we can convert the column vectors x and z into the standard form:

$$X=(0,0,x)^T, \qquad Z=(0,z,c_z)^T, \tag{19}$$

where x, z are real positive and c_z is generally complex. This fixes the phases of the two fields (named again as n_{L2} and n_{L3}) orthogonal to the massless mode (n_{L1}) but leaves the latter's phase free. To keep the partnership under $SU(2)_L$ between the neutral n_L and charged f_L fields generation by generation, the same transformation should be applied to f_{Lj} as well. This modifies the entries in y but does not alter its generality. We assume that this has been done already.

To reduce the y matrix to its minimal form, we proceed as follows. By a unitary transformation of f_{Rj}, we can cast y into the form:

$$y=\begin{pmatrix} r_1e^{i\alpha_1} & r_2e^{i\alpha_2} & r_3e^{i\alpha_3}\\ 0 & y_2 & c_2\\ 0 & 0 & y_3\end{pmatrix}, \tag{20}$$

where y_2 and y_3 are real positive and c_2 is complex. This fixes the phases of $f_{R3,R2}$ but leaves free that of f_{R1}. By rephasing further $f_{1L} \to e^{i\beta}f_{1L}$, which is augmented with $n_{1L} \to e^{i\beta}n_{1L}$ to preserve the $SU(2)_L$ partnership, the first row of y becomes effectively, $e^{-i\beta}(r_1e^{i\alpha_1}, r_2e^{i\alpha_2}, r_3e^{i\alpha_3})$. Now we choose $\beta=\alpha_2$ (or equally well, $\beta=\alpha_3$), which fixes the phase of f_{1L} and thus that of the massless n_{1L} as well, and then choose $\alpha_1=\beta$, which fixes the phase of f_{R1}. This leaves with us the final version of the y matrix:

$$Y=\begin{pmatrix} y_1 & y_4 & c_1\\ 0 & y_2 & c_2\\ 0 & 0 & y_3\end{pmatrix}, \tag{21}$$

where $y_{1,2,3,4}$ are real positive and $c_{1,2}$ are generally complex.

We have used up the degrees of freedom in defining fermion fields to reduce the number of parameters in the Yukawa sector to its minimum, i.e., in terms of independent physical parameters, without sacrificing generality. There are seven real positive parameters (m_Σ and x, $y_{1,2,3,4}$, z) and three complex ones ($c_{1,2,z}$). They will be traded for nine masses (of one doubly charged, four singly charged, and four neutral fermions), three CP phases, and a single independent mixing angle. It looks challenging at first sight for the model to accommodate the two large mixing angles measured in oscillation. But as we shall see later, all heavy fermions are nearly degenerate, which effectively saves parameters at our disposal. The mass matrices for the singly charged and neutral fermions are summarized as

$$\begin{aligned} M_1&=m_\Sigma\begin{pmatrix} \frac{\epsilon_2}{\sqrt{2}}Y & -\frac{\epsilon_2}{2\sqrt{3}}X\\ 0_{1\times3} & 1\end{pmatrix},\\ M_0&=m_\Sigma\begin{pmatrix} 0_{3\times3} & \frac{\epsilon_2}{\sqrt{6}}X & \frac{\epsilon_4}{2\sqrt{2}}Z\\ \frac{\epsilon_2}{\sqrt{6}}X^T & 0 & 1\\ \frac{\epsilon_4}{2\sqrt{2}}Z^T & 1 & 0\end{pmatrix}, \end{aligned} \tag{22}$$

where $\epsilon_2=\mathrm{v}_2/m_\Sigma$, $\epsilon_4=\mathrm{v}_4/m_\Sigma$ with ϵ_4 being necessarily tiny. The submatrices X, Y, Z for NH have been given in Eqs. (19) and (21), while for IH they are

$$X=\begin{pmatrix} x\\ 0\\ 0\end{pmatrix}, \qquad Z=\begin{pmatrix} c_z\\ z\\ 0\end{pmatrix}, \qquad Y=\begin{pmatrix} y_1 & 0 & 0\\ c_1 & y_2 & 0\\ c_2 & y_4 & y_3\end{pmatrix}. \tag{23}$$

Now we diagonalize the mass matrices M_0 and M_1 perturbatively exploiting the hierarchies in parameters. We describe how this is done for the NH case and will indicate how to obtain the analogous results for the IH case. To save writing, we put a prime to all parameters in the standardized Yukawa matrices shown in Eqs. (19), (21), and (23), and reserve the unprimed parameters to those that have been multiplied by a factor of $\epsilon_{2,4}$; namely

$$x = \frac{\epsilon_2}{\sqrt{6}} x', \qquad (z, c_z) = \frac{\epsilon_4}{2\sqrt{2}} (z', c_z'),$$
$$(y_j, c_i) = \frac{\epsilon_2}{\sqrt{2}} (y_j', c_i'). \tag{24}$$

The symmetric complex matrix M_0 is diagonalized to real nonnegative eigenvalues by a unitary matrix U:

$$U^T M_0 U = \mathrm{diag}(m_1, m_2, m_3, m_4, m_5), \tag{25}$$

where $m_1 = 0$ for NH while $m_3 = 0$ for IH. U is composed of the five column vectors $u(j)$ corresponding to the eigenvalues m_j, $U = (u(1), \ldots, u(5))$. Since ϵ_4 is tiny, we can solve the problem perturbatively in the z, c_z parameters (collectively denoted as ϵ) while keeping exact dependence on x. After some work, we obtain the eigenvalues for NH:

$$\begin{aligned}
m_1 &= 0,\\
\frac{m_2}{m_\Sigma} &= s_0[\sqrt{z^2 + c_0^2|c_z|^2} - c_0|c_z|] + O(\epsilon^3),\\
\frac{m_3}{m_\Sigma} &= s_0[\sqrt{z^2 + c_0^2|c_z|^2} + c_0|c_z|] + O(\epsilon^3),\\
\frac{m_4}{m_\Sigma} &= \sqrt{1 + x^2} - |c_z| c_0 s_0 + \frac{1}{2} c_0 [c_0^2 z^2\\
&\quad + (1 - 3c_0^2 s_0^2)|c_z|^2] + O(\epsilon^3),\\
\frac{m_5}{m_\Sigma} &= \sqrt{1 + x^2} + |c_z| c_0 s_0 + \frac{1}{2} c_0 [c_0^2 z^2\\
&\quad + (1 - 3c_0^2 s_0^2)|c_z|^2] + O(\epsilon^3),
\end{aligned} \tag{26}$$

corresponding to the light eigenvectors

$$\begin{aligned}
u(1) &= (1, 0, 0, 0, 0)^T,\\
u(2) &= e^{-i\beta_+} \begin{pmatrix} 0 \\ c_m \\ s_m(-c_0 e^{-i\alpha_z}) \\ s_m c_0\left(-c_0\sqrt{z^2 + c_0^2|c_z|^2} - s_0^2|c_z|\right) \\ s_m s_0 e^{-i\alpha_z} \end{pmatrix}\\
&\quad + O(\epsilon^2),\\
u(3) &= e^{-i\beta_-} \begin{pmatrix} 0 \\ s_m \\ c_m c_0 e^{-i\alpha_z} \\ c_m c_0\left(-c_0\sqrt{z^2 + c_0^2|c_z|^2} + s_0^2|c_z|\right) \\ c_m(-s_0 e^{-i\alpha_z}) \end{pmatrix}\\
&\quad + O(\epsilon^2),
\end{aligned} \tag{27}$$

and the heavy ones

$$\begin{aligned}
u(4) &= \frac{e^{-i\beta_-}}{\sqrt{2}} \begin{pmatrix} 0 \\ c_0^2 z \\ -e^{-i\alpha_z}[s_0 - \frac{1}{2} c_0^2(2c_0^2 - s_0^2)|c_z|] \\ [1 + \frac{1}{2} c_0^2 s_0 |c_z|] \\ -e^{-i\alpha_z}[c_0 - \frac{1}{2} c_0 s_0 (2s_0^2 - c_0^2)|c_z|] \end{pmatrix}\\
&\quad + O(\epsilon^2),\\
u(5) &= \frac{e^{-i\beta_+}}{\sqrt{2}} \begin{pmatrix} 0 \\ c_0^2 z \\ e^{-i\alpha_z}[s_0 + \frac{1}{2} c_0^2(2c_0^2 - s_0^2)|c_z|] \\ [1 - \frac{1}{2} c_0^2 s_0 |c_z|] \\ e^{-i\alpha_z}[c_0 + \frac{1}{2} c_0 s_0 (2s_0^2 - c_0^2)|c_z|] \end{pmatrix}\\
&\quad + O(\epsilon^2).
\end{aligned} \tag{28}$$

Here we have parametrized $c_z = |c_z| e^{i\alpha_z}$ and $e^{i(\alpha_z + 2\beta_\pm)} = \pm 1$. And the triangular functions are defined as follows:

$$\begin{aligned}
s_0 &= \frac{x}{\sqrt{1 + x^2}}, \qquad c_0 = \frac{1}{\sqrt{1 + x^2}};\\
s_m &= \sqrt{\frac{m_2}{m_2 + m_3}}, \qquad c_m = \sqrt{\frac{m_3}{m_2 + m_3}}.
\end{aligned} \tag{29}$$

One can tidy up the $O(\epsilon)$ terms in $u(2)$ and $u(3)$ in terms of mass ratios using

$$\begin{aligned}
&c_0[c_0\sqrt{z^2 + c_0^2|c_z|^2} + (-) s_0^2 |c_z|]\\
&\quad = \frac{1}{2s_0} \frac{1}{m_\Sigma} [m_{3(2)} + (c_0^2 - s_0^2) m_{2(3)}],
\end{aligned} \tag{30}$$

but no similar simplification occurs for the terms in $u(4)$ and $u(5)$.

We make a few comments on the above result. The light neutrinos gain a mass of order $m \sim x'(z'$ or $|c_z'|)\mathrm{v}_2 \mathrm{v}_4 / m_\Sigma$. For a scalar potential with $\mu_\Phi^2 \gg \mu_H^2$, corresponding to Φ particles much heavier than the SM Higgs, we have from the previous subsection that $\mathrm{v}_4 \sim |\kappa| \mathrm{v}_2^3 / \mu_\Phi^2$. This yields a neutrino mass that is triply suppressed by heavy scales as designed in Ref. [10]. Second, the two heavy neutrinos are almost degenerate with a splitting as small as the one in light neutrinos:

$$m_5 - m_4 \approx m_3 - m_2 \approx 2c_0 s_0 |c_z| m_\Sigma. \tag{31}$$

The significance of this at high energy colliders will be explored in the future work. Third, the result for the IH case whose Yukawa coupling matrices are parametrized as in Eq. (23) is obtained by first reshuffling the labels of the light solutions, $(m_1, m_2, m_3) \to (m_3, m_1, m_2)$ and $(u(1), u(2), u(3)) \to (u(3), u(1), u(2))$, in Eqs. (26) and (27), and then interchanging the first and third rows in all $u(j)$.

Now we diagonalize the mass matrix M_1 for the singly charged leptons by bi-unitary transformations, $U_L^\dagger M_1 U_R = \mathrm{diag}(m_e, m_\mu, m_\tau, m_\chi)$, where χ is the heavy lepton of charge $-|e|$. Since the matrix y is not related to new physics, its entries should be naturally small. At this stage we have no idea on how large the parameter x could be, and thus leave it free. We therefore diagonalize M_1 in two scenarios according to whether the x parameter (scenario A) or the y parameters (B) are treated perturbatively. As will be clear later, the x parameter is severely constrained by LFV transitions of the muon so that both scenarios serve as almost equally good approximations. We shall describe the diagonalization for the NH case and indicate at the end how to obtain the results for the IH case.

In scenario A we do perturbation in x. We first solve the zeroth order eigenvalue problems

$$\begin{aligned} &(u^L(e)u^L(\mu)u^L(\tau))^\dagger \mathcal{M}_L^2(u^L(e)u^L(\mu)u^L(\tau)) \\ &\quad = \mathrm{diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \\ &(u^R(e)u^R(\mu)u^R(\tau))^\dagger \mathcal{M}_R^2(u^R(e)u^R(\mu)u^R(\tau)) \\ &\quad = \mathrm{diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \end{aligned} \tag{32}$$

for the two 3×3 Hermitian matrices which share the same set of real positive eigenvalues:

$$\begin{aligned} \mathcal{M}_L^2 &= \begin{pmatrix} y_1^2 + y_4^2 + |c_1|^2 & y_2 y_4 + c_1 c_2^* & y_3 c_1 \\ \text{c.c.} & y_2^2 + |c_2|^2 & y_3 c_2 \\ \text{c.c.} & \text{c.c.} & y_3^2 \end{pmatrix}, \\ \mathcal{M}_R^2 &= \begin{pmatrix} y_1^2 & y_1 y_4 & y_1 c_1 \\ \text{c.c.} & y_2^2 + y_4^2 & y_2 c_2 + y_4 c_1 \\ \text{c.c.} & \text{c.c.} & |c_1|^2 + |c_2|^2 + y_3^2 \end{pmatrix}. \end{aligned} \tag{33}$$

We denote the light charged leptons by the beginning Greek letters α and β, and introduce the auxiliary variable and vectors:

$$\begin{aligned} \delta_\alpha &= (1 - \lambda_\alpha)^{-1}, \\ \xi^L &= -\sum_\beta \delta_\beta u_\tau^{L*}(\beta) u^L(\beta), \\ \xi^R &= -\sum_\beta \delta_\beta u_\tau^{R*}(\beta) u^R(\beta), \end{aligned} \tag{34}$$

where the subscript in $u_\alpha^{L,R}(\beta)$ denotes their α-th entry. The eigenvalues are found to be

$$\begin{aligned} \frac{m_\alpha^2}{m_\Sigma^2} &= \lambda_\alpha - \frac{x^2}{2}|u_\tau^L(\alpha)|^2 \delta_\alpha \lambda_\alpha + O(x^3) \\ &= \lambda_\alpha - \frac{x^2}{2} y_3^2 |u_\tau^R(\alpha)|^2 \delta_\alpha + O(x^3), \\ \alpha &= e, \mu, \tau \end{aligned} \tag{35}$$

$$\begin{aligned} \frac{m_\chi^2}{m_\Sigma^2} &= 1 + \frac{x^2}{2}\sum_\alpha |u_\tau^L(\alpha)|^2 \delta_\alpha + O(x^3) \\ &= 1 + \frac{x^2}{2} + \frac{x^2}{2}\sum_\alpha y_3^2 |u_\tau^R(\alpha)|^2 \delta_\alpha + O(x^3), \end{aligned} \tag{36}$$

where the two expressions from diagonalization are equivalent using $y_3^2|u_\tau^R(\alpha)|^2 = \lambda_\alpha |u_\tau^L(\alpha)|^2$, and the unitary matrices exact to $O(x)$ are

$$U_L = \begin{pmatrix} u^L(e) & u^L(\mu) & u^L(\tau) & \frac{x}{\sqrt{2}}\xi^L \\ \frac{x}{\sqrt{2}}\delta_e u_\tau^L(e) & \frac{x}{\sqrt{2}}\delta_\mu u_\tau^L(\mu) & \frac{x}{\sqrt{2}}\delta_\tau u_\tau^L(\tau) & 1 \end{pmatrix} \times \mathrm{diag}(1, 1, 1, p_L), \tag{37}$$

$$U_R = \begin{pmatrix} u^R(e) & u^R(\mu) & u^R(\tau) & \frac{xy_3}{\sqrt{2}}\xi^R \\ \frac{xy_3}{\sqrt{2}}\delta_e u_\tau^R(e) & \frac{xy_3}{\sqrt{2}}\delta_\mu u_\tau^R(\mu) & \frac{xy_3}{\sqrt{2}}\delta_\tau u_\tau^R(\tau) & 1 \end{pmatrix} \times \mathrm{diag}(1, 1, 1, p_R). \tag{38}$$

In the above, the $O(x)$ phases $p_{L,R}$ are not fixed completely, but $p_R p_L^*$ is fixed by requiring that the eigenvalue, m_χ, in $U_L^\dagger M_1 U_R$ be indeed real positive. The same arbitrariness also occurs in $u^{L,R}$, but the number of physical phases in $U_{L,R}$ is restricted to two by the phases appearing in M_1. Considering $\lambda_\alpha \ll 1$ in practice, we have to good precision, $\delta_\alpha = 1 + O(\lambda_\alpha)$, so that $\xi_\gamma^{L,R} = -\delta_{\tau\gamma} + O(\lambda_\alpha)$ from unitarity of u^L and u^R, which simplifies things a bit.

In scenario B we instead treat the parameters (y_j, c_i) (denoted collectively as η) perturbatively. By solving first the eigenvalue problems,

$$\begin{aligned} &(u^L(e)u^L(\mu)u^L(\tau))^\dagger \bar{\mathcal{M}}_L^2(u^L(e)u^L(\mu)u^L(\tau)) \\ &\quad = \mathrm{diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \\ &(u^R(e)u^R(\mu)u^R(\tau))^\dagger \bar{\mathcal{M}}_R^2(u^R(e)u^R(\mu)u^R(\tau)) \\ &\quad = \mathrm{diag}(\lambda_e, \lambda_\mu, \lambda_\tau), \end{aligned} \tag{39}$$

where

$$\bar{\mathcal{M}}_L^2 = \mathcal{M}_L^2|_{y_3 \to c_+ y_3}, \qquad \bar{\mathcal{M}}_R^2 = \mathcal{M}_R^2|_{y_3 \to c_+ y_3}, \tag{40}$$

we find the eigenvalues

$$\frac{m_\alpha^2}{m_\Sigma^2} = \lambda_\alpha + O(\eta^4), \qquad \alpha = e, \mu, \tau, \tag{41}$$

$$\frac{m_\chi^2}{m_\Sigma^2} = 1 + \frac{1}{2}x^2 + O(\eta^2), \tag{42}$$

and the diagonalization unitary matrices ($\zeta = y_3 s_+ c_+$):

$$U_L = \begin{pmatrix} u_e^L(e) & u_e^L(\mu) & u_e^L(\tau) & 0 \\ u_\mu^L(e) & u_\mu^L(\mu) & u_\mu^L(\tau) & 0 \\ c_+ u_\tau^L(e) & c_+ u_\tau^L(\mu) & c_+ u_\tau^L(\tau) & -s_+ \\ s_+ u_\tau^L(e) & s_+ u_\tau^L(\mu) & s_+ u_\tau^L(\tau) & c_+ \end{pmatrix} + O(\eta^2), \quad (43)$$

$$U_R = \begin{pmatrix} u_e^R(e) & u_e^R(\mu) & u_e^R(\tau) & 0 \\ u_\mu^R(e) & u_\mu^R(\mu) & u_\mu^R(\tau) & 0 \\ u_\tau^R(e) & u_\tau^R(\mu) & u_\tau^R(\tau) & -\zeta \\ \zeta u_\tau^R(e) & \zeta u_\tau^R(\mu) & \zeta u_\tau^R(\tau) & 1 \end{pmatrix} + O(\eta^2). \quad (44)$$

The triangular functions in this scenario are

$$s_+ = \frac{x/\sqrt{2}}{\sqrt{1+x^2/2}}, \qquad c_+ = \frac{1}{\sqrt{1+x^2/2}}. \quad (45)$$

When all parameters x and (y_i, c_i) are small in magnitude and treated on the same footing, both scenarios yield an identical result to the leading order. In both scenarios, the following mass relations among the heavy Σ particles hold:

$$\begin{aligned} m_{\Sigma_2} &= m_\Sigma, \\ m_\chi &= m_\Sigma\sqrt{1+x^2/2}, \\ m_{\nu_4} &\approx m_{\nu_5} = m_\Sigma\sqrt{1+x^2}. \end{aligned} \quad (46)$$

In our later numerical analysis we shall work with scenario B. The explicit results displayed above are for the NH case. For the IH case whose matrices are parametrized as in Eq. (23) and without changing the increasing order of the mass eigenvalues, $U_{L,R}$ are obtained from those for NH by interchanging the (1, 3) rows which are computed with the parameter interchanges $y_1 \leftrightarrow y_3$ and $c_1 \leftrightarrow c_2$ made.

D. Couplings of leptons

The gauge interactions of the leptons are

$$\mathcal{L}_g = g_2(j_W^{+\mu} W_\mu^+ + j_W^{-\mu} W_\mu^- + J_Z^\mu Z_\mu) + eJ_{\rm em}^\mu A_\mu. \quad (47)$$

The currents are written first in terms of weak eigenstates and then grouped into $\Psi_{1R,1L}$ and $\Psi_{0R,0L}$:

$$\begin{aligned} j_W^{+\mu} &= \bar\Psi_{0L} w_L^0 \gamma^\mu \Psi_{1L} + \bar\Psi_{0R} w_R^0 \gamma^\mu \Psi_{1R} + \bar\Psi_{1R} w_R^2 \gamma^\mu \Sigma_{2L}^C \\ &\quad + \bar\Psi_{1L} w_L^2 \gamma^\mu \Sigma_{2R}^C, \\ c_W J_Z^\mu &= \bar\Psi_{0L} z_L^0 \gamma^\mu \Psi_{0L} + \bar\Psi_{1L} z_L^1 \gamma^\mu \Psi_{1L} + \bar\Psi_{1R} z_R^1 \gamma^\mu \Psi_{1R} \\ &\quad - (1-2s_W^2)\bar\Sigma_2^C \gamma^\mu \Sigma_2^C, \\ J_{\rm em}^\mu &= -\bar\ell\gamma^\mu\ell - 2\bar\Sigma_2^C\gamma^\mu\Sigma_2^C, \end{aligned} \quad (48)$$

where ℓ stands for all four leptons of charge -1. The coupling matrices are

$$\begin{aligned} w_L^0 &= \begin{pmatrix} \frac{1}{\sqrt{2}} 1_3 & 0 \\ 0 & 0 \\ 0 & -1 \end{pmatrix}, \qquad w_R^0 = \begin{pmatrix} 0_3 & 0 \\ 0 & -1 \\ 0 & 0 \end{pmatrix}, \\ w_L^2 &= w_R^2 = \begin{pmatrix} 0_3 \\ -1 \end{pmatrix}, \qquad z_L^0 = \begin{pmatrix} \frac{1}{2} 1_3 & & \\ & -1 & \\ & & 1 \end{pmatrix}, \\ z_L^1 &= \begin{pmatrix} \left(-\frac{1}{2} + s_W^2\right) 1_3 & \\ & s_W^2 \end{pmatrix}, \qquad z_R^1 = s_W^2 1_4, \end{aligned} \quad (49)$$

with the usual notation $s_W = \sin\theta_W$ and $c_W = \cos\theta_W$. In terms of mass eigenstates, $\Psi_{0L} = U\nu_L$, $\Psi_{0R} = U^*\nu_R = U^*\nu_L^C$, $\Psi_{1L} = U_L\ell_L$, and $\Psi_{1R} = U_R\ell_R$, the weak currents finally become

$$\begin{aligned} j_W^{+\mu} &= \bar\nu_L \mathcal{W}_L^0 \gamma^\mu \ell_L + \bar\nu_R \mathcal{W}_R^0 \gamma^\mu \ell_R + \bar\ell_R \mathcal{W}_R^2 \gamma^\mu \Sigma_{2L}^C \\ &\quad + \bar\ell_L \mathcal{W}_L^2 \gamma^\mu \Sigma_{2R}^C, \\ c_W J_Z^\mu &= \bar\nu_L \mathcal{Z}_L^0 \gamma^\mu \nu_L + \bar\ell_L \mathcal{Z}_L^1 \gamma^\mu \ell_L + \bar\ell_R \mathcal{Z}_R^1 \gamma^\mu \ell_R \\ &\quad - (1-2s_W^2)\bar\Sigma_2^C \gamma^\mu \Sigma_2^C, \end{aligned} \quad (50)$$

where

$$\begin{aligned} &\mathcal{W}_L^0 = U^\dagger w_L^0 U_L, \qquad \mathcal{W}_R^0 = U^T w_R^0 U_R, \\ &\mathcal{W}_L^2 = U_L^\dagger w_L^2, \qquad \mathcal{W}_R^2 = U_R^\dagger w_R^2, \qquad \mathcal{Z}_L^0 = U^\dagger z_L^0 U, \\ &\mathcal{Z}_L^1 = U_L^\dagger z_L^1 U_L, \qquad \mathcal{Z}_R^1 = U_R^\dagger z_R^1 U_R = s_W^2 1_4. \end{aligned} \quad (51)$$

The upper-left 3×3 submatrix in $\sqrt{2}\mathcal{W}_L^{0\dagger}$ ($\sqrt{2}$ from our normalization convention for currents) corresponds to the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix in the seesaw limit, and will be called the effective PMNS matrix or $\bar V_{\rm PMNS}$. To help understand the procedure to be adopted in our later numerical analysis, we take a closer look at it. As both scenarios A and B are similar for a not large x parameter, we illustrate our discussion in the latter. The submatrix is

$$\bar V_{\rm PMNS} = U_{fL}^\dagger {\rm diag}(1, 1, c_+/c_0) U_n. \quad (52)$$

Here $U_{fL} = (u^L(e) u^L(\mu) u^L(\tau))$ is the unitary matrix that diagonalizes $\bar{\mathcal{M}}_L^2$, while U_n is the unitary matrix that diagonalizes the light neutrino mass in the seesaw limit upon choosing $e^{-i\beta_+} = e^{i\alpha_z/2}$ and $e^{-i\beta_-} = ie^{i\alpha_z/2}$. In other words, we have $V_{\rm PMNS} = U_{fL}^\dagger U_n$ in the seesaw limit, and $c_+/c_0 \approx 1 + x^2/4$ measures the small departure of $\bar V_{\rm PMNS}$ from unitarity when the mixing with the heavy particles is taken into account. As the unitarity has been checked at a precision not better than a percentage, it is safe if x assumes a value not larger than, say, 0.1. This will be fully respected in our numerical analysis. This result applies to scenario A as well where x is small by definition.

We briefly highlight some other features in gauge couplings relevant to LFV transitions of charged leptons. The light charged leptons have $O(z, c_z)$ suppressed couplings to heavy neutrinos in $\mathcal{W}_L^0$. The massless neutrino decouples from $\mathcal{W}_R^0$, while all light charged leptons are suppressed in it by a factor of xy_3. A similar situation occurs in the charged current involving singly and doubly charged leptons: the left-handed part of the light charged leptons is suppressed by x and the right-handed by xy_3. Finally, the neutral current of the singly charged leptons is dominantly flavor diagonal, with an $O(x)$ mixing between the light and heavy particles and an $O(x^2)$ mixing amongst the light leptons.

As most new gauge couplings between light and heavy particles are controlled by the x parameter, we shall consider its largest allowed value in numerical analysis. The spectrum of the light neutrinos then implies that the z parameters must be extremely small. We can therefore focus in the Yukawa sector on the x terms. Ignoring the tiny mixture between the doublet and four-plet scalars that is proportional to $\mathrm{v}_4/\mathrm{v}_2$, the terms relevant to our study are

$$\begin{aligned} -\mathcal{L}_{\rm Yuk} &\supset -\frac{x'h}{2\sqrt{3}}\bar{f}_{L3}\Sigma_1^C + \text{H.c.} \\ &= -\frac{x'h}{2\sqrt{3}}(U_L^*)_{3\alpha}(U_R)_{4\beta}\bar{\ell}_{L\alpha}\ell_{R\beta}h + \text{H.c.}, \end{aligned} \tag{53}$$

where the summation is over all singly charged leptons and h is the physical scalar in $H^0 = (h + iG^0)/\sqrt{2}$.

III. LEPTON FLAVOR VIOLATING TRANSITIONS

A. Radiative decays and electromagnetic dipole moments

A direct consequence of the neutrino mass and mixing mechanism in the last section is the lepton flavor violating transitions of the light charged leptons. We start with the radiative decay $\ell_\alpha \to \ell_\beta\gamma$, whose amplitude has the dipole form

$$\begin{aligned} \mathcal{A} &= \frac{\sqrt{2}eG_F}{(4\pi)^2}\bar{u}_\beta(p-q)(h_L P_L + h_R P_R) \\ &\quad \times i\sigma_{\mu\nu}u_\alpha(p)\epsilon^\mu(q)q^\nu, \end{aligned} \tag{54}$$

where p, $p-q$ are the momenta of the initial and final leptons, q, $\epsilon(q)$ are the photon's momentum and polarization vector, and $P_{L,R} = (1 \mp \gamma_5)/2$. All information on dynamics is stored in the form factors $h_{L,R}$. The decay width is

$$\Gamma(\ell_\alpha \to \ell_\beta\gamma) = \frac{m_\alpha^3 \alpha G_F^2}{2^9\pi^4}(|h_L|^2 + |h_R|^2), \tag{55}$$

where we have ignored the mass of ℓ_β in phase space.

For the model considered here, the form factors $h_{L,R}$ are contributed by the Feynman diagrams shown in Fig. 1. Figures 1(a)–1(c) involve the charged currents between

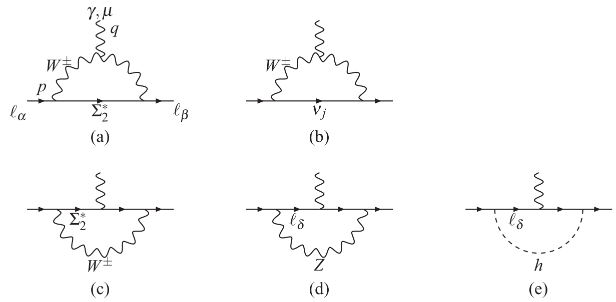

FIG. 1. Feynman diagrams for $\ell_\alpha \to \ell_\beta\gamma$.

the singly and doubly charged leptons, and between the neutral and singly charged leptons, respectively, while Figs. 1(d) and 1(e) originate from the flavor-changing neutral currents (FCNC) and physical Higgs exchange. For the gauge-boson mediated graphs we compute in the unitary gauge. This is simplest but caution must be exercised to cope with a technical point concerning constant terms, see the last paper in Ref. [30]. We have carefully done the Dirac algebra in d dimensions before the limit $d \to 4$ is taken, to avoid missing certain finite terms, and find consistent results with that work.

Ignoring again the terms suppressed by m_β and keeping terms up to the linear order in m_α, the form factors from each graph in Fig. 1 are

$$\begin{aligned} h_L(a) &= -2\mathcal{W}_{R\beta}^2[\mathcal{W}_{R\alpha}^{2*}m_\alpha\mathcal{F}(r_\Sigma) + \mathcal{W}_{L\alpha}^{2*}m_\Sigma\mathcal{G}(r_\Sigma)], \\ h_L(b) &= 2\mathcal{W}_{R,j\beta}^{0*}[\mathcal{W}_{R,j\alpha}^0 m_\alpha\mathcal{F}(r_j) + \mathcal{W}_{L,j\alpha}^0 m_j\mathcal{G}(r_j)], \\ h_L(c) &= 2\mathcal{W}_{R\beta}^2[\mathcal{W}_{R\alpha}^{2*}m_\alpha\mathcal{H}(r_\Sigma) + \mathcal{W}_{L\alpha}^{2*}m_\Sigma\mathcal{J}(r_\Sigma)], \\ h_L(d) &= \mathcal{Z}_{R,\beta\delta}^1[\mathcal{Z}_{R,\delta\alpha}^1 m_\alpha\mathcal{H}(s_\delta) + \mathcal{Z}_{L,\delta\alpha}^1 m_\delta\mathcal{J}(s_\delta)], \\ h_L(e) &= \frac{1}{\sqrt{2}G_F m_h^2}\frac{x'^2}{12}U_{R,4\beta}^*U_{L,3\delta}[U_{L,3\delta}^*U_{R,4\alpha}m_\alpha\mathcal{K}(t_\delta) \\ &\quad + U_{R,4\delta}^*U_{L,3\alpha}m_\delta\mathcal{L}(t_\delta)], \end{aligned} \tag{56}$$

where summation over the virtual lepton flavors is implied, and

$$\begin{aligned} h_R(a,b,c,d) &= h_L(a,b,c,d)|_{L\leftrightarrow R}, \\ h_R(e) &= h_L(e)|_{L\leftrightarrow R, 3\leftrightarrow 4}. \end{aligned} \tag{57}$$

We have denoted the ratios of the masses appearing in the loops as $r_\Sigma = m_\Sigma^2/m_W^2$, $r_j = m_j^2/m_W^2$, $s_\delta = m_\delta^2/m_Z^2$, $t_\delta = m_\delta^2/m_h^2$, where j and δ enumerate all virtual neutral and singly charged leptons, respectively. Some products of the coupling matrices in the above can be simplified using their explicit forms. For instance, the first term in $h_L(d)$ drops out since $\mathcal{Z}_R^1$ is diagonal; the matrices in the charged current involving the doubly charged lepton are $\mathcal{W}_{R(L)\alpha}^2 = -U_{R(L),4\alpha}^*$. And the loop functions are

$$\mathcal{F}(r)=\frac{1}{6(1-r)^4}[10-43r+78r^2-49r^3+4r^4+18r^3\ln r],$$
$$\mathcal{G}(r)=\frac{1}{(1-r)^3}[-4+15r-12r^2+r^3+6r^2\ln r],$$
$$\mathcal{H}(r)=\frac{1}{3(1-r)^4}[-8+38r-39r^2+14r^3-5r^4+18r^2\ln r],$$
$$\mathcal{J}(r)=\frac{2}{(1-r)^3}[4-3r-r^3+6r\ln r],$$
$$\mathcal{K}(r)=\frac{1}{12(1-r)^4}[2+3r-6r^2+r^3+6r\ln r],$$
$$\mathcal{L}(r)=\frac{1}{2(1-r)^3}[-3+4r-r^2-2\ln r]. \quad (58)$$

Related to the above radiative transition amplitudes are the anomalous magnetic moments and electric dipole moments of the singly charged light leptons. They are worked out to the linear order in the mass m_α of the considered lepton. The anomalous magnetic moment, defined as $a=(g-2)/2$, is

$$a(\ell_\alpha)=\frac{2\sqrt{2}G_F m_\alpha}{(4\pi)^2}[h(a)+h(b)+h(c)+h(d)+h(e)], \quad (59)$$

where

$$h(a)=-2[(|\mathcal{W}^2_{R\alpha}|^2+|\mathcal{W}^2_{L\alpha}|^2)m_\alpha\mathcal{F}(r_\Sigma) + \mathrm{Re}(\mathcal{W}^2_{R\alpha}\mathcal{W}^{2*}_{L\alpha})m_\Sigma\mathcal{G}(r_\Sigma)],$$
$$h(b)=2[(|\mathcal{W}^0_{R,j\alpha}|^2+|\mathcal{W}^0_{L,j\alpha}|^2)m_\alpha\mathcal{F}(r_j) + \mathrm{Re}(\mathcal{W}^{0*}_{R,j\alpha}\mathcal{W}^0_{L,j\alpha})m_j\mathcal{G}(r_j)],$$
$$h(c)=2[(|\mathcal{W}^2_{R\alpha}|^2+|\mathcal{W}^2_{L\alpha}|^2)m_\alpha\mathcal{H}(r_\Sigma) + \mathrm{Re}(\mathcal{W}^2_{R\alpha}\mathcal{W}^{2*}_{L\alpha})m_\Sigma\mathcal{J}(r_\Sigma)],$$
$$h(d)=(|\mathcal{Z}^1_{R,\alpha\delta}|^2+|\mathcal{Z}^1_{L,\alpha\delta}|^2)m_\alpha\mathcal{H}(s_\delta) + \mathrm{Re}(\mathcal{Z}^1_{R,\alpha\delta}\mathcal{Z}^1_{L,\delta\alpha})m_\delta\mathcal{J}(s_\delta),$$
$$h(e)=\frac{x'^2}{12\sqrt{2}G_F m_h^2}[(|U_{L,3\delta}U_{R,4\alpha}|^2+|U_{R,4\delta}U_{L,3\alpha}|^2)\times m_\alpha\mathcal{K}(t_\delta)+\mathrm{Re}(U^*_{R,4\alpha}U_{L,3\delta}U^*_{R,4\delta}U_{L,3\alpha})m_\delta\mathcal{L}(t_\delta)]. \quad (60)$$

Again some of the above can be simplified using explicit forms of the coupling matrices given in the last section.

The electric dipole moment of the fermion ψ is defined as $\mathcal{L}_{\rm edm}=-id/2\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi F^{\mu\nu}$, with $F^{\mu\nu}$ being the electromagnetic field tensor. It is evaluated to be

$$d(\ell_\alpha)=-\frac{2\sqrt{2}eG_F}{(4\pi)^2}[\bar{h}(a)+\bar{h}(b)+\bar{h}(c)+\bar{h}(d)+\bar{h}(e)], \quad (61)$$

where

$$\bar{h}(a)=-2\mathrm{Im}(\mathcal{W}^2_{R\alpha}\mathcal{W}^{2*}_{L\alpha})m_\Sigma\mathcal{G}(r_\Sigma),$$
$$\bar{h}(b)=2\mathrm{Im}(\mathcal{W}^{0*}_{R,j\alpha}\mathcal{W}^0_{L,j\alpha})m_j\mathcal{G}(r_j),$$
$$\bar{h}(c)=2\mathrm{Im}(\mathcal{W}^2_{R\alpha}\mathcal{W}^{2*}_{L\alpha})m_\Sigma\mathcal{J}(r_\Sigma),$$
$$\bar{h}(d)=\mathrm{Im}(\mathcal{Z}^1_{R,\alpha\delta}\mathcal{Z}^1_{L,\delta\alpha})m_\delta\mathcal{J}(s_\delta),$$
$$\bar{h}(e)=\frac{1}{\sqrt{2}G_F m_h^2}\frac{x'^2}{12}\mathrm{Im}(U^*_{R,4\alpha}U_{L,3\delta}U^*_{R,4\delta}U_{L,3\alpha})\times m_\delta\mathcal{L}(t_\delta). \quad (62)$$

Since $\mathcal{Z}^1_R$ is diagonal and $\mathcal{Z}_L$ is Hermitian, there is actually no contribution from the FCNC graph, $\bar{h}(d)=0$.

B. Purely leptonic transitions

Now we consider the purely leptonic transitions of the light charged leptons. These include the experimentally well-bounded decays $\ell_\delta\to\ell_\alpha\ell_\beta\bar{\ell}_\gamma$ and the muon-electron (μe) conversion in nuclei. The leading contributions in the model considered here arise from FCNC couplings of the Z boson. The Higgs exchange is suppressed by additional factors of x and a heavier Higgs mass, while we have verified that the photonic contribution is always subdominant. We do not consider LFV decays of the Z boson since they are experimentally much less constrained.

There are three types of decays corresponding to $\alpha=\beta=\gamma$, $\alpha=\gamma\neq\beta$, and $\alpha=\beta\neq\gamma$. Since the flavor-changing couplings carry a factor of x^2, only the transitions of the first two types are important while the third one is severely suppressed. We thus concentrate on the decays, $\ell_\alpha\to\ell_\beta\ell_\gamma\bar{\ell}_\beta$ with $\gamma=\beta$ or $\gamma\neq\beta$, whose leading terms come from diagrams shown in Fig. 2. Note that there is a relative minus sign between the two graphs and that for $\gamma=\beta$ one should attach a factor of $1/2$ in the total decay rate. Ignoring the final-state masses, the rate is given by

$$\frac{\Gamma(\ell_\alpha\to\ell_\beta\ell_\gamma\bar{\ell}_\beta)}{4\Gamma_0}=\frac{1}{1+\delta_{\beta\gamma}}[|\mathcal{Z}^1_{L,\gamma\alpha}\mathcal{Z}^1_{L,\beta\beta}|^2+|\mathcal{Z}^1_{L,\beta\alpha}\mathcal{Z}^1_{L,\gamma\beta}|^2 + 2\mathrm{Re}(\mathcal{Z}^1_{L,\gamma\alpha}\mathcal{Z}^1_{L,\beta\beta}\mathcal{Z}^{1*}_{L,\beta\alpha}\mathcal{Z}^{1*}_{L,\gamma\beta})+|\mathcal{Z}^1_{L,\gamma\alpha}\mathcal{Z}^1_{R,\beta\beta}|^2 + |\mathcal{Z}^1_{L,\beta\alpha}\mathcal{Z}^1_{R,\gamma\beta}|^2+(L\leftrightarrow R)], \quad (63)$$

with $\Gamma_0=G_F^2m_\alpha^5/(192\pi^3)$ being the decay rate for the dominant decay mode, $\ell_\alpha\to\nu_\alpha\ell_\beta\bar{\nu}_\beta$. Since $\mathcal{Z}^1_R$ is diagonal and $\alpha\neq\beta$, $\alpha\neq\gamma$, the term $(L\leftrightarrow R)$ actually drops out.

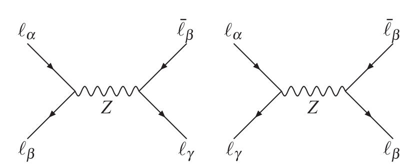

FIG. 2. Feynman diagrams for $\ell_\alpha\to\ell_\beta\ell_\gamma\bar{\ell}_\beta$.

A competitive process is the coherent μe conversion in nuclei, $\mu^- N \to e^- N$. It involves various atomic and nuclear effects in addition to the short-distance physics of lepton flavor violation. A comprehensive study has been given in Ref. [39] based on the method developed in Ref. [40], which improved over earlier efforts on various corrections [42,43] to the original calculations [44,45]. These corrections turn out to be particularly important for heavy nuclei.

The effective Lagrangian relevant to the coherent conversion via the leptonic FCNC couplings of the Z boson can be written as [39]

$$\mathcal{L}_{\mu e} = -\frac{G_F}{\sqrt{2}} \sum_{q=u,d,s} [(g_{LV(q)} \bar{e}\gamma^\mu P_L \mu + g_{RV(q)} \bar{e}\gamma^\mu P_R \mu) \times \bar{q}\gamma_\mu q + \text{H.c.}], \tag{64}$$

where $g_{LV(u)} = (2 - 16 s_W^2/3) Z^1_{L,e\mu}$, $g_{LV(d,s)} = (-2 + 8 s_W^2/3) Z^1_{L,e\mu}$, and $g_{RV(q)} = 0$ for the considered model. Then, the μe conversion rate is

$$\Gamma(\mu^- N \to e^- N) = 2G_F^2 [|\tilde{g}_{LV}^{(p)} V_N^{(p)} + \tilde{g}_{LV}^{(n)} V_N^{(n)}|^2 + |\tilde{g}_{RV}^{(p)} V_N^{(p)} + \tilde{g}_{RV}^{(n)} V_N^{(n)}|^2], \tag{65}$$

where $\tilde{g}_{LV}^{(p)} = 2 g_{LV(u)} + g_{LV(d)}$, $\tilde{g}_{LV}^{(n)} = g_{LV(u)} + 2 g_{LV(d)}$, and similarly for $\tilde{g}_{RV}^{(p)}$ and $\tilde{g}_{RV}^{(p)}$. $V_N^{(p)}$ and $V_N^{(n)}$ are overlap integrals of the μ, e with the protons and neutrons in the nucleus N, which have been numerically evaluated and cataloged in [39]. The above rate is usually normalized to the corresponding ordinary muon capture rate, ω_{capt}, to yield a branching ratio, $\text{Br}(\mu^- N \to e^- N)$, for the μe conversion on a particular nucleus N. Since the purely leptonic decays and the μe conversion in nuclei originate from the same FCNC couplings, the ratio of their branching ratios has the simple form,

$$\frac{\text{Br}(\mu^- N \to e^- N)}{\text{Br}(\mu \to e e \bar{e})} = \frac{G_F^2 [(2 - 8 s_W^2) V_N^{(p)} - 2 V_N^{(n)}]^2}{\omega_{\text{capt}} (1 - 4 s_W^2 + 6 s_W^4)}, \tag{66}$$

upon ignoring minor corrections in the diagonal element $Z^1_{L,ee}$. Namely, the relative importance of the two transitions rests on that of their experimental bounds.

IV. NUMERICAL ANALYSIS

As we shall see later, the Yukawa coupling x', or $x = x' v_2/(\sqrt{6} m_\Sigma)$, that couples the light and heavy fermions via the Higgs doublet is a central parameter that controls the overall scale of the LFV transition rates. We mentioned earlier that the parameter measures the unitarity violation in the effective PMNS matrix. Since the heavy fermions have a squared mass splitting proportional to x^2 [see Eq. (46)], it could also be sensitive to the violation of custodial symmetry measured by the parameter $\Delta\rho = m_W^2/(c_W^2 m_Z^2) - 1$. We have calculated the one-loop contribution due to the heavy fermions

$$\Delta\rho_\Sigma = \frac{4\sqrt{2} G_F m_\Sigma^2}{(4\pi)^2} \frac{19}{48} x^4 \approx 1.7 \times 10^{-5} x'^4 \frac{1\ \text{TeV}^2}{m_\Sigma^2}. \tag{67}$$

This is balanced by that of a nonvanishing VEV of the nondoublet scalar field, $v_4 \neq 0$, that occurs already at the tree level, $\Delta\rho_\Phi \approx -6 v_4^2/v_2^2$ (see also Ref. [10]). We noted in the above that half of the power in x^4 comes from the mass splitting of the heavy fermions. The other half originates from the mixing effect in the two vertices, which is essential for a contribution to $\Delta\rho$ since a vectorlike multiplet cannot contribute even if it is not degenerate. Since the ρ parameter is measured at a precision not better than 10^{-4}, we are on the safe side if x' is not larger than 0.7 even for a doubly charged fermion as light as 200 GeV. This is a much weaker constraint than we shall get below from LFV transitions.

Before we show our numerical results, we outline how the free parameters are manipulated based upon the formulas in Sec. III. Since the unitarity of the PMNS matrix has been verified to certain level, we can use it as a guide in browsing the parameter space. In this way we can cover the majority of the parameter space that is consistent with an almost unitary effective PMNS matrix, $\bar{V}_{\text{PMNS}}$. The PMNS matrix generally contains three mixing angles, one Dirac phase, and two Majorana phases in the standard form:

$$V_{\text{PMNS}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}u_\delta^* \\ -c_{23}s_{12} - s_{13}s_{23}c_{12}u_\delta & c_{23}c_{12} - s_{13}s_{23}s_{12}u_\delta & s_{23}c_{13} \\ s_{23}s_{12} - s_{13}c_{23}c_{12}u_\delta & -s_{23}c_{12} - s_{13}c_{23}s_{12}u_\delta & c_{23}c_{13} \end{pmatrix} \text{diag}(u_1, u_2, u_3), \tag{68}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, $u_j = \exp(i\alpha_j/2)$, and $u_\delta = \exp(i\delta)$. We use the measured values for those angles (while choosing some values for the phases which are not yet measured) and for the light neutrino masses $m_{1,2,3}$ in either NH or IH. Then, the heavy neutrino masses $m_{4,5}$ and the parameters z, $|c_z|$ are uniquely fixed once the parameters (x, m_Σ) are assigned a value. The diagonalization matrix U for the neutrinos is also fixed up to the phase of $c_z = |c_z| e^{i\alpha_z}$. In particular, the matrix U_n that is formed from the first three rows of the vectors in Eq. (27) (for the NH case and similarly for IH) is fixed up to a diagonal phase matrix, $U_z \equiv \text{diag}(1, e^{i\alpha_z/2}, e^{-i\alpha_z/2})$, multiplied from the left. Applying the definition $V_{\text{PMNS}} = U_{fL}^\dagger U_n$ and the first equation in Eq. (39) (for scenario B and similarly for scenario A), we have $U_z^\dagger \bar{\mathcal{M}}_L^2 U_z = (U_z^\dagger U_n) V_{\rm PMNS} {\rm diag}(\lambda_e, \lambda_\mu, \lambda_\tau) V_{\rm PMNS}^\dagger (U_n^\dagger U_z)$ where the right-hand side is completely known with the additional input of the light charged lepton masses. We observe that this procedure then determines uniquely all of the parameters $y_{1,2,3,4}$, $c_{1,2}$ as well as α_z on the left-hand side. The diagonalization matrix U_L is thus fixed. Once the Y matrix is known, we can follow the formalism in Sec. III to find the other diagonalization matrix U_R. In a final step we go back to check that the effective PMNS matrix $\bar{V}_{\rm PMNS}$ obtained from the matrices U and U_L determined above does not violate unitarity beyond the allowed level.

To get some feel about the branching ratios for the LFV transitions, we start with the results for the simplified case of tribimaximal neutrino mixing with all phases set to zero. We assume $m_\Sigma = 200$ GeV in the following discussions so that the heavy fermions are within the reach of LHC. In the upper panel of Fig. 3 we show the branching ratios for the muon decays and μe conversion in nuclei $^{197}_{79}$Au and $^{48}_{22}$Ti as a function of the x parameter for the NH case in scenario B, together with the current experimental bounds on them (horizontal lines). The lower panel depicts the branching ratios for the two τ decays, $\tau \to \mu\mu\bar{\mu}$, $\mu e\bar{e}$, for the same range of x, while other decays are severely suppressed. We see that the μe conversion on the heavy gold nucleus sets the most stringent constraint on the x parameter though it also inherits the largest uncertainty from nuclear physics. The bound from the conversion on titanium is comparable to that from the purely leptonic decay, $\mu \to 3e$. For such a small x the deviation from the SM values of the anomalous magnetic moments in Eq. (59) and the contribution to the electric dipole moments in Eq. (61) are too small to be relevant. In Fig. 4 we depict the corresponding results for the IH case in scenario B. Note that in this case the dominant LFV τ decays change to $ee\bar{e}$ and $e\mu\bar{\mu}$. Generally speaking, when assuming tribimaximal neutrino mixing, the stringent bounds on the muon imply that the tau decays are not likely to be observable in the near future for the majority of the parameter space.

We thus ask if there is a region in the parameter space where the muon decays and μe conversion in nuclei are significantly suppressed while the tau decays are not much below the current bounds. To help identify the interested region it is useful to work with the leading terms in the limits of infinite virtual heavy masses and vanishing virtual light masses. An inspection of Eqs. (56) and (63) augmented with the coupling matrices displayed in Sec. II tells us that in the NH case the radiative decay $\ell_\alpha \to \ell_\beta \gamma$ is dominated by the terms with the mixing matrices $u_\tau^{L*}(\ell_\beta) u_\tau^L(\ell_\alpha)$, $u_\tau^{R*}(\ell_\beta) u_\tau^L(\ell_\alpha)$, and $u_\tau^{L*}(\ell_\beta) u_\tau^R(\ell_\alpha)$, whose coefficients are less suppressed by a small x parameter, and that the rates for $\mu \to ee\bar{e}$ and μe conversion in nuclei are

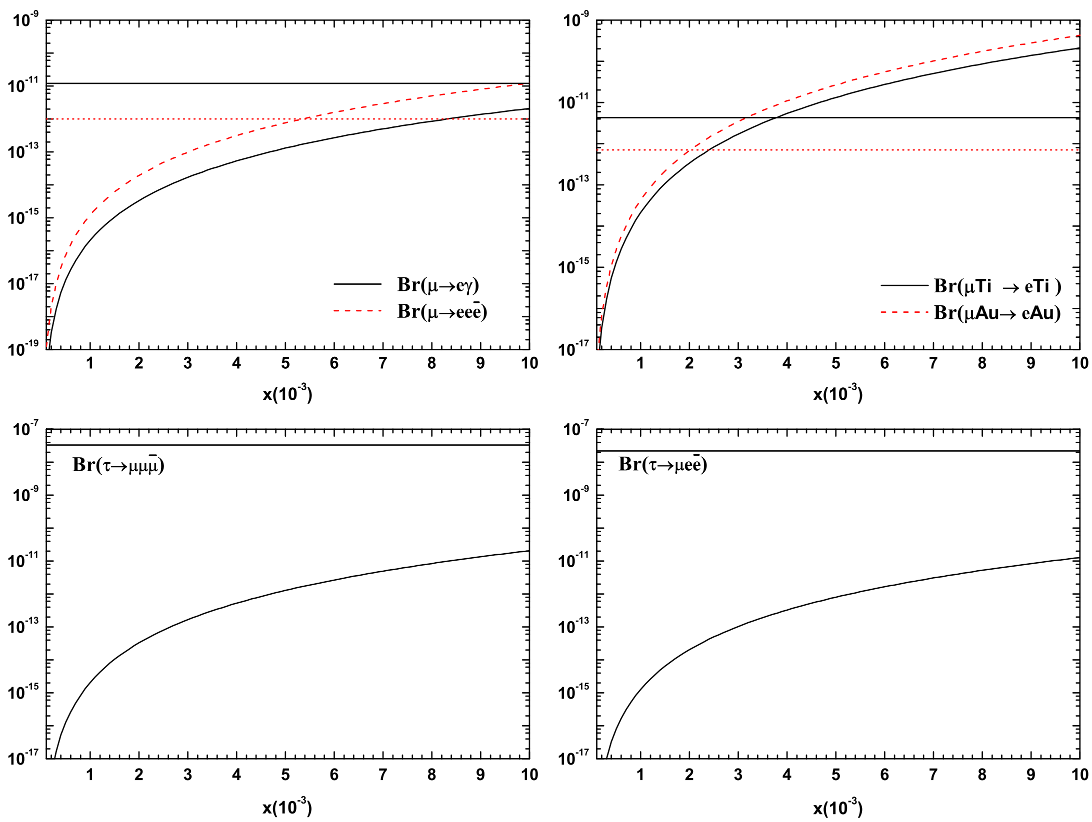

FIG. 3 (color online). Branching ratios as a function of x for NH in scenario B and tribimaximal mixing.

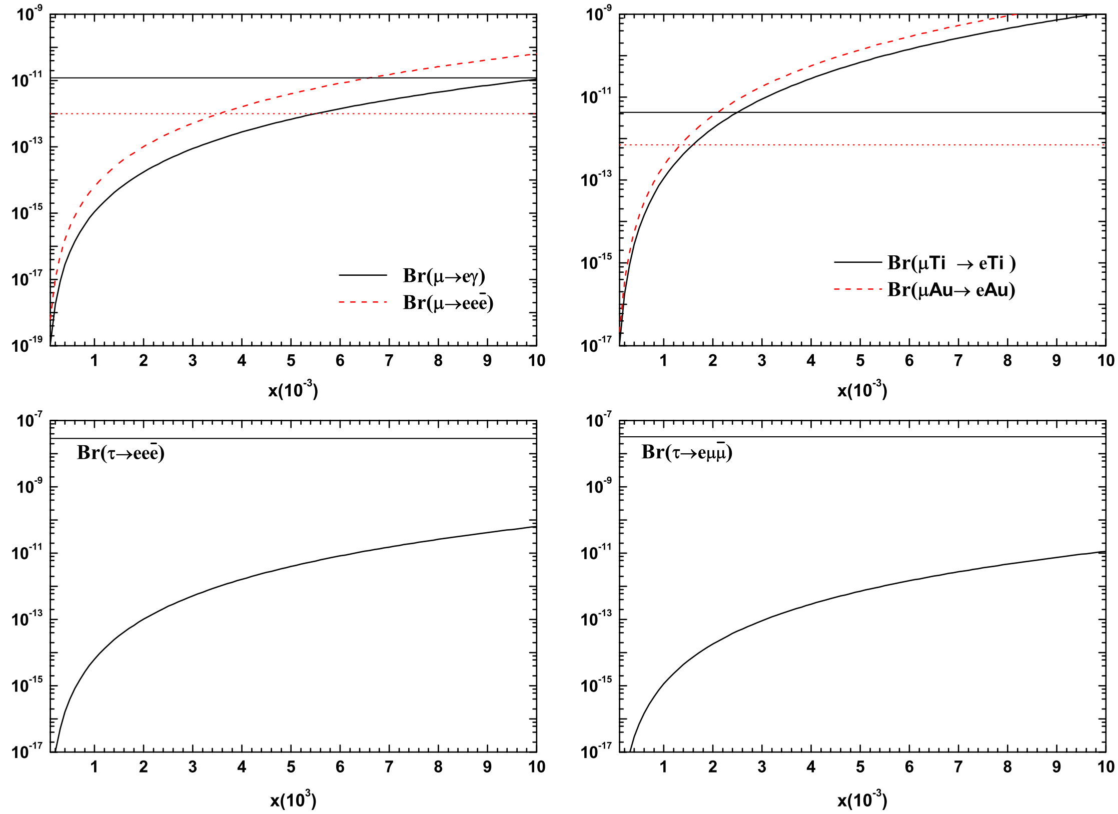

FIG. 4 (color online). Branching ratios as a function of x for IH in scenario B and tribimaximal mixing.

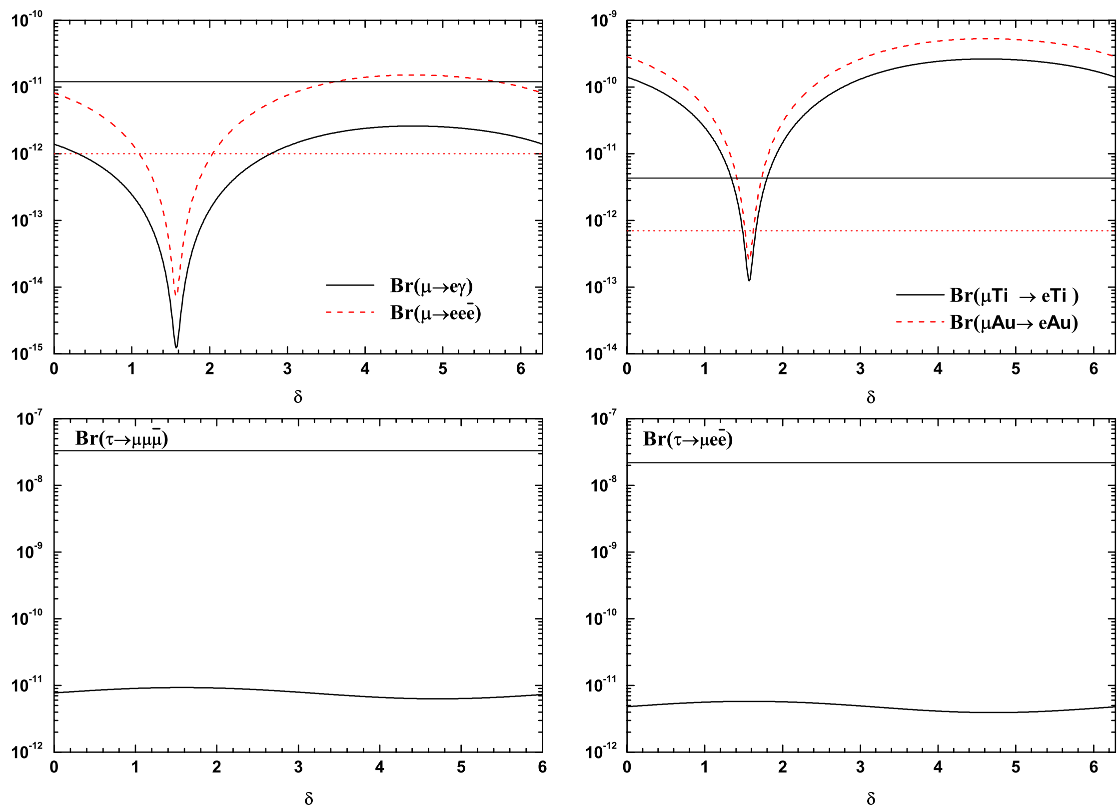

FIG. 5 (color online). Branching ratios as a function of δ for NH in scenario B and best-fit mixing angles.

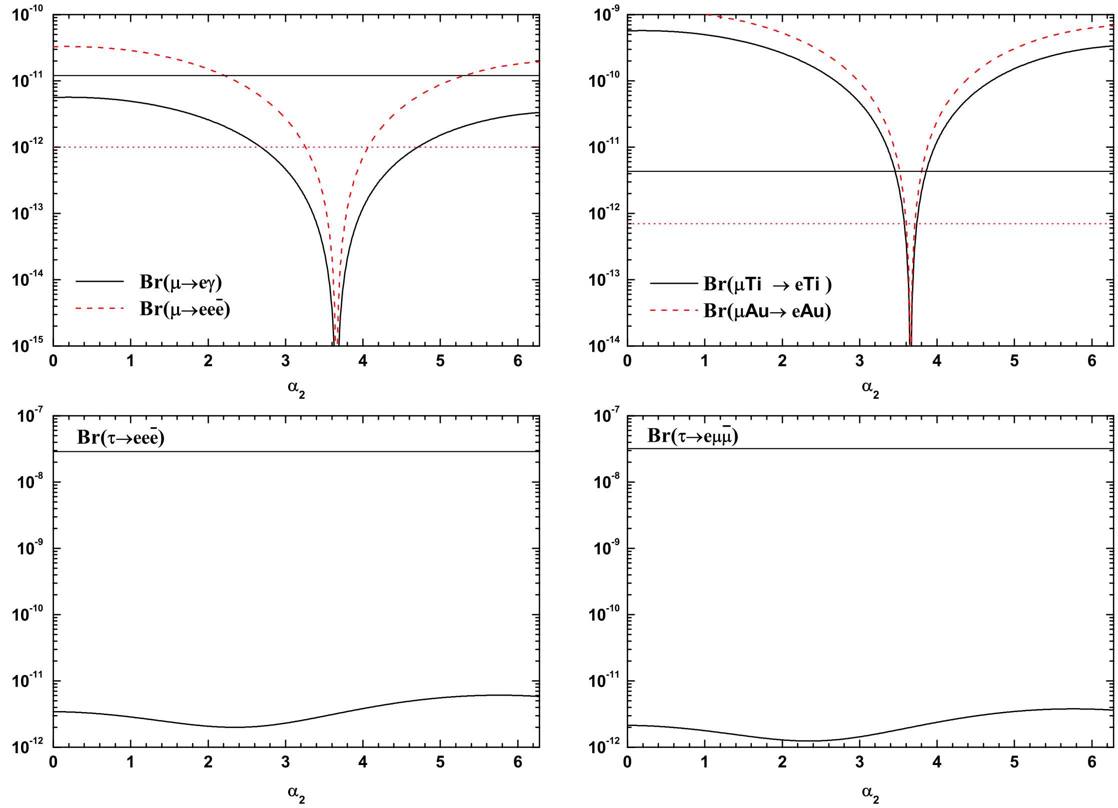

FIG. 6 (color online). Branching ratios as a function of α_2 for IH in scenario B and best-fit mixing angles.

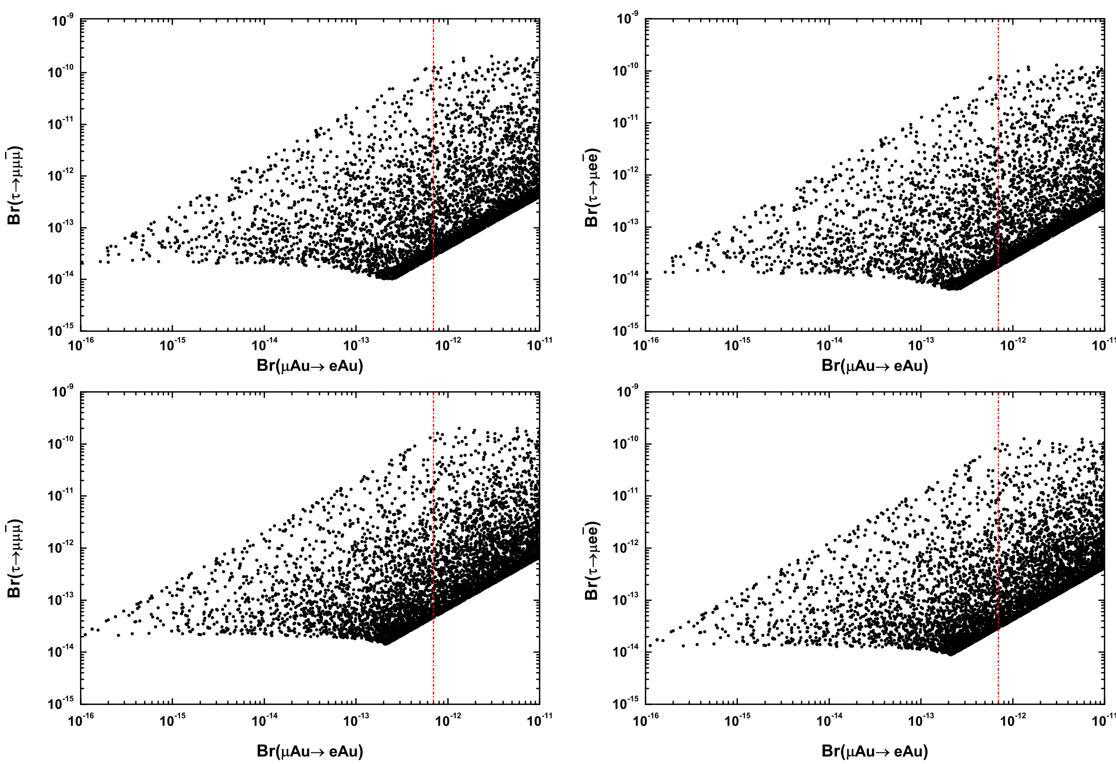

FIG. 7 (color online). Branching ratios sampled over (x, α_2) (upper panels) and (x, δ) (lower panels) for NH in scenario B and best-fit mixing angles.

proportional to $|u_\tau^{L*}(e)u_\tau^L(\mu)|^2$. Since for instance in scenario A, $|u_\tau^R(\ell_\alpha)| = |u_\tau^L(\ell_\alpha)|\sqrt{\lambda_\alpha}/y_3$ (attach a factor of c_+ to y_3 for scenario B), we see that the dominant terms for the LFV muon decays and μe conversion in nuclei are controlled by the combination $\xi_{e\mu} \equiv u_\tau^{L*}(e)u_\tau^L(\mu)$ [and similarly in the IH case by $\xi_{e\mu} \equiv u_e^{L*}(e)u_e^L(\mu)$]. We therefore seek for regions in which the combination $\xi_{e\mu}$ would be significantly diminished. For the mixing angles and the neutrino squared mass differences we use the central values from the global fit in Ref. [46]: $\sin^2\theta_{12} = 0.32$, $\sin^2\theta_{23} = 0.50$, $\Delta m_{21}^2 = 7.6 \times 10^{-5}\ \mathrm{eV}^2$, $|\Delta m_{31}^2| = 2.4 \times 10^{-3}\ \mathrm{eV}^2$, and set θ_{13} at its upper limit, $\sin^2\theta_{13} = 0.05$. We choose $\alpha_1 = \alpha_3 = 0$ while leaving the Dirac phase δ and Majorana phase α_2 free. For the SM parameters we use the numbers of the Particle Data Group. We find that $\xi_{e\mu}$ approaches its minimum at $(\delta, \alpha_2) \sim (\pi/2, 2\pi)$ for the NH case and at $(\delta, \alpha_2) \sim (0.64, 3.66)$ for IH. This result is independent of the x parameter (for x not too large in scenario B).

In Fig. 5 we show the branching ratios of the muon decays and μe conversion in nuclei and the largest tau decays in the NH case by scanning over the Dirac phase δ. We have assumed $x = 8 \times 10^{-3}$ and $\alpha_2 = 2\pi$. The corresponding result for the IH case is depicted in Fig. 6 as a function of α_2 at the same x parameter and $\delta = 0.64$. One sees from these two figures that without breaking the stringent bounds on the muon lepton the branching ratios for some of the leptonic tau decays can approach the level of 10^{-11} for almost the whole range of the scanned phase. This is much enhanced compared to the case of tribimaximal mixing shown in Figs. 3 and 4, but is still 3 orders of magnitude below the current sensitivity.

The above tendency encourages us to scan over a larger set of parameters. So we finally sample over the x parameter from 8×10^{-4} to 8×10^{-3} and one of the phases in its whole range while keeping the other phase fixed at the value that minimizes $\xi_{e\mu}$. Our results are shown in Fig. 7 for the NH and in Fig. 8 for the IH case, respectively. In both figures, the upper panel scans over x and the Majorana phase α_2 while the lower one is over x and the Dirac phase δ. We include only the most stringent μe conversion on gold and the largest tau decay that is available in each case. In the most optimistic situation, some τ decays can reach the level that is about 2 orders of magnitude below the current sensitivity. We notice that the two figures in the same panel have a similar pattern. This arises because the decay $\tau \to \ell_\alpha \ell_\beta \bar{\ell}_\beta$ with $\alpha \neq \beta$ is dominated by one Feynman graph which is almost the same as any of the two graphs for the decay $\tau \to \ell_\alpha \ell_\alpha \bar{\ell}_\alpha$.

V. CONCLUSION

The origin of tiny neutrino mass has remained mysterious after years of endeavor. From the viewpoint of

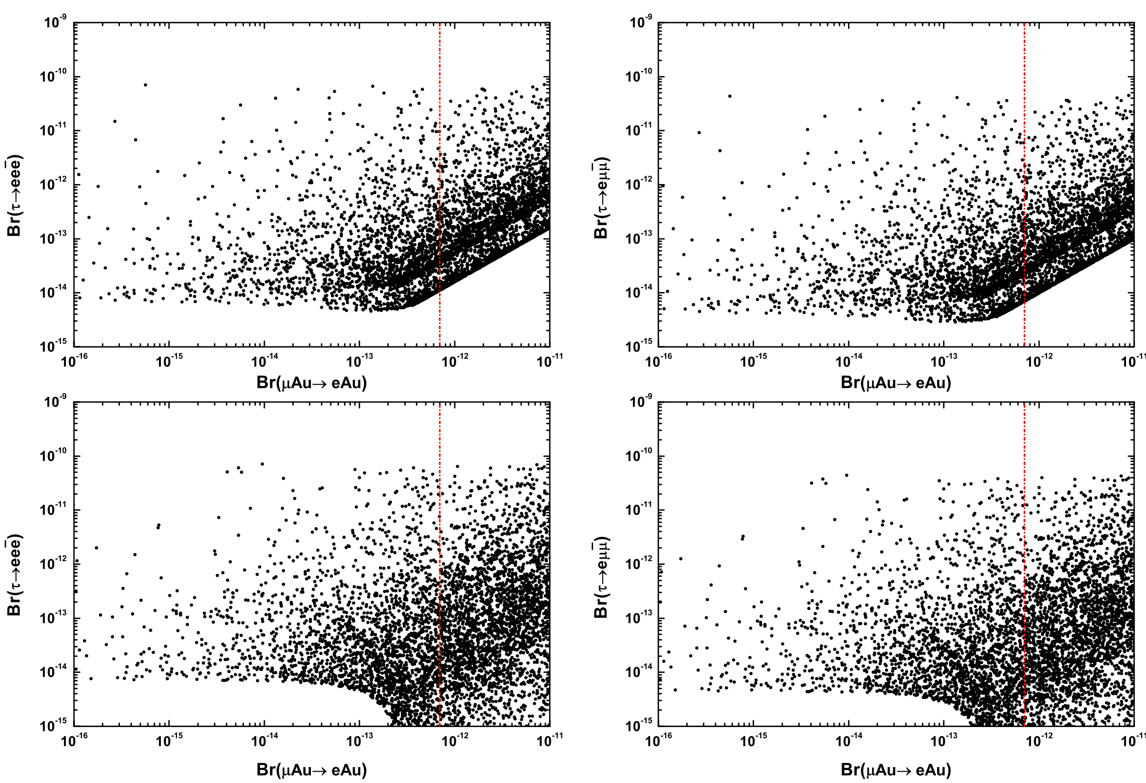

FIG. 8 (color online). Branching ratios sampled over (x, α_2) (upper panels) and (x, δ) (lower panels) for IH in scenario B and best-fit mixing angles.

effective field theory the tiny mass can be accommodated by the canonical seesaw mechanisms. But it is generally hard to explore in current experiments the physics that would be responsible for the mechanisms because the relevant physics scale is very high and the new interactions with light particles are generally too weak. It is thus highly desirable if there is any new mechanism that would predict accessible effects beyond the neutrino mass.

There are two basic approaches to relax the tension between the accessibility of new physics and the effectiveness in producing tiny neutrino mass. One can either attribute the mass to a higher order quantum effect or postpone its appearance to a higher-dimensional effective interaction. An explicit model has been recently attempted in the second approach [10]. The idea is to avoid the conventional dimension-five interaction by composing new fields in higher-dimensional representations so that the first contribution to the neutrino mass occurs at dimension seven. The new particles enjoy the SM gauge interactions, and thus if not very heavy would be produced at high energy colliders like Tevatron and LHC. The point that we want to emphasize here is that to establish the kinship of those particles to the origin of neutrino mass it would be necessary to detect their interactions with light leptons. These interactions are as usual shaped by the mixing effects between the light and heavy particles, and thus should also leave their fingerprints in precisely measured flavor-changing processes at low energy. The purpose of the current work has been to examine if there is any chance to look for the mixing effects in LFV transitions of the charged leptons.

We have made a systematic analysis of the model. In particular, we provided a convenient parametrization of the leptons' mass matrices in terms of independent physical parameters. By diagonalizing them explicitly the lepton flavor structure becomes transparent in interactions. The contributions of these interactions to the radiative, purely leptonic decays of the charged leptons, and the μe conversion in nuclei are then computed. We considered how the stringent constraints from the muon lepton affect the decay processes in the tau sector. Generally the current experimental bounds on the decay $\mu \to ee\bar{e}$ and the μe conversion in nuclei, in particular, are so strong that it is very difficult to observe the tau lepton decays. However, our sampling over the unknown phases and Yukawa coupling shows that there are small regions in the parameter space in which some tau decays have a branching ratio that is about 2 orders of magnitude below the current bounds. It will be challenging, if not hopeless, to observe in those decays the mixing effects related to the neutrino mass generation.

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