

Newly observed $D(2550)$, $D(2610)$, and $D(2760)$ as $2S$ and $1D$ charmed mesonsZhi-Feng Sun,^{1,2} Jie-Sheng Yu,^{1,2} and Xiang Liu^{1,2,*}¹*School of Physical Science and Technology, Lanzhou University, Lanzhou 730000, China*²*Research Center for Hadron and CSR Physics, Lanzhou University, and Institute of Modern Physics of CAS, Lanzhou 730000, China*Takayuki Matsuki[†]*Tokyo Kasei University, 1-18-1 Kaga, Itabashi, Tokyo 173-8602, Japan*

(Received 4 September 2010; published 1 December 2010)

We study three charmed resonances, $D(2550)$, $D(2610)$, and $D(2760)$, newly observed by the BABAR Collaboration, utilizing the mass spectra and investigating the strong decays. Our calculation indicates that $D(2610)$ is an admixture of 2^3S_1 and 1^3D_1 with $J^P = 1^-$. $D(2760)$ can be explained as either the orthogonal partner of $D(2610)$ or 1^3D_3 . Our estimate of the decay width for $D(2550)$, assuming it as 2^1S_0 , is far below the experimental value. The decay behavior of the remaining two $1D$ charmed mesons, i.e., 3D_2 and 1D_2 ($J^P = 2^-$) states, is predicted, which will help future experimental search for these missing D -wave charmed mesons.

DOI: 10.1103/PhysRevD.82.111501

PACS numbers: 14.40.Lb, 12.38.Lg, 13.25.Ft

Very recently, the BABAR Collaboration has observed three new charmed mesons, $D(2550)$, $D(2610)$, and $D(2760)$ [1]. The spectra of D mesons are very poorly known because higher states have been hindered by poor statistics and their relatively large widths. The BABAR analysis on these particles is that $2S$ and $1D$ are the most likely candidates when relying on the mass values predicted by the conventional nonrelativistic potential model [2] and the relativistic potential models [3–5]. By reanalyzing their data, especially by studying their decay widths with the successful 3P_0 model [6], we investigate whether the quark model prediction fits with the BABAR data.

In the $D^+ \pi^-$ invariant mass spectrum, two D mesons, $D(2610)^0$ and $D(2760)^0$ with neutral charge, have been observed along with confirming two established charmed mesons, $D_0^*(2400)^0$ and $D_2^*(2460)^0$. BABAR has also found the isospin partners $D(2610)^+$ and $D(2760)^+$ in the $D^0 \pi^+$ channel. By analyzing the $D^{*+} \pi^-$ invariant mass spectrum, three structures around 2533.0 MeV, 2619.0 MeV, and 2747.7 MeV have been released, which shows that BABAR has not only confirmed $D(2610)^0$ and $D(2760)^0$ in the $D^{*+} \pi^-$ channel but has also found a new charmed state $D(2550)^0$. The resonance parameters (in units of MeV) of $D(2550)$, $D(2610)$, and $D(2760)$ are summarized as [1]

$$M_{D(2550)^0} / \Gamma_{D(2550)^0} = 2539.4 \pm 4.5 \pm 6.8 / 130 \pm 12 \pm 13,$$

$$M_{D(2610)^0} / \Gamma_{D(2610)^0} = 2608.7 \pm 2.4 \pm 2.5 / 93 \pm 6 \pm 13,$$

$$M_{D(2760)^0} / \Gamma_{D(2760)^0} = 2763.3 \pm 2.3 \pm 2.3 / 60.9 \pm 5.1 \pm 3.6,$$

$$M_{D(2610)^+} / \Gamma_{D(2610)^+} = 2621.3 \pm 3.7 \pm 4.2 / 93,$$

$$M_{D(2760)^+} / \Gamma_{D(2760)^+} = 2769.7 \pm 3.8 \pm 1.5 / 60.9,$$

*xiangliu@lzu.edu.cn

†matsuki@tokyo-kasei.ac.jp

where the first error is statistical and the second is systematic.

As listed in the paper by the Particle Data Group (PDG) [7], there are six charmed mesons D , D^* , $D_0^*(2400)$, $D_1(2430)$, $D_1(2420)$, and $D_2^*(2460)$ with the established spin-parity assignments (see Fig. 1 for details). The newly observed charmed resonances, $D(2550)$, $D(2610)$, and $D(2760)$, are not only making the spectroscopy of charmed mesons abundant, but also stimulating our interest in revealing the underlying properties of these particles, and, of course, providing a good opportunity to test the existent theory of a heavy-light meson system and further enlarging our knowledge of nonperturbative QCD.

The BABAR Collaboration's analysis of angular distribution indicates that $D(2550)$ and $D(2610)$ may be identified as the first radial excitations of S -wave charmed

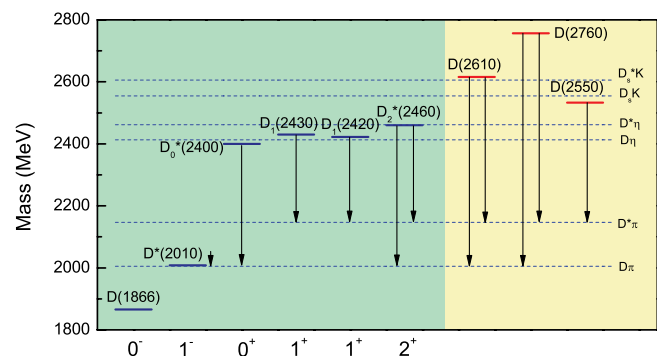


FIG. 1 (color online). The mass spectrum of six established charmed mesons in PDG [7] and three newly observed charmed mesons by the BABAR Collaboration [1], compared with the thresholds of $D^{(*)}\pi$, $D^{(*)}\eta$, $D_0\pi$, and $D_s^{(*)}K$. Here, the experimental decay channels corresponding to these charmed mesons are also drawn as down arrows. The decay channel $D(2550) \rightarrow D_0^*(2400) + \pi$ is omitted from the figure.

mesons while $D(2760)$ can be a D -wave state [1]. The quantum numbers J^P of $D(2550)$ and $D(2610)$ are assigned as 0^- and 1^- , respectively, which explains why BABAR has found $D(2550)$ only in the $D^*\pi$ channel and $D(2610)$ in both the $D\pi$ and $D^*\pi$ channels. As a candidate of the D -wave charmed meson, the spin-parity content of $D(2760)$ can be either 1^- or 3^- because the observed decay process $D(2760) \rightarrow DK$ fully excludes $J^P = 2^-$.

Spectroscopy: As shown in Table I, different theoretical groups [2–5] have carried out the systematic calculation of the mass spectra for charmed mesons. In addition to successful reproduction of $1S$ and $1P$ charmed mesons, they predict the masses of charmed mesons with quantum numbers $0^-(2^1S_0)$, $1^-(2^3S_1)$, $1^-(1^3D_1)$, and $3^-(1^3D_3)$. It is noted that the predicted theoretical mass of $D(2550)$ falls into the range 2.483–2.589 GeV, which supports the $0^-(2^1S_0)$ assignment to $D(2550)$. Because the predicted as well as observed masses of charmed mesons for $1^-(2^3S_1)$ and $1^-(1^3D_1)$ are close to each other [1–5], $D(2610)$ and $D(2760)$ can be regarded as an admixture of $1^-(2^3S_1)$ and $1^-(1^3D_1)$, which satisfies

$$\begin{pmatrix} |D(2610)\rangle \\ |D(2760)\rangle \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} |1^-(2^3S_1)\rangle \\ |1^-(1^3D_1)\rangle \end{pmatrix}, \quad (1)$$

where $|1^-(2^3S_1)\rangle$ and $|1^-(1^3D_1)\rangle$ are pure states and ϕ denotes the mixing angle. This situation is similar to that of charmonium $\psi(3770)$, which may be the mixing state of $1^-(2^3S_1)$ and $1^-(1^3D_1)$ charmonia as suggested in Ref. [8].

From the above analysis and Table I, one concludes in general that 2^1S_0 is a good candidate for a pseudoscalar meson $D(2550)$. $D(2610)$ may be a pure 2^3S_1 state or an admixture of 2^3S_1 and 1^3D_1 states. $D(2760)$ can be a pure 1^3D_1 or a pure 1^3D_3 , or an admixture of 2^3S_1 and 1^3D_1 as the orthogonal cousin of $D(2610)$.

Decay width: Further studies of the two-body strong decay will be helpful to distinguish the different structure

assignment to $D(2550)$, $D(2610)$, and $D(2760)$ using the quark pair creation (QPC) model [6,9,10], which is a successful phenomenological model for dealing with the Okubo-Zweig-Iizuka-allowed strong decays of hadron. The relevant decay channels of these particles are listed in Table II, applying the quantum number assignments to $D(2550)$, $D(2610)$, and $D(2760)$ discussed above. Defining the transition operator \mathcal{T} in the QPC model [6,9,10], the main task is to calculate the helicity amplitude $M^{M_{J_A} M_{J_B} M_{J_C}}(\mathbf{K})$ corresponding to the strong decay processes $A(c(1)\bar{q}(2)) \rightarrow B(c(1)\bar{q}(3)) + C(\bar{q}(2)q(4))$ shown in Table II, where the harmonic oscillator wave function $\Psi_{n_r \ell m}(\mathbf{k}) = \mathcal{R}_{n_r \ell}(R, \mathbf{k}) \mathcal{Y}_{\ell m}(\mathbf{k})$ is applied to calculate the spatial integral in the transition matrix element. (See Ref. [11] for more details.) The parameter R in the harmonic oscillator wave function is adjusted so that it reproduces the realistic root mean square radius. The root mean square is obtained by solving the Schrödinger equation with the potential in Refs. [12,13], which gives different R values corresponding to π/η , ρ/ω , K , D , D^* , D_s , D_s^* , $D_1(2430)$, $D_1(2420)$, $D_2^*(2460)$, $D(2^3S_1)$, and $D(1^3D_J)$, respectively. The remaining input parameters are the constituent quark masses of charm, up/down, and strange, i.e., 1.45 GeV, 0.33 GeV, and 0.55 GeV, respectively [12]. In addition, the strength of the QPC from the vacuum can be extracted by fitting the data. In this paper, we take $\gamma = 6.3$ [14]. The strength of $s\bar{s}$ creation satisfies $\gamma_s = \gamma/\sqrt{3}$ [9].

In Figs. 2 and 3, the decay behaviors of $D(2550)$, $D(2610)$, and $D(2760)$ with different quantum number assignments are presented.

$D(2550)$: The total decay width of $D(2550)$ mainly comes from its $D_0^*\pi^0$ and $D^*\pi$ contributions shown in Fig. 2, where the theoretical estimate of the width for $D(2^1S_0)$ is given by $\Gamma \sim 8$ MeV for the typical value $R = 3.6$ GeV $^{-1}$, which is *far below the observed value of the decay width* 127.6 MeV for $D(2550)$. In Fig. 2, we

TABLE I. Theoretical calculations for D mesons with quantum number $n^{2s+1}L_J$ and a comparison with experimental data. Here, we tentatively set the masses of $1^-(2^3S_1)$ and $1^-(1^3D_1)$ as those of $D(2610)$ and $D(2760)$, respectively. States with quotation marks are given by $|{}^{33}P_1''\rangle = \sqrt{2/3}|{}^3P_1\rangle - \sqrt{1/3}|{}^1P_1\rangle$ and $|{}^{11}P_1''\rangle = \sqrt{1/3}|{}^3P_1\rangle + \sqrt{2/3}|{}^1P_1\rangle$ [3], which correspond to the two 1^+ states in S and T doublets shown in the caption of Table II, respectively. The mass value 2318 MeV for Experiment is for neutral and 2403 MeV is for charged.

$J^P(n^{2s+1}L_J)$	Experiment [1,7]	GI [2]	MMS [3]	DiPE [4]	EGF [5]
$0^-(1^1S_0)$	1867	1880	1869	1868	1871
$1^-(1^3S_1)$	2008	2004	2011	2005	2010
$0^+(1^3P_0)$	$\begin{cases} 2318 \\ 2403 \end{cases}$	2400	2283	2377	2406
$1^+(1^{33}P_1'')$	2427	2490	2421	2417	2469
$1^+(1^{11}P_1'')$	2420	2440	2425	2460	2426
$2^+(1^3P_2)$	2460	2500	2468	2490	2460
$0^-(2^1S_0)$	2533	2580	2483	2589	2581
$1^-(2^3S_1)$	2619	2640	2671	2692	2632
$1^-(1^3D_1)$	2763	2820	2762	2795	2788
$3^-(1^3D_3)$		2830		2799	2863

NEWLY OBSERVED $D(2550)$, $D(2610)$, AND ...

TABLE II. The allowed decay channels (■) of $2S$ and $1D$ charmed mesons with the quantum number assignment to $D(2550)$, $D(2610)$, and $D(2760)$ discussed in this paper. For $D(2610)$, decays into $D\rho$, $D\omega$, $D^*\eta$, and D_s^*K are forbidden due to the limit of phase space. Here, $D_1(2430)$ and $D_1(2420)$ are the 1^+ states in the $S = (0^+, 1^+)$ and $T = (1^+, 2^+)$ doublets, respectively.

Modes	Channel	$0^-(2^1S_0)$	$1^-(2^3S_1)$	$1^-(1^3D_1)/3^-(1^3D_3)$
$0^- + 0^-$	$D\pi$		■	■
	$D\eta$		■	■
	$D_s K$		■	■
$0^- + 1^-$	$D\rho$			■
	$D\omega$			■
$1^- + 0^-$	$D^*\pi$	■	■	■
	$D^*\eta$		■	■
	D_s^*K			■
$0^+ + 0^-$	$D_0(2400)\pi$	■		
$1^+(S) + 0^-$	$D_1(2430)\pi$		■	■
$1^+(T) + 0^-$	$D_1(2420)\pi$		■	■
$2^+ + 0^-$	$D_2^*(2460)\pi$		■	■

also show the R dependence of the total decay width for $D(2^1S_0)$, whose shape results from the node effects of the higher radial wave function. In the range $3.4 \leq R \leq 3.8 \text{ GeV}^{-1}$, the upper limit of the theoretical width is still smaller than the experimental one for $D(2550)$. This discrepancy between the theoretical and experimental results may indicate that the quark model calculation might not be appropriate in this case, or the assignment of 2^1S_0 to $D(2550)$ might be inappropriate. The comparison of the resonance parameters for $D(2550)$ and $D(2610)$ in Eq. (1) may also be controversial because the width 127.6 MeV for

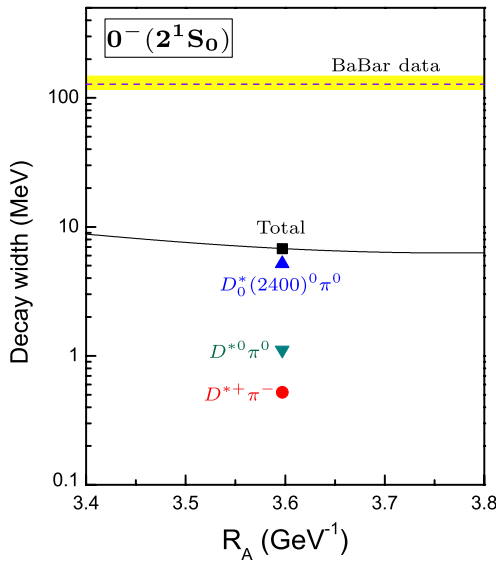


FIG. 2 (color online). The R value dependence and the typical values of the decay width for $D(2^1S_0)$. Here, the dashed line with the shaded (yellow) error band is the BABAR Collaboration's results of the width of $D(2550)$.

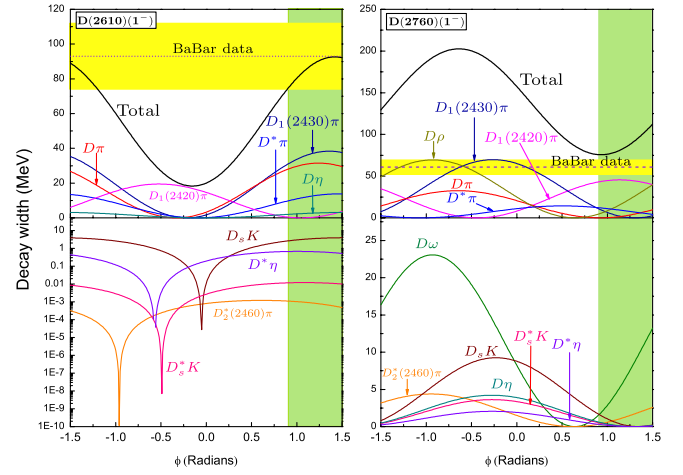
 PHYSICAL REVIEW D **82**, 111501(R) (2010)


FIG. 3 (color online). The variation of the total and partial decay widths for two 1^- states discussed in this paper with the mixing angle range $-1.5 \leq \phi \leq 1.5$ radians. Here, dashed lines with horizontal shaded (yellow) error bands are the BABAR Collaboration's results of the widths for $D(2610)$ and $D(2760)$.

$D(2550)$ is larger than 93 MeV for $D(2610)$, even though the number of decay channels for $D(2610)$ is larger than that for $D(2550)$ (see Table II), where both $D(2550)$ and $D(2610)$ with $J^P = 0^-$ and 1^- , respectively, can decay into $D^*\pi$ through the P -wave. Thus, if we assume $D(2550)$ as a $0^-(2^1S_0)$ charmed meson and that our method is appropriate, our theoretical estimate for $D(2550)$ becomes a narrow state.

Close and Swanson [13] have also studied the strong decay behavior of the 2^1S_0 charmed meson and have obtained the larger total strong decay width with a smaller discrepancy by experiment. However, the mass of the 2^1S_0 charmed meson in Ref. [13] is taken as 2.58 GeV (to be compared with our value in Table I). This value, 2.58 GeV, resulted in the decay width given in Ref. [13] being larger than ours. In our calculation, we follow the approach given by Ref. [15], which has already been tested by calculating the decay width for the process $D_{s2}(2573) \rightarrow DK + D^*K + D_s\eta$ [12] which is consistent with the corresponding experimental data [7].

$D(2610)$ and $D(2760)$: As shown in Fig. 3, assignment of $D(2610)$ and $D(2760)$ to pure 2^3S_1 and 1^3D_1 charmed mesons, respectively, can be fully excluded because their decay widths estimated by the QPC model cannot be fitted with the corresponding observed decay widths when setting the mixing angle $\phi = 0$. Instead, there exists unique possible assignment to the structure of $D(2610)$, i.e., an admixture of 2^3S_1 and 1^3D_1 charmed mesons discussed in Eq. (1). One finds an overlap region (vertical [green] shaded band) between the theoretical and experimental results for $D(2610)$ with the mixing angle $0.9 \leq \phi \leq 1.5$ radians, which strongly supports that $D(2610)$ and $D(2760)$ are orthogonal cousins. This explains why $D(2610)$ has been first observed in both $D\pi$ and $D^*\pi$

channels because the main decay modes of $D(2610)$ are $D_1(2430)\pi$, $D\pi$, $D^*\pi$, and $D\eta$, as shown in Fig. 3. In addition, the theoretical estimate for the several ratios $\mathcal{R}_1 = \Gamma(D^*\pi)/\Gamma(D\pi)$, $\mathcal{R}_2 = \Gamma(D\pi)/\Gamma(D_1(2430)\pi)$, and $\mathcal{R}_3 = \Gamma(D^*\eta)/\Gamma(D\eta)$ for the dominant decays of $D(2610)$ is calculated in this mixing angle region $0.9 \leq \phi \leq 1.5$:

$$\begin{aligned} 0.35 &\leq \mathcal{R}_1 \leq 0.47, \\ 0.78 &\leq \mathcal{R}_2 \leq 0.91, \\ 0.17 &\leq \mathcal{R}_3 \leq 0.33, \end{aligned}$$

which may provide a check to the validity of the assumption that the structure assignment to $D(2610)$ is given by Eq. (1). Measurement of the ratio $\Gamma(D^*\pi)/\Gamma(D\pi)$ for $D(2610)$ may be easiest to do because the final states $D^{(*)}$ and π can be easily detected.

D(2760) and D-waves: If we explain $D(2760)$ as 1^3D_3 , the calculated total decay width for $D(1^3D_3)$ is about half of the observed one when scanning the range $3.4 \leq R \leq 3.8 \text{ GeV}^{-1}$, where the total widths are 33.2, 32.9, 32.5, 31.9, and 31.3 MeV corresponding to $R = 3.4, 3.5, 3.6, 3.7$, and 3.8 GeV^{-1} , respectively, which indicates that the total decay width is not sensitive to R . Owing to the uncertainties coming from the QPC model and the present experiment, 1^3D_3 assignment to $D(2760)$ cannot be excluded. We especially notice that $D^*\pi$ and $D\pi$ are the dominant decay modes for $D(1^3D_3)$ with the ratio $\Gamma(D^*\pi)/\Gamma(D\pi) = 1.1$ with the typical value $R = 3.6 \text{ GeV}^{-1}$ (see Table III), which explains why $D^{(*)}\pi$ is first observed among many decay channels of $D(2760)$. In addition to this, $D(2760)$ can be the orthogonal partner of $D(2610)$ because the total decay width for $D(2760)$ calculated by the QPC model is close to the upper limit of the observed one with the same mixing angle range as $D(2610)$. This is shown in Fig. 3 with the predicted partial decay behaviors of $D(2760)$. One needs further precise measurements on the main decay channels of $D(2760)$, especially on $D^{(*)}\pi$ and $D_1(2420)$, because the two-body strong decays of $D(2760)$ for the above two different structure assignments display different behaviors as shown in Table III and illustrated in the right diagram of Fig. 3.

TABLE III. The partial decay width of the $3^-(1^3D_3)$ state (in units of MeV).

Decay channel	Decay width	Decay channel	Decay width
Total	32.47		
$D^*\pi$	12.78	$D\pi$	11.71
$D\rho$	3.49	$D_2^*(2460)\pi$	1.21
$D\omega$	1.07	$D_1(2420)\pi$	0.65
$D\eta$	0.55	$D_s K$	0.43
$D^*\eta$	0.30	$D_s^* K$	0.14
$D_1(2430)\pi$	0.12		

D(2⁻) states: The following discussion predicts the decay behaviors of two 2^- states in $1D$ charmed mesons, which are still missing in experiment. As described in Fig. 4, the total and partial decay widths for two 2^- states are given with the mass dependence because the spectra of two 2^- states are unknown. By increasing the mass of the 2^- charmed meson, more decay channels are open. As a result, the widths of two 2^- charmed mesons become broad, where $\underline{1}^-0^-$ and $\underline{2}^+1^-$ channels (with the underline marking the quantum number of the charmed meson) are the dominant decay modes of both 2^- charmed mesons for the whole mass region. Additionally, $\underline{0}^-1^-$ is also the dominant decay of the $2^-(1^{“3}D_2^{”})$ charmed meson once this channel is open. In Fig. 4, we have not included the decay mode $\underline{0}^+0^-$ because the process $2^- \rightarrow \underline{0}^+0^-$ through D -wave interaction is a smaller contribution than other channels. Thus, $D^*\pi$ and $D_2^*(2460)\pi$ could be two key decay channels when one experimentally searches for these two 2^- charmed mesons. If we take the typical value $m = 2.76 \text{ GeV}$ for these two 2^- charmed mesons, the total decay widths can reach up to 103 MeV and 143 MeV for the $2^-(1^{“3}D_2^{”})$ and $2^-(1^{“1}D_2^{”})$ states, respectively.

Summary: Three newly observed charmed resonances, $D(2550)$, $D(2610)$, and $D(2760)$, have been for the first time assigned as $2S$ and $1D$ charmed mesons by the analysis of mass spectra as well as the calculation of two-body strong decay. Concretely, $D(2610)$ should be the mixing of $D(2^3S_1)$ and $D(1^3D_1)$ charmed mesons with spin-parity $J^P = 1^-$, whereas pure $1^-(2^3S_1)$ state assignment to $D(2610)$ has been fully excluded in this paper. There exist two possible structure assignments for $D(2760)$, i.e., the orthogonal cousin of $D(2610)$ or $3^-(1^3D_3)$, which can be distinguished by further study

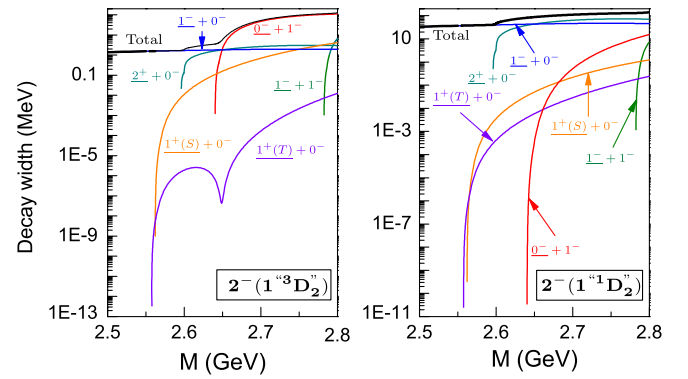


FIG. 4 (color online). The mass dependence of the total two-body and partial strong decay of two $1D$ charmed mesons with $J^P = 2^-$, which are a mixing of 1^1D_2 and 1^3D_2 states with definitions $|^{“3}D_2^{”}\rangle = \sqrt{3/5}|^3D_2\rangle - \sqrt{2/5}|^1D_2\rangle$ (left) and $|^{“1}D_2^{”}\rangle = \sqrt{2/5}|^2D_2\rangle + \sqrt{3/5}|^1D_2\rangle$ (right) [13,16]. Here, the mass range of these two 2^- charmed mesons is taken as 2.5–2.8 GeV, and the J^P quantum numbers of charmed mesons of final state are marked by underlines.

on the main decay channels of $D(2760)$ in future experiments. Although $D(2550)$ is seemingly explained as the 2^1S_0 state under the analysis of the mass spectra, we have found a discrepancy between theory and the experiment for the width of $D(2550)$. Our theoretical calculation of its decay width is far less than the one observed by BABAR. This discrepancy may be due either to our quark model assumption for $D(2550)$ or to the assignment of 2^1S_0 to $D(2550)$. Moreover, the decay behaviors of the remaining two $1D$ charmed mesons have also been predicted, which provides valuable information for future experiments in the search for all $1D$ charmed states.

Just because of the similarity between charmed and charmed-strange mesons, this study will shed light on the underlying properties of the observed charmed-strange states $D_{s1}(2710)$ and $D_{sJ}(2860)$ [17] because of two facts that may reflect the global flavor $SU(3)$ recovery [18]: (i) The mass gap between $D(2760)$ and $D(2610)$ surprisingly agrees with that of $D_{sJ}(2860)$ and $D_{s1}(2710)$, both of

which are about 150 MeV; and (ii) $D_{sJ}(2860)/D_{s1}(2710)$ have almost the same mixing angle between 1^3S_1 and 1^3D_1 states as that of charmed cousins $D(2760)/D(2610)$ if considering the mixing of two 1^- states to explain $D(2760)/D(2610)$ and $D_{sJ}(2860)/D_{s1}(2710)$ [12]. More abundant experimental phenomena together with further efforts on phenomenological study will contribute to our understanding of a heavy-light meson system.

This project is supported by the National Natural Science Foundation of China under Grants No. 10705001 and No. 11035006, the Ministry of Education of China (FANEDD) under Grant No. 200924, DPFIHE under Grant No. 20090211120029, NCET under Grant No. NCET-10-0442, and the Fundamental Research Funds for the Central Universities under Grant No. lzujbky-2010-69). T. M. expresses his sincere thanks to Professor Xiang Liu for the agreeable atmosphere and helpful discussion during his stay at Lanzhou University.

-
- [1] P. del Amo Sanchez *et al.* (The BABAR Collaboration), [arXiv:1009.2076](https://arxiv.org/abs/1009.2076).
- [2] S. Godfrey and N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [3] T. Matsuki, T. Morii, and K. Sudoh, *Prog. Theor. Phys.* **117**, 1077 (2007).
- [4] M. Di Piero and E. Eichten, *Phys. Rev. D* **64**, 114004 (2001).
- [5] D. Ebert, V. O. Galkin, and R. N. Faustov, *Eur. Phys. J. C* **66**, 197 (2010).
- [6] L. Micu, *Nucl. Phys.* **B10**, 521 (1969).
- [7] K. Nakamura *et al.* (Particle Data Group), *J. Phys. G* **37**, 075021 (2010).
- [8] J. L. Rosner, *Phys. Rev. D* **64**, 094002 (2001).
- [9] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Phys. Lett.* **72B**, 57 (1977).
- [10] A. Le Yaouanc, L. Oliver, O. Pene, and J. C. Raynal, *Hadron Transition in the Quark Model* (Gordon and Breach, New York, 1988), p. 311.
- [11] Z. G. Luo, X. L. Chen, and X. Liu, *Phys. Rev. D* **79**, 074020 (2009); B. Zhang, X. Liu, W. Z. Deng, and S. L. Zhu, *Eur. Phys. J. C* **50**, 617 (2007); Z. F. Sun and X. Liu, *Phys. Rev. D* **80**, 074037 (2009); X. Liu, Z. G. Luo, and Z. F. Sun, *Phys. Rev. Lett.* **104**, 122001 (2010).
- [12] D. M. Li and B. Ma, *Phys. Rev. D* **81**, 014021 (2010).
- [13] F. E. Close and E. S. Swanson, *Phys. Rev. D* **72**, 094004 (2005).
- [14] S. Godfrey and R. Kokoski, *Phys. Rev. D* **43**, 1679 (1991).
- [15] H. G. Blundell, Ph.D. thesis, Carleton University, 1996, [arXiv:hep-ph/9608473](https://arxiv.org/abs/hep-ph/9608473).
- [16] T. Matsuki, T. Morii, and K. Seo, *Prog. Theor. Phys.* **124**, 285 (2010).
- [17] B. Aubert *et al.* (The BABAR Collaboration), *Phys. Rev. D* **80**, 092003 (2009).
- [18] T. Matsuki, T. Morii, and K. Sudoh, *Phys. Lett. B* **659**, 593 (2008).