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Features and non-Gaussianity from inflationary particle production

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Interactions between the inflaton and any additional fields can lead to isolated bursts of particle production during inflation (for example from parametric resonance or a phase transition). Inflationary particle production leaves localized features in the spectrum and bispectrum of the observable cosmological fluctuations, via the Infra-Red (IR) cascading mechanism. We focus on a simple prototype interaction $g^2(\phi - \phi_0)^2\chi^2$ between the inflaton, ϕ, and isoinflaton, χ; extending previous work on this model in two directions. First, we quantify the magnitude of the produced non-Gaussianity by extracting the moments of the probability distribution function from lattice field theory simulations. We argue that the bispectrum feature from particle production might be observable for reasonable values of the coupling, g^2. Second, we develop a detailed analytical theory of particle production and IR cascading during inflation, which is in excellent agreement with numerical simulations. Our formalism improves significantly on previous approaches by consistently incorporating both the expansion of the universe and also metric perturbations. We use this new formalism to estimate the shape of the bispectrum from particle production, showing this to be distinguishable from other mechanisms that predict large non-Gaussianity.

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I. INTRODUCTION

The inflationary paradigm has become a cornerstone of modern cosmology. As measurements of the cosmic microwave background (CMB) radiation grow increasingly precise, it has become topical to look beyond the simplest single-field, slow-roll inflationary scenario. In particular, it is interesting to determine the extent to which nonminimal signatures, such as features in the primordial power spectrum or observable non-Gaussianities, can be accommodated by microscopically sensible inflation models. Efforts in this direction are valuable because they allow us to test our theoretical prejudices and provide observers with well-motivated templates for departures from the standard scenario. Finally, a detection of some nonminimal features might open a rare observational window into fundamental particle physics at extremely high energy scales. In this work, we will consider a very simple and well-motivated class of models, which predict novel observable signatures in the spectrum and bispectrum of the primordial curvature fluctuations.

In a variety of inflation models, the motion of the inflaton can trigger the production of some noninflaton (isocurvature) particle *during* inflation. Models of this type have attracted considerable interest recently; examples have been studied in which particle production occurs via parametric resonance [1–9], as a result of a phase transition [10–17], or otherwise [18]. Such constructions are novel for a variety of reasons:

(1) The produced isoinflaton particles may rescatter off the slow roll condensate and generate a significant contribution to the primordial curvature fluctuations through the process of infrared (IR) cascading [1]. This provides a new mechanism for generating cosmological perturbations that is *qualitatively* different from the standard mechanism [19], the curvaton [20,21] or modulated fluctuations [22,23].

(2) Particle production and IR cascading leads to a variety of novel observational signatures, including features in the primordial power spectrum and also non-Gaussianities [1,2].

(3) Particle production arises naturally in a number of microscopically realistic models of inflation, including examples from string theory [3] and supersymmetric (SUSY) field theory [24]. In particular, inflationary particle production is a generic feature of open string inflation models [2], such as brane/axion monodromy [25–28]. (See also [29].)

(4) Observable features in the primordial power spectrum, generated by particle production and IR cascading, offer a novel example of the nondecoupling of high scale physics in the CMB [4,30]. In the most interesting examples, the produced particles are extremely massive for (almost) the entire history of the universe, however, their effect cannot be integrated out due to the nonadiabatic time dependence of the isoinflaton mode functions during particle production. In [4], particle production during large field inflation was proposed as a possible probe of Planck-scale physics.

(5) The energetic cost of producing particles during inflation has a dissipative effect on the dynamics of the inflaton. Particle production may therefore slow the motion of the inflaton, even on a steep potential. This gives rise to a new inflationary mechanism, called *trapped inflation* [3,30], which

may circumvent some of the fine-tuning problems associated with standard slow-roll inflation. See [3] for an explicit string theory realization of trapped inflation and [31] for a generalization to higher dimensional moduli spaces and enhanced symmetry loci. The idea of using dissipative dynamics to slow the motion of the inflaton was exploited also for a very interesting mechanism (which predates trapped inflation) called *warm inflation* [24,32–34]. See also the variant of natural inflation [35] that was proposed recently by Anber and Sorbo [18].

In this article we study the impact of isolated bursts of inflationary particle production on the observable primordial curvature perturbations. In order to illustrate the basic physics, we focus on a very simple and general prototype model where the inflaton, ϕ, and isoinflaton, χ, fields interact via the coupling

$$\mathcal{L}_{\mathrm{int}} = -\frac{g^2}{2}(\phi - \phi_0)^2\chi^2. \tag{1}$$

On physical grounds, we expect that our results will generalize in a straightforward way to more complicated models, such as fermion isoinflaton fields, gauged interactions and (perhaps) inflationary phase transitions.

Scalar field interactions of the type (1) have also been studied recently in connection with nonequilibrium Quantum Field Theory (QFT) [36], in particular, with applications to the theory of preheating after inflation [37–42] and also moduli trapping [30,31] at enhanced symmetry points. Although our focus is on particle production *during* inflation (as opposed to during preheating, after inflation) some of our results nevertheless have implications for preheating, moduli trapping and also nonequilibrium QFT more generally. For example, in [1] analytical and numerical studies of rescattering and IR cascading during inflation made it possible to observe, for the first time, the dynamical approach to the turbulent scaling regime that was discovered in [43,44].

Let us now discuss briefly the physics of the model (1). At the moment when $\phi = \phi_0$ (which we assume occurs during the observable range of e-foldings of inflation) the χ particles become instantaneously massless and are produced by quantum effects. This burst of particle production drains energy from the condensate $\phi(t)$, temporarily slowing the motion of the inflaton background and violating slow roll. Shortly after this moment the χ particles become extremely nonrelativistic so that their number density dilutes as a^{-3}, and eventually the inflaton settles back onto the slow-roll trajectory.

The dominant effect of particle production on the observable spectrum of curvature fluctuations arises because the produced, massive χ particles can rescatter off the condensate to generate bremsstrahlung radiation of long-wavelength $\delta\phi$ fluctuations via diagrams such as Fig. 1. Multiple such rescatterings lead to a rapid cascade of power into the IR. The inflaton modes generated by this IR cascading freeze once their wavelength crosses the horizon and lead to a bump-like feature in the primordial power spectrum. This bump-like feature is accompanied by a localized, uncorrelated non-Gaussian feature in the bispectrum [1].

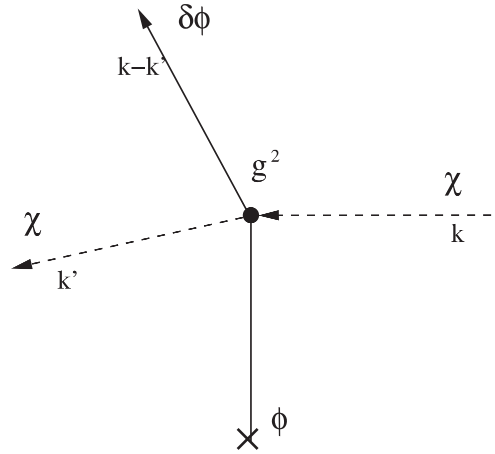

FIG. 1. Rescattering diagram.

In this paper, we extend previous work [1,2] on the model (1). First, we revisit the problem of quantifying the magnitude of the produced non-Gaussianity. Using lattice field theory simulations we compute numerically the skewness and kurtosis of the probability distribution function (PDF) of the primordial curvature fluctuations. By comparison to the more familiar local model of non-Gaussianity, we argue that the bispectrum associated with this mechanism may be observable in future missions.

Next, we provide a detailed analytical theory of the quantum production of χ particles and the subsequent rescattering off the slow-roll condensate for the model (1). This new formalism improves significantly upon previous efforts [1] by consistently incorporating both the expansion of the universe and also metric perturbations. We test our approach by comparison to fully nonlinear lattice field theory simulations, finding excellent agreement. We also use our formalism to estimate the shape of the bispectrum.

The outline of this paper is as follows. In Sec. II we review the key results of [1,2], describing heuristically the underlying mechanism of IR cascading and the resultant observational signatures. In Sec. III we characterize the size of the non-Gaussianity associated with particle production and IR cascading, relying primarily on lattice field theory simulations. In Sec. IV, we provide an analytical theory of inflationary particle production and IR cascading in the model (1), neglecting metric perturbations. Using this new formalism, we estimate the shape of the bispectrum in the model (1). In Sec. V, we reconsider our analytical approach, showing how metric perturbations can be consistently incorporated, and further, we demonstrate explicitly that their inclusion does not significantly alter the results of Sec. IV. Finally, in Sec. VI, we conclude.

II. OVERVIEW OF THE MECHANISM

In this section we provide a brief overview of the dynamics of particle production and IR cascading in the model (1), and we also summarize the key observational signatures. This section is largely a review of [1,2], the reader already familiar with those works may wish to skip ahead to the next section.

We consider the following model

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{g^2}{2}(\phi-\phi_0)^2\chi^2\right], \tag{2}$$

where R is the Ricci curvature constructed from the metric $g_{\mu\nu}$, ϕ is the inflaton field, and χ is the isoinflaton. As usual, we assume a flat Friedmann Robertson Walker space-time with scale factor $a(t)$

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)d\mathbf{x}^2 \tag{3}$$

and employ the reduced Planck mass $M_p \equiv (8\pi G_N)^{-1/2} \cong 2.43\times 10^{18}$ GeV. We leave the potential $V(\phi)$ driving inflation unspecified except to assume that it is sufficiently flat in the usual sense; that is, $\epsilon \ll 1$, $|\eta| \ll 1$ where

$$\epsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2, \qquad \eta \equiv M_p^2\frac{V''}{V} \tag{4}$$

are the usual slow-roll parameters.

The coupling $\frac{g^2}{2}(\phi-\phi_0)^2\chi^2$ in (2) is introduced to ensure that the isoinflaton field can become instantaneously massless at some point $\phi = \phi_0$ along the inflaton trajectory (which we assume occurs during the observable range of e-foldings of inflation). At this moment χ particles will be produced by quantum effects. (In Sec. IV we will discuss how particle production and rescattering are modified by the inclusion of a mass term $\mu^2\chi^2$ for the isoinflaton.)

A. Quantum production of χ particles

Let us first consider the homogeneous dynamics of the inflaton field, $\phi(t)$. Near the point $\phi = \phi_0$ we can generically expand

$$\phi(t) \cong \phi_0 + vt, \tag{5}$$

where $v \equiv \dot\phi(0)$, and we have arbitrarily set the origin of time so that $t = 0$ corresponds to the moment when $\phi = \phi_0$. The interaction (1) induces an effective (time-varying) mass for the χ particles of the form

$$m_\chi^2 = g^2(\phi-\phi_0)^2 \cong k_\star^4 t^2, \tag{6}$$

where we have defined the characteristic scale

$$k_\star = \sqrt{g|v|}. \tag{7}$$

It is straightforward to verify that the simple expression (6) will be a good approximation for $(H|t|)^{-1} \lesssim \mathcal{O}(\epsilon,\eta)$, which, in most models, will be true for the entire observable 60 e-foldings of inflation.

Note that, without needing to specify the background inflationary potential $V(\phi)$, we can write the ratio $k_\star/H$ as

$$\frac{k_\star}{H} = \sqrt{\frac{g}{2\pi\mathcal{P}_\zeta^{1/2}}}, \tag{8}$$

where $\mathcal{P}_\zeta^{1/2} = 5\times 10^{-5}$ is the usual amplitude of the vacuum fluctuations from inflation. In this work, we assume $k_\star > H$, which is easily satisfied for reasonable values of the coupling $g^2 > 10^{-7}$. In particular, for $g^2 \sim 0.1$ we have $k_\star/H \sim 30$.

The scenario we have in mind is the following. Inflation starts at some field value $\phi > \phi_0$, and the inflaton rolls toward the point $\phi = \phi_0$. Initially, the isoinflaton field is extremely massive $m_\chi \gg H$, and hence, it stays pinned in the vacuum, $\chi = 0$, and does not contribute to superhorizon curvature fluctuations. Eventually, at $t = 0$, the inflaton rolls through the point $\phi = \phi_0$, where $m_\chi = 0$ and χ particles are produced. To describe this burst of particle production one must solve the following equation for the χ-particle mode functions in an expanding universe

$$\ddot\chi_k + 3H\dot\chi_k + \left[\frac{k^2}{a^2} + k_\star^4 t^2\right]\chi_k = 0. \tag{9}$$

Equations of this type are well-studied in the context of preheating after inflation [38] and moduli trapping [30]. In the regime $k_\star > H$ particle production is fast compared to the expansion time[1] and one can solve (9) very accurately for the occupation number of the created χ particles

$$n_k = e^{-\pi k^2/k_\star^2}. \tag{10}$$

Very quickly after the moment $t = 0$, within a time $\Delta t \sim k_\star^{-1} \ll H^{-1}$, these produced χ particles become nonrelativistic ($m_\chi > H$), and their number density starts to dilute as a^{-3}.

Following the initial burst of particle production, there are two distinct physical effects that take place. First, the energetic cost of producing the gas of massive out-of-equilibrium χ particles drains energy from the inflaton condensate, forcing $\dot\phi$ to drop abruptly. This velocity dip is the result of the backreaction of the produced χ

[1] In the opposite regime, $k_\star \ll H$, the field χ will be light as compared to the Hubble scale for a significant portion of inflation. In this case, it is no longer consistent to treat the background dynamics as being effectively single-field, hence the scenario has changed considerably. We will not consider this possibility any further.

fluctuations on homogeneous condensate $\phi(t)$. The second physical effect is that the produced massive χ particles rescatter off the condensate via the diagram Fig. 1 and emit bremsstrahlung radiation of light inflaton fluctuations (particles). Backreaction and rescattering leave distinct imprints in the observable cosmological perturbations. Let us discuss each separately.

B. Backreaction effects

We first consider the impact of backreaction. This effect can be studied analytically using the mean field equation

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} + g^2(\phi - \phi_0)\langle\chi^2\rangle = 0, \tag{11}$$

where the vacuum average is computed following [30,38]

$$\langle\chi^2\rangle \cong \Theta(t)\frac{n_\chi a^{-3}}{g|\phi - \phi_0|}, \tag{12}$$

and $n_\chi = \int \frac{d^3k}{(2\pi)^3} n_k \sim k_\star^3$ is the total number density of produced χ particles. The Heaviside function $\Theta(t)$ in (12) enforces the fact that the backreaction effects become important only for $t > 0$, *after* the χ particles have been produced. The factor of a^{-3} in (12) reflects the usual volume dilution of nonrelativistic matter.

The solutions of (11) display the expected behavior: the energetic cost of the production of χ particles at $t = 0$ leads to an abrupt dip in the velocity $\dot{\phi}$, momentarily violating the smallness of the slow-roll parameter $\ddot{\phi}/(H\dot{\phi})$. Within a few e-foldings of the moments $t = 0$, the produced χ particles have become extremely massive and have been diluted away by the inflationary expansion of the universe. At this time, the inflaton must settle back onto the slow-roll trajectory, $\dot{\phi} \cong -V'/(3H)$.

Backreaction effects lead to a transient violation of slow-roll; and hence, we expect an associated ringing pattern (damped oscillations) in the primordial curvature fluctuations, similar to models with a sharp feature in the potential [14–16,45–50]. This effect can be seen by solving the well-known equation for the curvature perturbation on co-moving hypersurfaces, $\mathcal{R}$, in linear theory:

$$\mathcal{R}_k'' + 2\frac{z'}{z}\mathcal{R}_k' + k^2\mathcal{R}_k = 0. \tag{13}$$

Here, the prime denotes derivatives with respect to conformal time $\tau = \int^t a^{-1}(t')dt'$, and $z \equiv a\dot{\phi}/H$. Note that (13) is valid only in the absence of entropy perturbations. However, in our case the χ field is extremely massive, $m_\chi^2 \gg H^2$, for nearly the entire duration of inflation; hence, the direct isocurvature contribution to $\mathcal{R}$ is negligible.

In [1] the coupled system (11) and (13) was solved numerically and the expected ringing pattern in the power spectrum $P_\mathcal{R}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k|^2$ was obtained. (See also [7].) This effect is subdominant to the rescattering processes described in the next subsection; hence, we will not pursue backreaction any further in this work.

C. Rescattering effects

The second physical effect which takes place after the quantum production of χ particles in the model (1) is rescattering. This effect was considered for the first time in the context of inflationary particle production in [1]. Figure 1 illustrates the dominant process: bremsstrahlung emission of long-wavelength $\delta\phi$ fluctuations from rescattering of the produced χ particles off the condensate $\phi(t)$. The time scale for such processes is set by the microscopic scale, $k_\star^{-1}$, and is thus very short compared to the expansion time, H^{-1}. Moreover, the production of inflaton fluctuations $\delta\phi$ deep in the IR is extremely energetically inexpensive, since the inflaton is very nearly massless. The combination of the short time scale for rescattering, and the energetic cheapness of radiating IR $\delta\phi$ leads to a rapid build-up of power in long-wavelength inflaton modes: IR cascading. This effect leads to a bump-like feature in the power spectrum of inflaton fluctuations, very different from the ringing pattern associated with backreaction. The bump-like feature from rescattering dominates over the ringing pattern from backreaction for all values of parameters.

In [1] the model (2) was studied using lattice field theory simulations, without neglecting any physical processes (that is to say that full nonlinear structure of the theory, including backreaction and rescattering effects, was accounted for consistently). However, this same dynamics can be understood analytically by solving the equation for the inflaton fluctuations $\delta\phi$ in the approximation that all interactions are neglected, except for the diagram Fig. 1. The appropriate equation is

$$\delta\ddot{\phi} + 3H\delta\dot{\phi} - \frac{\vec{\nabla}^2}{a^2}\delta\phi + V_{,\phi\phi}\delta\phi \cong -g^2[\phi(t) - \phi_0]\chi^2. \tag{14}$$

The solution of (14) may be split into two parts: the solution of the homogeneous equation and the particular solution, which is due to the source term. Schematically, we have

$$\delta\phi(t,\mathbf{x}) = \underbrace{\delta\phi_{\rm vac}(t,\mathbf{x})}_{\rm homogeneous} + \underbrace{\delta\phi_{\rm resc}(t,\mathbf{x})}_{\rm particular}. \tag{15}$$

The former contribution is the homogeneous solution, which behaves as $\delta\phi_{\rm vac} \sim H/(2\pi)$ on large scales and, physically, corresponds to the usual scale-invariant vacuum fluctuations from inflation. The particular solution, $\delta\phi_{\rm resc}$, corresponds physically to inflaton fluctuations, which are generated by rescattering. The abrupt growth of χ inhomogeneities at $t = 0$ sources the particular solution $\delta\phi_{\rm resc}$, leading to the production of inflation fluctuations, which subsequently cross the horizon and become frozen.

A detailed analytical theory of Eq. (14) will be the subject of Secs IV and V. Here, we simply point out that the primordial power spectrum in the model (1) may, to good approximation, be described by a simple semianalytic fitting function [2]

$$P_{\mathcal{R}}(k) = A_s\left(\frac{k}{k_0}\right)^{n_s-1} + A_{\rm IR}\left(\frac{\pi e}{3}\right)^{3/2}\left(\frac{k}{k_{\rm IR}}\right)^3 e^{-(\pi/2)(k/k_{\rm IR})^2}, \tag{16}$$

where the first term corresponds to the usual vacuum fluctuations from inflation (with amplitude A_s and spectral index n_s); while the second term corresponds to the bump-like feature from particle production and IR cascading. The amplitude of this feature ($A_{\rm IR}$) depends on g^2, while the location ($k_{\rm IR}$) depends on ϕ_0.

In [2] the simple fitting function (16) was used to place observational constraints on inflationary particle production using a variety of cosmological data sets. Current data are consistent with rather large spectral distortions of the type (16). Features as large as $A_{\rm IR}/A_s \sim 0.1$ are allowed in the case that $k_{\rm IR}$ falls within the range of scales relevant for CMB experiments. A feature of this magnitude corresponds to a realistic coupling $g^2 \sim 0.01$. Even larger values of g^2 are allowed if the feature is localized on smaller scales. In [51] large-scale structure forecast constraints were considered for the model (16). It was shown that, for $k_{\rm IR} \lesssim 0.1\ {\rm Mpc}^{-1}$, the constraint on $A_{\rm IR}/A_s$ will be stengthened to the 0.5% level by Planck or 0.1% including also data from a Square Kilometer Array (SKA). With a Cosmic Inflation Probe (CIP), similar constraints could be achieved for $k_{\rm IR}$ as large as $1\ {\rm Mpc}^{-1}$.

D. Non-Gaussianity from particle production

The bump-like feature in $P(k)$, corresponding to the second term in (16), must be associated with a non-Gaussian feature in the bispectrum [1]. Indeed, it is evident already from inspection of Eq. (14) that the inflaton fluctuations generated by rescattering are significantly non-Gaussian; the particular solution of (14) is bilinear in the Gaussian field χ.

Non-Gaussian statistics have attracted a considerable amount of interest recently, owing to their potential as a tool for observationally discriminating between the plethora of inflationary models in the literature. Although the simplest single-field slow-roll models are known to produce negligible (primordial) non-Gaussianity [52–54], there are a currently a number of alternative models, which *may* predict an observable signature. Examples include models with preheating into light fields [10,55–57], nonlocal inflation [58], the curvaton mechanism [59], multifield models [60], constructions with a small sound speed [61] (such as DBI [62] inflation), trapped inflation [3], the gelaton [63], models with features or rapid oscillations in the inflaton potential [47,48], nonvacuum initial conditions [61,64–66], warm inflation [67], etc.

Non-Gaussianity is usually characterized in terms of the *bispectrum*, $B_\zeta(k_i)$, which is the 3-point correlation function of the Fourier transform of the primordial curvature fluctuation on uniform density hyper-surfaces, ζ. Explicitly, we define

$$\langle \zeta_{\mathbf{k}_1}\zeta_{\mathbf{k}_2}\zeta_{\mathbf{k}_3}\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_\zeta(k_i), \tag{17}$$

where $k_i \equiv |\mathbf{k_i}|$ and ζ_k is related to the variable $\mathcal{R}_k$ appearing in (13) as $\zeta_k \cong -\mathcal{R}_k$ on large scales $k \ll aH$. The delta function in (17) reflects translational invariance and ensures that $B_\zeta(k_i)$ depends on three wavenumbers which form a triangle: $\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}=0$. A general bispectrum $B_\zeta(k_i)$ may be characterized by specifying its size (amplitude of B_ζ), shape (whether B_ζ peaks on squeezed, equilateral or flattened triangles) and running (the dependence of B_ζ on the size of the triangle). The various non-Gaussian scenarios discussed above may be classified according to the size, shape and running of the bispectrum, see [68] for a more detailed review.

The non-Gaussian signature from IR cascading is very different from other models, such as the local, equilateral or enfolded shapes, which have been studied in the literature. IR cascading only influences modes leaving the horizon near the moment $\phi = \phi_0$ when particle production occurs; hence, we expect the bispectrum to be very far from scale invariant (this is also true for the model considered in [47,48]). The dominant contribution to $B_\zeta(k_i)$ should peak strongly for triangles with a characteristic size $\sim k_{\rm IR}$, corresponding to the location of the bump in the power spectrum (16). We will estimate the shape of the bispectrum from particle production and IR cascading in more detail in Sec. IV and revisit this issue also in an upcoming publication [69].

The unusual shape and strong scaling properties of the bispectrum from particle production makes it difficult to compare the magnitude of non-Gaussianity in this model to more familiar bispectra, such as the local shape, which are very close to scale invariant. We find it useful to quantify the magnitude of the non-Gaussianity in the model (1) by computing the moments of the PDF, $P(\zeta)$, which is the probability that the curvature perturbation has a fluctuations of size ζ. These moments carry information about the correlation functions of ζ integrated over all wavenumbers k_i and therefore provide a useful tool to compare models with very different shape/running properties [70]. (See also [71] for a related discussion and alternative methodology.)

Let us define the central moments of the PDF as

$$\langle \zeta^n\rangle = \int \zeta^n P(\zeta)d\zeta. \tag{18}$$

The n-th cummulant κ_n is the connected n-point function. For $\langle\zeta\rangle = 0$, the first few nonvanishing cummulants are:

$$\kappa_2 = \langle \zeta^2 \rangle \equiv \sigma_\zeta^2, \tag{19}$$

$$\kappa_3 = \langle \zeta^3 \rangle, \tag{20}$$

$$\kappa_4 = \langle \zeta^4 \rangle - 3\langle \zeta^2 \rangle^2, \tag{21}$$

$$\kappa_5 = \langle \zeta^5 \rangle - 10\langle \zeta^3 \rangle \langle \zeta^2 \rangle. \tag{22}$$

It is useful to introduce the dimensionless cummulants, defined as

$$\hat{\kappa}_n \equiv \frac{\kappa_n}{\langle \zeta^2 \rangle^{n/2}} \equiv \frac{\kappa_n}{\sigma_\zeta^n}. \tag{23}$$

For a Gaussian PDF, we have $\hat{\kappa}_n = 0$ for $n \geq 3$; hence, these quantify departures from gaussian statistics. When the non-Gaussianities are small, $|\hat{\kappa}_{n\geq 3}| \ll 1$, then the corrections to $P(\zeta)$ are well described by the Edgeworth expansion:

$$P(\zeta) = \frac{1}{\sqrt{2\pi}\sigma_\zeta} e^{-\zeta^2/(2\sigma_\zeta^2)} \left[1 + \frac{\hat{\kappa}_3}{3!} H_3\left(\frac{\zeta}{\sigma_\zeta}\right) + \cdots \right], \tag{24}$$

where $H_3(x) = x^3 - 3x$ is a Hermite polynomial, and the $\ldots$ denotes corrections of order $\hat{\kappa}_4$, $\hat{\kappa}_3^2$, and smaller. See [70,72,73] for more details and [74] for an alternative derivation.

E. Relation to other works

Before proceeding to study the model (2) in detail, it is worth commenting on the relationship between our analysis and previous works. Trapped inflation [3,30] is a very closely related model that uses multiple bursts of particle production, each similar to the event described in Sec. II A, in order to slow the motion of the inflaton on a steep potential. In that case, dissipation which results from the backreaction effects discussed in Sec. II B actually *dominate* over the friction term $3H\dot{\phi}$, which is due to the inflationary expansion of the universe. In contrast, here we assume that the inflaton potential $V(\phi)$ is sufficiently flat to support slow-roll inflation; see Eq. (4). In our scenario, dissipative effects on the homogeneous motion of $\phi(t)$ due to particle production (backreaction effects) are always negligible; see Ssec. II B. Nevertheless, we expect that many of our results and analytical techniques will also be applicable in the case of trapped inflation.

Trapped inflation was not the first model to attempt to exploit dissipative effects in order to assist in slowing the motion of the inflaton. Another interesting construction of this type is *warm inflation* [24,32–34], which employs a gas of particles in thermal equilibrium. Again, our analysis is distinguished from this mechanism since we assume that the homogeneous dynamics of the inflaton $\phi(t)$ are of the usual slow-roll type.

Our main focus in this work is to determine the observational consequences of isolated burst of particle production on cosmological observables such as the spectrum and non-Gaussianity of the primordial fluctluations. In this sense, the spirit of our investigation is more similar to works such as [4–7] than to warm inflation or trapped inflation. However, unlike those papers, we have consistently accounted for rescattering effects, which provide the dominant contribution to observables in the case at hand; see Sec. II C.

III. NON-GAUSSIANITY OF THE PROBABILITY DISTRIBUTION FUNCTION

In order to quantify the magnitude of the non-Gaussianity generated by particle production, let us now consider the PDF in the model (1). We proceed numerically, revisiting the lattice field theory simulations performed in [1] using the HLattice code [75]. For illustration, we assume the standard chaotic inflation model $V(\phi) = m^2\phi^2/2$ with $m \cong 10^{-6}\sqrt{8\pi}M_p$ and $\phi_0 = 3.2\sqrt{8\pi}M_p$. We consider three different choices of coupling, $g^2 = 1, 0.1, 0.01$. Our simulations are performed in a 512^3 box whose co-moving size is initially ~ 3 times the horizon size H^{-1}. We run our simulations for roughly 3 e-foldings from the moment when $\phi = \phi_0$, which is more than enough to see the feature from IR cascading freeze out as an observable, superhorizon density fluctuation. Our choice of ϕ_0 ensures that the feature will be frozen-in at scales slightly smaller than the current horizon. Note that our quantitative results do not depend sensitively on the choice of ϕ_0, nor on the details of the background inflationary potential $V(\phi)$; see [1] for further discussion.

We extract the PDF of $\delta\phi$ from our HLattice simulations by measuring the fraction of the simulation box, which contains the fluctuation field $\delta\phi$ at a particular value. Notice that this approach is completely nonperturbative: it does not rely on the validity of the Edgeworth expansion, nor does it assume anything about the size or ordering of the cummulants. This procedure implicitly puts an IR cutoff at the box size L and an UV cutoff at the lattice spacing, Λ^{-1}. Since the non-Gaussian effects in our model are strongly localized in Fourier space, our quantitative results are largely insensitive to L and Λ.

In Fig. 2 we plot our numerical result for the PDF of the inflaton fluctuations generated by rescattering and IR cascading. In order to make the physics of inflationary particle production clear, we have subtracted off the contribution coming from the usual vacuum fluctuations of the inflaton. That is, the PDF in Fig. 2 is associated only with the contribution $\delta\phi_{\mathrm{resc}}$ in Eq. (15).

We can understand physically the behavior of PDF plotted in Fig. 2. Shortly after the initial burst of particle production, the inflaton perturbations $\delta\phi$ are extremely non-Gaussian, due to the sudden appearance of the source term $J \propto \chi^2$ in the equation of motion (14). Very quickly, in less than an e-folding, nonlinear interactions begin to drive the system towards gaussianity. A very similar

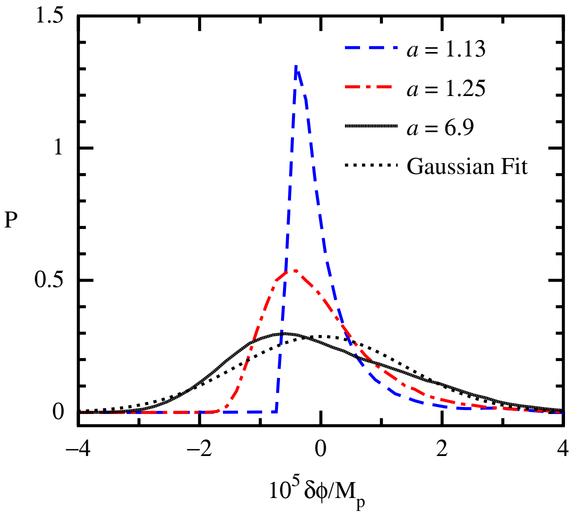

FIG. 2 (color online). The PDF of the inflaton fluctuations generated by rescattering and IR cascading, at a series of different values of the scale factor, a. The dotted black curve shows a Gaussian fit at late times and we have normalized the scale factor so that $a = 1$ at the moment when particle production occurs. For illustration, we have chosen $g^2 = 0.1$ and a standard chaotic inflation potential $V(\phi) = m^2\phi^2/2$.

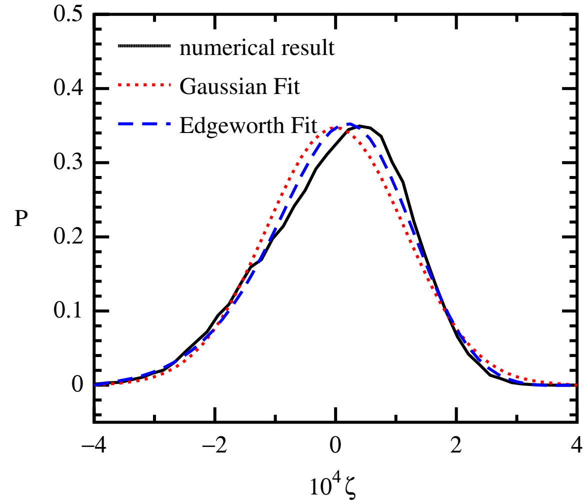

FIG. 3 (color online). The PDF of the total curvature fluctuation, ζ, at late times (well after all relevant modes have crossed the horizon and frozen). The solid black curve is the exact result from our HLattice simulations, and the dotted red curve is a gaussian fit. We have also plotted the leading correction to the gaussian result in the Edgeworth expansion, given explicitly by Eq. (24). For illustration, we have chosen $g^2 = 0.1$ and a standard chaotic inflation potential $V(\phi) = m^2\phi^2/2$.

behavior has been observed in lattice simulations of out-of-equilibrium interacting scalar fields during preheating [76,77]. In the case of rescattering during preheating, the system will eventually become Gaussian when the fields thermalize. However, in our case the universe is still inflating. As a result, non-Gaussian inflaton fluctuations generated by rescattering are stretched out by the quaside Sitter expansion and must freeze once their wavelength crosses the Hubble scale. Hence, at late times the PDF does not become completely gaussian, but rather freezes in with some nontrivial skewness. Within a few e-foldings from the moment of particle production, the time evolution of the PDF has become completely negligible.

In order to characterize the non-Gaussianity of the observable primordial fluctuations, we would like to construct the PDF for the curvature perturbation ζ, including *both* the contributions from the vacuum fluctuations of the inflaton and also from rescattering. To this end, we construct ζ using the naive relation $\zeta = -\frac{H}{\dot\phi}\delta\phi$ (see Sec. V for justification) and take into account both contributions to $\delta\phi$ in Eq. (15). In Fig. 3 we plot the full PDF obtained in this manner, evaluated at very late times, well after all relevant modes have crossed the horizon and become frozen.

Given our numerical results for the PDF of the total observable curvature fluctuation, that is Fig. 3, it is straightforward to compute dimensionless cummulants (23) such as the skewness ($\hat\kappa_3$), kurtosis ($\hat\kappa_4$), and noltosis ($\hat\kappa_5$) for various values of the coupling g^2. We have summarized our results in Table I. Note that for $g^2 = 0.01$ both $\hat\kappa_4$ and $\hat\kappa_5$ are too small to be measured accurately from our HLattice simulations.

In order to give some sense of the magnitude of the non-Gaussianity from particle production we have also computed an equivalent $f_{\rm NL}^{\rm local}$ defined by $5\hat\kappa_3/(18\sigma_\zeta)$, where the variance is $\sigma_\zeta \equiv \langle\zeta^2\rangle^{1/2} \sim 10^{-9/2}$. For a given g^2, this effective $f_{\rm NL}^{\rm local}$ is the magnitude of $f_{\rm NL}$, which would be necessary to reproduce the skewness $\hat\kappa_3$ of the IR cascading PDF using a local ansatz $\zeta = \zeta_g + \frac{3}{5}f_{\rm NI}[\zeta_g^2 - \langle\zeta_g^2\rangle]$.[2]

From Table I we see that IR cascading during inflation can generate significant non-Gaussianity. Even taking $g^2 = 0.01$ (which is compatible with cosmological data for any choice of ϕ_0 [2]) we still obtain a skewness $\hat\kappa_3 = -0.006$, which is the same value that would be produced by a local model with $f_{\rm NL} \sim -53$. This equivalent local non-Gaussianity is comparable to current observational bounds and is well within the expected accuracy of future missions. This suggests that non-Gaussian features from particle production during inflation might be observable for reasonable values of g^2.

The equivalent $f_{\rm NL}^{\rm local}$ values presented in Table I must be interpreted with care. We have included this information only to give a heuristic sense of the magnitude of non-Gaussianity in our model. It must be stressed that the PDF plotted in Fig. 3 is quite different from the analogous result for local-type non-Gaussianity. For example, the value of

[2] Our sign conventions for $f_{\rm NL}$ are consistent with Wilkinson Microwave Anisotropy Probe [78]. See [70] for a discussion of various conventions employed in the literature.

TABLE I. Moments of the Probability Distribution Function.

g^2	Skewness	Kurtosis	5-th moment	Equivalent
	$(\hat{\kappa}_3)$	$(\hat{\kappa}_4)$	$(\hat{\kappa}_5)$	$f_{\rm NL}^{\rm local}$
1	-0.51	0.2	1.2	-4500
0.1	-0.49	-0.1	1.5	-4300
0.01	-0.006	$<\mathcal{O}(10^{-3})$	$<\mathcal{O}(10^{-3})$	-53

the kurtosis (and higher moments) are different, as is the ordering of the cummulants. Moreover, we should emphasize that observational bounds on $f_{\rm NL}^{\rm local}$ *cannot* be directly applied to our model since the bispectrum in our case is uncorrelated with the vacuum fluctuations and is far from scale invariant. A detailed study of the detectability of non-Gaussianity from particle production will be the subject of a upcoming publication [69].

Depending on the value of ϕ_0, the model (1) may lead to a variety of observable signatures. As discussed previously, ϕ_0 controls the location of the feature in the primordial power spectrum (16). Non-Gaussian effects are also localized near the same characteristic scale, $k_{\rm IR}$. If $k_{\rm IR}$ corresponds to scales relevant for CMB experiments, then we predict a bump-like feature in the primordial power spectrum, $P_\zeta(k)$, and an associated feature in the bispectrum, $B_\zeta(k_i)$, with an unusual shape (that will be discussed in Sec. IV). A key question is whether the non-Gaussian feature can be observably large in a regime where the power spectrum feature is small enough to be compatible with current observations. Preliminary results are encouraging: for $g^2 = 0.01$ the power spectrum is consistent with all observational data [2] while the skewness of the PDF is rather large. A detailed investigation will require a simple, separable template for the bispectrum and will be discussed in a future publication [69].

On the other hand, we could imagine a scenario in which the feature from IR cascading shows up on smaller scales, relevant for LSS experiments [70,79–81]. In this case our scenario could be probed using higher order correlations of LSS probes (such as the galaxy bispectrum) or the abundance of collapsed objects (or voids). The latter possibility is interesting since the cluster/void abundance is determined the tails of the PDF and may be insensitive to the detailed shape of the bispectrum. Quantitative predictions for observable cluster/void abudances require the PDF of the evolved density field, smoothed on some relevant scale [69], rather than the PDF of the primordial curvature perturbation (which is plotted in Fig. 3). However, we can nevertheless describe the qualitative signatures that should be expected. Our model robustly predicts a negative skewness for both the curvature perturbation, ζ, and the density field, $\delta\rho/\rho$. Hence, we should expect a *decrease* in the abundance of the largest collapsed objects and an *increase* in the abundance of the largest voids [82,83]. Owing to the localized nature of the bispectrum feature, we expect that this effect should show up only when the density field is smoothed on a scale close to $k_{\rm IR}$.

It is worth mentioning that recent weak lensing measurement of the dark matter mass of the high-redshift galaxy cluster XMMUJ2235.3-2557 [84] have been construed as a possible hint of non-Gaussian initial conditions [85]. Unfortunately, our model does not produce the correct sign of skewness to explain such observations.

IV. ANALYTICAL FORMALISM

In [1] we studied particle production, rescattering and IR cascading using nonlinear lattice field theory simulations. In addition to this numerical approach, a cursory analytical formalism was also presented. Here, we revisit the analytical analytical theory of particle production, rescattering and IR cascading in an expanding universe, in order to better understand the results of [1] from a physical perspective.

A. The prototype model

We consider now the theory

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{1}{2}(\partial\chi)^2 - \frac{\mu^2}{2}\chi^2 - \frac{g^2}{2}(\phi-\phi_0)^2\chi^2\right], \tag{25}$$

which differs from our original model (2) by the inclusion of a mass term $\Delta\mathcal{L} = -\mu^2\chi^2/2$ for the isoinflaton. Such a term is not forbidden by any symmetry; and hence, one typically expects it to be generated by radiative corrections, even if the isoinflaton is classically massless at $\phi = \phi_0$. The new parameter μ has the effect of reducing the efficiency of the particle production effects discussed in Sec. II A; the time-varying mass of the isoinflaton

$$m_\chi^2 = \mu^2 + g^2(\phi - \phi_0)^2$$

does not vanish at $\phi = \phi_0$, but rather reaches a minimum value μ^2, making the adiabaticity condition more difficult to violate. A concern is the possibility that radiative corrections induce a large μ and suppress the observable effects associated with inflationary particle production. Indeed, it is well-known that fine-tuning may be required to keep the mass of any scalar field significantly below the cutoff scale associated with the validity of the effective field theory description (25). Following, we will show that the suppression of χ-particle production is not significant provided the following condition is satisfied

$$\mu^2 \ll k_\star^2, \tag{26}$$

where $k_\star \equiv \sqrt{g|v|}$. Depending on how (25) is embedded within a more complete framework, the constraint (26) may (or may not) require fine-tuning to satisfy. Below, we will show that the condition (26) is quite naturally satisfied for a large number of microscopically realistic constructions.

Our prototype model (25) has been chosen to elucidate the key physics and observational signatures of inflationary particle production in a simple framework wherein computations are tractable. We expect, however, that many of our qualitative results will carry over to more complicated scenarios. In particular, one might wish to supplement the action (25) by its SUSY completion; see the interesting work [24] for an explicit example. Such an embedding has the advantage that the flatness of the inflaton potential $V(\phi)$ may be protected from large radiative corrections coming from loops of the χ field. Moreover, a SUSY embedding of the model (25) also allows for some control over the quantum corrections to the mass scale μ.

For models obtained from string theory or supergravity (SUGRA), it is natural to have μ of order the Hubble scale[3] during inflation [86–88]; hence, we expect $\mu^2 \sim H^2$ for such models. In that case, the constraint (26) is automatically satisfied because $k_\star^2 \gg H^2$ whenever particle production is fast as compared to the expansion time (that is, for reasonable values of the coupling $g^2 > 10^{-7}$, which we assume throughout this work). Hence, there exists a very large class of realistic microscopic models in which radiative effects will not spoil the observational consequences of inflationary particle production and IR cascading.

Although the condition for the efficiency of particle production—that is Eq. (26)—can be easily satisfied for models coming from string theory or SUSY, we prefer to remain agnostic regarding how the prototype action (25) is embedded within a more complete framework. Throughout our analysis we will keep the inflaton potential $V(\phi)$ and the isoinflaton mass parameter μ more-or-less arbitrary. (We assume that the slow-roll conditions are satisfied, and also that $\mu^2 \geq 0$.) This phenomenological approach is not different from the philosophy that is employed in the majority of work on inflationary cosmology, since the slow-roll conditions (4) may be sensitive to UV physics whose detailed form is often not specified. The question of how the model (25), with a given choice of $V(\phi)$ and μ, arises from some complete model of particle physics is interesting. However, this question it is not the main focus of the current investigation. We refer the reader to [2] for several example microscopic embeddings within string theory and also SUSY (see also [3]).

[3]In the context of SUGRA, the finite energy density driving inflation breaks SUSY and induces soft scalar potentials with curvature of order $V''_{\rm soft} \sim \mu^2 \sim H^2$ [86]. In the case of string theory, many scalars are conformally coupled to gravity [87] through an interaction of the form $\delta\mathcal{L} = -\frac{1}{12}R\chi^2$, where the Ricci scalar is $R \sim H^2$ during inflation. More generally, any nonminimal coupling $\delta\mathcal{L} = -\frac{\xi}{2}R\chi^2$ between gravity and the isoinflaton will induce a contribution of order H to the effective mass of χ, as long as $\xi = \mathcal{O}(1)$.

Let us now proceed to develop an analytical formalism to study inflationary particle production in the model (25). The equations of motion that we wish to solve are

$$-\Box\phi + V'(\phi) + g^2(\phi - \phi_0)\chi^2 = 0, \tag{27}$$

$$-\Box\chi + [\mu^2 + g^2(\phi - \phi_0)^2]\chi = 0, \tag{28}$$

where $\Box = g_{\mu\nu}\nabla^\mu\nabla^\nu$ is the covariant d'Alembertian. It will be useful to work with conformal time τ, related to cosmic time t via $a d\tau = dt$. In terms of conformal time the metric takes the form

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}\cdot d\mathbf{x} = a^2(\tau)[-d\tau^2 + d\mathbf{x}\cdot d\mathbf{x}]. \tag{29}$$

We denote derivatives with respect to cosmic time as $\dot{f} \equiv \partial_t f$ and with respect to conformal time as $f' \equiv \partial_\tau f$. The Hubble parameter $H = \dot{a}/a$ has conformal time analogue $\mathcal{H} = a'/a$. For an inflationary (quaside Sitter) phase ($H \cong$ const), one has

$$a = -\frac{1}{H\tau}\frac{1}{1-\epsilon}, \qquad \mathcal{H} = -\frac{1}{\tau}\frac{1}{1-\epsilon} \tag{30}$$

to leading order in the slow-roll parameter $\epsilon \ll 1$.

As discussed in Sec. II, the motion of the homogeneous inflaton $\phi(t)$ leads to the production of a gas of χ particles at the moment $t = 0$ when $\phi = \phi_0$. The first step in our analytical computation is to describe this burst of particle production in an expanding universe. Following the initial burst, both backreaction and rescattering effects take place. Our formalism will focus on the latter effect, which has been shown to be much more important [1].

B. Particle production in an expanding universe

The first step in our scenario is the quantum mechanical production of χ-particles due to the motion of ϕ. To understand this effect we must solve the equation for the χ fluctuations in the rolling inflaton background. Approximating $\phi \cong \phi_0 + vt$ Eq. (28) gives

$$\ddot{\chi} + 3H\dot{\chi} - \frac{\vec{\nabla}^2}{a^2}\chi + [\mu^2 + k_\star^4 t^2]\chi = 0, \tag{31}$$

where $k_\star \equiv \sqrt{g|v|}$. We remind the reader that $k_\star \gg H$ for reasonable values of the coupling, see Eq. (8).

The flat-space analogue of Eq. (31) is very well understood from studies of broadband parametric resonance during preheating [38] and also moduli trapping at enhanced symmetry points [30]. One does not expect this treatment to differ significantly in our case since both the time scale for particle production Δt and the characteristic wavelength of the produced fluctuations λ are small compared to the Hubble scale. Hence, we expect that the occupation number of produced χ particles will not differ significantly from the flat-space result, at least on scales $k \gtrsim H$. Furthermore, notice that the χ field is extremely massive for most of inflation. Indeed, even in the case $\mu^2 = 0$, we have

$$\frac{m_\chi^2}{H^2} \cong \frac{k_\star^4 t^2}{H^2}. \tag{32}$$

Since $k_\star \gg H$, it follows that $m_\chi^2 \gg H^2$, except in a tiny interval $H|\Delta t| \sim (H/k_\star)^2$, which amounts to roughly 10^{-3} e-foldings for $g^2 \sim 0.1$. Therefore, we do not expect any significant fluctuations of χ to be produced on superhorizon scales $k \lesssim H$. (Allowing for $\mu^2 \neq 0$ only strengthens this conclusion.)

Let us now consider the solutions of Eq. (31). We work with conformal time τ and write the Fourier transform of the quantum field χ as

$$\chi(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{\xi_{\mathbf{k}}^\chi(\tau)}{a(\tau)} e^{i\mathbf{k}\cdot\mathbf{x}}. \tag{33}$$

Note the explicit factor of a^{-1} in (33), which is introduced to give $\xi_{\mathbf{k}}^\chi$ a canonical kinetic term. The q-number valued Fourier transform $\xi_{\mathbf{k}}^\chi(\tau)$ can be written as

$$\xi_{\mathbf{k}}^\chi(\tau) = a_{\mathbf{k}} \chi_k(\tau) + a_{-\mathbf{k}}^\dagger \chi_k^\star(\tau), \tag{34}$$

where the annihilation/creation operators satisfy the usual commutation relation

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}'), \tag{35}$$

and the c-number valued mode functions χ_k obey the following oscillatorlike equation

$$\chi_k''(\tau) + \omega_k^2(\tau)\chi_k(\tau) = 0. \tag{36}$$

The time-dependent frequency is

$$\begin{aligned} \omega_k^2(\tau) &= k^2 + a^2 m_\chi^2(\tau) - \frac{a''}{a} \\ &\cong k^2 + \frac{1}{\tau^2}\left[\frac{k_\star^4}{H^2} t^2(\tau) + \left(\frac{\mu}{H}\right)^2 - 2\right], \end{aligned} \tag{37}$$

where

$$m_\chi^2(\tau) = \mu^2 + g^2(\phi - \phi_0)^2 \cong \mu^2 + k_\star^4 t^2(\tau) \tag{38}$$

is the time-dependent effective mass of the χ particles, and

$$t(\tau) = \frac{1}{H} \ln\left(\frac{-1}{H\tau}\right) \tag{39}$$

is the usual cosmic time variable. We have arbitrarily set the origin of conformal time so that $\tau = -1/H$ corresponds to the moment when $\phi = \phi_0$.

It is useful to define the occupation number n_k of the χ of particles with momentum $\mathbf{k}$, defined as the energy of the mode $\frac{1}{2}|\chi_k'|^2 + \frac{1}{2}\omega_k^2|\chi_k|^2$ divided by the energy ω_k of each particle. Explicitly, we define

$$n_k = \frac{\omega_k}{2}\left[\frac{|\chi_k'|^2}{\omega_k^2} + |\chi_k|^2\right] - \frac{1}{2}, \tag{40}$$

where the term $-\frac{1}{2}$ comes from extracting the zero-point energy of the linear harmonic oscillator (see [38] for a review). Our definition (40) coinicides with the usual notion of particle number in the asymptotic adiabatic regimes ($|t| \gtrsim k_\star^{-1}$). During the very brief nonadiabatic period ($|t| \ll k_\star^{-1}$) our result coincides with the usual notion of quasiparticle number, obtained by instantaneous diagonalization of the Hamiltonian.

Let us now try to understand analytically the behavior of the solutions of (36). At early times $t \ll -k_\star^{-1}$, the frequency ω_k varies adiabatically

$$\left|\frac{\omega_k'}{\omega_k^2}\right| \ll 1. \tag{41}$$

In this in-going adiabatic regime, the modes χ_k are not excited; and the solution of (36) is well described by the adiabatic solution $\chi_k(\tau) = f_k(\tau)$, where

$$f_k(\tau) \equiv \frac{1}{\sqrt{2\omega_k(\tau)}} \exp\left[-i\int^\tau d\tau' \omega_k(\tau)\right]. \tag{42}$$

We have normalized (42) to be pure positive frequency so that the state of the isoinflaton field at early times corresponds to the adiabatic vacuum with no χ particles. (Inserting (42) into (40) one finds $n_k = 0$ for the adiabatic solution, as expected.)

The adiabatic solution (42) ceases to be a good approximation very close to the moment when $\phi = \phi_0$, that is at times $|t| \lesssim k_\star^{-1}$. In this regime the adiabaticity condition (41) is violated for modes with wave-number $H \lesssim k \lesssim \sqrt{k_\star^2 - \mu^2}$, and χ particles within this momentum band are produced. During the nonadiabatic regime we can still represent the solutions of (36) in terms of the functions $f_k(\tau)$ as

$$\chi_k(\tau) = \alpha_k(\tau) f_k(\tau) + \beta_k(\tau) f_k^\star(\tau). \tag{43}$$

This expression affords a solution of (36) provided the time-dependent Bogoliubov coefficients obey the following set of coupled equations

$$\alpha_k'(\tau) = \frac{\omega_k'(\tau)}{2\omega_k(\tau)} \exp\left[+2i\int^\tau d\tau' \omega_k(\tau')\right]\beta_k(\tau), \tag{44}$$

$$\beta_k'(\tau) = \frac{\omega_k'(\tau)}{2\omega_k(\tau)} \exp\left[-2i\int^\tau d\tau' \omega_k(\tau')\right]\alpha_k(\tau). \tag{45}$$

The Bogoliubov coefficients are normalized as $|\alpha_k|^2 - |\beta_k|^2 = 1$ and the assumption that no χ particles are present in the asymptotic past[4] fixes the initial conditions $\alpha_k = 1$, $\beta_k = 0$ for $t \to -\infty$. This is known as the adiabatic initial condition.

[4] This assumption is justified since any initial excitation of χ would have been damped out exponentially fast by the expansion of the universe.

From the structure of Eqs. (44) and (45), it is clear that violations of the condition (41) near $t = 0$ leads to a rapid growth in the $|\beta_k|$ coefficient. The time variation of β_k can be interpreted as a corresponding growth in the occupation number. Inserting (43) into (40) we find

$$n_k = |\beta_k|^2. \tag{46}$$

At late times ($t \gtrsim k_\star^{-1}$) adiabaticity is restored, and the growth of $n_k = |\beta_k|^2$ must saturate. By inspection of Eqs. (44) and (45), we can see that the Bogoliubov coefficients must tend to constant values in the out-going adiabatic regime. Therefore, within less than an e-folding from the moment of particle production the solution χ_k of Eq. (36) can be represented as a simple superposition of positive frequency f_k modes and negative frequency $f_k^\star$ modes. Our goal now is to derive an analytical expression for the modes χ_k, which is valid in this out-going adiabatic region.

First, we seek an expression for the Bogoliubov coefficients α_k, β_k in the out-going adiabatic regime $t \gtrsim k_\star^{-1}$. From (44) and (45) it is clear that the value of the Bogoliubov coefficients at late times can depends only on dynamics during the interval $|t| \lesssim k_\star^{-1}$ where the adiabaticity condition (41) is violated. This interval is tiny compared to the expansion time and we are justified in treating $a(\tau)$ as roughly constant during this phase. Hence, it follows that the flat-space computation of the Bogoliubov coefficients [30,38] must apply, at least for scales $k \gtrsim H$. To a very good approximation we therefore have the well-known result,

$$\alpha_k \cong \sqrt{1 + e^{-\pi\mu^2/k_\star^2} e^{-\pi k^2/k_\star^2}}, \tag{47}$$

$$\beta_k \cong -ie^{-\pi\mu^2/(2k_\star^2)} e^{-\pi k^2/(2k_\star^2)}, \tag{48}$$

in the out-going adiabatic regime. Equation (48) gives the usual expression[5] for the co-moving occupation number of particles produced by a single burst of broadband parametric resonance:

$$n_k = |\beta_k|^2 = e^{-\pi\mu^2/k_\star^2} e^{-\pi k^2/k_\star^2}. \tag{49}$$

Comparing Eqs. (49) and (10), we see that the mass parameter μ for the isoinflaton has the effect of suppressing the number density of produced χ particles by an amount $e^{-\pi\mu^2/k_\star^2} \leq 1$. This suppression reflects the reduced phase space of produced particles: the adiabaticity condition is violated only for modes with $k < \sqrt{k_\star^2 - \mu^2}$. Notice that the suppression of χ particle production is negligible when $\mu^2 \ll k_\star^2$, precisely the condition (26) that was alluded to earlier. For the remainder of this work we will assume that $\mu \lesssim k_\star$, since in the opposite regime the observational signatures of particle production effects are exponentially suppressed.

Next, we seek an expression for the adiabatic solution $f_k(\tau)$ in the out-going regine $t \gtrsim k_\star^{-1}$. We assume $\mu \lesssim k_\star$ and also focus on the interesting region of phase space, $H \lesssim k \lesssim \sqrt{k_\star^2 - \mu^2}$. In this case, the adiabatic solution (42) is very well approximated by

$$f_k(\tau) \cong \frac{1}{a^{1/2} k_\star \sqrt{2t(\tau)}} e^{-(i/2)k_\star^2 t^2(\tau)}, \tag{50}$$

where $t(\tau)$ is defined by (39). It is interesting to note that Eq. (50) is identical to the analogous flat-space result [1], except for the factor of $a^{-1/2}$. Taking into account also the explicit factor of a^{-1} in our definition of the Fourier transform (33), we recover the expected large-scale behavior for a massive field in de Sitter space, that is $\chi \sim a^{-3/2}$. This dependence on the scale factor is easy to understand physically, it simply reflects the volume dilution of non-relativistic particles: $\rho_\chi \sim m_\chi^2 \chi^2 \sim a^{-3}$. Notice that the parameter μ does not appear in (50). This is so because, for the time-varying mass of the χ field, Eq. (38), is dominated by the interaction term when $k_\star |t| \gtrsim 1$ and $\mu \lesssim k_\star$.

Finally, we arrive at an expression for the out-going adiabatic χ modes, which is accurate for interesting scales $H \lesssim k \lesssim \sqrt{k_\star^2 - \mu^2}$. Putting together the results (50) and (43) along with the well-known expressions (47) and (48), we arrive at

$$\begin{aligned}\chi_k(\tau) \cong{}& \sqrt{1 + e^{-\pi\mu^2/k_\star^2} e^{-\pi k^2/k_\star^2}} \frac{1}{a^{1/2} k_\star \sqrt{2t(\tau)}} e^{-(i/2)k_\star^2 t^2(\tau)} \\ & - ie^{-\pi\mu^2/(2k_\star^2)} e^{-\pi k^2/(2k_\star^2)} \frac{1}{a^{1/2} k_\star \sqrt{2t(\tau)}} e^{+(i/2)k_\star^2 t^2(\tau)}\end{aligned} \tag{51}$$

valid for $t \gtrsim k_\star^{-1}$. Equation (51) is the main result of this subsection. We will now justify that this expression is quite sufficient for our purposes.

For modes deep in the UV, $k \gtrsim k_\star$, our expression (51), is not accurate.[6] However, such high momentum particles are not produced, the condition (41) is always satisfied for $k \gg k_\star$. Note that the absence of particle production deep in the UV is built into our expression (51): as $k \to \infty$, this function tends to the vacuum solution $\chi_k \to f_k$.

Our expression (51) is also not valid deep in the IR, for modes $k < H$. To justify this neglect requires somewhat more care. Notice that, even very far from the point $\phi = \phi_0$ long-wavelength modes $k \ll H$ should not be

[5]Our result for the Bogoliubov coefficients is consistent with [30]. In that work μ was interpreted as an impact parameter for the motion of the modulus, whereas in our work we interpret this as a bare mass term. This distinction has no impact on the result for the occupation number because, in both cases, the parameter appears in the same way in the equation of motion for the fluctuations of χ.

[6]The expression (50) for the adiabatic modes f_k is not valid at high momenta, where $\omega_k \cong k$.

thought of as particlelike. The large-scale mode functions are not oscillatory but rather damp exponentially fast as $\chi \sim a^{-3/2}$. Hence, even if we started with some superhorizon fluctuations of χ at the beginning of inflation, these would be suppressed by an exponentially small factor before the time when particle production occurs. Any superhorizon fluctuation generated near $t = 0$ would need to be exponentially huge to overcome this damping. However, resonant particle production during inflation does *not* lead to exponential growth of mode functions.[7]

To verify explicitly that there is no significant effect for superhorizon fluctuations, let us consider solving Eq. (31) neglecting gradient terms. The equation we wish to solve, then, is

$$\partial_t^2(a^{3/2}\chi) + \left[k_\star^4 t^2 - \frac{9}{4}H^2\right](a^{3/2}\chi) = 0. \qquad (52)$$

(For simplicity, we take $\mu = 0$ and $\epsilon = 0$ for this paragraph, however, this has no effect on our results.) The solution of this equation may be written in terms of parabolic cylinder functions $D_\nu(z)$ as

$$\chi(t, \mathbf{x}) \sim \frac{1}{a^{3/2}}\Big(C_1 D_{-(1/2)+(9H^2/8k_\star^2)i}[(1+i)k_\star t] + C_2 D_{-(1/2)-(9H^2/8k_\star^2)i}[(-1+i)k_\star t]\Big). \qquad (53)$$

For our purposes the precise values of the coefficients C_1, C_2 are not important. Rather, it suffices to note that for $k_\star|t| \gtrsim 1$ the function (53) behaves as

$$\chi(t, \mathbf{x}) \sim |t|^{-1/2} e^{-3Ht/2} \times [\text{oscillatory}]. \qquad (54)$$

This explicit large-scale asymptotics confirms our previous claims that the superhorizon fluctuations of χ damp to zero exponentially fast, as $a^{-3/2} \sim e^{-3Ht/2}$. As discussed previously, this damping is easy to understand in terms of the volume dilution of nonrelativistic particles. (If we had included the parameter μ^2 in Eq. (52), then our conclusions would only be strengthened, since this parameter has the effect of making the isoinflaton even more massive.) We can also understand the power-law damping that appears in (54) from a physical perspective. The properly normalized modes behave as $a^{3/2}\chi \sim \omega_k^{-1/2}$, while on large scales we have $\omega_k \sim |m_\chi| \sim |t|$. Hence, the late-time damping factor $t^{-1/2}$, which appears in (54), reflects the fact that the χ particles become ever more massive as ϕ rolls away from the point ϕ_0.

Finally, it is straightforward to see that the function (53) does not display any exponential growth near $t = 0$. Hence, we conclude that there is no significant generation of superhorizon χ fluctuations due to particle production.[8]

In this subsection we have seen that the quantum production of χ particles in an expanding universe proceeds very much as it does in flat space. This is reasonable since particle production occurs on a time scale short compared to the expansion time and involves modes which are inside the horizon at the time of production.

C. Inflaton fluctuations

In Sec. IV B we studied the quantum production of χ particles, which occurs when ϕ rolls past the massless point $\phi = \phi_0$. Subsequently, there are two distinct physical processes that take place: backreaction and rescattering. As we discussed, the former effect has a negligible impact of the observable spectrum of cosmological perturbations and may be neglected.

In this subsection we study the rescattering of produced χ particle off the inflaton condensate. The dominant process to consider is the diagram illustrated in Fig. 1, corresponding to bremsstrahlung emission of $\delta\phi$ fluctuations (particles) in the background of the external field. (There is also a subdominant process of the type $\chi\chi \to \delta\phi\delta\phi$, which is phase space suppressed.) Take into account only the rescattering diagram illustrated in Fig. 1 is equivalent to solving the following equation for the q-number inflaton fluctuation

$$\left[\partial_t^2 + 3H\partial_t - \frac{\vec{\nabla}^2}{a^2} + m^2\right]\delta\phi = -g^2[\phi(t) - \phi_0]\chi^2, \quad (55)$$

where we have introduced the notation $m^2 \equiv V_{,\phi\phi}$ for the inflaton effective mass. (Note that we are not assuming a background potential of the form $m^2\phi^2/2$, only that $V_{,\phi\phi} \neq 0$ in the vicinity of the point $\phi = \phi_0$.)

Equation (55) may be derived by noting that (2) gives an interaction of the form $g^2(\phi - \phi_0)\delta\phi\chi^2$ between the inflaton and isoinflaton, in the background of the external field $\phi(t)$. Equivalently, one may construct this equation by a straightforward iterative solution of (27).

We work in conformal time and define the q-number Fourier transform $\xi_{\mathbf{k}}^\phi(\tau)$ of the inflaton fluctuation analogously to (33):

$$\delta\phi(\tau, \mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{\xi_{\mathbf{k}}^\phi(\tau)}{a(\tau)} e^{i\mathbf{k}\cdot\mathbf{x}}. \qquad (56)$$

[7] In this regard our scenario is very different from preheating at the end of inflation. In the latter case the inflaton passes *many* times through the massless point $m_\chi = 0$, and there are, correspondingly, many bursts of particle production. After many oscillations of the inflaton field, the χ-particle occupation numbers build-up to become exponentially large, and averaged over many oscillations of the background, the χ mode functions grow exponentially. However, in our case there is only a *single* burst of particle production at $t = 0$. The resulting occupation number (10) is always less than unity, and the solutions of (36) never display exponential growth.

[8] This is strictly true only in the linearized theory. It is possible that χ particles are generated by nonlinear effects such as rescattering. However, even such second order χ fluctuations will be extremely massive compared to the Hubble scale and must therefore suffer exponential damping $a^{-3/2}$ on large scales.

(To avoid potential confusion we again draw the attention of the reader to the explicit factor a^{-1} in our convention for the Fourier transform.) The equation of motion (55) now takes the form

$$\left[\partial_\tau^2 + k^2 + a^2 m^2 - \frac{a''}{a}\right]\xi_{\mathbf{k}}^{\phi}(\tau) = -g k_\star^2 a(\tau) t(\tau) \int \frac{d^3k'}{(2\pi)^{3/2}} \xi_{\mathbf{k}'}^{\chi} \xi_{\mathbf{k}-\mathbf{k}'}^{\chi}(\tau). \quad (57)$$

The solution of (57) consists of two parts: the solution of the homogeneous equation and the particular solution, which is due to the source. The former corresponds, physically, to the usual vacuum fluctuations from inflation. On the other hand, the particular solution corresponds physically to the secondary inflaton modes, which are generated by rescattering.

D. Homogeneous solution and green function

We consider first the homogeneous solution of (57). Since the homogeneous solution is a gaussian field, we may expand the q-number Fourier transform in terms of annihilation/creation operators $b_{\mathbf{k}}$, $b_{\mathbf{k}}^\dagger$ and c-number mode functions $\phi_k(\tau)$ as

$$\xi_{\mathbf{k}}^{\phi}(\tau) = b_{\mathbf{k}} \phi_k(\tau) + b_{-\mathbf{k}}^\dagger \phi_k^\star(\tau). \quad (58)$$

Here, the inflaton annihilation/creation operators $b_{\mathbf{k}}$, $b_{\mathbf{k}}^\dagger$ obey

$$[b_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = \delta^{(3)}(\mathbf{k} - \mathbf{k}') \quad (59)$$

and commute with the annihilation/creation operators of the χ field:

$$[a_{\mathbf{k}}, b_{\mathbf{k}'}] = [a_{\mathbf{k}}, b_{\mathbf{k}'}^\dagger] = 0. \quad (60)$$

Using (30) and (4) it is straightforward to see that the homogeneous inflaton mode functions obey the following equation

$$\partial_\tau^2 \phi_k + \left[k^2 - \frac{1}{\tau^2}\left(\nu^2 - \frac{1}{4}\right)\right]\phi_k = 0, \quad (61)$$

where we have defined

$$\nu \cong \frac{3}{2} - \eta + \epsilon. \quad (62)$$

The properly normalized mode function solutions are well-known and may be written in terms of the Hankel function of the first kind as

$$\phi_k(\tau) = \frac{\sqrt{\pi}}{2}\sqrt{-\tau} H_\nu^{(1)}(-k\tau). \quad (63)$$

This solution corresponds to the usual quantum vacuum fluctuations of the inflaton field during inflation.

In passing, let us compute the power spectrum of the quantum vacuum fluctuations from inflation. Using the solutions (63), we have

$$P_\phi^{\rm vac}(k) = \frac{k^3}{2\pi^2}\left|\frac{\phi_k}{a}\right|^2 \cong \frac{H^2}{(2\pi)^2}\left(\frac{k}{aH}\right)^{n_s - 1} \quad (64)$$

on large scales $k \ll aH$. The explicit factor of a^{-2} in (64) appears to cancel the a^{-1} in our definition of the Fourier transform (56). The spectral index is

$$n_s - 1 = 3 - 2\nu \cong 2\eta - 2\epsilon \quad (65)$$

using (62).

Given the solution (63) of the homogeneous equation, it is now trivial to construct the retarded Green function for Eq. (57). This may be written in terms of the free theory mode functions (63) as

$$\begin{aligned} G_k(\tau - \tau') &= i\Theta(\tau - \tau')[\phi_k(\tau)\phi_k^\star(\tau') - \phi_k^\star(\tau)\phi_k(\tau')] \\ &= \frac{i\pi}{4}\Theta(\tau - \tau')\sqrt{\tau\tau'}[H_\nu^{(1)}(-k\tau)H_\nu^{(1)}(-k\tau')^\star \\ &\quad - H_\nu^{(1)}(-k\tau)^\star H_\nu^{(1)}(-k\tau')]. \end{aligned} \quad (66)$$

E. Particular solution: Rescattering effects

We now consider the particular solution of (57). This is readily constructed using the Green function (66) as

$$\xi_{\mathbf{k}}^{\phi}(\tau) = -\frac{g k_\star^2}{(2\pi)^{3/2}} \times \int d\tau' d^3k' G_k(\tau - \tau') a(\tau') t(\tau') \xi_{\mathbf{k}'}^{\chi} \xi_{\mathbf{k}-\mathbf{k}'}^{\chi}(\tau'). \quad (67)$$

Notice that this particular solution is statistically independent of the homogeneous solution (58). In other words, the particular solution can be expanded in terms of the annihilation/creation operators $a_{\mathbf{k}}$, $a_{\mathbf{k}}^\dagger$ associated with the χ field, whereas the homogeneous solution is written in terms of the annihilation/creation operators $b_{\mathbf{k}}$, $b_{\mathbf{k}}^\dagger$ associated with the inflaton vacuum fluctuations. These two sets of operators commute with one another.

We will ultimately be interested in computing the n-point correlation functions of the particular solution (68). For example, carefully carrying out the Wick contractions, the connected contribution to the 2-point function is

$$\begin{aligned} \langle \xi_{\mathbf{k}_1}^{\phi} \xi_{\mathbf{k}_2}^{\phi}(\tau)\rangle &= \frac{2g^2 k_\star^4}{(2\pi)^3}\delta^{(3)}(\mathbf{k_1} + \mathbf{k_2}) \\ &\times \int d\tau' d\tau'' a(\tau') a(\tau'') t(\tau') t(\tau'') \\ &\times G_{k_1}(\tau - \tau') G_{k_2}(\tau - \tau'') \\ &\times \int d^3k' \chi_{k_1 - k'}(\tau') \chi_{k_1 - k'}^\star(\tau'') \chi_{k'}(\tau') \chi_{k'}^\star(\tau''). \end{aligned} \quad (68)$$

The power spectrum of $\delta\phi$ fluctuations generated by rescattering is then defined in terms of the 2-point function in the usual manner

$$\langle \xi^{\phi}_{\mathbf{k}}(t)\xi^{\phi}_{\mathbf{k}'}(\tau)\rangle \equiv \delta^{(3)}(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}a(\tau)^2 P^{\mathrm{resc}}_{\phi}. \tag{69}$$

(The explicit factor of a^2 in the definition (69) appears to cancel the factor of a^{-1} in our convention for Fourier transforms (56).)

The total power spectrum is simply the sum of the contribution from the vacuum fluctuations (64) and the contribution from rescattering (69):

$$P_{\phi}(k) = P^{\mathrm{vac}}_{\phi}(k) + P^{\mathrm{resc}}_{\phi}(k). \tag{70}$$

There are no cross-terms, owing to the fact $a_{\mathbf{k}}$ and $b_{\mathbf{k}}$ commute.

F. Renormalization

We now wish to evaluate the 2-point correlator (68). In principle, this is straightforward: first substitute the result (51) for the χ_k modes and the result (66) for the Green function into (68), next evaluate the integrals. However, there is a subtlety. The resulting power spectrum is formally infinite. Moreover, the 2-point correlation function (68) receives contributions from two distinct effects. There is a contribution from particle production, which we are interested in. However, there is also a contribution coming from quantum vacuum fluctuations of the χ field interacting nonlinearly with the inflaton. The latter contribution would be present even in the absence of particle production, when $\alpha_k = 1$, $\beta_k = 0$.

In order to isolate the effects of particle production on the inflaton fluctuations, we would like to subtract off the contribution to the 2-point correlation function (68), which is coming from the quantum vacuum fluctuations of χ. This subtraction also has the effect of rendering the power spectrum (69) finite, since it extracts the usual UV divergent contribution associated with the Minkowski-space vacuum fluctuations.

As a step towards renormalizing the 2-point correlation function of inflaton fluctuations from rescattering (68), let us first consider the simpler problem of renormalizing the 2-point function of the Gaussian field χ. We defined the renormalized 2-point function in momentum space as follows:

$$\langle \xi^{\chi}_{k_1}(t_1)\xi^{\chi}_{k_2}(t_2)\rangle_{\mathrm{ren}} = \langle \xi^{\chi}_{k_1}(t_1)\xi^{\chi}_{k_2}(t_2)\rangle - \langle \xi^{\chi}_{k_1}(t_1)\xi^{\chi}_{k_2}(t_2)\rangle_{\mathrm{in}}. \tag{71}$$

In (71) the quantity $\langle \xi^{\chi}_{k_1}(t_1)\xi^{\chi}_{k_2}(t_2)\rangle_{\mathrm{in}}$ is the contribution that would be present even in the absence of particle production, computed by simply taking the solution (43) with $\alpha_k = 1$, $\beta_k = 0$. Explicitly, we have

$$\langle \xi^{\chi}_{k_1}(t_1)\xi^{\chi}_{k_2}(t_2)\rangle_{\mathrm{in}} = \delta^{(3)}(\mathbf{k_1}+\mathbf{k_2})f_{k_1}(t_1)f^{\star}_{k_2}(t_2), \tag{72}$$

where f_k are the adiabatic solutions (42).

To see the impact of this subtraction, let us consider the renormalized variance for the isoinflaton field, $\langle \chi^2\rangle$. Employing the prescription (71) we have

$$\begin{aligned}\langle \chi^2(\tau,\mathbf{x})\rangle_{\mathrm{ren}} &= \int \frac{d^3k}{(2\pi)^3 a^2(\tau)}\left[|\chi_k(\tau)|^2 - \frac{1}{2\omega_k(\tau)}\right] \\ &= \langle \chi^2(\tau,\mathbf{x})\rangle - \delta_M,\end{aligned} \tag{73}$$

where δ_M is the contribution from the Coleman–Weinberg potential. This proves that our prescription reproduces the scheme advocated in [30]. The renormalized variance (73) is finite and may be computed explicitly using our solutions (51). We find

$$\langle \chi^2(t,\mathbf{x})\rangle_{\mathrm{ren}} \cong \frac{n_\chi a^{-3}}{g|\phi-\phi_0|}, \tag{74}$$

where

$$n_\chi \equiv \int \frac{d^3k}{(2\pi)^3} n_k \sim e^{-\pi\mu^2/k_\star^2} k_\star^3 \tag{75}$$

is the total co-moving number density of produced χ particles. The result (74) was employed in [7] to quantify the effect of backreaction on the inflaton condensate in the mean field treatment (11). Hence, the renormalization scheme (71) was implicit in that calculation also.

At the level of the 2-point function, our renormalization scheme is tantamount to assuming that Coleman–Weinberg corrections are already absorbed into the definition of the inflaton potential, $V(\phi)$. In general, such corrections might steepen $V(\phi)$ and spoil slow-roll inflation. Here, we assume that this problem has already been dealt with, either by fine-tuning the bare inflaton potential or else by including extended SUSY (which can minimize dangerous corrections). See also [30] for a related discussion. Note, also, that our renormalization procedure is equivalent to the quasiparticle normal ordering scheme described in [89].

Having established a scheme for remormalizing the 2-point function of the Gaussian field χ, it is now straightforward to consider higher order correlation functions. We simply rewrite the 4-point function as a product of 2-point functions using Wick's theorem. Next, each Wick contraction is renormalized as (71). Applying this prescription to (68) amounts to

$$\begin{aligned}&\langle \xi^{\phi}_{k_1}(\tau)\xi^{\phi}_{k_2}(\tau)\rangle_{\mathrm{ren}} \\ &\quad= \frac{2g^2k_\star^4}{(2\pi)^3}\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2})\int d\tau' d\tau'' t(\tau')t(\tau'')a(\tau')a(\tau'') \\ &\quad\times G_{k_1}(\tau-\tau')G_{k_2}(\tau-\tau'')\int d^3k'[\chi_{k_1-k'}(\tau')\chi^{\star}_{k_1-k'}(\tau'') \\ &\quad- f_{k_1-k'}(\tau')f^{\star}_{k_1-k'}(\tau'')]\times[\chi_{k'}(\tau')\chi^{\star}_{k'}(\tau'')-f_{k'}(t')f^{\star}_{k'}(\tau'')],\end{aligned} \tag{76}$$

where $f_k(t)$ are the adiabatic solutions defined in (42).

G. Power spectrum

We are now in a position to compute the renormalized power spectrum of inflation fluctuations generated by rescattering, $P_\phi^{\mathrm{resc}}(k)$. We renormalize the 2-point correlator of the inflaton fluctuations generated by rescatter according to (76) and extract the power spectrum by comparison to (69). We have relegated the technical details to Appendix A, and here we simply state the final result

$$
\begin{aligned}
P_\phi^{\mathrm{resc}}(k) = \frac{g^2 k^3 k_\star}{16\pi^5}\Bigg[&\frac{e^{-2\pi\mu^2/k_\star^2}e^{-\pi k^2/(2k_\star^2)}}{2\sqrt{2}}(I_2(k,\tau)^2\\
&+|I_1(k,\tau)|^2)+\bigg[e^{-\pi\mu^2/k_\star^2}e^{-\pi k^2/(4k_\star^2)}\\
&+\frac{e^{-2\pi\mu^2/k_\star^2}}{2\sqrt{2}}e^{-3\pi k^2/(8k_\star^2)}\bigg](I_2(k,\tau)^2\\
&-\mathrm{Re}[I_1(k,\tau)])+\bigg[\frac{8\sqrt{2}}{3\sqrt{3}}e^{-3\pi\mu^2/(2k_\star^2)}e^{-\pi k^2/(3k_\star^2)}\\
&+\frac{4\sqrt{2}}{5\sqrt{5}}e^{-5\pi\mu^2/(2k_\star^2)}e^{-3\pi k^2/(5k_\star^2)}\bigg]\\
&\times \mathrm{Im}[I_1(k,\tau)I_2(k,\tau)]\Bigg],
\end{aligned}
\tag{77}
$$

where the functions I_1, I_2 are the curved space generalization of the characteristic integrals defined in [1]. Explicitly we have

$$I_1(k,\tau) = \frac{1}{a(\tau)}\int d\tau' G_k(\tau-\tau')e^{ik_\star^2 t^2(\tau')}, \tag{78}$$

$$I_2(k,\tau) = \frac{1}{a(\tau)}\int d\tau' G_k(\tau-\tau'). \tag{79}$$

The characteristic integral I_2 can be evaluated analytically, however, the resulting expression is not particularly enlightening. Evaluation of the integral I_1 requires numerical methods. More details in Appendix A. Equation (77) is the main result of this section.

To test our analytical formalism, let us compare the result (77) with the output of fully nonlinear HLattice simulations. In Fig. 4 we plot our results for $P_\phi^{\mathrm{resc}}(k)$ as a function of k, for several time steps in the evolution. We have normalized $P_\phi^{\mathrm{resc}}(k)$ to the amplitude of the usual vacuum fluctuations from inflation, $P_\phi^{\mathrm{vac}}(k) \sim H^2/(2\pi)^2$. This figure illustrates the final stages of IR cascading; we see the peak of the bump-like feature slide to $k \sim e^{-3}k_\star$, at which point the associated mode functions $\delta\phi_k$ have crossed the horizon and become frozen. At later times in the evolution the peak of the feature and also the IR tail ($\sim k^3$) remain fixed. Modes associated with the UV end of the spectrum are still inside the horizon and continue to evolve as $\delta\phi_k \sim a^{-1}$, which explains the damping of the $k > e^{-2}k_\star$ part of the spectrum. At late times, the shape of the feature that is frozen outside the horizon can be very well approximated by the semianalytic fitting function (16).

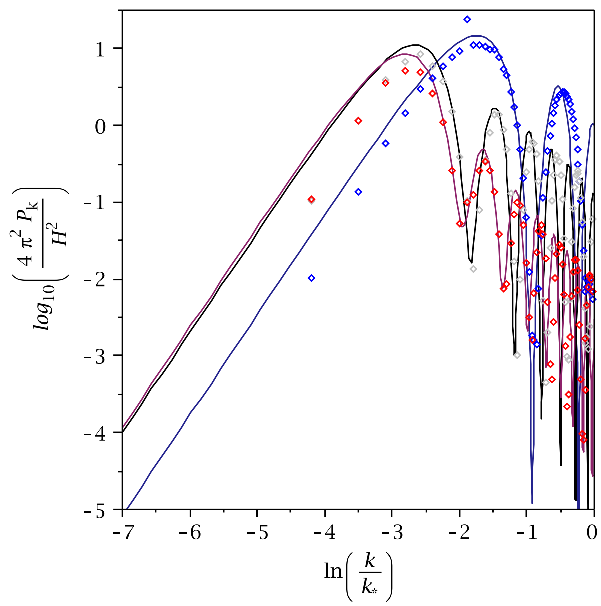

FIG. 4 (color online). The power spectrum of inflaton modes induced by rescattering. (normalized to the usual vacuum fluctuations) as a function of $\ln(k/k_\star)$, plotted for three representative time steps in the late-time evolution. For each time step we plot the analytical result (the solid line) and the data points obtained using lattice field theory simulations (diamonds). The agreement between these two independent results is evident. For illustration, we have set $\mu^2 = 0$.

The agreement between our analytical formalism and the exact numerical results is quite evident from Fig. 4 and provides a highly nontrivial check on our calculation.

H. The bispectrum

So far, we have shown how to compute analytically the power spectrum generated by particle production, rescattering, and IR cascading in the model (2). We found that IR cascading leads to a bump-like contribution to the primordial power spectrum of the inflaton fluctuations. However, this same dynamics must also have a nontrivial impact on non-Gaussian statistics, such as the bispectrum. Indeed, it is already evident from our previous analysis that the inflaton fluctuations generated by rescattering may be significantly non-Gaussian. From the expression (68) we see that the particular solution (due to rescattering) is bilinear is the Gaussian field χ.

We define the bispectrum of the inflaton field fluctuations in terms of the 3-point correlation function as

$$\langle \xi^\phi_{\mathbf{k_1}}\xi^\phi_{\mathbf{k_2}}\xi^\phi_{\mathbf{k_3}}(\tau)\rangle = (2\pi)^3 a^3(\tau)\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_\phi(k_i). \tag{80}$$

The factor a^3 appears in (80) to cancel the explicit factors of a^{-1} in our convention (56) for the Fourier transform. It is well-known that the non-Gaussianity associated with the usual quantum vacuum fluctuations of the inflaton is negligible [52–54]; therefore, when evaluating the bispectrum (80), we consider only the particular solution (67), which is due to rescattering. Carefully carrying out the Wick contractions, we find the following result for the renormalized 3-point function

$$\begin{aligned}\langle \xi^{\phi}_{\mathbf{k_1}} \xi^{\phi}_{\mathbf{k_2}} \xi^{\phi}_{\mathbf{k_3}}(\tau)\rangle_{\rm ren} &= \frac{4g^3 k_\star^6}{(2\pi)^{9/2}}\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) \\ &\times \prod_{i=1}^{3}\int d\tau_i t(\tau_i) a(\tau_i) G_{k_i}(\tau-\tau_i) \\ &\times \int d^3p[\chi_{k_1-p}(\tau_1)\chi^\star_{k_1-p}(\tau_2) - f_{k_1-p}(\tau_1)f^\star_{k_1-p}(\tau_2)] \\ &\times [\chi_{k_3+p}(\tau_2)\chi^\star_{k_3+p}(\tau_3) - f_{k_3+p}(\tau_2)f^\star_{k_3+p}(\tau_3)] \\ &\times [\chi_p(\tau_1)\chi^\star_p(\tau_3) - f_p(\tau_1)f^\star_p(\tau_3)] + (k_2 \leftrightarrow k_3),\end{aligned} \tag{81}$$

where the modes χ_k are defined by (43), and f_k are the adiabatic solutions (42). On the last line of (81) we have labeled schematically terms, which are identical to the preceding three lines, only with k_2 and k_3 interchanged. One may verify that this expression is symmetric under interchange of the momenta k_i by changing dummy variables of integration.

I. Estimating the shape of the bispectrum

It is straightforward (but tedious) to plug the expressions (50) and (51) into (81) and evaluate the integrals. The resulting expression is extremely cumbersome and not particularly enlightening. We are interested here in extracting some information about the shape of the bispectrum $B_\phi(k_i)$. For this purpose, it suffices to work in the flat-space limit, $H \to 0$. This will give a reasonable qualitative picture of the full result since the entire process of IR cascading occurs over a time scale somewhat shorter than the expansion time. In [1] this same approximation was employed to study the power spectrum from IR cascading and was found to reproduce the $H \neq 0$ results to good accuracy.

A detailed calculation of $B_\phi(k_i)$ has been relegated to Appendix B. Here we simply provide a representative contribution, in order to give a rough sense of the qualitative behavior:

$$B_\phi(k_i) \sim C\prod_{i=1}^{3} e^{-\pi k_i^2/(3k_\star^2)}\left[\frac{1-\cos(\sqrt{k_i^2+m^2}t)}{(k_i^2+m^2)}\right] \tag{82}$$

for some constant C. This expression captures some of the qualitative features of the full result, in particular, the dynamical cascading of non-Gaussianity into the IR to generate a localized bispectrum feature. It should be stressed that (82) is a heuristic estimate and *not* a fitting function nor a systematic approximation to the full result. Hence, Eq. (82) should not be used to make quantitative predictions of any kind.

As anticipated, our expression for $B_\phi(k_i)$ peaks only over when all wave numbers are close to the characteristic scale corresponding to the location of the bump in (16). Therefore, particle production and IR cascading leads to a localized non-Gaussian feature in the bispectrum, rather than the nearly scale-invariant signatures that are usually considered. We will discuss the phenomenology of this new type of non-Gaussianity in a forthcoming publication [69].

Now, we would like to attempt to characterize the shape of the non-Gaussianity from particle production and IR cascading. To this end we define a "shape function" $S(k_i)$ as follows

$$S(k_i) = N^{-1}(k_1k_2k_3)^2 B_\phi(k_i), \tag{83}$$

where N is a normalization factor which will not concern us.[9] The function $S(k_i)$ has the advantage that the strong k^6 running of the bispectrum is extracted. Hence, any residual scaling behavior displayed by $S(k_i)$ must be a result of nonlinear interactions; see also [47,48].

Symmetry of the bispectrum under permutations of momenta implies that we can focus only on the region $k_1 \geq k_2 \geq k_3$, to avoid counting the same configuration twice. Moreover, the triangle inequality implies that $1 - \frac{k_2}{k_1} \leq \frac{k_3}{k_1}$. Therefore, we can completely specify the shape of the bispectrum for a given size of triangle k by plotting $S(k, kx_2, kx_3)$ in the region $x_3 \leq x_2 \leq 1$ and $1 - x_2 \leq x_3$. (See also [90].) Because our bispectrum is very far from scale-invariant, it follows that this shape function is sensitive to the choice of k. Therefore, in Fig. 5 we choose several representative choices: $\ln(k/k_{\rm bump}) = -1, 0, 1, 2$.

We see that a rich array of shape are possible: for $k \lesssim k_{\rm bump}$, the bispectrum is qualitatively similar to the equilateral model; however, at slightly larger k, there is considerable support on flattened triangles also. Note that for $k \gtrsim 7.4 k_{\rm bump}$, the shape of the bispectrum is extremely unusual and is not easily comparable to any shape that has been proposed in previous literature.

V. COSMOLOGICAL PERTURBATION THEORY

In Sec. IV we developed an analytical theory of particle production and IR cascading during inflation which is in very good agreement with nonlinear lattice field theory simulations. However, this formalism suffers from a neglect of metric perturbations, and consequently, we were unable to rigorously discuss the gauge invariant curvature

[9]As we argued in Sec. III, the size of the non-Gaussianity in this model is most naturally quantified by evaluating the cumulants. Here we are interested *only* in discussing the shape of this novel type of non-Gaussianity.

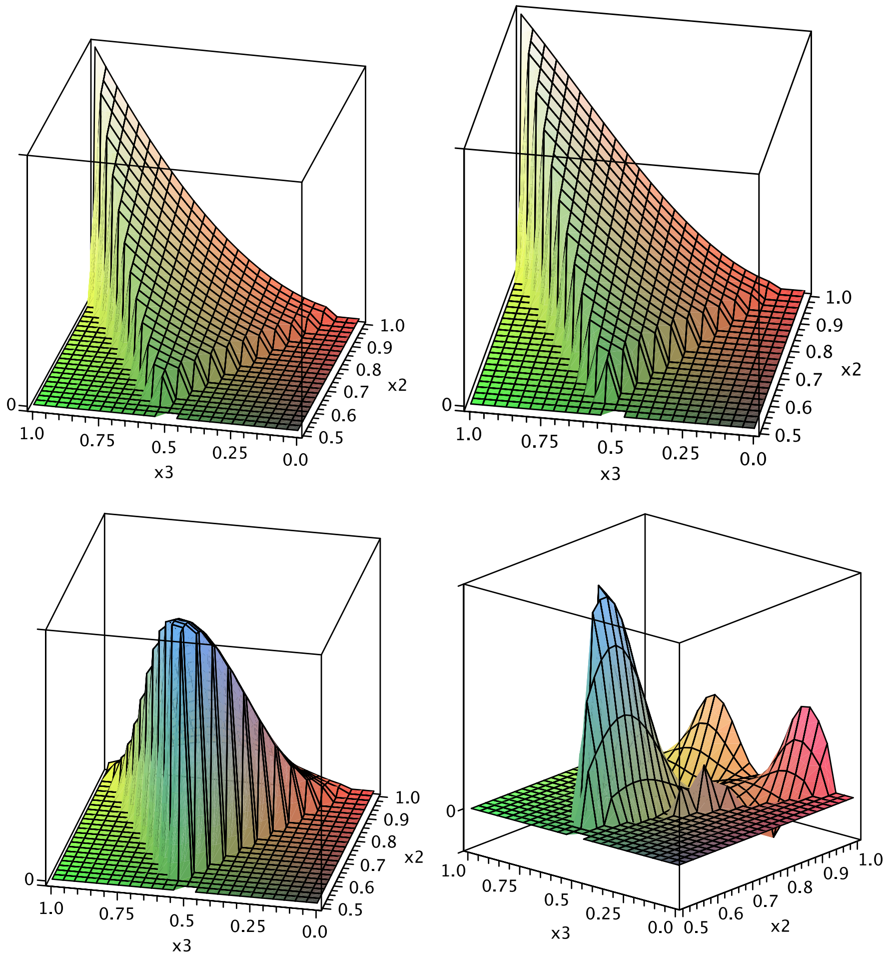

FIG. 5 (color online). The shape function $S(k, kx_2, kx_3)$, defined by (83), as a function of the dimensionless quantities x_2, x_3, which parametrize the shape of the triangle. The upper left panel corresponds to $k = e^{-1}k_{\rm bump}$, the upper right panel is $k = k_{\rm bump}$, the lower left panel is $k = e^{+1}k_{\rm bump}$, and the lower right panel is $k = e^{+2}k_{\rm bump}$. In the IR ($k \le k_{\rm bump}$) the shape of the bispectrum is similar to the equilateral shape, however, there is also some support on flattened triangles near $k \sim e^{+1}k_{\rm bump}$. At larger values of k the shape is unlike any other template proposed in the literature. For illustration we have chosen $\mu^2 = 0$.

perturbation ζ. Hence, the reader may be concerned about gauge ambiguities in our results. In this section we address such concerns, showing that metric perturbations may be incorporated in a straightforward manner and that their consistent inclusion does not change our results in any significant way. We will do so by showing explicitly that, with appropriate choice of gauge, Eqs. (55) and (31) for the fluctuations of the inflaton and isoinflaton still hold, to first approximation. We will also go beyond our previous analysis by explicitly showing that in this same gauge the spectrum of the curvature fluctuations, P_ζ, is trivially related to the spectrum of inflaton fluctuations, P_ϕ (and similarly for the bispectrum).

To render the analysis tractable, we would like to take full advantage of the results derived in the last section. To do so, we employ the Seery *et al.* formalism for working directly with the field equations [91] and make considerable use of results derived by Malik in [92,93]. (Note that our notations differ somewhat from those employed by Malik. The reader is therefore urged to take care in comparing our formulae.)

We expand the inflaton and isoinflaton fields up to second order in perturbation theory as

$$\phi(\tau, \mathbf{x}) = \phi(\tau) + \delta_1\phi(\tau, \mathbf{x}) + \frac{1}{2}\delta_2\phi(\tau, \mathbf{x}), \tag{84}$$

$$\chi(\tau, \mathbf{x}) = \delta_1\chi(\tau, \mathbf{x}) + \frac{1}{2}\delta_2\chi(\tau, \mathbf{x}). \tag{85}$$

The perturbations are defined to average to zero $\langle\delta_n\phi\rangle = \langle\delta_n\chi\rangle = 0$ so that $\langle\phi(t, \mathbf{x})\rangle = \phi(t)$ and $\langle\chi(t, \mathbf{x})\rangle = 0$. (The condition $\langle\chi\rangle = 0$ is ensured by the fact that $m_\chi \gg H$ for nearly the entire duration of inflation.)

We employ the flat slicing and threading throughout this section. With this gauge choice the perturbed metric takes the form

$$g_{00} = -a^2(1 + 2\psi_1 + \psi_2), \tag{86}$$

$$g_{0i} = a^2\partial_i\left[B_1 + \frac{1}{2}B_2\right], \tag{87}$$

$$g_{ij} = a^2\delta_{ij}, \tag{88}$$

so that spatial hyper-surfaces are flat. Note also that in this gauge the field perturbations $\delta_n\phi$, $\delta_n\chi$ coincide with the Sasaki-Mukhanov variables [94] at both first and second order.

A. Gaussian perturbations

In [92] Malik has derived closed-form evolution equations for the field perturbations $\delta_n\phi$, $\delta_n\chi$ at both first ($n = 1$) and second ($n = 2$) order in perturbation theory. Let us first study the gaussian perturbations. The closed-form Klein–Gordon equation for $\delta_1\phi$ derived in [92] can be written as

$$\delta_1\phi'' + 2\mathcal{H}\delta_1\phi' - \vec{\nabla}^2\delta_1\phi + \left[a^2m^2 - 3\left(\frac{\phi'}{M_p}\right)^2\right]\delta_1\phi = 0. \tag{89}$$

Following our previous analysis, we expand the first-order perturbation in terms of annihilation/creation operators as

$$\delta_1\phi(t, \mathbf{x}) = \int\frac{d^3k}{(2\pi)^{3/2}}\left[b_{\mathbf{k}}\frac{\delta_1\phi_k(\tau)}{a(\tau)}e^{i\mathbf{k}\cdot\mathbf{x}} + \text{H.c.}\right], \tag{90}$$

where H.c. denotes the Hermitian conjugate of the preceding term, and we draw the attention of the reader to the the explicit factor of a^{-1} in our definition of the Fourier transform. Working to leading order in slow-roll parameters we have

$$\delta_1\phi_k'' + \left[k^2 + \frac{1}{\tau^2}(-2 + 3\eta - 9\epsilon)\right]\delta_1\phi_k = 0. \tag{91}$$

This equation coincides exactly with (61) and the properly normalized solutions again take the form (63). The only difference is that the order of the Hankel function, ν, is now given by

$$\nu \cong \frac{3}{2} - \eta + 3\epsilon \tag{92}$$

rather than by Eq. (62). The power spectrum of the gaussian fluctuations is, again, given by (64). The correction to the order of the Hankel function ν translates into a correction to the spectral index: instead of (65) we now have

$$n_s - 1 = 2\eta - 6\epsilon, \tag{93}$$

which is precisely the standard result [95].

Thus, as far as the quantum vacuum fluctuations of the inflaton are concerned, the only impact of consistently including metric perturbations is an $\mathcal{O}(\epsilon)$ correction to the spectral index n_s.

Let us now turn our attention to the first-order fluctuations of the isoinflaton. The closed-form Klein–Gordon equation for $\delta_1\chi$ derived in [92] can be written as

$$\delta_1\chi'' + 2\mathcal{H}\delta_1\chi' - \vec{\nabla}^2\delta_1\chi + a^2[\mu^2 + k_\star^4 t^2(\tau)]\delta_1\chi = 0. \tag{94}$$

This coincides *exactly* with Eq. (31), which we have already solved. The fact that linear perturbations of χ do not couple to the metric fluctuations follows from the condition $\langle\chi\rangle = 0$.

B. Non-Gaussian perturbations

Now let us consider now the second order perturbation equations. The closed-form Klein–Gordon equation for $\delta_2\phi$ derived in [92] can be written as

$$\delta_2\phi'' + 2\mathcal{H}\delta_2\phi' - \vec{\nabla}^2\delta_2\phi + \left[a^2m^2 - 3\left(\frac{\phi'}{M_p}\right)^2\right]\delta_2\phi = J(\tau, \mathbf{x}). \tag{95}$$

As usual, the left-hand-side is identical to the first-order Eq. (89), while the source term J is constructed from a bilinear combination of the first-order quantities $\delta_1\phi$ and $\delta_1\chi$. In order to solve Eq. (95), we require explicit expressions for the Green function G_k and the source term J. The Green function is trivial for the case at hand; it is still given by our previous result (62), provided one takes into account the fact that the order of the Hankel functions ν is now given by (92), rather than (62). In other words, the Green function for the non-Gaussian perturbations (95) differ from the result obtained neglecting metric perturbations only by $\mathcal{O}(\epsilon)$ corrections.

Next, we would like to consider the source term, J, appearing in (95). Schematically, we can split the source into contributions bilinear in the Gaussian inflaton fluctuation $\delta_1\phi$ and contributions bilinear in the isoinflaton $\delta_1\chi$:

$$J = J_\phi + J_\chi. \tag{96}$$

The contribution J_ϕ would be present even in the absence of the isoinflaton. These correspond, physically, to the usual non-Gaussian corrections to the inflaton vacuum fluctuations coming from self-interactions. This contribution to the source is well-studied in the literature and is known to contribute negligibly to the bispectrum [91]. Thus, in what follows, we will ignore J_ϕ.

On the other hand, the contribution J_χ appearing in (96) depends only on the isoinflaton fluctuations $\delta_1\chi$. This contribution can be understood, physically, as generating non-Gaussian inflaton fluctuations $\delta_2\phi$ by rescattering of the produced χ particles off the condensate. Hence, the contribution J_χ may source large non-Gaussianity and is most interesting for us. It is straightforward to compute J_χ

explicitly for our model using the general results of [92]. We find

$$\begin{aligned} J_\chi &= -2a^2g^2(\phi-\phi_0)(\delta_1\chi)^2 \\ &\pm \frac{\sqrt{2\epsilon}}{M_p}\Big[-a^2(\mu^2+g^2(\phi-\phi_0)^2)(\delta_1\chi)^2 \\ &- \frac{1}{2}(\vec{\nabla}\delta_1\chi)^2 - \frac{1}{2}(\delta_1\chi')^2 + \nabla^{-2}(\partial_i(\delta_1\chi)\vec{\nabla}^2\partial^i(\delta_1\chi) \\ &+ \vec{\nabla}^2(\delta_1\chi)\vec{\nabla}^2(\delta_1\chi) + \delta_1\chi'\vec{\nabla}^2\delta_1\chi + (\vec{\nabla}\delta_1\chi')^2)\Big], \end{aligned} \tag{97}$$

where the upper sign is for $\phi' > 0$, the lower sign is for $\phi' < 0$. Notice that the contributions to J_χ on the fourth and fifth line of (97) contain the inverse spatial Laplacian ∇^{-2} and are thus nonlocal. These terms all contain at least as many gradients as inverse gradients, and hence, the large-scale limit is well-defined. In [96] it was argued that these terms nearly always contribute negligibly to the curvature perturbation on large scales.

Let us now examine the structure of the isoinflaton source J_χ, Eq. (97). The first line of (97) goes like $a^2g^2(\phi-\phi_0)(\delta_1\chi)^2$. This coincides exactly with the source term in Eq. (55), which was already studied in Sec. IV. On the other hand, the terms on the second, third, fourth, and fifth lines of (97) are new. These represent corrections to IR cascading, which result from the consistent inclusion of metric perturbations. We will now argue that these extra terms are negligible as compared to the first line. If we denote the energy density in Gaussian isoinflaton fluctuations as $\rho_\chi \sim m_\chi^2(\delta_1\chi)^2$; then, by inspection, we see that the first line of (97) is parametrically of order $\rho_\chi/|\phi-\phi_0|$ while the remaining terms are or order $\sqrt{\epsilon}\rho_\chi/M_p$. Hence, we expect the first term to dominate for the field values $\phi \cong \phi_0$, which are relevant for IR cascading. This suggests that the dominant contribution to J_χ is the term, which we have already taken into account in Sec. IV.

Let us now make this argument more quantitative. We assume that $\mu^2 \lesssim k_\star^2$, since otherwise particle production effects are exponentially suppressed. Inspection reveals that the only new contribution to (97) that has any chance of competing with the old term $a^2g^2(\phi-\phi_0)(\delta_1\chi)^2$ is the one proportional to $\sqrt{\epsilon}a^2g^2(\phi-\phi_0)^2(\delta_1\chi)^2/M_p$ (on the second line). This new correction has the possibility of becoming significant because it grows after particle production, as ϕ rolls away from ϕ_0. This growth, which reflects the fact that the energy density in the χ particles increases as they become more massive, cannot persist indefinitely. Within a few e-foldings of particle production, the isoinflaton source term must behave as $J_\chi \sim a^{-3}$, corresponding to the volume dilution of nonrelativistic particles. Hence, in order to justify the analysis of Sec. IV we must check that the term

$$J_{\rm new} \sim \frac{\sqrt{\epsilon}}{M_p}a^2g^2(\phi-\phi_0)^2(\delta_1\chi)^2 \tag{98}$$

does not dominate over the term, which we have already considered

$$J_{\rm old} \sim a^2g^2(\phi-\phi_0)(\delta_1\chi)^2 \tag{99}$$

during the relevant time $H\Delta t = \mathcal{O}(1)$ after particle production. It is straightforward to show that

$$\frac{J_{\rm old}}{J_{\rm new}} \sim \frac{M_p}{\sqrt{\epsilon}}\frac{1}{\phi-\phi_0} \sim \frac{M_pH}{\dot{\phi}\sqrt{\epsilon}}\frac{1}{N} \sim \frac{1}{\epsilon}\frac{1}{N}, \tag{100}$$

where $N = Ht$ is the number of e-foldings elapsed from particle production to the time when IR cascading has completed. Hence, $N = \mathcal{O}(1)$, and we conclude that the second, third, fourth, and fifth lines of (97) are (at least) slow roll suppressed as compared to the first line.

In summary, we have shown that consistent inclusion of metric perturbations yields corrections to the inflaton fluctuations $\delta\phi$, which fall into two classes:

(1) Slow-roll suppressed corrections to the inflaton vacuum fluctuations $\delta_1\phi$ (these amount to changing the definition of ν in the solution (63)). These corrections have two physical effects. First, they yield an $\mathcal{O}(\epsilon)$ correction to the spectral index. Second, they modify the propagator G_k by an $\mathcal{O}(\epsilon)$ correction.

(2) Corrections to the source J for the non-Gaussian inflaton perturbation $\delta_2\phi$. These corrections are the second, third, and fourth lines of (97)), which, as we have seen, are slow roll suppressed.

It should be clear that neither of these corrections alters our previous analysis in any significant way.

C. Correlators

So far, we have shown that a consistent inclusion of metric perturbations does not significantly alter our previous results for the field perturbations. Specifically, $\delta_1\chi$ is identical to our previous solution of Eq. (31) for the isoinflaton, while $\delta_1\phi$ coincides with the homogeneous solution of Eq. (55), up to slow-roll corrections. At second order in perturbation theory, we have seen that

$$\delta_2\phi = \int d^4x' G(x-x')J_\chi(x') + \mathcal{O}[(\delta_1\phi)^2].$$

To leading order in slow-roll $J_\chi \cong -2a^2g^2(\phi-\phi_0)\times(\delta_1\chi)^2$ and the first term coincides with our previous result for the particular solution of Eq. (55). The terms of order $(\delta_1\phi)^2$ represent non-Gaussian corrections to the vacuum fluctuations from inflation (coming from self-interactions of $\delta\phi$ and the nonlinearity of gravity). These would be present even in the absence of particle production and are known to have a negligible impact on the spectrum and bispectrum [91].

We are ultimately interested in the connected n-point correlation functions of $\delta\phi$. For example, the 2-point function $\langle(\delta\phi)^2\rangle$ gets a contribution of the form $\langle(\delta_1\phi)^2\rangle$, which gives the usual nearly scale-invariant large-scale power spectrum from inflation. The cross term $\langle\delta_1\phi\delta_2\phi\rangle$ is of order $\langle(\delta_1\phi)^4\rangle$ and represents a negligible loop correction to the scale-invariant spectrum from inflation. (The cross term does not involve the isoinflaton since $\delta_1\phi$ and $\delta_1\chi$ are statistically independent.) Finally, there is a contribution $\langle(\delta_2\phi)^2\rangle$, which involves terms of order $\langle\chi^4\rangle$ coming from rescattering and terms of order $\langle(\delta_1\phi)^4\rangle$, which represent (more) loop corrections to the scale-invariant spectrum from inflation. Thus, we can schematically write

$$P_\phi(k) = P_\phi^{\rm vac}(k)[1 + ({\rm loops})] + P_\phi^{\rm resc}(k).$$

Here, $P_\phi^{\rm vac} \sim k^{n_s-1}$ is the usual nearly scale-invariant spectrum from inflation, and $P_\phi^{\rm resc}$ is the bump-like contribution from rescattering and IR cascading, which we have studied in the previous section. The loop corrections to $P^{\rm vac}(k)$ have been studied in detail in the literature (see, for example, [97–100]) and are known to be negligible in most models.

We can also make a similar schematic decomposition of the bispectrum by considering the structure of the 3-point correlator $\langle(\delta\phi)^3\rangle$. Following our previous line of reasoning, it is clear that the dominant contribution comes from rescattering and is of order $\langle\chi^6\rangle$. The terms involving $\langle(\delta_1\phi)^3\rangle$, on the other hand, represent the usual non-Gaussianity generated during single-field slow-roll inflation and are known to be small [91].

D. The curvature perturbation

Ultimately, one wishes to compute not the field perturbations $\delta_n\phi$, $\delta_n\chi$, but rather the gauge invariant curvature fluctuation, ζ. We expand this in perturbation theory in the usual manner

$$\zeta = \zeta_1 + \frac{1}{2}\zeta_2. \tag{101}$$

In [93] Malik has derived expressions for the large-scale curvature perturbation in terms of the Sasaki–Mukhanov variables at both first and second order in perturbation theory. We remind the reader that in the flat slicing (which we employ) the Sasaki–Mukhanov variable for each field simply coincides with the field perturbation (*i.e.*—$Q_\phi = \delta\phi$ and $Q_\chi = \delta\chi$).

At first-order in perturbation theory, the isoinflaton does not contribute to the curvature perturbation (since $\langle\chi\rangle = 0$), and we have

$$\zeta_1 = -\frac{\mathcal{H}}{\phi'}\delta_1\phi. \tag{102}$$

At second order in perturbation theory the expression for the curvature perturbation is more involved. Using the results of [93] and working to leading order in slow-roll parameters we find[10]

$$\begin{aligned}\zeta_2 \cong& -\frac{\mathcal{H}}{\phi'}\left[\delta_2\phi - \frac{\delta_2\phi'}{3\mathcal{H}}\right] + \frac{1}{3(\phi')^2}[(\delta_1\chi')^2\\ &+ a^2(\mu^2 + g^2v^2t^2(\tau))(\delta_1\chi)^2] + \frac{1}{3(\phi')^2}[(\delta_1\phi')^2\\ &+ a^2m^2(\delta_1\phi)^2].\end{aligned} \tag{103}$$

Let us discuss the various contributions to this equation. The third line contributes to the non-Gaussianity of the vacuum fluctuations during inflation. These terms are known to be negligible [52–54,91] and, indeed, one may explicitly verify that (103) would predict $f_{\rm Nl} \sim \mathcal{O}(\epsilon, \eta)$ in the absence of particle production.

Next, we consider the second line of (103). This represents the direct contribution of the gaussian fluctuations $\delta_1\chi$ to the curvature perturbation. This contribution is tiny since the χ particles are extremely massive for nearly the entire duration of inflation and hence $\delta_1\chi \sim a^{-3/2}$ (see also [3] for a related discussion). The smallness of this contribution to ζ can be understood physically by noting that the superhorizon isocurvature fluctuations in our model are negligible.

Finally, let us consider the contribution on the first line of (103). This contribution is the most interesting. To make contact with observations, we must compute the curvature perturbation at late times and on large scales. In Sec. IV we have already shown that $\delta_n\phi$ is constant on large scales and at late times for both $n = 1$ and $n = 2$. This is the expected result: the curvature fluctuations are frozen far outside the horizon and in the absence of entropy perturbations [103].[11] Hence $\delta_2\phi'$ is completely negligible and the first term on the first line of (103) must dominate over the second term. We conclude that, at late times and on large scales, the second order curvature perturbation is very well approximated by

$$\zeta_2 \cong -\frac{\mathcal{H}}{\phi'}\delta_2\phi + \cdots \tag{104}$$

[10]We have dropped a spurious additive $2\zeta_1^2$, which stems from using the Malik and Wands [101] definition of the curvature perturbation, rather than the definition employed by Lyth and Rodriguez [102] and also by Maldacena [53]. (See also [10].)

[11]Note that, in some cases, the curvature fluctuations may evolve significantly after horizon exit [104,105]. (See also [106].) This is a concern in models where there are significant violations of slow-roll. In [1] we have already shown that the transient violation of slow-roll has a negligible effect on the curvature fluctuations in our model; see also [68]. Hence, the result $\zeta_n \sim \delta_n\phi \sim$ const far outside the horizon is consistent with previous studies.

In summary, we have shown that the power spectrum of curvature fluctuations from inflation in the model (2) is trivially related to the power spectrum of inflaton fluctuations

$$P_\zeta(k) \cong \frac{H^2}{\dot{\phi}^2} P_\phi(k) = \frac{1}{2\epsilon M_p^2} P_\phi(k) \qquad (105)$$

at both first and second order in cosmological perturbation theory. This relation is valid at late times and for scales far outside the horizon. The curvature spectrum (105) may be written as

$$P_\zeta(k) = P_\zeta^{\rm vac}(k)[1 + ({\rm loops})] + P_\zeta^{\rm resc}(k). \qquad (106)$$

The power spectrum of the inflaton vacuum fluctuations agrees with the usual result obtained in linear theory [95]

$$P_\zeta^{\rm vac}(k) \cong \frac{H^2}{8\pi^2 \epsilon M_p^2}\left(\frac{k}{aH}\right)^{2\eta - 6\epsilon}. \qquad (107)$$

In (106) we have schematically labeled the corrections arising from the third line of (103) and the source J_ϕ as loop. These are non-Gaussian corrections to the inflaton vacuum fluctuations arising from self-interactions of the inflaton and also the nonlinearity of gravity. Such corrections are negligible. The most interesting contribution to the power spectrum (106) is due to rescattering, $P_\zeta^{\rm resc}(k)$. This quantity is proportional to our previous result (77).

In passing, notice that the bispectrum B_ϕ (defined by (80)) of inflaton fluctuations will differ from the bispectrum B of the curvature fluctuations (defined by (17)) only by a simple rescaling:

$$B(k_i) \cong -\left(\frac{H}{\dot{\phi}}\right)^3 B_\phi(k_i) = -\frac{1}{(2\epsilon)^{3/2} M_p^3} B_\phi(k_i). \qquad (108)$$

The dominant contribution to B_ϕ comes from rescattering effects and scales as $\langle \delta_2 \phi^3 \rangle \sim \langle \delta_1 \chi^6 \rangle$.

The analysis of this section justifies our neglect of metric fluctuations in Sec. IV.

VI. CONCLUSIONS

In the context of a realistic microscopic frame-work, we might generically expect the inflaton to couple to a large number of fields whose energy density does not play any important role in driving inflation. Such couplings can lead to isolated bursts of particle production during inflation. The associated observational signatures provide a rare opportunity to learn about how ϕ couples to other species, as opposed to the self-coupling information, which is encoded in $V(\phi)$. In this paper we have considered a simple example of this effect, which is dynamically rich and derivable from realistic particle physics models, such as string theory.

Inflationary particle production leads to features in the primordial curvature fluctuations via the mechanism of IR cascading. This process is interesting in its own right: it is qualitatively different from other mechanisms in the literature (in that we do not rely on the quantum vacuum fluctuations of some light isocurvature fields) and the underlying dynamics are relevant for preheating, moduli trapping and nonequilibrium QFT more generally. Moreover, particle production and IR cascading lead to a variety of novel observable signatures, including localized features in both the spectrum and bispectrum of the cosmological fluctuations.

In this paper we have extended previous work [1,2] on inflationary particle production in two directions. First, we have developed an analytical theory of particle production and IR cascading during inflation, which is in excellent agreement with lattice field theory simulations. This formalism helps to clarify the underlying physics of the mechanism and provides a crucial cross-check on our numerical methods.

Our second main result has been a more detailed investigation of the non-Gaussian signature associated with particle production and IR cascading. The bispectrum in this model is rather unusual: it peaks only for triangles with a size comparable to some characteristic scale. We have argued that the magnitude of this type of non-Gaussianity is best characterized by studying the moments of the PDF. For realistic values of the coupling, the skewness of the PDF is quite large. For example, with $g^2 \sim 0.01$ the power spectrum for our model is compatible with all observational data [2] while the skewness of the PDF is equivalent to what would be produced in a local model with $f_{\rm NL}^{\rm equiv} \sim -53$. This value is somewhat larger than current observational bounds, suggesting that non-Gaussianity from inflationary particle production may be observable in future missions. However, we stress that the non-Gaussian signature in our model is quite different from what would be expected for a local model with $\zeta = \zeta_g + \frac{3}{5} f_{\rm NL}^{\rm equiv}[\zeta_g^2 - \langle \zeta_g^2 \rangle]$. In particular, the higher order cummulants (such as the kurtosis) are different, as are the shape and running of the bispectrum.

Note that, if it were to be detected, the non-Gaussian signature from IR cascading must be correlated with an observable feature in the power spectrum and also with signatures in polarization. Hence, it should be possible to robustly rule out the possibility that massive isocurvature particles were produced at some point during the observable range of e-foldings of inflation.

The non-Gaussian signature predicted by inflationary particle production is rather complicated as compared to the local or equilateral models. However, the underlying field theory description of our model is extremely simple and rather generic from the low-energy perspective. In order to obtain large non-Gaussianity it was not necessary to fine-tune the inflaton trajectory or appeal to resummation of an infinite series of high dimension operators. Indeed, the only tuning that is required for our signal to be observable is the requirement that $\phi = \phi_0$ during the observable range of e-foldings. We believe that this type of non-Gaussianity is very natural and merits further investigation from the observational perspective.

There are a variety of directions for future studies. From the theoretical perspective, it would be interesting to explicitly generalize our results to more complicated models with particle production during inflation (such as SUSY models, higher spin isoinflatons, and phase transitions). There are also a wide range of interesting phenomenological possibilities. Varying the location of the feature we can have a variety of possible signatures for the CMB and LSS. We expect that IR cascading will also have implications for the spectrum of gravity waves from inflation and also primordial black holes. We could imagine superposing multiple bursts of particle production to obtain an even richer variety of signatures. It would be interesting to construct a simple, separable estimator for the bispectrum from IR cascading, which can be confronted with observational data in order to obtain explicit constraints on the underlying model parameters. We leave these possibilities for future investigation.

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APPENDIX A: DETAILED COMPUTATION OF $P(k)$

In this appendix we discuss some of the technical details associated with the computation of the renormalized power spectrum (77). First, notice that using (50) and (51) we can write the quantity appearing in each renormalized Wick contraction as

$$\chi_k(\tau)\chi_k^\star(\tau') - f_k(\tau)f_k^\star(\tau') \cong \frac{1}{k_\star^2}\frac{1}{\sqrt{a(\tau)a(\tau')}}\frac{1}{\sqrt{t(\tau)t(\tau')}}\left[n_k\cos\left(\frac{k_\star^2 t^2(\tau)}{2} - \frac{k_\star^2 t^2(\tau')}{2}\right) + \sqrt{n_k}\sqrt{1+n_k}\sin\left(\frac{k_\star^2 t^2(\tau)}{2} - \frac{k_\star^2 t^2(\tau')}{2}\right)\right], \tag{A1}$$

where the occupation number n_k is defined by (49). Plugging (A1) into (77) we find

$$\begin{aligned}P_\phi(k) = \frac{g^2k^3}{8\pi^5}\Bigg[&\int d^3k' n_{k-k'}n_{k'} \times \int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)}\frac{G_k(\tau-\tau'')}{a(\tau)}\cos^2\left[\frac{k_\star^2 t^2(\tau')}{2} - \frac{k_\star^2 t^2(\tau'')}{2}\right]\\ &+ \int d^3k' \sqrt{n_{k-k'}n_{k'}}\sqrt{1+n_{k-k'}}\sqrt{1+n_{k'}} \times \int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)}\frac{G_k(\tau-\tau'')}{a(\tau)}\sin^2\left[\frac{k_\star^2 t^2(\tau')}{2} + \frac{k_\star^2 t^2(\tau'')}{2}\right]\\ &+ \int d^3k' (n_{k-k'}\sqrt{n_{k'}}\sqrt{1+n_{k'}} + n_{k'}\sqrt{n_{k-k'}}\sqrt{1+n_{k-k'}})\int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)}\frac{G_k(\tau-\tau'')}{a(\tau)}\\ &\times\cos\left[\frac{k_\star^2 t^2(\tau')}{2} - \frac{k_\star^2 t^2(\tau'')}{2}\right]\sin\left[\frac{k_\star^2 t^2(\tau')}{2} + \frac{k_\star^2 t^2(\tau'')}{2}\right]\Bigg].\end{aligned} \tag{A2}$$

Notice that the time and phase space integrations in (A2) decouple. This is the key simplification which makes an analytical evaluation of this expression tractable. Let us consider these integrations separately.

1. Time integrals

All of the integrals over conformal time that appear in (A2) can be expressed in terms of two characteristic integrals, which we call I_1 and I_2. Explicitly, these are defined as

$$I_1(k,\tau) = \frac{1}{a(\tau)}\int d\tau' G_k(\tau-\tau')e^{ik_\star^2 t^2(\tau')}, \tag{A3}$$

$$I_2(k,\tau) = \frac{1}{a(\tau)}\int d\tau' G_k(\tau-\tau'). \tag{A4}$$

The second characteristic integral, I_2, can be evaluated analytically. However, the resulting expression is not particularly enlightening. Evaluation of I_1, on the other hand, requires numerical methods.

Let us now show how the various integrals appearing in (A2) may be rewritten in terms of I_1, I_2. First, consider the first line of (A2), where the following integral appears:

$$\int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)}\frac{G_k(\tau-\tau'')}{a(\tau)}\cos^2\left[\frac{k_\star^2 t^2(\tau')}{2} - \frac{k_\star^2 t^2(\tau'')}{2}\right] = \frac{|I_1(k,\tau)|^2}{2} + \frac{I_2(k,\tau)^2}{2}. \tag{A5}$$

Next, consider the second line of (A2), where the following integral appears:

$$\int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)}\frac{G_k(\tau-\tau'')}{a(\tau)}\sin^2\left[\frac{k_\star^2 t^2(\tau')}{2} + \frac{k_\star^2 t^2(\tau'')}{2}\right] = -\frac{\mathrm{Re}[I_1(k,\tau)^2]}{2} + \frac{I_2(k,\tau)^2}{2}. \tag{A6}$$

Finally, consider the fourth line of (A2) where the following integral appears:

$$\int d\tau' d\tau'' \frac{G_k(\tau-\tau')}{a(\tau)} \frac{G_k(\tau-\tau'')}{a(\tau)} \cos\left[\frac{k_\star^2 t^2(\tau')}{2} - \frac{k_\star^2 t^2(\tau'')}{2}\right] \times \sin\left[\frac{k_\star^2 t^2(\tau')}{2} + \frac{k_\star^2 t^2(\tau'')}{2}\right] = \mathrm{Im}[I_1(k,\tau) I_2(k,\tau)]. \quad \text{(A7)}$$

In the expressions (A6) and (A7), the notations Re and Im denote the real and imaginary parts, respectively.

2. Phase space integrals

As a warm-up to the subsequent calculation consider the following integral:

$$\begin{aligned}\int d^3k' n^a_{k-k'} n^b_{k'} &= \int d^3k' \exp[-a\pi|\mathbf{k}-\mathbf{k}'|^2/k_\star^2] \\ &\quad \times \exp[-b\pi|\mathbf{k}'|^2/k_\star^2] \\ &= \frac{k_\star^3}{(a+b)^{3/2}} \exp\left[-\pi(a+b)\frac{\mu^2}{k_\star^2}\right] \\ &\quad \times \exp\left[-\frac{ab}{a+b}\frac{\pi k^2}{k_\star^2}\right]. \end{aligned} \quad \text{(A8)}$$

This formula is valid when a, b are positive real numbers. Notice that this expression is symmetric under interchange of a and b.

The phase space integral in the first line of (A2) is computed by a trivial application of the identity (A8):

$$\int d^3k' n_{k-k'} n_{k'} = \frac{k_\star^3}{2\sqrt{2}} e^{-2\pi\mu^2/k_\star^2} e^{-\pi k^2/(2k_\star^2)}. \quad \text{(A9)}$$

However, the remaining phase space integrals appearing in (A2) cannot be obtained exactly in closed form because they contain terms like $\sqrt{1+n_{k'}}$, where the gaussian factors appear under the square root. In order to deal with such expressions, we note because $n_k \ll 1$ over most of the domain of integration, it is reasonable to replace $\sqrt{1+n_{k'}} \cong 1 + n_{k'}/2$. Let us now proceed in this manner. The phase space integral on the second line of (A2) is:

$$\begin{aligned}&\int d^3k' \sqrt{n_{k-k'} n_{k'}} \sqrt{1+n_{k-k'}} \sqrt{1+n_{k'}} \\ &\cong \int d^3k' \left[n^{1/2}_{k-k'} n^{1/2}_{k'} + \frac{1}{2} n^{3/2}_{k-k'} n^{1/2}_{k'} + \frac{1}{2} n^{1/2}_{k-k'} n^{3/2}_{k'}\right] \\ &= k_\star^3 \left[e^{-\pi\mu^2/k_\star^2} \exp\left(-\frac{\pi k^2}{4k_\star^2}\right) \right. \\ &\quad \left. + \frac{e^{-2\pi\mu^2/k_\star^2}}{2\sqrt{2}} \exp\left(-\frac{3\pi k^2}{8k_\star^2}\right)\right]. \end{aligned} \quad \text{(A10)}$$

Finally, consider the phase space integral on the third line of (A2):

$$\begin{aligned}&\int d^3k' [n_{k-k'} \sqrt{n_{k'}} \sqrt{1+n_{k'}} + n_{k'} \sqrt{n_{k-k'}} \sqrt{1+n_{k-k'}}] \\ &\cong \int d^3k' \left[n_{k-k'} n^{1/2}_{k'} + n_{k'} n^{1/2}_{k-k'} + \frac{1}{2} n_{k-k'} n^{3/2}_{k'} \right. \\ &\quad \left. + \frac{1}{2} n_{k'} n^{3/2}_{k-k'}\right] \\ &= k_\star^3 \left[\frac{4\sqrt{2}}{3\sqrt{3}} e^{-3\pi\mu^2/(2k_\star^2)} \exp\left(-\frac{\pi k^2}{3k_\star^2}\right) \right. \\ &\quad \left. + \frac{2\sqrt{2}}{5\sqrt{5}} e^{-5\pi\mu^2/(2k_\star^2)} \exp\left(-\frac{3\pi k^2}{5k_\star^2}\right)\right]. \end{aligned} \quad \text{(A11)}$$

We have verified the formulae (A10) and (A11) numerically. In both cases that the numerical results agree with these semianalytical expressions up to the percent level.

We can now, finally, insert the results (A5)–(A7) and (A9)–(A11) into the expression (A2). Doing so, we arrive at our main analytical result, which is Eq. (77).

APPENDIX B: EVALUATION OF THE BISPECTRUM

In this appendix we discuss some of the technical details associated with the explicit evaluation of the renormalized bispectrum (77). To render the analysis tractable we will work in the flat-space limit $H \to 0$, which is sensible since the process of IR cascading takes only a single e-folding; and moreover, this approximation was shown to yield sensible results for the power spectrum in [1]. Neglecting the expansion of the universe we have $t = \tau$ and the retarded Green function (66) becomes

$$G_k(t-t') = \frac{\Theta(t-t')}{\Omega_k} \sin[\Omega_k(t-t')] \quad \text{(B1)}$$

with $\Theta(x)$ the Heaviside function and $\Omega_k \equiv \sqrt{k^2+m^2}$. In this limit the characteristic integrals $I_1(k,t)$, $I_2(k,t)$ defined by (78) and (79) can be computed analytically. For $I_1(k,t)$ we find

$$I_1(k,t) = \frac{\sqrt{\pi}}{2k_\star\Omega_k} e^{i\Omega_k t - i\Omega_k^2/(4k_\star^2) - i\pi/4} F(k,t), \quad \text{(B2)}$$

$$\begin{aligned}F(k,t) = \frac{1}{2}\bigg[&(1+e^{-2i\Omega_k t}) \mathrm{erf}\left(\frac{e^{-i\pi/4}}{2}\frac{\Omega_k}{k_\star}\right) \\ &- \mathrm{erf}\left(\frac{e^{-i\pi/4}}{2}\left(\frac{\Omega_k}{k_\star} - 2k_\star t\right)\right) \\ &- e^{-2i\Omega_k t} \mathrm{erf}\left(\frac{e^{-i\pi/4}}{2}\left(\frac{\Omega_k}{k_\star} + 2k_\star t\right)\right)\bigg], \end{aligned} \quad \text{(B3)}$$

while, for $I_2(k,t)$, we have

$$I_2(k,t) = \frac{1}{\Omega_k^2}[1 - \cos(\Omega_k t)]. \quad \text{(B4)}$$

(Note that our definition of I_1, I_2 differs from [1] by a factor of Ω_k^{-1}.) Finally, the renormalized Wick contraction (A1) also simplifies in the limit $H \to 0$:

$$\chi_k(t)\chi_k^\star(t') - f_k(t)f_k^\star(t') \cong \frac{1}{k_\star^2}\frac{1}{\sqrt{tt'}}\left[n_k \cos\left(\frac{k_\star^2 t^2}{2} - \frac{k_\star^2(t')^2}{2}\right) + \sqrt{n_k}\sqrt{1+n_k}\sin\left(\frac{k_\star^2 t^2}{2} - \frac{k_\star^2(t')^2}{2}\right)\right], \tag{B5}$$

where the occupation number n_k is defined by (10).

Inserting (B1) and (B5) into (81) we find the following expression for the renormalized 3-point correlation function:

$$\begin{aligned}\langle \xi^\phi_{\mathbf{k_1}}\xi^\phi_{\mathbf{k_2}}\xi^\phi_{\mathbf{k_3}}(t)\rangle = {}& \frac{4g^3}{(2\pi)^{9/2}}\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\prod_{j=1}^{3}\frac{dt_j}{\Omega_{k_j}}\sin[\Omega_{k_j}(t-t_j)]\int d^3p\Bigg[\Bigg[n_{k_1-p}\cos\left(\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_2)^2}{2}\right)\\ &+ n_{k_1-p}^{1/2}\sqrt{1+n_{k_1-p}}\sin\left(\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_2)^2}{2}\right)\Bigg]\Bigg[n_{k_3+p}\cos\left(\frac{(k_\star t_2)^2}{2} - \frac{(k_\star t_3)^2}{2}\right)\\ &+ n_{k_3+p}^{1/2}\sqrt{1+n_{k_3+p}}\sin\left(\frac{(k_\star t_2)^2}{2} + \frac{(k_\star t_3)^2}{2}\right)\Bigg]\Bigg[n_p\cos\left(\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_3)^2}{2}\right)\\ &+ n_p^{1/2}\sqrt{1+n_p}\sin\left(\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_3)^2}{2}\right)\Bigg]\Bigg].\end{aligned} \tag{B6}$$

It only remains to expand out the expression (B6) and evaluate the various integral that arise. As in the case of the 2-point function, the phase space and time integrals decouple, making an analytical evaluation tractable. Let us consider the various integrals that arise separately.

1. Phase space integrals

First, let us introduce a notation for the fundamental phase space integral that arises

$$\begin{aligned}K_{a,b,c} &= \int d^3p\, n^a_{k_1-p}n^b_{k_3+p}n^c_p\\ &= \frac{k_\star^3}{(a+b+c)^{3/2}}\exp\left[-\pi(a+b+c)\frac{\mu^2}{k_\star^2}\right]\\ &\quad\times\exp\left[-\pi\frac{(ack_1^2 + bck_3^2 + abk_2^2)}{k_\star^2(a+b+c)}\right],\end{aligned} \tag{B7}$$

where we have used the fact that $\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3} = 0$. To evaluate integrals containing radicals such as $\sqrt{1+n_p}$ we use the same trick as was employed for (A10). That is, we approximate:

$$\begin{aligned}&\int d^3p\, n_{k_1-p}n_{k_3+p}n_p^{1/2}\sqrt{1+n_p}\\ &\quad\cong \int d^3p\, n_{k_1-p}n_{k_3+p}n_p^{1/2} + \frac{1}{2}\int d^3p\, n_{k_1-p}n_{k_3+p}n_p^{3/2}\\ &\quad= K_{1,1,1/2} + \frac{1}{2}K_{1,1,3/2}\end{aligned} \tag{B8}$$

and similarly for the other combinations that arise in the expansion of (B6). We have checked numerically that this gives a good approximation to the exact result.

2. Time integrals

The evaluation of the time integrals appearing in (B6) is a straightforward generalization of the results presented in Appendix A. Let us introduce some notations for the various combinations of the fundamental integrals I_1 and I_2 that will appear in the final result:

$$\begin{aligned}A \equiv{}& \prod_j\int\frac{dt_j}{\Omega_{k_j}}\sin(\Omega_{k_j}(t-t_j))\times\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_2)^2}{2}\right]\\ &\times\cos\left[\frac{(k_\star t_2)^2}{2} - \frac{(k_\star t_3)^2}{2}\right]\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_3)^2}{2}\right]\\ ={}& \frac{1}{4}[I_2(k_1,t)I_2(k_2,t)I_2(k_3,t) + I_2(k_1,t)\mathrm{Re}[I_1(k_2,t)I_1(k_3,t)]\\ &+ (2\,\text{permutations})],\end{aligned} \tag{B9}$$

$$\begin{aligned}B \equiv{}& \prod_j\int\frac{dt_j}{\Omega_{k_j}}\sin(\Omega_{k_j}(t-t_j))\times\sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_2)^2}{2}\right]\\ &\times\sin\left[\frac{(k_\star t_2)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\\ ={}& \frac{1}{4}[-\mathrm{Im}[I_1(k_1,t)I_1(k_2,t)I_1(k_3,t)]\\ &+ I_2(k_1,t)I_2(k_2,t)\,\mathrm{Im}[I_1(k_3,t)] + (2\,\text{permutations})],\end{aligned} \tag{B10}$$

$$\begin{aligned}C_1 \equiv{}& \prod_j\int\frac{dt_j}{\Omega_{k_j}}\sin(\Omega_{k_j}(t-t_j))\times\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_2)^2}{2}\right]\\ &\times\cos\left[\frac{(k_\star t_2)^2}{2} - \frac{(k_\star t_3)^2}{2}\right]\sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\\ ={}& \frac{1}{4}[\mathrm{Im}[I_1(k_1,t)I_1^\star(k_2,t)I_1(k_3,t)]\\ &+ I_2(k_1,t)I_2(k_2,t)\,\mathrm{Im}[I_1(k_3,t)] + (2\,\text{permutations})],\end{aligned} \tag{B11}$$

$$C_2 \equiv \prod_j \int \frac{dt_j}{\Omega_{k_j}} \sin(\Omega_{k_j}(t-t_j)) \times \cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_2)^2}{2}\right] \times \sin\left[\frac{(k_\star t_2)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_3)^2}{2}\right] = \frac{1}{4}[\mathrm{Im}[I_1^\star(k_1,t)I_1(k_2,t)I_1(k_3,t)] + I_2(k_1,t)I_2(k_2,t)\,\mathrm{Im}[I_1(k_3,t)] + (2\,\text{permutations})], \tag{B12}$$

$$C_3 \equiv \prod_j \int \frac{dt_j}{\Omega_{k_j}} \sin(\Omega_{k_j}(t-t_j)) \times \sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_2)^2}{2}\right] \times \cos\left[\frac{(k_\star t_2)^2}{2} - \frac{(k_\star t_3)^2}{2}\right]\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_3)^2}{2}\right] = \frac{1}{4}[\mathrm{Im}[I_1(k_1,t)I_1(k_2,t)I_1^\star(k_3,t)] + I_2(k_1,t)I_2(k_2,t)\,\mathrm{Im}[I_1(k_3,t)] + (2\,\text{permutations})], \tag{B13}$$

$$D_1 \equiv \prod_j \int \frac{dt_j}{\Omega_{k_j}} \sin(\Omega_{k_j}(t-t_j)) \times \cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_2)^2}{2}\right] \times \sin\left[\frac{(k_\star t_2)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_3)^2}{2}\right] = \frac{1}{4}[I_2(k_1,t)I_2(k_2,t)I_2(k_3,t) + I_2(k_3,t)\,\mathrm{Re}[I_1(k_1,t)I_1^\star(k_2,t)] - I_2(k_2)\,\mathrm{Re}[I_1(k_1,t)I_1(k_3,t)] - I_2(k_1,t)\,\mathrm{Re}[I_1(k_2,t)I_1(k_3,t)]], \tag{B14}$$

$$D_2 \equiv \prod_j \int \frac{dt_j}{\Omega_{k_j}} \sin(\Omega_{k_j}(t-t_j)) \times \sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_2)^2}{2}\right] \times \cos\left[\frac{(k_\star t_2)^2}{2} - \frac{(k_\star t_3)^2}{2}\right]\sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_3)^2}{2}\right] = \frac{1}{4}[I_2(k_1,t)I_2(k_2,t)I_2(k_3,t) + I_2(k_1,t)\,\mathrm{Re}[I_1(k_2,t)I_1^\star(k_3,t)] - I_2(k_3)\,\mathrm{Re}[I_1(k_1,t)I_1(k_2,t)] - I_2(k_2,t)\,\mathrm{Re}[I_1(k_1,t)I_1(k_3,t)]], \tag{B15}$$

$$D_3 \equiv \prod_j \int \frac{dt_j}{\Omega_{k_j}} \sin(\Omega_{k_j}(t-t_j)) \times \sin\left[\frac{(k_\star t_1)^2}{2} + \frac{(k_\star t_2)^2}{2}\right] \times \sin\left[\frac{(k_\star t_2)^2}{2} + \frac{(k_\star t_3)^2}{2}\right]\cos\left[\frac{(k_\star t_1)^2}{2} - \frac{(k_\star t_3)^2}{2}\right] = \frac{1}{4}[I_2(k_1,t)I_2(k_2,t)I_2(k_3,t) + I_2(k_2,t)\,\mathrm{Re}[I_1(k_1,t)I_1^\star(k_3,t)] - I_2(k_3)\,\mathrm{Re}[I_1(k_1,t)I_1(k_2,t)] - I_2(k_1,t)\,\mathrm{Re}[I_1(k_2,t)I_1(k_3,t)]]. \tag{B16}$$

3. The full bispectrum

We are now finally in a position to write out an explicit expression for the renormalized 3-point function of the inflaton fluctuations generated by IR cascading. That expression is given below, in terms of the various that were defined explicitly in Eqs. (B7) and (B9)–(B16). As promised, the explicit result for the 3-point correlation function is cumbersome and not entirely enlightening.

$$\begin{aligned}\langle \xi^\phi_{\mathbf{k_1}}\xi^\phi_{\mathbf{k_2}}\xi^\phi_{\mathbf{k_3}}(t)\rangle = \frac{4g^3}{(2\pi)^{9/2}}\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})\bigg[&K_{1,1,1}A + \left[K_{1/2,1/2,1/2} + \frac{1}{2}(K_{3/2,1/2,1/2} + K_{1/2,3/2,1/2} + K_{1/2,1/2,3/2})\right]B \\ &+ \left[K_{1,1,1/2} + \frac{1}{2}K_{1,1,3/2}\right]C_1 + \left[K_{1,1/2,1} + \frac{1}{2}K_{1,3/2,1}\right]C_2 + \left[K_{1/2,1,1} + \frac{1}{2}K_{3/2,1,1}\right]C_3 \\ &+ \left[K_{1,1/2,1/2} + \frac{1}{2}(K_{1,3/2,1/2} + K_{1,1/2,3/2})\right]D_1 + \left[K_{1/2,1,1/2} + \frac{1}{2}(K_{3/2,1,1/2} + K_{1/2,1,3/2})\right]D_2 \\ &+ \left[K_{1/2,1/2,1} + \frac{1}{2}(K_{3/2,1/2,1} + K_{1/2,3/2,1})\right]D_3 + (k_2 \leftrightarrow k_3)\bigg].\end{aligned} \tag{B17}$$

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