Setting the scale of the pp and $p\bar{p}$ total cross sections using AdS/QCD

Sophia K. Domokos and Jeffrey A. Harvey
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

Nelia Mann
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637, USA
Reed College, 3203 SE Woodstock Boulevard, Portland, Oregon 97202, USA
(Received 3 September 2010; published 15 November 2010)

This paper is an addendum to earlier work where we computed the Pomeron contribution to pp and $p\bar{p}$ scattering in AdS/QCD. Our model for pp scattering in the Regge regime depends on four parameters: the slope and intercept of the Pomeron trajectory α_c', $\alpha_c(0)$, a mass scale M_d, which determines a form factor entering into matrix elements of the energy-momentum tensor, and a coupling $\lambda_{\mathcal{P}}$ between the lightest spin-two glueball and the proton, which sets the overall scale of the total cross section. Here we perform a more detailed computation of $\lambda_{\mathcal{P}}$ in the Sakai-Sugimoto model by using the construction of nucleons as instantons of the dual 5D gauge theory and an effective 5D fermion description of these nucleons which has been successfully used to compute a variety of nucleon-meson couplings. We find $\lambda_{\mathcal{P},\mathrm{SS}} \simeq 6.38\ \mathrm{GeV}^{-1}$, which is in reasonable agreement with the value $\lambda_{\mathcal{P},\mathrm{fit}} = 8.28\ \mathrm{GeV}^{-1}$ determined by fitting single Pomeron exchange to data.

DOI: 10.1103/PhysRevD.82.106007 PACS numbers: 11.25.Tq

I. INTRODUCTION

Many soft quantities in QCD can be successfully described using the ideas of Regge theory, in which scattering is dominated not by the exchange of single particles but rather by infinite towers of resonances whose mass squared and spin are linearly related as $J = \alpha(0) + \alpha' M^2$. This is closely connected to the idea of a string dual of QCD at large N_c. At very large energies total cross sections are dominated by the exchange of the trajectory with the largest intercept $\alpha(0)$, also known as the Pomeron. From a modern point of view the Pomeron is the leading Regge trajectory containing the lightest spin-two glueball. For an overview of this perspective, our conventions, and a more detailed list of references see [1]. Various connections between the Pomeron, Regge theory, and AdS/QCD have also been explored in [2–8].

The spin-two glueball field can be treated as a second-rank symmetric traceless tensor $q_{\mu\nu}$. General arguments as well as specific calculations indicate that this field should couple predominantly to the QCD stress tensor $T^{\mu\nu}$:

$$S_{\mathrm{int}} = \lambda_{\mathcal{P}} \int d^4x q_{\mu\nu} T^{\mu\nu}. \tag{1}$$

The coupling $\lambda_{\mathcal{P}}$ sets the scale of the total cross section for pp and $p\bar{p}$ scattering. For example, in the model of [1] the total cross section is given by

$$\sigma_{\mathrm{tot}} = c\pi\lambda_{\mathcal{P}}^2 \left(\frac{\alpha_c' s}{2}\right)^{\alpha_c(0)-1}, \tag{2}$$

where c is a constant of order one and the Pomeron Regge trajectory is $J = \alpha_c(0) + \alpha_c' M^2$. Fits to data give a value $\lambda_{\mathcal{P},\mathrm{fit}} \simeq 8.28\ \mathrm{GeV}^{-1}$ [9].

In [1] we assumed single Pomeron exchange and fit to data to obtain $\alpha_c(0) \simeq 1.09$. This behavior eventually violates the Froissart bound, although this does not happen until values of s much above those that are currently accessible. Other authors have argued that total cross sections are better fit by a $\log^2(s)$ behavior [10–13] at current values of s. If this is the case, multiple Pomeron exchange must already be important (corresponding to multiloop diagrams in the dual string description) and explicit calculations will be much more difficult. However, it still seems likely that the overall scale of the cross section will be set by the coupling $\lambda_{\mathcal{P}}$.

In [1] $\lambda_{\mathcal{P}}$ was calculated in a very simple approximation in which the proton was treated as a Skyrmion constructed out of the pion field with the result that $\lambda_{\mathcal{P},\mathrm{Skyrme}} \simeq 3.9\ \mathrm{GeV}^{-1}$, which is significantly smaller than the experimental value. However, the description of the proton in the dual model of [14] is known to be more complicated than this. In particular, the towers of vector and axial-vector mesons contribute significantly to the solution, a fact that has played an important role in the determination of nucleon-meson coupling constants [15–21]. In this paper we perform a more detailed calculation of $\lambda_{\mathcal{P}}$ in the model of [14] in the limit of large 't Hooft coupling and obtain $\lambda_{\mathcal{P},\mathrm{SS}} \simeq 6.38\ \mathrm{GeV}^{-1}$, which is within ~23% of the experimental value.

In Sec. II, we give a very brief review of the Sakai-Sugimoto model and of the structure of baryons in this model. In Sec. III, we describe the baryon as a Skyrmion, and in Sec. IV we introduce the effective fermion description of baryons and compute the lowest baryon wave functions. In Sec. V, we deduce the coupling of the glueball to baryons and compute the leading contribution to $\lambda_{\mathcal{P}}$ at large 't Hooft coupling. We end, in Sec. VI, by discussing the limitations of this work due to the assumptions about large 't Hooft coupling.

II. REVIEW OF THE SAKAI-SUGIMOTO MODEL

In the Sakai-Sugimoto model [14] we start with the dual of the pure glue sector of QCD constructed in [22]. Here, we have a stack of N_c $D4$-branes wrapping an S^1. Antiperiodic boundary conditions along this S^1 are imposed on the fermion fields to break supersymmetry. The $D4$-branes source a metric given by

$$ds^2 = \left(\frac{U}{R}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu + f(U)d\tau^2) + \left(\frac{R}{U}\right)^{3/2}\left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right), \tag{3}$$

where, in terms of the string length ℓ_s, string coupling g_s, and number of colors N_c, $R^3 = \pi g_s N_c \ell_s^3$. The function $f(U)$ is given by

$$f(U) = 1 - \frac{U_{KK}^3}{U^3}, \tag{4}$$

where U_{KK} is the minimal value of the radial coordinate $U \in [U_{KK}, \infty]$. The solution has topology $R^{3,1} \times D \times S^4$ with (U, τ) coordinates on the disk D and τ the (periodic) angular coordinate on the disk with identification

$$\tau \sim \tau + \frac{2\pi}{M_{KK}}. \tag{5}$$

M_{KK}, which governs the mass scale of states in the theory, is related to the parameters R and U_{KK} appearing in the metric via

$$M_{KK} = \frac{3U_{KK}^{1/2}}{2R^{3/2}}. \tag{6}$$

The dilaton is given by

$$e^{-\Phi} = \frac{1}{g_s}\left(\frac{R}{U}\right)^{3/4}, \tag{7}$$

and in addition the Ramond-Ramond field F_4 carries N_c units of flux on the S^4.

Quark degrees of freedom are included through the addition of N_f $D8$-branes [14]. While different embeddings of the $D8$-branes, described by profiles $\tau(U)$, are possible, we choose the embedding of the $D8$-branes for which the minimal value of U is $U_0 = U_{KK}$, for which $\tau(U)$ is constant. The $D8$-branes are then flat inside the ten-dimensional space. The induced metric on the $D8$-brane is

$$ds_{8+1}^2 = ds_{4+1}^2 + ds_4^2, \tag{8}$$

where

$$ds_{4+1}^2 = \left(\frac{U}{R}\right)^{3/2}(\eta_{\mu\nu}dx^\mu dx^\nu) + \left(\frac{R}{U}\right)^{3/2}\frac{1}{f(U)}dU^2 \tag{9}$$

and

$$ds_4^2 = \left(\frac{R}{U}\right)^{3/2}U^2 d\Omega_4^2, \tag{10}$$

with $d\Omega_4^2$ the metric on the unit S^4.

Define h_4 to be the determinant of the S_4 metric components and h_{4+1} the determinant of the $4+1$ metric components. We also define

$$V_4 = \int d\Omega_4 \equiv \frac{8\pi^2}{3} \tag{11}$$

and

$$\mathrm{Vol}_{S_4} = \int \sqrt{h_4}d\Omega_4 = R^3 U \times V_4. \tag{12}$$

A. Conformal coordinates and coordinate ranges

It is convenient to make a change of variables from U to w so that the $4+1$ metric is conformal to flat space. This requires that

$$\left(\frac{R}{U}\right)^{3/2}\frac{1}{f(U)}dU^2 = \left(\frac{U}{R}\right)^{3/2}dw^2, \tag{13}$$

which gives

$$w(U) = \int_{U_{KK}}^{U}\frac{R^{3/2}dU'}{\sqrt{U'^3 - U_{KK}^3}}. \tag{14}$$

Inverting this defines $U(w)$, and the induced metric on the $D8$-brane is then

$$ds_{8+1}^2 = \left(\frac{U(w)}{R}\right)^{3/2}(dw^2 + \eta_{\mu\nu}dx^\mu dx^\nu) + R^{3/2}U^{1/2}d\Omega_4^2. \tag{15}$$

Now, the coordinate U only covers half the D8-brane, and $w(U)$, as defined above, does the same. Therefore, we extend the range to $w \in [-w_{\max}, +w_{\max}]$ to cover the whole $D8$-brane. Wave functions are either even or odd in w, a fact directly related to the CP properties of the corresponding four-dimensional meson states. Glueballs arise as four-dimensional normalizable modes of the higher-dimensional metric perturbations and the lowest spin-two glueball has a wave function $\tilde{T}(w)$ which is an even function of w [23,24]. In what follows we write the relevant 5D metric in terms of w as

$$ds_{4+1}^2 = H(w)(dw^2 + \eta_{\mu\nu}dx^\mu dx^\nu), \tag{16}$$

with $H(w) \equiv (U(w)/R)^{3/2}$.

B. Action for gauge fields

The Dirac-Born-Infeld action, expanded to quadratic order in the gauge fields, leads to

$$S_{D8} = \frac{\mu_8(2\pi\alpha')^2}{4}\int d^9\xi e^{-\Phi}\sqrt{-\mathrm{Det}g_{ab}}\,\mathrm{Tr}F_{ab}F^{ab}, \tag{17}$$

with $a, b = 0, 1, \ldots 8$. Using the conformal metric this leads to

$$S_{D8} = \frac{\mu_8(2\pi\alpha')^2 R^3 V_4}{g_s} \times \int d^4x dw \frac{U(w)}{4} \eta^{MN}\eta^{PQ}\,\mathrm{Tr}F_{MP}F_{NQ}, \quad (18)$$

with $M, N = 0, 1, 2, 3, w$. In [21] this is written as

$$S_{D8} = \int d^4x dw \frac{1}{4e^2(w)} \eta^{MN}\eta^{PQ}\,\mathrm{Tr}F_{MP}F_{NQ}. \quad (19)$$

If the generators are normalized as

$$\mathrm{Tr}\, T^a T^b = \tfrac{1}{2}\delta^{ab}, \quad (20)$$

this leads to a nonstandard definition of the coupling. With the canonical factor of $1/4$ in front of each component of the gauge field action, the canonically defined coupling is $g_5(w) = \sqrt{2}e(w)$.

Using the previous definitions we can write

$$\frac{1}{e^2(w)} = \frac{\lambda N_c M_{KK} U(w)}{108\pi^3 U_{KK}}, \quad (21)$$

which agrees with the result quoted in [17,18,21]. Introducing a dimensionless version of $U(w)$ as $u(w) = U(w)/U_{KK}$,

$$\frac{1}{e^2(w)} = \frac{\lambda N_c M_{KK} u(w)}{108\pi^3}, \quad (22)$$

which highlights the fact that all ℓ_s dependence drops out of the leading gauge theory action once quantities are expressed in terms of the defining parameters of the dual field theory: λ, M_{KK}, and N_c.

III. DESCRIPTION OF THE SKYRMION/INSTANTON/BARYON

The baryon is described by a charge-one instanton of the $SU(2)$ gauge field on the flavor branes, which spans the (x, y, x, w) directions. The exact solution of the equations of motion is not known, but it is believed that one can take the flat space instanton as a reasonable approximation to the full solution. This solution has moduli consisting of the $SU(2)$ orientation, the location of the instanton in (x, y, z, w), and the scale size ρ. The scale size of the instanton is fixed by balancing the gauge action, which makes it want to shrink, and the coupling of the baryon current to the tower of ω mesons arising from the Chern-Simons term, which makes it want to grow. It becomes a spin-$1/2$ object after quantizing the collective coordinates, as is familiar from the original Skyrmion literature. For details see [15,17]. The location of the instanton in (x, y, z) is arbitrary as a consequence of 3D translational invariance. The nontrivial metric dependence on w, on the other hand, results in a w-dependent contribution to the energy with the minimum occurring at $w = 0$.

The baryon mass quoted in the literature is

$$m_B^{\mathrm{cl}} = \frac{\lambda N_c}{27\pi} M_{KK}\left(1 + \frac{\sqrt{2\cdot 3^5 \cdot \pi^2/5}}{\lambda} + \cdots\right). \quad (23)$$

For very large λ, $m_B^{\mathrm{cl}} \gg M_{KK}$ and the second term is subleading. When the instanton is displaced from $w = 0$, one finds that to quadratic order

$$m_B(w) = m_B^{\mathrm{cl}}(1 + \tfrac{1}{3}(wM_{KK})^2 + \cdots). \quad (24)$$

While this will turn out to be the correct expression for the 4D mass, it is important to distinguish it from the 5D mass. The instanton or baryon is a gauge field configuration depending on x^i and w but independent of x^0. It is localized in x^i and w and after integrating over these coordinates the action becomes

$$S = \int m_B^{\mathrm{cl}} dx^0, \quad (25)$$

and the mass m_B^{cl} is extracted from this expression [see e.g. Eq. (3.18) in [15]]. However in 5D curved space the action for a static particle of mass m_5 is expressed in terms of the proper time as

$$S = \int m_5 d\tau, \quad (26)$$

which leads to

$$m_5 = \frac{m_B^{\mathrm{cl}}}{\sqrt{g_{00}}}. \quad (27)$$

For the metric we are using $g_{00} = (U/R)^{3/2}$ and so the 5D mass is actually

$$m_5 (U/R)^{3/4} = m_B^{\mathrm{cl}}. \quad (28)$$

IV. EFFECTIVE FERMION DESCRIPTION OF BARYONS

Following the treatment of [17–21], which we examine in detail below, we describe the Skyrmion as an effective fermion field living in the D8 world volume. The basic idea is that the Skyrmion is an instanton constructed out of the flavor gauge fields, with a size that is much smaller than M_{KK} at large 't Hooft coupling λ. Since the baryon has spin $1/2$, it is reasonable to write its effective description in terms of a 5D fermion field. To compute what we call $\lambda_{\mathcal{P}}$, the coupling of the spin-two glueball to the proton, we will need to work out the coupling of the 5D metric to the 5D fermion and reduce this to a 4D coupling using mode expansions for the 5D metric and the 5D fermion field representing the baryon.

We saw above that the pure gauge part of this action can be derived by starting from a general coordinate-invariant action and reducing it using 5D conformal coordinates.

We follow a similar procedure for the 5D fermions. We begin with a general coordinate (and local Lorentz invariant) action in 5D, to which we can add metric perturbations and thus extract the coupling to the glueball. Let us start from a curved space action in 5D—unlike the flat, 4D effective action posited in [17–21]:

$$S_f[\bar{\psi}, \psi, g] = -i\mathcal{N}\int d^5x e^{-\Phi(w)}\mathrm{Vol}_{S^4}(w)\sqrt{h_{4+1}} \times [\bar{\psi} e^m_{\hat{a}}\Gamma^{\hat{a}} D_m\psi + m_5(w)\bar{\psi}\psi]. \quad (29)$$

Here m, n, and p are 5D world indices, $\hat{a}$, $\hat{b}$, and $\hat{c}$ are 5D tangent space indices, and the covariant derivative is (ignoring the gauge fields which will not enter into the rest of our calculation)

$$D_m\psi = \left(\partial_m - \frac{i}{4}\omega_m^{\hat{a}\hat{b}}\sigma_{\hat{a}\hat{b}}\right)\psi \quad (30)$$

with

$$\sigma_{\hat{a}\hat{b}} = \frac{i}{2}[\Gamma_{\hat{a}}, \Gamma_{\hat{b}}]. \quad (31)$$

The $\Gamma_{\hat{a}}$ are tangent space gamma matrices obeying

$$\{\Gamma_{\hat{a}}, \Gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}}. \quad (32)$$

The factors of the dilaton and the volume of the S^4 are as we would expect them to emerge from the Sakai-Sugimoto model after quantizing the collective coordinates of the instanton to obtain a spin-1/2 object. Note that

$$e^{-\Phi}\mathrm{Vol}_{S_4}\sqrt{h_{4+1}} = \frac{V_4}{g_s}U(w)^4. \quad (33)$$

Any normalization factors are included in the overall prefactor $\mathcal{N}$, which will later be absorbed by a redefinition of the fermion field. Other than the w-dependent prefactors this is the standard curved space action for a spin-1/2 fermion.

We can now evaluate the action for the particular background metric given in (16). We choose

$$e^{\hat{a}} = H(w)^{1/2}\delta^{\hat{a}}_m dx^m \quad (34)$$

and find that the structure equation for the spin connection is solved (modulo local Lorentz transformations and diffeomorphisms) by the spin connection

$$\omega^{\hat{\mu}}{}_{\hat{\nu}} = 0, \qquad \omega^{\hat{\mu}}{}_{\hat{w}} = \tfrac{1}{2}\partial_w \ln(H(w))dx^\mu, \quad (35)$$

with

$$D_w\psi = \partial_w\psi \quad (36)$$

and

$$-\frac{i}{4}\omega_\mu{}^{\hat{a}\hat{b}}\sigma_{\hat{a}\hat{b}} = \frac{1}{8}\omega_\mu{}^{\hat{a}\hat{b}}[\Gamma_{\hat{a}}, \Gamma_{\hat{b}}] = \frac{1}{4}\omega_\mu{}^{\hat{\mu}\hat{w}}[\Gamma_{\hat{\mu}}, \Gamma_{\hat{w}}] = \frac{\partial_w H}{4H(w)}\Gamma_{\hat{\mu}}\Gamma_{\hat{w}}. \quad (37)$$

We can take $\Gamma_{\hat{\mu}} = \gamma_\mu$ and $\Gamma_{\hat{w}} = \gamma_5$ where the flat space γ matrices are defined as

$$\gamma^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}, \qquad \gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (38)$$

and find

$$S = -i\mathcal{N}\int dwd^4x(\mathrm{Vol}_{S^4}(w)e^{-\phi(w)}\sqrt{h_{4+1}})H^{-1/2} \times\left(\bar{\psi}\left[\gamma^\mu\partial_\mu + \gamma^5\partial_w + \frac{\partial_w H}{H}\gamma^5\right]\psi + m_5(w)H^{1/2}\bar{\psi}\psi\right). \quad (39)$$

We can now reabsorb w-dependent factors into the fermion field ψ to obtain a simplified form for the action. Taking

$$\psi(x, w) \equiv \frac{1}{H(w)}\mathcal{B}(x, w), \quad (40)$$

the action becomes

$$S = -i\mathcal{N}\int dwd^4x(\mathrm{Vol}_{S^4}(w)e^{-\phi(w)}\sqrt{h_{4+1}}H^{-5/2}) \times(\bar{\mathcal{B}}[\gamma^\mu\partial_\mu + \gamma^5\partial_w]\mathcal{B} + m_5(w)H^{1/2}\bar{\mathcal{B}}\mathcal{B}). \quad (41)$$

The prefactor in round brackets is

$$(\mathrm{Vol}_{S^4}(w)e^{-\phi(w)}\sqrt{h_{4+1}}H^{-5/2}) = \frac{V_4 R^{15/4}}{g_s}U^{1/4}(w). \quad (42)$$

If we approximate this factor by its value at $w = 0$, and then absorb this constant as well as $\mathcal{N}$ into $\mathcal{B}$, then we reproduce the action posited in [17,21],

$$S_B = -i\int d^4xdw[\bar{\mathcal{B}}\gamma^m D_m\mathcal{B} + m_{\mathcal{B}}(w)\bar{\mathcal{B}}\mathcal{B} + \cdots] - \int d^4xdw\frac{1}{4e^2(w)}\mathrm{tr}F_{mn}F^{mn}, \quad (43)$$

with $m_B^{\mathrm{cl}}(w) = H^{1/2}(w)m_5(w)$. Note that the covariant derivative above only involves the gauge fields

$$D_m = \partial_m - iA_m \quad (44)$$

and that we have omitted higher terms coupling the baryon field to the gauge fields, though these are necessary for reproducing various interactions as well as the long-range tail of the Skyrmion configuration.

V. COUPLING OF THE FERMION TO THE GLUEBALL METRIC PERTURBATION

The glueball is described by a symmetric, traceless perturbation of the 3 + 1 part of the metric $g_{\mu\nu} \rightarrow g_{\mu\nu} - h_{\mu\nu}$. The effect of this perturbation is equivalent to writing

$$\bar{\mathcal{B}}\gamma^\mu\partial_\mu\mathcal{B} = \bar{\mathcal{B}}\eta_{\mu\nu}\gamma^\mu\partial^\nu\mathcal{B} = \bar{\mathcal{B}}H^{-1}(w)g_{\mu\nu}\gamma^\mu\partial^\nu\mathcal{B} \quad (45)$$

and replacing $g_{\mu\nu} \to g_{\mu\nu} - h_{\mu\nu}$ to obtain the coupling

$$H^{-1}(w)h_{\mu\nu}\mathcal{B}\gamma^\mu\partial^\nu\mathcal{B} = H^{-1}(w)h_{\mu\nu}\tfrac{1}{2}\mathcal{B}(\gamma^\mu\partial^\nu + \gamma^\nu\partial^\mu)\mathcal{B}, \quad (46)$$

which is equal to

$$H^{-1}(w)h_{\mu\nu}T^{\mu\nu}_{\mathcal{B}}, \quad (47)$$

where $T^{\mu\nu}_{\mathcal{B}}$ is the w-dependent fermion stress-energy tensor.

We are now ready to compute $\lambda_{\mathcal{P}}$ starting with the coupling we just derived:

$$\int dwd^4xH^{-1}(w)h_{\mu\nu}T^{\mu\nu}_{\mathcal{B}}. \quad (48)$$

We write the metric perturbation $h_{\mu\nu}$ in terms of the glueball wave function following the treatment in [23] to obtain

$$h_{\mu\nu} = \left(\frac{U(w)}{R}\right)^{3/2}\tilde{T}(U(w))q_{\mu\nu}, \quad (49)$$

where $q_{\mu\nu}$ is a canonically normalized spin-two field in four dimensions. The lightest mode of $h_{\mu\nu}(x, w)$ has 4D mass eigenvalue $m_{\rm glue} = (2/3)M_{KK}\sqrt{5.5} \simeq 1.47$ GeV.

We also write the 5D baryon in terms of w-dependent wave functions and 4D fermion fields. The 5D Dirac equation which follows from the action (43) is

$$\gamma^\mu\partial_\mu\mathcal{B} + \gamma^5\partial_w\mathcal{B} + m_{\mathcal{B}}(w)\mathcal{B} = 0. \quad (50)$$

To solve this we write $\mathcal{B}(x, w)$ in terms of γ^5 eigenstates:

$$\mathcal{B}(x^\mu, w) = \sum_n \begin{pmatrix} B_+^{(n)}(x)f_+^{(n)}(w) \\ B_-^{(n)}(x)f_-^{(n)}(w)\end{pmatrix}. \quad (51)$$

As was the case for the glueball, we are only interested in the lowest ($n = 1$) mode, which represents the physical nucleon doublet. Dropping the (n) superscript we then have for the lowest eigenmode

$$\begin{aligned} &f_-(w)\bar{\sigma}^\mu\partial_\mu B_-(x) + \partial_w f_+(w)B_+(x) \\ &\quad + m_{\mathcal{B}}(w)B_+(x)f_+(w) = 0, \\ &f_+(w)\sigma^\mu\partial_\mu B_+(x) - \partial_w f_-(w)B_-(x) \\ &\quad + m_{\mathcal{B}}(w)B_-(x)f_-(w) = 0. \end{aligned} \quad (52)$$

Comparing this to the equation for a massive 4D spinor with mass $m_{\mathcal{N}}$ we see that the eigenvalue equation is

$$\begin{aligned} (\partial_w + m_{\mathcal{B}}(w))f_+ &= m_{\mathcal{N}}f_-, \\ (-\partial_w + m_{\mathcal{B}}(w))f_- &= m_{\mathcal{N}}f_+, \end{aligned} \quad (53)$$

and to get a standard 4D action the wave functions should be normalized to

$$\int dw|f_\pm(w)|^2 = 1. \quad (54)$$

At large λ the baryons become very heavy compared to the scale M_{KK} and the width of the wave functions scales like $1/\sqrt{\lambda}$. Thus at very large λ we can approximate

$$|f_+(w)|^2 = |f_-(w)|^2 = \delta(w). \quad (55)$$

In this approximation the interaction takes the form

$$\lambda_{\mathcal{P}}\int d^4xq_{\mu\nu}T^{\mu\nu}_{\mathcal{B}}(x), \quad (56)$$

where

$$\mathcal{B}(x) = \begin{pmatrix} B_+(x) \\ B_-(x)\end{pmatrix} \quad (57)$$

is the 4D nucleon wave function and $T_{\mathcal{B}}(x)$ is the 4D energy-momentum tensor of the spin-1/2 object. The coupling constant is simply

$$\lambda_{\mathcal{P}} = \tilde{T}(U(0)). \quad (58)$$

Evaluating this at $w = 0$ gives

$$\lambda_{\mathcal{P}} = \tilde{T}(1) = \sqrt{\frac{3}{0.9\pi^2}}\frac{1}{f_\pi} = 6.38\ {\rm GeV}^{-1}. \quad (59)$$

VI. CONCLUSIONS AND FURTHER ISSUES

In this paper we have computed λ_P in the effective fermion picture from [17–21]. We found good agreement with the data; however, we must keep in mind the limitations of this treatment of baryons in the Sakai-Sugimoto model. The first two terms in the expansion of the baryon mass in powers of $1/\lambda$ are

$$m_B^{\rm classical} = \frac{\lambda N_c}{27\pi}M_{KK}\left(1 + \frac{\sqrt{2\cdot 3^5\cdot \pi^2/5}}{\lambda} + \cdots\right), \quad (60)$$

which comes about from balancing the Coulomb and "Pontragin" contributions to the energy. These are the first terms in an expansion in the inverse 't Hooft coupling $\lambda = g_{YM}^2N_c$. For very large λ, $m_B^{\rm classical} \gg M_{KK}$.

However, the actual values of the parameters needed to fit the ρ mass and f_π in this model are $M_{KK} = 0.94$ GeV and $\lambda = 17$. Note that the proton mass is 0.938 GeV, which is almost exactly M_{KK}. Using these values one finds

$$m_B^{\rm classical} = 0.6M_{KK}(1 + 1.82 + \cdots). \quad (61)$$

The first-order term is not a very accurate estimate of the baryon mass, and the second term, which is supposed to be subleading, is actually larger than the first. This $1/\lambda$ expansion should thus be employed with some care.

On the other hand, the leading approximation in λ to a variety of meson-nucleon couplings in this model does agree quite well with experimental data [16–18,21]. From the above calculation it seems that the same is true for the coupling of the spin-two glueball to the proton, a quantity which sets the scale of the total cross section for proton-proton scattering.

ACKNOWLEDGMENTS

This work was supported in part by NSF Grant No. PHY 0855039. We thank Piljin Yi for helpful conversations and the Aspen Center for Physics for providing a congenial atmosphere at the start of this work.

[1] S. K. Domokos, J. A. Harvey, and N. Mann, Phys. Rev. D **80**, 126015 (2009).
[2] R. C. Brower, J. Polchinski, M. J. Strassler, and C. I. Tan, J. High Energy Phys. 12 (2007) 005.
[3] R. C. Brower, M. Djuric, I. Sarcevic, and C. I. Tan, arXiv:1007.2259.
[4] L. Cornalba, M. S. Costa, and J. Penedones, J. High Energy Phys. 03 (2010) 133.
[5] T. Banks and G. Festuccia, J. High Energy Phys. 06 (2010) 105.
[6] O. Andreev, Phys. Rev. D **71**, 066006 (2005).
[7] R. A. Janik, Phys. Lett. B **500**, 118 (2001).
[8] S. J. Brodsky and G. F. de Teramond, Phys. Rev. D **78**, 025032 (2008).
[9] This formula differs by a factor of 4 from the original formula in [1] due to an algebraic error in [1].
[10] M. M. Block, E. M. Gregores, F. Halzen, and G. Pancheri, Phys. Rev. D **60**, 054024 (1999).
[11] J. R. Cudell *et al.*, Phys. Rev. D **65**, 074024 (2002).
[12] K. Igi and M. Ishida, Phys. Rev. D **66**, 034023 (2002).
[13] M. M. Block and F. Halzen, Phys. Rev. D **70**, 091901 (2004).
[14] T. Sakai and S. Sugimoto, Prog. Theor. Phys. **113**, 843 (2005).
[15] H. Hata, T. Sakai, S. Sugimoto, and S. Yamato, Prog. Theor. Phys. **117**, 1157 (2007).
[16] K. Hashimoto, T. Sakai, and S. Sugimoto, Prog. Theor. Phys. **120**, 1093 (2008).
[17] D. K. Hong, M. Rho, H. U. Yee, and P. Yi, Phys. Rev. D **76**, 061901 (2007).
[18] D. K. Hong, M. Rho, H. U. Yee, and P. Yi, J. High Energy Phys. 09 (2007) 063.
[19] D. K. Hong, M. Rho, H. U. Yee, and P. Yi, Phys. Rev. D **77**, 014030 (2008).
[20] J. Park and P. Yi, J. High Energy Phys. 06 (2008) 011.
[21] Y. Kim, S. Lee, and P. Yi, J. High Energy Phys. 04 (2009) 086.
[22] E. Witten, Adv. Theor. Math. Phys. **2**, 505 (1998).
[23] R. C. Brower, S. D. Mathur, and C. I. Tan, Nucl. Phys. **B587**, 249 (2000).
[24] N. R. Constable and R. C. Myers, J. High Energy Phys. 10 (1999) 037.