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Multiple M0-brane system in an arbitrary eleven-dimensional supergravity background

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The equations of motion of multiple M0-brane (multiple M-wave or mM0) systems in an arbitrary $D = 11$ supergravity superspace, which generalize the matrix model equations for the case of interaction with a generic 11-dimensional supergravity background, are obtained in the frame of the superembedding approach. We also derive the Bogomol'nyi-Prasad-Sommerfeld (BPS) equations for supersymmetric bosonic solutions of these mM0 equations and show that the set of $1/2$ BPS solutions contain a fuzzy sphere modeling M2 brane as well as that the Nahm equation appears as a particular case of the $1/4$ BPS equations.

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I. INTRODUCTION

More than 15 years ago the concept of M-theory, a hypothetical underlying theory unifying five consistent string models and 11-dimensional supergravity, appeared [1]. Since that time many interesting and important results have been obtained, including the unexpected applications of the ideas and methods of string/M-theory, first, to studying quantum gauge field theories [2], including calculating the viscosity of quark-gluon plasma [3], and, then, to condensed matter physics, superfluidity, and superconductivity [4].

However, the question on the fundamental degrees of freedom of M-theory still remains open. On several occasions the opinion (see e.g., [5]) has been expressed that the present indirect description of M-theory is the best description possible. This is done in terms of its perturbative and low energy limits, given, respectively, by the five consistent ten-dimensional string theories and 11-dimensional supergravity, by a chain of dualities relating these and by the set of supersymmetric extended objects, super-p-branes or, shorter, p-branes (strings for $p = 1$, membranes for $p = 2$, etc.; $p = 0$ corresponds to particles). From the M-theoretical perspective the most interesting are ten-dimensional fundamental strings (also called F1-branes) and Dirichlet p-branes (Dp-branes) and 11-dimensional M-branes, i.e., Mp-branes with $p = 0$, 2, 5 (see, however, [6] for recent interest in lower-dimensional branes). These can be described by the supersymmetric solutions of $D = 10$ and $D = 11$ supergravity [7], by the action functionals given by the integrals over their $(p + 1)$-dimensional worldvolumes W^{p+1} (worldvolume actions) [8–11] and also in the frame of the superembedding approach [12–16]. This, following the so-called Sorokin-Tkach-Volkov approach to superparticles and superstrings [17,18],[1] describes p-branes in terms of embedding of its *worldvolume superspace* ($\mathcal{W}^{(p+1|16)}$ for $D = 11$ and type II $D = 10$ p-branes) into the target superspace ($\Sigma^{(D|32)}$ for $D = 10$ type II and $D = 11$ p-branes).[2]

As far as an effective description of multiple brane systems is concerned, it was quickly appreciated that at very low energy the system of N nearly coincident Dp-branes (multiple Dp-brane or mDp system) is described by the maximally supersymmetric $d = (p + 1)$-dimensional $U(N)$ Yang-Mills model (SYM model) [20]. However, in the problem of constructing a complete (more complete) nonlinear supersymmetric action for multiple Dp-branes, posed in ninetieth [21], only a particular progress could be witnessed (see [22–24] for lower-dimensional and lower-codimensional branes as well as [25–27] discussed below).

The so-called ''dielectric brane action'' proposed by Myers [28], although widely accepted, is purely bosonic, is not Lorentz invariant, and resisted the attempts of its straightforward Lorentz covariant and supersymmetric generalization all these years.[3] Recently developed by Howe, Lindström, and Wulff, the *boundary fermion approach* [25,26] provides a supersymmetric and covariant description of Dirichlet branes, but on the ''pure classical'' (or ''minus one quantization'') level in the sense that, to arrive at the description of multiple D-brane system in terms of the variables similar to the ones in the standard action for a single Dp-brane [9,10] (usually considered as a classical or quasiclassical action), one has to perform a quantization of the boundary fermion sector. The complete quantization of the model [25,26] should produce not only worldvolume fields of the multiple Dp-brane system but

[1]See [15] for more references and [19] for related studies.

[2]Thirty-two (32) in the notation $\Sigma^{(11|32)}$ for the 11-dimensional target (or bulk) superspace is the number of components of the $SO(1, 10)$ spinor and 16 in the notation $\mathcal{W}^{(p+1|16)}$ above appears as just 32/2.

[3]This does not look so surprising if we recall that it was derived in [28] by a chain of dualities starting from the ten-dimensional non-Abelian Dirac-Born-Infeld action with a symmetric trace prescription [21], the supersymmetric generalization of which is also unknown.

also bulk supergravity and higher stringy modes. The partial quantization of only the boundary fermion sector (applying the original prescription [29] of replacing the boundary fermion variables by gamma matrices of corresponding internal symmetry group) allowed Howe, Lindström, and Wulff to reproduce the purely bosonic Myers action [28], but the Lorentz invariance was lost on that way. See [27,30] and the recent work in [31] for further discussion on the present status of the boundary fermion approach. Finally, a (possibly approximate but going beyond $U(N)$ SYM) superembedding description of mDp-brane system was proposed and developed for mD0 in [27].

The situation with the description of multiple M-brane systems was even more complicated: for many years it was even unclear what model provides the description of the multiple M2 (mM2) system at very low energy. The expected properties of such a model, playing for mM2 the same rôle as $U(N)$ SYM for mDp, were described in [32] where also problems hampering the way to its construction were analyzed. In the search for a solution to these problem, a new $\mathcal{N} = 8$ supersymmetric $d = 3$ Chern-Simons plus matter model based on Filippov 3 algebra [33] instead of Lie algebra (Bagger-Lambert-Gustavsson model) was constructed in [34]. However, presently the commonly accepted candidate for the low energy description of the mM2 system is a more conventional $SU(N) \times SU(N)$ invariant Aharony-Bergman-Jafferis-Maldacena model [35], although this possesses only $\mathcal{N} = 6$ manifest $d = 3$ supersymmetries. The search for a nonlinear generalization of the Bagger-Lambert-Gustavsson model resulted in a purely bosonic and Lorentz noncovariant action [36] generalizing the Myers proposition for a multiple bosonic membrane case. The counterpart of the Myers action for the purely bosonic limit of the multiple M0-brane system (also called multiple M-waves, mM0, or multiple gravitons) was constructed in [37].

The superembedding approach to the *multiple M0-brane (mM0)* system was proposed in [30]. In it the relative motion of mM0 constituents is described by the maximally supersymmetric $SU(N)$ gauge theory on $\mathcal{W}^{(1|16)}$ superspace with one bosonic and 16 fermionic directions, the embedding of which into the target 11-dimensional superspace $\Sigma^{(11|32)}$ is specified by the superembedding equation [see Eq. (3.3) below]. The latter produces, as its self-consistency conditions, the dynamical equations of motion for the mM0 center of energy degrees of freedom.

This superembedding approach to the mM0 system provides a covariant generalization of the matrix model equations with manifest 11-dimensional Lorentz invariance. In light of that a single M0-brane is dual to the single D0-brane [10], the superembedding description of the multiple M0 system, constructed and checked for consistency for the case of the flat target 11-dimensional superspace in [30], provides the restoration of 11-dimensional Lorentz invariance in the (originally ten dimensional) multiple D0 brane system as was described by the superembedding approach of [27].

The aim of the present paper is to derive the equations of motion for a multiple M0 system in a generic curved supergravity superspace in the framework of the superembedding approach of [30]. These equations describe the multiple M0 interaction with the 11-dimensional supergravity fluxes and provide the covariant generalization of the matrix model [38] for the case of the arbitrary supergravity background. The form of these equations has been briefly reported in [39], where the universality of their structure was emphasized: when written with indefinite coefficients, these equations can be reproduced (up to vanishing of a few of the above coefficients) from the requirement of $SO(1,1) \otimes SO(9)$ invariance and very few data on the basic fields describing the relative motion of mM0 constituents and on the fluxes which do interact with them.

Here we give the details of the derivation of the mM0 equations and perform a complete study of the consistency conditions for the superembedding approach. We show that these consistency conditions are obeyed due to the pullback of the supergravity equations of motion, namely, of the specific projections of the pullbacks of the Rarita-Schwinger and the Einstein equations to $\mathcal{W}^{(1|16)}$. This provides a counterpart of the known fact that the $D = 11$ and $D = 10$ supergravity superspace constraints, and hence the supergravity equations of motion, can be derived from the requirement of the κ symmetry of the worldvolume action for a single M-brane and D-brane or fundamental string, respectively.

We also use the superembedding approach to derive the Bogomol'nyi-Prasad-Sommerfeld (BPS) conditions for the supersymmetric pure bosonic solutions of the equations of motion. In particular, we present the explicit form of the $1/2$ BPS conditions and show that it has a fuzzy two-sphere solution describing the M2-brane as a $1/2$ BPS configuration of the multiple M0 system. We also show that the famous Nahm equation, which also has a fuzzy two-sphere-related (fuzzy funnel) solution, appears as a particular case of the $1/4$ BPS equation with vanishing 4-form flux.

The paper is organized as follows. In Sec. II we present the necessary details on the superspace formulation of $D = 11$ supergravity [40,41] in the notation close to [42]. Section III contains a brief review of the superembedding approach in its application to a single M0 brane, the equations of motion for this supersymmetric object and the description of the intrinsic and extrinsic geometry of the worldline superspace $\mathcal{W}^{(1|16)}$ embedded in the curved supergravity superspace $\Sigma^{(11|32)}$. Particularly, in Sec. III E we present some properties of the relevant projections of the pullbacks of target superspace "fluxes" (which are 4-form field strength superfield $F_{abcd} = F_{[abcd]}(Z)$, gravitino field strength superfield $T_{ab}{}^{\alpha} = T_{[ab]}{}^{\alpha}(Z)$ and Riemann tensor superfield $R_{ab}^{cd} = R_{[ab}{}^{[cd]}(Z))$ to the worldvolume superspace, including the relation between them which follows from the Rarita-Schwinger and Einstein equations of the $D = 11$ supergravity. In Sec. IV we formulate our proposal for the description of the mM0 system by the $SU(N)$ connection on the $d = 1$ $\mathcal{N} = 16$ worldline superspace $\mathcal{W}^{(1|16)}$. This superspace, the embedding of which to $\Sigma^{(11|32)}$ is restricted by the superembedding equation, describes the center of energy motion of the mM0 system, which we discuss in Sec. IV A. In Sec. IV B we present the constraints on the $d = 1$ $\mathcal{N} = 16$ $SU(N)$ connection ($1d$ $16\mathcal{N}$ SYM supermultiplet) and, in Sec. IV C, derive the dynamical equations of the relative motion of mM0 constituents which follow from these constraints. The BPS equations for supersymmetric bosonic solutions of the mM0 equations of motion are presented in Sec. V where we also describe the $1/2$ BPS fuzzy sphere solution modeling M2 brane by a configuration of the mM0 system with N constituents, and the appearance of the Nahm equation from the $1/4$ BPS equation of mM0. Some useful technical details are presented in the Appendices.

II. SUPERSPACE OF $D = 11$ SUPERGRAVITY

M-branes or M-theory super-p-branes are extended objects propagating in $D = 11$ supergravity superspace $\Sigma^{(11|32)}$. We denote local coordinates of $\Sigma^{(11|32)}$ by $Z^M = (x^m, \theta^{\check{\alpha}})$ ($\check{\alpha} = 1, \ldots, 32$, $m = 0, 1, \ldots, 9, 10$), with bosonic x^{μ} and fermionic $\theta^{\check{\alpha}}$,

$$x^{\mu} x^{\nu} = x^{\nu} x^{\mu}, \qquad x^{\mu} \theta^{\check{\alpha}} = \theta^{\check{\alpha}} x^{\mu}, \qquad \theta^{\check{\alpha}} \theta^{\check{\beta}} = -\theta^{\check{\beta}} \theta^{\check{\alpha}}.$$

The supergravity is described by the set of supervielbein 1-forms

$$E^A := dZ^M E_M{}^A(Z) = (E^a, E^{\alpha}), \tag{2.1}$$

including bosonic vectorial form E^a ($a = 0, 1, \ldots, 9, 10$) and fermionic spinorial form E^{α} ($\alpha = 1, \ldots, 32$), which satisfy the set of superspace constraints [40,41]. The most important of these constraints determine the bosonic torsion 2-form of $\Sigma^{(11|32)}$. This reads

$$T^a := DE^a = -iE^{\alpha} \wedge E^{\beta} \Gamma^a_{\alpha\beta}, \tag{2.2}$$

where $\Gamma^a_{\alpha\beta} = \Gamma^a_{\beta\alpha}$ are 11-dimensional Dirac matrices (see Appendix A), $\wedge$ denotes the exterior product of differential forms,

$$E^b \wedge E^a = -E^a \wedge E^b, \qquad E^b \wedge E^{\alpha} = -E^{\alpha} \wedge E^b,$$
$$E^{\beta} \wedge E^{\alpha} = E^{\alpha} \wedge E^{\beta},$$

and D denotes the covariant derivative, $DE^a = dE^a - E^b \wedge w_b{}^a$, where $w^{ba} = dZ^M w_M^{ba}(Z) = -w^{ab}$ is the superspace $SO(1, 10)$ connection 1-form (11-dimensional spin connection).

After imposing a set of conventional constraints, the study of Bianchi identities (see [40,41] and, e.g., [42] and references therein) fixes the form of the fermionic torsion to be

$$T^{\alpha} := DE^{\alpha} = -E^a \wedge E^{\beta} t_{a\beta}{}^{\alpha} + \tfrac{1}{2} E^a \wedge E^b T_{ba}{}^{\alpha}(Z), \tag{2.3}$$

where

$$t_{a\beta}{}^{\alpha} := \frac{i}{18}\left(F_{abcd} \Gamma^{bcd}{}_{\beta}{}^{\alpha} + \frac{1}{8} F^{bcde} \Gamma_{abcde\beta}{}^{\alpha} \right) \tag{2.4}$$

is expressed in terms of the fourth rank antisymmetric tensor superfield $F_{abcd} = F_{[abcd]}(Z)$ ("4-form flux") which obeys

$$D_{[a} F_{bcde]} = 0. \tag{2.5}$$

This indicates that the leading component ($\theta = 0$ value) of F_{abcd} can be identified with the field strength of the 3-form gauge field of the 11-dimensional supergravity.

Furthermore, the supergravity Bianchi identities also express the superspace Riemann tensor 2-form

$$\begin{aligned} R^{ab} := (d\omega - \omega \wedge \omega)^{ab} = {}& E^{\alpha} \wedge E^{\beta} \\ & \times \left(-\frac{1}{3} F^{abc_1c_2} \Gamma_{c_1c_2} + \frac{i}{3 \cdot 5!} (*F)^{abc_1\ldots c_5} \Gamma_{c_1\ldots c_5} \right)_{\alpha\beta} \\ & + E^c \wedge E^{\alpha} \left(-iT^{ab\beta} \Gamma_{c\beta\alpha} + 2iT_c{}^{[a\beta} \Gamma^{b]}{}_{\beta\alpha} \right) \\ & + \frac{1}{2} E^d \wedge E^c R_{cd}{}^{ab}(Z) \end{aligned} \tag{2.6}$$

in terms of the same antisymmetric tensor superfield $F_{abcd}(Z)$, the superspace generalization of the gravitino field strength $T_{ab}{}^{\alpha}(Z)$ ["fermionic flux" defined in Eq. (2.3)] and Riemann tensor superfield $R_{ab}{}^{cd} = R_{ab}{}^{cd}(Z) = -R_{ba}{}^{cd} = -R_{ab}{}^{dc}$ obeying

$$R_{[abc]}{}^d = 0. \tag{2.7}$$

To be convinced that the supervielbein and Lorentz connection obeying the above set of superspace constraints describe just the supergravity multiplet and no other fields are present, one notices that the supergravity Bianchi identities also express the fermionic covariant derivatives of the antisymmetric tensor superfield F_{abcd}, of the fermionic flux $T_{ab}{}^{\alpha}$ and of the Riemann tensor superfield $R_{cda}{}^b(Z)$ through the same set of superfields. In particular,

$$D_{\alpha} F_{abcd} = -6 T_{[ab}{}^{\beta} \Gamma_{cd]\beta\alpha}, \tag{2.8}$$

$$D_{\alpha} T_{ab}{}^{\beta} = -\tfrac{1}{4} R_{ab}{}^{cd} \Gamma_{cd\alpha}{}^{\beta} - 2(D_{[a} t_{b]} + t_{[a} t_{b]})_{\alpha}{}^{\beta}, \tag{2.9}$$

where $t_{a\alpha}{}^{\beta}$ is expressed through $F_{abcd}(Z)$ by Eq. (2.4).

Further study of Bianchi identities also shows that the superspace constraints (2.2) are *on shell*, i.e., that the supergravity equations of motion appear as their consequences. Those include Einstein equations

$$R_{ab} = -\tfrac{1}{3}F_{ac_1c_2c_3}F_b^{c_1c_2c_3} + \tfrac{1}{36}\eta_{ab}F_{c_1c_2c_3c_4}F^{c_1c_2c_3c_4},$$
$$R_{ab} := R_{acb}{}^c, \qquad \eta_{ab} = \mathrm{diag}(+, -, \ldots, -) \qquad (2.10)$$

and the Rarita-Schwinger equations $T_{bc}{}^{\beta}\Gamma^{abc}_{\beta\alpha} = 0$. It is convenient to write the latter in the equivalent form of

$$T_{ab}{}^{\beta}\Gamma^{b}_{\beta\alpha} = 0. \qquad (2.11)$$

III. SUPEREMBEDDING APPROACH TO THE SINGLE M0-BRANE AND GEOMETRY OF THE WORLDLINE SUPERSPACE $\mathcal{W}^{(p+1|16)}$

A. Superembedding equation

The standard formulation of Mp-branes deals with the embedding of a purely bosonic worldvolume W^{p+1} (worldline W^1 for the case of the M0-brane) into the *target superspace* $\Sigma^{(11|32)}$. The *superembedding approach* to M-branes [12,14] describes their dynamics in terms of the embedding of *worldvolume superspace* $\mathcal{W}^{(p+1|16)}$ with $d = p + 1$ bosonic and 16 fermionic directions into $\Sigma^{(11|32)}$. This embedding can be described in terms of coordinate functions $\hat{Z}^M(\zeta) = (\hat{x}^m(\zeta), \hat{\theta}^{\check{\alpha}}(\zeta))$, which are superfields depending on the local coordinates $\zeta^{\mathcal{M}}$ of $\mathcal{W}^{(p+1|16)}$,

$$\mathcal{W}^{(p+1|16)} \in \Sigma^{(11|32)}: Z^M = \hat{Z}^M(\zeta^{\mathcal{N}}). \qquad (3.1)$$

For $p = 0$, these are $\zeta^{\mathcal{N}} = (\tau, \eta^{\check{q}})$, where $\eta^{\check{q}}$ are 16 fermionic coordinates of the *worldline superspace* $\mathcal{W}^{(1|16)}$,

$$\mathcal{W}^{(1|16)} \in \Sigma^{(11|32)}: Z^M = \hat{Z}^M(\tau, \eta^{\check{q}}),$$
$$\eta^{\check{q}}\eta^{\check{p}} = -\eta^{\check{p}}\eta^{\check{q}}, \qquad \check{q} = 1, \ldots, 16, \qquad (3.2)$$

and τ is its bosonic coordinate generalizing the particle proper time.

To describe a super p-brane, the coordinate functions $\hat{Z}^M(\zeta^{\mathcal{N}})$ have to satisfy the *superembedding equation* which states that the pullback $\hat{E}^a := d\hat{Z}^M(\zeta)E^a_M(\hat{Z})$ of the bosonic supervielbein form $E^a := dZ^M E^a_M(Z)$ to the worldvolume superspace has no fermionic projection. In the case of the M0-brane this superembedding equation reads

$$\hat{E}_{+q}{}^{a} := D_{+q}\hat{Z}^M E_M{}^a(\hat{Z}) = 0, \qquad (3.3)$$

where D_{+q} is a fermionic covariant derivative of $\mathcal{W}^{(1|16)}$, $q = 1, \ldots, 16$ is a spinor index of $SO(9)$, and $+$ denotes the "charge" (weight) with respect to the local $SO(1,1)$ group. In our notation the superscript plus index is equivalent to the subscript minus, and vice versa, so that one can equivalently write $D_{+q} = D^-_q$.

We denote the supervielbein of $W^{(1|16)}$ by

$$e^{\mathcal{A}} = d\zeta^{\mathcal{M}}e_{\mathcal{M}}{}^{\mathcal{A}}(\zeta) = (e^{\#}, e^{+q}), \qquad (3.4)$$

and the only bosonic covariant derivative of $\mathcal{W}^{(1|16)}$ by $D_{\#} := D_{++}$ so that $D = e^{\mathcal{A}}D_{\mathcal{A}}$ with

$$D_{\mathcal{A}} = (D_{\#}, D_{+q}). \qquad (3.5)$$

B. Moving frame and spinor moving frame variables

To study the consequences of the superembedding equation, it is convenient to introduce the auxiliary *moving frame superfields* $u^{=}_a$, $u^{\#}_a$, u^i_b which obey

$$u^{=}_a u^{a=} = 0, \qquad u^{\#}_a u^{a\#} = 0, \qquad u^{\#}_a u^{a=} = 2,$$
$$u^{=}_a u^{ai} = 0, \qquad u^{\#}_a u^{ai} = 0, \qquad u^i_a u^{aj} = -\delta^{ij}. \qquad (3.6)$$

The above constraints imply that the 11×11 matrix constructed from the columns $u^{=}_a$, $u^{\#}_a$ and u^i_b (*moving frame matrix*) is Lorentz group valued,

$$U^{(a)}_m = \left(\frac{u^{=}_m + u^{\#}_m}{2}, u^i_m, \frac{u^{\#}_m - u^{=}_m}{2}\right) \in SO(1,10). \qquad (3.7)$$

To clarify the way these moving frame variables appear in the superembedding approach let us first notice that the superembedding equation (3.3) can be written in the form of

$$\hat{E}^{\,a} := d\hat{Z}^M(\zeta)E_M{}^a(\hat{Z}(\zeta)) = \tfrac{1}{2}e^{\#}u^{=a} \qquad (3.8)$$

with some 11-vector superfield $u^{=}_a = u^{=}_a(\zeta)$. The study of consistency conditions shows that this vector must be lightlike, $u^{=}_a u^{a=} = 0$, which allows for its identification with one of the lightlike components of the moving frame (3.7).

More precisely, the integrability conditions for the superembedding equation imply that

$$\delta_{qp}u^{=}_a = v^{-\alpha}_q\Gamma^a_{\alpha\beta}v^{-\beta}_p, \qquad (3.9)$$

where the set of 16 spinorial superfields $v^{-\alpha}_q$ appear as coefficients for the $\mathcal{W}^{(1|16)}$ fermionic supervielbein forms in the expressions for the pullbacks of the target superspace fermionic supervielbein forms,

$$\hat{E}^{\,\alpha} := d\hat{Z}^M(\zeta)E_M{}^{\alpha}(\hat{Z}(\zeta)) = e^{+q}v^{-\alpha}_q + e^{\#}\chi_{\#}{}^{-q}v^{+\alpha}_q. \qquad (3.10)$$

Then one can show that, as a consequence of (3.9), the 11-vector superfield $u^{=}_a$ is lightlike and finds that it can be completed up the complete moving frame (3.7).

In a theory with $SO(1,1) \times SO(9)$ symmetry, the variables $v^{-\alpha}_q$ obeying the constraints (3.9) parametrize the celestial sphere S^9 ($9 = D - 2$ for $D = 11$; see [43] for $D = 4$, 6, 10 and [44,45] for the $D = 11$ superparticle cases). They form a 32×16 matrix which can be completed until the 32×32 *spinor moving frame* matrix

$$V_{(\beta)}{}^{\alpha} = \begin{pmatrix} v^{+\alpha}_q \\ v^{-\alpha}_q \end{pmatrix} \in \mathrm{Spin}(1,10). \qquad (3.11)$$

[Notice that $v^{+\alpha}_q$ has been already used in Eq. (3.10)]. This spinor moving frame matrix is related to the moving frame matrix (3.7) by the constraints expressing the Lorentz invariance of the Dirac matrices,

$$V\Gamma_b V^T = u_b^{(a)}\Gamma_{(a)}, \tag{3.12}$$

$$V^T\tilde{\Gamma}^{(a)}V = \tilde{\Gamma}^b u_b^{(a)}, \tag{3.13}$$

and of the charge conjugation matrix,

$$VCV^T = C, \qquad V^TC^{-1}V = C^{-1}. \tag{3.14}$$

The relation (3.9) appears as a 16×16 block in the splitting of the 32×32 matrix of constraint (3.12). The constraint (3.14) allows us to express the elements of the inverse spinor moving frame matrix ($V_\alpha^{(\beta)} = (v_{\alpha q}{}^-, v_{\alpha q}{}^+) \in \mathrm{Spin}(1,10)$) in terms of the original moving frame variables (3.11)

$$v_{\alpha q}{}^{\mp} = \pm iC_{\alpha\beta}v_q^{\mp\beta}, \qquad v_q^{\pm\alpha} = \pm iC^{\alpha\beta}v_{\beta q}{}^{\pm}. \tag{3.15}$$

[In our case of $D = 11$ with our mostly plus notation the charge conjugation matrix is imaginary, hence the appearance of i in Eqs. (3.15)].

The moving frame and spinor moving frame variables are also used to construct the $SO(1,1)$ and $SO(9)$ connections on the worldvolume superspace $\mathcal{W}^{(1|16)}$. The simplest way to define this connection as induced by (super) embedding is to write the $SO(1,10)\times SO(1,1)\times SO(9)$ covariant derivatives (3.5) of the moving frame and spinor moving frame variables as follows:

$$Du_m^= = u_m^i\Omega^{=i}, \tag{3.16}$$

$$Du_m^\# = u_m^i\Omega^{\#i}, \tag{3.17}$$

$$Du_m^i = \tfrac{1}{2}u_m^\#\Omega^{=i} + \tfrac{1}{2}u_m^=\Omega^{\#i}, \tag{3.18}$$

$$Dv_q^{-\alpha} = -\tfrac{1}{2}\Omega^{=i}\gamma^i_{qp}v_p^{+\alpha}, \tag{3.19}$$

$$Dv_q^{+\alpha} = -\tfrac{1}{2}\Omega^{\#i}\gamma^i_{qp}v_p^{-\alpha}. \tag{3.20}$$

Here $\Omega^{=i}$ and $\Omega^{\#i}$ generalize the $\frac{SO(1,10)}{SO(1,1)\times SO(9)}$ Cartan forms for the case of curved target superspace.

Now the $SO(1,1)$ curvature, $r = d\omega^{(0)}$, of the worldline superspace $\mathcal{W}^{(1|16)}$ and the $SO(9)$ curvatures of the normal bundle over it, $\mathcal{G}^{ij}$, can be defined through the Ricci identities specified for the moving frame variables,

$$DDu_a^\# = 2d\omega^{(0)}u_a^\# + \hat{R}_a{}^b u_b^\#, \tag{3.21}$$

$$DDu_a^= = -2d\omega^{(0)}u_a^= + \hat{R}_a{}^b u_b^=, \tag{3.22}$$

$$DDu_a{}^i = u_a^j\mathcal{G}^{ji} + \hat{R}_a{}^b u_b{}^i. \tag{3.23}$$

Here $\hat{R}_a{}^b$ is the pullback of the target superspace Riemann curvature two form (2.6) to $\mathcal{W}^{(1|16)}$. Contracting Eq. (3.21) with $u^{=a}$ and Eq. (3.23) with u^{ja}, and denoting the moving frame projections of the Riemann curvature pullback $\hat{R}_a{}^b$ by

$$\begin{aligned}\hat{R}^{=\#} &:= \hat{R}^{ab}u_a^= u_b^\#, & \hat{R}^{ij} &:= \hat{R}^{ab}u_a^i u_b{}^j,\\ \hat{R}^{=j} &:= \hat{R}^{ab}u_a^= u_b{}^j, & \hat{R}^{\#j} &:= \hat{R}^{ab}u_a^\# u_b{}^j,\end{aligned} \tag{3.24}$$

one finds the following generalization of the Gauss and Ricci equations of the classical surface theory (see [12] for references):

$$d\omega^{(0)} = \tfrac{1}{4}\hat{R}^{=\#} + \tfrac{1}{4}\Omega^{=i}\wedge\Omega^{\#i}, \tag{3.25}$$

$$\mathcal{G}^{ij} = \hat{R}^{ij} - \Omega^{=[i}\wedge\Omega^{\#j]}. \tag{3.26}$$

One can also use (3.21) and (3.22) to obtain, as integrability conditions of Eqs. (3.16) and (3.17), the following generalization of the Peterson-Codazzi equations:

$$D\Omega^{=i} = \hat{R}^{=i}, \qquad D\Omega^{\#i} = \hat{R}^{\#i}. \tag{3.27}$$

More details on moving frame variables and their role in the superembedding approach can be found in Appendix B as well as in [16,30] in the case of the M0-brane and in [12,15,16] (and in references therein) in the general case.

C. Equations of motion of a single M0-brane from the superembedding approach

The superembedding Eq. (3.3) is *on shell* in the sense that it contains the M0-brane equations of motion among its consequences. We refer to [30] for the details on the derivation of these equations and just present the result. The fermionic equations of motion state the vanishing of the bosonic component of the pullback of the fermionic supervielbein of $\mathcal{W}^{(1|16)}$,

$$\chi_{\#p}{}^- := \hat{E}_\#{}^\alpha v_{\alpha p}{}^- = 0, \tag{3.28}$$

so that on the mass shell Eq. (3.10) simplifies to

$$\hat{E}^\alpha := d\hat{Z}^M(\zeta)E_M{}^\alpha(\hat{Z}(\zeta)) = e^{+q}v_q^{-\alpha}. \tag{3.29}$$

The bosonic equation of motion for the M0-brane reads

$$\Omega_\#^{=i} := -D_\# u^{=a}u_a{}^i = -D_\#\hat{E}_\#{}^a u_a{}^i = 0. \tag{3.30}$$

Equations (3.29) and (3.30) imply the differential form equation

$$\Omega^{=i} := -Du^{a=}u_a{}^i = 0 \tag{3.31}$$

stating vanishing of the 1-form in the right-hand sides (r.h.s.'s) of Eqs. (3.16) and (3.19). Hence, the dynamical equations of the M0-brane can be formulated as the condition that the lightlike moving frame vector $u_a^=$ and its square root [in the sense of Eq. (3.9)], the set of 16 constrained spinorial superfields $v_q^{-\alpha}$, are covariantly constants,

$$Du_a^= = 0, \qquad Dv_a^{-\alpha} = 0. \tag{3.32}$$

D. Geometry of the worldline superspace $\mathcal{W}^{(1|16)}$ and of the $SO(9)$ bundle over it

As far as the worldline superspace $\mathcal{W}^{(1|16)}$, whose embedding into the target 11-dimensional superspace $\Sigma^{(11|32)}$ will be used to describe the motion of the multiple M0 system, we will need some details on the geometry of $\mathcal{W}^{(1|16)}$ and of the normal bundle over it.

Taking into account Eqs. (3.28) and (3.30), one finds that, similarly to the case of flat superspace, the bosonic torsion two form of $\mathcal{W}^{(1|16)}$ is given by

$$De^{\#} = -2ie^{+q} \wedge e^{+q}, \tag{3.33}$$

and that the curvature of the $SO(1,1)$ connection on $W^{(1|16)}$ vanishes

$$d\omega^{(0)} = 0 \tag{3.34}$$

[see Gauss equation (3.25)]. Nevertheless, the geometry induced on $W^{(1|16)}$ by its embedding to $\Sigma^{(11|32)}$ is not trivial because the fermionic torsion 2-form is nonzero,

$$De^{+q} = -\tfrac{1}{72}e^{\#} \wedge e^{+p}\hat{F}_{\# ijk}\gamma^{ijk}_{pq}. \tag{3.35}$$

Here $i, j, k = 1, \ldots, 9$, $\gamma^{ijk} = \gamma^{[i}\gamma^{j}\gamma^{k]}$ is the antisymmetric product of the nine-dimensional Dirac matrices, $\gamma^{i}_{qp} = \gamma^{i}_{pq}$, obeying

$$\gamma^{i}\gamma^{j} + \gamma^{j}\gamma^{i} = \delta^{ij}I_{16\times 16}, \qquad i, j = 1, \ldots, 9 \tag{3.36}$$

(some useful properties of these can be found in Appendix A). Equation (3.35) expresses the fermionic torsion in terms of the projection

$$\hat{F}_{\# ijk} := F^{abcd}(\hat{Z})u_a{}^{=}u_b{}^{i}u_c{}^{j}u_d{}^{k} \tag{3.37}$$

of the pullback to $\mathcal{W}^{(1|16)}$ of the 4-form flux (4-form field strength superfield) $F^{abcd}(Z)$ of the 11-dimensional supergravity. This flux projection enters as well in the expression for the $SO(9)$ curvature of normal bundle over $\mathcal{W}^{(1|16)}$ determined by the Ricci equation (3.26),

$$\begin{aligned} G^{ij} &= \hat{R}^{ij} \\ &= e^{+q} \wedge e^{+p}\left(\frac{2i}{3}\hat{F}_{\# ijk}\gamma^{k}_{qp} + \frac{i}{18}\hat{F}_{\# klm}\gamma^{ijklm}_{qp}\right) \\ &\quad - ie^{\#} \wedge e^{+q}\gamma_{qp[i}\hat{T}_{\# j]+p}. \end{aligned} \tag{3.38}$$

The last term in (3.38) contains the projection

$$\hat{T}_{\# i+q} := T_{ab}{}^{\beta}(\hat{Z})v^{-}_{\beta q}u^{=}_{a}u^{i}_{b} \tag{3.39}$$

of the pullback to $W^{(1|16)}$ of the "fermionic flux" $T_{ab}{}^{\beta}(Z)$ [superfield generalization of the gravitino field strength, see Eq. (2.3)].

Here and below, to make the equations lighter, we identify upper and lower case $SO(9)$ vector indices; although our 11-dimensional metric is "mostly minus," $\eta^{ab} = \mathrm{diag}(+, -, \ldots, -)$, this should not create confusion as far as we never use contractions of "internal" indices with $\eta_{ij} = -\delta^{ij}$. We also conventionally replace the $=$ superscript by the # subscript in the notation for the contractions of the tensors with $u^{=}_{a}$.

Notice that, with the M0 equations of motion written in the form of Eq. (3.31), $\Omega^{=i} = 0$, the Peterson-Codazzi Eq. (3.27) results in

$$\hat{R}^{=i} := \hat{R}^{ab}u^{=}_{a}u_{b}{}^{i} = 0.$$

Calculating the pullback of the Riemann curvature 2-form (2.6) to $\mathcal{W}^{(1|16)}$, one sees that this relation is satisfied identically.

While $\Omega^{=i} = 0$ encodes the M0 equations of motion, the second set of $SO(1,1) \otimes SO(9) \otimes SO(1,10)$ covariant 1-forms $\Omega^{\# i}$ determining the $SO(1,1) \otimes SO(9) \otimes SO(1,10)$ covariant derivatives of $u^{\#}_{a}$ and $v^{+\alpha}_{q}$ in Eqs. (3.17) and (3.20), remains unspecified by the superembedding equations. This reflects the K_9 gauge symmetry of the massless superparticle dynamics; in our superembedding approach K_9 appears as a gauge symmetry leaving invariant $u^{=}_{a}$ and $v^{-\alpha}_{q}$ while acting on the remaining moving frame superfields by

$$\begin{aligned} \delta u^{\#}_{a} &= 2k^{\# i}u^{i}_{a}, \qquad \delta u^{i}_{a} = 2k^{\# i}u^{=}_{a}, \\ &\qquad \delta v^{+\alpha}_{q} = k^{\# i}\gamma^{i}_{qp}v^{-\alpha}_{q}. \end{aligned} \tag{3.40}$$

With respect to K_9 the 1-form $\Omega^{\# i}$ is not covariant but transforms as a connection; actually it can be considered as a part of the connection of a normal bundle over $\mathcal{W}^{(1|16)}$. The structure group of this normal bundle is nonstandard, $SO(9)(\times K_9$ (rather than, say $SO(10)$) because the bosonic body of $\mathcal{W}^{(1|16)}$ is a lightlike line in spacetime. However, for our purposes here it is sufficient to account for the $SO(9)$ part of the curvature of this normal bundle and to keep manifest only the $SO(1,1) \otimes SO(9) \otimes SO(1,10)$ gauge symmetry, thus leaving K_9 symmetry hidden.

E. Pullback of the fluxes to $\mathcal{W}^{(1|16)}$ and the supergravity equations of motion

Thus, the characteristics of the geometry of $\mathcal{W}^{(1|16)}$, induced by its embedding to $\Sigma^{(11|32)}$, and of the normal bundle over it, involve only definite projections (3.37) and (3.39) of the pullbacks to $\mathcal{W}^{(1|16)}$ of the covariant bosonic and fermionic superfields ("fluxes") of the 11-dimensional supergravity. Then, if some model is defined on $\mathcal{W}^{(1|16)}$, its interaction with background supergravity will be described by this projections of the fluxes and by their derivatives. This poses the problem of calculating the worldline covariant derivatives of superfields (3.37) and (3.39) which might seem to be quite involved. Fortunately, the properties of $\mathcal{W}^{(1|16)}$ simplify these calculations essentially.

First, let us observe that Eqs. (3.37) and (3.39) involve only $u^{=}_{a}$, $v^{-}_{\alpha q}$ and u^{i}_{a} moving frame superfields. Then, as was mentioned above, the equations of motion for the single M0-brane (which follow from superembedding equation) can be expressed by the statement that $Dv_{\alpha q}^{-}=0$ and $Du_a^{=}=0$, Eq. (3.32). Furthermore, due to the same equations which can be written in the form of Eq. (3.31), the derivative of the u_a^i superfield reads $Du_a^i=\frac{1}{2}\Omega^{\# i}u_a^{=}$ [see (3.18)]. It is important that $Du_a^i\propto u_a^{=}$ and, hence, do not contribute in the derivative of an expression constructed from an antisymmetric tensor of $SO(1,10)$ contracting one of its indices with u_a^i and another with $u_a^{=}$. The projections (3.37) and (3.39) of the bosonic and fermionic fluxes are just of this type so that the calculation of their worldline superspace fermionic covariant derivatives is basically reduced to the algebraic operation with the expressions for the background superspace spinorial derivatives of the corresponding superfields, Eqs. (2.8) and (2.9).

After some algebra using the properties of moving frame and spinor moving frame variables [Eqs. (B5) and (B6) in Appendix B], we find that Eq. (2.8) implies

$$D_{+q}\hat{F}_{\# ijk}=3i\gamma_{[ij|qp}\hat{T}_{\#|k]p} \tag{3.41}$$

and Eq. (2.9) results in

$$D_{+p}\hat{T}_{\# i+q}=\frac{1}{2}\hat{R}_{\# ij\#}\gamma^j_{pq}+\frac{1}{3}D_{\#}\hat{F}_{\# ijk}\Big(\delta^{i[j}\gamma^{kl]}_{pq}+\frac{1}{6}\gamma^{ijkl}_{pq}\Big)+\hat{F}_{\# j_1j_2j_3}\hat{F}_{\# k_1k_2k_3}\Sigma_{pq}^{i,j_1j_2j_3,k_1k_2k_3} \tag{3.42}$$

Here

$$\hat{R}_{\# ij\#}:=R_{dcba}(\hat{Z})u^{d=}u^{ci}u^{bj}u^{a=} \tag{3.43}$$

is the specific projection of Riemann tensor and the explicit form of the last term reads

$$\hat{F}_{\# j_1j_2j_3}\hat{F}_{\# k_1k_2k_3}\Sigma_{pq}^{i,j_1j_2j_3,k_1k_2k_3}=-\frac{1}{12}\gamma^j_{pq}\Big(\hat{F}_{\# ik_1k_2}\hat{F}_{\# jk_1k_2}+\frac{1}{9}\delta^{ij}(\hat{F}_{\# k_1k_2k_3})^2\Big)+\frac{1}{9}\gamma^{j_1j_2j_3}_{pq}\hat{F}_{\# ij_1k}\hat{F}_{\# kj_2j_3}+\frac{1}{72}\gamma^{k_1k_2k_3k_4k_5}_{pq}(\hat{F}_{\# ik_1k_2}\hat{F}_{\# k_3k_4k_5}+\delta^i_{[k_1}\hat{F}_{\# k_2k_3|j}\hat{F}_{\# j|k_4k_5]}). \tag{3.44}$$

Notice that the projection (3.43) of the Riemann tensor is symmetric as far as

$$\hat{R}_{\#[ij]\#}=\tfrac{3}{2}\hat{R}_{[\# ij]\#}=0 \tag{3.45}$$

due to Eq. (2.7), $R_{[abc]d}=0$. Furthermore, its trace (on $SO(9)$ vector indices) is expressed through the product of the projections (3.37) of the 4-form fluxes by

$$\hat{R}_{\# j\# j}+\tfrac{1}{3}(\hat{F}_{\# ijk})^2=0, \tag{3.46}$$

which is the $u_a^{=}u^{=b}$ projection of the pullback of the supergravity Einstein Eq. (2.10) to $\mathcal{W}^{(1|16)}$.

The contraction of the pullback to $\mathcal{W}^{(1|16)}$ of the supergravity Rarita-Schwinger Eqs. (2.11) with $u^{--a}v_q^{-\alpha}$ gives

$$\gamma^i_{qp}\hat{T}_{\# i+p}=0. \tag{3.47}$$

It should not be too surprising that the self-consistency condition for this equation is satisfied identically when the consequence (3.46) of the supergravity Einstein equation (2.10) is taken into account, $\gamma^i_{qs}D_{+p}\hat{T}_{\# i+s}=-\frac{1}{2}\delta_{qp}(\hat{R}_{\# j\# j}+\frac{1}{3}(\hat{F}_{\# ijk})^2)=0$.

Now we have all necessary details on the geometry of the worldline superspace $\mathcal{W}^{(1|16)}$ induced by its superembedding in $\Sigma^{(11|32)}$ and are ready to study the supersymmetric gauge theory on this superspace which we use to describe the relative notion of the constituents of the mM0 system.

IV. MULTIPLE M0 DESCRIPTION BY $SU(N)$ SYM ON $\mathcal{W}^{(1|16)}$ SUPERSPACE

The superembedding approach to multiple M0-brane system implies, in particular, a superfield description of the relative motion of M0 constituents. Our proposition is to describe the relative motion of M0 constituents by the maximally supersymmetric $SU(N)$ Yang-Mills gauge theory on $\mathcal{W}^{(1|16)}$ whose embedding into the target 11-dimensional superspace is specified by the superembedding equation (3.3) [30]. To motivate such a choice, we first notice that, as far as the M0-brane is dual to the type IIA D0-brane [10], it is natural to expect that the multiple M0 system is dual to the multiple D0-brane one. Then, the worldline superspace $SU(N)$ SYM description of the relative motion in the multiple M0-system is suggested by the superembedding description of the multiple D0's [27]. The suggestion to describe this by a $d=1$ $\mathcal{N}=16$ $SU(N)$ SYM model on the $\mathcal{W}^{(1|16)}$ superspace comes from the fact that at very low energy the gauge fixed description of the dynamics of the multiple Dp-brane system in flat target type II superspace can be provided by maximally supersymmetric $(p+1)$-dimensional $U(N)$ $(=SU(N)\otimes U(1))$ SYM model, i.e., by dimensional reduction of the corresponding $D=10$ SYM model with $U(N)$ gauge symmetry [20].

Now we have to specify the embedding of the "center of mass" (better to say, "center of energy") superspace $\mathcal{W}^{(1|16)}$ of the multiple M0 system into the target superspace $\Sigma^{(11|32)}$ of 11-dimensional supergravity. The natural proposition is to require this to be defined by the superembedding equation (3.3). The arguments in favor of such a choice include the universality of the superembedding equation and the difficulty one meets in an attempt to generalize it. Now we can also refer on that the approach based on the use of the center energy superspace $\mathcal{W}^{(1|16)}$ obeying the superembedding equation was checked on consistency for the multiple M0 system in flat $D = 11$ superspace [30]. However, it was clear from the very beginning that this superembedding approach is able to provide a covariant generalization of the matrix model equation valid in any curved 11-dimensional supergravity background. In this section we derive the explicit form of such equations describing the multiple M0 interaction with the 11-dimensional supergravity fluxes.

A. mM0 center of energy motion from superembedding of $\mathcal{W}^{(1|16)}$ into $\Sigma^{(11|32)}$

Thus, the center of energy superspace of the mM0 system is chosen to be $\mathcal{W}^{(1|16)}$, the counterpart of the worldline superspace of single M0, the embedding of which into the target superspace, an arbitrary 11-dimensional supergravity superspace $\Sigma^{(11|32)}$, is restricted by the superembedding equation (3.3). As far as the superembedding equation specifies completely the geometry of the worldline superspace, all the knowledge on the torsion forms and curvature of $\mathcal{W}^{(1|16)}$ and normal bundle over it, Eqs. (3.33), (3.34), (3.35), and (3.38), on its extrinsic geometry, Eq. (3.31), as well as on the pullbacks of fluxes to $\mathcal{W}^{(1|16)}$, Eqs. (3.37), (3.39), (3.40), (3.41), (3.42), (3.43), (3.44), (3.45), (3.46), and (3.47), are true for this center of energy superspace. In particular, the pullbacks of the target space supervielbein to $\mathcal{W}^{(1|16)}$ obey Eqs. (3.28) and (3.30), which encodes the dynamical equations of motion for single M0-brane [equivalent to Eqs. (3.32)],

$$\hat{E}^{\,a} = e^{\#}u^{a=}/2, \qquad Du^{a=} = 0, \qquad u^{a=}u_a^{=} = 0, \quad (4.1)$$

$$\hat{E}^{\,\alpha} = e^{+q}v_q^{-\alpha}, \quad Dv_q^{-\alpha} = 0, \quad v_q^{-}\Gamma^a v_p^{-} = \delta_{qp}u_a^{=}. \quad (4.2)$$

The fact that the equations of motion for the center of energy of the multiple p-brane system have the form of the equations for the single brane looks natural, in particular, when we are speaking about a system of particles. However, one has to stress that for the mM0 system, as far as the single M0 brane is a *massless* 11-dimensional superparticle, the statement that the dynamics of the center of energy is governed by a single M0 equations implies that the mM0 center of energy moves on a lightlike geodesic in the bosonic body of $\Sigma^{(11|32)}$. This fact, expressed by the third equation in (4.1) [or, equivalently, by $\hat{E}^a_{\#}\hat{E}_{\#a} = 0$], should not look surprising if we keep in mind the image of, for instance, a beam of light, which moves as a whole in a lightlike direction despite, say, gravitational interaction among photons.

One may also find this property natural for a generalization of the matrix model. Indeed, making a dimensional reduction of our mM0 system to ten dimensional, on the way similar to passing from single M0 to single D0 in [10] by generalized dimensional reduction, we will find a timelike motion of the center of mass of the ten-dimensional system, which would be the mD0 system in a type IIA supergravity background. Actually such a system, but in a simpler background, was the final "destination" of the DLCQ (discrete light-cone quantization) approach in [46,47][4]

B. Basic superfields describing relative motion of mM0 constituents and basic constraints for them

Thus, our center of energy superspace $\mathcal{W}^{(1|16)}$ is defined by the superembedding equation (3.3) imposed on the coordinate functions $\hat{Z}^M(\zeta) = (\hat{x}^m(\zeta), \hat{\theta}^{\check{\alpha}}(\zeta))$. This results in dynamical equations which formally coincide with the equations of motion of a single M0-brane, which implies, in particular, that the center of energy motion is lightlike. Our proposition is to describe the relative motion of the mM0 constituents by the $d = 1$, $\mathcal{N} = 16$ $SU(N)$ SYM model on this superspace. This is formulated in terms of a 1-form gauge potential $A = e^{\#}A_{\#} + e^{+q}A_{+q}$ the field strength of which,

$$\begin{aligned} G_2 &= dA - A \wedge A \\ &= 1/2 e^{+q} \wedge e^{+p} G_{+p+q} + e^{\#} \wedge e^{+q} G_{+q\#}, \end{aligned} \quad (4.3)$$

should be restricted by the set of constraints. The natural choice for the these constraints is

$$G_{+q+p} = i\gamma^i_{qp}\mathbf{X}^i, \quad (4.4)$$

where $\gamma^i_{qp} = \gamma^i_{pq}$ are nine-dimensional Dirac matrices (3.36) and $\mathbf{X}^i = -(\mathbf{X}^i)^\dagger$ is a nanoplet of $N \times N$ anti-Hermitian matrix superfields. The leading component of this, $\mathbf{X}^i|_{\eta^q=0}$, provides a natural candidate for the field describing the relative motion of the M0 constituents. As was stressed in [39], it is important that this superfield has the $SO(1,1)$ wait 2, the fact which we find convenient to present in the form $\mathbf{X}^i = \mathbf{X}^i_{\#} := \mathbf{X}^i_{++}$ [and which can be seen from Eq. (4.4)].

Let us notice that the essential constraint in (4.4) is $G_{+q+p}\gamma^{ijkl}_{pq} = 0$, while the vanishing of the $SO(9)$ singlet

[4]Certainly we appreciate differences between our approach and the DLQG reasonings of [46,47] which discusses the M-theory compactification on a lightlike circle, considering this as limit of spacial circle of radius R_s and restricting to the sector of fixed momentum along the circle $p = N/R_s$ which is argued to produce a theory of N D0 branes. The most evident difference is that in DLCQ the number of D0-branes is defined by the integer number characterizing the fixed value of the momentum in the compact direction, $p = N/R_s$, while in our construction the number of D0's in mD0 system is defined by the number of mM0 constituents in the prototype 11-dimensional system and the fixed momentum in compact (spacelike) dimension corresponds to the mass of the ten-dimensional mD0 system.

part of this, $G_{+q+q} = 0$, is the conventional constraint which determines $A_\#$ in terms of A_{+q} and its derivatives. One can also think about a more complicated set of constraints $G_{+q+p} = i\gamma^i_{qp}\mathbf{X}^i + i\gamma^{ijkl}_{qp}\mathbf{Y}^{ijkl}$ where $\mathbf{Y}^{ijkl}$ is constructed from $\mathbf{X}^i$ superfields and their covariant fermionic (and bosonic) derivatives. For the case of flat target superspace, this corresponds to a deformation of the $D = 10$ SYM model reduced to $d = 1$; such deformations do exist and were the subject of studies in [48] and more recently in [49]. Although the existence of their counterparts corresponding to the curved target superspace seems to be a reasonable conjecture, to our best knowledge no special study of that has been carried out as of yet. Furthermore, even if this conjecture were proven, so that it were natural to expect the appearance of such type of deformations in a multiple brane models, the study of such models would promise to be very complicated (up to not being practical, at least without the use of a computer programs like the one applied in [48]). So in this paper we restrict ourself by considering the model with the simplest constraints (4.4); if the above mentioned deformation were found, our results based on constraint (4.4) would provide at least a reasonable (manageable) approximation to such a more complete but much more complicated description.

Studying Bianchi identities $DG_2 = 0$ one finds that the self-consistency of the constraints (4.4) requires the matrix superfield $\mathbf{X}^i$ to obey the *superembedding-like equation* [30]

$$D_{+q}\mathbf{X}^i = 4i\gamma^i_{qp}\Psi_q, \tag{4.5}$$

where the anti-Hermitian fermionic spinor superfield $\Psi_q := \Psi_{+++q}$ with $SO(1,1)$ weight 3 is related to the Hermitian fermionic field strength in (4.3) by $\Psi_q = iG_{+q++}$.

As far as the SYM model is defined on the superspace $\mathcal{W}^{(1|16)}$ obeying the superembedding Eq. (3.3), its geometry is characterized by Eqs. (3.35), (3.38), and (3.34). This implies that

$$\{D_{+q}, D_{+p}\}\mathbf{X}^i = 4iD_\#\mathbf{X}^i\gamma^i_{qp} - i[\mathbf{X}^i,\mathbf{X}^j]\gamma^j_{qp} + \frac{4i}{3}\mathbf{X}^j\hat{F}_{\#k_1k_2k_3} \times\left(\delta^{i[k_1}\gamma^{k_2}_{pq}\delta^{k_3]j} - \frac{1}{12}\gamma^{ijk_1k_2k_3}_{pq}\right) \tag{4.6}$$

Using this anticommutation relation together with the superembedding-like equation (4.5), we find

$$D_{+p}\Psi_q = \frac{1}{2}\gamma^i_{pq}D_\#\mathbf{X}^i + \frac{1}{16}\gamma^{ij}_{pq}[\mathbf{X}^i,\mathbf{X}^j] - \frac{1}{12}\mathbf{X}^i\hat{F}_{\#jkl}\left(\delta^{i[j}\gamma^{kl]} + \frac{1}{6}\gamma^{ijkl}\right)_{pq}. \tag{4.7}$$

This equation shows that the set of physical fields of the $d = 1$, $\mathcal{N} = 16$ SYM model defined by constraints (4.4) is exhausted by the leading component of the bosonic superfield $\mathbf{X}^i$, providing the non-Abelian, $N\times N$ matrix generalization of the Goldstone field describing a single M0-brane in static gauge, and by its superpartner, the leading component of the fermionic superfield Ψ_q in (4.5), providing the non-Abelian, $N\times N$ matrix generalization of the fermionic Goldstone fields describing a single M0-brane. These can be extracted from the fermionic coordinate functions of a single M0-brane by fixing the gauge with respect to local fermionic κ symmetry. Notice that in our approach no non-Abelian counterpart of the κ symmetry is needed as far as the relative motion of the mM0 constituents is described by matrix counterpart of the physical Goldstone fields of a single brane rather then of the coordinate functions.

This also explains a specific way of realizing the manifest $SO(1,10)$ Lorentz symmetry in our model. The physical fields of a single brane model are usually extracted by fixing a Lorentz noncovariant gauge (with respect to κ symmetry and reparametrization symmetry) and, as a result, carry the indices of a subgroup of the $SO(1,10)$ Lorentz group, including $SO(9)\times SO(1,1)$ in the M0 case. Then our matrix valued fields, being a counterpart of these physical fields, carry the $SO(9)$ indices and definite $SO(1,1)$ weights, while they are inert under the $SO(1,10)$ Lorentz group which acts nontrivially on the variables describing the center of energy motion only.

C. Equations of motion and polarization of multiple M0 by flux

The next stage is to study the self-consistency condition of Eq. (4.7). Using the fermionic covariant derivative algebra we can present that in the form

$$\{D_{+q}, D_{+p}\}\Psi_r = 4iD_\#\Psi_r\delta_{qp} - i[\Psi_r,\mathbf{X}^j]\gamma^j_{qp} + \frac{i}{3}\hat{F}_{\#ijk}\Psi_s \times\left(\gamma^{[i}_{qp}\gamma^{jk]}_{sr} + \frac{1}{12}\gamma^{ijkk_1k_2}_{pq}\gamma^{k_1k_2}_{sr}\right) = D_{+(q}\left(\gamma^i D_\#\mathbf{X}^i + \frac{1}{8}\gamma^{ij}[\mathbf{X}^i,\mathbf{X}^j] - \frac{1}{6}\mathbf{X}^i\hat{F}_{\#jkl}\left(\delta^{i[j}\gamma^{kl]} + \frac{1}{6}\gamma^{ijkl}\right)\right)_{p)r}. \tag{4.8}$$

Then, using Eq. (3.41) and

$$[D_{+p}, D_\#]\mathbf{X}^i = i[\mathbf{X}^i,\Psi_p] + \frac{i}{18}F_{\#jkl}(\gamma^{jkl}\gamma^i)_{pq}\Psi_q - i\hat{T}_{\#[i|+q}\gamma_{|j]qp}\mathbf{X}^j, \tag{4.9}$$

we find, after some algebra, that the pq-trace part of Eq. (4.8) results in the interacting dynamical equation for the 16-plet of fermionic matrix (super)fields

$$D_\#\Psi_q = -\tfrac{1}{4}\gamma^i_{qp}[\mathbf{X}^i,\Psi_p] + \tfrac{1}{24}\hat{F}_{\#ijk}\gamma^{ijk}_{qr}\Psi_r - \tfrac{1}{4}\mathbf{X}^i\hat{T}_{\#i+q}. \tag{4.10}$$

We have simplified the final form of the fermionic Eq. (4.10) using the consequence (3.47) of the supergravity Rarita-Schwinger equation. Using this equation one can also check that the other irreducible parts of the self-consistency condition (4.8) are satisfied identically (the fact which can be considered as a nontrivial consistency check for our basic equations).

As usual in supersymmetric theories, the higher components in decomposition of the superfield version of the fermionic equations over the Grassmann coordinates of superspace gives the bosonic equations of motion. In the case of our multiple M0 system, using the commutation relations

$$[D_{+p}, D_{\#}]\Psi_q = -i\{\Psi_q, \Psi_p\} + \frac{1}{72}F_{\# ijk}\gamma^{ijk}_{pr}D_{+r}\Psi_q - \frac{i}{4}\Psi_s\gamma^{ij}_{sq}\gamma^{j}_{pr}\hat{T}_{\# i+r}, \quad (4.11)$$

as well as Eqs. (4.9), (3.41), and (3.42), we find the Gauss constraint

$$[\mathbf{X}^i, D_{\#}\mathbf{X}^i] = 4i\{\Psi_q, \Psi_q\} \quad (4.12)$$

and the proper bosonic equation of motion

$$D_{\#}D_{\#}\mathbf{X}^i = \tfrac{1}{16}[\mathbf{X}^j, [\mathbf{X}^j, \mathbf{X}^i]] + i\gamma^i_{qp}\{\Psi_q, \Psi_p\} + \tfrac{1}{4}\mathbf{X}^j\hat{R}_{\# j\# i} + \tfrac{1}{8}\hat{F}_{\# ijk}[\mathbf{X}^j, \mathbf{X}^k] - 2i\Psi_q\hat{T}_{\# i+q}. \quad (4.13)$$

Notice that the Gauss constraint comes from the trace ($\propto \delta_{qp}$) part of the equation $D_{+p}D_{\#}\Psi_q = [D_{+p}, D_{\#}]\Psi_q + D_{\#}(D_{+p}\Psi_q)$ written with the use of Eqs. (4.7) and (4.10). The second order equations of motion (4.10) is obtained from the $\propto \gamma^i_{qp}$ irreducible part of that equation, while the other irreducible parts ($\propto \gamma^{ij}_{qp}$, $\propto \gamma^{ijk}_{qp}$ and $\propto \gamma^{ijkl}_{qp}$) are satisfied identically when the consequence (3.47) of the supergravity Rarita-Schwinger equation is taken into account. This provides one more consistency check of our approach.

The bosonic Eq. (4.13) has an interesting structure, particularly in its part describing coupling to the generic supergravity background. The fourth term in the r.h.s. of this equation, $\hat{F}_{\# ijk}[\mathbf{X}^j, \mathbf{X}^k]$, is typical for "dielectric coupling" characteristic for the Emparan-Myers "dielectric brane effect" [28,50]. It is essentially non-Abelian as far as in the Abelian case this contribution vanishes. This is the case also for the first and the second terms in the r.h.s. of (4.13), which are also present in the case of flat background without fluxes and in $1d$-dimensional reduction of ten-dimensional SYM (which is clearly not the case for the other three terms describing interactions with fluxes of 11-dimensional supergravity).

The third term in the r.h.s. (4.13) is linear in $\mathbf{X}^j$ and thus gives rise to a mass term for this $su(N)$ valued matrix bosonic (super)field. The corresponding mass matrix is induced by fluxes, namely, it is expressed through the pullback of the specific projection of Riemann tensor, $\hat{R}_{\# ij\#}$ of Eq. (3.43). Notice that the latter is symmetric in its $SO(9)$ vector indices (3.45) which is important because otherwise Eq. (4.13) would look non-Lagrangian. Notice also that, due to the consequence (3.46) of the supergravity Einstein equation, when the multiple M0 system interacts nontrivially with the 4-form fluxes, the field $\mathbf{X}^j$, describing the relative motion of the mM0 constituents, is always massive as far as the trace of its mass matrix is nonvanishing.

V. BPS EQUATIONS AND SUPERSYMMETRIC BOSONIC SOLUTIONS OF THE MULTIPLE M0 EQUATIONS

A. Supersymmetry preservation by single M0

The presence of M0-brane breaks 1/2 of the spacetime supersymmetry. This can be easily seen from Eqs. (3.8) and (3.29) describing the on-shell superembedding of the M0 worldline superspace or of the center of energy superspace of the mM0 system, $\mathcal{W}^{(1|16)}$, into the target 11-dimensional superspace $\Sigma^{(11|32)}$. Indeed, in the superspace formulation the local supersymmetry transformations of a supergravity model can be identified with supertranslations in the fermionic directions,

$$\varepsilon^{\alpha} = \delta Z^M E_M^{\alpha}(Z) =: i_{\delta}E^{\alpha}. \quad (5.1)$$

Then Eq. (3.29) implies that, if a 32-component spinor parameter ε^{α} describes a supersymmetry preserved by some M0-brane, its pullback $\hat{\varepsilon}^{\alpha}$ to $\mathcal{W}^{(1|16)}$ is expressed through 16 parameters

$$\epsilon^{+q} = \delta\zeta^{\mathcal{M}}e_{\mathcal{M}}^{+q}(\zeta) =: i_{\delta}e^{+q} \quad (5.2)$$

of the local worldline supersymmetry,

$$\hat{\varepsilon}^{\,\alpha} = \epsilon^{+q}v_q^{-\alpha}. \quad (5.3)$$

Thus, in a completely supersymmetric background a solution of M0-brane equations can preserve 16 or less of 32 target (super)space supersymmetries. If the supergravity background preserves a part of supersymmetries, the situation becomes more complicated as the number of preserved supersymmetries may become dependent on details of M0-brane motion (see the recent work in [51] for the specific case of strings and branes in type IIA superspace describing the 3/4 supersymmetric $\mathrm{AdS}_4 \times CP^3$ background).

The superembedding approach allows to make some general statements about supersymmetry preservation by M0-brane motion in a purely bosonic background. In superspace such backgrounds are characterized by

$$T_{ab}{}^{\alpha}(x) = 0. \quad (5.4)$$

Then the pullback of this fermionic field strength to the worldvolume and its projections also vanish. Taking into account that only the projection (3.39) enters the description of the M0 worldline superspace geometry and, through that, its dynamics, one sees that the part of worldline supersymmetry (part of the $1/2$ of the target space supersymmetry) preserved by a certain M0 motion is characterized by parameters which obey

$$\epsilon^{+p}(D_{+p}\hat{T}_{\#i+q})|_0 = 0 \tag{5.5}$$

with $|_0 := |_{\eta^p=0}$.

In our approach this equation also determines the $\mathcal{W}^{(1|16)}$ supersymmetry (a part of the $1/2$ of the target space supersymmetry) preserved by the center of energy motion of the mM0 system. But in this case this is not the end of story as the supersymmetry preserved by the center of energy motion can be either preserved or broken by the relative motion of the mM0 constituents.

B. Supersymmetry preservation by multiple mM0 system

The supersymmetry transformation $\delta_{\rm susy}\psi_q(\tau)$ of the $N\times N$ matrix fermionic field $\psi_q(\tau) := \Psi_q(\tau, 0) \equiv \Psi_q|_0$ can be identified as $\delta_{\rm susy}\psi_q(\tau) = \epsilon^{+p}D_{+p}\Psi_q|_0$. Then the preservation of supersymmetry for bosonic solutions of the equations describing relative motion of mM0 implies

$$\epsilon^{+p}(D_{+p}\Psi_q)|_0 = 0. \tag{5.6}$$

Furthermore, using Eqs. (3.42), (3.44), and (4.7) one can present the system of Eqs. (5.5) and (5.6) for the parameter of supersymmetry preserved by the mM0 system in the following form:

$$\begin{aligned}\epsilon^{+p}\mathcal{N}_{ipq} &= 0,\\ \mathcal{N}_{ipq} &:= D_{+p}\hat{T}_{\#i+q}\\ &= \frac{1}{2}\gamma^j_{pq}\Big(\hat{R}_{\#ij\#} - \frac{1}{6}\hat{F}_{\#ik_1k_2}\hat{F}_{\#jk_1k_2} - \frac{1}{54}\delta^{ij}(\hat{F}_{\#k_1k_2k_3})^2\Big)\\ &\quad + \frac{1}{3}D_\#\hat{F}_{\#ijk}\Big(\delta^{i[j}\gamma^{kl]}_{pq} + \frac{1}{6}\gamma^{ijkl}_{pq}\Big) + \frac{1}{9}\gamma^{j_1j_2j_3}_{pq}\hat{F}_{\#ij_1k}\hat{F}_{\#kj_2j_3}\\ &\quad + \frac{1}{72}\gamma^{k_1k_2k_3k_4k_5}_{pq}(\hat{F}_{\#ik_1k_2}\hat{F}_{\#k_3k_4k_5} + \delta^i_{[k_1}\hat{F}_{\#k_2k_3|j}\hat{F}_{\#j|k_4k_5]}),\end{aligned} \tag{5.7}$$

$$\begin{aligned}\epsilon^{+p}\mathbf{M}_{pq} &= 0,\\ \mathbf{M}_{pq} &:= D_{+p}\Psi_q\\ &= \Big(\gamma^i_{pq}D_\#\mathbf{X}^i + \frac{1}{8}\gamma^{ij}_{pq}[\mathbf{X}^i,\mathbf{X}^j] - \frac{1}{6}\mathbf{X}^i\hat{F}_{\#jkl}\\ &\quad\times\Big(\delta^{i[j}\gamma^{kl]} + \frac{1}{6}\gamma^{ijkl}\Big)_{pq}\Big).\end{aligned} \tag{5.8}$$

Here and below we denote the leading component of superfield by the same symbol as the whole superfield, i.e., if treating equations in terms of superfield, we assume $|_0$ ($:= |_{\eta^p=0}$) symbol, but do not write this explicitly.

C. 1/2 BPS equations for single M0-brane

It is natural to begin with the study of $1/2$ BPS equations for the more conventional case of a single M0-brane. This preserves $1/2$ of the target space supersymmetry if Eq. (5.7) is satisfied for the arbitrary $SO(9)$ spinor ϵ^{+p}. Hence, the $1/2$ BPS equations for the single M0-brane are enclosed in the equation

$$\mathcal{N}_{ipq} = 0, \tag{5.9}$$

where $\mathcal{N}_{ipq}$ is defined in (5.7). Decomposing this on the irreducible parts, one finds the following set of the $1/2$ BPS equations for single M0-brane:

$$\hat{R}_{\#ij\#} + \tfrac{1}{6}\hat{F}_{\#ikl}\hat{F}_{\#klj} + \tfrac{1}{36}\delta^{ij}(\hat{F}_{\#k_1k_2k_3})^2 = 0, \tag{5.10}$$

$$D_\#\hat{F}_{\#ijk} = 0, \tag{5.11}$$

$$\hat{F}_{\#ij[k_1}\hat{F}_{\#k_2k_3]j} = 0, \tag{5.12}$$

$$\hat{F}_{\#j[k_1k_2}\hat{F}_{\#k_3k_4]j} = 0, \tag{5.13}$$

$$\hat{F}_{\#i[k_1k_2}\hat{F}_{\#k_3k_4k_5]} = 0. \tag{5.14}$$

Notice that Eq. (5.10) cannot be obtained from pullback of the Einstein equation of supergravity, Eq. (2.10), but its trace coincides with the consequence (3.46) of this Einstein equation.

Equation (5.11) implies that the pullback of the 4-form flux is essentially constant (independent of the proper time coordinate; notice that one can fix the gauge $A_\# = 0$). As far as the algebraic Eqs. (5.12), (5.13), and (5.14) are concerned, they are solved by

$$\hat{F}_{\#ijk} = 3/4 w^i_I w^j_J w^k_K \epsilon^{IJK}, \qquad \begin{cases} i = 1,\ldots,9\\ I = 1,2,3\end{cases}, \tag{5.15}$$

where $\epsilon^{IJK} = \epsilon^{[IJK]}$ is the Levi-Civita symbol, $\epsilon^{123} = 1$, and the 9×3 matrices w^i_I obey $D_\# w^i_I = 0$.

Thus, a certain M0 motion can preserve $1/2$ of 32 target spacetime supersymmetries if the projection of the pullback of the target superspace flux to the M0 worldline obeys (5.15).

D. 1/2 BPS equations for multiple M0-brane system

In our approach, a certain configuration of multiple M0 system can preserve 1/2 of the complete supersymmetry if the center of energy motion obeys Eq. (5.9) and the relative motion of mM0 constituents preserves all the 16 supersymmetries preserved by the center of energy motion. Thus, we have to assume that (5.8) is obeyed for an arbitrary ϵ^{+p} so that the system of 1/2 BPS equations for the multiple M0 system includes Eq. (5.9),

$$\mathcal{N}_{ipq} = \frac{1}{2}\gamma^{j}_{pq}\Big(\hat{R}_{\#ij\#} - \frac{1}{6}\hat{F}_{\#ik_1k_2}\hat{F}_{\#jk_1k_2} - \frac{1}{54}\delta^{ij}(\hat{F}_{\#k_1k_2k_3})^2\Big) + \frac{1}{3}D_{\#}\hat{F}_{\#ijk}\Big(\delta^{i[j}\gamma^{kl]}_{pq} + \frac{1}{6}\gamma^{ijkl}_{pq}\Big) + \frac{1}{9}\gamma^{j_1j_2j_3}_{pq}\hat{F}_{\#ij_1k}\hat{F}_{\#kj_2j_3} + \frac{1}{72}\gamma^{k_1k_2k_3k_4k_5}_{pq}(\hat{F}_{\#ik_1k_2}\hat{F}_{\#k_3k_4k_5} + \delta^{i}_{[k_1}\hat{F}_{\#k_2k_3|j}\hat{F}_{\#j|k_4k_5]}) = 0, \quad (5.16)$$

and

$$\mathbf{M}_{pq} := \gamma^{i}_{pq}D_{\#}\mathbf{X}^i + \frac{1}{8}\gamma^{ij}_{pq}\Big([\mathbf{X}^i, \mathbf{X}^j] - \frac{4}{3}F_{\#ijk}\mathbf{X}^k\Big) + \frac{1}{36}\mathbf{X}^i F_{\#jkl}\gamma^{ijkl}_{pq} = 0. \quad (5.17)$$

Furthermore, as the 1/2 BPS equations for the center of energy motion (5.16) is equivalent to the set of Eqs. (5.10), (5.11), and (5.15), one can search for 1/2 BPS solutions of the multiple M0 system on the basis of Eq. (5.17) with the projection of the pullback of the 4-form flux defined by Eq. (5.15) with $D_{\#}w^i_I = 0$.

Actually, it is instructive to take a step back and not use (5.15) from the very beginning. Decomposing Eq. (5.17) on the irreducible parts, we find

$$D_{\#}\mathbf{X}^i = 0, \quad (5.18)$$

$$[\mathbf{X}^i, \mathbf{X}^j] = \tfrac{4}{3}\hat{F}_{\#ijk}\mathbf{X}^k, \quad (5.19)$$

$$\mathbf{X}^{[i}\hat{F}_{\#}{}^{jkl]} = 0. \quad (5.20)$$

One-dimensional gauge connection is always trivial, and so is the bosonic part $e^{\#}A_{\#}$ of our $d = 1$ $\mathcal{N} = 16$ superspace connection $A = e^{\#}A_{\#} + e^{+q}A_{+q}$. Hence Eq. (5.18) means that for the 1/2 BPS configuration of mM0 the relative motion of the mM0 constituents is described by essentially constant $N \times N$ matrices obeying Eqs. (5.19) and (5.20). Now one notices that, as far as the pullback of the flux is a number, while our $\mathbf{X}^i$ are $N \times N$ matrices, the solutions of Eq. (5.20) have a nontrivial matrix structure only if the flux has the form $\hat{F}_{\#ijk} = 3/4 w^i_I w^j_J w^k_K \epsilon^{IJK}$, as in Eq. (5.15) dictated by the supersymmetry preservation by the center of energy motion. Equations (5.19) and (5.20) with such a flux are solved by the following fuzzy 2-sphere configuration:

$$\mathbf{X}^{i} = w^i_I T^I, \qquad [T^I, T^J] = \epsilon^{IJK}T^K. \quad (5.21)$$

Here the triplet of $N \times N$ matrices T^I provides an $N \times N$ representation of the $SU(2)$ generators. As in (5.15), w^i_I are 9×3 matrices ($i = 1, \ldots, 9$, $I = 1, 2, 3$) playing the rôle of a bridge between representations of the $SO(9)$ and $SO(3) = SU(2)$ groups.

A simple particular case of the above 1/2 BPS solution is the one with $w^i_I = f\delta^I_i$, which occurs when the projection of the flux has only one nonvanishing basic component $\hat{F}_{\#123}$ and the relative positions of mM0 constituents are described by the set of only three nonvanishing bosonic $N \times N$ matrices $\mathbf{X}^i = (fT^1, fT^2, fT^3, 0, 0, 0, 0, 0, 0)$,

$$\mathbf{X}^{i} = f\delta^i_I T^I, \qquad [T^I, T^J] = \epsilon^{IJK}T^K. \quad (5.22)$$

$$\hat{F}_{\#IJK} = 3/4 f^3 \epsilon^{IJK}, \qquad \hat{F}_{\#ijk} = 0 \quad \text{for} \quad (i, j, k) \neq \text{permutation of } 123 \quad (5.23)$$

As we have already stated, the configuration (5.22) is called a fuzzy two-sphere [52]. Similar configuration was shown to solve the purely bosonic equations for 'dielectric D0-branes' following from the $p = 0$ Myers action for a particular type IIA background [28]. In that case three nonvanishing $N \times N$ matrices $\mathbf{X}^I = fT^I$ define the set of eight $u(N)$ valued matrices $\mathbf{X}^{\hat{i}} = (fT^I, 0, 0, 0, 0, 0)$ describing both the relative motion and the motion of the center of mass of the purely bosonic Myers D0-branes. The issue of supersymmetry was not addressed in [28] as far as the supersymmetric generalization of the Myers action was not known.

In contrast, the fuzzy sphere solution of our multiple M0 equations is supersymmetric by construction and can be considered as modeling the M2-brane. Moreover, our approach allows to see explicitly the origin of the $SU(2)$ structure constant in the 4-form flux, Eq. (5.23), and that this form is essentially the only nonvanishing flux allowed by the conditions of preservation of 1/2 of the target space supersymmetry.

E. A particular class of 1/4 BPS states and the Nahm equation

The set of 1/4 BPS states of the mM0 system is split on different sectors. Indeed, for this case eight of 16 supersymmetries which might be preserved by the embedding of superspace $\mathcal{W}^{(1|16)}$ into $\Sigma^{(11|32)}$ are broken and, in the generic case, some number r of them are broken by the center of mass motion and $(8 - r)$—by the relative motion of the M0 constituents ($r \leq 8$ in the context of our

1/4 BPS discussion). In other words, r of the 16 components of $SO(9)$ spinor ϵ^p can be set to zero by Eq. (5.7), corresponding to center of energy motion, and the remaining part—$(8-r)$—by Eq. (5.8), characterizing the relative motion of mM0 constituents. Here we will restrict ourselves by considering one specific sector, with the 1/2 supersymmetric center of energy "motion" and additional 1/4 of supersymmetry broken by the relative motion of mM0 constituents.

In the assumption that the center of mass "motion" obeys the 1/2 BPS condition (5.9), the BPS states preserving 1/4 of target space supersymmetry should obey

$$\mathcal{P}_{pr}\mathbf{M}_{rq} = \tfrac{1}{2}(1-\bar{\gamma})_{pr}\mathbf{M}_{rq} = 0, \qquad (5.24)$$

where the 16×16 matrices $\mathbf{M}_{pq}$ are defined in Eq. (5.8) and $\mathcal{P}_{pq} = \frac{1}{2}(1-\bar{\gamma})_{pq}$ is the rank 8 projector, $\mathcal{P}\mathcal{P} = \mathcal{P}$, constructed from the matrix $\bar{\gamma}_{pq}$ which obeys

$$\bar{\gamma}^2 = I, \qquad \mathrm{tr}(\bar{\gamma}) := \bar{\gamma}_{qq} = 0. \qquad (5.25)$$

We are going to show now that the famous Nahm equation [53] appears as a particular $SO(3)$ invariant case of Eq. (5.24).

Let us set $\hat{F}_{\#ijk} = 0$, consider bosonic solutions with only three nonvanishing $N\times N$ matrices of nine,

$$\mathbf{X}^i = (\mathbf{X}^I, \underbrace{0,\ldots,0}_{6}) = (\mathbf{X}^1, \mathbf{X}^2, \mathbf{X}^3, 0, 0, 0, 0, 0, 0), \qquad (5.26)$$

and identify $\bar{\gamma} = i\gamma^{123}$. Then $\bar{\gamma}\gamma^{IJ} = -i\epsilon^{IJK}\gamma^K$ and Eq. (5.25) reads

$$((1-\bar{\gamma})\gamma^I)_{pq}\Big(D_\#\mathbf{X}^I + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^J, \mathbf{X}^K]\Big) = 0. \qquad (5.27)$$

This implies the Nahm equation [53]

$$D_\#\mathbf{X}^I + \frac{i}{8}\epsilon^{IJK}[\mathbf{X}^J, \mathbf{X}^K] = 0. \qquad (5.28)$$

The literal coincidence with the original form of the Nahm equation appears when we fix the gauge $A_\# = 0$ and set to zero the normal bundle connection, so that $D_\#\mathbf{X}^I = \dot{\mathbf{X}}^I := \frac{\partial \mathbf{X}^I}{\partial\tau}$.

Thus, the famous Nahm equation, which has a fuzzy-sphere-related fuzzy-funnel solution, appears as a particular case of a $SO(3)$ symmetric 1/4 BPS equation for our multiple M0 system in the background with vanishing 4-form flux. Hence, surprisingly enough, the origin of the Levi-Civita symbol ϵ^{IJK} in the Nahm equation for the mM0 system is not the 11-dimensional supergravity flux, as one might expect, but rather the requirement of the $SO(3)$ symmetry of the particular 1/4 BPS configurations described by three nonvanishing components of $\mathbf{X}^i$.

VI. CONCLUSION AND DISCUSSION

In this paper we have obtained equations of motion for the system of multiple M0-branes in an arbitrary supergravity background. These equations are derived in the frame of superembedding approach, defining $1d$ SYM connection restricted by the set of constraints on the $1d$ $\mathcal{N}=16$ superspace $\mathcal{W}^{(1|16)}$ the embedding of which in the generic 11-dimensional supergravity superspace $\Sigma^{(11|32)}$ is determined by the superembedding equation. The same superembedding equation defines the embedding of the worldvolume superspace of a single M0-brane, which is the massless 11-dimensional superparticle, and encodes its equations of motion which imply that the M0 worldline is light-like. In the case of mM0 the superembedding equations for the superspace $\mathcal{W}^{(1|16)}$ results in the equations describing the center of energy motion of the mM0 system, which is also characterized by a lightlike geodesic.

The equations for the relative motion of mM0 constituents follow from the constraints imposed on the field strength of the SYM connection on $\mathcal{W}^{(1|16)}$. These, together with the center of energy equations of motion, provide a generalization of the matrix model [38] for an arbitrary supergravity background. Notice that the matrix model has been known before only for a very few particular backgrounds, including the maximally supersymmetric pp-wave background [54]. Hence, the natural application of the present approach is to use our general equations to obtain the matrix model in physically interesting backgrounds. In particular, the equations for the matrix model in $AdS_4\times S^7$ and $AdS_7\times S^4$ backgrounds can be straightforwardly obtained in this manner.

This will be the subject of our subsequent study which will also include the derivation of our mM0 equations in a supersymmetric pp-wave background and a comparison of the result with the BMN (Berenstein-Maldacena-Nastase) matrix model [54]. In this respect we should mention that there exist some conjectures [55] that the BMN model actually provides the description of the matrix theory in an arbitrary background. The only comment we would like to make in this respect in the present paper is that, even if such a conjecture were proved to be correct, it would be a nontrivial problem to extract the information on a certain system in definite non-pp-wave background from it[5] so that, in our view, it would be certainly useful anyway to have an explicit form of the matrix model in an arbitrary supergravity background.

[5]In particular, as argued in [56], even the AdS $\times$ S background is completely determined by the *two* orthogonal Penrose limits: "Having only one limit does not determine the whole spacetime. Thus, the two orthogonal Penrose limits form a sort of classical holographic boundary for the background with D-2 commuting Killing directions"[56].

We should also notice that all the terms with background contributions to the r.h.s.'s of our mM0 equations are linear in fluxes which is in disagreement with expectations based on the study of the Myers-type actions [28,37]. Although the Myers action is purely bosonic and resisted all the attempts of its straightforward supersymmetric and Lorentz covariant generalization for 11 years (except for the cases of lower-dimensional and lower codimensional Dp-branes [22–24]), taking into account particular progress in this direction reached recently in the frame of the boundary fermion approach [25] and also evidences from the string amplitude calculations,[6] we should not exclude the possibility that the above mentioned discrepancy implies that our approach gives only an approximate description of the matrix model interaction with supergravity fluxes [but if so, it is Lorentz covariant, supersymmetric and going beyond the $U(N)$ SYM approximation].

If this is the case, a way to search for a more general interaction lays through modification of the basic equations of our superembedding approach, namely, the superembedding equation, defining the embedding of the mM0 center of energy superspace $\mathcal{W}^{(1|16)}$ into the target $D = 11$ supergravity superspace $\Sigma^{(11|32)}$, and the basic constraints of the $d = 1$, $\mathcal{N} = 16$ SYM model on the center of energy superspace. (Notice that modification of the basic superembedding-type equations in the boundary fermion approach was suggested recently in [31]). The problem of the deformation of the basic constraint determining the equations for the relative motion of mM0 constituents is the curved superspace generalization of the studies in [31,48,49]. However, unfortunately, if we allow consistent deformations of the basic equations of our superembedding approach, although most probably these exist, they would certainly make the equations very complicated up to being unpractical.

Interestingly enough, if we do not deform the superembedding equation, but allow for a deformation of the $d = 1$, $\mathcal{N} = 16$ $SU(N)$ SYM constraints on the center of mass superspace, the situation seems to be much more under control due to the rigid structure of the mM0 equations [39]. In this case the center of mass motion and supersymmetry of the corresponding superspace is influenced only by the projections (3.37), (3.39), and (3.43) of the supergravity fluxes, so that it is reasonable to assume that only these fluxes can enter the equations of relative motion of the mM0 constituents. Then, the requirement of $SO(1, 1) \times SO(9)$ symmetry leaves very few possibilities to add the new terms to the ones already present in the r.h.s.'s of the Eqs. (4.10), (4.11), (4.12), and (4.13) [39]. In particular, the only possible nonlinear term which might be added to the r.h.s. of the bosonic Eqs. (4.13) is[7] $\mathbf{X}^j \hat{F}_{\#jkl}\hat{F}_{\#ikl}$ describing the contribution proportional to the second power of the 4-form flux to the mass matrix of the $su(N)$-valued fields $\mathbf{X}^j$.

In this paper we have also used our superembedding approach to obtain BPS equations for supersymmetric solutions of the mM0 equations. As an example we have shown that the $1/2$ BPS equations in the presence of 4-form flux have the fuzzy sphere solution modeling M2-brane by a $1/2$ supersymmetric configuration of multiple M0. We also found that the Nahm equation [53] appears as a particular $SO(3)$ invariant case of the $1/4$ BPS equations in the absence of the four form flux.

The further study of our mM0 BPS equations and search for new solutions of the mM0 equations of motion is an interesting problem for future study. A particularly intriguing problem is to search for a description (better to say, modeling) of the M5 brane and/or M2-M5 system in this framework. The popular candidate for the description of this latter is the Basu-Harvey equation [63]

$$D_{\#}\mathbf{X}^{\tilde{I}} = \epsilon^{5\tilde{I}JKL}\mathbf{X}^5\mathbf{X}^J\mathbf{X}^K\mathbf{X}^L, \qquad \tilde{I} = 1, \ldots, 4.$$

We have not succeed in deducing this from the BPS conditions for supersymmetric solution of our mM0 equations, so that this remains an interesting problem for future. Notice that this problem is seen due to the manifest supersymmetry of our approach: it has not been excluded yet that the bosonic limit of our mM0 equations do have some solutions modeling a five brane, but if so, this would not be a model of M5 brane as far as this should be $1/2$ supersymmetric and thus should be a solution of the $1/2$ BPS equations. Probably the results of [37] on the necessity on the nonlinear terms for "matching Abelian and non-Abelian calculations" suggest to try to use modification of our basic equations resulting in nonlinear flux contributions into the r.h.s. of the mM0 equations of motion in search for solving this M5-brane problem.

It is also possible that the description of higher branes requires to pass from the one-dimensional matrix model to its higher-dimensional counterparts, beginning from the matrix string [57] (see [58] for arguments in favor of this). In this respect it is interesting to check a possibility to extend our superembedding approach, as it is developed

[6]One should be careful with using the string amplitude calculations and T duality as final motivation of definite, not Lorentz invariant interacting terms in the Myers action, as far as we do not expect an explicit breaking of the Lorentz symmetry in string theory (and spontaneous breaking by super-p-branes allows for the existence of the Lorentz invariant actions producing the Lorentz covariant equations, so that only the ground state solution of these breaks partially the Lorentz symmetry). If our present understanding of string theory is correct, if the stringy amplitude calculations predict some term in an effective action, then this should allow for a Lorentz invariant and supersymmetric form. Notice that in [24], after giving a supersymmetric but Lorentz noninvariant generalization of the Myers-type action for multiple D0 in flat superspace, the authors notice that, by omitting some terms, this action can be made Lorentz covariant, still preserving some nonlinearities.

[7]One might also propose the term $\mathbf{X}^i \hat{F}_{\#jkl}\hat{F}_{\#jkl} \equiv \mathbf{X}^i(\hat{F}_{\#jkl})^2$, but this reduces to $\mathbf{X}^i \hat{R}_{\#j\#j}$ by Eq. (3.46).

for mM0 and mD0, for the case of higher p multiple p-brane systems beginning from type IIB multiple D-strings or multiple (p, q) strings. This will be the subject of our future study.

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APPENDIX A: 11-DIMENSIONAL AND NINE-DIMENSIONAL GAMMA MATRICES

A convenient $SO(1,1)\otimes SO(9)$ invariant representations for the 11-dimensional gamma matrices and charge conjugation matrix read

$$\begin{aligned}
&(\Gamma^a)_{\underline{\alpha}}{}^{\underline{\beta}} \equiv \left(\frac{1}{2}(\Gamma^{\#}+\Gamma^{=}), \Gamma^i, \frac{1}{2}(\Gamma^{\#}-\Gamma^{=})\right),\\
&\qquad a = 0, 1, \ldots, 9, 10, \qquad i = 1, \ldots, 9,\\
&(\Gamma^{\#})_{\alpha}{}^{\beta} = \begin{pmatrix} 0 & 2i\delta_{pq} \\ 0 & 0 \end{pmatrix},\\
&(\Gamma^{=})_{\alpha}{}^{\beta} = \begin{pmatrix} 0 & 0 \\ -2i\delta_{pq} & 0 \end{pmatrix},\\
&(\Gamma^{i})_{\alpha}{}^{\beta} = \begin{pmatrix} -i\gamma^i_{pq} & 0 \\ 0 & i\gamma^i_{pq} \end{pmatrix},
\end{aligned} \tag{A1}$$

$$C_{\alpha\beta} = -C_{\beta\alpha} = \begin{pmatrix} 0 & i\delta_{pq} \\ -i\delta_{pq} & 0 \end{pmatrix} = (C^{-1})^{\alpha\beta} =: C^{\alpha\beta}. \tag{A2}$$

These are imaginary as far as we use the mostly minus metric convention so that the flat spacetime metric reads $\eta_{ab} = \mathrm{diag}(1, -1, \ldots, -1)$.

In (A1) γ^i_{pq} are 16×16 $d = 9$ Dirac matrices. These are symmetric $\gamma^i_{pq} = \gamma^i_{qp}$, and possesses the following properties

$$\gamma^{(i}\gamma^{j)} = \delta^{ij} I_{16\times 16}, \qquad \gamma^i_{pq} = \gamma^i_{qp} := \gamma^i_{(pq)}, \qquad \gamma^i_{(pq}\gamma^i_{r)s} = \delta_{(pq}\delta_{r)s}. \tag{A3}$$

The $d = 9$ charge conjugation matrix is also symmetric, which allows to chose its representation by Kronecker delta symbol δ_{qp} and do not distinguish upper and lower Spin(9) ($SO(9)$ spinor) indices. Notice that the matrices γ^{ij}_{qp} and γ^{ijk}_{qp} are antisymmetric so that the complete basis for the set of 16×16 symmetric matrices is provided by δ_{pq}, γ^i_{pq}, γ^{ijkl}_{pq},

$$\delta_{r(q}\delta_{p)s} = \frac{1}{16}\delta_{pq}\delta_{rs} + \frac{1}{16}\gamma^i_{pq}\gamma^i_{rs} + \frac{1}{16\cdot 4!}\gamma^{ijkl}_{pq}\gamma^{ijkl}_{rs}. \tag{A4}$$

In our conventions $\gamma^{123\,456\,789}_{qp} = \delta_{qp}$ and, consequently,

$$\gamma^{i_1\ldots i_7}_{qp} = -\tfrac{1}{2}\epsilon^{i_1\ldots i_7 jk}\gamma^{jk}_{qp}, \tag{A5}$$

$$\gamma^{i_1\ldots i_5}_{qp} = \frac{1}{4!}\epsilon^{i_1\ldots i_5 j_1\ldots j_4}\gamma^{j_1\ldots j_4}_{qp}. \tag{A6}$$

This, together with (A1), implies that our 11-dimensional Dirac matrices obey

$$\Gamma^0\Gamma^1\ldots\Gamma^9\Gamma^{(10)} = \tfrac{1}{2}\Gamma^{\#}\Gamma^{=}\Gamma^1\ldots\Gamma^9 = -iI_{32\times 32}. \tag{A7}$$

APPENDIX B: SOME PROPERTIES OF MOVING FRAME AND SPINOR MOVING FRAME VARIABLES ASSOCIATED TO THE MASSLESS SUPERPARTICLE

Here we collect some useful equations describing properties of moving frame and spinor moving frame variables (3.7) and (3.11).

Moving frame variables appropriate to the description of massless D-dimensional (super)particle were also called light-cone harmonics in [59] and Lorentz harmonics in [60]. They are defined as columns of the $D\times D$ Lorentz group matrix of which obey the constraints

$$\begin{aligned}
&U_b^{(a)} = (u_b^{=}, u_b^{\#}, u_b^i) \in SO(1, D-1)\\
&\Leftrightarrow \begin{cases} U\eta U^T = \eta & \Leftrightarrow \delta_a^b = \frac{1}{2}u_a^{\#}u^{b=} + \frac{1}{2}u_a^{=}u^{b\#} - u_a^i u^{bi} \\ U^T\eta U = \eta & \Leftrightarrow \begin{cases} u_a^{=}u^{a=} = 0, \qquad u_a^{\#}u^{a\#} = 0,\\ u_a^{=}u^{a\#} = 2, \qquad u_a^{=}u^{ai} = 0, u_a^{\#}u^{ai} = 0,\\ u_a^i u^{aj} = -\delta^{ij} \end{cases} \end{cases}\\
&b = 0, 1, \ldots, (D-2), (D-1), \qquad (a) = (\#, =, 1, \ldots, (D-2)).
\end{aligned} \tag{B1}$$

The spinor moving frame variables or *spinorial harmonics* (see [43–45,60–62]) are constrained spinors forming two rectangular blocks of the Spin $(1, D-1)$ valued matrix corresponding to (the "square root" of) the D-dimensional moving frame matrix (B1). Their definition is D and p dependent, i.e., different not only for different D's but also for different p-branes. The harmonics appropriate to the description of massless $D=11$ (super)particle are collected in spin $(1,10)$ valued matrix (3.11) obeying Eqs. (3.12), (3.13), and (3.14). Its inverse matrix [45]

$$V_\alpha^{(\beta)} = (v_{\alpha q}{}^-, v_{\alpha q}{}^+) \in \text{Spin}\,(1,10) \tag{B2}$$

obeys

$$V_{(\beta)}{}^\gamma V_\gamma^{(\alpha)} = \delta_{(\beta)}^{(\alpha)} \Leftrightarrow \begin{cases} v_q^{+\alpha} v_{\alpha p}{}^- = \delta_{qp}, & v_q^{+\alpha} v_{\alpha p}{}^+ = 0 \\ v_q^{-\alpha} v_{\alpha p}{}^- = 0, & v_q^{-\alpha} v_{\alpha p}{}^+ = \delta_{qp} \end{cases} \tag{B3}$$

and

$$V\Gamma^{(a)}V^T = \Gamma^b u_b^{(a)}, \qquad V^T\tilde{\Gamma}^{(a)}V = u_b^{(a)}\tilde{\Gamma}^b, \tag{B4a}$$

$$V^T C V = C, \qquad V C^{-1} V^T = C^{-1}. \tag{B4b}$$

The square root-type relation between spinor moving frame an moving frame variables encoded in the constraints (B4a) can be split further into

$$2v_{\alpha q}{}^- v_{\beta q}{}^- = \Gamma^a_{\alpha\beta} u_a^=, \tag{B5a}$$

$$2v_{\alpha q}{}^+ v_{\beta q}{}^+ = \Gamma^a_{\alpha\beta} u_a^\#, \tag{B5b}$$

$$2v_{(\alpha|q}{}^+ \gamma^i_{qp} v_{|\beta)q}{}^+ = \Gamma^a_{\alpha\beta} u_a{}^i, \tag{B5c}$$

$$v_q^- \tilde{\Gamma}_a v_p^- = u_a^= \delta_{qp}, \tag{B5d}$$

$$v_q^+ \tilde{\Gamma}_a v_p^+ = u_a^\# \delta_{qp}, \tag{B5e}$$

$$v_q^- \tilde{\Gamma}_a v_p^+ = u_a{}^i \gamma^i_{qp}. \tag{B5f}$$

The equations in (B4b), expressing the Lorentz invariance of the charge conjugation matrix C, allow to construct (explicitly) the elements of the inverse spinor moving frame matrix, as in (3.15),

$$v_{\alpha q}{}^{\mp} = \pm i C_{\alpha\beta} v_q^{\mp\beta}, \qquad v_q^{\pm\alpha} = \pm i C^{\alpha\beta} v_{\beta q}^{\pm}. \tag{B6}$$

In the massless superparticle model the set of 16 spinors $v_{\alpha p}^-$ in (B2) can be identified with the homogeneous coordinates of the celestial S^9 sphere given by the $SO(1,10)$ Lorentz group coset [43–45]

$$\{v_{\alpha p}{}^-\} = \frac{\text{Spin}\,(1,10)}{[\text{Spin}\,(1,1)\otimes\text{Spin}\,(9)](\times\mathbb{K}_9} = \mathbb{S}^9. \tag{B7}$$

In the dynamical system of the massless (super)particle these describe the angles defining the direction of the lightlike momentum so that one can consider $v_{\alpha p}{}^-$'s as carriers of all the momentum degrees of freedom but energy. The set of others 16 spinors, $v_{\alpha p}{}^+$, can be gauged away (but, of course, not set to zero) by the $K9$ transformations (3.40), so that, in principle, one can work with the set of constrained spinors $v_{\alpha p}{}^-$ only. However, it is often convenient to use the complete spinor moving frame and keep only the $SO(9)\otimes SO(1,1)$ symmetry as an equivalence relation on the set of $v_{\alpha q}{}^-$'s and $v_{\alpha q}{}^+$'s which satisfy the set of constraints in (B2). Then these constrained spinorial variables become homogeneous coordinates of the noncompact $SO(1,10)/[SO(9)\times SO(1,1)]$ coset, while $K9$ can be considered as a nonmanifest ("hidden") symmetry.

The $SO(1,10)$ covariant derivatives $(d+w)$ of the harmonic variables which do not break the kinematical constraints (B1) and (B2) (admissible derivatives) are expressed by

$$(d+w)u_a^= := du_a^= + w_a{}^b u_b^= = -2u_a^=\Omega^{(0)} + u_a^i\Omega^{=i}, \tag{B8}$$

$$(d+w)u_a^\# = +2u_a^\#\Omega^{(0)} + u_a^i\Omega^{\#i}, \tag{B9}$$

$$(d+w)u_a{}^i = \tfrac{1}{2}u_a^=\Omega^{\#i} + \tfrac{1}{2}u_a^\#\Omega^{=i} - u_a^j\Omega^{ji}, \tag{B10}$$

through the covariant 1-forms

$$\Omega^{(a)(b)} := U^{c(a)}(d+w)U_c^{(b)} = -\Omega^{(b)(a)} = \begin{pmatrix} 0 & \frac{1}{2}(\Omega^{\#i}+\Omega^{=i}) & 2\Omega^{(0)} \\ -\frac{1}{2}(\Omega^{\#i}+\Omega^{=i}) & \Omega^{ij} & -\frac{1}{2}(\Omega^{\#i}-\Omega^{=i}) \\ -2\Omega^{(0)} & \frac{1}{2}(\Omega^{\#i}-\Omega^{=i}) & 0 \end{pmatrix}, \tag{B11}$$

which generalize the $SO(1,10)/[SO(1,1)\otimes SO(9)]$ Cartan forms for the case of local $SO(1,10)$ symmetry (see [45]).

As reflected by the constraints in (B2), the spinor Lorentz harmonics V (B2) give the spinor representation of the Lorentz group element the vector representation of which is given by the moving frame vectors (B1). Then their admissible covariant derivatives [i.e., derivatives preserving the constraints (B2)] are expressed through the same generalized Cartan forms

$$V_{(\alpha)}^{\gamma}(d+w)V_{\gamma}^{(\beta)} = \tfrac{1}{4}\Omega^{(a)(b)}\Gamma_{(a)(b)(\alpha)}{}^{(\beta)} \in \mathrm{spin}\,(1,10),$$
$$\Omega^{(a)(b)} := U^{m(a)}(d+w)U_m^{(b)} \in so(1,10). \tag{B12}$$

This implies

$$(d+w)v_q^- = -\Omega^{(0)}v_q^- - \tfrac{1}{4}\Omega^{ij}v_p^-\gamma_{pq}^{ij} + \tfrac{1}{2}\Omega^{=i}\gamma_{qp}^i v_p^+, \tag{B13}$$

$$(d+w)v_q^+ = \Omega^{(0)}v_q^+ - \tfrac{1}{4}\Omega^{ij}v_p^+\gamma_{pq}^{ij} + \tfrac{1}{2}\Omega^{\#i}\gamma_{qp}^i v_p^- \tag{B14}$$

for the elements of the Spin (1, 10) valued matrix (B2).

Since the Cartan forms $\Omega^{(0)}$ and Ω^{ij} transform as connections under local $SO(1,1)$ and $SO(9)$ transformations, respectively, we can use them to define $SO(1,10)\otimes SO(1,1)\otimes SO(9)$ covariant exterior derivatives (covariant differentials) of the moving frame variables. Using such a covariant differential we can write Eqs. (B8), (B9), (B13), and (B14) in the form of

$$Du_m^{=} := (d+w)u_m^{=} + 2u_m^{=}\Omega^{(0)} = u_m^i\Omega^{=i},$$
$$Du_m^{\#} := (d+w)u_m^{\#} - 2u_m^{\#}\Omega^{(0)} = u_m^i\Omega^{\#i}, \tag{B15}$$

$$Du_m^i := (d+w)u_m^i + u_m^j\Omega^{ji} = \tfrac{1}{2}u_m^{\#}\Omega^{=i} + \tfrac{1}{2}u_m^{=}\Omega^{\#i}, \tag{B16}$$

$$Dv_q^- := (d+w)v_q^- + \Omega^{(0)}v_q^- + \tfrac{1}{4}\Omega^{ij}v_p^-\gamma_{pq}^{ij} = \tfrac{1}{2}\Omega^{=i}\gamma_{qp}^i v_p^+, \tag{B17}$$

$$Dv_q^+ := (d+w)v_q^+ - \Omega^{(0)}v_q^+ + \tfrac{1}{4}\Omega^{ij}v_p^+\gamma_{pq}^{ij} = \tfrac{1}{2}\Omega^{\#i}\gamma_{qp}^i v_p^-. \tag{B18}$$

To simplify notation in (B17) and (B18) we omit the spinorial indices; than it is not excessive to notice that in these equations we have presented the covariant derivatives of the element of inverse spinor moving frame matrix (B2) carrying lower Spin (1, 10) index, while the covariant derivatives of the initial spinor moving frame variables (3.11), with upper spin (1, 10) index, read

$$Dv_q^{-\alpha} := dv_q^{-\alpha} + \Omega^{(0)}v_q^{-\alpha} + \tfrac{1}{4}\Omega^{ij}v_p^{-\alpha}\gamma_{pq}^{ij} = -\tfrac{1}{2}\Omega^{=i}v_p^{+\alpha}\gamma_{pq}^i, \tag{B19}$$

$$Dv_q^{+\alpha} := dv_q^{+\alpha} - \Omega^{(0)}v_q^{+\alpha} + \tfrac{1}{4}\Omega^{ij}v_p^{+\alpha}\gamma_{pq}^{ij} = -\tfrac{1}{2}\Omega^{\#i}v_p^{-\alpha}\gamma_{pq}^i. \tag{B20}$$

Also the following algebraic equations were useful in our calculations

$$(v_q^-\Gamma_a)_\alpha = u_a^{=}v_\alpha^{+q} - u_a^i\gamma_{qp}^i v_\alpha^{-p},$$
$$(v_q^+\Gamma_a)_\alpha = u_a^{\#}v_\alpha^{-q} - u_a^i\gamma_{qp}^i v_\alpha^{+p}, \tag{B21}$$

$$(\tilde{\Gamma}_a v^{-q})^\alpha = u_a^{=}v_q^{+\alpha} + u_a^i\gamma_{qp}^i v_p^{-\alpha},$$
$$(\tilde{\Gamma}_a v^{+q})^\alpha = u_a^{\#}v_q^{-\alpha} + u_a^i\gamma_{qp}^i v_p^{+\alpha}, \tag{B22}$$

$$(v_q^-\Gamma_{ab}v_p^-) = 2u_{[a}^{=}u_{b]}^i\gamma_{qp}^i. \tag{B23}$$

APPENDIX C: SOME OTHER TECHNICAL DETAILS

In calculating the $SO(9)$ curvature (3.38) from (2.6) it is useful to notice that $\widehat{*F}_{\#ijk_1\ldots k_4} = \frac{1}{6}\epsilon^{ijk_1\ldots k_4 l_1 l_2 l_3}\hat{F}_{\#l_1l_2l_3}$. Here and below the $SO(1,1)$ and $SO(9)$ indices are obtained by contraction with the moving frame variables $u_a^{=}$, u_a^i, $v_q^{-\alpha}$, Eqs. (3.7) and (3.11).

Useful relations for the projections of the pullback of the tensor-spin tensor (2.4) to $\mathcal{W}^{(1|16)}$ are

$$(v_q^-\hat{t}^{=})^\alpha = \frac{1}{36}\hat{F}_{\#ijk}\gamma_{qp}^{ijk}v_p^{+\alpha},$$
$$(\hat{t}^{=}v_q^-)_\alpha = -\frac{1}{12}\hat{F}_{\#ijk}v_{\alpha p}{}^{+}\gamma_{pq}^{ijk}, \tag{C1}$$

$$(v_q^-\hat{t}^i v_p^-) = -\frac{1}{6}\hat{F}_{\#jkl}\left(\delta^{i[j}\gamma_{qp}^{kl]} + \frac{1}{6}\gamma_{qp}^{ijkl}\right). \tag{C2}$$

The explicit form of the tensor-spin tensor in the last term of Eq. (3.42) is

$$\Sigma_{pq}^{i,j_1j_2j_3,k_1k_2k_3} := \frac{1}{18\cdot 4!}\bigg(\gamma^{j_1j_2j_3}\left(\delta^{i[k_1}\gamma^{k_2k_3]} + \frac{1}{6}\gamma^{ik_1k_2k_3}\right) + 3\left(\delta^{i[k_1}\gamma^{k_2k_3]} + \frac{1}{6}\gamma^{ik_1k_2k_3}\right)\gamma^{j_1j_2j_3} + (j_{1,2,3}\leftrightarrow k_{1,2,3})\bigg). \tag{C3}$$

It obeys

$$\Sigma_{pp}^{i,j_1j_2j_3,k_1k_2k_3} = 0,$$
$$\Sigma_{qp}^{i,j_1j_2j_3,k_1k_2k_3}\gamma_{pq}^{jk} = 0,$$
$$\Sigma_{qp}^{i,j_1j_2j_3,k_1k_2k_3}\gamma_{pq}^{i} = -\frac{8}{3}\delta_{k_1}^{[j_1}\delta_{k_2}^{j_2}\delta_{k_3}^{j_3]},$$
$$\Sigma_{qp}^{i,j_1j_2j_3,k_1k_2k_3}\gamma_{pq}^{j} = -\frac{4}{27}(9\delta^{i[j_1}\delta_{[k_2}^{j_2}\delta_{k_3}^{j_3]}\delta_{k_1]}^{j} + \delta^{ij}\delta_{k_1}^{[j_1}\delta_{k_2}^{j_2}\delta_{k_3}^{j_3]}). \tag{C4}$$

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