Slavnov-Taylor identities for the 2 + 1 dimensional noncommutative CP^{N-1} model

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In the context of the $1/N$ expansion, the validity of the Slavnov-Taylor identity relating three- and two-point functions for the 2 + 1-dimensional noncommutative CP^{N-1} model is investigated, up to subleading $1/N$ order, in the Landau gauge.

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I. INTRODUCTION

Historically, the Slavnov-Taylor (ST) identities [1] have played an essential role in proving the renormalizability of non-Abelian gauge theories [2]. It is therefore important to know the limitations or even the validity of these identities whenever new structures as algebra deformations and space noncommutativity are introduced. Nowadays, this issue has aroused a great deal of attention particularly due to results that seem to indicate that at the Planck's scale the space may become noncommutative [3]. In this situation the coordinates should satisfy

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \tag{1}$$

where for the most studied case, called canonical noncommutativity, $\theta^{\mu\nu}$ is a constant, antisymmetric matrix (see also [4]). In general terms, the unleashing of noncommutativity signals not only for the breaking of Lorentz invariance but also leads to the appearance of an ultraviolet metamorphosis (some ultraviolet divergences are transmutated into infrared singularities), the so-called IR/UV mixing, which may destroy the perturbative scheme [5]. This mixing may also produce inconsistencies whenever the renormalization procedure requires a detailed balancing between Feynman amplitudes [6,7]. Besides these basic aspects the possible modifications of results linked to standard symmetries must also be investigated. It has been proved, for example, that CPT symmetry is preserved by the noncommutativity, in spite of its strong nonlocality [8]. Gauge symmetry seems also to be important to secure the presence of Goldstone bosons for spontaneously broken symmetries [6]. Concerning the ST identities, exploratory studies have been dedicated to the effects of the noncommutativity on the renormalization of the QED_4 [9] and also specific scattering processes in the tree approximation [10]. These studies were complemented by a systematic analysis at the one-loop level for QED_4 in Ref. [11]. Such studies are relevant particularly taking into account the incoming LHC experiments to test possible extensions of the standard model. Going further with these investigations, in this work we shall analyze the possible modifications on the ST identities due to the noncommutativity of the underlying space in the context of the three-dimensional CP^{N-1} model. When compared with QED_4, the new feature in this model is the absence of a kinetic term for the gauge field, which however is generated by quantum corrections. This study is also a natural sequel of an earlier work on the noncommutative CP^{N-1} model in which, up to the leading order of $1/N$, the absence of dangerous UV/IR mixing was proved [12].

The noncommutative CP^{N-1} model is defined by the action

$$\begin{aligned} S = \int d^3x \Big\{ & \partial^\mu \phi_a^\dagger \partial_\mu \phi_a - m^2 \phi_a^\dagger \star \phi_a \\ & + \lambda \star \Big(\phi_a \star \phi_a^\dagger - \frac{N}{g} \Big) + e^2 \phi_a^\dagger \star A^\mu \star A_\mu \star \phi_a \\ & + ie(\partial^\mu \phi_a^\dagger \star A_\mu \star \phi_a - \phi_a^\dagger \star A_\mu \star \partial_\mu \phi_a) \\ & - \frac{N}{2\alpha} (\partial^\mu A_\mu) \star (\partial^\nu A_\nu) \\ & + N \partial^\mu \bar{c} \star [\partial_\mu c - ie(c \star A_\mu - A_\mu \star c)] \Big\}, \end{aligned} \tag{2}$$

where ϕ_a $(a = 1, \ldots, N)$ is a N-tuple of charged scalar fields transforming in accord with the left fundamental representation of the $U_\star(1)$ group,

$$\begin{aligned} &\phi_a(x) \to U_\star(x) \star \phi_a(x), \\ &U_\star(x) = e_\star^{i\Lambda(x)} \equiv 1 + i\Lambda(x) - \tfrac{1}{2}\Lambda(x) \star \Lambda(x) + \cdots, \end{aligned} \tag{3}$$

the star symbol denoting the Moyal product (for a review about noncommutativity see [13])

$$f(x) \star g(x) = e^{(i/2)\Theta^{\mu\nu}\partial_{x_\mu}\partial_{y_\nu}} f(x) g(y)|_{x=y}. \tag{4}$$

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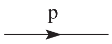
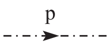
Besides the gauge field, the auxiliary field λ, which implements the constraint $\phi_a \star \phi_a^\dagger = \frac{N}{g}$, is taken in the adjoint representation of the gauge group, i.e.,

$$\lambda(x) \to \lambda'(x) = U_\star(x) \star \lambda(x) \star U_\star^{-1}(x). \tag{5}$$

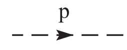
The great advantage of this choice is that λ and A_μ are then, in the leading $1/N$ order, independent fields. In the present situation, the propagators will be given by



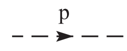
$$= \Delta^0(p) = \frac{i}{p^2 - m^2}, \tag{6a}$$



$$= S^0(p) = \frac{i}{Np^2}, \tag{6b}$$



$$\approx \Delta_\lambda^0(p) = \frac{8i\sqrt{-p^2}}{N}\left(1 + \frac{4m}{\pi}\frac{1}{\sqrt{-p^2}}\right). \tag{6d}$$



$$\approx \Delta_\lambda^0(p) = \frac{8i\sqrt{-p^2}}{N}\left(1 + \frac{4m}{\pi}\frac{1}{\sqrt{-p^2}}\right). \tag{6d}$$

Note that the propagators (6a) and (6b) are obtained directly from the action (2) considering the quadratic part of the fields ϕ and c, whereas the propagators for the gauge (6c) and auxiliary (6d) fields are obtained perturbatively, by considering large spacelike p behavior.

The vertices for the theory are the following:

$$ie(\partial^\mu \phi_a^\dagger \star A_\mu \star \phi_a - \phi_a^\dagger \star A_\mu \star \partial_\mu \phi_a) \leftrightarrow -ie(2k + p)_\mu e^{-ik\wedge p}, \tag{7a}$$

$$e^2 \phi_a^\dagger \star A^\mu \star A_\mu \star \phi_a \leftrightarrow 2ie^2 g^{\mu\nu} e^{-ik_1\wedge k_2} \cos(p_1 \wedge p_2), \tag{7b}$$

$$\lambda \star \phi_a \star \phi_a^\dagger \leftrightarrow ie^{-ik\wedge p}, \tag{7c}$$

$$-ieN\partial^\mu \bar{c} \star (c \star A_\mu - A_\mu \star c) \leftrightarrow 2eNk^\alpha \sin(p \wedge k), \tag{7d}$$

such that the graphical representation are given, respectively, in Fig. 1.

As explained in Ref. [12], the renormalization of the noncommutative CP^{N-1} model is greatly simplified with the help of the graphical identities of Fig. 2, first found for the commutative setting in Ref. [14]. In particular the identity in Fig. 2(b) implies that graphs containing the vertex (7b) cancel pairwise except by the one-loop graph contribution to the vector field propagator which has already been included in (6c).

Notice that, as indicated in the last line of (2), we are adopting a generic Lorentz gauge fixing whereas the calculations performed in [12] were restricted to the Landau gauge. Our gauge fixing together with the term for the ghost fields c and $\bar{c}$ signalize a formal symmetry associated with the invariance of the action under Becchi-Rouet-Stora-Tyutin (BRST) transformations which have the following form:

$$\phi_a \to \phi_a' = \phi_a + ic \star \phi_a \epsilon, \tag{8a}$$

$$\phi_a^\dagger \to \phi_a^{\dagger\prime} = \phi_a^\dagger - i\phi_a^\dagger \star c\epsilon, \tag{8b}$$

$$A_\mu \to A_\mu' = A_\mu - \frac{1}{e}\partial_\mu c\epsilon + i[c, A_\mu]_\star \epsilon, \tag{8c}$$

$$\lambda \to \lambda' = \lambda + i[c, \lambda]_\star \epsilon, \tag{8d}$$

$$c \to c' = c - ic \star c\epsilon, \tag{8e}$$

$$\bar{c} \to \bar{c}' = \bar{c} - \frac{1}{e\alpha}\partial^\mu A_\mu \epsilon, \tag{8f}$$

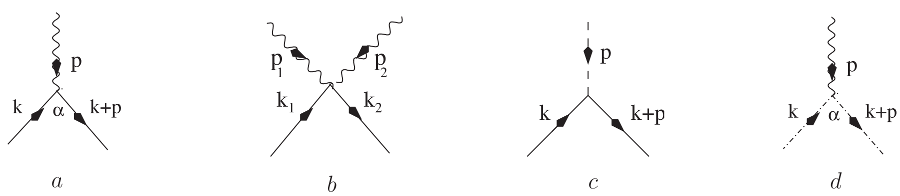


FIG. 1. Vertices associated to the action (3).

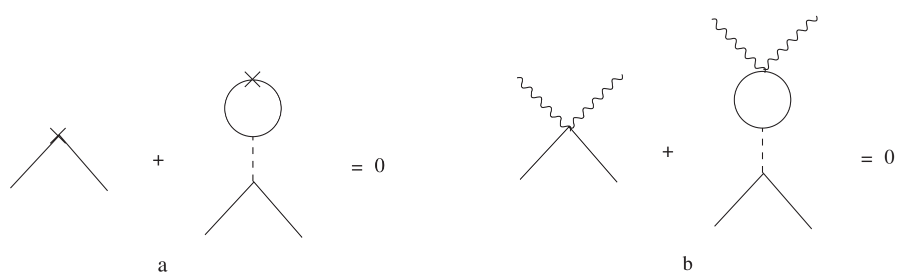


FIG. 2. Graphical identities for the CP^{N-1} model.

where ϵ is an infinitesimal Grassmannian parameter. Because of the presence of the Moyal product, the implications of this invariance have to be examined anew. In particular, we shall inspect the ST identities characteristics of this invariance but, as the leading contributions in $1/N$ involve both the one-loop and two-loop diagrams whose analytic expressions are very intricate, we will focus directly on the asymptotic behavior for high momenta of the relevant Green functions.

To derive the ST identities, as usual, we add to the source terms for the basic fields the source terms associated with the BRST transformations

$$\begin{aligned} S_{\text{source}} = \int d^3x \Big\{ & J_\mu \star A^\mu + \eta_a^\dagger \star \phi_a + \phi_a^\dagger \star \eta_a + \bar{\xi} \star c \\ & + \bar{c} \star \xi + \zeta \star \lambda + u \star (i[c, \lambda]_\star) \\ & + K_\mu \star \left(-\frac{1}{e} \partial^\mu c + i[c, A^\mu]_\star \right) + v \star (-ic \star c) \\ & + \omega_a^\dagger \star (ic \star \phi_a) + (-i\phi_a^\dagger \star c) \star \omega_a \Big\}. \end{aligned} \tag{9}$$

The invariance of the functional generator under the field transformations (8) formally allows for the ST identity

$$\begin{aligned} \int d^3x \Big\{ & J_\mu \star \frac{\delta W}{\delta K_\mu} + \eta_a^\dagger \star \frac{\delta W}{\delta \omega_a^\dagger} - \frac{\delta W}{\delta \omega_a} \star \eta_a \\ & + \bar{\xi} \star \frac{\delta W}{\delta v} + \frac{1}{e\alpha} \partial_\mu \frac{\delta W}{\delta J_\mu} \star \xi + \zeta \star \frac{\delta W}{\delta u} \Big\} = 0, \end{aligned} \tag{10}$$

where W is the functional generator for the connected Green functions. The above result together with the relation

$$\int d^3x \left(i\xi + Ne\partial_\mu \frac{\delta W}{\delta K_\mu} \right) = 0, \tag{11}$$

obtained from the invariance of $S + S_{\text{source}}$ under a general transformation $\delta \bar{c}$ of the ghost field, constitutes a powerful tool for the study of the UV behavior of field theories.

We begin the analysis of the above identities by proving that the longitudinal part of the gauge field propagator is not modified by radiative corrections, as it happens in [11]. In fact, by functionally deriving (10) with respect to the $J^\nu(y)$ and $\xi(z)$ sources, we get

$$\frac{\delta^2 W}{\delta \xi(z) \delta K^\nu(y)} \Bigg| + \frac{1}{e\alpha} \partial_z^\mu \frac{\delta^2 W}{\delta J^\nu(y) \delta J^\mu(z)} \Bigg| = 0, \tag{12}$$

where henceforth a vertical bar is used just to remember that the function immediately to its left must be calculated with all sources equal to zero. Now, from (11) it follows that

$$Ne\partial_x^\mu \frac{\delta^2 W}{\delta \xi(z) \delta K^\mu(x)} \Bigg| = -i\delta^3(x - z), \tag{13}$$

implying that

$$-\frac{i}{N} \delta^3(y - z) + \frac{1}{\alpha} \partial_y^\nu \partial_z^\mu \frac{\delta^2 W}{\delta J^\nu(y) \delta J^\mu(z)} \Bigg| = 0. \tag{14}$$

In momentum space, this equation becomes

$$k^\mu k^\nu D_{\mu\nu}(k) = -\frac{i\alpha}{N}, \tag{15}$$

so that the longitudinal part of the gauge propagator, which is proportional to $k_\mu k_\nu$, must be given by

$$D_{\mu\nu}^L(k) = -\frac{i\alpha}{N} \frac{k^\mu k^\nu}{(k^2)^2}. \tag{16}$$

Therefore, at any finite order of $1/N$, it is not affected by the noncommutativity. This result will be used in the forthcoming analysis of the ST identity.

We now consider the three-point function which involves the gauge and the charged fields. $\langle 0|TA^\mu \phi \phi^\dagger|0\rangle$ by deriving (10) with respect to the sources $\eta_a(x)$, $\eta_b^\dagger(y)$, and $\xi(z)$, we get

$$\begin{aligned} & \frac{\delta^3 W}{\delta \xi(z) \delta \eta_a(x) \delta \omega_b^\dagger(y)} \Bigg| - \frac{\delta^3 W}{\delta \xi(z) \delta \eta_b^\dagger(y) \delta \omega_a(x)} \Bigg| \\ & + \frac{1}{e\alpha} \partial_z^\mu \frac{\delta^3 W}{\delta \eta_b^\dagger(y) \delta \eta_a(x) \delta J^\mu(z)} \Bigg| = 0 \end{aligned} \tag{17}$$

or, equivalently,

$$\begin{aligned} \frac{1}{e\alpha} \partial_z^\mu \langle \mathrm{T}\phi_b(y) \phi_a^\dagger(x) A_\mu(z) \rangle &= i \langle \mathrm{T}\bar{c}(z) \phi_a^\dagger(x) c(y) \star \phi_b(y) \rangle \\ & - i \langle \mathrm{T}\bar{c}(z) \phi_b(y) \phi_a^\dagger(x) \star c(x) \rangle. \end{aligned} \tag{18}$$

It is convenient to write the above identity in terms of the one-particle irreducible vertex functions whose generating functional, Γ, is defined by

$$\begin{aligned} & W[J, \eta, \bar{\eta}, \xi, \bar{\xi}; K, v, \omega, \bar{\omega}] \\ & = \Gamma[A_{cl}, \phi_{cl}, \phi_{cl}^\dagger, C_{cl}, \bar{C}_{cl}; K, v, \omega, \bar{\omega}] \\ & + \int d^4x (J_\mu \star A_{cl}^\mu + \eta^\dagger \star \phi_{cl} + \phi_{cl}^\dagger \star \eta \\ & + \bar{\xi} \star C_{cl} + \bar{C}_{cl} \star \xi), \end{aligned} \tag{19}$$

where we have introduced the classical fields

$$\begin{aligned} A_{cl}^\mu &= \frac{\delta W}{\delta J_\mu}, & \phi_{cl} &= \frac{\delta W}{\delta \eta^\dagger}, \\ \phi_{cl}^\dagger &= -\frac{\delta W}{\delta \eta}, & C_{cl} &= \frac{\delta W}{\delta \bar{\xi}}, \\ \bar{C}_{cl} &= -\frac{\delta W}{\delta \xi}. \end{aligned} \tag{20}$$

Employing the momenta representation, it then follows that

$$\frac{i}{e\alpha}(p_3)_\mu D^{\mu\nu}(p_3)\Delta(p_2)\Delta(p_1)\Gamma_\nu(p_2,-p_1,p_3)$$
$$= i\int\frac{d^3k}{(2\pi)^3}e^{ik\wedge p_2}\Delta(p_1)\Delta(k)S(p_2-k)S(-p_3)$$
$$\times\Gamma_4(k,-p_1,p_2-k,p_3)$$
$$-i\int\frac{d^3k}{(2\pi)^3}e^{-ik\wedge p_1}\Delta(p_2)\Delta(k)S(-p_1+k)S(-p_3)$$
$$\times\Gamma_4(p_2,-k,-p_1+k,p_3), \quad (21)$$

where in a simplified notation $S(k)$ and $\Delta(k)$ represent the Fourier transforms of $S(x)$ and $\Delta(x)$, respectively, the matter field and the ghost field propagators. The Γ functions introduced above are the Fourier transforms of

$$\Gamma_\nu(a,x;b,y;z)=\frac{\delta^3\Gamma}{\delta\phi_a^\dagger(x)\delta\phi_b(y)\delta A_{cl}^\nu(z)}, \quad (22)$$

$$\Gamma_4(a,x;b,y;z;u)=\frac{\delta^4\Gamma}{\delta\phi_a^\dagger(x)\delta\phi_b(y)\delta\bar{c}_{cl}(z)\delta c_{cl}(u)}. \quad (23)$$

The steps leading to (21) are very formal but its validity can be directly verified as we shall do now, up to the subleading order of $1/N$. We note that this equation can be rewritten as

$$\frac{1}{Ne}\frac{p_3^\nu}{p_3^2}\Gamma_\nu(p_2,-p_1,p_3)=\Delta^{-1}(p_2)S(-p_3)H_2(p_1,p_2,p_3)$$
$$-\Delta^{-1}(p_1)S(-p_3)H_1(p_1,p_2,p_3), \quad (24)$$

where we have used the identity (16) for the longitudinal part of the gauge field propagator and, as suggested in an analysis of the ST identities for QCD [15], introduced the functions

$$H_1(p_1,p_2,p_3)=i\int\frac{d^3k}{(2\pi)^3}e^{-ik\wedge p_1}\Delta(k)S(-p_1+k)$$
$$\times\Gamma_4(p_2,-k,-p_1+k,p_3), \quad (25)$$

$$H_2(p_1,p_2,p_3)=i\int\frac{d^3k}{(2\pi)^3}e^{ik\wedge p_2}\Delta(k)S(p_2-k)$$
$$\times\Gamma_4(k,-p_1,p_2-k,p_3). \quad (26)$$

We will now check (24) up to subleading order of $1/N$. Note first that, including corrections up to $1/N$ order, the matter field propagator is given by

$$\Delta(p)=\frac{i}{p^2-m^2-\frac{i}{N}\Sigma_\phi(p)}. \quad (27)$$

From now on, we will work in the Landau gauge, $\alpha=0$. Adopting dimensional regularization with minimal subtraction, we have

$$\Sigma_\phi^{\rm unr}(p)=-N\int\frac{d^Dk}{(2\pi)^D}(k+2p)^\mu D^0_{\mu\nu}(k)(k+2p)^\nu$$
$$\times\Delta^0(k+p)-N\int\frac{d^Dk}{(2\pi)^D}\Delta^0(k+p)\Delta^0_\lambda(k)$$
$$=-\frac{20i}{\pi^2}\frac{1}{\epsilon}p^2+\text{finite terms}, \quad (28)$$

where the superscripts unr denote unrenormalized function. Notice that the one-loop graph containing the vertex (7b) does not contribute since, as discussed before, this type of diagram cancels pairwise due to the identity in Fig. 2(b). The convenient counterterm is $b\partial_\mu\phi_a^\dagger\partial^\mu\phi_a$, where the renormalization constant is $b=\frac{20}{N\pi^2}\frac{1}{\epsilon}$. As for the ghost propagator, we obtain

$$S(p_3)=\frac{i}{p_3^2[N-i\Sigma_c(p_3)]}. \quad (29)$$

The unrenormalized $\Sigma_c(p_3)$ is given by

$$\Sigma_c^{\rm unr}(p_3)=\left(\frac{1}{p_3^2}\right)\bigg[-(2eN)^2\int\frac{d^Dk}{(2\pi)^D}(k+p_3)^\mu D^0_{\mu\nu}(k)$$
$$\times p_3^\nu S^0(k+p_3)\sin^2(k\wedge p_3)\bigg]. \quad (30)$$

The result for the planar part is

$$\Sigma_c^{\rm unr}(p_3)=-\frac{32i}{3\pi^2}\frac{1}{\epsilon}+\text{finite terms}, \quad (31)$$

which may be renormalized by the counterterm $fN\partial_\mu\bar{c}\partial^\mu c$, with $f=\frac{32}{3N\pi^2}\frac{1}{\epsilon}$.

The unrenormalized three-point vertex Γ_ν and H_m functions have the following expansions:

$$\Gamma_\nu=\Gamma_\nu^0+\frac{1}{N}\Gamma_\nu^{1\rm unr} \quad (32)$$

and

$$H_m=H_m^0+\frac{1}{N}H_m^1, \quad (33)$$

up to $1/N$ order. We have verified that the H_m^1 functions are not UV divergent; therefore, no counterterms are needed. However, as shown in [12], $\Gamma_\nu^{1\rm unr}$ consists of divergent diagrams with one and two loops. In the two-loop case, the regularization is introduced just in the last integral. Thus, the total UV divergence is given by

$$\Gamma_\nu^{1unr}=\frac{28ie(2p_2+p_3)_\nu}{3\pi^2}\frac{1}{\epsilon}. \quad (34)$$

The numerical difference, a factor of 2, from Ref. [12], is due to a different regularization prescription adopted in that work. Therefore, the counterterm is

$$Bie(\partial^\mu \phi_a^\dagger \star A_\mu \star \phi_a - \phi_a^\dagger \star A_\mu \star \partial_\mu \phi_a), \qquad (35)$$

where the renormalization constant is $B = \frac{28}{3N\pi^2}\frac{1}{\epsilon}$.

Using the above notation, and allowing terms up to $1/N$ order, the identity (24) may be rewritten as

$$\frac{1}{Ne}\left(\frac{p_3^\nu}{p_3^2}\right)\left[\Gamma^0_{\ \nu} + \frac{1}{N}\Gamma^1_{\ \nu}\right][p_3^2(N - i\Sigma_c(p_3))] = \left\{\left[p_2^2 - m^2 - \frac{i}{N}\Sigma_\phi(p_2)\right]\left[H_2^0 + \frac{1}{N}H_2^1\right]\right\} - \left\{\left[p_1^2 - m^2 - \frac{i}{N}\Sigma_\phi(p_1)\right]\left[H_1^0 + \frac{1}{N}H_1^1\right]\right\}, \qquad (36)$$

where the renormalized functions are given by

$$\Gamma^1_\nu = \Gamma^{1\text{unr}}_\nu + NB\Gamma^0_\nu, \qquad (37a)$$

$$\Sigma_c = \Sigma_c^{\text{unr}} + iNf, \qquad (37b)$$

$$\Sigma_\phi = \Sigma_\phi^{\text{unr}} + iNb. \qquad (37c)$$

To obtain the ST identity at leading order we must consider the vertex function Γ^0_ν on the left-hand side of the expression (36). The right-hand side receives the contribution of H_1^0 and H_2^0, which are both equal to $ie^{-ip_2\wedge p_3}$. Replacing these results in (36), we get

$$\frac{1}{e}p_3^\nu(-ie)(2p_2 + p_3)_\nu e^{-ip_2\wedge p_3} = (p_2^2 - m^2)ie^{-ip_2\wedge p_3} - (p_1^2 - m^2)ie^{-ip_2\wedge p_3}, \qquad (38)$$

which is identically satisfied, as can be seen by using the momentum conservation $p_1 = p_2 + p_3$.

A less trivial result is obtained when we analyze the subleading order which receives loop corrections. As we will see, the identity in subleading order carries quantum corrections and establishes a relation among the renormalization constants. Therefore, from (36) we must have

$$\frac{1}{e}p_3^\nu[\Gamma^1_{\ \nu} - i\Gamma^0_{\ \nu}\Sigma_c(p_3)] = [(p_2^2 - m^2)H_2^1 - iH_2^0\Sigma_\phi(p_2)] - [(p_1^2 - m^2)H_1^1 - iH_1^0\Sigma_\phi(p_1)]. \qquad (39)$$

Replacing (37) into the above expression, the UV divergences of the unrenormalized functions shown in (37) cancel each other, which proves the validity of the noncommutative ST identities for the CP^{N-1} model. Furthermore, we obtain the relation involving the renormalization constants, $B + f = b$.

II. CONCLUSION

We have verified the ST identity in the $1/N$ expansion for the noncommutative CP^{N-1} model. As is known, the diagrams of $1/N$ order involve one and two loops which are very intricate. Therefore, we restricted ourselves to the verification of the matching of the UV divergent parts. Our result proves that the relation $B + f = b$, found by direct calculation, is an explicit consequence of the BRST invariance of the original action. Besides these UV parts, we have also infrared singular parts coming from nonplanar parts of the functions. However, in [12] it was shown that the leading IR singular parts are canceled due to diagramatic identities [14], leaving only logarithmic singularities, which are not problematic as they are integrable.

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