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Large dual transformations and the Petrov-Diakonov representation of the Wilson loop

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In this work, based on the Petrov-Diakonov representation of the Wilson loop average $\bar{W}$ in the $SU(2)$ Yang-Mills theory, together with the Cho-Fadeev-Niemi decomposition, we present a natural framework to discuss possible ideas underlying confinement and ensembles of defects in the continuum. In this language we show how for different ensembles the surface appearing in the Wess-Zumino term in $\bar{W}$ can be either decoupled or turned into a variable, to be summed together with gauge fields, defects, and dual fields. This is discussed in terms of the regularity properties imposed by the ensembles on the dual fields, thus precluding or enabling the possibility of performing the large dual transformations that would be necessary to decouple the initial surface.

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I. INTRODUCTION

Nowadays, one of the most important and interesting open problems in physics corresponds to understanding quark confinement. Although quantum chromodynamics is completely successful in describing high energy phenomena, where because of asymptotic freedom the main characters are quarks and gluons, a theoretical explanation for the confinement of these objects in colorless asymptotic states is still lacking.

With regard to gluon confinement, an important line of research corresponds to studying the effect of the Gribov horizon [1]on the gluon propagator. These ideas indicate that the inclusion of a Gribov-Zwanziger term in the pure Yang-Mills action, as to avoid Gribov copies, leads to infrared suppressed gluon and ghost propagators [2–6]. While the absence of the pole in the gluon propagator would explain why gluons cannot occupy asymptotic states, it is difficult to imagine an explanation for quark confinement in this framework, as the infrared suppression could not produce long range forces.

Therefore, among the possible frameworks for the confinement of (heavy) quarks in pure Yang-Mills theory, those based on the inclusion of a nonperturbative sector represented by magnetic defects become favored, and the problem turns out to be the identification of defects, their associated phases, and how they can imply an area law for the Wilson loop. Although these points have been studied for many years now, a closed theoretical understanding is still lacking [7–9].

For example, in the mechanism of dual superconductivity [10–13], the QCD vacuum is expected to behave as a superconductor of chromomagnetic charges, which implies the confinement of chromoelectric charges, in an analogous (dual) manner to what would happen with a type II superconductor, where magnetic monopoles would be confined because of the magnetic flux tube generated between them.

When implementing the Abelian projection [14], monopoles can appear as defects when a gauge fixing that diagonalizes a field that transforms in the adjoint representation of $SU(N)$ is considered.

Another possible manner to identify them is as defects when trying to implement the Cho-Faddeev-Niemi (CFN) decomposition, with the advantage that in this case no particular gauge fixing condition is invoked. For instance, the monopoles for $SU(2)$ are defects of the local direction $\hat{n}$ used to decompose the connection in color space (see [15–21], and references therein),

$$\vec{A}_\mu = A_\mu^{(n)}\hat{n} - \frac{1}{g}\hat{n}\times\partial_\mu\hat{n} + \vec{X}_\mu^{(n)}, \qquad \hat{n}\cdot\vec{X}_\mu^{(n)} = 0. \quad (1)$$

Besides monopoles, $Z(N)$ center vortices are also of great interest, as they could explain the string tension dependence on the representation of the subgroup $Z(N)$ of $SU(N)$ observed in the lattice (N-ality), a property that cannot be explained by the isolated effect of monopoles. In addition, when closed center vortices are included, an area law (confining phase) or perimeter law (deconfining phase) has also been observed, depending on whether these objects percolate or not [22–25].

Moreover, strong correlations between monopoles and center vortices are supported by recent results on the lattice, and they are quite promising in accommodating the different properties of the confining phase [26–28] (for a review, see also Ref. [7]).

The aim of this work is to present a natural framework to discuss possible ideas underlying confinement and ensembles of defects in the continuum.

In this regard, we have recently unified the description of monopoles and center vortices [29]as different types of defects of the complete local color frame $\hat{n}_a$, $a = 1, 2, 3$ used in the Cho-Faddeev-Niemi decomposition of the $SU(2)$ gauge fields, where $\hat{n}_3 = \hat{n}$ and

$$\vec{X}_\mu^{(n)} = X_\mu^1\hat{n}_1 + X_\mu^2\hat{n}_2. \quad (2)$$

When the element $\hat{n}$ contains monopolelike defects, localized on closed strings, the elements $\hat{n}_1$, $\hat{n}_2$ inevitably contain defects on open surfaces, and these can correspond to Dirac world sheets or to pairs of center vortex world sheets, attached to the monopoles. When we go close to and around an open center vortex (Dirac) world sheet, $\hat{n}_1$ and $\hat{n}_2$ rotate once (twice), corresponding to the flux $2\pi/g$ $(4\pi/g)$ carried by them.

In this manner, additional singular terms in the Yang-Mills action appear, due to the fact that derivatives do not commute when defects are present. These are either localized on Dirac world sheets or on thin center vortices.

In fact, these singular terms were missing in previous literature about the Cho-Faddeev-Niemi decomposition. In this respect, we would like to point out that effective Skyrme models have been constructed in terms of $\hat{n}$ [17,20,21,30,31], guided by the decomposition in Eq. (1). Then, although they capture information about monopoles without reference to unobservable Dirac world sheets, as expected in a well-defined effective model, the information about center vortices in the $\hat{n}_1$, $\hat{n}_2$ sector is lost in this heuristic process (for a discussion, see Refs. [29,32]).

In this article we will first give a representation for the Wilson loop average $\bar{W}$ in the $SU(2)$ Yang-Mills theory, similar to the one in Refs. [16,33], but including the singular terms for the monopole and the center vortex sectors. For this purpose we will use the Petrov-Diakonov (PD) representation of the Wilson loop [34–36], as the natural variables here are those used in the Cho-Faddeev-Niemi decomposition [16].

In particular, for a given gauge field $\vec{A}_\mu$, the Wilson loop order parameter $W(\mathcal{C})$ can be written as an integral over $U \in SU(2)$ containing an Abelian looking integrand that depends on $A^{(n)}_\mu$, the field that appears in the decomposition of $\vec{A}_\mu$ with respect to the local frame induced by U (for a brief review, see Sec. III). The important point is that this representation also includes a Wess-Zumino term, concentrated on a "Wilson surface" $S(\mathcal{C})$, whose border is the Wilson loop $\mathcal{C}$, although the usual representation for $W(\mathcal{C})$ contains no reference to a surface.

In the Petrov-Diakonov representation any surface $S(\mathcal{C})$ can be used, up to singular situations where it passes over the monopoles [35]. This raises the problem of how to deal with this arbitrary surface in the average over fields and ensembles of defects. In Ref. [37], this kind of problem has been discussed in the context of compact $QED(3)$ and $QED(4)$.

Using our representation for $\bar{W}$, we will discuss here how monopole and center vortex ensembles can render the surface appearing in the Wess-Zumino term a variable, to be summed together with gauge fields, defects, and dual fields. This occurs when the regularity properties imposed by the associated physical phases on the dual fields preclude the implementation of large dual field transformations in the path integral, a necessary step that should be considered in order to decouple the initial Wilson surface and show it is an unobservable object.

In general, using our arguments in three dimensions ($3D$) or four dimensions ($4D$), prior to the ensemble integration, we will obtain a representation evidencing the decoupling of the initial Wilson surface or its replacement by a "Wilson surface variable," depending on the assumed closure properties for the dual fields.

In $3D$, as center vortices are stringlike, we will also be able to propose the general form of an effective action describing the interaction among gauge, vortex, and dual fields, as well as Wilson surfaces. Therefore, the relationship between deconfining/confining ensembles and closure/nonclosure properties of the large dual transformations will be clear in this case.

Of course, which is the correct ensemble of defects associated with Yang-Mills theories is the fundamental part of the problem of confinement. In particular, how can the dressing of thin defects lead to dimensional parameters characterizing thick objects that condense. This is outside the scope of this article, which is organized in the following manner.

In Secs. II and III, we review how to describe monopoles and center vortices in terms of the defects of the complete local color frame used to decompose the gauge fields, as well as the Petrov-Diakonov representation of the Wilson loop W. Section IV is dedicated to a brief discussion of the representation for the average $\bar{W}$, including a general ensemble of monopoles and center vortices.

In Sec. V, we discuss the arbitrary Wilson surface $S(\mathcal{C})$ in connection with the integrand of $\bar{W}$. In Sec. VI, we present possible effective models that describe chains of correlated monopoles and center vortices, and discuss how they could preclude the implementation of large dual changes of variables.

In Sec. VII, we show how to decouple the Wilson surface $S(\mathcal{C})$ in favor of its border, in the case where the dual fields are closed under large dual transformations. In the opposite case, we show how $S(\mathcal{C})$ is replaced by a Wilson surface variable, also including a discussion of generalized multivalued dual fields in continuum $4D$ theories.

Finally, we present our conclusions in Sec. VIII.

II. DEFECTS OF THE LOCAL COLOR FRAME

When studying Abelian projection scenarios, the gauge fields are generally separated into "diagonal" fields, living in the Cartan subalgebra of $SU(N)$, and "off-diagonal" charged fields. For instance, in the case of $SU(2)$, the uncharged sector can be chosen along the $\hat{e}_3$ direction in color space, while the components along $\hat{e}_1$ and $\hat{e}_2$ correspond to the charged sector.

In the CFN decomposition, this separation into charged and uncharged sectors is also implemented, with the advantage that it is naturally done along a general $\hat{n}_3 = \hat{n}$ local direction in color space.

In Ref. [29], we have unified monopoles and center vortex world sheets as different classes of defects in the local color frame $\hat{n}_a = R\hat{e}_a$, $R \in SO(3)$, used in the CFN decomposition. While it is well known that monopolelike defects are associated with a nontrivial Π_2 for the space of directions $\hat{n}$, we can also think of thin center vortices as the natural defects of a frame, due to the nontrivial fundamental group $\Pi_1 = Z(2)$ of $SO(3)$.

The possibility of matching general nontrivial configurations containing monopoles and center vortices is evidenced by parametrizing the gauge fields in terms of the CFN decomposition, based on a class of frames $\hat{n}_a$,

$$(VU)T^a(VU)^{-1} = \hat{n}_a \cdot \vec{T}, \qquad \hat{n}_a = R(VU)\hat{e}_a, \tag{3}$$

where U is single-valued along any closed loop, defining a frame $\hat{m}_a$,

$$UT^aU^{-1} = \hat{m}_a \cdot \vec{T}, \qquad \hat{m}_a = R(U)\hat{e}_a, \tag{4}$$

such that $\hat{m}_3 = \hat{m}$ is a topologically nontrivial mapping that encodes the monopole sector. The V part is multivalued and enables the description of the center vortex sector.

Let us consider, for example, a gauge field whose decomposition is given by

$$\vec{a}_\mu \cdot \vec{T} = -\left(C^{(n)}_\mu \hat{n} + \frac{1}{g}\hat{n} \times \partial_\mu \hat{n}\right) \cdot \vec{T}, \qquad C^{(n)}_\mu = -\frac{1}{g}\hat{n}_1 \cdot \partial_\mu \hat{n}_2. \tag{5}$$

In the case where $V \equiv I$, and taking $U = \bar{U} = e^{-i\varphi T_3}e^{-i\theta T_2}e^{+i\varphi T_3}$, where φ and θ are the polar angles defining $\hat{r}$, Eq. (5) corresponds to a nontrivial "gauge" transformation $\frac{i}{g}\bar{U}\partial_\mu \bar{U}^{-1}$ introducing an antimonopole [16]. Note that no singularity is present at $\theta \approx 0$, where $\bar{U} \approx I$. The Dirac string is placed at $\theta = \pi$; when we go close and around the negative z axis, the elements $\hat{n}_1$, $\hat{n}_2$ rotate twice. A monopole is obtained with the replacement $\theta \to \pi - \theta$, $\varphi \to \varphi + \pi$.

More generally, a field decomposed according to Eq. (1), with $V \equiv I$, can be written as a nontrivial transformation of a regular background $\vec{\mathcal{A}}_\mu$,

$$\vec{A}_\mu \cdot \vec{T} = \vec{\mathcal{A}}^{\bar{U}}_\mu \cdot \vec{T} = \bar{U}\vec{\mathcal{A}}_\mu \cdot \vec{T}\bar{U}^{-1} + \frac{i}{g}\bar{U}\partial_\mu \bar{U}^{-1}. \tag{6}$$

As is well known, the field strength for $\vec{\mathcal{A}}^{\bar{U}}_\mu$ is

$$\vec{\mathcal{F}}^{\bar{U}}_{\mu\nu} \cdot \vec{T} = \bar{U}\vec{\mathcal{F}}_{\mu\nu} \cdot \vec{T}\bar{U}^{-1} + \frac{i}{g}\bar{U}[\partial_\mu, \partial_\nu]\bar{U}^{-1}. \tag{7}$$

That is, the fields $\vec{A}_\mu$ and $\vec{\mathcal{A}}^{\bar{U}}_\mu$ are not physically equivalent, because of the second term in Eq. (7) which is concentrated on a Dirac world sheet, namely, the two-dimensional surface where $\bar{U}$ is singular.

Now, by considering in Eq. (5) a local frame defined by $U \equiv I$ and $V = \bar{V} = e^{i\varphi T_3}$, we obtain

$$\vec{a}_\mu \cdot \vec{T} = \frac{1}{g}\partial_\mu \varphi \delta^{a3}T^a, \tag{8}$$

that is, a thin center vortex placed on the two-dimensional surface formed by the z axis, for every Euclidean time. As the transformation $\bar{V} = e^{i\varphi T_3}$ is not single valued, we have

$$\frac{1}{g}\partial_\mu \varphi \delta^{a3}T^a = \frac{i}{g}\bar{V}\partial_\mu \bar{V}^{-1} - \text{ideal vortex}, \tag{9}$$

where the additional term (the so-called ideal vortex) is localized on the three-volume where the transformation is discontinuous. For a general discussion of thin and ideal center vortices in the continuum, see Refs. [38,39]. Then, unlike monopoles, center vortices can only be written in the form $\frac{i}{g}\bar{V}\partial_\mu \bar{V}^{-1}$ on a region outside the above mentioned three-volume.

Furthermore, if on the monopole ansatz after Eq. (5), $V \equiv I$ were replaced by $\bar{V} = e^{-i\varphi\hat{m}\cdot\vec{T}}$, we would have $\bar{V}\bar{U} = e^{-i\varphi T_3}e^{-i\theta T_2}$. Then, instead of a monopole attached to a Dirac world sheet placed at $\theta = \pi$, one attached to a pair of center vortices at $\theta = 0$ and $\theta = \pi$ would be obtained. In this case, when we go close and around the positive and negative z axis, the elements $\hat{n}_1$, $\hat{n}_2$ rotate once, with different orientations. In general, any configuration containing monopoles and center vortices (correlated or not) can be written in terms of three Euler angles $\bar{V}\bar{U} = e^{-i\alpha T_3}e^{-i\beta T_2}e^{+i(\alpha-\gamma)T_3}$ that correspond to a single-valued $\bar{U} = e^{-i\alpha T_3}e^{-i\beta T_2}e^{+i\alpha T_3}$, and a rotation $\bar{V} = e^{-i\gamma\hat{m}\cdot\vec{T}} = \bar{U}e^{-i\gamma T_3}\bar{U}^{-1}$, leaving $\hat{m} = \hat{n}$ fixed.

III. PETROV-DIAKONOV REPRESENTATION

The usual representation for the non-Abelian Wilson loop order parameter is given by

$$W(\mathcal{C}) = (1/2)\mathrm{tr}P\exp\left(ig \oint dx_\mu \vec{A}_\mu \cdot \vec{T}\right). \tag{10}$$

There is an alternative representation, due to Petrov and Diakonov [34–36]. For quarks in the fundamental representation, it is given by

$$W(\mathcal{C}) = (1/2)\int[\mathcal{D}U(\tau)]e^{(i/2)g\int_0^1 d\tau\, \mathrm{tr}[\tau^3(U^{-1}AU+(i/g)U^{-1}(d/d\tau)U)]}, \tag{11}$$

$$A(\tau) = \frac{dx_\mu}{d\tau}\vec{A}_\mu \cdot \vec{T}. \tag{12}$$

Here, the Wilson loop $\mathcal{C}$ has been parametrized as $x_\mu = x_\mu(\tau)$, $\tau \in [0, 1]$, $x_\mu(0) = x_\mu(1)$. The integration measure is

$$\int[\mathcal{D}U(\tau)] = \int dU \int_{U(0)=U}^{U(1)=U} \mathcal{D}U(\tau), \tag{13}$$

which means that the functional integral is done over U transformations that are single valued along the Wilson loop.

Considering that on a given loop it is always possible to write

$$A(u) = \frac{i}{g} Q^{-1} \frac{d}{d\tau} Q, \qquad Q(u) = \exp\left(-ig \int_0^u du' A(u')\right), \tag{14}$$

it results in [34–36]

$$\begin{aligned} W(\mathcal{C}) &= (1/2) \int dU \int_{U(0)=U}^{U(1)=U} \mathcal{D}U(\tau) e^{(i/2)\int_0^1 d\tau \, \mathrm{tr}[\tau^3 (i(QU)^{-1}(d/d\tau)(QU))]}, \\ &= (1/2) \sum_\alpha D^{(1/2)}_{\alpha\alpha}(Q^{-1}(1)Q(0)). \end{aligned} \tag{15}$$

Of course, the Wilson variable generally takes a nontrivial value; that is, $Q(1)$ is generally not $Q(0) = 1$.

To see how these expressions work, let us recall that closed center vortices are usually defined as defects in the connection such that $W(\mathcal{C})$ changes sign when the defect is linked and is otherwise left unchanged.

As is well known, considering a line $x(\tau)$ which lives on a simply connected region outside a closed vortex, where it is possible to write $\vec{A}_\mu = \vec{\mathcal{A}}_\mu^{\bar{V}\bar{U}}$, and then taking the limit where their end points are joined to form the loop $\mathcal{C}$, the usual representation for $W(\mathcal{C})$ gives $e^{iq\pi} W_{\mathcal{A}}(\mathcal{C})$, where $W_{\mathcal{A}}(\mathcal{C})$ is the Wilson loop for the field $\mathcal{A}_\mu$.

Now, we can use the PD representation. From Eq. (12), we have

$$\begin{aligned} A(\tau) &= \frac{dx_\mu}{d\tau} \vec{A}_\mu \cdot \vec{T} \\ &= \left[(\bar{V}\bar{U}) \mathcal{A}(\tau) (\bar{V}\bar{U})^{-1} + \frac{i}{g} (\bar{V}\bar{U}) \frac{d}{d\tau} (\bar{V}\bar{U})^{-1} \right], \end{aligned} \tag{16}$$

where we have defined $\mathcal{A}(\tau) = \frac{dx_\mu}{d\tau} \vec{\mathcal{A}}_\mu \cdot \vec{T}$. Recalling that on the loop we can always write $\mathcal{A}(\tau) = \frac{i}{g} \mathcal{Q}^{-1} \frac{d}{d\tau} \mathcal{Q}$, we get $Q = \mathcal{Q} \bar{U}^{-1} \bar{V}^{-1}$. Then, using in Eq. (15) the cyclic property of the trace, and considering that $D^{(1/2)}$ is an odd function, the previous result is reobtained,

$$\begin{aligned} W(\mathcal{C}) &= (1/2) \sum_\alpha D^{(1/2)}_{\alpha\alpha} (\mathcal{Q}(0) \bar{U}_i^{-1} \bar{V}_i^{-1} \bar{V}_f \bar{U}_f \mathcal{Q}^{-1}(1)) \\ &= e^{iq\pi} W_{\mathcal{A}}(\mathcal{C}). \end{aligned} \tag{17}$$

It is important to emphasize that the second part in the exponent of Eq. (11) is a Wess-Zumino term, and can be rewritten not in terms of a line but in terms of a surface integral [34–36]. Therefore, in general, we have

$$W(\mathcal{C}) = (1/2) \int [\mathcal{D}U(\tau, \xi)] e^{i(g/2) \int d^4x s_{\mu\nu} (f^{(m)}_{\mu\nu} + h^{(m)}_{\mu\nu})}, \tag{18}$$

where the source $s_{\mu\nu}$ is concentrated on a surface $S(\mathcal{C})$ whose border is the Wilson loop $\mathcal{C}$, and is constructed by requiring $\int d^4x s_{\mu\nu} (f^{(m)}_{\mu\nu} + h^{(m)}_{\mu\nu})$ to be the flux of $f^{(m)}_{\mu\nu} + h^{(m)}_{\mu\nu}$ through $S(\mathcal{C})$. This surface can be parametrized by $x(\tau, \xi)$, and $s_{\mu\nu}$ must satisfy

$$\begin{aligned} j_\mu(\mathcal{C}) &= \epsilon_{\mu\nu\rho\sigma} \partial_\nu s_{\rho\sigma}, \\ j_\mu(\mathcal{C}) &= \int d\tau \frac{dx_\mu}{d\tau} \delta(x - x(\tau)), \end{aligned} \tag{19}$$

where $x(\tau) = x(\tau, 1)$ is a parametrization of $\mathcal{C}$. In Eq. (18), we also have

$$\begin{aligned} f^{(m)}_{\mu\nu} &= f^{(m)}_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} \partial_\nu A^{(m)}_\sigma, \\ h^{(m)}_{\mu\nu} &= -\frac{1}{2g} \epsilon_{\mu\nu\rho\sigma} \hat{m} \cdot (\partial_\rho \hat{m} \times \partial_\sigma \hat{m}), \end{aligned} \tag{20}$$

where the connection is decomposed by using a frame $\hat{m}_a$, defined on $S(\mathcal{C})$, and induced by $U(\tau, \xi)$, namely,

$$U T_a U^{-1} = \hat{m}_a \cdot \vec{T}, \tag{21}$$

$$\vec{A}_\mu = A^{(m)}_\mu \hat{m} - \frac{1}{g} \hat{m} \times \partial_\mu \hat{m} + \vec{X}^{(m)}_\mu. \tag{22}$$

We also note that the possibility of writing,

$$\int_0^1 d\tau \frac{i}{g} \mathrm{tr}\left[\tau^3 U^{-1} \frac{d}{du} U\right] = \int d^4x s_{\mu\nu} h^{(m)}_{\mu\nu}, \tag{23}$$

depends on the single valuedness of $U(\tau)$ (see Ref. [35]). This condition is met precisely because of the integration measure in Eq. (13).

IV. WILSON LOOP AVERAGE

Now we will work with thin objects defined on the whole Euclidean spacetime, taking into account the singular terms arising from the color frame defects. Let us consider the Wilson loop average,

$$\begin{aligned} \bar{W}(\mathcal{C}) = \frac{1}{2\mathcal{N}} \int [\mathcal{D}\vec{A}] F_{gf} e^{-S_{\mathrm{YM}}[\vec{A}]} \, \mathrm{tr} P \\ \times \exp\left(ig \oint dx_\mu \vec{A}_\mu \cdot \vec{T}\right), \end{aligned} \tag{24}$$

$$\mathcal{N} = \int [\mathcal{D}\vec{A}] F_{gf} e^{-S_{\mathrm{YM}}[\vec{A}]}, \tag{25}$$

where F_{gf} is the part of the measure that fixes the gauge, including in general auxiliary fields.

Using the PD representation, we have

$$\begin{aligned} \bar{W}(\mathcal{C}) = \frac{1}{2\mathcal{N}} \int [\mathcal{D}\vec{A}] \\ \times [\mathcal{D}U(\tau, \xi)] F_{gf} e^{-S_{\mathrm{YM}}[\vec{A}]} e^{(i/2) g \int d^4x s_{\mu\nu} (f^{(m)}_{\mu\nu} + h^{(m)}_{\mu\nu})}. \end{aligned} \tag{26}$$

In fact, as we are interested in discussing the Wilson loop globally, for any closed loop and any associated surface, we will have to consider the extension $U(x)$, defined on the whole Euclidean spacetime, up to possible singularities, such that $U(x(\tau, \xi)) = U(\tau, \xi)$.

Now, as the Wilson loop is written in terms of the CFN variables, it is convenient to change to these variables in the path integral [16,33]. The procedure is to include the integration over the extended U's, which amounts to introducing a product of group volumes, and then performing a change (with unit Jacobian) to the variables $A_\mu^{(m)}$, $\vec{X}_\mu^{(m)}$ ($m = 1, 2$) in the decomposition of $\vec{A}_\mu$ with respect to the basis induced by $U(x)$.

An important point to be emphasized is that after the change, $\vec{A}_\mu$ configurations containing monopoles will be represented by U's inducing frames with monopolelike defects in $\hat{m}$. In addition, as U configurations are single valued, thin center vortices will be manifested as defects in the components of the charged fields $\vec{X}_\mu^{(m)}$. For convenience, the ensemble integration over these defects can be replaced by the integration over a V sector, which according to Eq. (3) rotates $\hat{m}_1$, $\hat{m}_2$ to $\hat{n}_1$, $\hat{n}_2$, leaving $\hat{m} = \hat{n}$ fixed. This is done in order to identify monopoles and center vortices with singular frames. Then, we have

$$\bar{W}(\mathcal{C}) = \frac{1}{2\mathcal{M}} \int [\mathcal{D}A][\mathcal{D}X][\mathcal{D}U] \times [\mathcal{D}V] F_{gf} e^{-S_{\rm YM}[\hat{n}_a, A^{(n)}, X^{(n)}]} e^{(i/2)g \int d^4x s_{\mu\nu}(f_{\mu\nu}^{(n)} + h_{\mu\nu}^{(n)})}, \tag{27}$$

$$\mathcal{M} = \int [\mathcal{D}A][\mathcal{D}X][\mathcal{D}U][\mathcal{D}V] F_{gf} e^{-S_{\rm YM}[\hat{n}_a, A^{(n)}, X^{(n)}]}. \tag{28}$$

A fundamental ingredient to be taken into account is regarding the nontrivial singular terms associated with the frame defects. In Ref. [29], we have identified two types, which were missing in the field strength tensor computed in Refs. [16–19]. The first one depends on defects of the third component $\hat{n}_3 \equiv \hat{n}$ and occurs in the charged sector of the field strength tensor. In Ref. [29], this type of term has been nullified by considering $\hat{n}$ configurations that have at most monopole defects. In this case, $S_{\rm YM}$ results,

$$S_{\rm YM} = \int d^4x \left[\frac{1}{4}(f_{\mu\nu}^{(n)} + h_{\mu\nu}^{(n)} + k_{\mu\nu})^2 + \frac{1}{2}\bar{g}^{\mu\nu} g^{\mu\nu}\right], \tag{29}$$

where

$$g^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}[\partial_\rho + ig(A_\rho^{(n)} + C_\rho^{(n)})]\Phi_\sigma, \qquad C_\mu^{(n)} = -\frac{1}{g}\hat{n}_1.\partial_\mu \hat{n}_2, \tag{30}$$

$$\Phi_\mu = \frac{1}{\sqrt{2}}(X_\mu^1 + iX_\mu^2), \qquad k_{\mu\nu} = \frac{g}{2i}\epsilon_{\mu\nu\rho\sigma}(\bar{\Phi}_\rho \Phi_\sigma - \Phi_\rho \bar{\Phi}_\sigma), \tag{31}$$

$$f_{\mu\nu}^{(n)} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho A_\sigma^{(n)}, \qquad h_{\mu\nu}^{(n)} = -\frac{1}{2g}\epsilon_{\mu\nu\rho\sigma}\hat{n} \cdot (\partial_\rho \hat{n} \times \partial_\sigma \hat{n}). \tag{32}$$

The second type occurs when trying to express the monopole part $h_{\mu\nu}^{(n)}$ of the dual field strength in terms of the monopole potential $C_\mu^{(n)}$. In this case, we obtain

$$h_{\mu\nu} = \tilde{h}_{\mu\nu}^{(n)} + d_{\mu\nu}^{(n)}, \qquad \tilde{h}_{\mu\nu}^{(n)} = \epsilon_{\mu\nu\rho\sigma}\partial_\rho C_\sigma^{(n)}, \tag{33}$$

where the singular terms $d_{\mu\nu}^{(n)}$ are concentrated on the frame defects. If not for this difference, the surface integral in the Wess-Zumino term of the PD representation could be converted into a line integral.

Now we can proceed as we did for the partition function in Ref. [29]. Introducing real and complex Lagrange multipliers, $\lambda_{\mu\nu}$ and $\Lambda_{\mu\nu}$, we get

$$\bar{W}(\mathcal{C}) = \frac{1}{2\mathcal{M}} \int [\mathcal{D}\lambda][\mathcal{D}\Psi][\mathcal{D}U][\mathcal{D}V] e^{-S_c - \int d^4x (1/4)\lambda_{\mu\nu}\lambda_{\mu\nu}} \times e^{i\int d^4x[(1/2)\lambda_{\mu\nu}(f_{\mu\nu}^{(n)} + h_{\mu\nu}^{(n)} + k_{\mu\nu}) - J_c^\mu (A_\mu^{(n)} + C_\mu^{(n)}) + (g/2)s_{\mu\nu}(f_{\mu\nu}^{(n)} + h_{\mu\nu}^{(n)})]}. \tag{34}$$

where we have defined $[\mathcal{D}\Psi] = [\mathcal{D}A^{(n)}][\mathcal{D}\Phi][\mathcal{D}\Lambda]\tilde{F}_{gf}$. Here, we have the action for the charged fields,

$$S_c = \int d^4x \left[\frac{1}{2}\bar{\Lambda}^{\mu\nu}\Lambda^{\mu\nu} - \frac{i}{2}(\bar{\Lambda}^{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\partial_\rho \Phi_\sigma + \Lambda^{\mu\nu}\epsilon^{\mu\nu\rho\sigma}\partial_\rho \bar{\Phi}_\sigma)\right], \tag{35}$$

minimally coupled to the $U(1)$ color current $J_c^\mu = J^\mu + K^\mu$,

$$J^\mu = -\frac{i}{2}g\epsilon^{\mu\nu\rho\sigma}\bar{\Lambda}_{\nu\rho}\Phi_\sigma + \frac{i}{2}g\epsilon^{\mu\nu\rho\sigma}\Lambda_{\nu\rho}\bar{\Phi}_\sigma. \tag{36}$$

The terms K_μ and $\tilde{F}_{gf}$ appear when fixing an extended maximally Abelian gauge,

$$\partial_\mu (A_\mu^{(n)} + C_\mu^{(n)}) = 0. \tag{37}$$

$$[\partial_\mu + ig(A_\mu^{(n)} + C_\mu^{(n)})]\Phi_\mu = 0, \qquad [\partial_\mu - ig(A_\mu^{(n)} + C_\mu^{(n)})]\bar{\Phi}_\mu = 0. \tag{38}$$

More precisely,

$$F_{gf} = \tilde{F}_{gf} e^{-i\int d^4x (A_\mu^{(n)} + C_\mu^{(n)})K^\mu}, \tag{39}$$

where $\tilde{F}_{gf}$ is independent of $A_\mu^{(n)}$, and contains the integration measure for Lagrange multipliers, ghosts, and auxiliary fields, while K^μ depends on these fields, as well as on Φ_μ.

Because of the $A_\mu^{(n)}$ path integration, a constraint is implicit here,

$$J_c^\mu = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu(\lambda_{\rho\sigma} + g s_{\rho\sigma}), \tag{40}$$

so that we finally get,

$$\bar{W}(\mathcal{C}) = \int [\mathcal{D}\lambda][\mathcal{D}\Psi][\mathcal{D}U][\mathcal{D}V] e^{-S_c - \int d^4x (1/4)\lambda_{\mu\nu}\lambda_{\mu\nu}} \times e^{i\int d^4x \{((1/2)\epsilon_{\mu\nu\rho\sigma}\partial_\nu(\lambda_{\rho\sigma}+gs_{\rho\sigma}) - J_\mu^c)A_\mu^{(n)} + (1/2)\lambda_{\mu\nu}k_{\mu\nu} + (1/2)(\lambda_{\mu\nu}+gs_{\mu\nu})d_{\mu\nu}^{(n)}\}}. \tag{41}$$

It will also be convenient to discuss the representation in $3D$, derived by following the same steps, namely,

$$\bar{W}(\mathcal{C}) = \int [\mathcal{D}\lambda][\mathcal{D}\Psi][\mathcal{D}U][\mathcal{D}V] e^{-S_c - \int d^3x (1/2)\lambda_\mu\lambda_\mu} \times e^{i\int d^3x \{(\epsilon_{\mu\nu\rho}\partial_\nu(\lambda_\rho + (g/2)s_\rho) - J_\mu^c)A_\mu^{(n)} + \lambda_\mu k_\mu + (\lambda_\mu + (g/2)s_\mu)d_\mu^{(n)}\}}, \tag{42}$$

$$S_c = \int d^3x [\bar{\Lambda}^\mu \Lambda^\mu - i(\bar{\Lambda}^\mu \epsilon^{\mu\nu\rho}\partial_\nu \Phi_\rho + \Lambda^\mu \epsilon^{\mu\nu\rho}\partial_\nu \bar{\Phi}_\rho)]. \tag{43}$$

In the total charge current $J_c^\mu = J^\mu + K^\mu$, the term K^μ receives contributions from the charged fields of the gauge fixing sector and

$$J^\mu = ig\epsilon^{\mu\nu\rho}\bar{\Lambda}_\nu \Phi_\rho - ig\epsilon^{\mu\nu\rho}\Lambda_\nu \bar{\Phi}_\rho, \qquad k_\mu = \frac{g}{2i}\epsilon_{\mu\nu\rho}(\bar{\Phi}_\nu \Phi_\rho - \Phi_\nu \bar{\Phi}_\rho). \tag{44}$$

The source s_μ is concentrated on $S(\mathcal{C})$ and is such that $\int d^3x s_\mu (f_\mu + h_\mu)$ gives the flux of $(f_\mu + h_\mu)$. Also in Eq. (42), we have the implicit constraint,

$$J_\mu^c = \epsilon_{\mu\nu\rho}\partial_\nu\left(\lambda_\rho + \frac{g}{2}s_\rho\right), \qquad \epsilon_{\mu\nu\rho}\partial_\nu s_\rho = j_\mu(\mathcal{C}). \tag{45}$$

Finally, $d_\mu^{(n)}$ is concentrated on the defects and is obtained from

$$h_\mu^{(n)} = \tilde{h}_\mu^{(n)} + d_\mu^{(n)}, \tag{46}$$

$$h_\mu^{(n)} = -\frac{1}{2g}\epsilon_{\mu\nu\rho}\hat{n}\cdot(\partial_\nu \hat{n} \times \partial_\rho \hat{n}), \qquad \tilde{h}_\mu^{(n)} = \epsilon_{\mu\nu\rho}\partial_\mu C_\rho^{(n)}. \tag{47}$$

For a monopole/antimonopole correlated with a pair of center vortices, the terms representing the defects in Eqs. (46) and (33) are given by [29]

$$d_\mu^{(n)} = d_\mu^{(1)} + d_\mu^{(2)}, \qquad d_{\mu\nu}^{(n)} = d_{\mu\nu}^{(1)} + d_{\mu\nu}^{(2)}, \tag{48}$$

$$d_\mu^{(\alpha)} = \frac{2\pi}{g}\int d\sigma \frac{dx_\mu^\alpha}{d\sigma}\delta^{(3)}(x - x^\alpha(\sigma)). \tag{49}$$

$$d_{\mu\nu}^{(\alpha)} = \frac{2\pi}{g}\int d^2\sigma_{\mu\nu}\delta^{(4)}(x - x^\alpha(\sigma_1, \sigma_2)), \tag{50}$$

Here, $x^\alpha(\sigma)$ $[x^\alpha(\sigma_1, \sigma_2)]$, $\alpha = 1, 2$, is a pair of open center vortex world lines (world sheets) with the same boundaries at x^+, x^- (C^+, C^-), where the monopole and antimonopole are localized. That is,

$$\partial_\mu d_\mu^{(\alpha)} = \frac{2\pi}{g}(\delta^{(3)}(x - x^+) - \delta^{(3)}(x - x^-)), \tag{51}$$

$$\partial_\nu d_{\mu\nu}^{(\alpha)} = \frac{2\pi}{g}\left(\oint_{C^+} dy_\mu \delta^{(4)}(x - y) - \oint_{C^-} dy_\mu \delta^{(4)}(x - y)\right). \tag{52}$$

For uncorrelated objects, we can write $d_\mu^{(n)} = d_\mu^{(m)} + d_\mu^{(v)}$, $d_{\mu\nu}^{(n)} = d_{\mu\nu}^{(m)} + d_{\mu\nu}^{(v)}$ [29], where the first part comes from defects in $\hat{n}_1$, $\hat{n}_2$ concentrated on open Dirac strings or world sheets, while the second part comes from defects localized on closed center vortex world lines or world sheets, thus satisfying

$$\partial_\mu d_\mu^{(v)} = 0, \qquad \partial_\nu d_{\mu\nu}^{(v)} = 0. \tag{53}$$

V. WILSON SURFACES AND FRAME DEFECTS

Up to now, we have seen how to represent the Wilson loop average in the continuum, by considering an ensemble of thin defects. In fact, in Yang-Mills theories, these defects are expected to be dressed by quantum fluctuations, gaining dimensional properties such as the vortex thickness and stiffness. This is the difficult part of the problem of confinement; however, we can assume this scenario and analyze its feedback on the structure of the theory.

That is, we can replace the measure over the monopole and vortex sectors $[\mathcal{D}U][\mathcal{D}V]$ by another one $[\mathcal{D}\mathrm{mon}]\times[\mathcal{D}\mathrm{vor}] = [\mathcal{D}U][\mathcal{D}V]e^{-S_d}$, including an action S_d for the physical part of the defects, characterizing the ensemble. The ensemble integration in Eqs. (41) and (42) can be separated to define an effective contribution $S_{v,m}$,

$$e^{-S_{v,m}[\bar{\lambda}_\mu]} = \int [\mathcal{D}\mathrm{mon}][\mathcal{D}\mathrm{vor}] e^{i(2\pi/g)\sum \int dx_\mu \bar{\lambda}_\mu}, \qquad \bar{\lambda}_\mu = \lambda_\mu + \frac{g}{2}s_\mu, \tag{54}$$

$$e^{-S_{v,m}[\bar{\lambda}_{\mu\nu}]} = \int [\mathcal{D}\text{mon}][\mathcal{D}\text{vor}] e^{i(\pi/g)\sum \int d^2\sigma_{\mu\nu}\bar{\lambda}_{\mu\nu}},$$
$$\bar{\lambda}_{\mu\nu} = \lambda_{\mu\nu} + g s_{\mu\nu}. \tag{55}$$

For correlated defects, with center vortices forming chains of monopoles and antimonopoles, the sum in the integrand would be performed over open center vortices attached in pairs to the corresponding monopoles and antimonopoles. In case of uncorrelated defects, the sum would be over closed center vortices plus the sum over open Dirac strings (in $3D$) or Dirac world sheets (in $4D$).

It is still an open problem which ensemble is associated with $SU(2)$ Yang-Mills theory. In the next section, we will discuss some possibilities in the framework provided by the CFN decomposition and the PD representation in the presence of defects.

Note that in the representation for $\bar{W}$, in Eqs. (41) and (42), the terms containing $\epsilon_{\mu\nu\rho}\partial_\nu s_\rho$, $\epsilon_{\mu\nu\rho\sigma}\partial_\nu s_{\rho\sigma}$, according to Eqs. (45) and (19), depend only on the Wilson loop $\mathcal{C}$. However, because of the Wess-Zumino term in the PD representation and the presence of defects, $\bar{W}$ contains a reference to the initially considered $S(\mathcal{C})$, although the usual non-Abelian Wilson loop representation contains no reference to a surface.

Terms in $d_\mu^{(n)}$, $d_{\mu\nu}^{(n)}$ associated with closed center vortices contribute with a flux $\pm 2\pi/g$ for each center vortex crossing the surface. For a fixed Wilson loop $\mathcal{C}$, this contribution is independent of the surface $S(\mathcal{C})$ considered, given a factor $(-1)^{\text{link}}$ that depends on the total linking number between the closed center vortices and $\mathcal{C}$. When vortices percolate, this linking gives an area law that displays N-ality [7].

As we have previously seen, monopoles can be joined by Dirac defects or by pairs of open center vortices.

In the first case, for a surface crossed by a Dirac defect the flux is $\pm 4\pi/g$, while for a surface that is not crossed the flux is zero. Both situations contribute with a trivial phase $\pm 2\pi$, or zero, respectively.

In the second case, consider, for example, a given monopole/antimonopole configuration joined by a pair of center vortices. If the loop $\mathcal{C}$ is "linked" by the chain, the flux contribution will be $+2\pi/g$ or $-2\pi/g$, depending on which center vortex in the pair crosses the surface $S(\mathcal{C})$. In both cases the Wilson loop gains a -1 factor.

However, we see that when considering the ensemble integration over defects, there are singularities when the monopoles pass over $S(\mathcal{C})$. This leads to the problem of how to obtain a representation of the Wilson loop average with no reference to the initially considered Wilson surface $S(\mathcal{C})$. The answer will depend on the type of ensemble. Initially we will discuss in the CFN-PD framework how, when the magnetic defects proliferate, the different phases can enable or preclude the possibility of performing large dual transformations.

VI. POSSIBLE ENSEMBLES AND THE ASSOCIATED CLOSURE PROPERTIES OF THE DUAL FIELDS

As already discussed, the usual representation of the Wilson loop contains no reference to a surface, so that the Petrov-Diakonov representation of the Wilson loop average should be invariant under the change of initial Wilson surface $S(\mathcal{C})$.

The consideration of a different $S(\mathcal{C})$ can be written as the addition of a closed surface $\partial\vartheta$, written as the border of a three-volume ϑ: $S(\mathcal{C}) \to S(\mathcal{C}) \circ \partial\vartheta$. This change can also be written in terms of the new sources, $s_\mu + \Delta s_\mu$, $s_{\mu\nu} + \Delta s_{\mu\nu}$, where, as $\partial\vartheta$ has no border, the additional pieces verify

$$\epsilon_{\mu\nu\rho}\partial_\nu \Delta s_\rho = 0, \qquad \epsilon_{\mu\nu\rho\sigma}\partial_\nu \Delta s_{\rho\sigma} = 0, \tag{56}$$

so that in $3D$ and $4D$ we can write

$$\frac{g}{2}\Delta s_\mu = \partial_\mu \omega^{(3)}, \qquad g\Delta s_{\mu\nu} = \partial_\mu \omega_\nu^{(4)} - \partial_\nu \omega_\mu^{(4)}. \tag{57}$$

Note that as long as x is not on the closed surface $\partial\vartheta$, we have $\partial_\mu \omega^{(3)} = 0$, $\partial_\mu \omega_\nu^{(4)} - \partial_\nu \omega_\mu^{(4)} = 0$. That is, $\omega^{(3)}(x)$ is piecewise constant. It takes the value $\pm g/2$, when x is inside ϑ, and is zero outside. The plus or minus sign depends on whether the normal to $\partial\vartheta$ has an internal or external orientation.

In $4D$, the solution to Eq. (57) is $\omega_\mu^{(4)} = \partial_\mu \omega^{(4)}$, where $\omega^{(4)}$ is a multivalued phase. That is, when a path linking the surface $\partial\vartheta$ is followed, $\omega^{(4)}$ changes by an amount $\pm g/2$, while it does not change otherwise.

Now it is obvious that for a given λ_μ, $\lambda_{\mu\nu}$ in the integrand of Eqs. (41) and (42), the configurations

$$\lambda_\mu + \partial_\mu \omega, \qquad \lambda_{\mu\nu} + \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \tag{58}$$

with ω and ω_μ smooth well-defined fields, always correspond to another possible field configuration, so that we can operate with the associated changes of variables as usual. Then we are tempted to always consider

$$\lambda_\mu \to \lambda_\mu + \partial_\mu \omega^{(3)}, \qquad \lambda_{\mu\nu} \to \lambda_{\mu\nu} + \partial_\mu \omega_\nu^{(4)} - \partial_\nu \omega_\mu^{(4)}, \tag{59}$$

as an acceptable change of variables. In terms of the Hodge decomposition,

$$\lambda_\mu = \partial_\mu \phi + B_\mu, \qquad \lambda_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu + B_{\mu\nu}, \tag{60}$$

$$\partial_\mu B_\mu = 0, \qquad \partial_\mu \phi_\mu = 0, \qquad \partial_\nu B_{\mu\nu} = 0, \tag{61}$$

we are asking about the possibility of considering changes of variables,

$$\phi \to \phi + \omega^{(3)}, \qquad \phi_\mu \to \phi_\mu + \omega_\mu^{(4)} = \phi_\mu + \partial_\mu \omega^{(4)}. \tag{62}$$

As we will see, this is not always possible and will depend on how the symmetries are realized in the effective description for the Yang-Mills theory. In the next subsections we will discuss some effective models; to simplify, we will consider the partition functions, obtained by setting the sources s_μ, $s_{\mu\nu}$ equal to zero in Eqs. (41) and (42).

A. Correlated monopoles and center vortices in 3D

Center vortices have been discussed in the $SU(N)$ Georgi-Glashow model in 3D [10]. Classically, this model contains vortices with topological charge $Z(N)$. At the quantum level, the vortex sector can be represented by means of vortex operators associated with the monopole singularities in Euclidean spacetime, where the vortices are created or destroyed. The relevant Green's functions are incorporated by means of an effective Lagrangian for a vortex field,

$$\partial_\mu \bar{V} \partial_\mu V + \mu^2 \bar{V} V + \alpha(\bar{V}V)^2 + \beta(V^N + \bar{V}^N), \qquad (63)$$

which displays a global $Z(N)$ symmetry. When the vortex is an elementary excitation ($\mu^2 > 0$), there is no spontaneous symmetry breaking (SSB). If vortices condense, SSB occurs ($\mu^2 < 0$) and the formation of a domain wall between a heavy quark-antiquark pair leads to an area law for the Wilson loop [10].

Let us discuss the relationship between our representation and the effective model in Eq. (63). In the phase where the vortex is an elementary excitation with mass μ, center vortex world lines can be associated with the propagation of pointlike particles. Because of the coupling $e^{i(2\pi/g)\sum\int dx_\mu \lambda_\mu}$, when representing this ensemble of world lines in terms of an effective complex field $V(x)$, the vector field λ_μ in $S_{v,m}[\lambda_\mu]$ [cf. Equation (54)] should be coupled through the covariant derivative,

$$D_\mu V = [\partial_\mu + i(2\pi/g)\lambda_\mu]V.$$

In order to determine the possible terms in $S_{v,m}[\lambda_\mu]$, let us consider a transformation $\lambda_\mu \to \lambda_\mu + \partial_\mu \omega$, with smooth ω. In this case, the integrand in Eq. (54) would gain a nontrivial factor,

$$e^{i(2\pi/g)\sum\int dx_\mu \partial_\mu \omega} = e^{i(4\pi/g)\sum(\omega(x_i^+) - \omega(x_j^-))}. \qquad (64)$$

Here, we used that center vortices are always attached in pairs to monopoles (antimonopoles) located at x_i^+ (x_j^-). Therefore, when center vortices concatenate monopoles to form closed chains, we see that the presence of the monopoles should lead to an explicit ω-symmetry breaking in $S_{v,m}$. On the other hand, the possible terms in $S_{v,m}$ must be constrained by a symmetry, that in the phase where $\mu^2 > 0$ is expected to be displayed by the vacuum of the theory. When performing the $\omega^{(3)}$ transformation in Eq. (59), the associated factor in Eq. (64) is $e^{i(4\pi/g)\sum(\omega^{(3)}(x_i^+) - \omega^{(3)}(x_j^-))} = e^{\pm i(4\pi/g)(N_+ - N_-)(g/2)} = 1$, where N_+ (N_-) is the number of monopoles (antimonopoles) in ϑ.

Therefore, the natural result for the ensemble integration over chains is of the form

$$S_{v,m} = \overline{D_\mu V} D_\mu V + \mu^2 \bar{V} V + \alpha(\bar{V}V)^2 + \beta(V^2 + \bar{V}^2) + S_0[\tilde{F}_\mu], \qquad (65)$$

where $\tilde{F}_\mu = \epsilon_{\mu\nu\rho}\partial_\nu \lambda_\rho$. This $S_{v,m}$ enjoys the desired properties, as the ω symmetry is explicitly broken by the V^2, $\bar{V}^2$ terms. In addition, it displays a local $Z(2)$ symmetry $V \to e^{-i(2\pi/g)\omega^{(3)}} V$, $\lambda_\mu \to \lambda_\mu + \partial_\mu \omega^{(3)}$. This comes about as $\omega^{(3)}$ is given by $\pm g/2$ inside ϑ, while it is zero outside. Then, this transformation changes the sign of V, $\bar{V}$ inside ϑ, thus leaving the V^2, $\bar{V}^2$ terms invariant. The term S_0 is also invariant; this can be seen from the property $\epsilon_{\mu\nu\rho}\partial_\nu\partial_\rho\omega^{(3)} = 0$, implied from Eqs. (56) and (57). For a discussion of local discrete transformations in 3D gauge theories, when matter fields in the fundamental representation are present, see Refs. [40,41].

The effective contribution in Eq. (65) can also be obtained by direct ensemble integration based on polymer field theory techniques, considering a phase where center vortices are flexible, characterized by a small stiffness, and tensile, weighted by a factor $e^{-\mu L}$ [42].

Then, taking into account the other terms in Eq. (42) and the integral over $[\mathcal{D}\Psi]$, the effective model for the partition function in $SU(2)$ Yang-Mills, including the effect of chains, would be of the form (for a discussion of the $[\mathcal{D}\Psi]$ integration, see Ref. [29], and references therein),

$$S_{\rm eff} = \overline{D_\mu V} D_\mu V + \mu^2 \bar{V} V + \alpha(\bar{V}V)^2 + \beta(V^2 + \bar{V}^2) + S[\tilde{F}_\mu] + \gamma \lambda_\mu \lambda_\mu, \qquad (66)$$

where the term $\lambda_\mu \lambda_\mu$ explicitly breaks the $\omega^{(3)}$ symmetry in $S_{\rm eff}$, preserving a global $Z(2)$. Now, in a phase where this global $Z(2)$ symmetry is spontaneously broken ($\mu^2 < 0$), we have a topological structure, whose existence depends on the consideration of well-behaved continuous fields. In particular, we will have finite action domain walls where $V(x)$ will continuously change from $+V_0$ to $-V_0$, accompanied by a well-behaved continuous λ_μ. As we go across the wall, either the phase of $V(x)$ must change continuously from 0 to π or we can have a π discontinuity at a thin surface S inside the thick wall, as long as $V(x)$ vanishes for points $x \in S$. These kinds of walls have been discussed in Ref. [40].

In other words, when the global $Z(2)$ is spontaneously broken, changes of variables of the form $V \to e^{-i(2\pi/g)\omega^{(3)}} V$ or $\lambda_\mu \to \lambda_\mu + \partial_\mu \omega^{(3)}$ are not acceptable, as the fields produced will no longer correspond to well-behaved continuous fields. On the other hand, in the phase where the global $Z(2)$ symmetry is not spontaneously broken ($\mu^2 > 0$), these requirements are no longer applicable, and the large dual transformations are acceptable.

It is also interesting to note that if $S[\tilde{F}_\mu]$ were dominated by a Maxwell term (see Ref. [29]), λ_μ would be a massive vector field. Then, depending on the generated mass scale, λ_μ would be suppressed and the model in Eq. (63) would be obtained. In addition, because of Eq. (44), the off-diagonal current is given by $\epsilon_{\mu\nu\rho}\partial_\nu\lambda_\rho$ (in this subsection we are considering $s_\mu = 0$) so that this suppression would correspond to Abelian dominance [43,44].

B. Loop-like monopoles in 4D

In 4D, the problem concerning the closure properties of large dual field transformations can easily be understood in the simpler context of ensembles of uncorrelated monopoles and center vortices. In this case, the ensemble integration is of the form $S_{v,m}[\lambda_{\mu\nu}] = S_v[B_{\mu\nu}] + S_m[\phi_\mu]$,

$$e^{-S_v[B_{\mu\nu}]} = \int[\mathcal{D}\mathrm{vor}]e^{i(\pi/g)\sum_v \oint d^2\sigma_{\mu\nu}B_{\mu\nu}}, \qquad (67)$$

$$e^{-S_m[\phi_\mu]} = \int[\mathcal{D}\mathrm{mon}]e^{i(4\pi/g)\sum_{ij}(\oint_{C_j^+} dy_\mu\phi_\mu - \oint_{C_i^-} dy_\mu\phi_\mu)}, \qquad (68)$$

where we have used that unobservable Dirac world sheets can be decoupled in favor of their borders (see Ref. [32]).

As the dual vector field ϕ_μ is minimally coupled with closed stringlike objects, the action $S_{v,m}$ originated from the ensemble integration will be gauge invariant under regular gauge transformations $\phi_\mu \to \phi_\mu + \partial_\mu\omega$ and will contain a complex field Φ representing the monopole sector minimally coupled through the covariant derivative (for a review, see Ref. [45]),

$$[\partial_\mu + i(4\pi/g)\phi_\mu]\Phi.$$

Now, in the corresponding effective action for $SU(2)$ Yang-Mills, the $\lambda_{\mu\nu}\lambda_{\mu\nu}$ term and the $\mathcal{D}\Psi$ integration in Eq. (41) will give additional gauge invariant terms, depending on $\partial_\mu\phi_\nu - \partial_\mu\phi_\nu$.

In a phase where the $U(1)$ gauge symmetry is spontaneously broken, we will again have a topological structure, whose existence depends on the consideration of well-behaved continuous fields. For instance, the phase in $\Phi(x)$ can be ill defined only in places of false vacuum. Therefore, when SSB is present, changes of variables with multivalued phase $\omega^{(4)}$ cannot be accepted, as in general $e^{-i(4\pi/g)\omega^{(4)}}\Phi$ would be ill defined on the closed surface $\partial\vartheta$.

This discussion, together with the minimal coupling with ϕ_μ, leads to the impossibility of considering $\phi_\mu \to \phi_\mu + \partial_\mu\omega^{(4)}$ as an acceptable change of variables in the path integral for a SSB phase. A similar situation occurs with the spacetime independent phase transformations, in the SSB phase, where the boundary condition imposed on Φ at infinity is not closed under them.

In more formal language, according to the Elitzur theorem [46], gauge transformations cannot be spontaneously broken. That is, at the nonperturbative level, in the canonical version of the quantized theory, there is no gauge variant operator with a nonzero expectation value (for a discussion in the context of confinement, see Refs. [8,9]).

What can be spontaneously broken is the subgroup of ‘‘global’’ gauge transformations that remains after a gauge fixing is implemented. An order parameter to explore the possible realizations must be something invariant under gauge transformations and variant under global transformations. This can be constructed for different gauge fixings. In the dual $\hat{\phi}_\mu$ theory it could be considered of the form

$$\hat{O} = e^{i(4\pi/g)\int d^4x'\partial_\mu\hat{\phi}_\mu(x')D(x'-x)}\hat{\Phi}(x), \qquad (69)$$

where $D(x)$ is the Green function for the Laplacian operator. This order parameter is invariant under local regular phase transformations $\hat{\phi}_\mu \to \hat{\phi}_\mu + \partial_\mu\alpha(x)$, $\hat{\Phi} \to e^{-i(4\pi/g)\alpha(x)}\hat{\Phi}$, while under spacetime independent ones it transforms as $\hat{O} \to e^{i\alpha}\hat{O}$.

We also note that $\omega^{(4)}$ satisfies $\partial_\mu\partial_\mu\omega^{(4)} = 0$ (see Sec. VII A), so that the order parameter in Eq. (69) also transforms under the operation $\hat{\phi}_\mu \to \hat{\phi}_\mu + \partial_\mu\omega^{(4)}$, $\hat{\Phi} \to e^{-i(4\pi/g)\omega^{(4)}}\hat{\Phi}$, according to $\hat{O} \to e^{-i(4\pi/g)\omega^{(4)}}\hat{O}$.

Then, when the spacetime independent phase transformations are spontaneously broken, the large dual transformations are also spontaneously broken; that is, the vacuum is not invariant under them.

C. Correlated monopoles and center vortices in 4D

For chains of monopoles and antimonopoles, we have [cf. Equations (50) and (52)]

$$e^{-S_{v,m}[B_{\mu\nu},\phi_\mu]} = \int[\mathcal{D}\mathrm{vor}][\mathcal{D}\mathrm{mon}]e^{i(\pi/g)\sum_v\int d^2\sigma_{\mu\nu}B_{\mu\nu} + i(4\pi/g)\sum_{ij}(\oint_{C_j^+} dy_\mu\phi_\mu - \oint_{C_i^-} dy_\mu\phi_\mu)}, \qquad (70)$$

where $B_{\mu\nu}$ is integrated over open vortex world sheets with their borders attached in pairs, so as to form the associated monopole or antimonopole loops at C_j^+, C_i^-.

For each vortex world sheet, we have a contribution in the integrand of the form

$$V(C^+)V(C^-)e^{i(\pi/g)\int_\Sigma d^2\sigma_{\mu\nu}B_{\mu\nu}},$$
$$V(C^\pm) = e^{\pm i(2\pi/g)\oint_{C^\pm} dy_\mu\phi_\mu}, \qquad (71)$$

where $\Sigma = \Sigma(C^+, C^-)$ is a surface with borders at C^+ and C^-.

This configuration represents the creation, propagation, and annihilation of a loop, minimally coupled to $B_{\mu\nu}$, so that $V(C)$ can be compared to the disorder operator introduced in Ref. [10] for the Yang-Mills theory.

If center vortex world sheets were closed objects propagating stringlike excitations, characterized by a finite tension, $S_{v,m}[\lambda_{\mu\nu}]$ in Eq. (55) would be invariant under the transformations $\lambda_{\mu\nu} \to \lambda_{\mu\nu} + \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$, including the large ones, $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$, so that a typical effective action for this sector would be of the form [47–50],

$$S_{\text{c.v.}} = S_{\text{c.v.}}[\tilde{H}_\mu], \qquad \tilde{H}_\mu = \epsilon_{\mu\nu\rho\sigma}\partial_\nu \lambda_{\rho\sigma} \tag{72}$$

[note that due to Eqs. (56) and (57), $\epsilon_{\mu\nu\rho\sigma}\partial_\nu\partial_\rho \times \partial_\sigma \omega^{(4)} = 0$]. When these center vortex world sheets concatenate monopoles, we can see from Eq. (70) that the presence of the latter explicitly breaks the ω_μ symmetry in $S_{v,m}[\lambda_{\mu\nu}]$. However, this contribution will be symmetric under the regular $\phi_\mu \to \phi_\mu + \partial_\mu \omega$ transformations and, as in Eq. (70) the loop variables appear in the form $V^2(C^\pm)$, it is expected to be symmetric under the large ones, $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$. Then, $S_{v,m}$ can be written as $S^{(4)} + S_0[\tilde{H}_\mu]$, where $S^{(4)}$ is only symmetric under $\omega^{(4)}$ transformations, breaking the ω_μ symmetry. This part would be analogous to the V-dependent terms in Eq. (64); however, the problem of presenting effective models for $S^{(4)}$ is highly nontrivial, as in $4D$ the vortex field $V(x)$ is replaced by a loop variable $V(C)$.

Taking into account the other terms in the representation and the $[\mathcal{D}\Psi]$ integration [29], in this case, the effective action for Yang-Mills is expected to be of the form

$$S_{\text{eff}} = S^{(4)} + S[\tilde{H}_\mu] + \gamma \lambda_{\mu\nu}\lambda_{\mu\nu}, \tag{73}$$

where the $\lambda_{\mu\nu}\lambda_{\mu\nu}$ term explicitly breaks the $\omega^{(4)}$ symmetry present in the first two terms. Again, we could expect a phase for the ensemble of chains where the associated regularity requirements imposed on $\lambda_{\mu\nu}$ could disallow the changes of variables $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$, as occurs in $3D$ with the $\mu^2 < 0$ phase and the changes of variables $\phi \to \phi + \omega^{(3)}$ (see Sec. VI A).

VII. WILSON SURFACE DECOUPLING VS WILSON SURFACE VARIABLES

The discussion about how a surface whose border is the Wilson loop can become observable in Yang-Mills theory is a key point in understanding the possible mechanisms underlying confinement and its associated properties.

In Ref. [10], the possible observability of Wilson surfaces or center vortex world sheets has been analyzed as follows. The algebra between the Wilson loop operator $\hat{W}(\mathcal{C}, t)$ and the disorder operator $\hat{V}(C', t)$ is

$$\hat{W}(\mathcal{C}, t)\hat{V}(C', t) = \hat{V}(C', t)\hat{W}(\mathcal{C}, t)(-1)^{\text{link}}, \tag{74}$$

where C and C' are defined at a given time t, and the right-hand side contains the linking number between them. Then, a family $C'(a)$ in R^4, $a \in [0, 1]$ is considered, continuously changing from C'_0, passing by an intermediate C'_i, and then returning to C'_0, both curves living on the constant time t hyperplane where C is contained. As we are in R^4, this family can be chosen with C'_0 (C'_i) unlinked (linked) with C, and such that $C'(a)$ never comes close to C. In these conditions, a declustering property was used,

$$\langle W(\mathcal{C})V(C'_a)\rangle \approx \langle W(\mathcal{C})\rangle\langle V(C'_a)\rangle e^{i\alpha(\mathcal{C},C'_a)}, \tag{75}$$

where the phase is required in order to be consistent with Eq. (74), which implies that $e^{i\alpha(\mathcal{C},C'_a)}$ must change from $+1$ to -1 and then back to $+1$ in this process. If massless modes exist in Yang-Mills theory, $\alpha(\mathcal{C}, C'_a)$ could be a smoothly varying function. On the other hand, when $Z(2)$-invariant Higgs fields are switched on, it has been argued that a sudden change in the phase must exist, and as the pairs of curves are always maintained far apart, an observable surface must be attached to the Wilson loop or to the half-charge magnetic loop.

In Ref. [37], the Wilson loop average $\bar{W}$ has been analyzed in confining models such as compact $QED(3)$ and $QED(4)$, the latter regularized on the lattice. In that reference, considering the dual field ϕ defined on the interval $[-\infty, +\infty]$, a representation based on axion fields, with multivalued action, has been obtained, and a series of approximations led to an explicit dependence of the resulting $\bar{W}$ on the arbitrary $S(\mathcal{C})$ appearing in its definition. Then it has been conjectured that this problem would be resolved if all the branches of the multivalued action were considered in the calculation, and that this would be equivalent to considering the integration over all Wilson surfaces (that now become dynamical) and dual $\tilde{\phi}$'s with an appropriate jump at the associated surface.

Because of the Wess-Zumino term in the PD representation, our expressions in Eqs. (41) and (42) for $\bar{W}$ in Yang-Mills theory also have an arbitrary surface $S(\mathcal{C})$ attached to the Wilson loop from the beginning. However, the representation must be independent of $S(\mathcal{C})$. In the next subsections, we will discuss how to obtain, in general, a Wilson loop representation with no reference to the initially considered $S(\mathcal{C})$.

The answer will depend on the underlying realization of symmetries in the effective models describing the Yang-Mills theory, which according to the discussion in Sec. VI will determine whether changes of variables $\phi \to \phi + \omega^{(3)}$, $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$ are acceptable or not.

In $3D$, we have seen that in the phase without global $Z(2)$ SSB, the change $\phi \to \phi + \omega^{(3)}$ is acceptable; this corresponds to single-valued possibly discontinuous ϕ's defined on the interval $[-\infty, +\infty]$. In this case, we will be able to decouple the initial Wilson surface following treatment I. On the other hand, in the SSB phase, the change is not acceptable, and the ϕ's will have to be considered as continuous multivalued angles. Here, the reference to the arbitrary initial $S(\mathcal{C})$ will also disappear, but giving place to an integral over all the Wilson surfaces and the above mentioned $\tilde{\phi}$'s. These two possibilities for the class of ϕ's and their consequences will also be extended to classes of ϕ_μ's in $4D$ theories in the continuum.

A. Dealing with Wilson surfaces I

Let us consider ϕ, ϕ_μ as single-valued fields, so that the large dual transformations, adding the single-valued pieces $\omega^{(3)}$, $\partial_\mu \omega^{(4)}$, can be performed. Of course, in this case, the Wilson surface should be an unobservable object, but the question is, how can we use the large dual transformations in order to decouple $S(\mathcal{C})$ in favor of $\mathcal{C}$, thus evidencing the unobservability of $S(\mathcal{C})$?

For this aim, let us follow a procedure similar to the one we implemented in Ref. [32], where we discussed how to decouple unobservable Dirac defects in favor of their borders, in the CFN representation of the Yang-Mills partition function.

Considering the auxiliary fields ζ_μ, $\zeta_{\mu\nu}$, and a change of variables $\lambda_\mu + \frac{g}{2}s_\mu \to \lambda_\mu$, $\lambda_{\mu\nu} + g s_{\mu\nu} \to \lambda_{\mu\nu}$, we have

$$\bar{W}(\mathcal{C}) = \int [\mathcal{D}\zeta][\mathcal{D}\lambda][\mathcal{D}\Psi] e^{-S_c - \int d^3x (1/2)\zeta_\mu \zeta_\mu} \times e^{i\int d^3x \{(\lambda_\mu - (g/2)s_\mu)(\zeta_\mu + k_\mu) + (\epsilon_{\mu\nu\rho}\partial_\nu \lambda_\rho - J^c_\mu)A^{(n)}_\mu + \lambda_\mu d^{(n)}_\mu\}}, \tag{76}$$

$$\bar{W}(\mathcal{C}) = \int [\mathcal{D}\zeta][\mathcal{D}\lambda][\mathcal{D}\Psi] e^{-S_c - \int d^4x (1/4)\zeta_{\mu\nu}\zeta_{\mu\nu}} \times e^{i\int d^4x \{(1/2)(\lambda_{\mu\nu} - g s_{\mu\nu})(\zeta_{\mu\nu} + k_{\mu\nu}) + ((1/2)\epsilon_{\mu\nu\rho\sigma}\partial_\nu \lambda_{\rho\sigma} - J^c_\mu)A^{(n)}_\mu + (1/2)\lambda_{\mu\nu} d^{(n)}_{\mu\nu}\}}. \tag{77}$$

The path integrals in $[\mathcal{D}\lambda]$ can be done over the fields defined in Eq. (60), with ϕ, ϕ_μ single-valued. Including the conditions in Eq. (61), in $3D$ we must consider the replacement,

$$[\mathcal{D}\lambda] \to [\mathcal{D}B][\mathcal{D}\phi][\mathcal{D}\xi] e^{i\int d^4x \xi \partial_\mu B_\mu}, \tag{78}$$

while in $4D$, we have

$$[\mathcal{D}\lambda] \to [\mathcal{D}B][\mathcal{D}\phi][\mathcal{D}\xi] \times [\mathcal{D}\gamma] e^{i\int d^4x \xi_\mu \partial_\nu B_{\mu\nu}} e^{i\int d^4x \gamma \partial_\mu \phi_\mu}. \tag{79}$$

Therefore, using Eq. (57) and considering in Eqs. (76) and (77) the large dual transformations, with trivial Jacobian,

$$\phi \to \phi - \omega^{(3)}, \qquad \phi_\mu \to \phi_\mu - \partial_\mu \omega^{(4)}, \tag{80}$$

the terms in Eqs. (76) and (77), containing, respectively, $d^{(n)}_\mu$, $d^{(n)}_{\mu\nu}$, gain a phase which is a trivial multiple of 2π; the second term is invariant, while the first term gives a change in the surface. In the $4D$ case, it is important to emphasize that the explicit form for $\partial_\mu \omega^{(4)}$ is

$$\partial_\mu \omega^{(4)} = \pm \frac{g}{2} \int_\vartheta d^3\tilde{\sigma}_\nu (\delta_{\mu\nu}\partial^2 - \partial_\mu \partial_\nu) D(x - \bar{x}(\sigma)), \tag{81}$$

$$d^3\tilde{\sigma}_\mu = \frac{1}{2}\epsilon_{\mu\alpha\beta\gamma}\epsilon_{ijk} \frac{\partial \bar{x}_\alpha}{\partial \sigma_i} \frac{\partial \bar{x}_\beta}{\partial \sigma_j} \frac{\partial \bar{x}_\gamma}{\partial \sigma_k} d\sigma_1 d\sigma_2 d\sigma_3. \tag{82}$$

Using Stokes's theorem, this can be written only in terms of $\partial\vartheta$, the manifold where the added closed Wilson surface is placed (for a discussion in the context of thin center vortices and Dirac world sheets, see Refs. [32,38,39]). Therefore, the index structure in Eq. (81) implies $\partial_\mu \partial_\mu \omega^{(4)} = 0$, and $\partial_\mu \phi_\mu$ in the measure given in Eq. (79) is invariant under the change of variables in Eq. (80).

Summarizing, in $3D$ and $4D$ we can deform the Wilson surface by means of a change of variables, with trivial Jacobian, keeping its border $\mathcal{C}$ fixed.

Now let us consider a Hodge decomposition,

$$\begin{aligned} \zeta_\mu + k_\mu &= \partial_\mu \psi + C_\mu, \\ \zeta_{\mu\nu} + k_{\mu\nu} &= \partial_\mu \psi_\nu - \partial_\nu \psi_\mu + C_{\mu\nu}, \end{aligned} \tag{83}$$

with

$$\partial_\mu C_\mu = 0, \qquad \partial_\nu C_{\mu\nu} = 0, \qquad \partial_\mu \psi_\mu = 0, \tag{84}$$

that permits the identification of C_μ, $C_{\mu\nu}$ as fields only coupled to the Wilson loop $\mathcal{C}$, while the fields ψ, ψ_μ are the ones coupled with the whole surface $S(\mathcal{C})$.

We will show that the Wilson surface can be decoupled by means of an appropriate change of variables, leaving only the effect of its border. For this purpose we leave the integration over ζ_μ, $\zeta_{\mu\nu}$ and the charged fields present in k_μ, $k_{\mu\nu}$ until the end, and analyze the integral over λ_μ, $\lambda_{\mu\nu}$ first. Let us consider the term coupling ψ, ψ_μ,

$$J_{S(\mathcal{C})} = \begin{cases} \int d^3x s_\mu \partial_\mu \psi \\ \int d^4x s_{\mu\nu}(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu). \end{cases} \tag{85}$$

For the initial Wilson surface, and sources s_μ, $s_{\mu\nu}$, we can assume $J_{S(\mathcal{C})} > 0$ without loss of generality. In addition, we can assume that a closed surface $\partial\vartheta$ exists, such that

$$J_{[\partial\vartheta]} = \begin{cases} \int d^3x \Delta s_\mu \partial_\mu \psi \\ \int d^4x \Delta s_{\mu\nu}(\partial_\mu \psi_\nu - \partial_\nu \psi_\mu) \end{cases} \tag{86}$$

is nonzero. In this regard, it suffices to consider a small ϑ, as in this case $J_{[\partial\vartheta]}$ is given by the local value of $\partial^2\psi$, $\partial^2\psi_\mu$. If this value were zero for any $\partial\vartheta$, we would have

$\psi \equiv 0$, $\psi_\mu \equiv 0$, and the term coupling the surface would be automatically zero.

Now let us include m times the closed surface $\partial\vartheta$ and define the sources s'_μ, $s'_{\mu\nu}$, concentrated on the surface $S'(\mathcal{S}) = S(\mathcal{C}) \circ [\partial\vartheta]^m$. This amounts to the transformation,

$$\phi \to \phi - m\omega^{(3)}, \qquad \phi_\mu \to \phi_\mu - \partial_\mu(m\omega^{(4)}). \tag{87}$$

Then, we have

$$J_{S'(\mathcal{C})} = J_{S(\mathcal{C})} + mJ_{[\partial\vartheta]}. \tag{88}$$

Now, we can take $\partial\vartheta$ oriented such that

$$J_{[\partial\vartheta]} < 0, \tag{89}$$

so that $J_{S'(\mathcal{C})}$ can be rendered negative for a large enough value of m. As $S'(\mathcal{C})$ can be continuously deformed into $S(\mathcal{C})$, by shrinking $\partial\vartheta$ to zero, an intermediate surface $S_0(\mathcal{C})$ must exist in this process such that $J_{S_0(\mathcal{C})} = 0$ is verified. This suggests that it is always possible to make a large dual transformation that changes the initial $S(\mathcal{C})$ into $S_0(\mathcal{C})$, thus nullifying the terms coupling the Wilson surface with ψ, ψ_μ. Then, in practice, the prescription in this case for obtaining a representation for $\bar{W}$ with no reference to the initial $S(C)$ is simply to disregard the above mentioned terms in Eqs. (76) and (77).

B. Dealing with Wilson surfaces II

Now the question is what to do in the case where the ensemble of defects requires regular fields ϕ, ϕ_μ in the Hodge decomposition (60), so that large dual changes of variables are no longer acceptable.

In order to answer this question, let us first consider the $3D$ case, denoting the fields in the decomposition for λ_μ, with the properties used in the previous subsection, as $\phi^{\rm I}$ and $B^{\rm I}_\mu$. That is, $\phi^{\rm I}$ is single valued and defined on the interval $[-\infty, +\infty]$. Now, considering a smooth λ_μ, adding and substracting a source $s_\mu(\tilde{\Sigma})$ concentrated on a general Wilson surface $\tilde{\Sigma}$ whose border is $\mathcal{C}$, we can also write a decomposition using fields $\phi^{\rm II}$ and $B^{\rm II}_\mu$, with $\phi^{\rm II}$ being a multivalued field, when we go around the Wilson loop $\mathcal{C}$. That is,

$$\lambda_\mu = \partial_\mu\phi^{\rm I} + B^{\rm I}_\mu = \partial_\mu\phi^{\rm II} + B^{\rm II}_\mu, \tag{90}$$

$$\partial_\mu\phi^{\rm II} = \partial_\mu\tilde{\phi} - \frac{g}{2}s_\mu(\tilde{\Sigma}), \qquad \tilde{\phi} = \phi^{\rm I} + \frac{g}{2}\partial^{-2}(\partial\cdot s(\tilde{\Sigma})), \tag{91}$$

$$B^{\rm II}_\mu = B^{\rm I}_\mu - \frac{g}{2}\epsilon_{\mu\nu\rho}\partial_\nu\partial^{-2}j_\rho(\mathcal{C}). \tag{92}$$

Note that, when computing $\partial_\mu\phi^{\rm II}$, the derivative of the discontinuity in the second term of $\tilde{\phi}$ is canceled by the $-\frac{g}{2}s_\mu(\tilde{\Sigma})$ term, so that the defined $\phi^{\rm II}$ is a continuously changing multivalued field, satisfying $\epsilon_{\mu\nu\rho}\partial_\nu\partial_\rho\phi^{\rm II} = -\frac{g}{2}j_\mu(\mathcal{C})$.

On the other hand, the class of fields λ_μ generated by the single-valued, possibly discontinuous, $\phi^{\rm I}$'s is different from the class of fields λ_μ generated by the continuous multivalued $\phi^{\rm II}$'s. In the first case, there is no problem in summing $\phi^{\rm I}$ and $\omega^{(3)}$ to obtain another possible configuration; in the second case, summing the multivalued $\phi^{\rm II}$ and $\omega^{(3)}$ does not make any sense.

In a similar way, in $4D$, we will have type I and type II dual fields ϕ_μ, the former are the single-valued fields used in the previous subsection, the latter being appropriate for describing situations where $\omega^{(4)}$ changes of variables are not acceptable.

Then in this section, we will introduce a decomposition in terms of type II fields in $3D$ and $4D$, enjoying the properties

$$\epsilon_{\mu\nu\rho}\partial_\nu\partial_\rho\phi = -\frac{g}{2}j_\mu(\mathcal{C}),$$
$$\epsilon_{\mu\nu\rho\sigma}\partial_\nu\partial_\rho\phi_\sigma = -\frac{g}{2}j_\mu(\mathcal{C}). \tag{93}$$

In three dimensions, the integral of $\epsilon_{\mu\nu\rho}\partial_\nu\partial_\rho\phi$ over an open surface with border $\mathcal{P}$, crossed by the Wilson loop $\mathcal{C}$, gives $\pm g/2$. Then, using Stokes's theorem, the integral of $\partial_\mu\phi$ along $\mathcal{P}$ gives $\Delta\phi = \pm g/2$, while this change is zero on a path that does not link $\mathcal{C}$. We have already seen that the multivalued ϕ can be written in terms of $\tilde{\phi}(x)$, discontinuous at some surface $\tilde{\Sigma}$ whose border is the Wilson loop $\mathcal{C}$, such that $\partial_\mu\phi = \partial_\mu\tilde{\phi} - \frac{g}{2}s_\mu(\tilde{\Sigma})$.

In four dimensions, ϕ_μ must be considered as a vector field that cannot be globally defined on the closed surfaces $\mathcal{S}$ linked by the Wilson loop. It can be differently defined on two hemispheres meeting on a closed path $\mathcal{P}$, where the difference between ϕ_μ continued from each one of the hemispheres is $\partial_\mu\alpha$, with α multivalued. This can be visualized by considering, for example, the Wilson loop contained in the $x^0 = 0$ hyperplane (a three-volume). If we stay on this hyperplane, the loop $\mathcal{C}$ is seen to be linked by path $\mathcal{P}$. If we continuously move to other hyperplanes with $x^0 \neq 0$, the Wilson loop will no longer be seen, while the former path $\mathcal{P}$ will be seen to continuously shrink to a point, mapping both hemispheres in four dimensions, for positive or negative x_0, forming the closed surface linked by $\mathcal{C}$.

Precisely because of Eq. (93), the integral of $\epsilon_{\mu\nu\rho\sigma}\partial_\nu\partial_\mu\phi_\nu$, over an open three-volume with border $\mathcal{S}$, gives $\pm g/2$ and can be equated via Gauss's theorem with the integral of $\epsilon_{\mu\nu\rho\sigma}\partial_\rho\phi_\sigma$ over the closed surface $\mathcal{S}$ linked by $\mathcal{C}$. This surface integral can be done on the two hemispheres A and B, sharing the same border $\mathcal{P}$, where ϕ_μ takes the values ϕ^A_μ and ϕ^B_μ, respectively. Now, we can use Stokes's theorem to write the surface integral as the line integral of $\phi^A_\mu - \phi^B_\mu = \partial_\mu\alpha$ over the closed path $\mathcal{P}$, thus obtaining $\Delta\alpha = \pm g/2$.

Then, in Eq. (60), the multivalued field ϕ_μ can be replaced by $\tilde{\phi}_\mu(x)$, defined on the whole Euclidean spacetime as a function of point x and discontinuous at some surface $\tilde{\Sigma}$, whose border is the Wilson loop $\mathcal{C}$. Again, the derivatives of ϕ_μ cannot contain any singular term on $\tilde{\Sigma}$, so that the replacement must by done as follows:

$$\partial_\mu \phi_\nu - \partial_\nu \phi_\mu = \partial_\mu \tilde{\phi}_\nu - \partial_\nu \tilde{\phi}_\mu - g s_{\mu\nu}(\tilde{\Sigma}), \tag{94}$$

where the second term is concentrated on $\tilde{\Sigma}$ and compensates the δ distribution on $\tilde{\Sigma}$ that originated when taking the derivatives of the discontinuous vector field $\tilde{\phi}_\mu(x)$.

Because of the multivalued character of the fields, the factors containing the defects in Eqs. (41) and (42) become

$$e^{i\int d^3x(\lambda_\mu + (g/2)s_\mu)d_\mu^{(n)}} = e^{i\int d^3x(\partial_\mu \tilde{\phi} + B_\mu)d_\mu^{(n)}}, \tag{95}$$

$$e^{i\int d^4x(1/2)(\lambda_{\mu\nu} + g s_{\mu\nu})d_{\mu\nu}^{(n)}} = e^{i\int d^4x(1/2)(\partial_\mu \tilde{\phi}_\nu - \partial_\nu \tilde{\phi}_\mu + B_{\mu\nu})d_{\mu\nu}^{(n)}}, \tag{96}$$

where we used

$$\frac{g}{2}\int d^3x(s_\mu - s_\mu(\tilde{\Sigma}))d_\mu^{(n)} = 2n\pi, \tag{97}$$

$$\frac{g}{2}\int d^4x(s_{\mu\nu} - s_{\mu\nu}(\tilde{\Sigma}))d_{\mu\nu}^{(n)} = 2n\pi. \tag{98}$$

In addition, the implicit constraints in Eqs. (40) and (45) become

$$J_\mu^c = \epsilon_{\mu\nu\rho}\partial_\nu\left(B_\rho + \frac{g}{2}[s_\rho - s_\rho(\tilde{\Sigma})]\right) = \epsilon_{\mu\nu\rho}\partial_\nu B_\rho, \tag{99}$$

$$\begin{aligned} J_c^\mu &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu(B_{\rho\sigma} + g[s_{\rho\sigma} - s_{\rho\sigma}(\tilde{\Sigma})]) \\ &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma}, \end{aligned} \tag{100}$$

where we used that the sources s_μ, $s_{\mu\nu}$ are concentrated on $S(\mathcal{C})$, sharing the same border $\mathcal{C}$ with $\tilde{\Sigma}$.

Therefore, using the above results when considering multivalued dual fields ϕ, ϕ_μ, we can represent the Wilson loop in Eqs. (41) and (42) according to

$$\begin{aligned} \bar{W}(\mathcal{C}) = &\int [\mathcal{D}\tilde{\Sigma}][\mathcal{D}\mathcal{F}(\tilde{\Sigma})]e^{-S_c - \int d^3x(1/2)(\partial_\mu\tilde{\phi} - (g/2)s_\mu(\tilde{\Sigma}) + B_\mu)^2} \\ &\times e^{i\int d^3x\{(\epsilon_{\mu\nu\rho}\partial_\nu B_\rho - J_\mu^c)A_\mu^{(n)} + \lambda_\mu k_\mu + (\partial_\mu\tilde{\phi} + B_\mu)d_\mu^{(n)}\}}, \end{aligned} \tag{101}$$

$$\begin{aligned} \bar{W}(\mathcal{C}) = &\int [\mathcal{D}\tilde{\Sigma}][\mathcal{D}\mathcal{F}(\tilde{\Sigma})]e^{-S_c - \int d^4x(1/4)(\partial_\mu\tilde{\phi}_\nu - \partial_\nu\tilde{\phi}_\mu - g s_{\mu\nu}(\tilde{\Sigma}) + B_{\mu\nu})^2} \\ &\times e^{i\int d^4x\{((1/2)\epsilon_{\mu\nu\rho\sigma}\partial_\nu B_{\rho\sigma} - J_\mu^c)A_\mu^{(n)} + (1/2)\lambda_{\mu\nu}k_{\mu\nu} + (1/2)(\partial_\mu\tilde{\phi}_\nu - \partial_\nu\tilde{\phi}_\mu + B_{\mu\nu})d_{\mu\nu}^{(n)}\}}, \end{aligned} \tag{102}$$

$[\mathcal{D}\mathcal{F}(\tilde{\Sigma})] = [\mathcal{D}B][\mathcal{D}\tilde{\phi}][\mathcal{D}\Psi]F_{gf}^B F_{gf}^{\tilde{\phi}}$, where F_{gf}^B is the part of the measure fixing the condition for B_μ, $B_{\mu\nu}$, and in four dimensions $F_{gf}^{\tilde{\phi}}$ is the part fixing the condition for $\tilde{\phi}_\mu$.

In this manner, $\bar{W}(\mathcal{C})$ no longer refers to the particular surface $S(\mathcal{C})$, initially introduced in the PD representation. In turn, the path integral over multivalued fields is equivalent to the integral over all the surfaces $\tilde{\Sigma}$ with border $\mathcal{C}$, together with the path integral over the fields $\tilde{\phi}$, $\tilde{\phi}_\mu$, with a given jump at $\tilde{\Sigma}$.

VIII. CONCLUSIONS

In this work we have presented a natural framework for discussing possible ideas underlying confinement and ensembles of defects in $3D$ and $4D$ $SU(2)$ Yang-Mills theory in the continuum.

Initially, we have considered a representation for the Wilson loop average $\bar{W}$, based on the Petrov-Diakonov representation of the non-Abelian Wilson loop W, combined with the Cho-Faddeev-Niemi decomposition of $SU(2)$ gauge fields, which permits one to write the average $\bar{W}$ as a path integral over $SU(2)$ mappings. These mappings induce local frames $\hat{n}_a$ in color space, whose defects represent not only the monopole sector, but also a $Z(2)$ center vortex sector.

The interesting point is that the integrand of $\bar{W}$ contains an arbitrary surface $S(\mathcal{C})$, whose border is the Wilson loop, originated from the Wess-Zumino term in the Petrov-Diakonov representation. On the other hand, the usual representation for $W(\mathcal{C})$ only refers to $\mathcal{C}$. Then, the problem is how the representation for $\bar{W}$ can be worked out so as to implement the independence on the initial choice for $S(\mathcal{C})$.

In other words, when defects proliferate, the natural question that arises is how and under what conditions the surface $S(\mathcal{C})$ can be decoupled, in favor of its border, or it becomes a Wilson surface variable.

On the other hand, the discussion about how a surface can become observable is a key point to understanding the possible mechanisms underlying confinement and its associated properties.

In Ref. [10], this has been analyzed by means of the peculiar declustering properties of correlators involving the Wilson loop operator $\hat{W}(\mathcal{C})$ and the disorder operator $\hat{V}(\mathcal{C}')$.

In Ref. [37], the Wilson loop average $\bar{W}$ has been considered in the context of compact $QED(3)$ and compact $QED(4)$ regularized on the lattice. There, considering in

$3D$ the dual field ϕ defined on the interval $[-\infty, +\infty]$, a representation based on axion fields, with multivalued action, has been obtained, and a series of approximations led to an explicit dependence of the resulting $\bar{W}$ on the arbitrary $S(\mathcal{C})$ appearing in its definition. Then, it has been conjectured that this problem would be resolved if all the branches of the multivalued action were considered in the calculation, and that this would be equivalent to considering an integration over all Wilson surfaces, and dual fields $\tilde{\phi}$ with a given jump at the corresponding surface.

In this article we have discussed this kind of problem in terms of the regularity properties imposed on the dual fields by the different ensembles of defects, and the associated closure properties under large dual transformations.

Our representation for $\bar{W}$ contains an integral over the ensemble of defects, a path integral over the diagonal and off-diagonal gluon fields, including a gauge fixing, and one over dual fields $\lambda_\mu = \partial_\mu \phi + B_\mu$, $\lambda_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu + B_{\mu\nu}$, minimally coupled to the center vortex world lines or world sheets, in three and four dimensions, respectively.

In terms of the effective action $S_{v,m}$ originated from the ensemble integration, the effective model for the Yang-Mills partition function is of the form

$$
\begin{aligned}
S_{\rm eff} &= S_{v,m}[\lambda_\mu] + S[\tilde{F}_\mu] + \gamma \lambda_\mu \lambda_\mu, \\
\tilde{F}_\mu &= \epsilon_{\mu\nu\rho} \partial_\nu \lambda_\rho, \\
S_{\rm eff} &= S_{v,m}[\lambda_{\mu\nu}] + S[\tilde{H}_\mu] + \gamma \lambda_{\mu\nu} \lambda_{\mu\nu}, \\
\tilde{H}_\mu &= \epsilon_{\mu\nu\rho\sigma} \partial_\nu \lambda_{\rho\sigma},
\end{aligned}
$$

in $3D$ and $4D$, respectively.

For example, in $3D$, we have argued that for chains of monopoles attached in pairs to center vortices, $S_{v,m}$ is naturally associated with a vortex field $V(x)$, minimally coupled with λ_μ, displaying a local $Z(2)$ symmetry. This symmetry is also present in the second term $S[\tilde{F}_\mu]$. However, because of the last term $\lambda_\mu \lambda_\mu$, the $Z(2)$ symmetry in $S_{\rm eff}$ is only global, and the effective model is expected to be a generalization of the well-known vortex model of Ref. [10].

Moreover, in a phase where the global $Z(2)$ is spontaneously broken, the effective theory contains domain walls, a topological structure whose existence depends on the consideration of well-behaved continuous fields $V(x)$, λ_μ. Then, the change of variables associated with the local $Z(2)$ transformations $\phi \to \phi + \omega^{(3)}$, adding to λ_μ a source localized on a closed Wilson surface, cannot even be accepted in this case, as these transformations are not closed. On the contrary, if there is no SSB, the regularity requirement is no longer valid, and this change of variables becomes acceptable.

Similarly for monopole chains in $4D$, the $\lambda_{\mu\nu}\lambda_{\mu\nu}$ term in $S_{\rm eff}$ would be the noninvariant part under large dual transformations $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$, adding a source localized on a closed Wilson surface $\partial\vartheta$. Here the discussion about a possible topological structure for the effective theory is highly nontrivial, as the vortex field $V(x)$ in $3D$ is replaced by a loop variable $V(C)$. Nevertheless, we can assume that different phases could exist, where the associated regularity requirements on $\lambda_{\mu\nu}$ could lead us to consider the changes of variables $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$ as acceptable or not.

For example, this discussion already occurs in $4D$ when looking at the monopole part of $S_{v,m}$, in the simpler situation where monopoles are uncorrelated with center vortices. As is well known, this part is typically represented by a complex field $\Phi(x)$, minimally coupled with ϕ_μ. In a phase where the dual $U(1)$ is spontaneously broken, the model has a topological structure, whose existence depends on the consideration of well-behaved continuous fields $\Phi(x)$, $\lambda_{\mu\nu}$, thus precluding the $\phi_\mu \to \phi_\mu + \partial_\mu \omega^{(4)}$ transformations.

In canonical language, this corresponds to the fact that an order parameter must be invariant under regular gauge transformations. For the condition $\partial_\mu \phi_\mu = 0$, such an order parameter turns out to be variant not only under spacetime independent phase transformations, but also under multivalued $\omega^{(4)}$ transformations. Then, if the global $U(1)$ is spontaneously broken, the large dual transformations will also display SSB.

For these reasons, in the last part of this work, we were led to analyze the representation for $\bar{W}$ in two possible scenarios, before considering an effective model for the ensemble integration.

In the representation of $\bar{W}$ in $3D$, we have discussed two alternatives for the class of fields λ_μ. They are generated by $\phi^{\rm I}$, general single-valued fields defined on the interval $[-\infty, +\infty]$, or by the fields $\phi^{\rm II}$, multivalued when we go around the Wilson loop $\mathcal{C}$. While in the former case changes of variables $\phi^{\rm I} \to \phi^{\rm I} + \omega^{(3)}$ are acceptable, in the latter, the addition of $\phi^{\rm II}$ with $\omega^{(3)}$ is meaningless.

These alternatives have been generalized to $4D$, where the class of fields $\lambda_{\mu\nu}$ can be generated by two types of fields. The first type is closed under the transformation $\phi^I_\mu \to \phi^I_\mu + \partial_\mu \omega^{(4)}$, with $\omega^{(4)}$ a multivalued phase when we go around $\partial\vartheta$. For the second type, this transformation does not make any sense, as the fields cannot be globally defined on the closed surfaces linked by the Wilson loop $\mathcal{C}$.

In general, if in $3D$ or $4D$ the required fields are type I, we have shown that it is possible to perform changes of variables in the representation for $\bar{W}$ so as to decouple the Wilson surface $S(\mathcal{C})$.

In the second case, the integral over type II multivalued fields were replaced by an integral over all possible surfaces $\tilde{\Sigma}$ whose border is $\mathcal{C}$, and dual fields $\tilde{\phi}$, $\tilde{\phi}_\mu$, functions of point x on the Euclidean spacetime, with an appropriate jump at $\tilde{\Sigma}$. In this manner, any reference to the initial arbitrary $S(\mathcal{C})$ also disappeared, but in a different way; the initial surface has become a Wilson surface variable.

Summarizing, for $SU(2)$ Yang-Mills theories, we introduced a framework to discuss the coupling between gauge fields containing defects, surfaces attached to the Wilson loop, and dual fields. We have discussed some effective models, the implied regularity requirements, and the associated inequivalent manners to represent the Wilson loop without reference to the initial Wilson surface considered. This general framework could prove useful as a starting point to understand the promising scenario associated with correlated monopoles and center vortices in continuum 4D Yang-Mills theories.

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