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Distinguishing *k*-defects from their canonical twins

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We study *k*-defects—topological defects in theories with more than two derivatives and second-order equations of motion—and describe some striking ways in which these defects both resemble and differ from their analogues in canonical scalar field theories. We show that, for some models, the homotopy structure of the vacuum manifold is insufficient to establish the existence of *k*-defects, in contrast to the canonical case. These results also constrain certain families of Dirac-Born-Infeld instanton solutions in the 4-dimensional effective theory. We then describe a class of *k*-defect solutions, which we dub ''doppelgängers,'' that precisely match the field profile and energy density of their canonical scalar field theory counterparts. We give a complete characterization of Lagrangians which admit doppelgänger domain walls. By numerically computing the fluctuation eigenmodes about domain wall solutions, we find different spectra for doppelgängers and canonical walls, allowing us to distinguish between *k*-defects and the canonical walls they mimic. We search for doppelgängers for cosmic strings by numerically constructing solutions of Dirac-Born-Infeld and canonical scalar field theories. Despite investigating several examples, we are unable to find doppelgänger cosmic strings, hence the existence of doppelgängers for defects with codimension >1 remains an open question.

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I. INTRODUCTION

Topological defect solutions to classical field theories have applications in many areas in physics, and, in particular, may have important implications for the evolution of the Universe [1–4]. In the early Universe, such defects may have formed as the Universe cooled and various gauge or global symmetry groups were broken. Some defects, such as grand unified theory monopoles, can lead to potential cosmological problems which historically inspired the development of the theory of cosmic inflation. Other defects, such as cosmic strings, are potentially observable in the present day, for example, by affecting the spectrum of perturbations observed in the microwave background and matter distributions (although defects cannot play the dominant role in structure formation). Further, the microphysics of such objects may be important in some circumstances, such as weak scale baryogenesis [5–8]. Another interesting possibility arises if the strings are superconducting, as originally pointed out by Witten [9], since the evolution of a network of such superconducting cosmic strings can differ from a nonsuperconducting one. In particular, the supercurrent along loops of string can build up as the loop radiates away its energy, affecting the endpoint of loop evolution. This supercurrent can become large enough to destabilize the loop or may compete with the tension of the string loop and result in stable remnants, known as *vortons* [10], with potentially important consequences for cosmology [11,12].

In this article, we investigate new features of topological defects in scalar field theories with noncanonical kinetic terms. In particular, we study kinetic terms with more than two derivatives, but which lead to second-order equations of motion. These scalar field theories are similar to those employed in *k*-essence models which have been studied in the context of cosmic acceleration and were introduced in [13–15]. Kinetic terms of this general type also play an important role in other interesting models, such as those of ghost condensation [16] or Galileon [17] fields. The topological defects present in this general class of theories are often termed ''*k*-defects,'' and some aspects of these objects have been studied in earlier works [18–27].

In this work we report on some surprising aspects of *k*-defects, especially *k*-domain walls and their associated instantons. We find that there are very reasonable choices for the *k*-defect kinetic term—such as the Dirac-Born-Infeld (DBI) form—for which there are no static defect solutions in a range of parameters, despite the fact that the potential may have multiple minima. Thus, unlike canonical scalar field theories, knowledge of the homotopy groups of the vacuum manifold is sometimes insufficient to classify the spectrum of topological defects. Because of the close connection between domain walls and instantons, this result also constrains certain instanton solutions to noncanonical 4-dimensional effective theories.

Perhaps more surprisingly, it is also possible for *k*-defects to masquerade as canonical scalar field domain walls. By this, we mean the following: given a scalar field ϕ with a canonical kinetic term and potential $V(\phi)$, then,

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up to rigid translations $x \rightarrow x + c$, the field profile $\phi(x)$ and energy density $E(x)$ are uniquely determined for a solution containing a single wall. We show that there always exists a class of k-defect Lagrangians which generate precisely the same field profile and energy density profile as the unique canonical defect. To an observer who measures the field profile and energy density of the configuration, any k-defect in this class precisely mimics the canonical domain wall. Nevertheless, despite having identical defect solutions, we show that these two theories are not reparametrizations of each other, since the fluctuation spectra about the walls are different.

Most of our analytical work is carried out for scalar field theories with domain wall solutions. In order to study the generalization to other topological defects, we carry out a numerical investigation of global cosmic string k-defect solutions. For the natural generalization of the DBI kinetic term, we show that it is possible to match either the field profile or energy density of the canonical global string, but not both simultaneously. Thus, while we are unable to find an analogue of the doppelgänger domain walls in this case, we cannot conclusively show they do not exist.

This paper is organized as follows. In Sec. II we describe the general theory of k-defects and use the specific example of the DBI action to illustrate how the question of the existence of defects is more complicated than the canonical case. We also discuss instanton solutions to the DBI action and compare our conclusions to existing discussions in the literature. Section III introduces the idea of doppelgänger domain walls, which can precisely mimic the field profile and energy density of a canonical domain wall. We establish conditions for the existence of doppelgängers, and discuss the fluctuation spectra about doppelgänger and canonical walls. In Sec. IV we employ numerical methods to search for doppelgänger cosmic strings, but are unsuccessful. We conclude in Sec. V.

II. EXISTENCE AND PROPERTIES OF k -DEFECTS AND INSTANTONS

Our discussion focuses on two families of models involving a scalar field. The first family consists of canonical scalar field theories is of the form

$$S = \int \left[-\frac{1}{2}(\partial\phi)^2 - V(\phi) \right] d^4x, \tag{1}$$

where we use the $(-+++)$ metric signature, set $\hbar = c = 1$, and define $(\partial\phi)^2 \equiv \eta^{\mu\nu}(\partial_\mu\phi)\partial_\nu\phi$. Although we focus our discussion on four spacetime dimensions, essentially all of our conclusions regarding domain walls apply in any spacetime dimension >2 since all but one of the spatial dimensions are spectators.

The second family of models generalizes the canonical scalar field theory by including additional derivatives of ϕ. This family is described by actions of the form

$$S = \int [P(X) - V(\phi)] d^4x, \tag{2}$$

where we define

$$X = (\partial\phi)^2 = -\dot{\phi}^2 + (\nabla\phi)^2. \tag{3}$$

We refer to a Lagrangian of the form (2) as a "$P(X)$ Lagrangian." (Note that there are multiple conventions for the definition of X in the literature.) The canonical scalar field theory corresponds to $P(X) = -X/2$. While there are more than two derivatives of ϕ in the Lagrangian, by assuming that the Lagrangian depends only on X and ϕ as in (2) we guarantee that the resulting equations of motion are second order.

In this section, we show that static domain walls need not exist for all parameter ranges of a wide variety of $P(X)$ theories, even when the potential in (2) possesses multiple disconnected minima. We demonstrate this result using a specific form of $P(X)$, corresponding to the DBI kinetic term. We then adapt these results to study the properties of Coleman-de Luccia–type instantons in 4-dimensional effective theories with DBI kinetic terms.

A. Domain walls in naive DBI

A simple and well-motivated form of $P(X)$ is contained in the DBI action, given by

$$P(X) = M^4 - M^2\sqrt{M^4 + (\partial\phi)^2}, \tag{4}$$

where M is a mass scale associated with the kinetic term, which we will refer to as the "DBI mass scale." When $(\partial\phi)^2 \ll M^4$, this kinetic term reduces to the canonical one. In what follows, we set $M = 1$, and hence normalize all mass scales to the DBI mass scale. A kinetic term of the form (4) can arise naturally in various ways: for example, it is the four-dimensional effective theory describing the motion of a brane with position ϕ in an extra dimension. Often these kinetic terms appear along with additional functions of ϕ, known as "warp factors." These do not influence our conclusions in an essential way and so, for now, we will use the simple form (4) to illustrate our conclusions, and return to the case with warp factors in Sec. II B.

We refer to the $P(X)$ Lagrangian defined by (4) as the "naive" DBI theory since one is merely adding a potential function $V(\phi)$ to the DBI kinetic term (4). There are other, and in some respects better, ways to generalize a pure DBI term and include interactions. We will discuss one such extension extensively in Sec. III. Nonetheless, the $P(X)$ Lagrangian defined by (4) is commonly employed in the literature, and will provide an instructive example of k-defects possessing a number of interesting properties, as we now discuss.

1. The canonical wall

As a warm-up, we first study the canonical domain wall profile. We assume that all fields depend on only one spatial coordinate z, and are independent of time. With these assumptions, there exists a conserved quantity J with $dJ/dz = 0$, defined by

$$J = \phi' \frac{\partial L}{\partial \phi'} - L = -\frac{1}{2}\phi'^2 + V(\phi), \tag{5}$$

where L is the Lagrangian density. We assume that the potential is positive semidefinite and has discrete zero-energy minima at $\phi = \phi_\pm$, such that $V(\phi_\pm) = 0$, with $\phi_- < \phi_+$. Assuming boundary conditions where $\phi = \phi_\pm$ at $z = \pm\infty$, we have that $V = \phi' = 0$ at $z = \pm\infty$. Therefore, $J = 0$ at infinity, and since it is conserved, it vanishes everywhere. This implies that (5) can be rewritten as

$$\phi'^2 = 2V(\phi), \tag{6}$$

which can be straightforwardly integrated to yield the usual domain wall solution.

To compute the energy density of the solution, we use the fact that

$$H = \dot{\phi}\frac{\partial L}{\partial \dot{\phi}} - L = -L, \tag{7}$$

where H is the Hamiltonian density and the second equality follows from our assumption that the configuration is static. Using (6) we have that the energy density $E(\phi)$ is given by

$$H = E(\phi) = 2V(\phi). \tag{8}$$

In general, the energy density cannot be expressed as a function of the field only, but must include the gradient. A relation like (8) is only true because we have a conserved quantity for static configurations, which relates the field value and its gradient. Thus, all of the physics of the static canonical domain wall is encoded in the conserved quantity J.

2. The naive DBI wall

We can carry out a similar derivation for the DBI wall in a $P(X)$ theory defined by (2) and (4). Recalling we have set $M = 1$, the conserved quantity J is given by

$$J = \frac{1}{\sqrt{1+\phi'^2}} - 1 + V(\phi). \tag{9}$$

As in the canonical case described in Sec. II A 1, we assume that the potential is positive semidefinite and has discrete zero-energy minima at $\phi = \phi_\pm$, such that $V(\phi_\pm) = 0$, with $\phi_- < \phi_+$. We also assume the same boundary conditions, so that $\phi = \phi_\pm$ at $z = \pm\infty$. Since $V = \phi' = 0$ at $z = \pm\infty$, J must vanish everywhere. Hence, inverting (9) yields

$$\phi'^2 = \frac{1}{[1 - V(\phi)]^2} - 1. \tag{10}$$

This expression is the analogue of (6), and can be integrated to give the field profile once $V(\phi)$ is specified. Given a static configuration, the energy density is then given by

$$E(\phi) = \frac{V(\phi)[2 - V(\phi)]}{1 - V(\phi)}, \tag{11}$$

where we have used (9) and the fact that $J = 0$ everywhere.

Unlike the canonical case, it is apparent that problems may arise when integrating (10). In the canonical case, so long as $V(\phi)$ is bounded for $\phi \in [\phi_-, \phi_+]$, we have ϕ' finite everywhere. This is no longer the case with (10). If there is any $\phi_1 \in [\phi_-, \phi_+]$ such that $V(\phi_1) > 1$, then (10) implies that ϕ' is undefined. The problem can be traced back to (9), in which the first two terms on the right-hand side can sum to any number between zero (when ϕ' vanishes) and -1 (when $|\phi'|$ is infinite). Thus, at any point where $V(\phi) > 1$, there is simply no value of ϕ' which will allow the requirement that $J = 0$ everywhere to be satisfied. We conclude that there are no nontrivial static solutions to the theory defined by (4) if $V(\phi) > 1$ at any $\phi \in [\phi_-, \phi_+]$.

To study the nature of the singularity, suppose we have integrated (10) from $\phi = \phi_-$ at $z = -\infty$ and have encountered a value $\phi = \phi_1$ at which $V(\phi_1) = 1$. Assume that this value is reached at $z = z_1$. For a generic function $V(\phi)$ we have

$$V(\phi_1 + \Delta\phi) = 1 + v'\Delta\phi + \mathcal{O}(\Delta\phi^2), \tag{12}$$

where $v' = V'(\phi)|_{\phi=\phi_1}$. Retaining only terms up to first order in $\Delta\phi$ and using (10) leads to

$$\phi' = -\frac{1}{v'\Delta\phi}, \tag{13}$$

which has the solution

$$\phi(z) = \phi_1 + \sqrt{-\frac{2(z - z_1)}{v'}}. \tag{14}$$

Hence, ϕ is well defined when $z < z_1$, before the singularity is reached. It is not defined for $z > z_1$, and at $z = z_1$ there is cusp-type singularity in the field, at which the field value is finite but the gradient and all higher derivatives become infinite.

It is natural to ask whether this singularity is integrable; that is, whether the solution can be continued past the singular point at $z = z_1$. We now show that the solution cannot be continued, and hence there are no global solutions to (4) with the desired boundary conditions. We prove this claim for the simple case in which there is only one connected interval of field space between the minima for which $V(\phi) > 1$ [the generalization to the case where there are multiple disconnected regions where $V(\phi) > 1$ is straightforward].

The relevant region of field space is naturally divided into three intervals

$$I_- \equiv [\phi_-, \phi_1), \qquad I_0 \equiv (\phi_1, \phi_2), \qquad I_+ \equiv (\phi_2, \phi_+].$$

The intervals $I_\pm$ include the minima of $V(\phi)$ and all field values for which $V(\phi) < 1$. The interval I_0 includes the field values for which $V(\phi) > 1$. At the boundary points ϕ_1 and ϕ_2 of I_0, $V(\phi) = 1$ and ϕ' reaches $\pm\infty$. We have shown that solutions of (4) with the desired boundary conditions can be constructed which take values in $I_\pm$, but now claim that these solutions cannot be continued into I_0.

The key to proving our claim is to employ the quantity J, which must be conserved by the equations of motion, and is well-defined for any value of ϕ' (even $\phi' = \pm\infty$). First, suppose that we have a candidate continuation of the solution on I_- into I_0. Using this continuation, we choose any point z_* for which $\phi(z_*) \in I_0$, and use $\phi(z_*)$ and $\phi'(z_*)$ to evaluate J. Since $V(\phi) > 1$ at z_*, then by inspection of (9), we conclude that $J > 0$ at z_*. Since J is conserved by the equations of motion, then J must assume this same positive definite value for all points in I_0. Inspection of (9) reveals that, when $J > 0$, ϕ' is finite when $V(\phi) = 1$. Hence, if we approach ϕ_1 while remaining in I_0, then the limiting value of ϕ' at ϕ_1 is finite. On the other hand, we have already shown that $J = 0$ in I_-, and when $J = 0$ we have that $\phi' = \pm\infty$ when $V(\phi) = 1$. Thus, if we approach ϕ_1 while remaining in I_- we have $\phi' = \pm\infty$ at $\phi = \phi_1$.

Thus, if there were a global solution, then ϕ' would approach a finite value from one side of ϕ_1, and an infinite value from the other side. This means that the purported global solution would not match smoothly across the singularity at ϕ_1; a contradiction. Hence we conclude that global solutions do not exist.

While the above statements are strictly correct within the context of the specific Lagrangian we have used, there are potential problems in treating the DBI Lagrangian as an effective field theory near the singularity at ϕ_1. Expanding the Lagrangian L about a static background solution $\phi(z)$ gives terms of the form

$$\delta_2 L \supset -\frac{\delta\phi'(z)^2}{2(1 + \phi'(z)^2)^{3/2}} \tag{15}$$

at quadratic order in the fluctuation $\delta\phi(z)$. Hence the kinetic term for fluctuations vanishes as we approach the point z_1 where $\phi = \phi_1$ and $\phi'(z) \to \infty$. Near the singularity, the effective theory is strongly coupled, quantum corrections to (4) are large, and the precise functional form of (4) is not trustworthy. Whether these corrections invalidate our conclusions is an open question. Nonetheless, our analysis shows that the topological structure of the vacuum is not enough to guarantee the existence of topological defects in models with extra derivatives.

B. Application to instantons

Domain wall solutions are closely related to the solutions to Euclidean field theories employed in constructing instantons. This is because the lowest-energy Euclidean configurations typically depend on a single coordinate, and thus have essentially the same structure as domain wall solutions. Although there are some differences, the correspondence becomes exact in the thin-wall limit. For example, to study the Coleman-de Luccia instanton occurring in a canonical field theory one considers the Euclidean action

$$S_E = 2\pi^2 \int \left[\frac{1}{2}\phi'^2 + V(\phi)\right]\rho^3 d\rho, \tag{16}$$

where ρ is the Euclidean radial coordinate, and in this subsection only we take $\phi' \equiv \partial\phi/\partial\rho$. Instantons are solutions of the equations of motion of this action. The main difference between the action (16) and the canonical domain wall action is the presence of the ρ^3 factor in the integration measure. When the thickness of the wall is much smaller than ρ—the "thin-wall limit"—the measure factor can be neglected, and the instanton problem reduces to the domain wall problem. Thanks to this correspondence, we can apply some of our domain wall techniques to the study of instantons in higher-derivative theories.

The properties of instanton solutions for DBI actions of the form (4) have been studied previously. In particular, in [28] a generalization of (4) was considered, of the form

$$S = \int [f(\phi)^{-1}(1 - \sqrt{1 + f(\phi)(\partial\phi)^2}) - V(\phi)]d^4x, \tag{17}$$

where the function $f(\phi)$ is the "warp factor." The corresponding Euclidean action is

$$S_E = 2\pi^2 \int [f(\phi)^{-1}(-1 + \sqrt{1 + f(\phi)\phi'^2}) + V(\phi)]\rho^3 d\rho. \tag{18}$$

The authors of [28] pointed out that solutions for ϕ develop cusplike behavior once $V(\phi)$ became large. It was argued that this corresponded to instantons where the field profile is multivalued, and the graph of $(z, \phi(z))$ traces out an S curve, as illustrated in Fig. 2 of [28,29]. Geometrically, if ϕ is interpreted as the position of a brane in an extra dimension, this would correspond to the brane doubling back upon itself. However, if we treat the action (18) as a 4-dimensional effective theory, then, as we shall explain below, these solutions only exist for special choices of the functions $f(\phi)$ and $V(\phi)$.

To apply our previous results, we must generalize them to include the measure factor and the warp factor. Since we are concerned entirely with the Euclidean equations of motion arising from (18), which are not affected by constants multiplying the Lagrangian, it is convenient to absorb a factor of $-2\pi^2$ into S_E, and thus consider the Euclidean Lagrangian

$$L_E = [f(\phi)^{-1}(1 - \sqrt{1 + f(\phi)\phi'^2}) - V(\phi)]\rho^3 \equiv \hat{L}_E \rho^3. \tag{19}$$

The Lagrangian L_E incorporates the effects of the warp factor and the measure factor, while $\hat{L}_E$ incorporates warp factor effects alone. For static solutions, the conserved quantity corresponding to $\hat{L}_E$ is

$$\hat{J} = f(\phi)^{-1}\left[\frac{1}{\sqrt{1 + \phi'^2 f(\phi)}} - 1 + V(\phi)\right], \tag{20}$$

which may be compared to (9). It is important to stress that (20) is not precisely conserved: $\hat{J}$ arises from $\hat{L}_E$, whereas the full equations of motion arise from L_E, which contains the measure factor ρ^3. The full equations of motion imply that

$$\frac{\partial \hat{J}}{\partial z} = \left(\frac{3}{\rho}\right)\frac{\phi'^2}{\sqrt{1 + f(\phi)\phi'^2}}, \tag{21}$$

and this nonconservation of $\hat{J}$ describes important physics. Just as in the canonical instanton, this is what enables tunneling between minima of $V(\phi)$ with different vacuum energies, an essential feature of the Coleman-de Luccia instanton. However, in the thin-wall limit, where the width of the instanton solution is much less than ρ, the total change in $\hat{J}$ will be very small across the instanton wall. Hence, if we focus only on the instanton wall itself, $\hat{J}$ is effectively conserved.

The approximate conservation of $\hat{J}$ enables us to employ some of our domain wall techniques from Sec. II A 2 to the instanton problem, and to show that there is no solution to the Euclidean equations of motion in which ϕ curls back on itself. Suppose such a solution did, in fact, exist. Folding back upon itself would occur when $\phi' = \infty$, and we denote the value of ϕ at which this occurs as ϕ_*, and the corresponding value of ρ by ρ_*. Using (20) and working backwards, we find this defines a value of $\hat{J}$ given by

$$\hat{J}_* = \frac{V(\phi_*) - 1}{f(\phi_*)}. \tag{22}$$

Approximate conservation of $\hat{J}$ means that we can take $\hat{J} = \hat{J}_*$ when dealing with physics in the vicinity of the wall. Despite the fact that the point $\phi = \phi_*$ is in some sense singular, $\hat{J}$ must be the same on either side of this point. This is because, clearly, $\hat{J}$ is approximately conserved away from singular points (such as ϕ_*). If we denote $\hat{J}_\pm$ as the value of $\hat{J}$ for $\phi < \phi_*$ and $\phi > \phi_*$, respectively, then the only way to ensure that$\mathrm{Lim}_{\phi\to\phi_*^+} = \infty$ and $\mathrm{Lim}_{\phi\to\phi_*^-} = \infty$ is to have $\hat{J}_+ = \hat{J}_- = \hat{J}_*$.

We now focus on a closed interval I_ϵ in ϕ, of radius ϵ, and centered on $\phi = \phi_*$, so $I_\epsilon = [\phi_* - \epsilon, \phi_* + \epsilon]$. Assuming $f(\phi)$ is smooth, given any $\delta > 0$ we can choose $\epsilon > 0$ so that

$$\frac{1}{f(\phi)\sqrt{1 + \phi'^2 f(\phi)}} \leq \delta, \qquad \forall\ \phi \in I_\epsilon. \tag{23}$$

Conservation of $\hat{J}$ then implies

$$\left|\frac{V(\phi) - 1}{f(\phi)} - \hat{J}_*\right| \leq \delta, \qquad \forall\ \phi \in I. \tag{24}$$

Using the definition (22) and taking the $\delta \to 0$ limit, we can rewrite this condition as

$$\frac{f'(\phi_*)}{f(\phi_*)} = \frac{V'(\phi_*)}{V(\phi_*) - 1}. \tag{25}$$

If this condition is not satisfied, it is impossible to continue the solution through the singular point. Any deviation from (25) leads to a singular solution, and no fold is possible. For generic functions f and V, the condition (25) is not satisfied, and hence the required instanton solutions do not exist.

To illustrate these results, we can consider the case $f(\phi) = 1$, corresponding to the naive DBI action studied in Sec. II A. The cusp is located at $\phi = \phi_*$ where $V(\phi_*) = 1$, and hence $\hat{J}_* = 0$. In order to fold back upon itself, ϕ must be greater than ϕ_* on one branch of the solution, and less than ϕ_* on the other. Hence $V(\phi) > 1$ on one branch, and $V(\phi) < 1$ on the other, for generic $V(\phi)$. However, from (20) it is clear that there is no solution for ϕ' when $V(\phi) > 1$, and hence the solution cannot be continued through the fold. This ultimately arises because the condition (25) cannot be satisfied if we take $f(\phi) = 1$.

III. DOPPELGÄNGER DOMAIN WALLS

In Sec. II A 2, we showed that domain walls in $P(X)$ theories can be very different from those in canonical scalar field theories. However, in this section, we show that in a particular class of higher-derivative theories, the walls can actually be remarkably similar to their canonical counterparts. Indeed, the background solution for these walls is completely indistinguishable from the canonical wall, with the same energy density and field profile. As we shall see, the two solutions differ only in their fluctuation spectra.

A. An example: Masquerading DBI

1. Motivating the action

Rather than diving immediately into a general analysis, it is instructive to begin with a simple and physically motivated example—the DBI action. One way of deriving the DBI kinetic term is to consider ϕ to be the coordinate of an extended object in an extra-dimensional space. Such objects can be described by the Nambu-Goto (NG) action, which is simply their surface area multiplied by the tension. If we take the higher-dimensional space to have coordinates X^N with $N = 0, \ldots 4$ then the action is

$$S_{\rm NG} = -T \int \sqrt{-\det\left[\eta_{MN}\frac{\partial X^M}{\partial x^\mu}\frac{\partial X^N}{\partial x^\nu}\right]} d^4x, \qquad (26)$$

where T is the tension, and η_{MN} is the metric in the full five-dimensional space, which we take to be Minkowskian. Taking the embedding defined by

$$X^N = x^N, \qquad N = 0, \ldots 3, \qquad X^4 = \phi(x^\mu), \qquad (27)$$

leads precisely to the $P(X)$ in (4), modulo a constant which only serves to set the energy of the vacuum to zero.

This extra-dimensional setup provides a useful geometrical picture for the origin of the DBI kinetic term. However, it is not clear how the simple addition of a potential $V(\phi)$, as we have done in Sec. II A 2, can be interpreted in this picture. If we hew to the extra-dimensional picture, it would seem that any new terms we add to the DBI action should correspond to geometrical quantities, such as the surface area of the membrane in the higher-dimensional space. Such an approach also ensures that these additional terms will be compatible with the coordinate reparametrization symmetry of the action (26).

Guided by these considerations, we study actions in which the tension T is promoted to a function of the spacetime coordinates X^M, so that (26) becomes

$$S_{\rm NG} = -\int T(X)\sqrt{-\det\left[\eta_{MN}\frac{\partial X^M}{\partial x^\mu}\frac{\partial X^N}{\partial x^\nu}\right]} d^4x. \qquad (28)$$

Descending to the four-dimensional theory, we find that such a system cannot be described by a $P(X)$-type Lagrangian (2) because of the way in which X and ϕ are coupled. The resulting action is

$$S = \int [1 - (1 + U(\phi))\sqrt{1 + (\partial\phi)^2}] d^4x, \qquad (29)$$

where, as in Sec. II A 2, we have set $M = 1$, where M is the mass scale associated with the DBI kinetic term. We have also added a constant to the Lagrangian density in order to ensure that the energy density vanishes when $\phi' = 0$ and $U(\phi) = 0$. When gradients are small and $(\partial\phi)^2 \ll M^4$, the Lagrangian is approximately

$$L = 1 - (1 + U(\phi))\sqrt{1 + (\partial\phi)^2} \sim \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}(\nabla\phi)^2 - U(\phi), \qquad (30)$$

and hence $U(\phi)$ is analogous to the potential in the canonical theory. However, as we shall see below, it plays a somewhat different role in the full theory.

2. Dirac-Born-Infeld doppelgängers

We are now ready to study defect solutions corresponding to the action (29). For this action, the conserved quantity J is

$$J = \frac{1 + U(\phi)}{\sqrt{1 + \phi'^2}} - 1. \qquad (31)$$

As before, we assume that $U(\phi)$ has two discrete minima $\phi_\pm$ where $U(\phi_\pm) = 0$ and take boundary conditions where $\phi = \phi_\pm$ at $z = \pm\infty$. Thanks to the boundary conditions, $J = 0$ at infinity, and therefore J vanishes everywhere because it is conserved. Inverting (31) gives

$$\phi'^2 = U(\phi)[U(\phi) + 2], \qquad (32)$$

which can be integrated to find the field profile for the defect. The Hamiltonian energy density of the defect is given by

$$E(\phi) = -1 + [1 + U(\phi)]\sqrt{1 + \phi'^2} = U(\phi)[U(\phi) + 2], \qquad (33)$$

where in the second equality we have used the expression (31) and the fact that J vanishes.

The curious properties of the doppelgänger walls arise from the fact that the right-hand sides of (32) and (33) are identical: the energy density is equal to ϕ'^2. The only other case we have seen thus far with this property was the canonical domain wall, as seen in (6) and (8). This property was not shared by the naive DBI domain wall, as can be seen from (10) and (11). This means that, for static solutions arising from the action (29), we can define an effective potential function $\hat{V}(\phi)$ for the DBI wall by

$$\hat{V}(\phi) \equiv \tfrac{1}{2}U(\phi)[U(\phi) + 2]. \qquad (34)$$

Note that minima of $U(\phi)$ where $U(\phi) = 0$ are also minima of $\hat{V}(\phi)$ where $\hat{V}(\phi) = 0$. With the identification (34) Eqs. (32) and (33) are precisely the same as the analogous equations for the canonical domain wall (6) and (8), but with the substitution $V \to \hat{V}$. By inverting (34), we find

$$U(\phi) = -1 + \sqrt{1 + 2\hat{V}(\phi)}. \qquad (35)$$

So, we conclude with the somewhat surprising result, given below.

Given a canonical scalar field theory with a positive semidefinite potential $V(\phi) \geq 0$ which supports domain wall solutions, there exists a choice for $U(\phi)$ in the DBI theory (29), given by setting $\hat{V} = V$ in (35), which guarantees domain walls with precisely the same field profile and energy density.

In the next two subsections, we present two pieces of evidence which support the idea that our claim is somewhat surprising. First, we show that a claim of this nature cannot be made for arbitrary theories with extra derivatives: generically, there is no way to choose a potential function so that the higher-derivative wall mimics the canonical one. We reinforce this argument by deriving an explicit condition for the existence of doppelgänger defects. Second, we numerically compute the fluctuation spectra about the background domain wall solution, and find they are very different for the canonical wall and the DBI one. This shows that the DBI theory (29) is not a rewriting of the canonical scalar field theory, despite having solutions with identical field profiles and energy density.

B. When do doppelgänger defects exist?

1. A Counter example—other $P(X)$ theories

While we have shown that the action (29) possesses doppelgänger solutions, this is not a generic property of theories with higher derivatives. The $P(X)$ theory with a DBI kinetic term studied in Sec. II A already provides one example where a $P(X)$-type theory always leads to domain wall solutions which differ from those of a canonical field theory, with either a different field profile or a different energy density (or both). The DBI wall with a $P(X)$ action of the type (4) can never mimic a canonical domain wall because, for a canonical wall, we always have that

$$\phi'^2 = E(\phi). \tag{36}$$

In the $P(X)$ DBI case, this would require the expressions on the right-hand side of (10) and (11) to be equal. A quick calculation shows that this can only happen if $V(\phi) = 0$, and hence the $P(X)$ DBI wall can never mimic a canonical wall.

As another example, we consider a different $P(X)$ theory defined by

$$P(X) = -\tfrac{1}{2}X + \alpha X^2, \tag{37}$$

where α is a real parameter with dimensions of $[\text{mass}]^{-4}$. When $X \ll \alpha$, this reduces to the canonical scalar field theory. Following the techniques used previously, we find that this theory possesses a conserved quantity J given by

$$J = -\tfrac{1}{2}\phi'^2 + 3\alpha\phi'^4 + V(\phi), \tag{38}$$

where $V(\phi)$ is the potential associated with the theory. One might suppose that, since this theory is a deformation of the canonical one, a deformation of the potential would suffice to mimic the canonical wall. Again assuming that the potential is positive semidefinite and has discrete zero-energy minima at $\phi = \phi_\pm$, with $\phi_- < \phi_+$, so that $V(\phi_\pm) = 0$, and assuming boundary conditions where $\phi = \phi_\pm$ at $\pm\infty$, we find the first integral

$$\phi'^2 = \frac{1 - \sqrt{1 - 48\alpha V(\phi)}}{12\alpha}, \tag{39}$$

whereas

$$E(\phi) = \phi'^2 - 4\alpha\phi'^4. \tag{40}$$

Since $E(\phi) \neq \phi'^2$, we see that there is no choice of the potential for which the theory defined by (37) mimics a canonical wall, so long as $\alpha \neq 0$.

2. Conditions for doppelgänger defects in more general actions

The discussion in the previous section does not imply the absence of other doppelgänger actions. As we now show, there are infinitely many higher-derivative actions which can mimic canonical domain walls. However, these other actions are "rare" in the sense that they are technically nongeneric in the space of all scalar field actions. We make this statement more precise below.

Consider the family of scalar field actions which have second-order equations of motion. Such an action is defined by a Lagrangian which is a function of both $X = (\partial\phi)^2$ and ϕ,

$$L = L(X, \phi), \tag{41}$$

containing the much smaller family of $P(X)$ actions as a special case. We denote the canonical action by L_0, so that

$$L_0(X, \phi) = -\tfrac{1}{2}X - V(\phi). \tag{42}$$

The conserved quantity for the general Lagrangian (41) is given by

$$J = 2X\frac{\partial L}{\partial X} - L, \tag{43}$$

whereas for the canonical action $J_0 = -X + V(\phi)$. Without loss of generality we assume that the domain wall boundary conditions are such that $J = 0$ everywhere. This can always be enforced by shifting L by a constant $L(X, \phi) \to L(X, \phi) + c$, which does not affect the equations of motion and only shifts the zero point of the energy density. For the canonical action, this implies that we can impose $V(\phi_{\min}) = 0$ for the global minima $\phi_{\min}$ of V.

What is required of a higher-derivative action so that it can mimic a canonical scalar field action? The first requirement is that both actions must have the same field profile $\phi_0(z)$ as a solution to their respective equations of motion. The second requirement is that the energy density of this field profile be the same when evaluated using the Hamiltonians associated with their respective actions.

We employ a geometrical construction to investigate these requirements. Instead of viewing L and L_0 as functions, it is helpful to think of them as surfaces hovering over the (X, ϕ) plane, with a height given by $L(X, \phi)$ or $L_0(X, \phi)$, respectively. These surfaces are referred to as the "graphs" of the functions L and L_0.

We first consider the second requirement, that the field profile $\phi_0(z)$ has the same Hamiltonian energy densities in the two theories. Suppose we have already established that the same field profile $\phi_0(z)$ is a solution to both actions. We denote by ϕ_- the value of ϕ at $z = -\infty$ and by ϕ_+ the value at $z = +\infty$ for this solution. The specified solution traces out a curve C on the (X, ϕ) plane given in parametric form by

$$C\colon z \mapsto (X_0(z), \phi_0(z)). \tag{44}$$

Since the configurations are static, the energy density is simply $-L$. Hence we can satisfy the first requirement if and only if

$$L(X, \phi) = L_0(X, \phi) \quad \text{on } C. \tag{45}$$

L and L_0 need not agree everywhere, but they must agree when evaluated on points on C. Geometrically, (45) means that the graphs of L and L_0 must intersect, and the projection of this intersection on to the (X, ϕ) plane must contain C.

We next consider the first requirement, that the equations of motion for either action admit the specified field profile $\phi_0(z)$ as a solution. We assume that $\phi_0(z)$ is a solution to the canonical theory, and derive the requirement that it also be a solution to L. Recall that, for static configurations, actions of the form (41) always admit a first integral obtained by solving the equation $J = 0$ for ϕ'^2. Hence, J must vanish when evaluated on the solution to the canonical theory. That is, $\phi_0(z)$ will be a solution to the higher-derivative scalar field theory if and only if

$$2X\frac{\partial L}{\partial X} - L = 2X\frac{\partial L_0}{\partial X} - L_0 \quad \text{on } C \tag{46}$$

which, using (45), yields

$$\frac{\partial L}{\partial X} = \frac{\partial L_0}{\partial X} \quad \text{on } C. \tag{47}$$

Hence, we require that the derivatives of L and L_0 with respect to X agree on C. Note that we never need to match derivatives with respect to ϕ—while $\partial L/\partial\phi$ does enter the equations of motion, it does not enter our conserved quantity and hence is not required to find a solution.

We conclude that:

An action $L(X, \phi)$ mimics a domain wall $\phi_0(z)$ of the canonical scalar field theory L_0 (that is, has the same field profile and energy density) if and only if the graphs of L and L_0 intersect above the curve C: $z \mapsto (X_0(z), \phi_0(z))$ in the (X, ϕ) plane, and if $\partial L/\partial X = \partial L_0/\partial X$ along the intersection.

This geometrical picture, when combined with the two constraints (45) and (47), allows us to make a powerful statement about how rare doppelgänger actions are. The graphs of L and L_0 are codimension-1 surfaces in the same three-dimensional space. Hence, they will generically intersect along a one-dimensional curve. Thus, we should not be surprised if two actions satisfy the constraint (45), which is essentially the statement that the graphs intersect along a one-dimensional curve. However, two codimension-1 manifolds will generically intersect "transversely"—the span of their tangent spaces will equal the tangent space of the manifold at the intersection ($\mathbf{R}^3$ in this case). The condition (47) implies that the graphs of L and L_0 do not intersect transversely. Thus, the existence of doppelgänger walls depends on constructing graphs in $\mathbf{R}^3$ which intersect nongenerically. This geometrical interpretation is illustrated in Fig. 1, where we have compared a canonical action with $V(\phi) = (1/4)(\phi^2 - 1)^2$ and its doppelgänger Lagrangian.

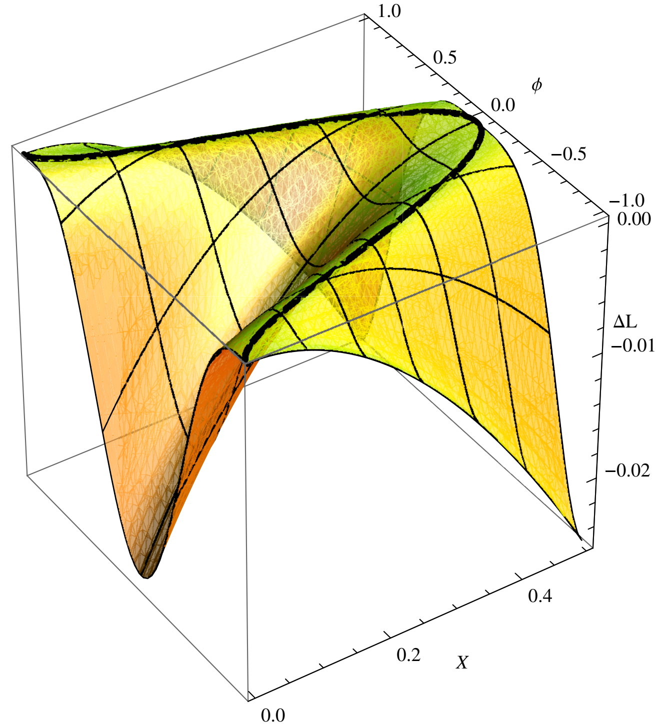

FIG. 1 (color online). An illustration of the geometrical interpretation of doppelgänger actions. The graphs of $L(X, \phi)$ and $L_0(X, \phi)$ intersect along a single curve, whose projection on to the (X, ϕ) plane is the curve C discussed in the text. Here we plot the graph of $L_0 - L$ for the DBI action and the curve C (in black). The intersection of the graphs of L and L_0 is nongeneric, since the first derivatives of the $L - L_0$ surface vanish along C.

Another way of putting this result is that, given any function $\Delta L(X, \phi)$, such that

$$\Delta L(X, \phi) = 0 \quad \text{on } C \quad \text{and} \quad \frac{\partial \Delta L}{\partial X} = 0 \quad \text{on } C, \tag{48}$$

then we can construct another action

$$L(X, \phi) = L_0(X, \phi) + \Delta L(X, \phi), \tag{49}$$

which will have the same domain wall solution as L_0. Clearly there are infinitely many functions ΔL satisfying (48), though they are nongeneric in the same sense as nontransversely intersecting pairs of surfaces are nongeneric.

C. DNA tests for defects: Fluctuation spectra for doppelgängers

The existence of doppelgänger defects raises the question of whether such objects are merely a reparametrization of the original, canonical scalar field wall. As we shall demonstrate here, the fluctuation spectra of the doppelgänger walls are distinctly different from those of canonical walls. Among other differences, when the doppelgänger walls are deeply in the DBI regime (V_0/M^4 large), they have far more bound states than the canonical wall. Since the fluctuation spectra are different, the two theories cannot be reparametrizations of each other.

We find the action and equation of motion for the fluctuations by taking

$$\phi(t, z) = \phi_0(z) + \delta\phi(t, z), \tag{50}$$

where $\phi_0(z)$ is a static background solution to the equations of motion and $\delta\phi(t, z)$ the fluctuation. We then expand the Lagrangian to quadratic order in $\delta\phi$. The term linear in $\delta\phi$ vanishes since $\phi_0(z)$ satisfies the equations of motion, and the purely quadratic piece is of the form

$$\delta_2 L = A(z)\delta\dot{\phi}^2 + B(z)\delta\phi^2 + C(z)\delta\phi'^2 + D(z)\delta\phi\delta\phi'. \tag{51}$$

For the canonical action, $A = 1/2$, $B = -V''(\phi_0(z))/2$, $C = -1/2$, and $D = 0$. For other cases, these coefficients depend on the particular background solution $\phi_0(z)$ and on the specific action used.

Since the action is independent of t, different frequencies do not mix and we can study an individual mode with frequency ω by taking

$$\delta\phi(t, z) = e^{-i\omega t}\delta\phi(z). \tag{52}$$

This leads to the quadratic action

$$\delta_2 L = (\omega^2 A(z) + B(z))\delta\phi^2 + C(z)\delta\phi'^2 + D(z)\delta\phi\delta\phi', \tag{53}$$

yielding the equation of motion

$$\frac{C}{A}\delta\phi'' + \frac{C'}{A}\delta\phi' + \left[\frac{D' - 2B}{2A}\right]\delta\phi = \omega^2\delta\phi. \tag{54}$$

Finding the energies of the fluctuation modes amounts to finding values of ω so that (54) is satisfied by a normalizable function $\delta\phi$.

The problem (54) is an eigenvalue problem of the Sturm-Liouville–type. Ideally, it would be in the form of a Schrödinger equation, which would allow us to readily identify free and bound states by analogy to the corresponding quantum mechanical system. Unfortunately, in general (54) is not of Schrödinger-type, thanks to the presence of the $\delta\phi'$ term. However, in many interesting cases the quantity

$$E_0 \equiv \frac{D' - 2B}{2A} \tag{55}$$

tends to a constant far away from the wall. Hence, evaluating (55) far away from the wall defines an analogue to the "binding energy" of various fluctuation modes. We call modes with $\omega^2 < E_0$ the "bound states," and modes with $\omega^2 > E_0$ "free states." This definition gives reasonable agreement with our expectations for bound and free states, as we discuss below.

The eigenvalue problem (54) can be solved numerically using a simple finite element approach. We have computed the lowest-lying eigenmodes for a canonical wall with potential

$$V(\phi) = \frac{V_0}{4}(\phi^2 - \phi_0^2)^2 \tag{56}$$

and some of its doppelgänger walls, assuming periodic boundary conditions with periodicity much larger than the wall width. Some of these solutions are shown in Fig. 2. These figures show the energies ω^2 of these fluctuation modes, normalized to the binding energy E_0 defined in (55), which is itself shown by the black horizontal line in the figure. As can be seen, our definition of bound states is reasonable, since the eigenmodes possess the properties one would expect of bound states (such as compact support) when their energies are below E_0, and the properties of free states (such as oscillatory behavior)

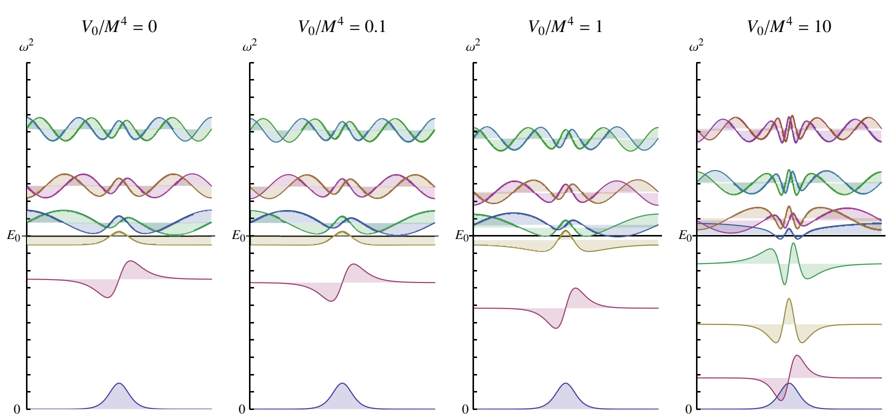

FIG. 2 (color online). The lowest-lying fluctuation eigenmodes for various domain walls. The vertical position of each eigenmode is the eigenvalue ω^2 normalized by the binding energy E_0. Shown are the spectra for a canonical scalar field wall with $V_0/M^4 = 0$ (leftmost panel) and then some of its doppelgängers with $V_0/M^4 = 0.1$, 1, and 10, respectively. As the ratio V_0/M^4 increases, the wall possesses more bound states. The lowest-lying state is identical for each wall, reflecting the fact that these walls share a background field profile.

when their energies are above E_0. Since the eigenspectra are different, we can conclude that the two theories, while possessing an identical background solution, are in fact distinct theories.

The figures also show that there are many more bound states for the doppelgänger wall when we increase the mass scale of the potential relative to the DBI scale. These bound states are possible because the DBI action "weights" gradient energy much less in the interior of the domain wall, and hence even highly oscillatory fluctuation modes can remain as bound states. Physically, the presence of these bound states means that the doppelgänger wall possesses additional oscillation modes which the canonical wall does not.

IV. *k*-STRINGS

It is natural to ask whether it is also possible to find doppelgängers of other defect solutions, such as global strings or monopoles. This question is somewhat difficult to answer since higher codimension defects are generally less analytically tractable than the domain wall. In particular, the existence of the conserved quantity J in the codimension-one (domain wall) case allowed us to find the field profile and energy density and construct a doppelgänger existence proof. No analogous quantity is available for higher codimension defects, such as global strings or monopoles.

In this section, we generalize the one-field DBI action to a two-field system, and investigate some properties of the correponding global string solutions. Since we have no conserved quantity, we take a numerical approach and directly integrate the equations of motion. Using our two-field DBI model, we find no doppelgänger global string solutions. Nevertheless, since we cannot treat the two-field system analytically, we cannot prove a "no-go" theorem and hence the existence of higher codimension doppelgänger defects remains an open question.

The canonical global string solution can be found by starting from the action with two real scalar fields

$$S = \int\left[-\frac{1}{2}(\partial\phi_1)^2 - \frac{1}{2}(\partial\phi_2)^2 - V(\phi_1, \phi_2)\right]d^4x, \quad (57)$$

where the potential $V(\phi_1, \phi_2)$ respects a global $O(2)$ symmetry, corresponding to rotations in the (ϕ_1, ϕ_2) plane. To study string solutions, we assume the field configuration is static and cylindrically symmetric, employ polar coordinates (r, θ) in real space, and use the rotational symmetry to decompose the fields in terms of new functions ϕ and Θ as

$$\begin{aligned}\phi_1(r, \theta) &= \phi(r)\cos\Theta(N\theta),\\ \phi_2(r, \theta) &= \phi(r)\sin\Theta(N\theta),\end{aligned} \quad (58)$$

where $N \in \mathbf{Z}$ is the winding number of the string. Restricting ourselves to strings of unit winding number $N = 1$, the entire action may then be written in terms of the single function $\phi(r)$. The equation of motion for this field is

$$\phi'' + \frac{\phi'}{r} - \frac{\phi}{r^2} - \frac{\partial V}{\partial\phi} = 0, \quad (59)$$

where $\phi' = \partial\phi/\partial r$. Given a potential $V(\phi)$ which admits a defect solution, that is, $V(0) \neq 0$ and there exists $\phi_0 > 0$ such that $V(\phi_0) = 0$ is a minimum, the string solution is subject to the boundary conditions that $\phi(0) = 0$ and $\phi \to \phi_0$ as $r \to \infty$. It is then straightforward to solve for the string field profile using the relaxation method.

There are many multifield generalizations of the basic DBI kinetic term (4) which appear in the literature. Typically these generalizations reduce to the usual DBI kinetic term when there is only a single field. Based on our experience with the doppelgänger solutions, the best-motivated generalization is analogous to (28), based on a generalization of the Nambu-Goto action with two extra dimensions given by

$$S_{\rm NG} = -\int T(X)\sqrt{-\det\left[\eta_{MN}\frac{\partial X^M}{\partial x^\mu}\frac{\partial X^N}{\partial x^\nu}\right]}d^4x, \quad (60)$$

where, as before, the tension T is a function of the embedding coordinates. We depart from (28) by taking six-dimensional embedding coordinates X^N, with $N = 0\ldots5$ and

$$X^N = x^N\!: N = 0, \ldots 3, \qquad X^4 = \phi_1(x^\mu), \qquad X^5 = \phi_2(x^\mu). \quad (61)$$

Hence, the four-dimensional theory contains two real scalar fields $\phi_{1,2}$ with an $O(2)$ global symmetry. With a suitable choice of tension $T(X)$, we can construct DBI generalizations of the usual global string.

At this point, we can follow a similar procedure to that carried out in the case of the canonical global string. The reduction of the fields in the case of the unit winding number string proceeds exactly as before, with the same decomposition defined by (58). If we use this decomposition in (60) we find

$$S_{\rm NG} = 2\pi\int[r - (1 + U(\phi))\sqrt{(r^2 + \phi^2)(1 + \phi'^2)}]dr, \quad (62)$$

where, as before, we have rewritten $T = 1 + U(\phi)$ and added a constant to the Lagrangian so that the energy is zero when $\phi' = 0$ and $U(\phi) = 0$. Note that there is no factor of r next to the differential, since the action (60) already correctly accounts for the volume measure in four dimensions.

To investigate whether doppelgänger strings can be constructed, we assume a symmetry-breaking potential $U(\phi) = U_0(\phi^2 - 1)^2$ in the DBI theory and solve via the relaxation method for the DBI field profile. Given the field

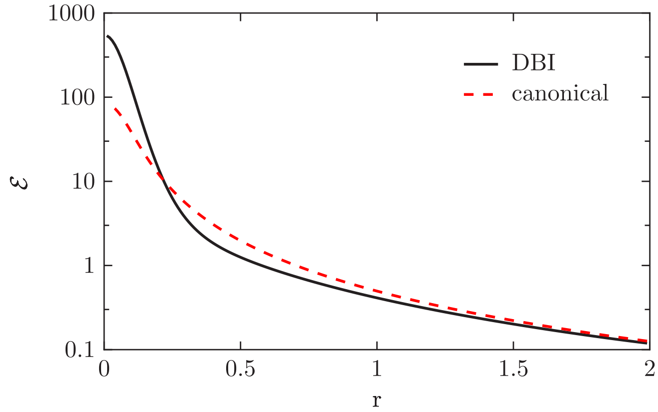

FIG. 3 (color online). Energy density as a function of radius for a DBI string and a canonical string with identical field profiles. The DBI potential is given by $U(\phi) = 10(\phi^2 - 1)^2$.

profile $\phi(r)$ of the DBI string, we solve numerically for the potential in the canonical scalar field theory which gives the same field profile. With this potential function, we compute the energy density in the canonical theory. In the examples we study, we find that the energy densities are different in the two theories. Analogous results hold if we match energy densities between the DBI and canonical theory—we find the field profile does not match. Hence we do not find any doppelgänger defects.

When taking the field profiles to be equal, we can construct a potential such that the DBI field profile is a solution to the canonical equations of motion by integrating the canonical equation of motion for ϕ, setting the potential to be 0 at large r:

$$V(\phi) = \int_{\phi}^{\phi_0} \left(\tilde{\phi}'' + \frac{\tilde{\phi}'}{r} - \frac{\tilde{\phi}}{r^2} \right) d\tilde{\phi} \tag{63}$$

For the examples we have studied, this leads to a total energy density which differs from the DBI energy density, as shown in Fig. 3.

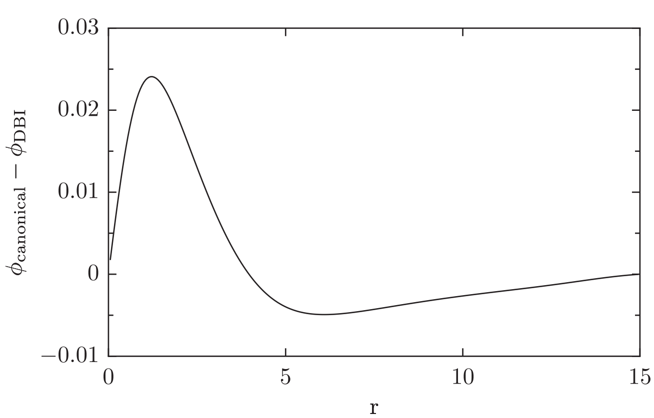

FIG. 4. The difference in field values for a DBI string and a canonical string with identical energy densities. The DBI potential is given by $U(\phi) = \frac{1}{4}(\phi^2 - 1)^2$.

We also consider the case where the energy densities are constrained to be equal. In this case, after solving for the field profile and energy density of the DBI string, we then similarly solve for the field profile of the canonical string while maintaining the canonical potential as $V = \mathcal{E}_{\rm DBI} - \frac{1}{2}(\phi'^2_{\rm canonical} + \frac{\phi^2_{\rm canonical}}{r^2})$. The results are shown in Fig. 4.

The two approaches, both constraining the field profiles to be equal and constraining the energy densities to be equal, yield a DBI string which is observably different from the canonical string for the examples we have studied. Thus we have found no examples of doppelgänger solutions for cosmic strings.

V. DISCUSSION

Nonperturbative field configurations such as topological defects may be formed during phase transitions in the early Universe, and their interactions and dynamics can have significant effects on cosmic evolution. In the case of a scalar field with a canonical kinetic term, the behavior of such configurations has been understood for some time. The resulting constraints on the types and scales of symmetry breaking are well understood, and the possibilities for interesting cosmological phenomena have been thoroughly investigated.

However, in recent years, particle physicists and cosmologists have become interested in noncanonical theories, such as those that might drive k inflation and k essence. Ghost-free and stable examples of such theories can be constructed, and as such one may take them seriously as microphysical models. Several authors have then studied the extent to which the properties of topological defects are modified by the presence of a more complicated kinetic term.

In this paper we have studied k-defect solutions to the DBI theory in some detail, discussing walls and strings, and clarifying the existence criteria and the behavior of instantons in these theories. Furthermore, we have addressed the question of whether k-defects, and in particular k walls and global k strings, can mimic canonical defects. We have demonstrated that given a classical theory with a canonical kinetic term and a spontaneously broken symmetry with a vacuum manifold admitting domain wall solution, there exists a large family of general Lagrangians of the $P(\phi, X)$ form which admit domain wall solutions with the same field profiles and same energy per unit area. These doppelgänger defects can mimic the field profile and energy density of canonical domain walls. Nevertheless, we have also shown that the fluctuation spectrum of a doppelgänger is different from its canonical counterpart, allowing one in principle to distinguish a canonical defect from its doppelgänger.

In the case of cosmic strings we have been unable to prove a similar result. Despite investigating several examples for the potential function in the DBI theory, we have been unable to find cases where there is a canonical theory which results in a matching energy density and field profile. However, since we have less analytic control in the case of defects of higher codimension, we have not been able to prove a no-go theorem. Hence the existence of doppelgänger defects for strings or monopoles remains an open question.

ACKNOWLEDGMENTS

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