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Singularity-free solutions for anisotropic charged fluids with Chaplygin equation of state

Farook Rahaman*
Department of Mathematics, Jadavpur University, Kolkata 700 032, West Bengal, India

Saibal Ray†
Department of Physics, Government College of Engineering and Ceramic Thechnology, Kolkata 700 010, West Bengal, India

Abdul Kayum Jafry‡
Department of Physics, Shibpur Dinobundhoo Institution, Howrah 700 102, West Bengal, India

Kausik Chakraborty§
Department of Physics, Government Training College, Hooghly 712103, West Bengal, India
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We extend the Krori-Barua analysis of the static, spherically symmetric, Einstein-Maxwell field equations and consider charged fluid sources with anisotropic stresses. The inclusion of a new variable (tangential pressure) allows the use of a nonlinear, Chaplygin-type equation of state with coefficients fixed by the matching conditions at the boundary of the source. Some physical features are briefly discussed.

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I. INTRODUCTION

A major issue of the static, spherically symmetric Einstein field equations in general relativity is to find out interior solutions which are free of singularity. It has been shown that an uncharged incompressible fluid sphere of mass m cannot be held in equilibrium below certain radius $a = \frac{9m}{4}$ and even demands a larger value for a related to a physically reasonable equation of state (EOS) [1]. Regarding stability of the model, Stettner [2] argued that a fluid sphere of uniform density with a net surface charge is more stable than without charge. Therefore, a general mechanism, adopted by the relativists to overcome this singularity due to gravitational collapsing of a static, spherically symmetric fluid sphere, is to include charge to the neutral system. It is observed that in the presence of charge either gravitational attraction is counterbalanced by the electrical repulsion in addition to the pressure gradient [3] or inhibits the growth of space-time curvature which has a great role to avoid singularities [4]. According to Ivanov [5] the presence of the charge function serves as a safety valve, which absorbs much of the fine-tuning, necessary in the uncharged case. However, in connection to this we would like to mention here a special kind of mechanism to avert singularity as used by Trautman [6] under Einstein-Cartan theory where the physical entity spin torsion is supposed to act as an agent of repulsive effect.

A large amount of works on charged fluid spheres are available in the literature [an exhaustive discussion with various classification schemes regarding sources for the Reissner-Nordström (RN) space-time can be obtained in Ref. [5]]. However, in connection to singularity we would like to mention here that Efinger [7], Kyle and Martin [8], and Wilson [9] have found relativistic internal solutions for static charged spheres, but none of these solutions is absolutely free from singularities. On the other hand, spheres of charged dust have been investigated by Bonnor [10], Bonnor and Wickramasuriya [11], and Raychaudhuri [12]. Among all these investigations it is observed that in Efinger's solution the metric has a singularity at the origin ($r = 0$), whereas the solutions due to Kyle and Martin [8] and Wilson [9] do not have any interior singularities. However, it is argued by Junevicus [13] that in both the above cases the metrics may have singularities at points other than the origin so that restrictions have to be imposed on the sphere to avoid them. According to him the fluid sphere solutions of Kyle and Martin [8], Wilson [9], Kramer and Neugebauer [14], and Krori and Barua (KB) [15] are of special interest since, with the imposition of suitable conditions, they are completely free of metric singularities and satisfy physical considerations (for the discussion and analysis of stability see Refs. [16,17], respectively).

We would like to note here, especially, the works of KB [15] and Junevicus [13] which are the basis of our present investigation. KB [15] constructed static, spherically symmetric solutions of the Einstein-Maxwell equations based on a particular choice of the metric components g_{00} and g_{11} in curvature coordinates. Assuming that the source is a charged fluid with isotropic stresses, the three independent Einstein equations were reduced to linear algebraic equations for the energy density $\rho(r)$, pressure $p(r)$, and the square of the electric field, $E(r)^2$. In this approach the independent Maxwell equation is used to obtain the charge

*farook_rahaman@yahoo.com
†saibal@iucaa.ernet.in
‡akjafry@yahoo.com
§kchakraborty28@yahoo.com

density from the predetermined form of $E(r)$. A special feature of KB [15] solutions is that they are singularity free. A thorough analysis of this singularity-free KB [15] solution has been done by Junevicus [13]. The main aspect of his investigation is to fix up the constants involved in the KB [15] metric in terms of the physical constants of mass, charge, and radius of the source. He also investigated the conditions for physical relevance leading to a functional dependence of the ratio of mass-to-radius on the ratio of charge-to-mass and also to upper and lower limits on these ratios. In his recent work on static charged perfect fluid spheres in general relativity Ivanov [5] also observed that the solution of KB [15] which is fixed by the junction conditions is nonsingular and the positivity conditions are satisfied.

In connection with the above discussion on the KB solution [15] it is to be mentioned regarding the very recent work on these solution by Varela *et al.* [18]. The work deals with self-gravitating, charged and anisotropic fluids to solve the Einstein-Maxwell equations. In order to discuss analytical solutions they [15] extend the KB method [15] to include pressure anisotropy and linear or nonlinear equations of state. The obtained solutions satisfy the energy conditions of general relativity and have the following features: (1) spheres with vanishing net charge contain fluid elements with unbounded proper charge density located at the fluid-vacuum interface; (2) inward-directed fluid forces caused by pressure anisotropy may allow equilibrium configurations with larger net charges and electric field intensities than those found in studies of charged isotropic fluids; (3) links of these results with charged strange quark stars as well as models of dark matter including massive charged particles are possible; and (4) the van der Waals equation of state leading to matter densities constrained by cubic polynomial equations is considered.

Our present investigation of static, spherically symmetric charged fluid sphere distribution is a continuation of the above work of an anisotropic fluid source [18]. This means that the radial and tangential pressures are, in general, unequal so that the simplest relation between them may be assumed as $p_t = np_r$ $(n \neq 1)$ [19]. However, there are several other forms of anisotropic relationship between pressures that can be noted in the literature; e.g., in connection to the electromagnetic mass model Herrera and Varela [20] introduced a condition of anisotropy in the form $p_t - p_r = gq^2r^2$, where g is a constant having nonzero value whereas Barreto *et al.* [21] define the degree of local anisotropy induced by charge as $p_t - p_r = \frac{E^2}{4\pi}$, where E is the local electric field intensity, to consider self-similar and charged radiating fluid spheres as anisotropic sources.

Recently, scientists show great interest on the Chaplygin gas EOS in order to explain the accelerating phase of the present Universe as well as to unify the dark energy and dark matter. As the Chaplygin gas EOS is a specific form of polytropic EOS, it looks promising to describe dark energy spherically symmetric charged objects, generally termed as *dark stars* in the literature [22–25]. As a possible mechanism of formation it is argued by several investigators that the first stars to form in the Universe, at redshifts $z \sim 10$–50, may be powered by dark matter annihilation for a significant period of time rather than nuclear fusion [26–29]. On the other hand, it is believed that dark energy exerts a repulsive force on its surrounding and this repulsive force, likewise electric charge, may prevent the star from gravitational collapse. Therefore, people have speculated that a massive star does not simply collapse to form a black hole, instead to the formation of a dark energy star with a final configuration without neither singularities nor horizons [30–33].

However, among the above mentioned dark star models we are especially interested in the work of Bertolami and Páramos [22] where they, like us, have used the generalized Chaplygin gas (GCG) EOS with special reference to anisotropic pressure, though our motivation and approach to solve the spherically symmetric gas model is quite different from theirs. The scheme of the present investigation is therefore as follows: we write down the four independent Einstein-Maxwell equations. By allowing the radial (p_r) and tangential (p_t) pressures to be different, we have found out the six variables p_r, p_t, ρ, $\epsilon = E(r)^2$, λ, and ν, where the other parameters are, respectively, matter-energy density, electric field intensity, and metric potentials (Sec. II). Ivanov [5] has explained the usual difficulties that generally arise when we combine equations of state (even a linear one) with the field equations. Interestingly, we are here dealing with a nonlinear EOS and are able to find solutions using an algebraic method (we do not solve differential equations). Adding the nonlinear Chaplygin gas EOS $p_r = H\rho - \frac{K}{\rho}$ (where H and K are two arbitrary constants) and using the KB ansatz for λ and ν, we get four algebraic equations for ρ, p_r, p_t, and $\epsilon = E(r)^2$. Using the independent Maxwell equation we determine the charge density σ from ϵ (Sec. III). We present and discuss the necessary matching of the solutions and the related boundary conditions in Sec. IV which allow us to find out the expressions for H and K with their physical features through the graphical plots. Also, energy conditions have been discussed in detail (Sec. V). Some concluding remarks are made in Sec. VI.

II. BASIC EQUATIONS

The KB [15] metric is given by

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (1)$$

where $\lambda(r) = Ar^2$ and $\nu(r) = Br^2 + C$ with arbitrary constants A, B, and C.

The most general energy momentum tensor compatible with spherical symmetry is

$$T^{\mu}_{\nu} = (\rho + p_r)u^{\mu}u_{\nu} - p_r g^{\mu}_{\nu} + (p_t - p_r)\eta^{\mu}\eta_{\nu} \quad (2)$$

with

$$u^{\mu}u_{\mu} = -\eta^{\mu}\eta_{\mu} = 1.$$

The Einstein-Maxwell equations are

$$e^{-\lambda}\left[\frac{\lambda'}{r} - \frac{1}{r^2}\right] + \frac{1}{r^2} = 8\pi\rho + E^2, \quad (3)$$

$$e^{-\lambda}\left[\frac{1}{r^2} + \frac{\nu'}{r}\right] - \frac{1}{r^2} = 8\pi p_r - E^2, \quad (4)$$

$$\frac{1}{2}e^{-\lambda}\left[\frac{1}{2}(\nu')^2 + \nu'' - \frac{1}{2}\lambda'\nu' + \frac{1}{r}(\nu' - \lambda')\right] = 8\pi p_t + E^2, \quad (5)$$

and

$$(r^2E)' = 4\pi r^2\sigma e^{\lambda/2}. \quad (6)$$

Equation (6) can equivalently be expressed in the form

$$E(r) = \frac{1}{r^2}\int_0^r 4\pi r^2\sigma e^{\lambda/2}dr = \frac{q(r)}{r^2}, \quad (7)$$

where $q(r)$ is the total charge of the sphere under consideration.

III. SOLUTIONS

Now, we consider the KB ansatz:

$$\lambda(r) = Ar^2, \qquad \nu(r) = Br^2 + C, \quad (8)$$

where, as mentioned earlier, A, B, and C are some arbitrary constants. It is of interest to note that these constants were determined by Junevicus [13] in terms of the physical quantities mass, charge, and radius of the source.

Along with the above ansatz let us also use the generalized Chaplygin gas EOS for the charged fluid as [34]

$$p_r = H\rho - \frac{K}{\rho}, \quad (9)$$

where H and K are two positive constants.

Equations (3) and (4) imply

$$(\rho + p_r) \equiv f(r) = \frac{(A + B)}{4\pi}e^{-Ar^2}. \quad (10)$$

From Eqs. (9), (8), and (10), we get the following solution set:

$$\rho = \left(\frac{f + \sqrt{f^2 + 2hK}}{h}\right), \quad (11)$$

$$p_r = f - \left(\frac{f + \sqrt{f^2 + 2hK}}{h}\right), \quad (12)$$

$$p_t = \frac{1}{8\pi}\left[2e^{-Ar^2}(B - A)(2 + Br^2) - \frac{1}{r^2}(1 - e^{-Ar^2}) + 8\pi\left(\frac{f + \sqrt{f^2 + 2hK}}{h}\right)\right], \quad (13)$$

$$E^2 = 2Ae^{-Ar^2} + \frac{1}{r^2}(1 - e^{-Ar^2}) - 8\pi\left(\frac{f + \sqrt{f^2 + 2hK}}{h}\right), \quad (14)$$

$$q^2 = 2Ar^4e^{-Ar^2} + r^2(1 - e^{-Ar^2}) - 8\pi r^4\left[\frac{f + \sqrt{f^2 + 2hK}}{h}\right], \quad (15)$$

where $h = 2(1 + H)$. We observe that for finite values of the physical parameters $h \neq 0$, so that $H \neq -1$.

IV. BOUNDARY CONDITIONS

The RN metric [35,36] is given by

$$ds^2 = -\left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)dt^2 + \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (16)$$

To match our interior metric with the above exterior one we impose only the continuity of g_{tt}, g_{rr}, and $\frac{\partial g_{tt}}{\partial r}$ across a surface, S, at $r = a$. This yeilds the following equations:

$$1 - \frac{2m}{a} + \frac{Q^2}{a^2} = e^{Ba^2+C}, \quad (17)$$

$$1 - \frac{2m}{a} + \frac{Q^2}{a^2} = e^{Aa^2}, \quad (18)$$

$$\frac{m}{a^2} - \frac{Q^2}{a^3} = Bae^{Ba^2+C}. \quad (19)$$

Therefore, from the above equations, one can easily get

$$A = -\frac{1}{a^2}\ln\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right], \quad (20)$$

$$B = \frac{1}{a^2}\left[\frac{m}{a} - \frac{Q^2}{a^2}\right]\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right]^{-1}, \quad (21)$$

$$C = \ln\left[1 - \frac{2m}{a} + \frac{Q^2}{a^2}\right] - \frac{\frac{m}{a} - \frac{Q^2}{a^2}}{[1 - \frac{2m}{a} + \frac{Q^2}{a^2}]}. \quad (22)$$

Note that no extra assumption on the value of $\epsilon(r)$ at $r = a$ is required here and we therefore obtain $\epsilon(a) = \frac{Q^2}{a^4}$. This result was expected as a consequence of the matching conditions at $r = a$ (absence of the thin shell). On the other hand, the electric field at $r = 0$ is zero. Therefore, the energy conditions at $r = 0$ involve only the central value of density, as well as for the radial and tangential pressures. As a thin shell exists, we could not use the boundary condition for ϵ at a which means one cannot get $\epsilon(a) = \frac{Q^2}{a^4}$.

Let us now impose the boundary conditions

$$p_r(a) = 0, \qquad \epsilon(0) = E(0) = 0. \tag{23}$$

We obtain two independent equations which are readily solved for H and K as functions of the source parameters. We note that $E(0) = 0$ implies

$$\rho_0 = \frac{3A}{8\pi} = \left[\frac{Fe^{Aa^2} + \sqrt{(Fe^{Aa^2})^2 + 2hK}}{h}\right] \tag{24}$$

and $p_r(a) = 0$ implies

$$F - \left[\frac{F + \sqrt{F^2 + 2hK}}{h}\right] = 0, \tag{25}$$

where $F = f(a) = \frac{(A+B)}{4\pi} e^{-Aa^2}$.

From the above two equations (24) and (25), one could find the values of two unknowns H and K in terms of A, B, and a, in other words, in terms of mass, charge, and radius of the spherically symmetric charged objects. Therefore, through a simple mathematical exercise, we have the expressions for the constants as follows:

$$H = \frac{3A(B-A)}{[9A^2 - 4(A+B)^2 e^{-2Aa^2}]}, \tag{26}$$

$$K = \left(\frac{(A+B)e^{-Aa^2}}{4\pi}\right)^2 \times \left[2\left(1 + \frac{3A(B-A)}{[9A^2 - 4(A+B)^2 e^{-2Aa^2}]}\right)^2 - 1\right]. \tag{27}$$

One can note that at $r = 0$, $E(0) = 0$, $\rho(0) = \frac{3A}{8\pi}$, and $p_r(0) = \frac{1}{2} p_t(0) = \frac{2B-A}{8\pi}$. Also, the curve profiles (Figs. 1–3)

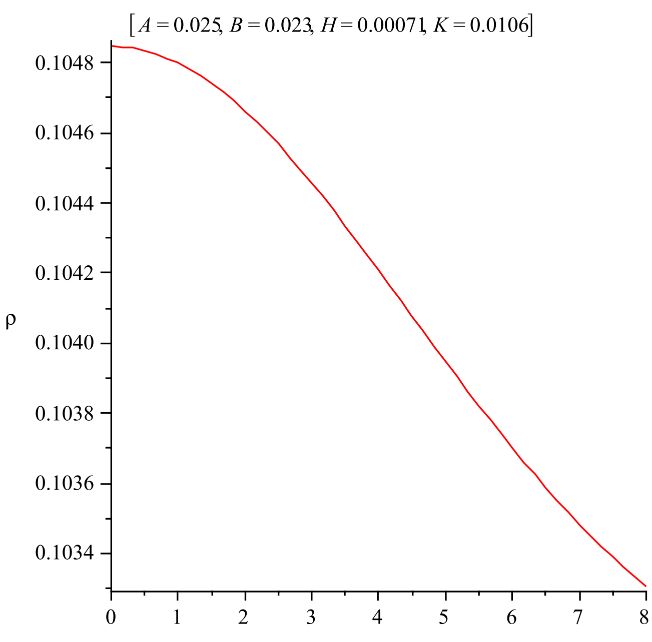

FIG. 1 (color online). The density parameter ρ is shown against r.

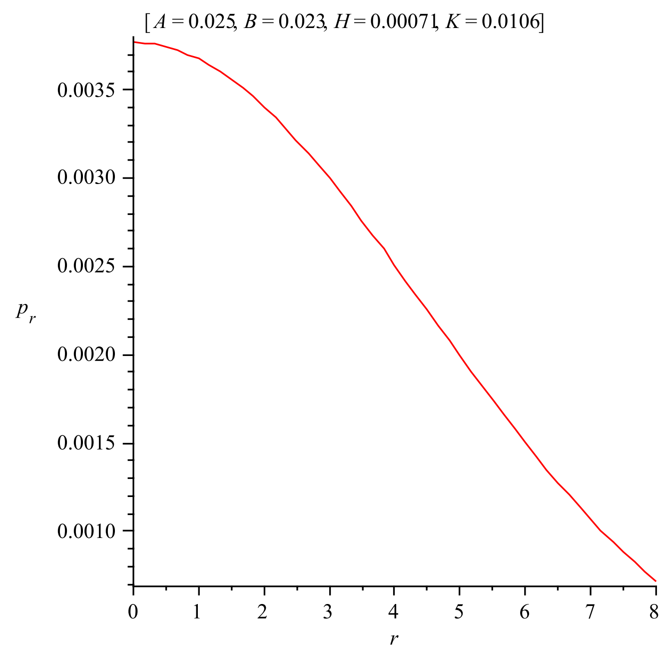

FIG. 2 (color online). The radial pressure p_r is shown against r.

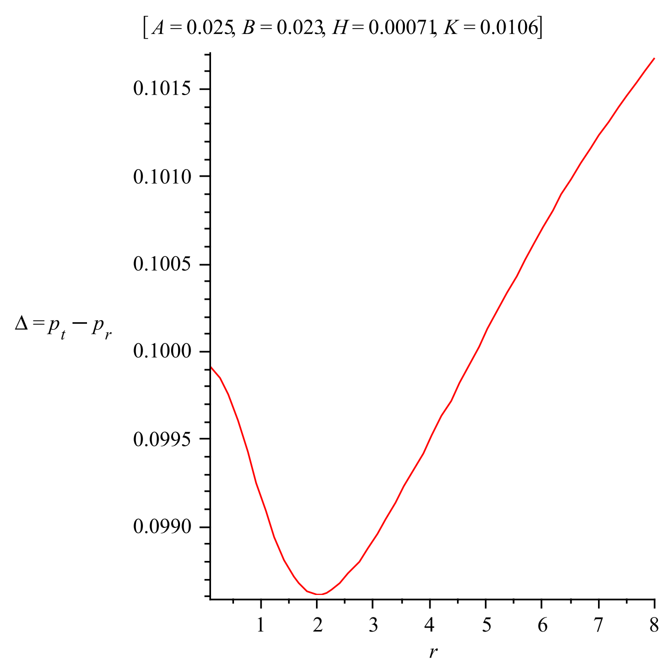

FIG. 3 (color online). The transverse pressure p_t is shown against r.

for the parameters, ρ, p_r, etc. indicate no singularity presence inside the star.

V. TOV EQUATIONS

The generalized Tolman-Oppenheimer-Volkov (TOV) equation as presented by Ponce de León [37] is

$$-\frac{M_G(\rho+p_r)}{r^2}e^{(\lambda-\nu)/2}-\frac{dp_r}{dr}+\sigma\frac{q}{r^2}e^{\lambda/2}+\frac{2}{r}(p_t-p_r)=0, \tag{28}$$

where $M_G = M_G(r)$ is the effective gravitational mass inside a sphere of radius r and $q = q(r)$ is given by (15). The effective gravitational mass is given by the expression

$$M_G(r)=\tfrac{1}{2}r^2e^{(\nu-\lambda)/2}\nu', \tag{29}$$

derived from the Tolman-Whittaker formula and the Einstein-Maxwell equations.

It is important to note that the above equation describes the equilibrium condition for charged fluid elements subject to gravitational, hydrostatic, and electric forces, plus another force due to the anisotropy factor which is a measure of the pressure anisotropy of the fluid comprising the charged body. Combined with (11)–(13) and (15), the above equation takes the form

$$F_g+F_h+F_e+F_a=0, \tag{30}$$

where

$$F_g=-Br(\rho+p_r), \tag{31}$$

$$F_h=-\frac{dp_r}{dr}, \tag{32}$$

$$F_e=\sigma E e^{Ar^2/2}, \tag{33}$$

$$F_a=\frac{2}{r}(p_t-p_r). \tag{34}$$

The profiles of F_g, F_h, F_e, and F_a for sources are shown in Fig. 4. This figure indicates that F_h is comparatively small. Thus the hydrostatic force has a negligible effect in spite of the fact that static equilibrium is attainable due to pressure anisotropy, gravitational, and electric forces.

Although several specific equations of state for $p_r(\rho)$ are used in the literature, very little is known for the much less intuitive second equation of state $p_t(\rho)$. The equation of state parameter $\omega_t \equiv \frac{p_t}{\rho}$ for the anisotropic object can be obtained directly from Eqs. (11) and (13), which is given by

$$\frac{p_t}{\rho}\equiv\omega_t=\frac{[2e^{-Ar^2}(B-A)(2+Br^2)-\frac{1}{r^2}(1-e^{-Ar^2})+8\pi(\frac{f+\sqrt{f^2+2hK}}{h})]}{8\pi(\frac{f+\sqrt{f^2+2hK}}{h})}. \tag{35}$$

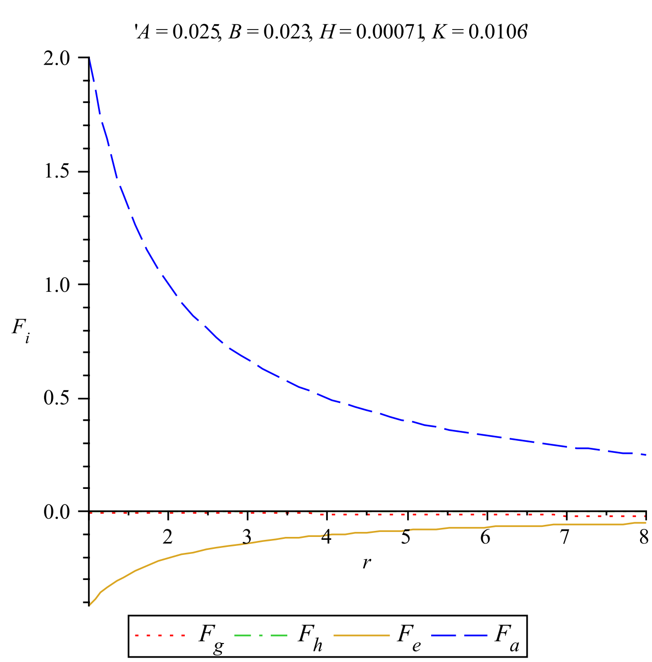

FIG. 4 (color online). Four different forces acting on fluid elements in static equilibrium are shown against r.

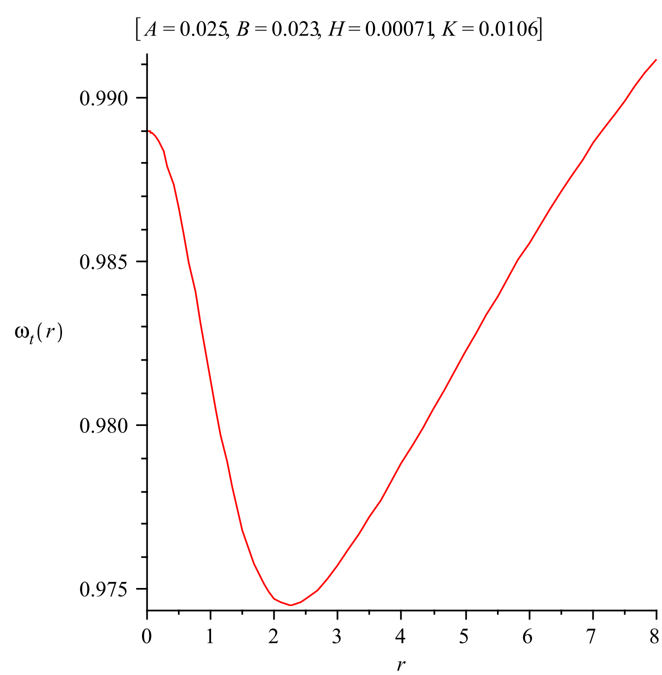

FIG. 5 (color online). The variation of the equation of state parameter ω_t is shown against r.

Figure 5 shows the variation of ω_t against r. The equation of state for anisotropic charged fluid is positive and confined within $0 \leq \omega_t \leq 1$, i.e., charged fluid is nonexotic in nature.

VI. ENERGY CONDITIONS

It is well known for the charged fluid that the null energy condition (NEC), weak energy condition (WEC), and strong energy condition (SEC) will be satisfied if and only if the following inequalities hold simultaneously at every point within the source:

$$\tilde{\rho} + \frac{\tilde{E}^2}{8\pi} \geq 0, \quad (36)$$

$$\tilde{\rho} + \tilde{p}_r \geq 0, \quad (37)$$

$$\tilde{\rho} + \tilde{p}_t + \frac{\tilde{E}^2}{4\pi} \geq 0, \quad (38)$$

$$\tilde{\rho} + \tilde{p}_r + 2\tilde{p}_t + \frac{\tilde{E}^2}{4\pi} \geq 0. \quad (39)$$

Direct plotting of the left sides of (9)–(12) shows that these inequalities are satisfied as well at every r (see Fig. 6).

At this point we feel it is required to determine whether specific choices of mass, charge, and radius lead to solutions satisfying the above energy conditions at $r = 0$. A close observation of the equations (20) and (21) suggests to us to adopt here some adimensional quantities which can be defined as $\alpha = a^2 A$ and $\beta = a^2 B$. We restrict our attention to solutions satisfying $\alpha \geq 0$, $\beta \geq 0$, and $\beta \leq 2\alpha$. These solutions satisfy the four energy conditions of general relativity, viz., the NEC, the WEC, the SEC, and the dominant energy condition (DEC).

For a sphere of radius a, mass M, and charge Q, Eqs. (2.13)–(2.15) in [13] [Eqs. (20)–(22) in our present case] are alternatively expressed in the adimensional forms

$$\alpha = -\ln(1 - 2\mu + \chi^2), \quad (40)$$

$$\beta = \frac{\mu - \chi^2}{1 - 2\mu + \chi^2}, \quad (41)$$

$$C = \ln(1 - 2\mu + \chi^2) - \frac{\mu - \chi^2}{1 - 2\mu + \chi^2}, \quad (42)$$

where $\mu = \frac{M}{a}$, and $\chi = \frac{|Q|}{a}$. It is very important that the field equations can eventually be expressed in terms of these adimensional constants, the adimensional variables $\tilde{\rho} = a^2\rho$, $\tilde{p}_r = a^2 p_r$, $\tilde{p}_t = a^2 p_t$, and $\tilde{\epsilon} = a^2\epsilon$, and the adimensional radial coordinate $x = \frac{r}{a}$. We have seen that particular values of the adimensional parameters μ, χ determine the adimensional KB constants α, β, and C, which in turn determine λ and ν at every $x \in [0, 1]$. The values of μ and χ are restricted by the condition that no horizon is included in the external region described by the RN metric.

We consider all possible roots of the equation $g_{00} = 0$. The radius of the charged sphere a is big enough so that no horizons are included in the external RN metric. The three possible cases follow.

A. Two real roots

$\mu^2 > \chi^2$.

We choose $1 > \mu + \sqrt{\mu^2 - \chi^2}$. Therefore, $\chi < 1$ and μ satisfies $\chi < \mu < \frac{1+\chi^2}{2}$.

B. One real root

$\mu = \chi$.

We choose $1 > \mu = \chi$.

C. No real roots

$\mu < \chi$, otherwise arbitrary.

The selected values of μ and χ determine values of α and β, which should satisfy the energy conditions. Another acceptability condition is that $\tilde{\epsilon}(x) \geq 0$ for every $x \in [0, 1]$.

The arising expressions for ρ, p_r, and p_t can be evaluated at $r = 0$. Hence we find the energy density and pressures at $r = 0$ as

$$\rho_0 = \frac{3A}{8\pi} = \frac{3\alpha}{8\pi a^2}, \quad (43)$$

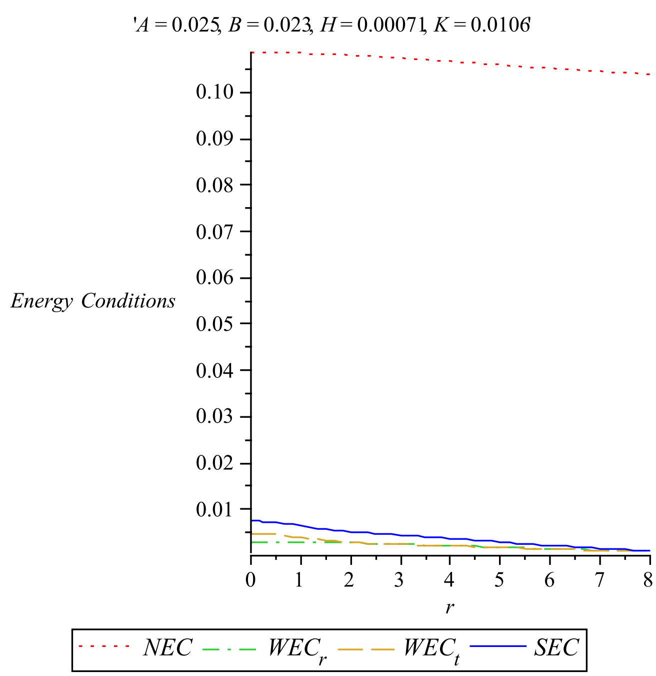

FIG. 6 (color online). The variations of the left-hand side of the expressions of energy conditions are shown against r.

$$p_{r0} = \frac{1}{2}p_{t0} = \frac{2B - A}{8\pi} = \frac{2\beta - \alpha}{8\pi a^2}. \qquad (44)$$

Now, the energy conditions [38] at the center can be written as follows:

(i) NEC: $p_{r0} + \rho_0 \geq 0 \Rightarrow \alpha + \beta \geq 0$.

(ii) WEC: $p_{r0} + \rho_0 \geq 0 \Rightarrow \alpha + \beta \geq 0$ and $\rho_0 \geq 0 \Rightarrow \alpha \geq 0$.

(iii) SEC: $p_{r0} + \rho_0 \geq 0 \Rightarrow \alpha + \beta \geq 0$ and $3p_{r0} + \rho_0 \geq 0 \Rightarrow \beta \geq 0$.

(iv) DEC: $\rho_0 > |p_{r0}| \Rightarrow 2\alpha \geq \beta \geq \alpha$.

The characterization of dark energy fluid is the violation of one of the SEC, more specifically, the one related to the Raychaudhuri equation [25,39]. If the second equation of the WEC is violated, we have a phantom dark energy fluid.

Now, the EOS at $r = 0$ is

$$\frac{p_{r0}}{\rho_0} = m, \qquad (45)$$

where

$$m = \frac{2\beta - \alpha}{3\alpha} \leq \frac{\beta}{2\alpha} \leq 1. \qquad (46)$$

Notice that ρ and p_r are decreasing functions of r (these can be shown by plotting the graphs of ρ and p_r or one can find $\frac{d\rho}{dr} < 0$ and $\frac{dp_r}{dr} < 0$, i.e., ρ and p are decreasing functions of r). At $r = 0$, they assume fixed values ρ_0 and p_{r0}. So, ρ and p have a maximum at $r = 0$. We have checked that $\rho_0' = 0$, $p_{r0}' = 0$ and $\rho_0'' < 0$, $p_{r0}'' < 0$.

VII. STABILITY

Bertolami and Páramos [22] argue that if one assumes that the GCG tends to a smooth distribution over space then most density perturbations tend to be flattened within a time scale related to their initial size and the characteristic speed of sound.

One of the important "physical acceptability conditions" for anisotropic matter are the squares of radial and tangential sound speeds, defined by

$$v_{sr}^2 = \frac{dp_r}{d\rho} = H + \frac{K}{[\frac{f+\sqrt{f^2+2hK}}{h}]^2}, \qquad (47)$$

and

$$v_{st}^2 = \frac{dp_t}{d\rho} = 1 - \frac{\sqrt{f^2+2hK}}{4\rho r A(A+B)} \times \left[\frac{2}{r^2}e^{Ar^2} - 4rA(B-A)(1+Br^2) - \frac{2}{r^3} - \frac{2A}{r}\right], \qquad (48)$$

should be less than the speed of light [40,41].

From the above equation (47), an important aspect can be observed that the squared of radial sound velocity is always positive irrespective of matter density and hence this is always positive even in the case of exotic matter. Figures 7 and 8 show that these parameters satisfy the inequalities $0 \leq v_{sr}^2 \leq 1$ and $0 \leq v_{st}^2 \leq 1$ everywhere within the charged fluid.

Now, we use Herrera's approach [40] to identify potentially unstable or stable anisotropic matter configurations

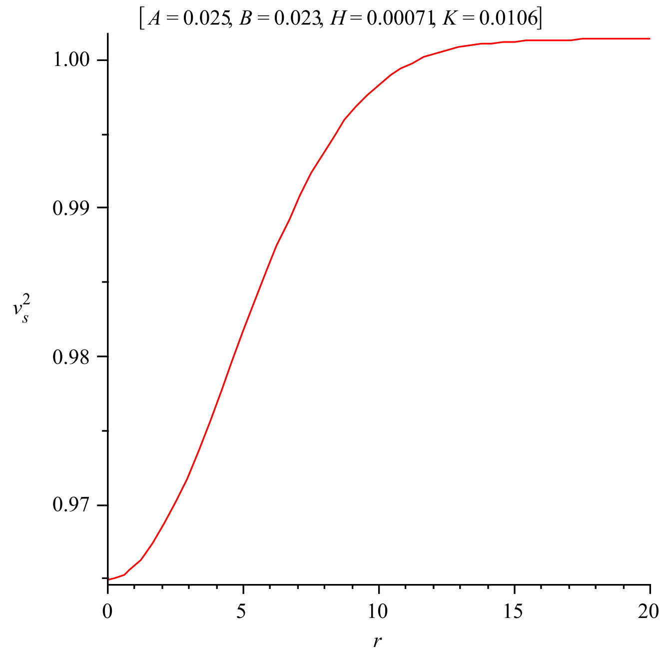

FIG. 7 (color online). The variation of radial sound speed v_{sr}^2 is shown against r.

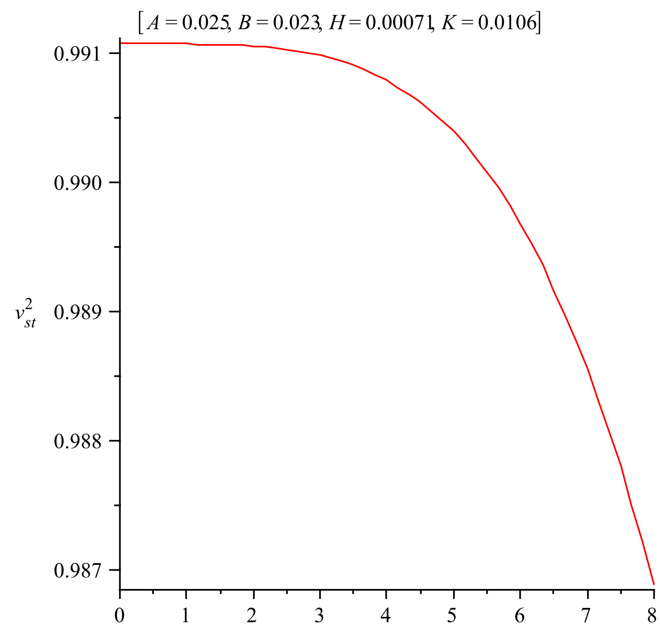

FIG. 8 (color online). The variation of tangential sound speed v_{st}^2 is shown against r.

known as the concept of cracking (or overturning). Since, $0 \leq v_{\rm sr}^2 \leq 1$ and $0 \leq v_{\rm st}^2 \leq 1$, therefore, according to [40,42], $| v_{\rm st}^2 - v_{\rm sr}^2 | \leq 1$. Figure 9 of the model also supports this.

Now,

$$-1 \leq v_{\rm st}^2 - v_{\rm sr}^2 \leq 1$$

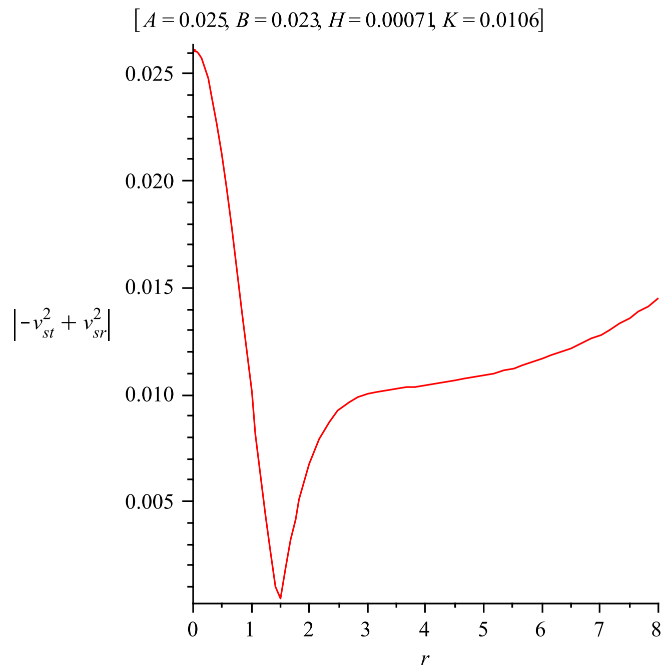

FIG. 9 (color online). The variation of $| v_{\rm st}^2 - v_{\rm sr}^2 |$ is shown against r.

implies

$$-1 \leq v_{\rm st}^2 - v_{\rm sr}^2 \leq 0, \quad \text{potentially stable},$$

$$0 < v_{\rm st}^2 - v_{\rm sr}^2 \leq 1, \quad \text{potentially unstable}.$$

One can note that the region for which $v_{\rm st}^2 < v_{\rm sr}^2$ is a potentially stable region and the region for which $v_{\rm st}^2 > v_{\rm sr}^2$ is a potentially unstable region. If $v_{\rm st}^2 - v_{\rm sr}^2$ keeps the same sign everywhere within a matter distribution, no cracking will occur. The curve profile (Fig. 10) for $v_{\rm st}^2 - v_{\rm sr}^2$ indicates that there is a change of sign and thus alternating a potentially unstable to a stable region within the distribution.

VIII. MINIMUM MASS-RADIUS RELATION

The above analysis indicates that our model is very much unstable within radius 1.5 unit. But, the configuration is stable within $1.5 < r \leq 8$. In a recent paper, Andréasson [42] discovered a surprising result as

$$\sqrt{M} < \frac{\sqrt{R}}{3} + \sqrt{\frac{R}{9} + \frac{Q^2}{3R}}$$

for a lower bound on the radius R of a charged sphere with mass M and charge Q.

The inequality is shown to hold for any solution which satisfies $p_r + 2p_t \leq \rho$.

The plot (Fig. 11) for $p_r + 2p_t - \rho$ against r indicates that in the region $1.5 < r \leq 8$, $p_r + 2p_t - \rho$ is negative. Since our model is stable within $1.5 < r \leq 8$, Andréasson's relation holds good for our model.

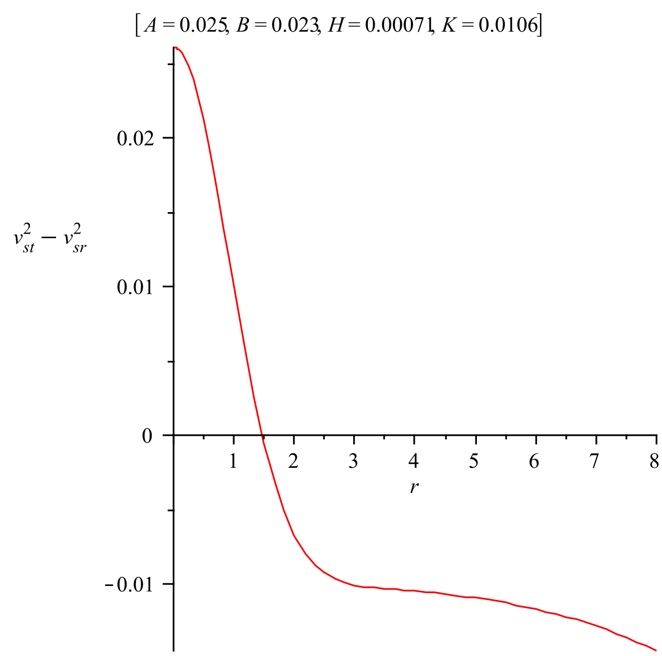

FIG. 10 (color online). The variation of $v_{\rm st}^2 - v_{\rm sr}^2$ is shown against r.

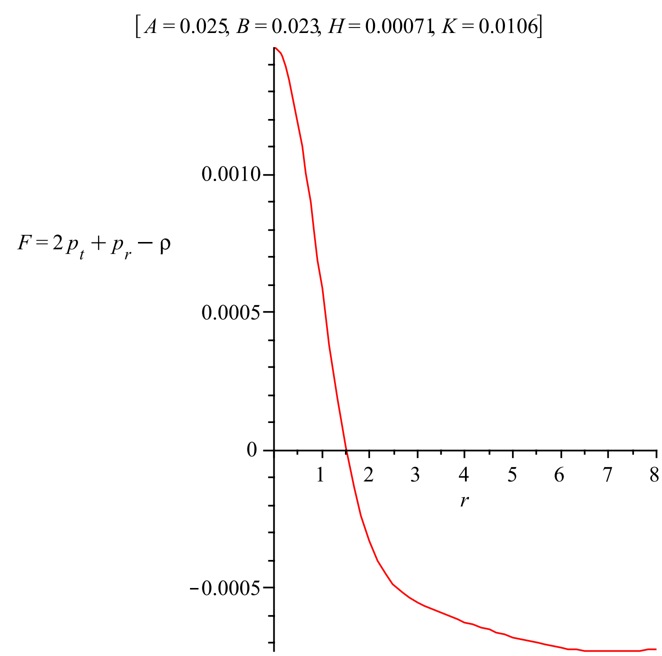

FIG. 11 (color online). The variation of $2p_t + p_r - \rho$ is shown against r.

It would be interesting to make some comments regarding opposite situations with the maximum mass-radius relation. By using the static spherically symmetric gravitational field equations Buchdahl [1] has obtained an absolute constraint of the maximally allowable mass radius for isotropic fluid spheres of the form $\frac{2M}{R} < \frac{8}{9}$ (for a generalized expression we refer to the work of Mak *et al.* [43]).

It is worthwhile to calculate effective gravitational mass which is due to the contribution of the energy density ρ of the matter and the electric energy density $\frac{E^2}{8\pi}$ and can be expressed as

$$M_{\rm effective} = 4\pi \int_0^R \left[\rho + \frac{E^2}{8\pi}\right] r^2 dr = \frac{1}{2}R + \frac{1}{\sqrt{A}}\gamma\left(\frac{3}{2}, AR^2\right) - \frac{1}{2\sqrt{A}}\gamma\left(\frac{1}{2}, AR^2\right), \qquad (49)$$

where $\gamma(\frac{3}{2}, AR^2)$ is the lower incomplete gamma function. In Fig. 12, we plot the mass-radius relation. We also plot $\frac{M_{\rm effective}}{R}$ against R (see Fig. 13) which shows that the ratio $\frac{M_{\rm effective}}{R}$ is decreasing even if the radius is increasing with the mass.

According to Ponce de León [37] the energy conditions require

$$e^{-\lambda} \leq 1, \qquad \nu' \geq 0.$$

These relations lead to a maximum charge as follows:

$$R \geq \frac{q^2}{M},$$

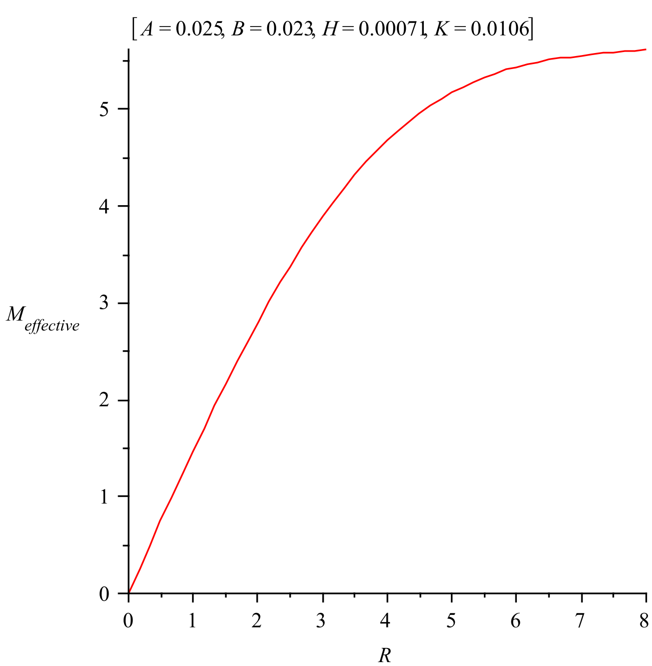

FIG. 12 (color online). The variation of $M_{\rm effective}$ is shown against R.

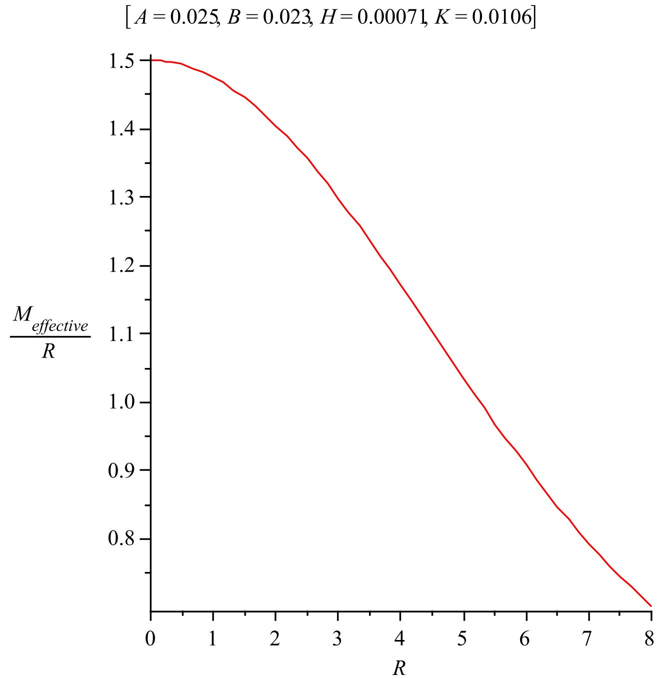

FIG. 13 (color online). The variation of $\frac{M_{\rm effective}}{R}$ is shown against R.

where M and q represent the total mass and charge of the charged sphere of radius R.

Assuming $R = 8$ km and $\frac{M}{R} = 0.5$, we find from Eq. (15) as $q^2 = 0.203\,72$. Thus our model satisfies Ponce de León's condition [37].

In this connection we add that the mass-radius-charge relation for compact astrophysical objects plays an important role in many physical processes. The strong gravitational field due to the density of the matter inside the stars indicates that a strong electric field due to the electric charge is possible. The effects of electric charge in compact stars assuming that the charge distribution is proportional to the mass density were studied recently by several authors [16,44–46].

IX. CONCLUSIONS

In this paper we are checking the energy conditions only at the center of the charged sphere. It would be convenient to extend the analysis to other points within the sphere. A series method like the one used in Eqs. (32) and (34) by KB [15] might be useful.

Unlike the work of Bertolami and Páramos [22], where they have used the generalized Chaplygin gas EOS in special reference to anisotropic pressure we generate the solutions for KB metric under Einstein-Maxwell spacetime. So a natural question would be, does the result go over into the solution of Bertolami *et al.* for isotropic stresses? The straightforward answer is no. This is because we have extended the KB approach assuming a singularity-free form of the metric ansatz to charged anisotropic source with a nonlinear, Chaplygin-type equation of state. Therefore, whether our solution corresponds to a Chaplygin dark star needs special verification, specifically whether a charged Chaplygin dark star exists demands further investigation. In a similar fashion one may raise the question, does the result go over into an exact solution for an EOS $p = H\rho$ ($K = 0$)? The answer this time is affirmative, as our solution coincides with the solution obtained by Varela *et al.* [18] for an EOS $p = H\rho$ ($K = 0$). One can easily see that our results go over into the expressions obtained by Varela *et al.* with $\alpha_1 = 0$. Also, it may be interesting to extrapolate the present investigation to the astrophysical bodies, especially quark or strange stars with radius around 8 km.

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APPENDIX: ANALYSIS OF JUNCTION CONDITIONS

We note that the metric coefficients are continuous at the junction, i.e., at S, where $r = a$. However, this does not mean that the metric coefficients are differentiable at the junction. The affine connection may be discontinuous there. The above statement can be quantified in terms of a second fundamental form of the boundary.

The second fundamental forms associated with the two sides of the shell are [47–50]

$$K_{ij}^{\pm} = -n_{\nu}^{\pm}\left[\frac{\partial^2 X_{\nu}}{\partial\xi^i\partial\xi^j} + \Gamma_{\alpha\beta}^{\nu}\frac{\partial X^{\alpha}}{\partial\xi^i}\frac{\partial X^{\beta}}{\partial\xi^j}\right]\Bigg|_S, \tag{A1}$$

where $n_{\nu}^{\pm}$ are the units normal to S and can be given by

$$n_{\nu}^{\pm} = \pm\left|g^{\alpha\beta}\frac{\partial f}{\partial X^{\alpha}}\frac{\partial f}{\partial X^{\beta}}\right|^{-(1/2)}\frac{\partial f}{\partial X^{\nu}} \tag{A2}$$

with $n^{\mu}n_{\mu} = 1$. Here, ξ^i are the intrinsic coordinates on the shell where $f = 0$ is the parametric equation of the shell S and $-$ and $+$ corresponds to interior (ours) and exterior (RN). It is to be noted that since the shell is infinitesimally thin in the radial direction there is no radial pressure. Using Lanczos equations [47–50], one can find the surface energy term Σ and surface tangential pressures $p_{\theta} = p_{\phi} \equiv p_t$ as

$$\Sigma = -\frac{1}{4\pi a}[\sqrt{e^{-\lambda}}]_{-}^{+}, \tag{A3}$$

$$p_t = \frac{1}{8\pi a}\left[\left(1 + \frac{a\nu'}{2}\right)\sqrt{e^{-\lambda}}\right]_{-}^{+}. \tag{A4}$$

The metric functions are continuous on S, and then one finds

$$\Sigma = 0, \tag{A5}$$

$$p_t = \frac{1}{8\pi a}\left[\left(\frac{M}{a} - \frac{Q^2}{a^2}\right)\sqrt{1 - \frac{2M}{a} + \frac{Q^2}{a^2}} - Aa^2\sqrt{1 - \frac{2M}{a} + \frac{Q^2}{a^2}}\right]. \tag{A6}$$

Hence one can match our interior solution with an exterior RN solution in the presence of a thin shell. The whole space-time is given by our metric and the RN metric which are smoothly joined.

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