PHYSICAL REVIEW D **82,** 104032 (2010)

Broken symmetry as a stabilizing remnant

Sandy S. C. Law[1,*] and Kristian L. McDonald[2,3,†]

[1]*Department of Physics, Chung Yuan Christian University, Chung-Li, Taiwan 320, Republic of China*
[2]*TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia, V6T 2A3, Canada*
[3]*Max-Planck-Institut für Kernphysik, Postfach 10 39 80, 69029 Heidelberg, Germany*
(Received 2 September 2010; published 16 November 2010)

The Goldberger-Wise mechanism enables one to stabilize the length of the warped extra dimension employed in Randall-Sundrum models. In this work we generalize this mechanism to models with multiple warped throats sharing a common ultraviolet brane. For independent throats this generalization is straightforward. If the throats possess a discrete interchange symmetry like Z_n, the stabilizing dynamics may respect the symmetry, resulting in equal throat lengths, or they may break it. In the latter case the ground state of an initially symmetric configuration is a stabilized asymmetric configuration in which the throat lengths differ. We focus on two- (three-) throat setups with a Z_2 (Z_3) interchange symmetry and present stabilization dynamics suitable for either breaking or maintaining the symmetry. Though admitting more general application, our results are relevant for existing models in the literature, including the two-throat model with Kaluza-Klein parity, and the three-throat model of flavor with a broken Z_3 symmetry.

DOI: 10.1103/PhysRevD.82.104032 PACS numbers: 04.50.Cd, 12.15.Ff, 14.60.Pq

I. INTRODUCTION

The ultraviolet (UV) sensitivity of the Higgs mass in the standard model (SM) makes it difficult to understand how the Higgs can remain light in the presence of heavy new physics. Mass corrections resulting from top quark loops can dominate the Higgs mass, dragging it up to the cutoff scale and thereby necessitating a fine-tuning to preserve a light Higgs. This "hierarchy problem" provides perhaps our best indication that new physics is likely to appear at the TeV scale and it is hoped that the LHC will soon shed light on this matter.

The Randall-Sundrum (RS) model [1] provides a candidate solution to the hierarchy problem. This framework allows one to generate natural scale hierarchies as the infrared (IR) scale is realized as a warped down incarnation of the Planck scale, $M_{\rm IR} \sim e^{-kL} M_{\rm Pl}$, where $k(L)$ is the curvature (length) of the warped extra dimension. Provided one can naturally realize the hierarchy $kL \simeq \mathcal{O}(10)$, the weak scale can be generated with $M_{\rm IR} \sim$ TeV and the Higgs mass protected from large corrections. The relationship between the curvature and L is determined by the dynamics that stabilize the extra dimension and the question of whether the RS model naturally realizes the weak/Planck hierarchy translates into the need for stabilization dynamics that naturally generate $kL \simeq \mathcal{O}(10)$.

Goldberger and Wise (GW) [2] have presented a mechanism that successfully generates $kL \simeq \mathcal{O}(10)$ without the need for input parameter hierarchies, thereby showing that the RS model provides a genuine solution to the hierarchy problem. The GW mechanism employs a bulk scalar field with brane localized potentials that force the scalar to acquire distinct nonzero values at the branes. The resulting interplay between the shearing energy (which prefers the extra dimension to be large) and the potential energy (which tends to shrink the extra dimension) of the background scalar solution stabilizes the extra dimension at a fixed finite value. Alternative methods for stabilizing L have also appeared [3].

Besides the Planck and weak scales there may be other scales present in nature. Examples of such scales occur generically in models with gauge UV completions of the type $\mathcal{G}_{\rm UV} \supset \mathcal{G}_{\rm SM}$, where $\mathcal{G}_{\rm UV}$ may be a grand unified gauge group, or some other gauge extension of the SM, that is broken to $\mathcal{G}_{\rm SM}$ at a posited high energy scale. The flavor sector of the SM is also suggestive of new scales in nature if the Yukawa couplings emerge as powers of a dimensionless ratio $\langle \phi_f \rangle / \Lambda$ for some flavon fields ϕ_f and a cutoff Λ. Further scales may exist if the dark or hidden sector of the Universe is not directly connected to the weak scale. The dark matter itself may acquire its mass by a distinct means to the SM fields, but even if the dark matter obtains a weak scale mass it can couple to forces whose mass scale is much lighter [4].

If the RS approach to the weak/Planck hierarchy is realized in nature it is natural to ask if additional scales can also be accommodated in this framework. The gravitational background of the RS model can be referred to as a warped throat and one can extend the RS gravitational background by considering multiple warped throats glued together at a common UV brane [5]. If the IR scales of the distinct throats differ the warping that realizes the

*slaw@cycu.edu.tw
†Kristian.McDonald@mpi-hd.mpg.de

weak/Planck hierarchy in RS models can also be employed to obtain additional mass scales.[1] Such setups may have a number of applications, including electroweak [7], flavor [8,9], leptogenesis [10], axion [11], and hidden sector physics [12,13]. If the IR scales of multiple throats are related by symmetry one can also motivate dark matter candidates [14]. As in RS models one must ensure that the throat lengths are suitably stabilized to naturally generate multiple hierarchical scales in multithroat models.

In this work we generalize the GW mechanism to models with multiple warped throats sharing a common ultraviolet brane. We consider both independent throats[2] and throats possessing a discrete interchange symmetry like Z_n. In the former case the generalization is straightforward. In the latter case the stabilizing dynamics may respect the symmetry, resulting in equal throat lengths, or they may break it, producing an asymmetric configuration in which the throat lengths differ. We shall focus on two- (three-) throat setups with a Z_2 (Z_3) interchange symmetry and present stabilization dynamics suitable for either breaking or maintaining the symmetry. Though admitting more general application, our results are relevant for existing models in the literature and we shall draw attention to some of these as appropriate.

One of the main points that we seek to bring to the readers attention is the new approach to discrete symmetry breaking afforded by our constructs. Models with discrete symmetries in four dimensions typically break the symmetry spontaneously with a weakly coupled Higgs or via strong dynamics at some energy scale. This symmetry breaking scale usually maps to some IR scale in the theory, like the top mass or some other fermion mass scale in models with discrete flavor symmetries, but must also be sufficiently shielded from SM fields to ensure compatibility with observations. One interesting aspect of discrete symmetry breaking via GW scalars is that the symmetry breaking can occur entirely at a high energy scale, like the Planck scale, through UV localized dynamics and subsequently feed into the IR through the emergence of distinct IR scales. This is interesting in the case of a discrete flavor symmetry, like that recently discussed in [8], as the SM fields need not couple directly to the source of symmetry breaking in the UV. They may instead couple to IR flavor fields in a flavor symmetric way and yet exist as part of a flavor asymmetric theory in the IR. We suspect that our ideas may admit interesting applications to flavor model building and, in particular, geometric throat arrangements may provide an alternative extra dimensional approach to discrete flavor symmetries to that presented in [15].

[1]See [6] for an alternative way to realize a sub-TeV hidden sector scale in addition to the weak/Planck hierarchy in the RS framework.

[2]In this work an "independent throat" refers to a throat that is part of a multithroat background but is not related to any of the other throats by an interchange symmetry.

Before proceeding we note that the field-theoretic approach to multithroat models [5] can be motivated by the fact that string realizations of the RS model can contain additional warped throats [16]. In the string picture these emerge from the compact space that acts as the UV brane in the RS approach and the notion of multiple throats glued together in the UV serves to model this string picture. Earlier phenomenological applications of the multithroat setup can be found in [17].

The organization of this paper is as follows. In Sec. II we present a single throat calculation to remind the reader of the GW methodology and set our notations. We generalize the GW mechanism to two independent throats in Sec. III. We consider two throats related by a Z_2 symmetry in Sec. IV and three throats related by a Z_3 in Sec V. In both cases we present symmetry preserving and symmetry breaking GW mechanisms. Section VI contains some comments on models with $n > 3$ throats and the paper concludes in Sec. VII.

II. GW MECHANISM IN A SINGLE THROAT

We would like to present an example calculation to demonstrate the approach of GW. Rather than summarizing the single throat calculation whose details can be found in [2] we present a slightly modified calculation which, despite failing to successfully stabilize the length of the extra dimension, serves to demonstrate the methodology and sets our notations. It is also useful in helping us understand features that emerge in some of the multithroat calculations that follow.

The metric we employ is defined by the interval

$$ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2 \equiv G_{MN}dx^M dx^N, \qquad (1)$$

where $M, N, \ldots$ ($\mu, \nu, \ldots$) are the 5D (4D) Lorentz indices, the extra dimension is labeled by $y \in [0, L]$, and the UV (IR) brane is located at $y = 0$ ($y = L$). GW considered a bulk scalar in the above background with bulk action [2]

$$S_B = \frac{1}{2}\int d^4x dy \sqrt{G}(G^{MN}\partial_M\phi\partial_N\phi - m^2\phi^2), \qquad (2)$$

and brane localized actions

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\bar{\lambda}(\phi^2 - u^2)^2, \qquad (3)$$

$$S_{\rm IR} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm ir}}\lambda(\phi^2 - v^2)^2, \qquad (4)$$

where $g^{\rm uv}_{\mu\nu}$ and $g^{\rm ir}_{\mu\nu}$ are the restrictions of $G_{\mu\nu}$ to $y = 0$ and $y = L$, respectively. Note that the boundary actions do not possess any odd terms in the field ϕ and consequently the entire action is invariant under a Z_2 symmetry $\phi \to -\phi$. Also, we work with $m^2 > 0$ here and in the generalizations that follow.

We shall consider the case where the bulk and IR brane actions retain their form S_B and S_{IR} but modify the UV brane potential to[3]

$$S_{UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{uv}}\bar{\lambda}(\phi^2 + u^2)^2, \tag{5}$$

with $u^2 > 0$. Variation of S_B produces the bulk equation of motion for ϕ as

$$\partial_N[\sqrt{G}G^{MN}\partial_M\phi] + \sqrt{G}m^2\phi = 0, \tag{6}$$

which for $\partial_\mu\phi = 0$ has the solution

$$\phi(y) = Ae^{\beta_+ ky} + Be^{\beta_- ky}, \tag{7}$$

where

$$\beta_\pm = 2 \pm \sqrt{4 + \frac{m^2}{k^2}} \equiv 2 \pm \nu. \tag{8}$$

The bulk solution must satisfy the following boundary conditions (BCs):

$$\partial_y\phi(0) - 2\bar{\lambda}(\phi^2(0) + u^2)\phi(0) = 0, \tag{9}$$

$$\partial_y\phi(L) + 2\lambda(\phi^2(L) - v^2)\phi(L) = 0, \tag{10}$$

which, in the limit of large λ, $\bar{\lambda}$, are

$$\phi(0) = \delta\phi(0), \tag{11}$$

$$\phi(L) = v + \delta\phi(L), \tag{12}$$

where[4] $\delta\phi = \mathcal{O}(\lambda^{-1})$.

Momentarily ignoring the $\delta\phi$ corrections the leading order IR boundary contributions to the potential for the length of the extra dimension vanish and $V(L)$ can be written as

$$V(L) = V_B(L) + V_{UV}, \tag{13}$$

where $V_{UV} = \bar{\lambda}u^4/2 + \mathcal{O}(\lambda^{-1})$ and the bulk piece is

$$V_B(L) = \frac{k}{2}[(\nu+2)A^2(e^{2\nu kL} - 1) + (\nu - 2)B^2 (1 - e^{-2\nu kL})]. \tag{14}$$

Enforcing the BCs on the bulk solution gives

$$A = -B \simeq ve^{-\beta_+ kL}, \tag{15}$$

and for $m^2/k^2 < 1$ we may follow [2] and write $\nu = 2 + \epsilon$ where $\epsilon \simeq m^2/4k^2$, giving

$$V(L) = 2kA^2(e^{2\nu kL} - 1) + \mathcal{O}(\epsilon), \tag{16}$$

$$\simeq 2kv^2e^{-4kL} + \cdots, \tag{17}$$

where the dots denote the constant piece and terms of $\mathcal{O}(\lambda^{-1})$ and $\mathcal{O}(\epsilon)$. This is the leading order potential L. We would like to also determine the $\mathcal{O}(\lambda^{-1})$ corrections to the potential. The corrections to the BCs are found to be

$$\delta\phi(0) = \frac{k}{2\bar{\lambda}u^2}(A\beta_+ + B\beta_-) \simeq \frac{2k}{\bar{\lambda}u^2}e^{-(4+\epsilon)kL}v, \tag{18}$$

$$\delta\phi(L) = -\frac{k}{4\lambda v^2}(A\beta_+e^{\beta_+kL} + B\beta_-e^{\beta_-kL}) \simeq -\frac{k}{\lambda v^2}v, \tag{19}$$

which give the following corrections to A, B:

$$\delta A = \frac{(\delta\phi(L) - \delta\phi(0)e^{\beta_-kL})}{e^{\beta_+kL} - e^{\beta_-kL}} \simeq -e^{-\beta_+kL}v\frac{k}{\lambda v^2}, \tag{20}$$

$$\delta B = \frac{(\delta\phi(0)e^{\beta_+kL} - \delta\phi(L))}{e^{\beta_+kL} - e^{\beta_-kL}} \simeq e^{-\beta_+kL}v\left[\frac{k}{\lambda v^2} + \frac{2k}{\bar{\lambda}u^2}\right]. \tag{21}$$

This produces a correction to the bulk potential for L:

$$\delta V_B(L) \simeq -4kv^2e^{-4kL}\left[\frac{k}{\lambda v^2}\right] + \cdots, \tag{22}$$

and results in a nonzero contribution to $V(L)$ from the boundary potentials:

$$\delta V_{IR}(L) \simeq 2kv^2e^{-4kL}\left[\frac{k}{\lambda v^2}\right] + \cdots, \tag{23}$$

$$V_{UV}(L) \simeq \frac{\bar{\lambda}u^4}{2} + 4kv^2e^{-4kL}\left[\frac{k}{\bar{\lambda}u^2}e^{-(4+2\epsilon)kL}\right] + \cdots. \tag{24}$$

Putting all these results together the complete potential for L through $\mathcal{O}(\lambda^{-1})$ is given by

$$V(L) = V_B + \delta V_B + \delta V_{IR} + V_{UV}$$
$$\simeq 2ke^{-4kL}v^2\left[1 - \frac{k}{\lambda v^2}\right] + \frac{\bar{\lambda}u^4}{2} + \cdots, \tag{25}$$

the minimum of which corresponds to $L \to \infty$. Thus we learn that if the GW scalar has a potential on the UV brane whose minimum corresponds to a vanishing brane vacuum expectation value (VEV), the potential for the length of the extra dimension does not stabilize L at a finite value. Instead the radius runs away and the extra dimension is

[3]Note that the minimum of this UV potential is nonzero and will therefore contribute to the UV brane tension. In RS models the UV brane tension must be related to the bulk cosmological constant (CC) to ensure a vanishing 4D CC. In the presence of the UV action (5) one should shift the usual RS brane tension $V_{uv} \to V_{uv} - \Delta V_{uv}$ to cancel out this additional boundary tension and retain the standard RS solution. This constitutes a modified version of the usual tuning of the 4D CC present in RS models. It will be understood in the present work that this shift of the UV brane tension has been undertaken whenever the UV potential has a nonvanishing minimum so the usual RS background solution holds.

[4]We generically refer to quartic parameters by λ so that $\mathcal{O}(\lambda^{-1})$ also stands for terms of $\mathcal{O}(\bar{\lambda}^{-1})$. This applies throughout the paper.

not compactified. This feature will occur in some of the multithroat scenarios below.

If one instead uses the UV action (3) as in [2] the calculation carries through in the same fashion, however instead of (25) one obtains

$$V(L) \simeq 2ke^{-4kL}(v - ue^{-\epsilon kL})^2\left[1 - \frac{k}{\lambda v^2}\right] + \cdots. \quad (26)$$

This potential differs from (25) in an important way since the minimum is now at

$$L = \frac{1}{\epsilon k}\ln\left(\frac{u}{v}\right), \quad (27)$$

and the extra dimension is stabilized at a finite value. This is the GW result. Notice that values of $Lk \sim \mathcal{O}(10)$ are easily obtained, an important feature that is necessary for the IR scale to be naturally much less than the Planck scale, $M_{\rm IR} \sim e^{-kL}M_{\rm Pl} \ll M_{\rm Pl}$, as required to solve the hierarchy problem [1].

For future reference we note that the observed failure of the GW scalar to stabilize the extra dimension for a UV potential minimized by $\phi(0) = 0$ holds more generally for nonzero $\phi(0) = u$ if $u < v$. If the energy density of the scalar profile is dominated in the IR a runaway solution is preferred as increasing the size of the extra dimension redshifts this energy. The stable solution therefore requires $u/v > 1$.

Following [2] we have neglected the backreaction of the GW scalar on the metric in the above analysis. This is acceptable provided $u^2/M_*^3 \ll 1$ and $v^2/M_*^3 \ll 1$, where the 5D gravity scale M_* satisfies $M_{\rm Pl}^2 \simeq M_*^3/k$. The backreaction has been considered for a modified version of the GW analysis in [18,19] and more recently a partial inclusion of the backreaction in the GW scenario has appeared [20]. Related analysis can also be found in [21]. We shall not consider the backreaction in the generalizations that follow and one should keep in mind that appropriate (and obvious) generalizations of the conditions u^2/M_*^3, $v^2/M_*^3 \ll 1$ must also hold.

We note that variations of the GW mehcanism for soft-wall models have appeared [22] and that a 4D interpretation of the GW mechanism, based on AdS/CFT, exists [23]. In the dual 4D picture GW stabilization corresponds to perturbing the CFT by introducing an "almost marginal" operator that explicitly breaks conformal invariance in the UV. The (running) coupling of this operator is determined by the profile for the bulk scalar in the 5D picture, and is such that it runs slowly until reaching some critical value in the IR, at which point it triggers a further spontaneous breaking of the conformal symmetry. In multithroat models the separate throats are dual to distinct CFT's that couple via a common UV source of conformal symmetry breaking (the common UV brane). Models with interchange symmetries among throats in 5D are therefore dual to 4D theories with a discrete interchange symmetry among distinct CFT's, with the almost marginal operators that perturb the CFT's in the UV being identical. Models with distinct throat lengths correspond to CFT's with different UV perturbations whose couplings therefore reach the critical value at which spontaneous symmetry breaking occurs at different scales in the IR.

III. GW MECHANISM FOR TWO INDEPENDENT THROATS

In this section we shall generalize the GW mechanism to models with two independent warped throats glued together at a common UV brane. We label the two throats as $i = 1, 2$, with the metric in the ith throat defined by

$$ds_i^2 = e^{-2ky_i}\eta_{\mu\nu}dx^\mu dx^\nu - dy_i^2 \equiv G^i_{MN}dx_i^M dx_i^N, \quad (28)$$

where $x_i^\mu \equiv x^\mu$ are the 4D coordinates and the warped extra dimensions are labeled by $x_i^5 \equiv y_i \in [0, L_i]$. The IR branes are located at $y_i = L_i$ and the common UV brane sits at $y_i = 0,\ \forall\ i$. The gravitational sources necessary to realize this background have been discussed in [5] and we refer the reader there for details. In this section and throughout we consider IR-UV-IR–type constructs. The kinetic term for the massless radion in such setups has the correct sign so the stability issues associated with the UV-IR-UV setups are not present. Also note that we consider a common bulk cosmological constant for the two throats so that the curvature k is the same in both throats.

We consider a GW scalar ϕ_i in each throat with the bulk action in the ith throat given by

$$S_B^i = \frac{1}{2}\int d^4x dy_i \sqrt{G^i}(G_i^{MN}\partial_M\phi_i\partial_N\phi_i - m_i^2\phi_i^2). \quad (29)$$

The IR brane localized actions are

$$S_{\rm IR}^i = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm ir}^i}\lambda_i(\phi_i^2 - v_i^2)^2, \quad (30)$$

and the action on the common UV brane is

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\left\{\sum_i\bar{\lambda}_i(\phi_i^2 - u_i^2)^2 + \kappa\phi_1^2\phi_2^2\right\}, \quad (31)$$

where $g^{\rm uv}_{\mu\nu}$ and $(g^{\rm ir}_{\mu\nu})^i$ are the restrictions of $G^i_{\mu\nu}$ to $y_i = 0$ and $y_i = L_i$, respectively. The total scalar action is thus

$$S = \sum_i(S_B^i + S_{\rm IR}^i) + S_{\rm UV}, \quad (32)$$

which does not possess any odd terms in the fields ϕ_i and is therefore invariant under two independent Z_2 symmetries:

$$Z_2^{(i)}:\ \phi_i \to -\phi_i. \quad (33)$$

These symmetries generalize the discrete symmetry of the GW mechanism and serve primarily to simplify the calculation.

Variation of S_B^i produces the bulk equation of motion for ϕ_i in the ith throat:

$$\partial_N[\sqrt{G^i}G_i^{MN}\partial_M\phi_i] + \sqrt{G^i}m_i^2\phi_i = 0, \tag{34}$$

which for $\partial_\mu\phi_i = 0$ has the solution

$$\phi_i(y_i) = A_i e^{\beta_+^i k y_i} + B_i e^{\beta_-^i k y_i}, \tag{35}$$

where

$$\beta_\pm^i = 2 \pm \sqrt{4 + \frac{m_i^2}{k^2}} \equiv 2 \pm \nu_i = 2 \pm (2 + \epsilon_i), \tag{36}$$

and $\epsilon_i \simeq m_i^2/4k^2$ for $m_i < k$. The demand that the variation of the action vanish on the boundaries leads to the BCs:

$$\partial_y\phi_i(0) - 2\bar{\lambda}_i(\phi_i^2(0) - u_i^2)\phi_i(0) - \kappa\phi_j^2(0)\phi_i(0) = 0, \quad i \neq j, \tag{37}$$

$$\partial_y\phi_i(L_i) + 2\lambda_i(\phi_i^2(L_i) - v_i^2)\phi_i(L_i) = 0. \tag{38}$$

The potential for the throat lengths L_i is a sum of contributions from both the bulks and the branes and may be written as

$$V(L_1, L_2) = \sum_i[V_B^i(L_i) + V_{\rm IR}^i(L_i)] + V_{\rm UV}(L_1, L_2). \tag{39}$$

Inserting the solution (35) into the ith bulk action and integrating over the extra dimension determines the bulk contribution to the potential,

$$V_B^i(L_i) = \frac{k}{2}[(\nu_i + 2)A_i^2(e^{2\nu_i k L_i} - 1) + (\nu_i - 2)B_i^2(1 - e^{-2\nu_i k L_i})]. \tag{40}$$

As in the GW case we consider large λ, $\bar{\lambda}$, so the leading order BCs are $\phi_i(0) = u_i$ and $\phi_i(L_i) = v_i$, giving

$$A_i \simeq e^{-\beta_+^i k L_i} v_i - u_i e^{-2\nu_i k L_i}, \tag{41}$$

$$B_i \simeq u_i(1 + e^{-2\nu_i k L_i}) - v_i e^{-\beta_+^i k L_i}, \tag{42}$$

so that

$$V_B^i(L_i) = 2kA_i^2(e^{2\nu_i k L_i} - 1) + \mathcal{O}(\epsilon_i), \tag{43}$$

$$\simeq 2ke^{-4kL_i}(v_i - u_i e^{-\epsilon_i k L_i})^2 + \cdots. \tag{44}$$

To leading order the IR brane contributions vanish and the UV brane contribution is a constant so that $V(L_1, L_2)$ is minimized at

$$L_i = \frac{1}{\epsilon_i k}\ln\left(\frac{u_i}{v_i}\right). \tag{45}$$

As one would expect this matches the GW result as we have effectively neglected the coupling term $\kappa\phi_1^2\phi_2^2$ on the UV to leading order. To find the corrections induced by κ we write

$$\phi_i(0) = u_i + \delta\phi_i(0), \tag{46}$$

$$\phi_i(L_i) = v_i + \delta\phi_i(L_i), \tag{47}$$

and find that

$$\delta\phi_i(L_i) \simeq -\frac{k}{\lambda_i v_i^2}(v_i - u_i e^{-\epsilon_i k L_i}), \tag{48}$$

and

$$\delta\phi_i(0) \simeq \frac{k}{\bar{\lambda}_i u_i^2}\left[(v_i - u_i e^{-\epsilon_i k L_i})e^{-\beta_+^i k L_i} - \frac{\kappa}{4k}u_j^2 u_i\right], \quad i \neq j. \tag{49}$$

Enforcing the corrected BCs on the bulk solution results in the following corrections to A, B:

$$\delta A_i \simeq -e^{-\beta_+^i k L_i}\left\{(v_i - u_i e^{-\epsilon_i k L_i})\frac{k}{\lambda_i v_i^2} - \frac{\kappa}{4}\frac{u_j^2}{\bar{\lambda}_i u_i}e^{-\epsilon_i k L_i}\right\}, \quad i \neq j, \tag{50}$$

$$\delta B_i \simeq e^{-\beta_+^i k L_i}(v_i - u_i e^{-\epsilon_i k L_i})\left[\frac{k}{\lambda_i v_i^2} + \frac{k}{\bar{\lambda}_i u_i^2}\right] - \frac{\kappa}{4}\frac{u_j^2}{\bar{\lambda}_i u_i}, \quad i \neq j. \tag{51}$$

With these results we can calculate the $\mathcal{O}(\lambda^{-1})$ corrections to $V(L_1, L_2)$. These include corrections to the bulk potentials (δV_B^i), the IR boundary potentials ($\delta V_{\rm IR}^i$), and the UV potential, giving

$$V(L_1, L_2) = \sum_i[V_B^i + \delta V_B^i + \delta V_{\rm IR}^i] + V_{\rm UV} \simeq \sum_i 2ke^{-4kL_i}(v_i - u_i e^{-\epsilon_i k L_i})\left\{(v_i - u_i e^{-\epsilon_i k L_i}) \times\left(1 - \frac{k}{\lambda_i v_i^2}\right) + \frac{\kappa u_j^2}{2\bar{\lambda}_i u_i}e^{-\epsilon_i k L_i}\right\} + \cdots, \quad j \neq i, \tag{52}$$

where the dots denote subdominant terms and a constant piece. To leading order in λ^{-1} the minimum of $V(L_1, L_2)$ is given by

$$L_i = \frac{1}{\epsilon_i k}\left\{\ln\left(\frac{u_i}{v_i}\right) + \ln\left(1 - \frac{\kappa u_j^2 v_i^2 \lambda_i}{2\bar{\lambda}_i u_i^2(v_i^2\lambda_i - k)}\right)\right\}, \quad j \neq i. \tag{53}$$

Thus the presence of independent GW scalars in each throat, with a common UV brane coupling, generates a potential for the throat lengths whose minimum is set by finite values of $L_{1,2}$. As in GW one may obtain $L_i k \simeq \mathcal{O}(10)$ without fine-tuning. The minimizing throat lengths return to the usual GW result in each throat in the limit $\kappa \to 0$, which is to be expected as the stabilization dynamics of the two throats decouple in this limit. For finite κ there is a correction to the GW result.

In general the lengths (53) are expected to differ in each throat. As such this generalization of the GW method is useful for models with two independent throats with distinct IR scales, as in [12,13] in which the SM resides in one throat with the usual order TeV IR scale and a hidden sector resides in a second throat with an independent IR scale. Reference [13] employed a hidden IR scale of order a GeV, which corresponds $L_1/L_2 \sim 0.8$, and such a difference is easily obtained with Eq. (53). Our results could also be employed in the scenario of [7] in which the third generation is sequestered in a separate throat with an independent IR scale relative to that in which the lighter generations reside.

IV. GW MECHANISM FOR TWO THROATS WITH A Z_2

We would like to consider the interesting case where the two throats are related by a Z_2 interchange symmetry. Such a gravitational background has already been employed in the literature [14] and is of interest because the interchange symmetry can affect dynamics and motivate phenomenological applications. The action of the interchange symmetry is

$$Z_2\colon\ y_1 \leftrightarrow y_2, \tag{54}$$

with a corresponding action on the field content of each throat:

$$\mathcal{F}_1(x^\mu, y_1) \leftrightarrow \mathcal{F}_2(x^\mu, y_2). \tag{55}$$

The fields $\mathcal{F}_i$ could denote the wave function in the ith throat of a field that propagates in both throats, an example being a SM gauge boson propagating in two throats that are subject to a "UED-like" [24] reflection parity. This scenario was considered in [14] where it was shown that the resulting KK-parity ensures stability of the lightest odd KK mode, thereby motivating a dark matter candidate for RS models. Alternatively the SM may be confined to one throat with a hidden sector residing in the other, as would occur in a multithroat version of the mirror matter models [25–27]. Our interests here are primarily in the generalization of the GW mechanism to such a symmetric throat arrangement, regardless of the specific application.

We label the throats as $i = 1, 2$ where the metric in the ith throat is again defined by

$$ds_i^2 = e^{-2ky_i}\eta_{\mu\nu}dx^\mu dx^\nu - dy_i^2 \equiv G^i_{MN}dx_i^M dx_i^N, \tag{56}$$

and we consider a GW scalar ϕ_i in each throat with the action of the interchange symmetry being

$$Z_2\colon\ \phi_1 \leftrightarrow \phi_2. \tag{57}$$

The bulk action in the ith throat is

$$S_B^i = \frac{1}{2}\int d^4x dy_i \sqrt{G^i}(G_i^{MN}\partial_M\phi_i\partial_N\phi_i - m^2\phi_i^2), \tag{58}$$

and the IR brane localized actions are

$$S^i_{\rm IR} = -\frac{1}{2}\int d^4x\sqrt{-g^i_{\rm ir}}\lambda(\phi_i^2 - v^2)^2, \tag{59}$$

where $(g^{\rm ir}_{\mu\nu})^i$ is the restriction of $G^i_{\mu\nu}$ to $y_i = L_i$. Note that the interchange symmetry requires equality of the bulk masses m and the IR brane parameters λ and v for scalars in distinct throats. For $\partial_\mu\phi_i = 0$ the solution to the bulk equations of motion are

$$\phi_i(y_i) = A_i e^{\beta_+ ky_i} + B_i e^{\beta_- ky_i}, \tag{60}$$

where $\beta_\pm$ is defined as

$$\beta_\pm = 2 \pm \sqrt{4 + \frac{m^2}{k^2}} \equiv 2 \pm \nu = 2 \pm (2 + \epsilon), \tag{61}$$

with $\epsilon \simeq m^2/4k^2$. The IR BC is

$$\partial_y\phi_i(L_i) + 2\lambda(\phi_i(L_i)^2 - v^2)\phi_i(L_i) = 0, \tag{62}$$

and we consider large λ so that

$$\phi_i(L_i) = v, \qquad i = 1, 2, \tag{63}$$

to leading order. We consider three distinct cases for the UV brane potential in what follows. Each case has different consequences for the structure of the resulting gravitational background.

A. Preservation of the Z_2 symmetry

First we shall present a generalized GW mechanism to stabilize the two throats while preserving the Z_2 interchange symmetry. In this case we write the action on the common UV brane as

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\{\lambda_+(\phi_1^2 + \phi_2^2 - 2u^2)^2 + \lambda_-(\phi_1^2 - \phi_2^2)^2\}, \tag{64}$$

where $g^{\rm uv}_{\mu\nu}$ is the restriction of $G^i_{\mu\nu}$ to $y_i = 0$. As in the previous section the entire action is invariant under two independent Z_2 symmetries whose actions are defined by

$$Z_2^{(i)}\colon\ \phi_i \to -\phi_i. \tag{65}$$

The corresponding UV BC is

$$[\partial_y\phi_i - 2\lambda_+(\phi_i^2 + \phi_j^2 - 2u^2)\phi_i - 2\lambda_-(\phi_i^2 - \phi_j^2)\phi_i]|_{\rm UV} = 0, \qquad i \neq j, \tag{66}$$

and for large $\lambda_\pm > 0$ the leading order UV BCs are

$$\phi_i(0) = u, \qquad i = 1, 2. \tag{67}$$

Imposing the BCs on the bulk solutions and integrating out the extra dimension generates the following potential for $L_{1,2}$:

$$V(L_1, L_2) \simeq \sum_i 2ke^{-4kL_i}(v - ue^{-\epsilon kL_i})^2 + \cdots, \quad (68)$$

which is minimized at the usual GW value:

$$L_1 = L_2 = \frac{1}{\epsilon k} \ln\left(\frac{u}{v}\right). \quad (69)$$

Equality of L_1 and L_2 implies preservation of the Z_2 interchange symmetry in the presence of the stabilizing dynamics. One might wonder however if the $\mathcal{O}(\lambda^{-1})$ corrections modify this equality; inclusion of these corrections gives

$$V(L_1, L_2) = \sum_i [V_B^i + \delta V_B^i + \delta V_{\rm IR}^i] + V_{\rm UV} \simeq \sum_i 2ke^{-4kL_i}(v - ue^{-\epsilon kL_i})^2\left[1 - \frac{k}{\lambda v^2}\right] + \cdots, \quad (70)$$

and the minimum remains at (69).

This type of stabilization mechanism, in which an initially Z_2 symmetric throat configuration remains Z_2 symmetric as the GW scalars reach their minimum and fix the value of the throat lengths, can be employed in, e.g., the warped dark matter model of [14]. As the Z_2 symmetry is preserved any field properties dependent on this symmetry, like the stability of the dark matter candidate, remain in tact.

B. Breaking the Z_2 symmetry

In this section we write the action on the common UV brane as[5]

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}^i}\{\lambda_a(\phi_1^2 + \phi_2^2 - u^2)^2 + \lambda_b\phi_1^2\phi_2^2\}, \quad (71)$$

which is also invariant under the two independent symmetries $\phi_i \to -\phi_i$. The UV BC is

$$[\partial_y\phi_i - 2\lambda_a(\phi_i^2 + \phi_j^2 - u^2)\phi_i - \lambda_b\phi_j^2\phi_i]|_{\rm UV} = 0, \quad i \neq j. \quad (72)$$

For large $\lambda_{a,b} > 0$ the leading order UV BCs correspond to just one scalar acquiring a boundary VEV,

$$\phi_1(0) = u, \qquad \phi_2(0) = 0, \quad (73)$$

where we label the field with a nonzero boundary VEV as $i = 1$. Imposing the BCs gives

$$A_1 \simeq ve^{-\beta_+ kL_1} - ue^{-2\nu kL_1},$$
$$B_1 \simeq u(1 + e^{-2\nu kL_1}) - ve^{-\beta_+ kL_1}, \quad (74)$$
$$A_2 = -B_2 \simeq e^{-\beta_+ kL_2}v$$

and to leading order the potential for L_i is

$$V_B^i \simeq 2k \times \begin{cases} e^{-4kL_1}(v - ue^{-\epsilon kL_1})^2 + \cdots, & i = 1, \\ e^{-4kL_2}v^2 + \cdots, & i = 2. \end{cases} \quad (75)$$

Including the $\mathcal{O}(\lambda^{-1})$ corrections the full potential is

$$V(L_1, L_2) = \sum_i [V_B^i + \delta V_B^i + \delta V_{\rm IR}^i] + V_{\rm UV} \simeq 2ke^{-4kL_1}(v - ue^{-\epsilon kL_1})^2\left[1 - \frac{k}{\lambda v^2}\right] + 2ke^{-4kL_2}v^2\left[1 - \frac{k}{\lambda v^2}\right] + \cdots, \quad (76)$$

yielding the following minimizing throat lengths:

$$L_1 = \frac{1}{\epsilon k}\ln\left(\frac{u}{v}\right) \quad \text{and} \quad L_2 \to \infty. \quad (77)$$

Observe that the Z_2 symmetry has been broken as $L_1 \neq L_2$; however only L_1 remains finite with L_2 running away to infinity. This runaway is consistent with our example calculation for a single throat in Sec. II.

C. Breaking the Z_2 symmetry with finite throat lengths

We would like to find a stabilization configuration that breaks the Z_2 interchange symmetry of the two throats and yet stabilizes both throats at finite lengths. In the preceding sections the scalar action possessed two additional discrete symmetries, $Z_2^{(i)}$: $\phi_i \to -\phi_i$, which generalize the discrete symmetry employed in [2]. These symmetries prove too restrictive if one seeks to stabilize both throats at finite lengths as the asymmetric minimum of the resulting UV potential induces a nonzero boundary VEV for only one of the scalars. As discussed in Sec. II for the GW scalar, the UV VEV must dominate the IR VEV in order to stabilize the throat length. In this section we relax the symmetries $Z_2^{(i)}$ to permit an asymmetric minimum to the UV potential which permits both scalars to take nonzero VEVs.

We employ the following UV action:

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\{\lambda_a(\phi_1^2 + \phi_2^2 - u^2)^2 + \lambda_b\phi_1^2\phi_2^2 - \mu\phi_1\phi_2 - \kappa(\phi_1^3\phi_2 + \phi_1\phi_2^3)\}, \quad (78)$$

where the μ and κ terms break the symmetries $Z_2^{(i)}$ but admit the following diagonal symmetry:

$$Z_2^D: \phi_{1,2} \to -\phi_{1,2}. \quad (79)$$

Retaining (58) and (59) for the bulk and IR actions, respectively, we see that Z_2^D is a symmetry of the entire action.

[5]This is a simple rewriting of the potential used in Sec. IVA with the parameters $\lambda_{a,b}$ and u related to the parameters used in that section; see [28,29] for a discussion of a related potential.

With the above UV action one obtains the following BCs:

$$[\partial_y \phi_i - 2\lambda_a(\phi_i^2 + \phi_j^2 - u^2)\phi_i - \lambda_b \phi_j^2 \phi_i + \frac{\mu}{2}\phi_j + \frac{\kappa}{2}\phi_j(\phi_j^2 + 3\phi_i^2)]|_{\mathrm{UV}} = 0, \qquad i \neq j.$$

We again consider the limit where the coupling constants in the UV Lagrangian are large and the derivative piece is subdominant. However to simplify the calculation we consider the case where the $\lambda_{a,b}$ terms also dominate the μ and κ terms. This hierarchy of parameters[6] is technically natural as the symmetry of the action is enhanced from Z_2^D to $Z_2^{(1)} \times Z_2^{(2)}$ in the limit μ, $\kappa \to 0$. We arrive at the following leading order BCs:

$$\phi_1(0) = u, \qquad \phi_2(0) = \frac{\mu + \kappa u^2}{2\lambda_b u^2} u, \tag{80}$$

where terms of order λ^{-2} are neglected.[7]

Based on what we have seen in the preceding sections we can immediately deduce that the leading order potential for the throat lengths is

$$V_B(L_1, L_2) \simeq 2ke^{-4kL_1}(v - ue^{-\epsilon kL_1})^2 + 2ke^{-4kL_2}\left(v - ue^{-\epsilon kL_2} \times \frac{\mu + \kappa u^2}{2\lambda_b u^2}\right)^2 + \cdots,$$

the minimization of which gives

$$L_1 = \frac{1}{\epsilon k} \ln\left(\frac{u}{v}\right), \tag{81}$$

$$L_2 = \frac{1}{\epsilon k}\left\{\ln\left(\frac{u}{v}\right) + \ln\left(\frac{\mu + \kappa u^2}{2\lambda_b u^2}\right)\right\}. \tag{82}$$

Thus the throat interchange symmetry is broken by the stabilization dynamics and both throats acquire finite lengths. With λ_b, μ, $\kappa > 0$ the second logarithm is negative and one has $L_2 < L_1$. Including the $\mathcal{O}(\lambda^{-1})$ corrections from the derivative pieces in the BCs gives

$$\begin{aligned} V(L_1, L_2) &= \sum_i [V_B^i + \delta V_B^i + \delta V_{\mathrm{IR}}^i] + V_{\mathrm{UV}} \\ &\simeq 2ke^{-4kL_1}(v - ue^{-\epsilon kL_1})^2\left[1 - \frac{k}{\lambda v^2}\right] \\ &\quad + 2ke^{-4kL_2}\left\{v - ue^{-\epsilon kL_2} \times \frac{\mu + \kappa u^2}{2\lambda_b u^2}\right\}^2 \\ &\quad \times \left[1 - \frac{k}{\lambda v^2}\right] + \cdots, \end{aligned} \tag{83}$$

and the leading order expressions for $L_{1,2}$ hold. This shows that the generalized GW mechanism can successfully break the interchange symmetry and fix the throat lengths at finite values. Such a mechanism would be of use in a flavor model based on a broken Z_2 symmetry or in a warped realization of the broken mirror model discussed in [26].

Note that there is a sense in which symmetry breaking occurs both in the UV (different GW VEVs) and in the IR (different throat lengths) in the present example. Ultimately it is the UV behavior of the GW scalars that triggers the symmetry breaking, which then manifests in the IR in the form of distinct IR scales. However, it is interesting that the emergence of multiple energy scales in the IR encodes information about the UV.

It is also important to make some comments on the above solution. Stability of the solution requires the UV boundary VEV to be larger than the IR VEV and therefore the solution in the second throat necessitates

$$\left(\frac{\mu + \kappa u^2}{2\lambda_b u^2}\right) > \frac{v}{u}. \tag{84}$$

Accordingly one cannot take $(\mu + \kappa u^2)/(2\lambda_b u^2)$ to be "too small" as the limit $\lambda_b \to \infty$ corresponds to $\phi_2(0) = 0$ and results in the runaway solution $L_2 \to \infty$ found in the previous section. If one demands that u/v does not greatly exceed 10 based on naturalness arguments then there is only a small window of parameter space in which our results hold and our approximations can be trusted. More generally there exists parameter space such that the asymmetric minima of the UV boundary potential in Eq. (78) has both $\phi_1(0) \sim u$ and $\phi_2(0) \sim u$. These solutions require that the $Z_2^{(i)}$ symmetry breaking terms are not subdominant. To see this note that in the limit $\kappa = 0$ the minima of the UV boundary potential must satisfy

$$\phi_1 \phi_2|_{\mathrm{UV}} = \frac{\mu}{2\lambda_b}, \tag{85}$$

where no approximation has been made. Furthermore the parameters must also satisfy $\mu/\lambda_b u^2 < 1$. Therefore the solutions with both $\phi_1(0) \sim u$ and $\phi_2(0) \sim u$ require $(\mu/\lambda_b u^2) \sim 1$ and lie outside our approximation. There is no difficulty of principle in employing these solutions, however the calculation becomes considerably more cumbersome. For this reason we have considered the above approximations which in any case demonstrate an ability to stabilize the throat lengths via Z_2 symmetry breaking dynamics.

Before concluding this section we note that one can break the Z_2 interchange symmetry without introducing $Z_2^{(i)}$ breaking terms at the expense of an extended field content. Introducing a UV localized scalar χ that is odd under Z_2 permits a UV term $\propto \chi(\phi_1^2 - \phi_2^2)$. If χ develops a VEV the Z_2 symmetry is broken and $\phi_{1,2}$ acquire distinct

[6]As $\lambda_{a,b}$ and κ have different mass dimension to μ this statement must be made with reference to dimensionless quantities by employing appropriate powers of a fixed reference scale like the curvature.

[7]We have checked that this critical point of the UV potential is indeed stable with $\lambda_{a,b} \gg \mu$, k, and $\lambda_{a,b} > 0$.

UV boundary values resulting in distinct finite throat lengths.

V. GW MECHANISM FOR THREE THROATS WITH A Z_3

Next we consider three throats related by a cyclic Z_3 interchange symmetry, the action of which is given by

$$Z_3\colon\ y_i \to y_{i+1}, \tag{86}$$

where $i = 1, 2, 3$ labels the throats, y_i labels the warped extra dimension in each throat, and $i + 1$ is defined mod 3 so that $y_3 \to y_1$. The metric in the ith throat is defined as per Eq. (56) and the action of Z_3 on the field content of each throat is

$$\mathcal{F}_i(x^\mu, y_i) \to \mathcal{F}_{i+1}(x^\mu, y_{i+1}). \tag{87}$$

To generalize the approach of GW to the three-throat background we consider a set of GW scalars ϕ_i, one in each throat, that transform under Z_3 as $\phi_i \to \phi_{i+1}$. The bulk action in the ith throat is

$$S_B^i = \frac{1}{2}\int d^4x dy_i \sqrt{G^i}(G_i^{MN}\partial_M\phi_i\partial_N\phi_i - m^2\phi_i^2), \tag{88}$$

where mass equality is enforced by the Z_3 symmetry. The IR brane localized actions take the standard form

$$S_{\rm IR}^i = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm ir}^i}\lambda(\phi_i^2 - v^2)^2, \tag{89}$$

where $(g_{\mu\nu}^{\rm ir})^i$ is the restriction of $G_{\mu\nu}^i$ to $y_i = L_i$ and equality of the constants λ and v on the different branes is again dictated by symmetry. With these actions the IR BC in the ith throat is

$$[\partial_y\phi_i + 2\lambda(\phi_i^2 - v^2)\phi_i]|_{\rm IR} = 0. \tag{90}$$

For large λ one has $\phi_i(L_i) = v$ to leading order. A determination of the potential for L_i requires specification of the UV action. In what follows we first consider a UV action that preserves the Z_3 symmetry and then present others that break it.

A. Preservation of the Z_3 symmetry

In this section we present a generalized GW mechanism for three throats subject to a Z_3 interchange symmetry that stabilizes the throats with identical lengths and therefore preserves the Z_3 symmetry. Applications for a symmetric three-throat configuration have been discussed in [5]. These include a discrete family symmetry based on the interchange of three identical throats and a geometric realization of the trinification model [30]. The Z_3 preserving system presented here provides an appropriate gravitational background for the examples discussed in [5].

To obtain a Z_3 preserving configuration we employ the following UV action:

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\Big\{-\mu^2\sum_i\phi_i^2 + \bar\lambda\Big[\sum_i\phi_i^2\Big]^2 + \tilde\lambda\sum_i\phi_i^4\Big\}, \tag{91}$$

where $g_{\mu\nu}^{\rm uv}$ is the restriction of $G_{\mu\nu}$ to $y_i = 0$. As this action does not contain any odd terms in the fields ϕ_i the entire action is invariant under three independent Z_2 symmetries whose actions are defined by

$$Z_2^{(i)}\colon\ \phi_i \to -\phi_i. \tag{92}$$

These symmetries generalize the discrete $\phi \to -\phi$ symmetry of [2].

Demanding that the variation of the action vanishes on the UV brane gives the UV BC:

$$\Big[\partial_y\phi_i + \mu^2\phi_i - 2\bar\lambda\Big(\sum_j\phi_j^2\Big)\phi_i - 2\tilde\lambda\phi_i^3\Big]_{\rm UV} = 0. \tag{93}$$

For large $\bar\lambda$, $\tilde\lambda$, $\mu > 0$ the leading order UV BCs are the same for each scalar:

$$\phi_i^2(0) = \frac{\mu^2}{2(3\bar\lambda + \tilde\lambda)} \equiv u^2, \qquad i = 1, 2, 3. \tag{94}$$

Imposing the BCs and calculating the potential gives

$$V(\{L_i\}) = \sum_i 2kA_i^2(e^{2\nu kL_i} - 1) + \mathcal{O}(\epsilon), \tag{95}$$

$$\simeq \sum_i 2ke^{-4kL_i}(v - ue^{-\epsilon kL_i})^2 + \cdots, \tag{96}$$

which is minimized at the usual GW value:

$$L_i = \frac{1}{\epsilon k}\ln\Big(\frac{u}{v}\Big), \qquad i = 1, 2, 3. \tag{97}$$

Equality of L_1, L_2, and L_3 preserves the Z_3 interchange symmetry to leading order. Including the $\mathcal{O}(\lambda^{-1})$ corrections modifies the potential to

$$V(\{L_i\}) = \sum_i[V_B^i + \delta V_B^i + \delta V_{\rm IR}^i] + V_{\rm UV} \simeq \sum_i 2ke^{-4kL_i}(v - ue^{-\epsilon kL_i})^2\Big[1 - \frac{k}{\lambda v^2}\Big] + \cdots, \tag{98}$$

but the minimum remains at (97) and the Z_3 symmetry is preserved.

B. Breaking the Z_3 symmetry

For alternative applications of a three-throat background one may prefer a Z_3 symmetric action to produce three throats with different lengths as a result of stabilization; see [8] for example. We can achieve this by rewriting the UV action as

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\bigg\{\lambda_a\bigg(\sum_i \phi_i^2 - u^2\bigg)^2 + \lambda_b(\phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2)\bigg\},$$

and taking $\lambda_{a,b} > 0$. The absence of odd terms in the fields ϕ_i ensures that the symmetries $Z_2^{(i)}$ still hold. For large $\lambda_{a,b} > 0$ the leading order UV BCs that minimize the UV potential are

$$\phi_1(0) = u, \qquad \phi_2(0) = \phi_3(0) = 0, \tag{99}$$

which break the Z_3 symmetry and stabilize one throat (which we label as $i = 1$) at

$$L_1 = \frac{1}{\epsilon k}\ln\bigg(\frac{u}{v}\bigg). \tag{100}$$

However the vanishing UV boundary values for $\phi_{2,3}$ to leading order result in the $i = 2$ and $i = 3$ throat lengths running away, $L_{2,3} \to \infty$. This is similar to the broken Z_2 symmetry case of Sec. IV B. Although this successfully breaks the Z_3 interchange symmetry of the throats the runaway nature of the $i = 2$, 3 throats means such a solution is likely of limited phenomenological utility.

In order to stabilize $L_{2,3}$ at finite values one should introduce $Z_2^{(i)}$ breaking terms in the UV action. A suitable action is

$$\begin{aligned}S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g^i_{\rm uv}}\bigg\{&\lambda_a\bigg(\sum_i \phi_i^2 - u^2\bigg)^2 + \lambda_b(\phi_1^2\phi_2^2 \\ &+ \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2) - \mu(\phi_1\phi_2 + \phi_2\phi_3 + \phi_3\phi_1) \\ &- \kappa(\phi_1\phi_2\phi_3^2 + \phi_2\phi_3\phi_1^2 + \phi_3\phi_1\phi_2^2), -\alpha(\phi_1\phi_3^3 \\ &+ \phi_2\phi_1^3 + \phi_3\phi_2^3) - \beta(\phi_1\phi_2^3 + \phi_2\phi_3^3 + \phi_3\phi_1^3)\bigg\},\end{aligned} \tag{101}$$

where the terms with coefficients μ, κ, α, and β break the symmetries $Z_2^{(i)}$ but do not break the throat interchange symmetry Z_3. The UV action remains invariant under the diagonal discrete symmetry,

$$Z_2^D\colon\ \phi_{1,2,3} \to -\phi_{1,2,3}, \tag{102}$$

which is also preserved by the bulk and IR actions. With this action the UV BCs are

$$\begin{aligned}\bigg[\partial_y\phi_i - \bigg\{2\lambda_a\bigg(\sum_\ell \phi_\ell^2 - u^2\bigg) + \lambda_b(\phi_j^2 + \phi_k^2)\bigg\}\phi_i \\ + \frac{\mu}{2}(\phi_j + \phi_k) + \frac{\kappa}{2}(\phi_j\phi_k^2 + \phi_k\phi_j^2 + 2\phi_i\phi_j\phi_k) \\ + \frac{\alpha}{2}(\phi_k^3 + 3\phi_j\phi_i^2) + \frac{\beta}{2}(\phi_j^3 + 3\phi_k\phi_i^2)\bigg]_{\rm UV} = 0,\end{aligned} \tag{103}$$

where $i \neq j \neq k \neq i$. For computational simplicity we consider the limit where the boundary potential terms dominate the derivative piece in this BC. We further take the $\lambda_{a,b}$ terms to be larger than the $Z_2^{(i)}$ symmetry breaking terms, a technically natural limit. In this case the leading order BCs are[8]

$$\begin{aligned}&\phi_1(0) = u, \qquad \phi_2(0) = \frac{\mu + \alpha u^2}{2\lambda_b u^2}u \equiv u_2, \\ &\phi_3(0) = \frac{\mu + \beta u^2}{2\lambda_b u^2}u \equiv u_3,\end{aligned} \tag{104}$$

and to leading order the potential for $L_{1,2,3}$ is

$$V(\{L_i\}) = \sum_i 2kA_i^2(e^{2\nu k L_i} - 1) + \mathcal{O}(\epsilon), \tag{105}$$

$$\simeq 2ke^{-4kL_1}(v - ue^{-\epsilon k L_1})^2 + \sum_{i=2,3} 2ke^{-4kL_i}(v - u_i e^{-\epsilon k L_i})^2 + \cdots. \tag{106}$$

The minimum of this potential occurs at

$$\begin{aligned}&L_1 = \frac{1}{\epsilon k}\ln\bigg(\frac{u}{v}\bigg), \qquad L_2 = \frac{1}{\epsilon k}\bigg\{\ln\bigg(\frac{u}{v}\bigg) + \ln\bigg(\frac{\mu + \alpha u^2}{2\lambda_b u^2}\bigg)\bigg\}, \\ &L_3 = \frac{1}{\epsilon k}\bigg\{\ln\bigg(\frac{u}{v}\bigg) + \ln\bigg(\frac{\mu + \beta u^2}{2\lambda_b u^2}\bigg)\bigg\}.\end{aligned} \tag{107}$$

Note that all three throat lengths are finite and in general $L_i \neq L_j$ for all $i \neq j$. Including the $\mathcal{O}(\lambda^{-1})$ corrections from the derivative pieces in the UV BCs the leading terms of Eq. (106) pick up a factor of $(1 - k/\lambda v^2)$ and the minimum agrees with (107) to $\mathcal{O}(\lambda^{-1})$.

The fact that all three throat lengths are different reflects the complete breaking of the Z_3 interchange symmetry. Such a GW scenario is relevant for, e.g., the work of [8] in which a three-throat configuration with a Z_3 interchange symmetry was considered. In that work each generation of SM fermions was confined to a different throat so the interchange symmetry of the throats also served as a flavor symmetry. The Z_3 flavor symmetry was posited to be broken by unspecified dynamics that result in different throat lengths. The calculations of this section provide a concrete realization of the gravitational background employed in that work. We note that the Z_3 symmetry breaking structure obtained here is not precisely that envisioned in [8]. While they sought to have the Z_3 symmetry broken by different throat lengths they also sought to have the UV brane remain Z_3 symmetric. Though our approach differs, the UV symmetry breaking inherent in our methodology may not significantly alter their conclusions. As the GW scalars are odd under Z_2^D the coupling of SM fields to the Z_3 breaking parameters may be sufficiently sequestered to retain the Z_3 flavor symmetry to good approximation in the

[8]As in the Z_2 case, the Hessian analysis indicates that this set of solutions corresponds to a stable critical point as long as $\lambda_{a,b} > 0$ is dominant.

UV, provided the SM fermions are not charged under this symmetry. It would be interesting to consider alternative approaches to determine if a purely symmetric UV sector can be found in conjunction with Z_3 breaking in the IR. We also note that ideas similar to those discussed here may be relevant to the three-throat configuration of [10].

The comments made at the end of Sec. IV C for the broken Z_2 case also apply here. It is important that one does not take the arguments of the second logarithms in the expressions for $L_{2,3}$ (107) to be too small compared to v/u as one returns to the runaway solutions $L_{2,3} \to \infty$ in the limit $\lambda_b \to \infty$. Solutions exist with $\phi_i(0) \sim u, \ \forall \ i$, but these require the $Z_2^{(i)}$ symmetry breaking terms to be as "large" as the $Z_2^{(i)}$ preserving terms. Calculations become somewhat more cumbersome in this range of parameter space.

For more general model building purposes one may wish to break a discrete symmetry among multiple throats to a discrete subgroup; that is one may desire some but not all throat lengths to be equal after stabilization. In the limit $\alpha = \beta$ the present example provides a demonstration of precisely this scenario. Observe from (107) that for $\alpha = \beta$ one has $L_2 = L_3$. Thus instead of breaking Z_3 completely the breaking pattern $Z_3 \to Z_2$ would result. The equality of α and β can be motivated by symmetry as in this limit the $Z_3 \times Z_2^D$ symmetry of the UV potential is enhanced to $S_3 \times Z_2^D$. Thus the present example can be employed to either break Z_3 completely or to break it partially to a Z_2 subgroup depending on the relation between α and β.

VI. MORE THAN THREE THROATS

The generalization of some of the preceding results to $n > 2$ throats is straightforward. For n independent throats one considers n GW scalars with bulk and IR actions matching those of the two independent throat analysis in Sec. III. The common UV action is generalized to

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\bigg\{\sum_{i=1}^{n}\bar{\lambda}_i(\phi_i^2 - u_i^2)^2 + \sum_{i\neq j}\kappa_{ij}\phi_i^2\phi_j^2\bigg\}. \tag{108}$$

In the limit where the $\bar{\lambda}_i$ terms dominate the BCs the resulting potential for the throat lengths $V(\{L_i\})$ is the sum of n decoupled pieces, each of which match the GW result to leading order. Thus the leading order expression for the throat lengths that minimize $V(\{L_i\})$ is

$$L_i = \frac{1}{\epsilon_i k}\ln\left(\frac{u_i}{v_i}\right). \tag{109}$$

As in the two-throat case these expressions receive $\mathcal{O}(\kappa_{ij}/\bar{\lambda})$ corrections as a result of the UV localized interactions among scalars.

For n throats related by a Z_n symmetry the results of Secs. IV A and V A can also be generalized. In those sections, two- and three-throat systems that preserve Z_2 and Z_3 throat interchange symmetries were presented. This is generalized by employing bulk scalars in each throat with bulk and IR actions matching those in Sec. V A. The Z_n symmetric UV action is generalized to[9]

$$S_{\rm UV} = -\frac{1}{2}\int d^4x\sqrt{-g_{\rm uv}}\bigg\{-\mu^2\sum_{i=1}^{n}\phi_i^2 + \bar{\lambda}\bigg[\sum_{i=1}^{n}\phi_i^2\bigg]^2 + \tilde{\lambda}\sum_{i=1}^{n}\phi_i^4\bigg\}. \tag{110}$$

For large $\bar{\lambda}$, $\tilde{\lambda}$, $\mu > 0$ the leading order UV BCs are

$$\phi_i^2(0) = \frac{\mu^2}{2(n\bar{\lambda} + \tilde{\lambda})} \equiv u^2, \qquad i = 1, 2, \ldots, n, \tag{111}$$

and the throat lengths are fixed at

$$L_i = \frac{1}{\epsilon k}\ln\left(\frac{u}{v}\right), \qquad i = 1, 2, \ldots, n, \tag{112}$$

thereby preserving the Z_n symmetry and generalizing the earlier results. Just as the $n = 3$ case permits a geometric realization of the trinification model the symmetric $n = 4$ case would similarly admit a geometric realization of the quartification group [32]. Following the methodology we have presented in the previous sections symmetry breaking scenarios for $n > 3$ throats can also be obtained.

VII. CONCLUSION

In this work we have generalized the GW mechanism for stabilizing the single warped throat of the RS model to multithroat backgrounds in which distinct warped throats share a common UV brane. We have shown that, due to a combination of IR and UV dynamics, the throat lengths can be stabilized at finite values in such setups. We considered independent throats for which the GW results generalize in a straightforward way and throats related by a discrete interchange symmetry. In the latter case we provided examples where the stabilization dynamics can either preserve or break the interchange symmetry. Our results are applicable to a broad class of multithroat models and have direct relevance for existing models in the literature.

ACKNOWLEDGMENTS

The authors thank D. P. George, S. Gopalakrishna, and R. Volkas for helpful discussions and D. Morrissey for comments on the manuscript. They also thank the theory

[9]For a discussion of a similar potential in a different context see [31].

groups at the Nationaal Instituut voor Subatomaire Fysica (NIKHEF) and the University of Melbourne for hospitality while parts of this work were completed. S. L. was supported in part by the National Science Council of Taiwan (Grant No. NSC-99-2811-M-033-013) and in part by the National Center for Theoretical Sciences. K. M. was supported by the National Science and Engineering Research Council of Canada.

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[2] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).

[3] J. Garriga and A. Pomarol, Phys. Lett. B **560**, 91 (2003).

[4] N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, and N. Weiner, Phys. Rev. D **79**, 015014 (2009); M. Pospelov and A. Ritz, Phys. Lett. B **671**, 391 (2009).

[5] G. Cacciapaglia, C. Csaki, C. Grojean, and J. Terning, Phys. Rev. D **74**, 045019 (2006).

[6] T. Gherghetta and B. Harling, J. High Energy Phys. 04 (2010) 039.

[7] G. Cacciapaglia, C. Csaki, C. Grojean, M. Reece, and J. Terning, Phys. Rev. D **72**, 095018 (2005).

[8] S. Abel and J. Barnard, J. High Energy Phys. 08 (2010) 039.

[9] F. Plentinger and G. Seidl, Phys. Rev. D **78**, 045004 (2008).

[10] A. Bechinger and G. Seidl, Phys. Rev. D **81**, 065015 (2010).

[11] T. Flacke, B. Gripaios, J. March-Russell, and D. Maybury, J. High Energy Phys. 01 (2007) 061; T. Flacke and D. Maybury, J. High Energy Phys. 03 (2007) 007.

[12] B. Gripaios, Nucl. Phys. **B768**, 157 (2007); **B830**, 390(E) (2010).

[13] K. L. McDonald and D. E. Morrissey, J. High Energy Phys. 05 (2010) 056.

[14] K. Agashe, A. Falkowski, I. Low, and G. Servant, J. High Energy Phys. 04 (2008) 027.

[15] G. Altarelli, F. Feruglio, and Y. Lin, Nucl. Phys. **B775**, 31 (2007); A. Adulpravitchai, A. Blum, and M. Lindner, J. High Energy Phys. 07 (2009) 053; T. Kobayashi, H. P. Nilles, F. Ploger, S. Raby, and M. Ratz, Nucl. Phys. **B768**, 135 (2007).

[16] H. L. Verlinde, Nucl. Phys. **B580**, 264 (2000); I. R. Klebanov and M. J. Strassler, J. High Energy Phys. 08 (2000) 052; S. B. Giddings, S. Kachru, and J. Polchinski, Phys. Rev. D **66**, 106006 (2002); S. Kachru, R. Kallosh, A. Linde, and S. P. Trivedi, Phys. Rev. D **68**, 046005 (2003); F. Brummer, A. Hebecker, and E. Trincherini, Nucl. Phys. **B738**, 283 (2006).

[17] S. Dimopoulos, S. Kachru, N. Kaloper, A. E. Lawrence, and E. Silverstein, Phys. Rev. D **64**, 121702 (2001); S. Dimopoulos, S. Kachru, N. Kaloper, A. E. Lawrence, and E. Silverstein, Int. J. Mod. Phys. A **19**, 2657 (2004); N. Barnaby, C. P. Burgess, and J. M. Cline, J. Cosmol. Astropart. Phys. 04 (2005) 007; A. Hebecker and J. March-Russell, Nucl. Phys. **B781**, 99 (2007); B. v. Harling, A. Hebecker, and T. Noguchi, J. High Energy Phys. 11 (2007) 042; B. v. Harling and A. Hebecker, J. High Energy Phys. 05 (2008) 031.

[18] O. DeWolfe, D. Z. Freedman, S. S. Gubser, and A. Karch, Phys. Rev. D **62**, 046008 (2000).

[19] C. Csaki, M. L. Graesser, and G. D. Kribs, Phys. Rev. D **63**, 065002 (2001).

[20] T. Konstandin, G. Nardini, and M. Quiros, Phys. Rev. D **82**, 083513 (2010).

[21] J. Lesgourgues and L. Sorbo, Phys. Rev. D **69**, 084010 (2004); A. Dey, D. Maity, and S. SenGupta, Phys. Rev. D **75**, 107901 (2007).

[22] J. A. Cabrer, G. von Gersdorff, and M. Quiros, New J. Phys. **12**, 075012 (2010); S. M. Aybat and D. P. George, J. High Energy Phys. 09 (2010) 010; T. Gherghetta and N. Setzer, Phys. Rev. D **82**, 075009 (2010).

[23] N. Arkani-Hamed, M. Porrati, and L. Randall, J. High Energy Phys. 08 (2001) 017; R. Rattazzi and A. Zaffaroni, J. High Energy Phys. 04 (2001) 021.

[24] T. Appelquist, H. C. Cheng, and B. A. Dobrescu, Phys. Rev. D **64**, 035002 (2001).

[25] R. Foot, H. Lew, and R. R. Volkas, Phys. Lett. B **272**, 67 (1991).

[26] Z. G. Berezhiani and R. N. Mohapatra, Phys. Rev. D **52**, 6607 (1995).

[27] L. B. Okun, Phys. Usp. **50**, 380 (2007).

[28] R. Foot, H. Lew, and R. R. Volkas, J. High Energy Phys. 07 (2000) 032.

[29] R. Barbieri, T. Gregoire, and L. J. Hall, arXiv:hep-ph/0509242.

[30] A. de Rujula, H. Georgi, and S. Glashow, in *Proceedings of the Workshop on Grand Unification*, edited by K. Kang *et al.* (World Scientific, Singapore, 1984). For extra dimensional models, see C. D. Carone and J. M. Conroy, Phys. Rev. D **70**, 075013 (2004); A. Demaria and R. R. Volkas, Phys. Rev. D **71**, 105011 (2005); J. E. Kim, Phys. Lett. B **591**, 119 (2004); K. S. Choi and J. E. Kim, Phys. Lett. B **567**, 87 (2003); I. Gogoladze, Y. Mimura, and S. Nandi, Phys. Rev. D **69**, 075006 (2004).

[31] R. Foot and R. R. Volkas, Phys. Lett. B **645**, 75 (2007).

[32] G. C. Joshi and R. R. Volkas, Phys. Rev. D **45**, 1711 (1992); K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. D **69**, 051301 (2004); A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **72**, 075007 (2005); **73**, 079902 (E) (2006); A. Demaria and K. L. McDonald, Phys. Rev. D **75**, 056006 (2007); K. S. Babu, T. W. Kephart, and H. Pas, Phys. Rev. D **77**, 116006 (2008).