Black stars induced by matter on a brane: Exact solutions

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New exact asymptotically flat solutions of five-dimensional Einstein equations with horizon are found to describe multidimensional black stars generated by matter on the brane, conceivably on high energy colliders. The five-dimensional space-time is realized as an orbifold against reflection of a special extra-space coordinate and matter on the brane is induced by tailoring of the five-dimensional Schwarzschild-Tangherlini black hole metric.

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The attractive opportunity to discover mini black holes on colliders has been within the scope of recent theoretical investigations [1]. Black hole creation may be a consequence of strong gravity at short distances [2] attainable in high energy experiments if our space is realized on a hypersurface—three-brane in a multidimensional space-time [3,4]. One of the serious difficulties to predict these processes [5,6] is related to correct (or better, exact) description of black hole geometry when the matter universe is strictly situated on the three-dimensional brane but gravity propagates into extra-space dimensions. So far, several attempts have been undertaken to find such a description [7,8] in which, however, a control on leaking the matter into extra dimensions either was absent or only approximate at asymptotically large distances (see review in [6,9]). Another problem is in the appearance of deltalike singularities in matter distribution hidden under horizon for static locally stable black holes. But, in fact, black objects must be produced on colliders from quarks and gluons in the high energy density evolution [10] so that matter always remains smoothly distributed on the brane. Therefore, one expects that rather black stars are created with matter both inside and outside an event horizon in a finite brane-surface volume.

In this work, we search for new exact solutions of five-dimensional Einstein equations with horizons to describe multidimensional black stars generated by matter on the brane. We restrict ourselves to the construction of brane-world black stars with relatively small horizons as compared to the size of extra dimension. The exact solutions are found for one infinite extra dimension and can be used as guiding ones to unravel the properties of black hole objects to be created on ultrahigh energy colliders (LHC). Different ways to allocate matter are analyzed by means of tailoring two five-dimensional black holes in special coordinates and cutting-and-pasting their parts on the brane under design. Special attention is paid to the stress-energy tensor from the bulk viewpoint vs an effective stress-energy tensor on the brane defined with the help of Einstein-SMS equations [11].

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Let us outline how to implement matter distribution on a brane in order to obtain an exact solution of five-dimensional Einstein equations. We assume that the matter is localized solely on a brane and is not spread out to the bulk. To build a brane, we search for a metric $g_{AB}(x, y)$ which is a bulk vacuum solution of the Einstein equations with an event horizon. Let us choose a hypersurface parameterized by coordinates x_μ while y is a fifth coordinate taking a constant value along this surface. The indices A, $B = (0, 1, 2, 3), 5$; μ, $\nu = 0, 1, 2, 3$. Suppose that: a) the induced metric $g_{\mu\nu}(x, y)$ is asymptotically flat for any hypersurface $y = \text{const}$ and inherits the horizon; b) the normal vector, orthogonal to a hypersurface $y = \text{const}$ is spacelike everywhere across a horizon; c) in the chosen coordinate systems $g_{5B}(x, y) = 0$ and the remaining metric components provide orbifold geometry $g_{AB}(x, y) = g_{AB}(x, -y)$. These metrics are compatible with the Einstein equations.

In order to generate a brane filled by matter, let us cut a part of space while preserving the orbifold geometry,

$$g_{AB}(x, y) \Rightarrow g_{AB}(x, |z| + a), \tag{1}$$

where a is an arbitrary real constant. The brane is generated at $z = 0$. In this approach the metric $g_{AB}(x, z; a)$ remains a solution of five-dimensional Einstein equations whereas the matter is induced according to the Israel-Lanczos junction conditions [12],

$$[g_{\mu\nu}K - K_{\mu\nu}]^{+0}_{-0} = \kappa_5\tau_{\mu\nu}, \tag{2}$$

with $\kappa_5 = 1/M_*^3$ and M_* is a Planck scale in five dimensions. Then the metric $g_{\mu\nu}(x, a)$ is a metric projected on the brane $z = 0$ and the induced stress-energy tensor is located on this brane. $[K_{\mu\nu}]^{+0}_{-0} = 2K_{\mu\nu}|^{+0}$ represents the extrinsic curvature tensor discontinuity [13] defined by two limits from both sides of the brane. We notice that the requirement of orbifolding makes a mean value of extrinsic curvature on the brane vanishing and therefore the

Regge-Teitelboim equations [14] for passive brane dynamics [15] to be trivially satisfied [16]. Accordingly, the shape of a brane is rigid.

Thus, in general, the Einstein equations in the bulk read,

$$ {}^{(5)}G_{AB} = \kappa_5 T_{AB}, \tag{3} $$

where the stress-energy tensor is taken as,

$$ T_{AB} = \delta_A^\mu \delta_B^\nu \tau_{\mu\nu} \delta(z). \tag{4} $$

In terms of the extrinsic curvature tensor for an orbifold space-time, one can reduce the five-dimensional Einstein equations up to the Shiromizu-Maeda-Sasaki ones [11] (SMS) to calculate the metric solely on the brane,

$$ {}^{(4)}G_{\mu\nu} \equiv G_{\mu\nu} = \kappa_5^2 \Sigma_{\mu\nu} - E_{\mu\nu} \equiv \kappa_4 S_{\mu\nu}, \qquad \kappa_4 \equiv \frac{1}{M_{Pl}^2}, \tag{5} $$

where

$$ \Sigma_{\mu\nu} = \frac{1}{24}(-2\tau\tau_{\mu\nu} + 6\tau_\mu^\sigma \tau_{\sigma\nu} + g_{\mu\nu}(-3\tau^{\sigma\rho}\tau_{\sigma\rho} + \tau^2)), \tag{6} $$

where $\tau \equiv \tau_\mu^\mu$. In order to define the conformal tensor projection $E_{\mu\nu}$ we introduce the normal vector $(n_A) = (0, 0, 0, 0, -\sqrt{-g_{55}})$ orthogonal to the brane, its covariant counterpart $n^A = g^{AB} n_B$, $n^A n_A = -1$ and the projector on the brane $q_A^B = \delta_A^B + n_A n^B$ for the signature $(+, -, -, -, -)$. Then the above tensor is related to the conformal Weyl tensor [13] projected on the brane with the help of q_A^B,

$$ E_{\mu\nu} = {}^{(5)}C^A_{BCD} n_A n^C q_\mu^B q_\nu^D; \qquad E_\mu^\mu = 0. \tag{7} $$

The SMS equations are taken in the form compatible with asymptotic flatness of brane metrics.

The effective stress-energy tensor $S_{\mu\nu}$ as seen by an observer on the brane is different from the bulk one being quadratic in $\tau_{\mu\nu}$ and is also determined by the gravitational energy flow from the bulk. However, we remind the reader that the scalar curvature does not depend on the bulk gravity flow $E_{\mu\nu}$,

$$ {}^{(4)}R = \frac{\kappa_5^2}{12}(3\tau_\nu^\mu \tau_\mu^\nu - 2\tau^2). $$

To prepare a suitable coordinate system, we start from the metric describing a five-dimensional static neutral black hole [17,18] in Schwarzschild coordinates $\{t, r, \theta_1, \theta_2, \theta\}$,

$$ g_{AB} = -\mathrm{diag}\left[-U(r), \frac{1}{U(r)}, R^2, R^2\cos^2\theta_1, r^2\right], \tag{8} $$

where $U(r) = 1 - \frac{M}{r^2}$, M is related to the Schwarzschild-Tangherlini radius $M \equiv r^2_{\text{Sch-}T}$ and for brevity $R = r\cos\theta$ is introduced. These coordinates run over the following intervals $-\infty < t < \infty$, $0 < r < \infty$, $-\pi/2 \le \theta_1 \le \pi/2$, $0 \le \theta_2 \le 2\pi$, $-\pi/2 \le \theta \le \pi/2$. Let us define the Gaussian normal coordinates in respect to the hypersurface $\theta = 0$. The vector orthonormal to this hypersurface $n^A = [0, 0, 0, 0, 1/r]$ and therefore, the required change of coordinates involves two variables $r = r(\rho, y)$, $\theta = \theta(\rho, y)$. These functions satisfy the geodesic equations and their solutions can be presented in the integral form,

$$ |y| = \int_\rho^r \frac{\mathrm{sign}(r-\rho)x^2}{\sqrt{(x^2 - M)(x^2 - \rho^2)}} dx, \tag{9} $$

$$ \theta = \rho \int_0^y (r(x, \rho))^{-2} dx = \int_\rho^r \frac{\mathrm{sign}((r-\rho)y)}{\sqrt{(x^2 - M)(x^2 - \rho^2)}} dx, \tag{10} $$

where inside the horizon $r < \rho < \sqrt{M}$ and outside the horizon $\sqrt{M} < \rho < r$. This coordinate system is well prepared to fulfill the above formulated requirements supporting an orbifold geometry $g_{AB}(x, y) = g_{AB}(x, -y)$. If the normal coordinate y is taken as an arbitrary real number, the respective range of variation for the coordinate ρ will not normally coincide with the entire semiaxis. Its acceptable values are thoroughly analyzed below.

We notice that the integrals in (9) and (10) can be expressed through the standard elliptic integrals. In particular, inside the horizon,

$$ \begin{aligned} |y| &= \sqrt{M}\left[K\left(\frac{\rho}{\sqrt{M}}\right) - F\left(\frac{r}{\rho}, \frac{\rho}{\sqrt{M}}\right) - E\left(\frac{\rho}{\sqrt{M}}\right) + E\left(\frac{r}{\rho}, \frac{\rho}{\sqrt{M}}\right)\right], \\ \theta &= \mathrm{sign}(y)\frac{\rho}{\sqrt{M}}\left[K\left(\frac{\rho}{\sqrt{M}}\right) - F\left(\frac{r}{\rho}, \frac{\rho}{\sqrt{M}}\right)\right], \end{aligned} \tag{11} $$

and outside the horizon,

$$ \begin{aligned} |y| &= \rho\left[K\left(\frac{\sqrt{M}}{\rho}\right) - F\left(\frac{\rho}{r}, \frac{\sqrt{M}}{\rho}\right) - E\left(\frac{\sqrt{M}}{\rho}\right) + E\left(\frac{\rho}{r}, \frac{\sqrt{M}}{\rho}\right)\right] + \frac{\sqrt{(r^2-\rho^2)(r^2-M)}}{r}, \\ \theta &= sgn(y)\left[K\left(\frac{\sqrt{M}}{\rho}\right) - F\left(\frac{\rho}{r}, \frac{\sqrt{M}}{\rho}\right)\right]. \end{aligned} $$

The metric in new coordinates reads,

$$ g_{AB} = -\mathrm{diag}\left[-U(r), \frac{r^2 r_\rho^2}{\rho^2 U(r)}, R^2, R^2\cos^2\theta_1, 1\right], \tag{12} $$

where $r = r(\rho, y)$, $\theta = \theta(\rho, y)$, $R = r\cos\theta$, $r_\rho \equiv \partial r/\partial\rho$.

From Eq. (11) at $r = 0$ one finds the minimal value $\rho_{\min}$ which exists for any y, lies in the interval $0 < \rho_{\min} < \sqrt{M}$ and grows monotonously with increasing $|y|$. Thus, there is a lower bound for variations of ρ. However, this lower bound in general is not equal to $\rho_{\min}$. Indeed let us analyze the limits for variation of the angular variable θ in (11) in the limit $r = 0$, and insert $\rho = \rho_{\min}$. It can be shown that at fixed y the variable $|\theta|$ monotonously decreases with increasing ρ thus, one can derive $\theta_{\max}$,

$$\theta_{\max} \equiv \mathrm{sign}(y) \frac{\rho_{\min}}{\sqrt{M}} K\left(\frac{\rho_{\min}}{\sqrt{M}}\right). \tag{13}$$

Meanwhile the variable, θ, runs between $-\pi/2$ and $\pi/2$. Thus, for large y there exists a critical value of $\rho_{\min}$ for which $|\theta_{\max}| = \pi/2$. We denote this value as ρ_c. It satisfies the following equation,

$$\frac{\pi}{2} = \frac{\rho_c}{\sqrt{M}} K\left(\frac{\rho_c}{\sqrt{M}}\right); \qquad \rho_c \simeq 0.79272\sqrt{M}. \tag{14}$$

From (11) at $r = 0$ one obtains the critical value

$$y_c \simeq 0.69868\sqrt{M}. \tag{15}$$

Any hypersurface of constant $y = a$ with $|a| < y_c$ contains the origin $r = 0$ and therefore the stress-energy tensor T_{AB} generated by (1) reveals a pointlike delta-singularity at the origin. For $|a| > y_c$, the hypersurfaces intersect $\theta = \pi/2$ and do not pass $r = 0$, see Fig. 1. On such hypersurfaces, the minimal value of ρ is larger than $\rho_{\min}$. Further on we will study only hypersurfaces which do not intersect the singularity at $r = 0$, i.e. for $|a| > y_c$, to avoid deltalike singularities in matter distributions. Thus, we presuppose that black stars created on colliders must possess a smooth matter-density.

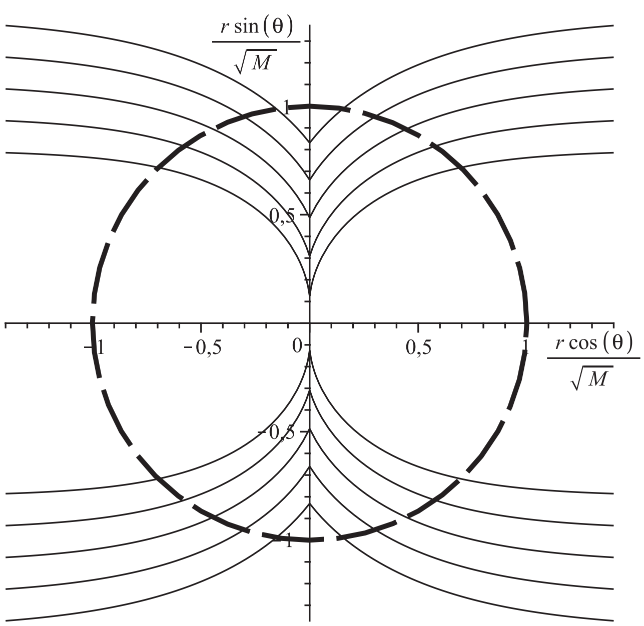

FIG. 1. Pairs of hypersurfaces symmetric in respect to the horizontal axis to be glued into a brane are shown by solid curves. The circle of horizon in dim= 5 is depicted by the long dashes.

Let us elucidate how the horizon looks in new coordinates. The variable ρ runs between r and $\sqrt{M}$, therefore the horizon $r = \sqrt{M}$ corresponds to $\rho = \sqrt{M}$. From (10) at $r = \rho = \sqrt{M}$ it follows that on the horizon $\theta = \frac{y}{\sqrt{M}}$, see Fig. 1. Taking into account the range of variation of θ, one concludes that in new variables the horizon is a part of cylinder-type surface with radius $\rho = \sqrt{M}$ and height $-\frac{\pi}{2\sqrt{M}} < y < \frac{\pi}{2\sqrt{M}}$.

Now we examine a hypersurface of constant $y = a$ and choose on it the following coordinates t, ρ, θ_1, θ_2. In accordance with (12) the metric induced on the hypersurface can be obtained by projection ${}^{(5)}g_{AB} \to {}^{(4)}g_{\mu\nu}$ with μ, $\nu = 0, 1, 2, 3$.

In normal Gaussian coordinates the extrinsic curvature is determined by,

$$K_{\mu\nu} = -\frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial y}. \tag{16}$$

Its trace,

$$K = \frac{-3r_y r_\rho \cos\theta - \cos\theta r r_{\rho,y} + 2r_\rho r \sin\theta \theta_y}{\cos\theta r_\rho r}, \tag{17}$$

where $r_\rho \equiv \partial r/\partial\rho$, $r_y \equiv \partial r/\partial y$, $\theta_\rho \equiv \partial\theta/\partial\rho$, $r_{\rho,y} \equiv \partial^2 r/\partial\rho\partial y$. Let us generate a brane following the tailoring construction (1). Then quite remarkably the timelike component of the stress-energy tensor τ_0^0 is positive for a positive shift a which follows from the Israel matching conditions (2). The sign of K is also correlated to the sign of a and coincides with the sign of τ for $a > 0$. Thereby the positivity of a is required for realization of energy conditions.

At large ρ its asymptotics reads,

$$K = \frac{4M^2y^3}{3\rho^8}\left(1 + O\left(\frac{y^2}{\rho^2}\right)\right), \tag{18}$$

thereby confirming the asymptotic flatness of the brane. The additional evidence for the flatness is given by the asymptotics of scalar curvature,

$${}^{(4)}R = \frac{4M^2y^2}{\rho^8}\left(1 + O\left(\frac{y^2}{\rho^2}\right)\right). \tag{19}$$

On the horizon it remains finite and continuous,

$$K|_{\rho=\sqrt{M}} = \frac{-2sgn(y)\sqrt{\frac{B}{B+1}}\cos\frac{y}{\sqrt{M}} + 2\sin\frac{y}{\sqrt{M}}}{\sqrt{M}\cos\frac{y}{\sqrt{M}}}, \tag{20}$$

where the constant B represents the following limit,

$$B \equiv \lim_{\rho\to\sqrt{M}} \frac{r-\rho}{\rho-\sqrt{M}} = \frac{1}{2}\left(\cosh\left(\frac{2y}{\sqrt{M}}\right) - 1\right). \tag{21}$$

In terms of this constant, the values of other geometrical quantities on the horizon can be expressed. In particular, the scalar curvature on the horizon takes the following value,

$$ {}^{(4)}R = -2\frac{B + 1 - \cos^2 y - 4|\sin y|\sqrt{1+B}\sqrt{B}\cos y}{(1+B)\cos^2 y} $$

Let us discuss the matter-density radial distribution using the Komar integral representation [13] for total mass of a black star. The total mass in 4 + 1 dimension is given by,

$$ \mathcal{M} = \frac{3}{16\pi\kappa_5}\int_{t=\text{const}} d^{(4)}V^{(5)}R_{AB}\xi^A m^B \equiv \int_0^\infty dR f_5(R), \tag{22} $$

where

$$ f_5(R) = -\frac{3}{8\pi\kappa_5}\int d\theta_1 d\theta_2 \sqrt{|{}^{(4)}g|}K^0_0|_{y=a} = -\frac{3}{2\kappa_5}\frac{r^3\cos^2\theta r_\rho}{\rho(r_\rho\cos\theta - r\theta_\rho\sin\theta)}K^0_0|_{y=a}, \tag{23} $$

and the radial coordinate $R \equiv r(\rho, a)\cos(\rho, a)$ is chosen. In Eq. (22) the Killing vector, $\xi = \partial_t$ and the vector m orthonormal to the hypersurface $t = \text{const}$ are used.

A different mass distribution is seen from the brane viewpoint as being generated by the effective stress-energy tensor $S_{\mu\nu}$ in (5). We again follow the Komar integral representation which is based on the timelike component of the Ricci tensor ${}^{(4)}R^0_0$

$$ {}^{(4)}R^0_0 = -\frac{M\rho(2r_\rho\rho\cos\theta - r\cos\theta + 2\rho r\sin\theta\theta_\rho)}{r^6 r_\rho\cos\theta}. \tag{24} $$

From (24) it can be derived that near singularity at $\theta(\rho, y) = \pi/2$ this component is positive whereas at the infinity it is negative. The exact calculations show that the 3-dim Komar integral [13],

$$ \mathcal{M} = \int_0^\infty dR f_4(R); \quad f_4(R) = \frac{8\pi r^3\cos^2\theta r_\rho {}^{(4)}R^0_0}{\kappa_4\rho(r_\rho\cos\theta - r\theta_\rho\sin\theta)}\Bigg|_{y=a} \tag{25} $$

on the brane vanishes, which is in accordance with the tidal character [8] of the black star metric $\sim 1/R^2$ at large radius.

The comparison of two radial density distributions are presented in Fig. 2

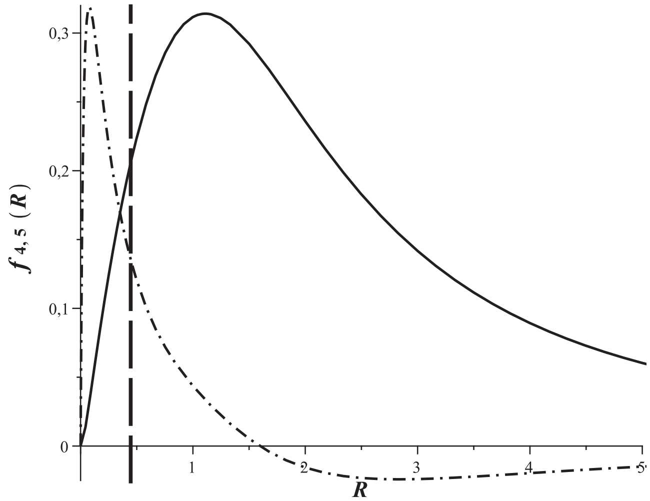

FIG. 2. The matter-density radial distribution $f_5(R)$ on the brane with $a = 1.1$, $M = 1$ is presented for $\kappa_5 = 1$ by a solid curve. The effective matter-density $f_4(R)$ is shown by the dashed dotted line for the value $\kappa_4 = 50$ to compare with $f_5(R)$. The horizon is indicated by the long dashed line.

We see that in spite of the presence of a singularity in ${}^{(4)}R^0_0$, this function is integrable in (25) and moreover, this singularity is completely suppressed by the volume factor.

Thus, we have shown that by cut-and-paste methods in special Gaussian normal coordinates one can build the exact geometry of a multidimensional black star with a *horizon*, generated by a *smooth* matter distribution in our universe.

In our approach, for a given total mass, the profiles of available configurations for matter distribution are governed by the parameter a which is presumably related to the collision kinematics when a black object (''black hole'') is created by partons on colliders. When $a > y_c$ the very distribution does not reveal any deltalike singularity at the origin and therefore the density contribution in its small vicinity is subdominant. For a larger a, the matter-density happens to be more diluted approaching the normal nuclear one and for $a > \pi\sqrt{M}/2$ the horizon disappears. The apparent singularity at the origin $R(a) = 0$ does not lead to acausal dynamics due to finiteness of corresponding components of metric and affine connections. Therefore, it is not a harmful naked singularity. To resume, one could think of the presented solution as a better approximation for describing mini black hole creation on high energy colliders than the more often used Schwarzschild-Tangherlini one.

Certainly the tailoring method used to build black objects with matter localized on branes can be generalized to the cases of charged and rotated black stars as well as black rings [19]. However, more work must be done to extend the solutions found on compact extra dimensions [16,20] and warped geometries [21].

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