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Primordial non-Gaussianity from the DBI Galileons

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We study primordial fluctuations generated during inflation in a class of models motivated by the DBI Galileons, which are extensions of the DBI action that yield second-order field equations. This class of models generalizes the DBI Galileons in a similar way with K inflation. We calculate the primordial non-Gaussianity from the bispectrum of the curvature perturbations at leading order in the slow-varying approximations. We show that the estimator for the equilateral-type non-Gaussianity, $f_{\rm NL}^{\rm equil}$, can be applied to measure the amplitude of the primordial bispectrum even in the presence of the Galileon-like term although it gives a slightly different momentum dependence from K-inflation models. For the DBI Galileons, we find $-0.32/c_s^2 < f_{\rm NL}^{\rm equil} < -0.16/c_s^2$ and large primordial non-Gaussianities can be obtained when c_s is much smaller than 1 as in the usual DBI inflation. In G-inflation models, where a de Sitter solution is obtained without any potentials, the nonlinear parameter is given by $f_{\rm NL}^{\rm equil} = 4.62 r^{-2/3}$, where r is the tensor to scalar ratio, giving a stringent constraint on the model.

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I. INTRODUCTION

The DBI inflation model [1] is one of the most interesting possibilities to realize large non-Gaussianity of the cosmic microwave background (CMB) temperature fluctuations. Non-Gaussianity of the curvature perturbation in DBI inflation has been studied extensively [2–22] (see also [23,24] for reviews).

Recently, a very interesting extension of the DBI inflation model, the so-called "DBI Galileons" was proposed by de Rham and Tolley [25] (see also [26]). This is based on the relativistic extension of the Galileon model [27–29] (for studies of cosmology based on the Galileon field, see Refs. [30–40]). The simplest example is a single field model that arises from a probe brane action in the five-dimensional spacetime. Let us consider the following four-dimensional induced action on the probe brane:

$$S = \int d^4x\sqrt{-g}(\lambda - M_5^3 K), \tag{1}$$

where λ is a tension of the brane, M_5 is the five-dimensional Planck constant, $g_{\mu\nu}$ is the induced metric on the brane,

$$g_{\mu\nu} = \eta_{\mu\nu} + \partial_\nu\pi\partial_\mu\pi, \tag{2}$$

and K is a trace of the extrinsic curvature $K_{\mu\nu}$,

$$K_{\mu\nu} = -\frac{\partial_\mu\partial_\nu\pi}{\sqrt{1+(\partial\pi)^2}}. \tag{3}$$

Here π is a modulus describing the position of the brane. Using the fact that the inverse metric is given by $g^{\mu\nu} = \eta^{\mu\nu} - \gamma^2\partial^\mu\pi\partial^\nu\pi$, where γ is the Lorentz factor $\gamma = 1/\sqrt{1+(\partial\pi)^2}$, the brane action is written as

$$S = -\lambda\int d^4x\sqrt{1+(\partial\pi)^2} + M_5^3 \times \int d^4x(\Box\pi - \gamma^2\partial_\mu\partial_\nu\pi\partial^\mu\pi\partial^\nu\pi). \tag{4}$$

By integrating by part and discarding the total derivative terms, this action can be rewritten as

$$S = -\lambda\int d^4x\sqrt{1+(\partial\pi)^2} + \frac{M_5^3}{2}\int d^4x(\gamma^2(\partial\pi)^2\Box\pi + \partial_\mu(\gamma^2)(\partial^\mu\pi)(\partial\pi)^2). \tag{5}$$

The first term is the usual DBI action. The higher order terms look to contain the higher derivatives but the equation of motion is at most second order in derivatives. In the nonrelativistic limit $\gamma \to 1$, this reduces the Galileon model where the action is invariant under the Galileon symmetry $\partial_\mu\pi \to \partial_\mu\pi + c_\mu$. It is also possible to include two more higher order terms in the action, but in this paper we focus our attention on the leading order cubic order terms.

Recently, Refs. [41,42] considered a generalization of this model. This generalization is based on the extension of DBI inflation to K inflation. The generalized action is given by

$$S = \int d^4x\sqrt{-g}(P(X,\phi) - G(X,\phi)\Box\phi), \tag{6}$$

where ϕ is the same degree of freedom as π, but it is defined so that ϕ has a dimension of mass and $X = -(1/2)(\partial\phi)^2$. In Eq. (6), $P(\phi, X)$ and $G(\phi, X)$ are arbitrary functions of ϕ and X. Precisely speaking, this generalization does not include the action (5). However, in the DBI inflation, the Lorentz factor, γ, varies very slowly and at

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leading order in the slow-varying parameters, which is usually used to calculate the leading order contribution to non-Gaussianity, the last term in (5) does not play a role. Thus the action (6) is general enough to include the case of the DBI Galileons. Again the equation of motion is at most second order in derivatives.

In this paper, we study primordial fluctuations generated during inflation described by the action (6). We calculate the power spectrum of the curvature perturbation as well as the bispectrum of the curvature perturbations. Two examples of the model are considered: one is the DBI Galileons described by the action (5). The other is G-inflation models proposed by Ref. [42]. This model is based on a specific choice of the functions $P(X)$ and $G(X)$ that realizes a de Sitter solution without any potentials.

This paper is organized as follows. In Sec. II, we introduce a model studied in this paper. The power spectrum is calculated in Sec. II and the bispectrum is discussed in Sec. III. In Sec. IV, we consider two examples, the DBI Galileons and G inflation. Section V is devoted to conclusions.

II. MODEL

Assuming that ϕ described by the action (6) is minimally coupled to gravity, we consider a class of models described by the following action:

$$S = \frac{1}{2}\int d^4x\sqrt{-g}[M_{\rm pl}^2 R + 2P(\phi, X) - 2G(\phi, X)\Box\phi], \tag{7}$$

where $M_{\rm pl}$ is the Planck mass. In the background, we are interested in flat, homogeneous, and isotropic Friedmann-Robertson-Walker universes described by the line element

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j, \tag{8}$$

where $a(t)$ is the scale factor. It can be shown that the energy density and pressure of the field are given by

$$\rho = 2P_{,X}X - P + 6G_{,X}H\dot\phi X - 2G_{,\phi}X, \tag{9}$$

$$p = P - 2(G_{,\phi} + G_{,X}\ddot\phi)X, \tag{10}$$

where $H = \dot a/a$ is the Hubble parameter, the dot represents a derivative with respect to cosmic time t, and the subscripts $,X$ and $,\phi$ denote derivatives with respect to X and ϕ, respectively. The Friedmann equation and the field equation are given by

$$3M_{\rm pl}^2H^2 = \rho, \tag{11}$$

$$\begin{aligned} &P_{,X}(\ddot\phi + 3H\dot\phi) + 2P_{,XX}X\ddot\phi + 2P_{,X\phi}X - P_{,\phi} \\ &\quad - 2G_{,\phi}(\ddot\phi + 3H\dot\phi) - 2G_{,X\phi}X(\ddot\phi - 3H\dot\phi) \\ &\quad + 6G_{,X}[(HX)^{\cdot} + 3H^2X] - 2G_{,\phi\phi} + 6G_{,XX}HX\dot X = 0. \end{aligned} \tag{12}$$

It is useful to define a slow-varying parameter

$$\epsilon \equiv -\frac{\dot H}{H^2} = \frac{XP_{,X} + 3G_{,X}H\dot\phi X}{M_{\rm pl}^2H^2}, \tag{13}$$

where for the second equality, we have assumed the quantities $|\ddot\phi/(H\dot\phi)|$ and $|G_{,\phi}\dot\phi/(GH)|$ are much smaller than 1. Since we are interested in fluctuations generated during inflation, we will consider the background that satisfies the slow-varying conditions which are given by $|\epsilon| \ll 1$, together with $|\ddot\phi/(H\dot\phi)| \ll 1$ and $|G_{,\phi}\dot\phi/(GH)| \ll 1$.

III. POWER SPECTRUM

We are interested in the primordial curvature perturbation on uniform density hypersurfaces, ζ, on large scales, which is directly related to temperature anisotropies in the CMB. In order to calculate the statistical quantities of ζ at leading order in the slow-varying approximations, we first calculate the bispectrum of the fluctuations of inflaton ϕ in the flat gauge where the three-dimensional metric takes the form $h_{ij} = a^2\delta_{ij}$, and then relate it to that of ζ using the relation obtained from the delta-N formalism [43,44]

$$\zeta = -\frac{H}{\dot\phi}Q. \tag{14}$$

In Eq. (14), ϕ is the background value and Q is the perturbation in the flat gauge. In this gauge, at leading order in the slow-varying approximations, the second-order action is expressed as

$$\begin{aligned} S_2 = \int dtd^3xa^3\Big[&P_{,X}\delta X^{(2)} + \frac{1}{2}P_{,XX}(\delta X^{(1)})^2 \\ &- G_{,X}\delta X^{(2)}\Box\phi^{(0)} - G_{,X}\delta X^{(1)}\Box\phi^{(1)} \\ &- \frac{1}{2}G_{,XX}(\delta X^{(1)})^2\Box\phi^{(0)}\Big], \end{aligned} \tag{15}$$

with

$$\begin{aligned} &\delta X^{(1)} = \dot\phi\,\dot Q, \qquad \delta X^{(2)} = \frac{1}{2}\dot Q^2 - \frac{1}{2a^2}\partial^i Q\partial_i Q, \\ &\Box\phi^{(0)} = -3H\dot\phi, \qquad \phi^{(1)} = -\ddot Q - 3H\dot Q + \frac{1}{a^2}\partial^i\partial_i Q. \end{aligned} \tag{16}$$

Introducing the sound speed for the scalar perturbations

$$c_s^2 = \frac{P_{,X} + 4\dot\phi HG_{,X}}{P_{,X} + 2XP_{,XX} + 6H\dot\phi(G_{,X} + XG_{,XX})}, \tag{17}$$

and integrating by parts, we can write the second-order action as

$$S_2 = \int dt d^3x \frac{a^3}{2c_s^2}(P_{,X} + 4\dot{\phi}HG_{,X})\Big[\dot{Q}^2 - \frac{c_s^2}{a^2}\partial^i Q \partial_i Q\Big]. \tag{18}$$

The perturbations in the interaction picture are promoted to the quantum operators as

$$Q(\tau, \mathbf{x}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{k} Q(\tau, \mathbf{k})e^{i\mathbf{k}\cdot\mathbf{x}}, \quad Q(\tau, \mathbf{k}) = u(\tau, \mathbf{k})a(\mathbf{k}) + u^*(\tau, -\mathbf{k})a^\dagger(-\mathbf{k}), \tag{19}$$

where $a(\mathbf{k})$ and $a^\dagger(-\mathbf{k})$ are the annihilation and creation operators, respectively. They satisfy the usual commutation relations

$$[a(\mathbf{k_1}), a^\dagger(\mathbf{k_2})] = (2\pi)^3\delta^{(3)}(\mathbf{k_1} - \mathbf{k_2}), [a(\mathbf{k_1}), a(\mathbf{k_2})] = [a^\dagger(\mathbf{k_1}), a^\dagger(\mathbf{k_2})] = 0. \tag{20}$$

From the second-order action (18), the solution for the mode functions is given by

$$u(\tau, \mathbf{k}) = \frac{H}{\sqrt{2c_s(P_{,X} + 4\dot{\phi}HG_{,X})}}\frac{1}{k^{3/2}}(1 + ikc_s\tau)e^{-ikc_s\tau}. \tag{21}$$

It is convenient to introduce the following parameters:

$$\nu \equiv \frac{G_{,X}\dot{\phi}X}{M_{pl}^2 H}, \qquad \tilde{\epsilon} \equiv \epsilon + \nu, \tag{22}$$

where $\tilde{\epsilon}$ coincides with ϵ when there is no Galileon-like term. $\tilde{\epsilon}$ is also much smaller than 1 for $\nu \ll 1$. Then, the power spectra of Q and ζ are given by

$$\langle Q(\mathbf{k_1})Q(\mathbf{k_2})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1} + \mathbf{k_2})\mathcal{P}_Q\frac{2\pi^2}{k_1^3}, \quad \mathcal{P}_Q = \frac{X}{4\pi^2 M_{\rm pl}^2 c_s\tilde{\epsilon}}, \tag{23}$$

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1} + \mathbf{k_2})\mathcal{P}_\zeta\frac{2\pi^2}{k_1^3}, \quad \mathcal{P}_\zeta = \frac{1}{8\pi^2 M_{\rm pl}^2}\frac{H^2}{c_s\tilde{\epsilon}}, \tag{24}$$

which are evaluated at the time of the sound horizon exit, $c_s k = aH$. It is worth noting that it is $\tilde{\epsilon}$ and not ϵ which appears in Eq. (24), which gives the behavior of the power spectrum that is different from the usual K-inflation model as we will see below. Defining additional slow-varying parameters

$$\tilde{\eta} \equiv \frac{\dot{\tilde{\epsilon}}}{\tilde{\epsilon}H}, \qquad s \equiv \frac{\dot{c}_s}{c_s H}, \tag{25}$$

the spectral index of the primordial power spectrum is given by

$$n_s - 1 = \frac{d\ln\mathcal{P}_\zeta(k)}{d\ln k} = -2\epsilon - \tilde{\eta} - s. \tag{26}$$

We need to require ϵ, $\tilde{\eta}$, and s to be very small in order to realize the almost scale invariant power spectrum. We have confirmed that this result is consistent with the one obtained in Ref. [42] when the conditions $|\ddot{\phi}/(H\dot{\phi})| \ll 1$ and $|G_{,\phi}\dot{\phi}/(GH)| \ll 1$ are satisfied. Notice that $\tilde{\eta}$ is different from the usual η defined by $\eta \equiv \dot{\epsilon}/(\epsilon H)$.

The power spectrum and spectral index of tensor perturbations are given by the usual expression

$$\mathcal{P}_T = \frac{2H^2}{\pi^2 M_{\rm pl}^2}, \qquad n_T = -2\epsilon. \tag{27}$$

In the usual K inflation, n_T and the tensor to scalar ratio $r \equiv \mathcal{P}_T/\mathcal{P}_\zeta$ are not independent, and there is a so-called "consistency relation" $r = -8c_s n_T$ [45]. However, it is clear that this relation does not hold in the presence of the Galileon-like term. Instead, we have

$$r = -8c_s(n_T - 2\nu), \tag{28}$$

where ν is given by Eq. (22).

IV. BISPECTRUM

The third order action can be obtained in the same way as

$$S_3 = \int dt d^3x \frac{a^3}{\dot{\phi}}\Big[C_1\dot{Q}^3 + \frac{C_2}{a^2}\dot{Q}\partial^i Q\partial_i Q + \frac{C_3}{a^4 H}\partial^i Q\partial_i Q\partial^j\partial_j Q + \frac{C_4}{a^2 H}\dot{Q}\partial^i\dot{Q}\partial_i Q\Big], \tag{29}$$

where

$$\begin{aligned} C_1 &= \tfrac{2}{3}X^2 P_{,XXX} + XP_{,XX} + 2H\dot{\phi}X^2 G_{,XXX} + 5H\dot{\phi}XG_{,XX} + H\dot{\phi}G_{,X}, \\ C_2 &= -(XP_{,XX} + 3H\dot{\phi}XG_{,XX} + H\dot{\phi}G_{,X}), \\ C_3 &= \tfrac{1}{2}H\dot{\phi}G_{,X}, \qquad C_4 = 2H\dot{\phi}G_{,X} + 2H\dot{\phi}XG_{,XX}. \end{aligned} \tag{30}$$

The vacuum expectation value of the three point operator in the interaction picture is written as [43,46]

$$\langle Q(t, \mathbf{k_1})Q(t, \mathbf{k_2})Q(t, \mathbf{k_3})\rangle = -i\int_{t_0}^t d\tilde{t}\langle[Q(t, \mathbf{k_1})Q(t, \mathbf{k_2})Q(t, \mathbf{k_3}), H_I(\tilde{t})]\rangle, \tag{31}$$

where t_0 is some early time during inflation when the field's vacuum fluctuation are deep inside the sound horizon and t is some time after the sound horizon exit. If one uses a conformal time, it is a good approximation to perform the integration from $-\infty$ to 0 because $\tau \approx -(aH)^{-1}$. H_I denotes the interaction Hamiltonian and it is given by $H_I = -L_3$, where L_3 is the Lagrangian obtained from the action (29). Using the solution for the mode function and the commutation relations for the creation and annihilation operators, we get

$$\langle Q(\mathbf{k_1})Q(\mathbf{k_2})Q(\mathbf{k_3})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) \times \frac{H^5}{(P_{,X}+4\dot{\phi}HG_{,X})^3\dot{\phi}} \times \frac{1}{\Pi_{i=1}^3 k_i^3}\mathcal{A}_\phi, \quad (32)$$

where

$$\mathcal{A}_\phi = 3\left(C_1 - \frac{C_4}{c_s^2}\right)\mathcal{A}_1 + \frac{C_2}{2c_s^2}\mathcal{A}_2 + \frac{C_3}{c_s^4}\mathcal{A}_3, \quad (33)$$

and C_1, C_2, C_3, C_4, and C_5 are given by Eq. (30). In Eq. (33), we have introduced the shape functions $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$ as

$$\mathcal{A}_1 = \frac{k_1^2k_2^2k_3^2}{K^3}, \quad (34)$$

$$\mathcal{A}_2 = \frac{k_1^2\mathbf{k_2}\cdot\mathbf{k_3}}{K}\left(1+\frac{k_2+k_3}{K}+2\frac{k_2k_3}{K^2}\right)+2\,\text{perms}, \quad (35)$$

$$\mathcal{A}_3 = \frac{k_3^2\mathbf{k_1}\cdot\mathbf{k_2}}{K}\left(1+\frac{k_1k_2+k_2k_3+k_3k_1}{K^2}+3\frac{k_1k_2k_3}{K^3}\right) + 2\text{ perms}, \quad (36)$$

where $K = k_1 + k_2 + k_3$. The shapes $\mathcal{A}_1$ and $\mathcal{A}_2$ appear in the usual K-inflation models [7] and their amplitudes can be measured by the estimator for the equilateral-type non-Gaussianity, $f_{\rm NL}^{\rm equil}$ [47]. On the other hand, the shape $\mathcal{A}_3$ is completely new that arises from the Galileon-like term.

For the bispectrum of ζ, making use of Eqs. (14) and (24), we obtain the following expression:

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}) \times (\mathcal{P}_\zeta)^2F(k_1,k_2,k_3), \quad (37)$$

where

$$F(k_1,k_2,k_3) = \frac{(2\pi)^4}{\Pi_{i=1}^3k_i^3}\sum_{j=1}^3 f^{(j)}\mathcal{A}_j, \quad (38)$$

$$f^{(1)} = \frac{-3(C_1c_s^2-C_4)}{(P_{,X}+4\dot{\phi}HG_{,X})},\quad f^{(2)} = -\frac{C_2}{2(P_{,X}+4\dot{\phi}HG_{,X})},\quad f^{(3)} = -\frac{C_3}{(P_{,X}+4\dot{\phi}HG_{,X})c_s^2}. \quad (39)$$

In the following, we will study the momentum dependence of the bispectrum. Especially, we will check the validity of using the estimator for equilateral-type non-Gaussianity, $f_{\rm NL}^{\rm equil}$, for the bispectrum even in the presence of the new shape $\mathcal{A}_3$. The estimator is defined by

$$F^{\rm equil}(k_1,k_2,k_3) = (2\pi)^4\left(\frac{9}{10}f_{\rm NL}^{\rm equil}\right) \times\left(-\frac{1}{k_1^3k_2^3}-\frac{1}{k_1^3k_3^3}-\frac{1}{k_2^3k_3^3}-\frac{2}{k_1^2k_2^2k_3^2} + \frac{1}{k_1k_2^2k_3^3}+(5\text{ perms})\right), \quad (40)$$

where the permutations act only on the last term in parentheses. This shape is factorizable and it is possible to construct a fast optimal estimator that can be applied to the CMB map. For this purpose, it is useful to define shape functions $F^{(i)}(k_1,k_2,k_3)$, $i = 1, 2, 3$ corresponding to the shapes $\mathcal{A}_i$ in Eq. (38). As mentioned before, the shape of the bispectrum in K inflation is characterized by the sum of $F^{(1)}(k_1,k_2,k_3)$ and $F^{(2)}(k_1,k_2,k_3)$ with $f^{(1)}$ and $f^{(2)}$ depending on $P(X)$. For example, in DBI inflation, the relation $f^{(2)}/f^{(1)} = -2/3$ holds. However, since functions $F^{(1)}(k_1,k_2,k_3)$ and $F^{(2)}(k_1,k_2,k_3)$ are not factorizable, $F^{\rm equil}(k_1,k_2,k_3)$ is usually used to approximate the shape functions $F^{(1)}(k_1,k_2,k_3)$ and $F^{(2)}(k_1,k_2,k_3)$.

In Fig. 1, we compare $F^{\rm equil}(k_1,k_2,k_3)$ with $F^{(1)}(k_1,k_2,k_3)$, $F^{(2)}(k_1,k_2,k_3)$, and $F^{(3)}(k_1,k_2,k_3)$ with appropriate normalizations. From this, we see that not only $F^{(1)}(k_1,k_2,k_3)$ and $F^{(2)}(k_1,k_2,k_3)$, but also the function $F^{(3)}(k_1,k_2,k_3)$ has a very similar shape with $F^{\rm equil}(k_1,k_2,k_3)$ and it can be expected that $f_{\rm NL}^{\rm equil}$ provides a good measure of the bispectrum even in the presence of the new shape function $F^{(3)}$. We can prove this quantitatively by the following shape correlator $\mathcal{C}$ for two different shapes characterized by F and F' introduced by Ref. [48] (see also [49,50]),

$$\mathcal{C}(F,F') = \frac{\mathcal{F}(F,F')}{\sqrt{\mathcal{F}(F,F)\mathcal{F}(F',F')}}, \quad (41)$$

where the overlap function $\mathcal{F}$ is given by

$$\mathcal{F}(F,F') = \int d\mathcal{V}_kF(k_1,k_2,k_3)F'(k_1,k_2,k_3) \times \Pi_{i=1}^4k_i^4w_B(k_1,k_2,k_3). \quad (42)$$

In Eq. (42) the integration is performed for the region where the triangle condition for (k_1, k_2, k_3) holds and weight function w_B is given by

$$w_B = \frac{1}{k_1+k_2+k_3}. \quad (43)$$

Table I shows that the shapes $F^{\rm equil}(k_1,k_2,k_3)$ and $F^{(3)}(k_1,k_2,k_3)$ are almost completely anticorrelated, which means that it is very reasonable to adopt the estimator $f_{\rm NL}^{\rm equil}$

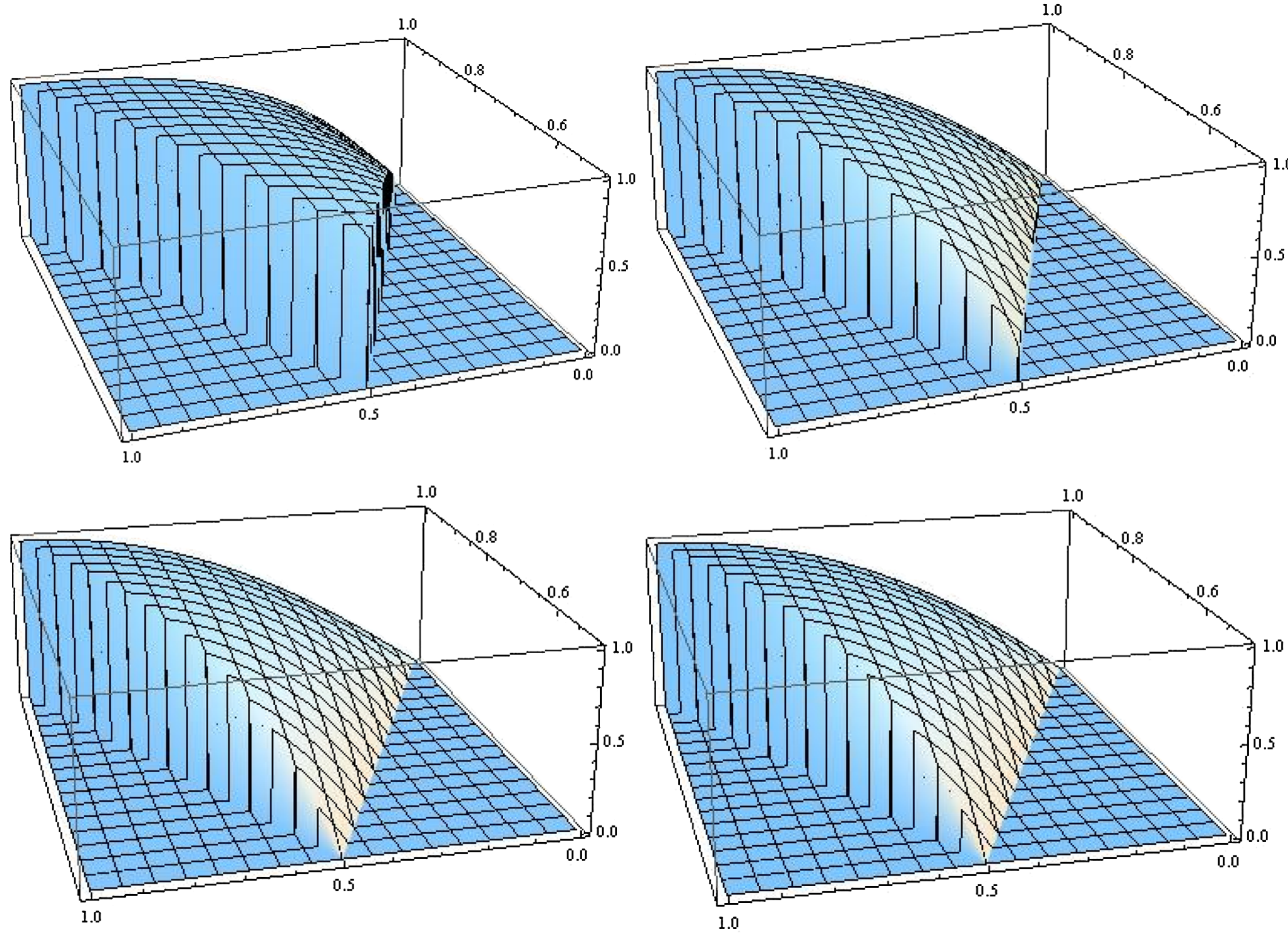

FIG. 1 (color online). In this group of figures, we plot the shape functions $F(1, k_2/k_1, k_3/k_1)(k_2/k_1)^2(k_3/k_1)^2$ as functions of $(k_2/k_1, k_3/k_1)$. The figures are normalized to have values of 1 for equilateral configurations $k_2/k_1 = k_3/k_1 = 1$ and set to zero outside the region $1 - k_2/k_1 \leq k_3/k_1 \leq k_2/k_1$. We plot $F^{(1)}(k_1, k_2, k_3)$, $F^{(2)}(k_1, k_2, k_3)$, $F^{(3)}(k_1, k_2, k_3)$, and $F^{\rm equil}(k_1, k_2, k_3)$ for upper left, upper right, lower left, and lower right, respectively.

to measure the amplitude of the bispectrum even in the presence of the shape $F^{(3)}(k_1, k_2, k_3)$.

Now that the validity of using the estimator for the equilateral-type non-Gaussianity, $f_{NL}^{\rm equil}$ for the bispectrum corresponding to the new shape $\mathcal{A}_3$ is confirmed, we will obtain $f_{\rm NL}^{\rm equil}$ for the bispectrum given by Eq. (38). We match the two different shapes (38) and (40) so that these two shapes have the same amplitudes at the equilateral configuration $k_1 = k_2 = k_3$. This gives

$$f_{\rm NL}^{\rm equil} = \frac{10}{243} f^{(1)} - \frac{85}{81} f^{(2)} - \frac{65}{81} f^{(3)}. \qquad (44)$$

V. EXAMPLES

A. The DBI Galileons

For the first example, we consider the DBI Galileons [25] described by the action (5). Here we extended the original DBI Galileon model by introducing $V(\phi)$, $f(\phi)$,

TABLE I. Shape correlations between the factorizable equilateral shape $F^{\rm equil}(k_1, k_2, k_3)$ and the shapes of primordial bispectra characterized by functions $F^{(1)}(k_1, k_2, k_3)$, $F^{(2)}(k_1, k_2, k_3)$, and $F^{(3)}(k_1, k_2, k_3)$. For comparison, we have also shown the correlation between the equilateral shape and that obtained in DBI inflation which is given by $F^{(1)}(k_1, k_2, k_3) + F^{(2)}(k_1, k_2, k_3)$ satisfying $f^{(2)}/f^{(1)} = -2/3$.

	$F^{(1)}(k_1, k_2, k_3)$	$F^{(2)}(k_1, k_2, k_3)$	$F^{(3)}(k_1, k_2, k_3)$	$F^{\rm DBI}(k_1, k_2, k_3)$
Overlap	0.936	−0.995	−0.999 89	−0.993

and $g(\phi)$ which depend on ϕ weakly so that the slow-varying parameters are sufficiently small.

$$P(\phi, X) = -f(\phi)^{-1}\sqrt{1-2Xf(\phi)} + f(\phi)^{-1} - V(\phi),$$
$$G(\phi, X) = \frac{g(\phi)X}{1-2Xf(\phi)}. \tag{45}$$

It is worth mentioning that only particular choices of the functions $f(\phi)$ and $g(\phi)$ come from genuine higher-dimensional symmetries, although the equations of motion are still kept to be second order.

In order to analyze this model, it is convenient to define $c_D \equiv 1/P_{,X}$ which corresponds to the sound speed in the DBI model in the absence of the Galileon-like term. Since it is known that $f_{\rm NL}^{\rm equil} \propto 1/c_D^2$ in DBI inflation and we are interested in the case where the large non-Gaussianity is generated, we consider only the case with $c_D \ll 1$.

We assume that the inflation is driven by the potential term, that is, $V \gg X/c_D$, $gH\dot{\phi}X$ in Eq. (11) as in usual DBI inflation. During inflation, the field equation (12) becomes

$$\frac{3}{c_D}H\dot{\phi} + \frac{18gH^2X}{c_D^4} + V_{,\phi} = 0. \tag{46}$$

The value of c_D is specified from the background equations once $V(\phi)$ and $f(\phi)$ are given. We assume $\dot{c}_D/c_DH \ll 1$ so that, at leading order, c_D is constant [it is also possible to construct a model where c_D is constant by choosing a functional form of $f(\phi)$ for a given $V(\phi)$, see Ref. [51] for the details]. We define a parameter $b_D \equiv gH\dot{\phi}/c_D^3$ so that the first two terms in Eq. (46) are comparable when b_D is of order 1. It is worth noting that b_D can be also expressed as $b_D = \nu/(\epsilon - 3\nu)$. Making use of the concrete forms of $P(X, \phi)$ and $G(X, \phi)$ in Eq. (45) as well as the definition of b_D, the coefficients C_1, C_2, C_3, and C_4 in the third order action (30) are given by

$$C_1 = \frac{1}{2c_D^5}(1+24b_D), \qquad C_2 = -\frac{1}{2c_D^3}(1+12b_D),$$
$$C_3 = -\frac{1}{2c_D}b_D, \qquad C_4 = \frac{4}{c_D^3}b_D, \tag{47}$$

where we have used $XP_{,XX} \sim 1/(2c_D^3)$, $X^2P_{,XXX} \sim 3/(4c_D^5)$, $G_{,X} \sim g/c_D^4$, $XG_{,XX} \sim 2g/c_D^6$, $X^2G_{,XXX} \sim 6g/c_D^8$, and $c_D \ll 1$.

Then, the parameters $f^{(j)}$ in Eq. (39) become

$$f^{(1)} = -\frac{3}{2c_D^2}\frac{(1+20b_D)}{(1+4b_D)(1+12b_D)},$$
$$f^{(2)} = \frac{(1+12b_D)}{4(1+4b_D)c_D^2},$$
$$f^{(3)} = -\frac{(1+12b_D)b_D}{2(1+4b_D)^2c_D^2}. \tag{48}$$

From Eq. (44) we obtain

$$f_{\rm NL}^{\rm equil} = -\frac{5}{324c_s^2}\frac{(21+546b_D+3776b_D^2+6048b_D^3)}{(1+4b_D)(1+12b_D)^2}, \tag{49}$$

which becomes $-0.16/c_s^2$ for $b_D \gg 1$ and $-0.32/c_s^2$ for $b_D \to 0$. To obtain Eq. (49), we have used that c_D and c_s are related as

$$c_D^2 = \frac{(1+12b_D)}{(1+4b_D)}c_s^2. \tag{50}$$

The nonlinear parameter $f_{\rm NL}^{\rm equil}$ scales as $\propto 1/c_s^2$ and can be detectable by future experiments such as PLANCK for sufficiently small c_s.

Especially, in the case of $\tilde{\eta} = s = 0$, we find that the following relation holds:

$$r = 8c_s\frac{1+4b_D}{1+3b_D}(1-n_s). \tag{51}$$

Combined with Eqs. (49) and (51), we can express $f_{\rm NL}^{\rm equil}$ in terms of n_s and r as

$$f_{\rm NL}^{\rm equil} \simeq -20\frac{(1-n_s)^2}{r^2}, \tag{52}$$

where the coefficient is almost independent of b_D for this setup. This relation suggests that these Galileon-like terms do not help embed the DBI inflation into string theory [52,53].

B. G inflation

The second example is the recently proposed G inflation where an exact de Sitter solution is realized without introducing a potential [42]. In this model the functions $P(X)$ and $G(X)$ are chosen as

$$P(X) = -X + \frac{X^2}{2M^3\mu}, \qquad G(X) = \frac{1}{M^3}X. \tag{53}$$

The de Sitter solution is obtained when $P_{,X} + 3H\dot{\phi}/M^3 = 0$ is satisfied in Eq. (11). The solutions are obtained as

$$X = \mu M^3 x,$$
$$H^2 = \frac{M^3}{18\mu}\frac{(1-x)^2}{x}, \quad \text{where } \frac{1-x}{x\sqrt{1-x/2}} = \sqrt{6}\frac{\mu}{M_{\rm pl}}. \tag{54}$$

Here as in Ref. [42], we only consider simple cases where $\mu \ll M_{\rm pl}$. In this situation, $(1-x) \simeq \sqrt{3}\mu/M_{\rm pl} \ll 1$ and μ is related to $M_{\rm pl}$, M, and H as $\mu = 6M_{\rm pl}^2H^2/M^3$.

Then, making use of the concrete forms of $P(X, \phi)$ and $G(X, \phi)$ in Eq. (53), the fine-tuning condition $P_{,X} + 3H\dot{\phi}/M^3 = 0$ and Eq. (54), the primordial power spectrum and tensor to scalar ratio are given by

$$\mathcal{P}_\zeta = \frac{\sqrt{6}H^2}{16\pi^2 M_{\rm pl}^2}\frac{1}{(1-x)^{3/2}},$$
$$r = \frac{16\sqrt{6}}{3}(1-x)^{3/2}, \tag{55}$$

where we neglected the correction to the tensor to scalar ratio, $r = 16c_s\tilde{\epsilon}$, that arises if we allow a small deviation from the pure de Sitter inflation. As was pointed out in Ref. [42], r becomes nonzero even in the pure de Sitter solution, i.e. for $\epsilon = 0$. On the other hand, $n_s - 1$ becomes 0 for the pure de Sitter inflation, but again if we allow a small deviation for it parametrized by the slow-varying parameters, n_s is given by Eq. (26).

For the primordial bispectrum, the parameters $f^{(j)}$ in Eq. (39) become

$$f^{(1)} = \frac{9}{2},$$
$$f^{(2)} = \frac{3}{2(1-x)}, \tag{56}$$
$$f^{(3)} = -\frac{3}{(1-x)},$$

where we have set $x = 1$ unless it appears in the form of $1 - x$. Then from Eq. (44) we obtain

$$f_{\rm NL}^{\rm equil} = \frac{5}{6(1-x)}. \tag{57}$$

Especially, from Eq. (55) and assuming the pure de Sitter inflation, $\epsilon = 0$, $f_{\rm NL}^{\rm equil}$ is related to r as

$$f_{\rm NL}^{\rm equil} = 4.62\frac{1}{r^{2/3}}, \tag{58}$$

which gives a strong constraint on this model. For example, if $r = 0.17$, which can be detected by the PLANCK satellite [54], $f_{\rm NL}^{\rm equil} = 15.1$. Therefore, a detection or nondetection of the tensor mode and equilateral-type non-Gaussianity by PLANCK will tightly constrain the model.

VI. CONCLUSION

The DBI inflation model [1] has been extensively studied recently for both theoretical and phenomenological reasons. Especially, it is well known that the DBI inflation model can give large non-Gaussianity of the CMB temperature fluctuations. Recently, de Rham and Tolley [25] proposed a new model so-called the DBI Galileons based on a probe brane action in the higher-dimensional spacetime. Interestingly, this model naturally provides a connection between the DBI model and the relativistic generalization of the Galileon model [27], where the equation of motion is at most second order in derivatives due to the Galileon symmetry $\partial_\mu\pi \to \partial_\mu\pi + c_\mu$. Since the DBI inflation is supposed to be driven by the dynamics of the brane in the higher-dimensional bulk, it is interesting to study the effect of the Galileon-like terms in DBI inflation

In this paper, motivated by the DBI Galileons, we studied primordial fluctuations generated during inflation described by the action (6) which is obtained by generalizing the action of the DBI Galileons. This generalization is done in a similar way to the extension of the DBI inflation to the K inflation. In order to calculate the statistical quantities of ζ, the curvature perturbation on uniform density hypersurfaces on large scales, at leading order in the slow-varying approximations, we have adopted the simple procedure [44] to first calculate the bispectrum of the fluctuations of inflaton ϕ in the flat gauge and then relate it to that of ζ using the delta-N formalism (14).

For the linear perturbations, we have confirmed that, owing to the Galileon-like term, the expression of the sound speed c_s for the scalar perturbations is modified from the usual K-inflation model [Eq. (17)]. We also provided general expressions for the power spectrum $\mathcal{P}_\zeta$, spectral index n_s, and tensor to scalar ratio r at leading order in the slow-varying approximations. In these expressions, $\tilde{\epsilon}$ defined by Eq. (22), not $\epsilon \equiv -\dot{H}/H^2$, plays an important role [Eqs. (24), (26), and (28)]. Because of this, the consistency relation between the tensor to scalar ratio and the tensor spectrum index is broken if there exists the Galileon-like term.

We calculated the bispectrum at the leading order in the slow-varying variables. The Galileon-like term gives a new shape $\mathcal{A}_3$ in addition to the shapes $\mathcal{A}_1$ and $\mathcal{A}_2$ which arise in the usual K inflation [Eqs. (34)–(36)]. For the new shape $\mathcal{A}_3$, we checked the validity of using the estimator for the equilateral-type non-Gaussianity, $f_{\rm NL}^{\rm equil}$, based on the shape correlator introduced by Ref. [48] and showed that the overlap is at about 99.99% level, which justifies the use of this estimator to measure the amplitude of the bispectrum even in the presence of the Galileon-like term. We obtained the general expression for the amplitude of the bispectrum in Eq. (44).

For the concrete examples, we have considered two models: one is the DBI Galileons described by the action (5). The other is the G-inflation model proposed by Ref. [42]. For the DBI Galileons, in the small sound speed limit, $f_{\rm NL}^{\rm equil}$ is given by Eq. (49) and written in terms of the sound speed c_s and b_D that is related to the amplitude of the Galileon-like term. Since it scales as $f_{\rm NL}^{\rm equil} \propto 1/c_s^2$, large primordial non-Gaussianities can be obtained when c_s is much smaller than 1, similar to the usual DBI inflation. It is worth mentioning that for a given c_s, the b_D dependence of $f_{\rm NL}^{\rm equil}$ is weak and we obtained $-0.32/c_s^2 < f_{\rm NL}^{\rm equil} < -0.16/c_s^2$. For G inflation where an exact de Sitter solution is obtained without any potential terms, we found $f_{\rm NL}^{\rm equil}$ and the tensor to scalar ratio was related as $f_{\rm NL}^{\rm equil} = 4.62r^{-2/3}$. Although a small deviation from the de Sitter solution could give a correction to the tensor to scalar ratio, this relation gives a stringent constraint on the model by a detection or nondetection of the equilateral-type non-Gaussianity and the tensor to scalar ratio.

In this paper, we considered the cubic order interaction in the Galileon theory and its relativistic generalization. As is shown in [25], it is possible to add two more higher order interactions which again arise from the probe brane action in a five-dimensional spacetime with the Gauss-Bonnet term. It is also possible to generalize these terms in the same way as generalizing Eq. (5) to Eq. (6). It would be interesting to study phenomenology of this generalization. The single field model arises from a probe brane action in the five-dimensional spacetime. If the DBI Galileons have some connections to string theory, the DBI Galileons should be naturally a multifield model as in the DBI inflation where the position of the brane in each compact direction is described by a scalar field. The multifield Galileon model has been extensively studied recently [55–60] and the relativistic extension of the model has been proposed [55]. We leave the study of multifield DBI Galileons, the relativistic generalization of the multifield Galileon, for a forthcoming paper.

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