Properties of bare strange stars associated with surface electric fields

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In this paper we investigate the electrodynamic surface properties of bare strange quark stars. The surfaces of such objects are characterized by the formation of ultrahigh electric surface fields which might be as high as $\sim 10^{19}$ V/cm. These fields result from the formation of electric dipole layers at the stellar surfaces. We calculate the increase in gravitational mass associated with the energy stored in the electric dipole field, which turns out to be only significant if the star possesses a sufficiently strong *net* electric charge distribution. In the second part of the paper, we explore the intriguing possibility of what happens when the electron layer (sphere) rotates with respect to the stellar strange matter body. We find that in this event magnetic fields can be generated which, for moderate effective rotational frequencies between the electron layer and the stellar body, agree with the magnetic fields inferred for several central compact objects. These objects could thus be comfortably interpreted as strange stars whose electron atmospheres rotate at frequencies that are moderately different ($\sim$ 10 Hz) from the rotational frequencies of the strange star itself.

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I. INTRODUCTION

The properties of hypothetical strange quark stars (strange stars for short) have been extensively investigated in the literature [1–6]. Strange stars are compact astrophysical objects composed by absolutely stable strange quark matter [7,8], which has a zero-pressure point at finite baryon density $n_* \geq n_0$, where $n_0 = 0.15\ \mathrm{fm}^{-3}$ is the equilibrium nuclear density. This matter consists of a roughly equal number of up, down, and strange quarks and a relatively small number of electrons. Toward the quark star surface, however, the number of strange quarks drops and a higher number of electrons is required to maintain electric charge neutrality [1–4]. The electrons, not being bound by the strong interaction, are displaced to the outside of the star and an electric dipole layer is formed at the surface of a quark star [1,9,10]. In the region between the core surface and the electron layer a static electric field of the order of 10^{17-19} V/cm is formed [1,9,10]. The actual value of the electric field will depend on electrostatic effects, including Debye screening, the surface tension of the interface between the vacuum and quark matter [11,12], on whether or not the matter is in a superconducting state [13–17], and on the degree of sharpness of the surface [18]. This electric field may support a thin nuclear crust (thickness $\sim$100 m, [1,3,6,19]) consisting of a lattice of ordinary atomic nuclei below neutron drip density. Alternatively, strange stars may also lack a nuclear crust in which case they are referred to as bare strange stars. In this paper we will consider the latter possibility.

In the first part of this paper we will focus on the electrostatic properties of strange stars, treated within the framework of general relativity. We extend the results presented in [20,21], where the general relativistic structure of compact stars with a net electric surface charge was analyzed. As shown there, the energy density associated with the electric surface field acts as a source of curvature and thus contributes to the gravitational masses of strange stars. In the present paper we extend the results of [20] to include the electron layer. We will show that even under the most extreme physical conditions possible, the energy density associated with the electric dipole layer at the surface of a strange star does not lead to a substantial change of the star's gravitational mass. Only when the star possesses a net electric charge, as considered in [20], one obtains a non-negligible increase in the gravitational mass as a result of the electric field.

In the second part of the paper we consider the consequences of stellar rotation for the electric fields. We allow the strange star and the electron layer surrounding it to rotate at different frequencies. Under these circumstances, electric currents will exist at the surface of the star. The associated magnetic field is calculated. This field turns out to be uniform inside the star but dipolar outside of the star. It is shown that for a certain range of frequencies and electrostatic properties, the computed magnetic field is in agreement with those inferred for three central compact objects (CCOs) (see [22,23] and references therein). The emission from CCOs is characterized by a steady photon flux in the X-ray range and the absence of emission radio and optical counterparts. As shown in [23], several of these objects are slowly rotating (see Table I) and possess relatively low magnetic fields. The findings of this paper could

TABLE I. Observed magnetic fields and frequencies of three CCOs. (The data is from [23] and references therein.)

CCO	Ω (Hz)	B (10^{11} G)
RX J0822.0-4300	8.928	<9.8
1E 1207.4-5209	2.3584	<3.3
CXOU J185 238.6 + 004 020	9.5238	3.1

indicate that these CCOs are not neutron stars but rather strange stars whose surrounding electron layers rotate slowly relative to the strange matter cores. There is evidence that CCOs have unusually small projected emitting areas, in the range of 0.3–5 km [22], which, if confirmed, would support the interpretation of CCOs as strange stars.

This paper is organized as follows. In Sec. II we review the structure equations of electrically charged compact stars. The change of the gravitational mass by ultrastrong electric dipole fields on quark stars is computed in Sec. III. This is followed in Sec. IV by an investigation of the magnetic surface properties of strange stars. A summary and conclusions are provided in Sec. VII.

II. STRUCTURE OF ELECTRICALLY CHARGED COMPACT STARS

Electrically charged compact stars are described by an energy-momentum tensor of the following type [20,21,24]:

$$T^{\mu}_{\nu} = (P + \epsilon c^2)u_{\nu}u^{\mu} + P\delta^{\mu}_{\nu} + \frac{1}{4\pi}\left(F^{\mu l}F_{\nu l} - \frac{1}{4}\delta^{\mu}_{\nu}F_{kl}F^{kl}\right), \tag{1}$$

where $F^{\nu\mu}$ is the electromagnetic field tensor which enters the Maxwell equations as

$$[(-g)^{1/2}F^{\nu\mu}]_{,\mu} = 4\pi j^{\nu}(-g)^{1/2}. \tag{2}$$

The quantity j^{ν} is the electromagnetic four-current and $g \equiv \det(g_{\nu\mu})$ with the metric tensor given by

$$g_{\nu\mu} = \begin{pmatrix} -e^{2\Phi(r)} & 0 & 0 & 0 \\ 0 & e^{2\Lambda(r)} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2\sin^2\theta \end{pmatrix}. \tag{3}$$

For static stellar configurations, as considered in this paper, the only nonvanishing component of the four-current is j^0. Because of symmetry reasons, the four-current is only a function of radial distance, r, and all components of the electromagnetic field tensor vanish, with the exception of F^{01} and F^{10}, which describe the radial component of the electric field. Upon writing the energy-momentum tensor (1) as

$$T^{\mu}_{\nu} = \begin{pmatrix} -\left(\epsilon + \frac{Q^2}{8\pi r^4}\right) & 0 & 0 & 0 \\ 0 & P - \frac{Q^2}{8\pi r^4} & 0 & 0 \\ 0 & 0 & P + \frac{Q^2}{8\pi r^4} & 0 \\ 0 & 0 & 0 & P + \frac{Q^2}{8\pi r^4} \end{pmatrix}, \tag{4}$$

and substituting this expression into Einstein's field equation, $G^{\mu}_{\nu} = (8\pi/c^2)T^{\mu}_{\nu}$, one finds for the pressure gradient inside electrically charged stars

$$\frac{dP}{dr} = -\frac{G(m + \frac{4\pi r^3}{c^2}(P - \frac{Q^2}{4\pi r^4 c^2}))}{c^2 r^2(1 - \frac{2Gm}{c^2 r} + \frac{GQ^2}{r^2 c^4})}(P + \epsilon) + \frac{Q}{4\pi r^4}\frac{dQ}{dr}, \tag{5}$$

and for their gravitational masses

$$\frac{dm}{dr} = \frac{4\pi r^2}{c^2}\epsilon + \frac{Q}{c^2 r}\frac{dQ}{dr}. \tag{6}$$

The quantity Q denotes the electric charge located inside a region of radius r and is given by Gauss' law,

$$Q(r) = 4\pi \int_0^r r'^2 \rho_{\mathrm{ch}}(r') e^{\Lambda(r')} dr'. \tag{7}$$

The quantities ρ_{ch} and Λ in Eq. (7) denote the local electric charge distribution and the star's radial metric function, respectively [20]. Equations (5) and (6) are generalizations of the standard Tolman-Oppenheimer-Volkoff equation which describe the global properties of electrically uncharged compact stars in the framework of general relativity theory. The standard Tolman-Oppenheimer-Volkoff equation is obtained from Eqs. (5) and (6) in the limit $Q \to 0$. The occurrence of the Q dependent terms in Eqs. (5) and (6) account for the Coulomb interaction among the charged particles that are part of the star as well as for the extra curvature produced by the energy density of the electric field associated with the charges [20,21,24]. In contrast to the Coulomb interaction, which can be either attractive or repulsive, depending on the electric charge, the additional contribution to curvature increases the pull of gravity on the stellar matter.

As discussed in [20], electric fields are only relevant for the structure of strange stars if $Q^2/8\pi r^4 \sim P$ [see also Eq. (4)]. This implies electric fields that are on the order of $E \sim 10^{19-20}$ V/cm. Electric fields of this magnitude, located at the surface of strange stars, increase their gravitational masses and radii by up to 15% and 5%, respectively, [20].

III. STELLAR MASS

We now turn our attention to the stellar mass equation (6). Integrating this equation leads to (making $c = 1$)

$$m(r) = 4\pi \int_0^r r'^2 \epsilon(r') dr' + m_{\mathcal{E}}, \tag{8}$$

where

$$m_{\mathcal{E}} = \int_0^r \frac{Q(r')^2}{2r'^2} dr' + \frac{Q(r)^2}{2r}. \tag{9}$$

The first term on the right-hand side of Eq. (8) is the standard expression for the gravitational mass of electrically uncharged stars. The second term on the right-hand side, $m_{\mathcal{E}}$, accounts for the mass increase due to electric charges. This increase consists of two contributions, the first one, $\int_0^r (Q(r')^2/2r'^2)dr'$, represents the local mass energy of the electric field inside the star, and the second, $Q(r)^2/2r$, represents the total mass energy required to assemble the charges on the stellar configuration. An object with global charge neutrality must have $Q(R) = 0$, where R is the stellar radius. This implies that any increase in the stellar mass originates from the energy density of local electric fields.

As discussed in Refs. [1–3,18], the surface of a bare strange star consists of a positively charged layer surrounded by an electron layer immediately outside of the star. This feature is caused by the diminishing quark chemical potential toward the stellar surface, which renders the (negatively charged) strange quarks less abundant. This, in turn, requires a higher number of electrons (to achieve global charge neutrality). Since the electrons do not feel the strong interaction that binds a strange star, they extend beyond the stellar surface and form an electric dipole layer, with a positively charged stellar surface. Thomas-Fermi calculations [1,2,9,18] indicate that such a configuration will produce electric fields of up to 10^{18} V/cm, and a dipole region that extends $\sim 10^3$ fm beyond the star's surface. We now investigate, in the framework of general relativity, under which conditions such a configuration will have a significant impact on the stellar mass. In order to do that, first we need to make a suitable ansatz for the electric charge distribution ($\rho_{\rm ch}$). As mentioned just above, the dipole layer is of the order of $\sim 10^3$ fm. Therefore, on a macroscopic scale we can safely assume that both the positive and negative electric layers are described by delta functions. Adopting spherical coordinates, we have

$$\rho_{\rm ch} = +K \frac{\delta(r - R^+)}{4\pi r^2} - K \frac{\delta(r - R^-)}{4\pi r^2}. \tag{10}$$

Such a charge distribution is shown schematically in Fig. 1. Integrating Eq. (7), with $\rho_{\rm ch}$ given by (10), we obtain

$$Q(r) = \begin{cases} +Q_0 & \text{for } R^+ \le r \le R^- \\ 0 & \text{elsewhere} \end{cases}, \tag{11}$$

where $Q_0 = K \times \exp(\Lambda(R))$. To obtain this result, we made use of the fact that the metric is essentially constant over the relevant region, $\sim 10^3$ fm at the star's surface, R.

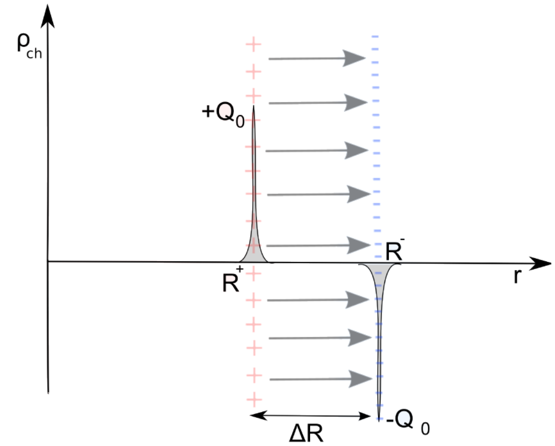

FIG. 1 (color online). Schematic representation of the electric charge distribution on the surface of a bare strange star. The core surface (R^+) becomes positively charged as the electrons (R^-) extend beyond the star's surface, giving rise to an electric dipole layer of width $\sim 10^3$ fm [1–3,9,18].

The electrostatic contribution to the mass of the star can then be written as

$$M_{\mathcal{E}} = \frac{Q_0^2}{2} \int_{R_+}^{R^-} \frac{dr}{r^2} = \frac{Q_0^2}{2} \frac{\Delta R}{R^+ R^-}, \tag{12}$$

where $\Delta R \equiv R^- - R^+$. This expression coincides with the energy of a spherical capacitor with charge Q_0. To estimate the increase in gravitational mass caused by the presence of an electric dipole layer, we first estimate the corresponding dipole charge using Gauss' law. Using the expression for the electric field outside of a spherically symmetric charge distribution we obtain

$$Q_0 = 4\pi\epsilon_0 r^2 \times E(r). \tag{13}$$

Since we are only dealing with a very narrow region near the surface, the metric functions can safely be assumed to be constant there, and thus the metric corrections can be incorporated in the constant Q_0.

Using expressions (12) and (13), and upon performing the appropriate unit transformation we obtain

$$\frac{M_{\mathcal{E}}}{M_\odot} = 3.1210^{-61} R_s^4 E^2 \frac{\Delta R}{R^+ R^-}, \tag{14}$$

with E in V/cm and R_s, ΔR, R^+, and R^- in km. In Fig. 2 we plot Eq. (14) for a strange star with 10 km radius and with the surface electric fields indicated. Figure 2 indicates that the increase in gravitational mass of strange stars, resulting from the energy associated with the electric dipole layer on the surface, is very small. We can conclude then that only when the star possess a net charge one might find a significant contribution to the gravitational mass, due to a macroscopic extension of the electric field outside of the star.

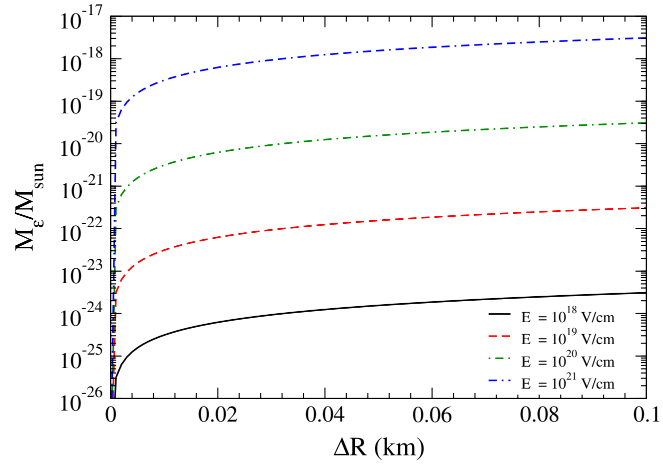

FIG. 2 (color online). Increase in gravitational mass due to electric dipole energy (M_ϵ is the electrostatic contribution to the mass of the star), as a function of the dipole width (ΔR). The calculations are performed for a strange star with radius of 10 km and for the surface electric fields indicated.

IV. POSSIBLE MECHANISM OF MAGNETIC FIELD GENERATION

In Sec. III we calculated the increase in gravitational mass that is caused by the energy density of the surface dipole layer. These results are calculated for a static strange star. In this section we will extend this study to dipole fields on rotating strange stars.

The most obvious effect that needs to be considered when taking rotation into account is the formation of a magnetic field. Let us assume that the star is rotating around the z axis. Thus, if the stellar core and the exterior electron layer are rotating at different velocities, an electric current at the surface of the star will be created, which leads to the formation of a magnetic field. This situation is

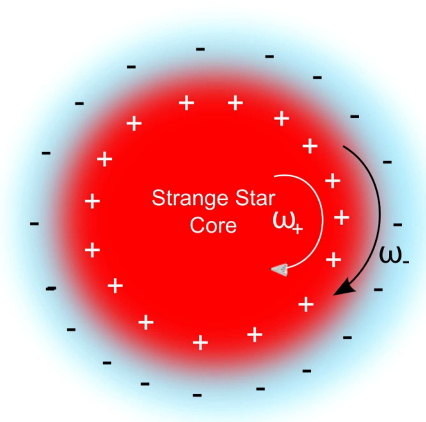

FIG. 3 (color online). Schematic illustration of the formation of electric currents at the surface of a rotating strange star. ω_+ and ω_- are the frequencies at which the core and electron layer are rotating, respectively.

schematically illustrated in Fig. 3. The electric surface current can be expressed as

$$I = \sigma(\omega_+ - \omega_-), \tag{15}$$

where σ is the surface charge density and ω_+ and ω_- are the frequencies at which the core and electron layer are rotating, respectively. Obviously, if the core and electron layer are rotating at the same frequency the surface currents are zero and no magnetic field is formed.

The surface charge density can be estimated by using the result of Sec. III. The positive charge of the core of a strange star has been estimated in Eq. (13). Using this result, we can express σ as

$$\sigma = \frac{Q_0}{A} = \frac{4\pi\epsilon_0 R^2 E(R)}{4\pi R^2} = \epsilon_0 E(R), \tag{16}$$

with $E(R)$ the electric field at the surface of strange stars (predicted for the case where the core and electron layer are rotating at the same frequency).

The magnetic field of a rotating, electrically charged sphere can be easily calculated [25]. The result is a uniform magnetic field pointing in the $+\hat{z}$ direction for the inside of the sphere, and a dipole field for the outside. The exterior field can then be written as

$$\vec{B} = \frac{1}{3}\mu_0\sigma(\omega_+ - \omega_-)\frac{R^4}{r^3}(2\cos\theta\hat{r} + \sin\theta\hat{\theta}). \tag{17}$$

Where θ is the polar angle, and $\hat{r}$ and $\hat{\theta}$ are unit vectors for the radial and polar directions. After the appropriate unit conversions we can write the magnetic field at the pole and at the equator as

$$B_{\rm p} = E(\omega_+ - \omega_-)R \times 7.4104 \times 10^{-9}\ \text{G}, \tag{18}$$

$$B_{\rm eq} = E(\omega_+ - \omega_-)R \times 3.7052 \times 10^{-9}\ \text{G}, \tag{19}$$

with R in km, E in V/cm, and ω in Hz. It is important to note that the predicted magnetic field is proportional to both the surface electric field, and the effective rotation frequency. In Fig. 4 we show the polar magnetic field as a function of the effective frequency ($\omega_{\rm eff} = \omega_+ - \omega_-$) for strange quark stars with different electric fields. Figure 4 shows that for typical frequencies expected for compact stars, the resulting magnetic field can be rather high. For the extreme case with effective frequency ~700 Hz, the magnetic field might be as high as 10^{15} G, for static electric fields of 10^{19} V/cm. Magnetic fields of this magnitude are expected for magnetars. These objects, however, have relatively low rotational frequencies, and therefore an effective frequency of the order of 700 Hz is unlikely. For a more moderate effective frequency of, say, 10 Hz, one obtains magnetic fields in the range of $B \sim 10^{10}$–10^{11}G. Such B values and frequencies are in agreement with observations made for some CCOs. These objects are characterized by a steady flux predominately in the X-ray range and the lack of optical and radio counterparts. There

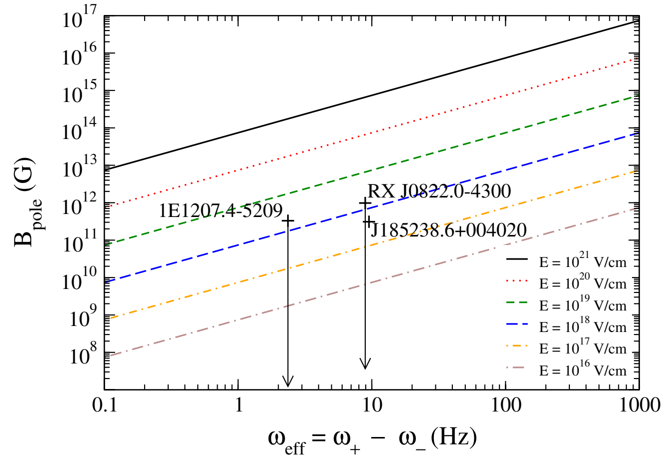

FIG. 4 (color online). Polar magnetic field ($B_{\rm pole}$) for strange quark stars as a function of the difference in frequency ($\omega_{\rm eff}$) between the positively charged core and negatively charged electron layer. Different electric fields indicate the value of the static electric field the star would have in the case that both the core and the electron layer are rotating at the same frequency ($\omega_{\rm eff} = 0$). Also plotted is the observed magnetic field and period for 3 CCOs (see [23] and references therein).

have been observations of the magnetic fields and frequencies for three of these objects (see [23] and references therein). Their quantities are listed in Table I and indicated in Fig. 4. Assuming that the observed frequency of these objects equals the effective differential frequency (i.e. the difference between the frequencies of the core and electron layer), we find that the predicted magnetic fields are in good agreement with the observed ones. Figure 4 shows that stars whose static electric fields are up to 10^{18} V/cm can generate the magnetic fields observed for some CCOs. This finding may indicate that these objects may be strange stars rather than neutron stars. More detailed studies are necessary, however, to put this conjecture on a firm basis.

V. A POSSIBLE HEATING MECHANISM

As discussed in Sec. IV, we allow the strange quark core and the electron layer to be differentially rotating. Considering that the electron layer is separated from the core by distances of the order of 1000 fm, one should expect some frictional heating in this region. A similar scenario is discussed in Refs. [26,27], where a layer of superfluid matter at the inner crust is differentially rotating with respect to the rest of the star with average differential frequency between 0.1 and 5 Hz. Following the footsteps of Refs. [26,27], we estimate the frictional heating for our model, in which the electron layer is differentially rotating with respect to the strange star core. For this we hereafter assume that the core is at rest ($\omega_+ = 0$) with respect to the electron layer, which is rotating with a frequency of $\omega_- = 10$ Hz. The frictional heating is given by the difference between the change in rotational energy, and the rate at which the torque that is causing the layer to spin down, is doing work,

$$H = -\frac{d}{dt}\left(\frac{I_- \omega_-^2}{2}\right) - |\tau|\omega_-. \tag{20}$$

Where $H(t)$ is the heating, I_- is the moment of inertia of the electron layer, and τ is the torque, which is given by

$$\tau = I_- \dot{\omega}_-. \tag{21}$$

In Eq. (21), $\dot{\omega}_-$ is the spin-down frequency for the electron layer. Combining Eq. (21) with (20) we get the following expression for the heating:

$$H(t) = 2I_- \omega_- |\dot{\omega}_-|. \tag{22}$$

We use the Newtonian approximation to calculate the moment of inertia of a spherical shell, and using the observed properties for object J1852 [23], which are $\dot{\omega} = -7.88 \times 10^{-16}\ \mathrm{s}^{-2}$ and $\omega = 9.5238$ Hz, we get the following heating rate:

$$H = 5.279 \times 10^9\ \mathrm{erg\,s^{-1}}. \tag{23}$$

We are now in position to estimate the time it would take for the electron layer to dissipate through friction, all of its rotational energy. Assuming a constant spin-down rate, we find that the time needed to convert the total rotation energy of the layer into heat is $\sim 5.21 \times 10^7$ yr, which is the same time scale of the cooling of a compact star. This figure constitutes an order-of-magnitude estimate only. Carrying out detailed microscopic calculations (see [28], for example) on the reheating of strange stars by differentially rotating electron spheres (and the associated time scales) is beyond the scope of this work, but will be carried out in a future study.

VI. ROLE OF COLOR SUPERCONDUCTIVITY

In the discussions of Sec. II, III, and IV, the possibility of pairing in quark matter was not addressed. If strange quark matter stars exist, they are probably in a superconducting state. The plausible condensation pattern of such matter at densities $\gg \rho_0$ (ρ_0 being the nuclear saturation density) is the color flavor locked (CFL) phase [14]. The interior of stellar CFL matter, which is subject to the conditions of chemical equilibrium and electric charge neutrality, is characterized by equal numbers of u, d, and s quarks and, thus, the total absence of electrons. The situation is different at the surface of stellar CFL matter [9,29] where the number of s quarks is reduced with respect to u and d quarks because of changes in the density of states. This leads to the presence of electrons at the surface of CFL matter and the formation of an electric dipole layer at the surface of a CFL strange quark star. (Usov has shown [9] that the electric field generated at the surface of a CFL strange star is even higher than the one at the surface of a nonsuperconducting strange quark matter, reaching values of $\sim 10^{18}$ V/cm.)

For intermediate densities ($\sim 2\rho_0$) the condensation pattern of strange quark matter is less clear. Model calculations indicate that for such densities strange quark matter may be in the 2SC phase [29] where only the u and d quarks of two colors are paired. In this case electrons will be present throughout the stellar CFL strange quark matter to maintain electric charge neutrality. The same is the case if strange quark matter were to form a crystalline color superconductor [30–32]. In the latter event the momenta of the quark pairs do not add up to zero, requiring the presence of electrons in such matter as well.

The bottom line of all this is that, independent of the specific condensation pattern of color superconducting strange quark matter, there will always exist electron dipole layers at the surfaces of CFL strange quark matter stars whose physical consequences are discussed in this paper.

VII. CONCLUSIONS

In this paper we have analyzed the surface properties of bare strange stars, focusing on their electromagnetic properties. We extended the work presented in [20,21,24], where the bulk properties of compact stars possessing a net electric charge were investigated. Here we consider a bare strange star with zero net electric charge. For such stars, as already shown in [1,2,9], because of the displacements of electrons, the stellar quark core becomes positively charged and the region outside of it becomes negatively charged, leading to an electric dipole layer. We used the general relativistic stellar structure equations of electrically charged compact stars [20,21,24] to calculate the increase in gravitational mass that originates from the energy of the electric dipole. We found that even for macroscopic (unrealistically large) dipole widths (on the order of a 100 m), the increase in gravitational mass is negligible ($\sim 10^{-20} M_\odot$). We can thus safely conclude that only strange stars which possess net electric charges, as considered in [20], may lead to distinct increases in gravitational mass.

The second part of this paper deals with electromagnetic effects at the surfaces of electrically charged strange stars, which emerge if the star and the electron sphere surrounding the star should rotate at different frequencies. In this event electric currents would be created at the stellar surfaces of the star. The strength of these currents is determined by the magnitude of the net electric charge available and by the difference in the rotational frequencies of the stellar core and the electron layer. The magnetic field of such a configuration was found to be uniform inside the star, and a dipole type outside. We also found that, depending on the electric field and the relative (effective) frequency between the star and the electron layer, the magnetic field may be as high as 10^{16} G. Such a strong magnetic field can only be achieved for very high static electric fields on the order of $\sim 10^{20}$–10^{21} V/cm and effective frequencies of $\sim$700–1000 Hz. For small effective rotational frequencies of say $\sim$10 Hz and more moderate static electric fields of $\sim 10^{16}$–10^{18} V/cm one obtains magnetic fields on the order of 10^9–10^{11} G. This is a very intriguing result because such magnetic field values and rotational frequencies are in good agreement with the observed magnetic fields and frequencies of three CCOs. These objects have relatively long rotational periods, and for the three cases for which data exists, small magnetic fields of $\sim 10^{11}$ G [23]. These objects could thus be comfortably interpreted as strange stars whose electron atmosphere rotates at a frequency that is slightly different from the strange star. It is important to stress that the model proposed by us establishes a connection between CCOs and strange quark matter stars that may possibly be in the CFL superconducting phase. There is also the possibility that the strange star presents chromomagnetic instabilities if the matter is in a 2SC phase [30]. This issue needs to be addressed carefully, and we will leave this topic for future investigations.

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