Charged spinning black holes as particle accelerators

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(Received 17 June 2010; revised manuscript received 27 September 2010; published 11 November 2010)

It has recently been pointed out that the spinning Kerr black hole with maximal spin could act as a particle collider with arbitrarily high center-of-mass energy. In this paper, we will extend the result to the charged spinning black hole, the Kerr-Newman black hole. The center-of-mass energy of collision for two uncharged particles falling freely from rest at infinity depends not only on the spin a but also on the charge Q of the black hole. We find that an unlimited center-of-mass energy can be approached with the conditions: (1) the collision takes place at the horizon of an extremal black hole; (2) one of the colliding particles has critical angular momentum; (3) the spin a of the extremal black hole satisfies $\frac{1}{\sqrt{3}} \leq \frac{a}{M} \leq 1$, where M is the mass of the Kerr-Newman black hole. The third condition implies that to obtain an arbitrarily high energy, the extremal Kerr-Newman black hole must have a large value of spin, which is a significant difference between the Kerr and Kerr-Newman black holes. Furthermore, we also show that, for a near-extremal black hole, there always exists a finite upper bound for center-of-mass energy, which decreases with the increase of the charge Q.

DOI: 10.1103/PhysRevD.82.103005 PACS numbers: 97.60.Lf, 04.70.−s

I. INTRODUCTION

Recently, Bañados, Silk and West (BSW) [1] showed that spinning black holes can play the role of particle accelerators. Compared with terrestrial accelerators, they have a fascinating and important property that two particles (for example, the dark matter particles) falling freely from rest at infinity can collide with arbitrarily high center-of-mass (CM) energy at the horizon of an extremal Kerr black hole, which could provide a visible probe of Planck-scale physics. However, fine-tunings arise, namely, the black hole must be a maximally spinning one and one of the particles should have orbital angular momentum per unit rest mass $l = 2$ corresponding to marginally bound geodesics. Subsequently, in [2,3], the authors further elucidated the mechanism for the results of BSW. They pointed out that there must exist a practical limitation on the achievable CM energy from the astrophysical limitations, i.e., the maximal spin, backreaction effects or gravitational radiation. For example, the spin a of astrophysical black holes should not exceed $\frac{a}{M} = 0.998$ (M is the mass of an astrophysical black hole) according to the work of Thorne [4]. Denoting the deviation of the spin from its maximal value as $\epsilon = 1 - a$, Jacobson and Sotiriou got the maximal CM energy [3]

$$\frac{E_{\mathrm{cm}}^{\max}}{m_0} \sim 4.06\epsilon^{-1/4} + \mathcal{O}(\epsilon^{1/4}), \qquad (1)$$

where, m_0 is the rest mass of the colliding particles. Taking $\frac{a}{M} = 0.998$ as a limit, one will obtain the maximal CM energy per unit rest mass 19.20, which is a finite value. Lake also showed that the CM energy of collision at the inner horizon of a nonextremal Kerr black hole is limited [5]. In Ref. [6], scattering of particles in gravitational field and extraction of energy from a rotating black hole was investigated.

It is known that the motion of a particle traveling in the background of a charged spinning black hole depends not only on the spin but also on the charge of the black hole. Therefore, the CM energy of collision will also depend both on the spin and charge. Note that there is no work focusing on the CM energy for the collision in the background of a charged spinning black hole. So, it is worthwhile to study the detailed behavior of the CM energy for the collision in the background of a charged spinning black hole. For the purpose, we will study the CM energy in the background of a Kerr-Newman (KN) black hole. Besides the spin a, the black hole has another parameter, the charge Q, which should affect the CM energy. On the other hand, it is generally thought that the black holes are surrounded by relic cold dark matter density spikes and there exists no electromagnetic interactions between the cold dark matter and other matters. So, it provides a strong motivation for us to consider the collision of two uncharged particles in the background of a KN black hole. With the motivation, we find the CM energy can still be unlimited for a pair of uncharged particles falling freely from rest at infinity and colliding at the horizon of an extremal black hole with some fine-tunings. For the near-extremal black hole, we also give a numerical exploration on the CM energy. The result implies that there always exists a finite upper bound for the CM energy, which decreases with the increase of the charge Q. In this paper, we neglect the effects of gravitational waves and the backreaction.

The paper is organized as follows. In Sec. II, we will give a detailed study on the equations of motion for

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particles. In Sec. III, employing the equations of motion for particles, we will obtain the CM energy for two colliding particles falling freely from rest at infinity in the background of a KN black hole. The results show that the CM energy at the horizon can be unlimited if one of the colliding particles has the critical angular momentum and the spin a of the black hole satisfies $\frac{1}{\sqrt{3}} \leq \frac{a}{M} \leq 1$. It is also shown that for a near-extremal black hole there always exists a finite upper bound of CM energy and the bound decreases with the increase of the charge Q. The final section is devoted to a brief summary. We use the units $c = G = 1$ in this paper.

II. MOTION EQUATIONS OF PARTICLES IN THE BACKGROUND OF A KERR-NEWMAN BLACK HOLE

In this section, we would like to study the equations of motion for a particle in the background of a KN black hole. First, let us give a brief review of the black hole background we dealt with. The KN black hole is described by the metric with the Boyer-Lindquist coordinates (where we have set the mass M of the black hole to 1)

$$ds^2 = \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta^2 + \frac{\sin^2\theta}{\rho^2}[adt - (r^2 + a^2)d\phi]^2 - \frac{\Delta}{\rho^2}[dt - a\sin^2\theta d\phi]^2, \tag{2}$$

where

$$\Delta = r^2 - 2r + a^2 + Q^2, \tag{3}$$

$$\rho^2 = r^2 + a^2\cos^2\theta. \tag{4}$$

Q is the charge of the black hole, and a is its angular momentum per unit rest mass and $0 \leq a \leq 1$. In the case $Q = 0$, the metric (2) describes a Kerr black hole. And in the case $a = Q = 0$, it describes a Schwarzschild black hole. The 4-dimensional electromagnetic potential reads

$$A_a = -\frac{Qr}{\rho^2}[(dt)_a - a\sin^2\theta(d\phi)_a]. \tag{5}$$

The horizons for the KN black hole are given by

$$r_\pm = 1 \pm \sqrt{1 - (a^2 + Q^2)}. \tag{6}$$

Here, the positive sign denotes the outer horizon and the negative one denotes the inner one. The existence of the horizons requires

$$a^2 + Q^2 \leq 1, \tag{7}$$

where "=" corresponds to the extremal black hole with one degenerate horizon.

Next, we would like to study the equations of motion for a test particle with mass μ and charge q in the background of a KN black hole. The motion of a particle is described by the Lagrangian

$$\mathcal{L} = \frac{1}{2}g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + qA_\mu\dot{x}^\mu, \tag{8}$$

where a dot over a symbol denotes ordinary differentation with respect to an affine parameter λ. The affine parameter λ is related to the proper time by $\tau = \mu\lambda$, which is equivalent to the normalizing condition

$$g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -\mu^2. \tag{9}$$

For an uncharged particle, $\mu^2 = 1, 0, -1$ are corresponded to timelike, null or spacelike geodesics, respectively. For a massive particle, we have $\mu^2 = 1$. The momenta is

$$P_\mu = \frac{\partial\mathcal{L}}{\partial\dot{x}^\mu} = g_{\mu\nu}\dot{x}^\nu + qA_\mu. \tag{10}$$

Thus the Hamiltonian is given by

$$H = P_\mu\dot{x}^\mu - \mathcal{L} = \frac{1}{2}g^{\mu\nu}(P_\mu - qA_\mu)(P_\nu - qA_\nu). \tag{11}$$

With the help of (11), the Hamilton-Jacobi equation can be expressed as

$$\frac{\partial S}{\partial\lambda} = H = \frac{1}{2}g^{\mu\nu}(P_\mu - qA_\mu)(P_\nu - qA_\nu) \tag{12}$$

with S the Jacobi action. To solve the Hamilton-Jacobi equation, we separate the Jacobi action as

$$S = -\frac{1}{2}\lambda - Et + l\phi + S_r(r) + S_\theta(\theta), \tag{13}$$

where the parameter E is the energy of the charged particle, and l is the angular momentum per unit rest mass of the particle in the ϕ direction as measured by an observer at rest at infinity. S_r and S_θ are, respectively, functions of r and θ. Inserting (13) in (12), we obtain

$$S_\theta^2 + (aE\sin\theta - l\sin^{-1}\theta)^2 + a^2\cos^2\theta = -\Delta S_r^2 + \Delta^{-1}((a^2 + r^2)E - al - qQr)^2 - r^2. \tag{14}$$

From the above form, we can see that the left-hand side is only the function of θ and the right-hand side is only the function of r. Thus, both sides must be equal to a constant denoted by $\mathcal{K}$. So, we have

$$S_\theta^2 = \mathcal{K} - (aE\sin\theta - l\sin^{-1}\theta)^2 - a^2\cos^2\theta, \tag{15}$$

$$\Delta S_r^2 = -\mathcal{K} + \Delta^{-1}((a^2 + r^2)E - al - qQr)^2 - r^2. \tag{16}$$

Using the relations $P_r = \frac{\partial S}{\partial r}$, $P_\theta = \frac{\partial S}{\partial\theta}$ and the Eq. (10), we have [7,8]

$$\frac{d\theta}{d\tau} = \sigma_\theta\frac{\sqrt{\Theta}}{\rho^2}, \tag{17}$$

$$\frac{dr}{d\tau} = \sigma_r \frac{\sqrt{R}}{\rho^2} \tag{18}$$

with

$$\Theta = \mathcal{K} - (l - aE)^2 - \cos^2\theta(a^2(1 - E^2) + l^2\sin^{-2}\theta), \tag{19}$$

$$R = P^2 - \Delta(r^2 + \mathcal{K}), \tag{20}$$

$$P = E(r^2 + a^2) - la - qQr. \tag{21}$$

The sign functions $\sigma_r = \pm$ and $\sigma_\theta = \pm$ are independent from each other. Using the relations $P_t = \frac{\partial S}{\partial t}$, $P_\phi = \frac{\partial S}{\partial \phi}$ and Eq. (10), we get

$$-E = g_{tt}\dot{t} + g_{t\phi}\dot{\phi} + qA_t, \tag{22}$$

$$l = g_{\phi t}\dot{t} + g_{\phi\phi}\dot{\phi} + qA_\phi. \tag{23}$$

Solving these equations, we get [7,8]

$$\frac{dt}{d\tau} = -\frac{a}{r^2}(aE\sin^2\theta - l) + \frac{(r^2 + a^2)}{\rho^2\Delta}P, \tag{24}$$

$$\frac{d\phi}{d\tau} = -\frac{(aE\sin^2\theta - l)}{\rho^2\sin^2\theta} + \frac{a}{\rho^2\Delta}P. \tag{25}$$

Here, we have obtain equations of motion for a particle. On the equatorial plane ($\theta = \frac{\pi}{2}$), the equations are reduced to

$$\begin{aligned} \frac{dt}{d\tau} &= \frac{-a(aE - l)\Delta + (r^2 + a^2)P}{r^2\Delta}, \\ \frac{dr}{d\tau} &= -\frac{\sqrt{R}}{r^2}, \\ \frac{d\theta}{d\tau} &= 0, \\ \frac{d\phi}{d\tau} &= \frac{(l - aE)\Delta + aP}{r^2\Delta}, \end{aligned} \tag{26}$$

where we take $\sigma_r = -1$. Note that the motion of a particle on the equatorial plane in the KN metric is completely determined by Eq. (26).

III. CENTER-OF-MASS ENERGY FOR A KERR-NEWMAN BLACK HOLE

In this section, we will study the CM energy of the collision for two particles moving on the equatorial plane of a KN black hole. Let us now consider that two charged particles with the same rest mass m_0 are at rest at infinity ($E = m_0$), then they approach the black hole and collide at some radius r. We assume that the two particles have angular momenta and charges (l_1, q_1) and (l_2, q_2), respectively. Taking into account that the background is curved, the energy in the center-of-mass frame for this collision should be computed with [1]

$$E_{\rm cm} = \sqrt{2}m_0\sqrt{1 - g_{\mu\nu}U^\mu_{(1)}U^\nu_{(2)}}, \tag{27}$$

where $U^\mu_{(1)}$ and $U^\nu_{(2)}$ are the 4-velocities of the two particles, which can be straightforwardly calculated from (26) and are

$$U^\mu_{(1)} = \left(\frac{a(l_1 - a)\Delta + (r^2 + a^2)P(q_1, l_1)}{r^2\Delta}, -\frac{\sqrt{R}}{r^2}, 0, \frac{(l_1 - a)\Delta + aP(q_1, l_1)}{r^2\Delta}\right), \tag{28}$$

$$U^\mu_{(2)} = \left(\frac{a(l_2 - a)\Delta + (r^2 + a^2)P(q_2, l_2)}{r^2\Delta}, -\frac{\sqrt{R}}{r^2}, 0, \frac{(l_2 - a)\Delta + aP(q_2, l_2)}{r^2\Delta}\right). \tag{29}$$

Here, we take $E = 1$ for simplicity. With the help of (27), we obtain the CM energy for the collision:

$$\left(\frac{E_{\rm cm}}{\sqrt{2}m_0}\right)^2 = -\frac{H}{r^2\Delta} \tag{30}$$

with H given by

$$\begin{aligned} H = &-2r^4 + r^3[2 + Q(q_1 + q_2)] - r^2(2a^2 + Q^2 - l_1 l_2 \\ &+ Q^2q_1q_2) - 2a^2r + 2r[a(l_1 + l_2) - l_1l_2] \\ &+ Q^2(a - l_1)(a - l_2) + aQr[a(q_1 + q_2) \\ &- (l_2q_1 + l_1q_2)] \\ &+ \sqrt{(a^2 + r^2 - al_1 - Qrq_1)^2 - \Delta(r^2 + (a - l_1)^2)} \\ &\times \sqrt{(a^2 + r^2 - al_2 - Qrq_2)^2 - \Delta(r^2 + (a - l_2)^2)}. \end{aligned} \tag{31}$$

Note that (30) is invariant under the interchange $l_1 \leftrightarrow l_2$ and $q_1 \leftrightarrow q_2$. On the other hand, in the case $Q = q_1 = q_2 = 0$, the CM energy (30) for two charged particles in the background of a KN black hole will reduce to the one for two uncharged particles in the background of a Kerr black hole given in [1], as it is expected. In fact, we need only the condition $Q = 0$ to obtain the CM energy in the background of a Kerr black hole, which indicates that the charge of the collision particles has no influence on the CM energy in the background of an uncharged black hole. We keep in mind that black holes are surrounded by relic cold dark matter density spikes. The CM energy of massive cold dark matter particles colliding near the black hole may reach a high CM energy, which could provide a probe to the high energy physics. It is also thought that the cold dark matter has no electromagnetic interactions with other matters. So, we consider that two uncharged cold dark matter particles collide in the background of a KN black hole. Thus, we take $q_1 = q_2 = 0$. Then the CM energy can be read from (30):

$$\left(\frac{E_{\rm cm}}{\sqrt{2}m_0}\right)^2 = -\frac{K}{r^2\Delta}, \tag{32}$$

where K is

$$\begin{aligned} K = &-2r^4 + 2r^3 - r^2(2a^2 + Q^2 - l_1 l_2) - 2a^2 r \\ &+ 2r[a(l_1 + l_2) - l_1 l_2] + Q^2(a - l_2)(a - l_2) \\ &+ \sqrt{(a^2 + r^2 - a l_1)^2 - \Delta[r^2 + (a - l_1)^2]} \\ &\times \sqrt{(a^2 + r^2 - a l_2)^2 - \Delta[r^2 + (a - l_2)^2]}. \end{aligned} \tag{33}$$

Clearly, the result confirms that the charge Q of the black hole indeed has influence on the CM energy.

Next, it is worthwhile to study the properties of (32) as the radius r approaches to the horizon r_+ of an extremal black hole. Since the black hole considered here is an extremal one, we have $Q = \sqrt{1 - a^2}$. Note that the horizon is always at $r_+ = 1$ for any spin a. It is clear that the denominator of $E_{\rm cm}$ in (32) vanishes at $r = r_+$. Then it is naive to obtain the result that $E_{\rm cm}$ diverges at the horizon. In fact, the numerator also vanishes at that point. The limiting value of $E_{\rm cm}$ at $r = r_+$ can be calculated as follows:

$$E_{\rm cm}(r \to r_+) = 2m_0\sqrt{1 + \frac{(l_1 - l_2)^2}{(l_1 - l_c)(l_2 - l_c)}\frac{l_c}{4a}}, \quad (a^2 + Q^2 = 1). \tag{34}$$

Clearly, the value of $E_{\rm cm}$ is indeed finite for generic values of l_1 and l_2. However, when l_1 or l_2 takes the critical angular momentum

$$l_c = \frac{1 + a^2}{a}, \tag{35}$$

the CM energy $E_{\rm cm}$ will be unlimited, which means that the particles can collide with arbitrarily high CM energy at the horizon. Thus, the result may provide an effective way to probe the Planck-scale physics in the background of an extremal KN black hole. Compared with the result for the Kerr black hole [1], the spin a of the black hole here can deviate from its maximal value to obtain an arbitrarily high CM energy. However, we need to make sure that the particle with critical angular momentum l_c can reach the horizon.

As we mentioned before, to obtain an arbitrarily high CM energy, one of the colliding particles should have critical angular momentum l_c. Here we first examine the critical angular momentum (35). When the spin $a = 1$, we get $l_c = 2$, which is just the critical angular momentum in the case of an extremal Kerr black hole [1]. However, for the case $a = 0$, l_c is divergent and the CM energy is

$$E_{\rm cm}(r \to r_+) = 2m_0\sqrt{1 + \frac{(l_1 - l_2)^2}{4}}, \tag{36}$$

which implies that, in order to get a very high CM energy, one of the colliding particles should have very large angular momentum. However, a particle with very large angular momentum cannot reach the horizon if it falls freely from rest at infinity. So there must exist a range for the spin a to ensure that the particle with critical angular momentum l_c reaches the horizon of the black hole. Next, we will determine the range of the spin a by the effective potential method. The effective potential for a particle with critical angular momentum l_c on the equatorial plane of an extremal black hole is

$$V_{\rm eff} = -\frac{1}{2}\left(\frac{dr}{d\tau}\right)^2 = -\frac{(r - 1)^2(r - r_c)}{r^4} \tag{37}$$

with $r_c = \frac{1-a^2}{2a^2}$. As expected, the effective potential $V_{\rm eff}$ approaches 0 at infinity. Here, we can get a condition for the particle falling freely from rest at infinity to reach the horizon:

$$V_{\rm eff} \leq 0 \quad \text{for any } r \geq 1, \tag{38}$$

which is equivalent to

$$r_c \leq 1. \tag{39}$$

Solving Eq. (39), we get the range for the spin a of the black holes:

$$\frac{1}{\sqrt{3}} \leq a \leq 1, \tag{40}$$

which means that for an extremal black hole with the spin $a \in (\frac{1}{\sqrt{3}}, 1)$, the particle with critical angular momentum l_c can reach the horizon of the black hole. We can also determine the range of the angular momentum for a fixed spin a with the same method. However, it cannot be written in a closed form for an arbitrary spin a. Here, we give some results: the range of the angular momentum is $(-3.9539, 2.6471)$ for $a = 0.4$, $(-4.2185, 2.3094)$ for $a = \frac{1}{\sqrt{3}}$ and $(-4.6864, 2.0111)$ for $a = 0.9$. So, for a fixed spin $a \in (\frac{1}{\sqrt{3}}, 1)$, if $l_1 = l_c$ and l_2 is in a proper range, the CM energy will be unlimited. We plot the effective potential $V_{\rm eff}$ in Fig. 1(a) for the spin $a = 0.4$, $\frac{1}{\sqrt{3}}$ and 0.9, respectively. Clearly, for $a = 0.4$, the effective potential $V_{\rm eff}$ is positive near the horizon $r_+ = 1$, so the particle cannot reach the horizon in this case. For $a = \frac{1}{\sqrt{3}}$ and 0.9, the effect potential $V_{\rm eff}$ is negative when $r > r_+ = 1$. So, the particle can reach the horizon in both cases. We also plot the CM energy $E_{\rm cm}$ of collision in Fig. 1(b) for $l_1 = l_c$ and $l_2 = -2$. For the case $a = 0.4 < \frac{1}{\sqrt{3}}$, the CM energy only exists for $r > 2.625$. This is because the collision for the two colliding particles with angular momenta $l_1 = l_c$ and $l_2 = -2$ cannot take place at $r < 2.625$. For the spin $a = \frac{1}{\sqrt{3}}$ and 0.9, the CM energy is divergent at the horizon $r_+ = 1$.

As noted above, we show that an arbitrarily high CM energy can be obtained when the collision takes place at

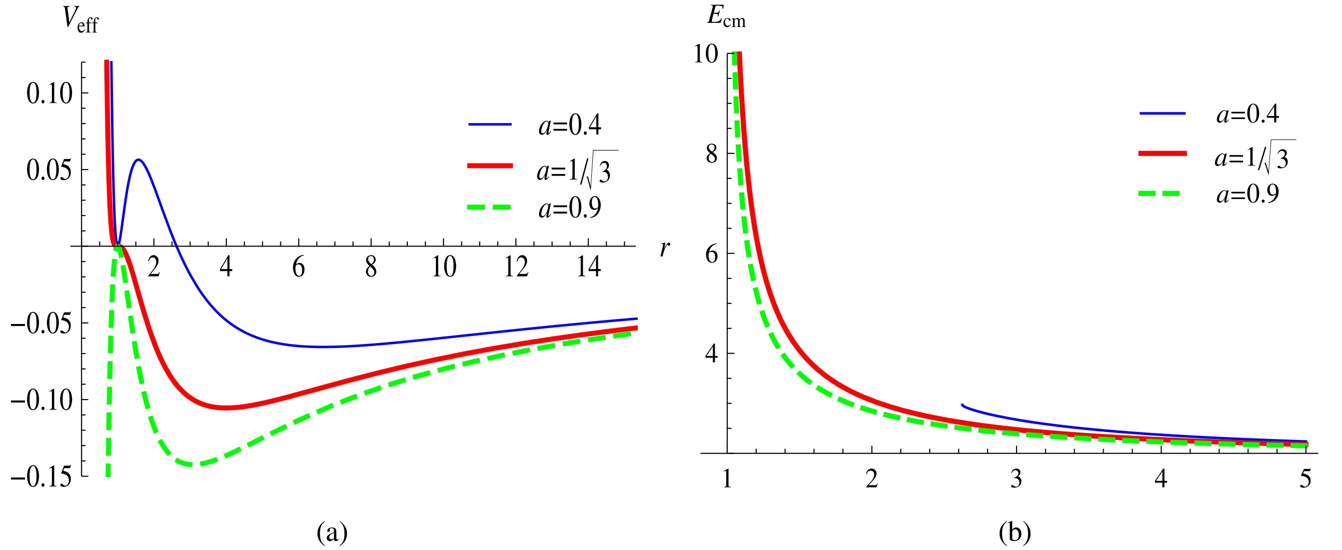

FIG. 1 (color online). For an extremal KN black hole (a) shows the effective potential $V_{\rm eff}$ vs r with angular momentum $l = l_c = \frac{1+a^2}{a}$. (b) shows the behavior of the CM energy $E_{\rm cm}$ with $m_0 = 1$ and $l_1 = l_c$, $l_2 = -2$.

the horizon of an extremal KN black hole with $l = l_c$ and $a \in (1/\sqrt{3}, 1)$. This scenario is an idealized one, because the proper time for a particle with the critical angular momentum l_c to approach the horizon of an extremal black hole from infinity is infinite. Thus this collision process does not take place in the real world. However, for the case of a near-extremal black hole, the proper time for the particle to reach the horizon is a finite value even though it is very large. So, it seems worthwhile to consider a near-extremal black hole. The CM energy at the outer horizon r_+ for a near-extremal black hole is found to be

$$E_{\rm cm}(r \to r_+) = 2m_0\sqrt{1 + \frac{(l_1 - l_2)^2}{(l_1 - l_c')(l_2 - l_c')}\frac{l_c'}{4a}}, \qquad (41)$$

where

$$l_c' = \frac{2 + 2\sqrt{1 - a^2 - Q^2} - Q^2}{a}. \qquad (42)$$

The form is the same as (34) with the replacement $l_c \to l_c'$. Here, we denote the small parameter $\epsilon = a_{\rm max} - a$ with $a_{\rm max} = \sqrt{1 - Q^2}$. For fixed charge Q and ϵ, the range $(l_{\rm min}, l_{\rm max})$ of angular momentum for the particles to reach the horizon can be determined numerically with the effective potential $V_{\rm eff}$ for a near-extremal black hole. For a angular momentum $l \in (l_{\rm min}, l_{\rm max})$, we can get a negative $V_{\rm eff}(l)$ for $r > r_+$. However, for arbitrary charge Q and ϵ, we find that, within a small range near the horizon r_+, the effective potential $V_{\rm eff}(l_c')$ is always positive, which means the angular momentum l_c' does not lie in the range $(l_{\rm min}, l_{\rm max})$. So the CM energy $E_{\rm cm}$ in (41) is not divergent. Thus the CM energy is finite for arbitrary charge Q and spin a. Considering that one of the colliding particles has the maximum angular momentum $l_{\rm max}$ and another one has the minimum angular momentum $l_{\rm min}$, we obtain the CM energy per unit rest mass for different Q and ϵ. The result is shown in Table I. From it, we can see that for a KN black hole with spin a less than $a_{\rm max}$ there will be an upper bound for the CM energy. It is also suggested that the CM energy grows very slowly as the maximally spinning case ($\epsilon \to 0$) is approached. For fixed parameter ϵ, the value of CM energy decreases with the increase of the charge Q. For the case $Q = 0$, it describes a Kerr black hole and the result shown in Table I is exactly consistent with [3].

In order to obtain a Planck-scale CM energy $E_{\rm Pl} \sim 10^{19}$ GeV, we would like to study how much the tolerances on the critical angular momentum l_c and the black hole parameter are allowed. First, we consider the case that the black hole is still an extremal one, but there exists a small tolerance δl on the critical angular momentum l_c. For simplicity, we choose $l_1 = l_c - \delta l$ and $l_2 = 0$. The rest mass for the colliding particle is considered to $m_0 \sim 1$ GeV, just like the mass of a neutron. Then with the help of (34), we get a approximate δl:

TABLE I. The CM energy per unit rest mass $\frac{E_{\rm cm}}{m_0}$ for a KN black hole with spin $a = a_{\rm max} - \epsilon$ and $l_1 = l_{\rm max}$, $l_2 = l_{\rm min}$.

	$\epsilon = 0.1$	$\epsilon = 0.05$	$\epsilon = 0.01$	$\epsilon = 0.001$	$\epsilon = 0.0001$
$Q = 0$	6.901	8.244	12.54	22.63	40.49
$Q = 0.1$	6.894	8.234	12.51	22.59	40.40
$Q = 0.2$	6.875	8.203	12.45	22.44	40.12
$Q = 0.3$	6.842	8.150	12.33	22.19	39.64
$Q = 0.4$	6.794	8.073	12.16	21.82	38.93
$Q = 0.5$	6.730	7.967	11.93	21.30	37.94
$Q = 0.6$	6.647	7.826	11.60	20.57	36.54
$Q = 0.7$	6.539	7.636	11.14	19.49	34.44
$Q = 0.8$	6.398	7.367	10.42	17.57	30.35

$$\delta l \approx \frac{l_c^2}{a}\left(\frac{m_0}{E_{\rm Pl}}\right)^2 \sim 10^{-37}. \tag{43}$$

Note that we have considered that $a \in (\frac{1}{\sqrt{3}}, 1)$. Now, we would like to estimate the tolerance on the extremal black hole parameters to achieve the Planck-scale energy. Here we consider that one of the colliding particles has the critical angular momentum l_c, but the black hole is a near-extremal one. Here, we denote the tolerance $\epsilon = a_{\rm max} - a \ll 1$, and suppose that $l_1 = l_c$ and $l_2 = 0$. Then, for $m_0 \sim 1$ GeV, with the CM energy (41), we have

$$\epsilon \approx \frac{l_c^4}{8a}\left(\frac{m_0}{E_{\rm Pl}}\right)^4 \sim 10^{-76}. \tag{44}$$

Here, we have shown that to achieve the Planck-scale energy, if the black hole is an extremal one, then the tolerance on the critical angular momentum is $\delta l \sim 10^{-37}$, and if one of the colliding particles has the critical angular momentum l_c but the black hole is a near-extremal one, then the tolerance on the black hole parameter is $\epsilon \sim 10^{-76}$. Replacing Planck-scale energy $E_{\rm Pl}$ with an arbitrary energy $E_{\rm cm}$, (44) can be reexpressed as

$$\left(\frac{E_{\rm cm}}{m_0}\right) \sim \frac{l_c}{\sqrt[4]{8a}}\epsilon^{-1/4}. \tag{45}$$

Comparing with (1), we find they have the same order $\epsilon^{-1/4}$. However, our formula (45) is only an approximation; the more exact result can be found in Table I for different charge Q and ϵ.

IV. SUMMARY

In this paper, we have investigated the collision of two uncharged particles (which could be thought to be the cold dark matter particles) falling freely from rest at infinity in the background of a KN black hole. It is pointed out by BSW [1] that the CM energy of collision for two particles in the background of an extremal Kerr black hole can approach to an arbitrarily high value if one of the particles has angular momentum $l = 2$. Our results show that when extended to the KN black hole case, an unlimited CM energy requires three conditions: (1) the collision takes place at the horizon of an extremal black hole; (2) one of the colliding particles has critical angular momentum $l = \frac{1+a^2}{a}$; (3) the spin a of the extremal black hole satisfies $\frac{1}{\sqrt{3}} \le a \le 1$. Compared with the Kerr black hole, to obtain an arbitrarily high CM energy, besides the conditions that the black hole is an extremal black hole and one of the colliding particles has critical angular momentum, there still exists a restriction on the value of the spin a of the KN black hole, which is a significant difference between the two black holes. For a near-extremal black hole, we also find that there always exists an upper bound for the CM energy, which decreases with the increase of the charge Q. For an extremal black hole, in order to obtain the Planck-scale energy, the tolerance on the critical angular momentum should be $\delta l \sim 10^{-37}$. On the other hand, if one of the colliding particles has the critical angular momentum l_c, then the tolerance on the black hole parameter is $\epsilon \sim 10^{-76}$. However, if the particle does not fall freely from rest at infinity [6,9,10], the unlimited CM energy may be approached. In future work, we will explore the CM energy for the collision of charged particles taking place in a nonextremal black hole background.

ACKNOWLEDGMENTS

This work was supported by the Program for New Century Excellent Talents in University, the Huo Ying-Dong Education Foundation of Chinese Ministry of Education (No. 121106), the National Natural Science Foundation of China (No. 11075065), and the Fundamental Research Funds for the Central Universities (No. lzujbky-2009-54 and No. lzujbky-2009-163).

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