Infrared propagators of Yang-Mills theory from perturbation theory

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We show that the correlation functions of ghosts and gluons for the pure Yang-Mills theory in Landau gauge can be accurately reproduced for all momenta by a one-loop calculation. The key point is to use a massive extension of the Faddeev-Popov action. The agreement with lattice simulation is excellent in $d = 4$. The one-loop calculation also reproduces all the characteristic features of the lattice simulations in $d = 3$ and naturally explains the peculiarities of the propagators in $d = 2$.

The infrared (IR) physics of strong interaction is well described today by lattice simulations of quantum chromodynamics (QCD). This tool is now commonly used to determine the spectrum of particles, cross sections, and other physical observables (see for example [1]). The analytical (or semianalytical) approaches have not reached such a high level of development, mainly because the standard perturbative approach breaks down at low energies. This calls for more sophisticated techniques, and a good guiding principle in developing them is to compare their predictions with lattice simulations. Unfortunately, the simplest quantities that can be computed in analytical approaches are nongauge-invariant and therefore require to fix the gauge. For this reason, a considerable amount of work has been performed in the last few years to study gauge-fixed versions of QCD in the lattice.

The simplest quantities that can be analytically studied are the 2-point correlation functions in Landau gauge, and most of the gauge-fixed simulations have focused on these quantities. In the case of pure gluodynamics (with no quarks), some facts are now clearly established. First, the gluon propagator does not diverge in the IR but tends to a positive constant for $d > 2$ [2–4] and to zero in $d = 2$ [5,6]. The ghost propagator is divergent in the IR limit with an enhancement when compared to the free propagator: the dressing function (i.e. the propagator times momentum squared) is monotonically decreasing with momentum. It seems to approach a finite positive constant in the IR for $d > 2$ and diverges in this limit for $d = 2$. It is also well documented that the Källén-Lhemann spectral function associated with the gluon propagator is not definite positive [7,8].

Let us now recall the various analytical approaches that have been used to determine these correlation functions. The standard perturbation theory in the framework of Faddeev-Popov (FP) gauge fixing, as is well known, is unable to access the IR limit of the theory because it presents a Landau pole. This may be related to the fact that the FP procedure does not fix completely the gauge because of the Gribov ambiguity: there exists in general

several gauge transformed configurations (Gribov copies) that satisfy a gauge condition [9]. A line of investigation has been developed to restrict the functional integral in order to take into account only a subset of the Gribov copies (hopefully, only one). This leads to the Gribov-Zwanziger model [9–11] and some variants of it [12]. The IR propagators have also been studied by using Schwinger-Dyson and nonperturbative renormalization group equations. In these approaches, one solves a truncated version of an infinite set of coupled equations for the vertex functions. Depending on how one implements these ideas, two families of solutions have been found: (i) the scaling solution [13–16], where the gluon propagator goes to zero in all dimensions and the ghost propagator is more singular than the bare one in the IR, and (ii) the decoupling solution [17,18] where the propagators have behaviors in qualitative agreements with the lattice simulations. We note at this level that all these approaches lead to quite involved calculations, with in some cases an important numerical part.

In this letter we take a more pragmatic point of view. We do not try to find a gauge-fixed theory that would be justified from first principles, but propose a minimal modification of the FP action that can account for the lattice simulation results. Of course, this phenomenological approach can only be motivated *a posteriori*, if it describes in a satisfactory way the simulation results. Our main guide is the observation that the gluon propagator tends to a finite positive value in the IR for $d > 2$. We propose to impose this property at the tree level by adding a mass term for the gluon in the FP action [19]. We do not change the ghost sector since the ghost propagator is found to be IR divergent in the simulations. This leads us to consider the Landau-gauge FP Euclidean Lagrangian supplemented with a gluon mass term:

$$\mathcal{L} = \frac{1}{4}(F_{\mu\nu}^a)^2 + \partial_\mu \bar{c}^a (D_\mu c)^a + ih^a \partial_\mu A_\mu^a + \frac{m^2}{2}(A_\mu^a)^2 \quad (1)$$

where $(D_\mu c)^a = \partial_\mu c^a + g f^{abc} A_\mu^b c^c$ and the field strength $F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu + g f^{abc} A_\mu^b A_\nu^c$ are expressed in

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terms of the coupling constant g. The Lagrangian (1) corresponds to a particular case of the Curci-Ferrari model [20]. At tree level, the gluon propagator is massive and transverse in momentum space:

$$G_{\mu\nu}^{ab}(p) = \delta^{ab} P_{\mu\nu}^{\perp}(p) \frac{1}{p^2 + m^2} \qquad (2)$$

with $P_{\mu\nu}^{\perp}(p) = \delta_{\mu\nu} - p_{\mu} p_{\nu}/p^2$. It is interesting to note that the spectral density associated with the propagator (2) is positive and therefore there is no violation of positivity at the tree level. We conclude that violations of positivity, if they exist in this model, are caused by fluctuations.

Actually, the gluon propagator observed in the lattice is not compatible with the bare propagator (2) and we will show below that, by including the one-loop corrections, one obtains propagators for gluons and ghosts that are in impressive agreement with those obtained in the lattice in $d = 4$ (including positivity violations) and that reproduce at the qualitative level the results for $d = 3$. Let us mention that a mass term has been used to improve perturbative QCD results in order to reproduce the phenomenology of strong interactions [21]. Moreover, there are successful confinement models [22] that use actions including a gluon mass term.

When analyzing the model described above, we must face the problem that the mass term breaks the Becchi-Rouet-Stora-Tyutin (BRST) symmetry [23] which is very important in the perturbative analysis. This symmetry has the form

$$\delta A_{\mu}^a = \eta(D_{\mu}c)^a, \qquad \delta c^a = -\eta \frac{g}{2} f^{abc} c^b c^c,$$
$$\delta \bar{c}^a = \eta i h^a, \qquad \delta i h^a = 0, \qquad (3)$$

where η is a global Grassmanian parameter. The BRST symmetry is in general used to prove the renormalizability of the theory but the breaking of this symmetry by the mass term does not spoil renormalizability [20]. The BRST symmetry is also used to reduce the state space to the physical space, in which the theory is unitary. Since the BRST symmetry is broken by the mass term, we cannot address the problem of unitarity as usually done. This issue is actually common to essentially all methods that try to go beyond the standard perturbation theory (as the Gribov-Zwanziger model) because they all break the standard

BRST symmetry. In this respect, the model considered here is not in a worse position than other approaches considered in the field. We must stress that this model is equivalent to the standard FP model in the ultraviolet limit $p \gg \Lambda_{QCD}$ if $m \sim \Lambda_{QCD}$. This means that in the domain of validity of standard perturbation theory, the model is as unitary as QCD. The unitarity of the model in other momentum regimes is of course an important open problem, as it is in all gauge fixings in which standard BRST symmetry is broken.

The model with Lagrangian (1), as a particular case of the Curci-Ferrari model, has a pseudo-BRST symmetry (not nilpotent) that has the same form as the standard BRST (3) except for the h variation which reads $\delta i h^a = \eta m^2 c^a$. On top of this symmetry, the Lagrangian has all the standard symmetries of the FP action for the Landau gauge. This includes the shift in antighost $\bar{c} \to \bar{c} + cst.$, a symplectic group [24], and four gauged supersymmetries recently found [25]. As a consequence, the mass [26] and coupling constant [27] renormalization factors (even in presence of the mass term [25]) are fixed in terms of gluon and ghost field renormalizations.

We present now the one-loop calculation of the propagators, which requires the calculation of four Feynman diagrams. It is convenient to parametrize the gluon $G_{\mu\nu}^{ab}(p)$ and the ghost $G^{ab}(p)$ propagators in the form:

$$G^{ab}(p) = \delta_{ab} F(p)/p^2,$$
$$G_{\mu\nu}^{ab}(p) = P_{\mu\nu}^{\perp}(p)\delta_{ab} G(p). \qquad (4)$$

The $F(p)$ is known as the ghost dressing function and the scalar function $G(p)$ will be referred to as the gluon propagator below. We choose the following renormalization conditions:

$$G(p = 0) = 1/m^2,$$
$$G(p = \mu) = 1/(m^2 + \mu^2), \qquad (5)$$
$$F(p = \mu) = 1.$$

We use the gluon-ghost vertex in the Taylor scheme [27] for the coupling constant g.

We consider first the 4-dimensional case. The one-loop result for the renormalized functions $F(p)$ and $G(p)$ [imposing the renormalization prescriptions (5)] are

$$G^{-1}(p)/m^2 = s + 1 + \frac{g^2 N s}{384\pi^2} \Big\{ 111s^{-1} - 2s^{-2} + (2 - s^2)\log s + 2(s^{-1} + 1)^3(s^2 - 10s + 1)\log(1 + s)$$
$$+ (4s^{-1} + 1)^{3/2}(s^2 - 20s + 12)\log\Big(\frac{\sqrt{4 + s} - \sqrt{s}}{\sqrt{4 + s} + \sqrt{s}}\Big) - (s \to \mu^2/m^2)\Big\}$$

$$F^{-1}(p) = 1 + \frac{g^2 N}{64\pi^2}\{-s\log s + (s + 1)^3 s^{-2}\log(s + 1) - s^{-1} - (s \to \mu^2/m^2)\} \qquad (6)$$

where $s = p^2/m^2$.

FIG. 2 (color online). Four-dimensional gluon propagator for $SU(3)$ gauge group times p^2. The results of the present work (red curve) are compared with lattice data of [3] (green open circles) and [4] (green crosses). The (black) dashed curve is the ultraviolet-improved curve obtained by the renormalization group.

FIG. 1 (color online). Four-dimensional correlation functions for $SU(2)$ gauge group. The results of the present work (red curve) are compared with lattice data of [2] (green points). Top figure: gluon propagator. Bottom figure: ghost dressing function.

In Fig. 1, we compare these expressions for the $SU(2)$ gauge group with the lattice simulations of [2]. The best choice of parameter is $g = 7.5$ and $m = 0.68$ GeV when normalization prescriptions are imposed at $\mu = 1$ GeV. One observes that both gluon and ghost propagators can be fitted with the same choice of parameters in a very satisfactory way. Note that the normalization conditions of the lattice simulations are not compatible with (5) so that we have to introduce a global multiplicative renormalization factor when comparing the curves.

We have also compared our results with the data of two different lattice studies [3,4] for the $SU(3)$ group. The two data sets have different overall momentum scale and we have rescaled the momenta of the data of [3] for superimposing them with those of [4]. We represent in Fig. 2 the dressing function of the gluon instead of the propagator in order to make visible the ultraviolet regime. The best choice of parameters is $g = 4.9$ and $m = 0.54$ GeV (again with $\mu = 1$ GeV) and it leads to a very satisfying agreement for momenta $p \lesssim 2$ GeV. It is important to stress that expressions (6) are one-loop results obtained from a fixed coupling constant calculation in a fixed renormalization point. It is well-known that in order to analyze the regime
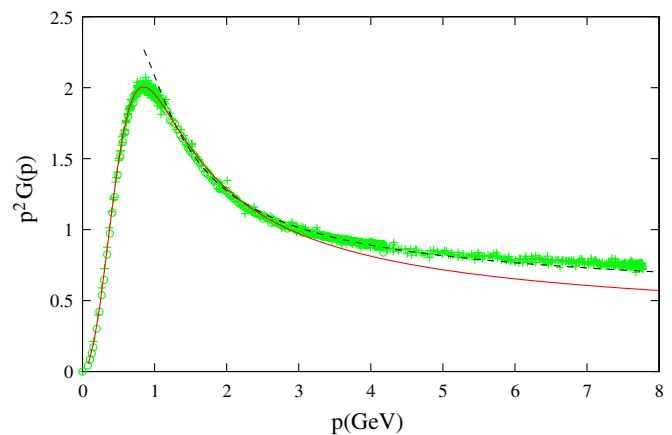
$p \gg m$, one must take into account renormalization group effects and, in particular, the running of the coupling. The corresponding procedure is standard and once it is implemented (see [28] for details), the agreement is essentially within error bars for $p > m$ as is also shown in Fig. 2. A very good agreement is also obtained for the ghost dressing function [28]. In any case, it is obvious that when $p \gg m$, the model (1) reproduces correctly the high momentum regime once renormalization group effects are taken into account.

An interesting feature of the one-loop gluon propagator is that it is nonmonotonous in the IR. In fact, the inverse propagator behaves at small momenta as $m^2 + Ng^2p^2/(192\pi^2) \log(p^2/m^2) + \mathcal{O}(p^2)$. This prediction of our calculation is very small for $d = 4$ and it is not visible in Fig. 1 but appears clearly in $d = 3$, see below.

An important property of the propagators measured on the lattice is the violation of positivity. One way to extract it is to calculate the quantity:

$$C(t) = \int_{-\infty}^{\infty} \frac{dp}{2\pi} e^{ipt} G(p). \tag{7}$$

It can be shown (see for instance [7]) that the positivity of the spectral function implies the positivity of $C(t)$. In Fig. 3, we plot a numerical Fourier transform of the $SU(3)$ gauge group gluon propagator for the best parameters described above; this shows a clear violation of positivity. We observe that the curve of $C(t)$ is very similar to the one of [7]: it is strongly positive for $t \lesssim 1$ fm and slightly negative beyond.

Let us now consider the three-dimensional case, where the one-loop calculation can be done explicitly again. The details of the calculations and the final expressions will be presented in a future publication [28]. We only mention
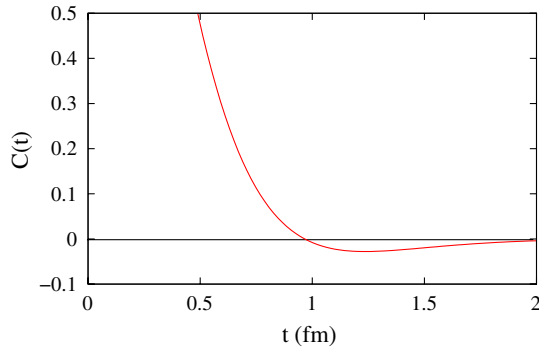
FIG. 3 (color online). Four-dimensional real space propagator $C(t)$ for $SU(3)$ gauge group. The curves grow when t tends to zero, saturating at the value $C(0) \simeq 2.1$.

here that the model (1) is able to account for the main features of gluon and ghost propagators found in lattice simulations. In Fig. 4 the results of the present model with the best fit parameters $g = 3.7 \sqrt{\text{GeV}}$ and $m = 0.89$ GeV for $\mu = 1$ GeV are compared with $d = 3$ simulations performed with the gauge group $SU(2)$ [2]. We observe that the best fit for gluon and ghost propagators are not as good as in $d = 4$. This is probably related to the fact that higher loop corrections are not very small. It is worth mentioning that the results improve if one imposes the normalization conditions at a larger momentum scale (for $\mu = 11$ GeV the best parameters are $g = 1.6 \sqrt{\text{GeV}}$ and $m = 0.35$ GeV). Such a large scheme dependence indicates that higher loop corrections give significant contributions. In any case, our calculation reproduces the finite IR gluon propagator and ghost dressing function. It also reproduces the nonmonotonic behavior of the gluon propagator in the IR. An expansion of the inverse propagator at low momentum leads to $m^2 - Ng^2p/64 + \mathcal{O}(p^2)$.

Within the model, the difference between $d = 2$ and $d > 2$ that is observed in the lattice (see above) also appears natural. In $d = 2$, we find that the gluon mass m and ghost dressing function at zero momentum $F(0)$ develop logarithmic IR divergences. Such divergences exclude the possibility of controlling the one-loop calculation as was used above for $d > 2$. A proper treatment of the $d = 2$ case requires a renormalization group approach adapted to the IR regime [28] and goes beyond the scope of the present letter.

The model presented here reproduces surprisingly well the $d = 4$ propagators of pure gluodynamics for $SU(2)$ and $SU(3)$ and describes in a simple way the main characteristics of those propagators in $d = 3$. The specificities of the $d = 2$ case result from the IR divergences that appear in
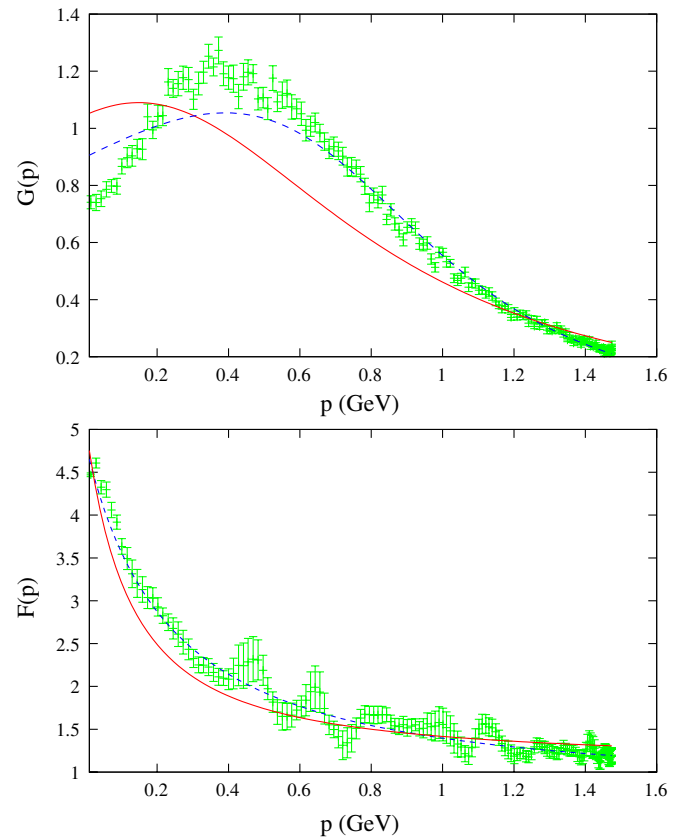

FIG. 4 (color online). Three-dimensional functions for $SU(2)$ gauge group: Comparison of present results (plain red curve for $\mu = 1$ GeV and dashed blue curve for $\mu = 11$ GeV) with lattice data of [2] (green bars). Top figure: gluon propagator. Bottom figure: ghost dressing function.

this dimension. Given the technical simplicity of this approach, this work opens the door to many subsequent applications in strong interactions physics (for example, the inclusion of quarks, a study of the dependence on the gauge fixing, three- and four-point correlation functions, quark-anti-quark static potential).

Considering the surprisingly good agreement between the one-loop calculation and the simulations, it is tempting to think that the model is not just a good phenomenological description and one should try to justify the use of this action from first principles. The issue of unitarity should be also explored.

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