

Unparticle contributions to B_s - \bar{B}_s mixing

Jong-Phil Lee*

Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea
and Division of Quantum Phases & Devices, School of Physics, Konkuk University, Seoul 143-701, Korea
 (Received 20 September 2010; published 16 November 2010)

The unparticle effects on the B_s - \bar{B}_s mixing is revisited. Taking into account the unitarity constraints on the unparticle operators, we find that the contribution of the vector unparticle is very suppressed compared to that of the scalar unparticle. This is due to the fact that the lower bound of the scaling dimension of the vector-unparticle operator is larger. It is also shown that the mixing phase from the scalar unparticle is negative, and unparticles can produce a large mixing phase.

DOI: 10.1103/PhysRevD.82.096009

PACS numbers: 12.90.+b, 14.40.Nd

A few years ago Georgi proposed a totally different type of new physics called “unparticles” [1]. In this scenario, there is a scale-invariant hidden sector which couples to the standard model (SM) particles very weakly at high-energy scale Λ_U . When seen at low energy, the hidden sector behaves in different ways from ordinary particles, hence dubbed as *unparticles*. In a word, unparticles behave like a fractional number of particles.

We have many reasons and clues to conclude that the SM of particle physics is only an effective theory at low energy, and there must be some new physics behind it. Many kinds of new physics—supersymmetry or extra dimensions, etc.—involve some new sets of *particles*, thus the discovery of the unparticle would be one of the most spectacular phenomena ever seen. With the reoperation and the first high-energy collision of the Large Hadron Collider (LHC) at CERN very recently, we anticipate unparticles as well as other new physics signals to be seen sooner or later.

Suppose that at some high energy $\sim M_U$, there is an ultraviolet (UV) theory in the hidden sector with the infrared (IR)-stable fixed point. The interaction between the UV theory and the SM sector can be described by an effective theory formalism. Below M_U , a UV operator \mathcal{O}_{UV} interacts with an SM operator \mathcal{O}_{SM} through $\mathcal{O}_{SM}\mathcal{O}_{UV}/M_U^{d_{SM}+d_{UV}-4}$. Here $d_{UV(SM)}$ is the scaling dimension of $\mathcal{O}_{UV(SM)}$. The renormalization flow enables one to go down along the scale, until a new scale Λ_U is met. It appears through the dimensional transmutation where the scale invariance emerges. Below Λ_U the theory is matched onto the above interaction with the new unparticle operator \mathcal{O}_U as

$$C_U \frac{\Lambda_U^{d_{UV}-d_U}}{M_U^{d_{SM}+d_{UV}-4}} \mathcal{O}_{SM} \mathcal{O}_U, \quad (1)$$

where d_U is the scaling dimension of \mathcal{O}_U and C_U is the matching coefficient. Because of the scale invariance, d_U does not have to be integers. This unusual behavior of unparticles is reflected on the phase space of \mathcal{O}_U .

To see it, consider the spectral function of the unparticle which is given by the two-point function of \mathcal{O}_U :

$$\begin{aligned} \rho_U(P^2) &= \int d^4x e^{iP \cdot x} \langle 0 | \mathcal{O}_U(x) \mathcal{O}_U^\dagger(0) | 0 \rangle \\ &= A_{d_U} \theta(P^0) \theta(P^2) (P^2)^{d_U-2}, \end{aligned} \quad (2)$$

where

$$A_{d_U} = \frac{16\pi^2 \sqrt{\pi}}{(2\pi)^{2d_U}} \frac{\Gamma(d_U + \frac{1}{2})}{\Gamma(d_U - 1) \Gamma(2d_U)}, \quad (3)$$

is the normalization factor. The corresponding phase space is

$$\begin{aligned} d\Phi_U(P) &= \rho_U(P^2) \frac{d^4P}{(2\pi)^4} \\ &= A_{d_U} \theta(P^0) \theta(P^2) (P^2)^{d_U-2} \frac{d^4P}{(2\pi)^4}. \end{aligned} \quad (4)$$

Since d_U is not constrained to be integers, $d\Phi_U$ looks like a phase space for a fractional number of particles.

After Georgi, there have been a lot of researches on unparticles [2,3]. Among them are the unparticle effects on B physics and meson mixing [4–10]. Especially, the B_s - \bar{B}_s mixing has much attention after the first observation by CDF and D0 [11]. Recently, the D0 Collaboration announced the evidence for the charge asymmetry of the like-sign dimuon events [12]. For more discussions about B_s - \bar{B}_s mixing, see [13,14] and references therein.

For simplicity we only consider the left-handed currents coupled to scalar (\mathcal{O}_U) and vector (\mathcal{O}_U^μ) unparticles as follows:

$$\frac{c_S}{\Lambda_U^{d_U}} \bar{q}' \gamma_\mu (1 - \gamma_5) q \partial^\mu \mathcal{O}_U + \frac{c_V}{\Lambda_U^{d_U-1}} \bar{q}' \gamma_\mu (1 - \gamma_5) q \mathcal{O}_U^\mu, \quad (5)$$

where $c_{S,V}$ are dimensionless coefficients. We assume that $c_{S,V}$ are real numbers. The above interactions provide flavor-changing neutral currents at tree level, which contribute to the B_s - \bar{B}_s mixing. The propagators of scalar and vector unparticles are given by [1,15]

*jplee@kias.re.kr

$$\begin{aligned} & \int d^4x e^{iPx} \langle 0 | T \mathcal{O}_U(x) \mathcal{O}_U(0) | 0 \rangle \\ &= \frac{iA_{d_U}}{2 \sin d_U \pi} \frac{e^{-i\phi_U}}{(P^2 + i\epsilon)^{2-d_U}}, \end{aligned} \quad (6)$$

and

$$\begin{aligned} & \int d^4x e^{iPx} \langle 0 | T \mathcal{O}_U^\mu(x) \mathcal{O}_U^\nu(0) | 0 \rangle = \frac{iA_{d_U}}{2 \sin d_U \pi} \\ & \times \frac{e^{-i\phi_U}}{(P^2 + i\epsilon)^{2-d_U}} \left[-g^{\mu\nu} + \frac{2(d_U - 2)}{d_U - 1} \frac{P^\mu P^\nu}{P^2} \right], \end{aligned} \quad (7)$$

respectively, and $\phi_U = (d_U - 2)\pi$. Note that the relative size of the coefficients of $-g^{\mu\nu}$ and $P^\mu P^\nu/P^2$ in Eq. (7) is not unity, but some function of d_U [15]. It is due to the unitarity constraints. This point is not reflected in the literature. Another point which is erroneously used so far is that the scaling dimension d_U is commonly used for \mathcal{O}_U and \mathcal{O}_U^μ . Obviously this is not true; in general they can be independent variables. Furthermore, [15] has shown that the scalar-unparticle dimension has a lower bound $d_U^S \geq 1$ while for the vector-unparticle dimension, $d_U^V \geq 3$ from unitarity [15]. Thus in what follows, we will distinguish $d_U^S \equiv d_S$ and $d_U^V = d_V$. As will be seen later, the unitarity bound for d_V has a significant meaning for the B_s - \bar{B}_s mixing.

In general, the B_s - \bar{B}_s mixing is parametrized by the quantity M_{12}^s defined by

$$2M_{B_s} M_{12}^s = \langle \bar{B}_s^0 | \mathcal{H}_{\text{eff}}^{\Delta B=2} | B_s^0 \rangle, \quad (8)$$

where $\mathcal{H}_{\text{eff}}^{\Delta B=2}$ is the effective Hamiltonian for the $\Delta B = 2$ transitions. The SM contribution to M_{12}^s is given by the box diagrams, resulting in

$$M_{12}^s = \frac{G_F^2 M_W^2}{12\pi^2} (V_{ts}^* V_{tb})^2 M_{B_s} B_{B_s} f_{B_s}^2 \hat{\eta}_{B_s} S_0(x_t), \quad (9)$$

where $S_0(x_t \equiv m_t^2/M_W^2)$ is the Inami-Lim function [16] and $\hat{\eta}_{B_s}$ is the QCD correction factor. The mass difference ΔM_s is then $\Delta M_s = 2|M_{12}^s|$, and the experimentally measured value is [11]

$$\Delta M_s^{\text{exp}} = 17.77 \pm 0.12 \text{ ps}^{-1}. \quad (10)$$

If there is a new interaction of Eq. (5), it contributes to the B_s - \bar{B}_s mixing through the s and t channels at tree level. Explicitly, one gets

$$\begin{aligned} M_{12}^U &= \frac{A_{d_s} e^{-i\phi_{u_s}}}{8 \sin d_s \pi} \left(\frac{f_{B_s}^2}{M_{B_s}} \right) c_s^2 \left(\frac{M_{B_s}^2}{\Lambda_U^2} \right)^{d_s} \frac{m_b^2}{M_{B_s}^2} \frac{5}{3} RB_2 \\ &+ \frac{A_{d_V} e^{-i\phi_{u_V}}}{8 \sin d_V \pi} \left(\frac{f_{B_s}^2}{M_{B_s}} \right) c_V^2 \left(\frac{M_{B_s}^2}{\Lambda_U^2} \right)^{d_V-1} \\ &\times \left[-\frac{8}{3} B_1 + \frac{2(d_V - 2)}{d_V - 1} \frac{m_b^2}{M_{B_s}^2} \frac{5}{3} RB_2 \right], \end{aligned} \quad (11)$$

where

$$R \equiv \left(\frac{M_{B_s}}{m_b + m_s} \right)^2. \quad (12)$$

Here $B_{1,2}$ are the bag parameters for the relevant operators as follows:

$$\langle \bar{B}_s | Q_1 | B_s \rangle = \frac{8}{3} M_{B_s}^2 f_{B_s}^2 B_1, \quad (13)$$

$$\langle \bar{B}_s | Q_2 | B_s \rangle = -\frac{5}{3} M_{B_s}^2 f_{B_s}^2 R B_2, \quad (14)$$

where

$$Q_1 = \bar{b}_\alpha \gamma_\mu (1 - \gamma_5) s_\alpha \bar{b}_\beta \gamma^\mu (1 - \gamma_5) s_\beta, \quad (15)$$

$$Q_2 = \bar{b}_\alpha (1 - \gamma_5) s_\alpha \bar{b}_\beta (1 - \gamma_5) s_\beta. \quad (16)$$

The new physics effects on B_s - \bar{B}_s mixing can be nicely encoded in the following manner [17]:

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^U \equiv M_{12}^{\text{SM}} \cdot \Delta. \quad (17)$$

The phase of M_{12} is

$$\phi_s = \phi_s^{\text{SM}} + \phi_s^\Delta, \quad (18)$$

where $\Delta = |\Delta| e^{i\phi_s^\Delta}$. With the help of Eq. (11), one can easily obtain (for simplicity we put $m_b = M_{B_s}$, and $B_{1,2} = R = 1$)

$$\begin{aligned} \Delta &= 1 + \frac{M_{12}^U}{M_{12}^{\text{SM}}} \\ &= [1 + c_s^2 f_s(d_s) \cot d_s \pi + c_V^2 f_V(d_V) \cot d_V \pi] \\ &\quad - i[c_s^2 f_s(d_s) + c_V^2 f_V(d_V)], \end{aligned} \quad (19)$$

where

$$\begin{aligned} f_s(d_s) &\equiv \frac{1}{M_{12}^{\text{SM}}} \left(\frac{f_{B_s}^2}{M_{B_s}} \right) \frac{2\pi^{5/2}}{(2\pi)^{2d_s}} \frac{\Gamma(d_s + \frac{1}{2})}{\Gamma(d_s - 1)\Gamma(2d_s)} \\ &\quad \times \left(\frac{M_{B_s}^2}{\Lambda_U^2} \right)^{d_s} \frac{5}{3}, \end{aligned} \quad (20)$$

$$\begin{aligned} f_V(d_V) &\equiv \frac{1}{M_{12}^{\text{SM}}} \left(\frac{f_{B_s}^2}{M_{B_s}} \right) \frac{2\pi^{5/2}}{(2\pi)^{2d_V}} \frac{\Gamma(d_V + \frac{1}{2})}{\Gamma(d_V - 1)\Gamma(2d_V)} \\ &\quad \times \left(\frac{M_{B_s}^2}{\Lambda_U^2} \right)^{d_V-1} \frac{2(d_V - 6)}{3(d_V - 1)}. \end{aligned} \quad (21)$$

Note that the power of $M_{B_s}^2/\Lambda_U^2$ is different for f_s and f_V . If $d_s = d_V$, then f_s is suppressed by a factor of $M_{B_s}^2/\Lambda_U^2$, which amounts to $\sim 3 \times 10^{-5}$ for $\Lambda_U = 1$ TeV. But if we consider the unitarity constraints, $d_s \geq 1$ and $d_V \geq 3$. For simplicity we may set $d_s = 1 + \epsilon$, $d_V = 3 + \epsilon$. In this case, on the contrary to the previous estimation, f_V is much more suppressed by the factor of

$$\sim \frac{1}{(2\pi)^4} \left(\frac{M_{B_s}^2}{\Lambda_{\mathcal{U}}^2} \right) = 1.8 \times 10^{-8}, \quad (22)$$

for $\Lambda_{\mathcal{U}} = 1$ TeV.

The experimental values for ΔM_s and ϕ_s^Δ constrain the new physics parameters. Figure 1 shows the allowed region of c_S and ϵ when $c_V = 0$. We use one of the latest values of $\phi_s = -0.79 \pm 0.24$ [18] which fits the new D0 anomalous dimuon asymmetry, and $\phi_s^{\text{SM}} = (4.7^{+3.5}_{-3.1}) \times 10^{-3}$ [14]. Even in the case of $c_V \neq 0$, the effect of $c_V \sim \mathcal{O}(1)$ is negligible because of the suppression by Eq. (22). If we switch off c_S and turn on c_V , we have no overlaps for ΔM_s^{exp} and ϕ_s^Δ , at least for moderate ranges of c_V and ϵ . In other words, the coupling c_V must be enormous to compensate the kinematic suppression (22).

The expression Eq. (19) also has important meanings for the phase, ϕ_s^Δ . The imaginary part of Δ is

$$c_S^2 f_S + c_V^2 f_V = -|\Delta| \sin \phi_s^\Delta = -\left(\frac{\Delta M_s}{\Delta M_s^{\text{SM}}} \right) \sin \phi_s^\Delta. \quad (23)$$

Since f_V is highly suppressed, the left-hand side remains positive (for moderate values of c_V) and thus $-\pi < \phi_s^\Delta < 0$. Note that our M_{12}^U is the same as that of [10], and different from [7] by a factor of $(i/2)$. For this reason, the $\cot(d_{S,V}\pi)$ term enters the imaginary part of Δ in [7] and the phase can have both positive and negative values with the variation of $d_{S,V}$. In our calculation this is not true. Figure 2 shows ΔM_s vs ϕ_s^Δ for various values of c_S . In this figure, we only consider the scalar contribution.

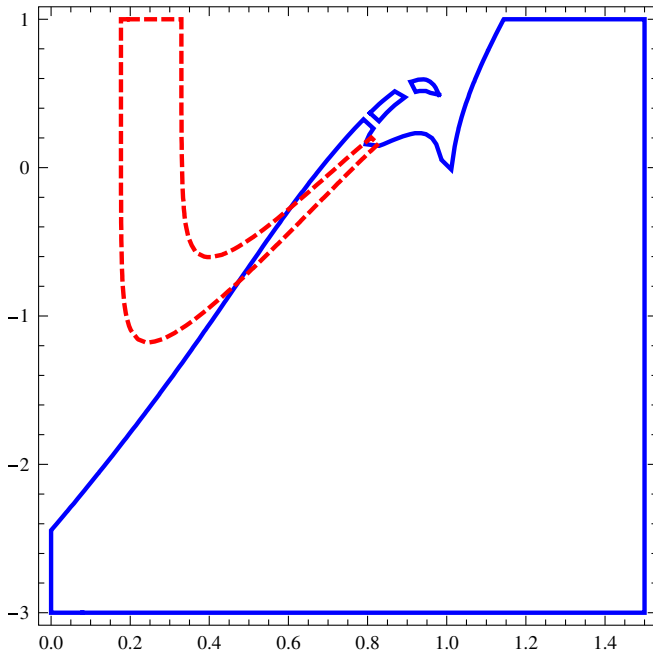


FIG. 1 (color online). Allowed region of c_S (vertical, in log scale) and ϵ (horizontal) from experimentally measured ΔM_s and ϕ_s^Δ for $c_V = 0$. The blue region (solid line) is from ΔM_s^{exp} ($1 - \sigma$), while the red one (dashed line) is from ϕ_s^Δ ($1 - \sigma$).

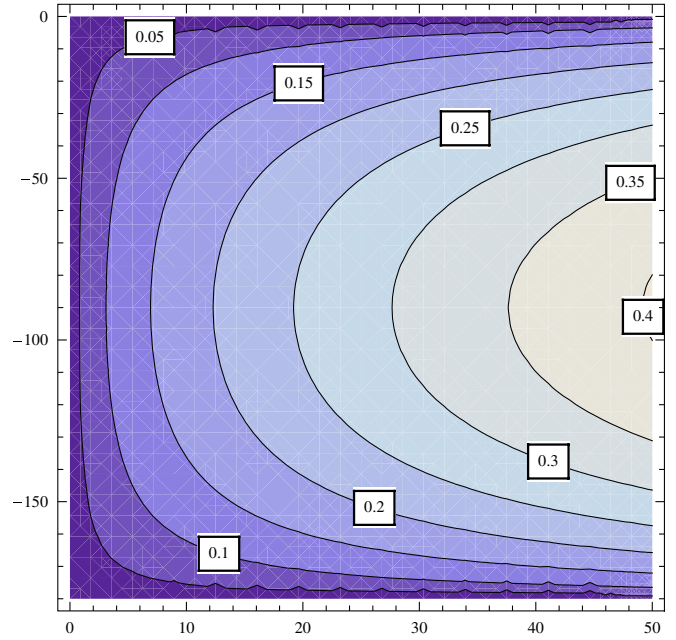


FIG. 2 (color online). Contour plots for ΔM_s (horizontal, in ps^{-1}) and ϕ_s^Δ (vertical, in degree) for $\epsilon = 0.5$. We put $c_V = 0$. The numbers in the boxes are the values of c_S .

For $c_V = 0$, the imaginary part of Δ is definitely negative, so we expect that the scalar unparticles induce negative $\sin \phi_s^\Delta$. As one can easily find in the figure, the scalar unparticle can produce a large phase.

If ϕ_s^Δ turned out to be positive, then one could expect $c_S = 0$ and $c_V \neq 0$. Note that for $\epsilon < 3$ the function f_V is negative. But the suppression is very severe, and the coupling c_V should be of order $\sim \mathcal{O}(10^8)$. So in this case one can conclude that the unparticle contributions cannot explain the positive ϕ_s^Δ for moderate values of the couplings.

In conclusion, we investigated the unparticle effects on the $B_s\text{-}\bar{B}_s$ mixing. Scalar and vector unparticles can contribute to the mixing at tree level via s and t channels of the unparticle exchange. The effects were already studied in the literature, but the previous studies did not consider the unitarity constraints of [15]. We found that the unitarity constraints play a crucial role in the analysis. If the scaling dimensions of the unparticle operators are universal as is common in the literature, the vector-unparticle contribution is dominant. But the unitarity condition puts different lower bounds for the dimensions of the unparticle operators. When we take into account this point, the vector-unparticle contribution is highly suppressed by a factor of $\sim \mathcal{O}(10^{-8})$, compared to the scalar-unparticle contribution (assuming that the couplings are of the same order). According to [15], the tensor structure of the propagator of the vector unparticles is slightly different from that of the ordinary vector particles. But since the vector contribution to the $B_s\text{-}\bar{B}_s$ mixing is negligible, it is very hard to notice the differences. We also found that the phase ϕ_s^Δ from the scalar unparticle is negative definite. This is

compatible with the current experimental data. Fortunately, the scalar unparticle can produce large mixing phase.

It might be also interesting to examine the unparticle effects on $B_s \rightarrow J/\psi \phi$ and $B_s \rightarrow \phi \phi$, as analyzed in [6]. With the unitarity constraints, the fact that the mixing-induced CP asymmetry for $B_s \rightarrow J/\psi \phi$ can be large by unparticles would not be changed, but the contribution would be dominated by scalar unparticles. And the transition amplitude of $B_s \rightarrow \phi \phi$ from vector unparticles is much more suppressed compared to the result of [6] since the amplitude is proportional to $(m_{B_s}/\Lambda_U)^{2d_U-2}$.

As a final remark, possible new physics effects on the decay matrix element Γ_{12}^s have received much attention

recently after the $D0$ anomaly. There have been lots of works considering new physics effects on Γ_{12}^s . Since Γ_{12}^s is the absorptive part of the effective Hamiltonian and unparticles can be seen as an infinite tower of massless particles [19], one could expect that there is a sizable contribution from unparticles [20]. Dedicated works to this issue will appear elsewhere. In the current analysis we simply assumed that new physics contributes only to M_{12} .

This work was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Korean Ministry of Education, Science and Technology (No. 2009-0088396).

-
- [1] H. Georgi, *Phys. Rev. Lett.* **98**, 221601 (2007); *Phys. Lett. B* **650**, 275 (2007).
- [2] K. Cheung, W. Y. Keung, and T. C. Yuan, *Phys. Rev. Lett.* **99**, 051803 (2007); *Phys. Rev. D* **76**, 055003 (2007).
- [3] M. Luo and G. Zhu, *Phys. Lett. B* **659**, 341 (2008); Y. Liao, *Phys. Rev. D* **76**, 056006 (2007); T. Kikuchi and N. Okada, *Phys. Lett. B* **661**, 360 (2008); **665**, 186 (2008); T. Kikuchi, N. Okada, and M. Takeuchi, *Phys. Rev. D* **77**, 094012 (2008); N. V. Krasnikov, *Int. J. Mod. Phys. A* **22**, 5117 (2007); *Mod. Phys. Lett. A* **23**, 3233 (2008); F. Sannino and R. Zwicky, *Phys. Rev. D* **79**, 015016 (2009); D. C. Dai and D. Stojkovic, *Phys. Rev. D* **80**, 064042 (2009); J.-P. Lee, [arXiv:0710.2797](https://arxiv.org/abs/0710.2797); *AIP Conf. Proc.* **1078**, 626 (2009); *Phys. Rev. D* **79**, 076002 (2009).
- [4] C. H. Chen and C. Q. Geng, *Phys. Rev. D* **76**, 115003 (2007).
- [5] X. Q. Li and Z. T. Wei, *Phys. Lett. B* **651**, 380 (2007).
- [6] R. Mohanta and A. K. Giri, *Phys. Rev. D* **76**, 075015 (2007).
- [7] A. Lenz, *Phys. Rev. D* **76**, 065006 (2007).
- [8] S. L. Chen, X. G. He, X. Q. Li, H. C. Tsai, and Z. T. Wei, *Eur. Phys. J. C* **59**, 899 (2009).
- [9] R. Mohanta and A. K. Giri, *Phys. Lett. B* **660**, 376 (2008).
- [10] J. K. Parry, *Phys. Rev. D* **78**, 114023 (2008).
- [11] A. Abulencia *et al.* (CDF Collaboration), *Phys. Rev. Lett.* **97**, 242003 (2006); A. Abulencia *et al.* (CDF-Run II Collaboration), *Phys. Rev. Lett.* **97**, 062003 (2006); V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. Lett.* **97**, 021802 (2006).
- [12] V. M. Abazov *et al.* (D0 Collaboration), *Phys. Rev. D* **82**, 032001 (2010); *Phys. Rev. Lett.* **105**, 081801 (2010).
- [13] J. P. Lee and K. Y. Lee, *Phys. Rev. D* **78**, 056004 (2008); [arXiv:0809.0751](https://arxiv.org/abs/0809.0751).
- [14] A. Lenz *et al.*, [arXiv:1008.1593](https://arxiv.org/abs/1008.1593).
- [15] B. Grinstein, K. A. Intriligator, and I. Z. Rothstein, *Phys. Lett. B* **662**, 367 (2008).
- [16] T. Inami and C. S. Lim, *Prog. Theor. Phys.* **65**, 297 (1981); **65**, 1772(E) (1981).
- [17] A. Lenz and U. Nierste, *J. High Energy Phys.* **06** (2007) 072.
- [18] C. W. Bauer and N. D. Dunn, [arXiv:1006.1629](https://arxiv.org/abs/1006.1629).
- [19] M. A. Stephanov, *Phys. Rev. D* **76**, 035008 (2007).
- [20] See, for example, X. G. He, B. Ren, and P. C. Xie, [arXiv:1009.3398](https://arxiv.org/abs/1009.3398).