

Lepton flavor violating signals of the littlest Higgs model with T parity via e^+e^- and $\gamma\gamma$ collisions at the ILC

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Taking into account the constraints on the free parameters of the littlest Higgs model with T parity (called the LHT model) from some rare decay processes, such as $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$, we consider the contributions of the LHT model to the lepton flavor violating (LFV) processes $e^+e^- \rightarrow l_i\bar{l}_j$ and $\gamma\gamma \rightarrow l_i\bar{l}_j$ ($i \neq j$). We find that the LHT model can indeed produce significant contributions to these LFV processes and its LFV signal might have a chance of being observed in the future International Linear Collider experiments.

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I. INTRODUCTION

During the past decade, neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are massive particles mixing with each other [1]. Moreover, their masses are extremely small, while mixing is nearly maximal, which means that the lepton flavor violating (LFV) processes are allowed. However, it is well known that, in the standard model (SM), neutrinos are massless and the LFV processes are not allowed at tree level. Thus, the exciting experimental fact opens a window to new physics beyond the SM [2]. In fact, lepton flavor symmetry is an accidental symmetry at low energy, and it may be violated beyond the SM. Many kinds of popular specific models, like supersymmetry, technicolor, and little Higgs models, indicate the possibility of large LFV. Therefore, an LFV signal in the charged lepton sector would be a clear hint for new physics beyond the SM. Experimental detection of the LFV phenomenon can provide an evidence of new physics.

Searching for new physics beyond the SM is one of the most important issues of current particle physics. The CERN Large Hadron Collider (LHC) can generate very massive new particles and will essentially enlarge the possibilities of testing for new physics effects. However, the LHC is a hadronic machine and precision measurements will be quite hard to undertake there. Also, the existence of large backgrounds at the LHC may hinder discoveries of new physical phenomena already possible at the energies that this accelerator will achieve. Thus, the next generation e^+e^- International Linear Collider (ILC) with the center of mass (c.m.) energy $\sqrt{s} = 0.5\text{--}1$ TeV and the typical integrated luminosity $\mathcal{L}_{\text{int}} = 0.5\text{--}1$ ab $^{-1}$ is currently being designed [3,4]. Because of its rather clean environment and high luminosity, the ILC will allow unambiguous precision measurements. In such a collider, in addition to e^+e^- collision, one can also realize $\gamma\gamma$ collision with the photon beams generated by the backward

Compton scattering of incident electron- and laser-beams. The $\gamma\gamma$ collision offers a unique opportunity to explore new physics effects through production mechanisms which are not accessible in leptonic or hadronic machines [5].

It is well known that many popular models beyond the SM predict the presence of new particles, such as new gauge bosons, new fermions, and new scalars, which can generally enhance the branching ratios for some LFV decay processes, for instance $l_i \rightarrow l_j\gamma$, $l_i \rightarrow l_j l_k \bar{l}_l$, and $Z \rightarrow l_i \bar{l}_j$. The upper experimental limits for some of these LFV decay processes can give severe constraints on the corresponding new physics models. Nevertheless, it is possible that the LFV signals may be observed at the ILC. This fact has led to lot of work to study the LFV processes $e^+e^- \rightarrow l_i\bar{l}_j$ and $\gamma\gamma \rightarrow l_i\bar{l}_j$ in the framework of specific models beyond the SM [6,7] and in a model-independent manner [8]. Taking into account the constraints of the upper experimental limits for some LFV decay processes on the littlest Higgs model with T parity (called the LHT model) [9], in this paper, we will focus our attention on its contributions to the LFV processes $e^+e^- \rightarrow l_i\bar{l}_j$ and $\gamma\gamma \rightarrow l_i\bar{l}_j$ ($i \neq j$). We calculate the production cross sections of these processes induced by the LHT model and discuss the possibility of detecting the LFV signals of the LHT model via e^+e^- and $\gamma\gamma$ collisions at the ILC.

The rest of this paper is organized as follows. In Sec. II, we briefly review the essential features of the LHT model, which are related our calculation. A simple discussion about the constraints on the LHT model from some LFV decay processes is also given in this section. The production cross sections of the LFV processes $e^+e^- \rightarrow l_i\bar{l}_j$ and $\gamma\gamma \rightarrow l_i\bar{l}_j$ are calculated in Secs. III and IV, respectively. Finally, the conclusions are given in Sec. V.

II. THE ESSENTIAL FEATURES OF THE LHT MODEL

Little Higgs theory [10] was proposed as an alternative solution to the hierarchy problem of the SM, which

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provides a possible kind of electroweak symmetry breaking (EWSB) mechanism accomplished by a naturally light Higgs boson. In order to make the littlest Higgs model consistent with electroweak precision tests and simultaneously having the new particles of this model in the reach of the LHC, a discrete symmetry, T parity, has been introduced, which forms the LHT model. The detailed description of the LHT model can be found for instance in Refs. [9,11,12], and here we just want to briefly review its essential features, which are related to our calculation.

The LHT model is based on an $SU(5)/SO(5)$ global symmetry breaking pattern. A subgroup $[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2$ of the $SU(5)$ global symmetry is gauged, and at the scale f it is broken into the SM electroweak symmetry $SU(2)_L \times U(1)_Y$. T -parity exchanges the $[SU(2) \times U(1)]_1$ and $[SU(2) \times U(1)]_2$ gauge symmetries. The T -even combinations of the gauge fields are the SM electroweak gauge bosons W_μ^a and A_μ . The T -odd combinations are T -parity partners of the SM electroweak gauge bosons.

After taking into account EWSB, at the order of v^2/f^2 , the masses of the T -odd set of the $SU(2) \times U(1)$ gauge bosons are given as

$$\begin{aligned} M_{B_H} &= \frac{g_1 f}{\sqrt{5}} \left[1 - \frac{5v^2}{f^2} \right], \\ M_{Z_H} &\approx M_{W_H} = g_2 f \left[1 - \frac{v^2}{8f^2} \right], \end{aligned} \quad (1)$$

where $v = 246$ GeV is the electroweak scale and f is the scale parameter of the gauge symmetry breaking of the LHT model, g_1 and g_2 are the SM $U(1)_Y$ and $SU(2)_L$ gauge coupling constants, respectively.

A consistent implementation of T parity also requires the introduction of mirror fermions—one for each quark and lepton species. The masses of the T -odd (mirror) fermions can be written in a unified manner:

$$M_{F_i} = \sqrt{2} k_i f, \quad (2)$$

where k_i are the eigenvalues of the mass matrix k and their values are generally dependent on the fermion species i . These new fermions (T -odd quarks and T -odd leptons) have new flavor violating interactions with the SM fermions mediated by the new gauge bosons (B_H , W_H^\pm and Z_H) and at higher order by the triplet scalar Φ . These interactions are governed by new mixing matrices V_{Hd} and V_{Hl} for down-type quarks and charged leptons, respectively. The corresponding matrices in the up-type quarks (V_{Hu}) and neutrino ($V_{H\nu}$) sectors are obtained by means of the relations:

$$V_{Hu}^+ V_{Hd} = V_{CKM}, \quad V_{H\nu}^+ V_{Hl} = V_{PMNS}. \quad (3)$$

Where the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_{CKM} is defined through flavor mixing in the down-type quark sector, while the PMNS matrix V_{PMNS} is defined through neutrino mixing.

The Feynman rules of the LHT model have been studied in Ref. [12] and the corrected Feynman rules of Ref. [12] are given in Refs. [13,14]. To simplify our paper, we do not list them here.

From the above discussions, we can see that the flavor structure of the LHT model is much richer than the one of the SM, mainly due to the presence of three doublets of mirror quarks and leptons and their interactions with the ordinary quarks and leptons. Such new flavor changing interactions can induce that the LHT model generates contributions to some flavor changing processes. The contributions of the LHT model to the LFV decay processes have been extensively studied and compared with current experimental limits in the literature [14–16]. It has been shown that the LHT model can enhance the SM prediction values by several orders of magnitude and the experimental measurement data for some LFV decay processes can give severe constraints on the free parameters of the LHT model.

The new particles predicted by the LHT model can generate significant contributions to the LFV decay process $\mu \rightarrow 3e$ via box diagrams and the effective vertices $\gamma(Z)e\bar{e}$ generated by γ - and Z -penguins, while only via γ -penguin for the LFV decay process $\mu \rightarrow e\gamma$ [14,15]. In order to suppress the values of the branching ratios $\text{Br}(\mu \rightarrow e\gamma)$ and $\text{Br}(\mu \rightarrow 3e)$ predicted by the LHT model below the present experimental upper bounds, the relevant mixing matrix V_{Hl} must be rather hierarchical or mass splitting for the first and second T -odd lepton masses is very small. Reference [15] has shown that, for $f = 1$ TeV, the mass splitting $\Delta M = |M_{l_H^1} - M_{l_H^2}|$ must satisfy $\Delta M \leq 40$ GeV and 60 GeV for assuming $V_{Hl} = V_{PMNS}$ and $V_{Hl} = V_{CKM}$, respectively, in which $M_{l_H^i}$ is the mass of the i -th generation T -odd lepton. It should be noted that the value of ΔM increases as the scale parameter f increases. However, the present experimental upper bounds for $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$ cannot give significant constraints on the mass of the third T -odd lepton generation. Thus, in this paper, we will assume $M_{l_H^1} = M_{l_H^2} = M_2$ and $M_{l_H^3} = M_3$. In our following numerical estimation, we take the mass parameters M_2 and M_3 , and the scale parameter f as free parameters.

III. THE LFV PROCESSES $e^+e^- \rightarrow l_i\bar{l}_j$ IN THE LHT MODEL

From above discussions we can see that the LHT model can contribute to the LFV processes $e^+e^- \rightarrow l_i\bar{l}_j$ ($i \neq j$) via the effective vertices $\gamma(Z)l_i\bar{l}_j$ and the box diagrams. The Feynman diagrams for the LFV processes $e^+e^- \rightarrow l_i\bar{l}_j$ are depicted in Fig. 1, in which $\omega^{\pm,0}$ and η represent the would-be Goldstone bosons arising from the LHT model. It is obvious that the contributions of the LHT model to this process mainly come from the effective vertices $\gamma(Z)l_i\bar{l}_j$ [Fig. 1(a)–1(k)], which are related the LFV processes

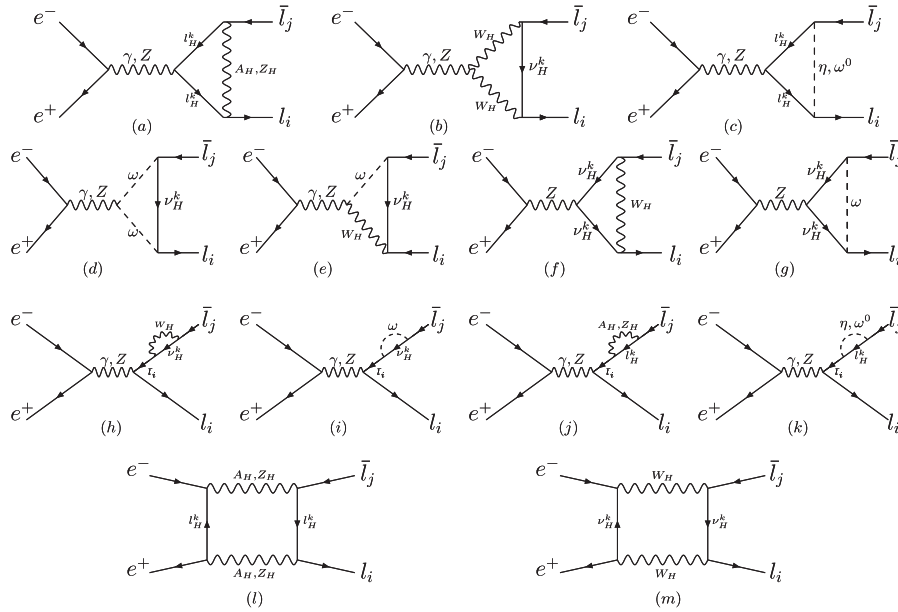


FIG. 1. The Feynman diagrams for the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$ in the LHT model.

$\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$. Thus, the production cross sections of the processes $e^+e^- \rightarrow l_i \bar{l}_j$ should also be constrained by the present experimental upper bounds of these processes. However, as shown in Refs. [14,15], as long as $\Delta M \approx 0$, the LHT model can satisfy the constraints from $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$. The constraints on other free parameters, such as the mass M_3 of the third T -odd lepton generation, the mixing matrix elements $(V_{HI})_{ij}$, etc. are very weak. So, it is possible that the LHT model can give sizable cross sections for the processes $e^+e^- \rightarrow l_i \bar{l}_j$. Our calculation has shown that it is indeed this case.

The one-loop calculation can be carried out by summing all of these one-loop diagrams and the results will be finite and gauge invariant. Each loop diagram is composed of some scalar loop functions, which are calculated by using *LoopTools* [17]. In the following sections, we will use the tt Hooft-Feynman gauge to calculate their production cross sections. Because the calculation of the loop diagrams is too tedious and the analytical expressions are lengthy, we will not present them here.

It is obvious that, except for the SM input parameters $\alpha = 1/128.8$, $S_W^2 = 0.2315$, and $M_Z = 91.187$ GeV [18], the production cross sections σ_{ij} for the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$ are dependent on the model dependent parameters $(V_{HI})_{ij}$, f , and the T -odd leptons' masses. The matrix elements $(V_{HI})_{ij}$ can be determined through $V_{HI} = V_{H\nu} V_{\text{PMNS}}$. To avoid any additional parameters introduced and to simplify our calculations, we take $V_{HI} = V_{\text{PMNS}}$, which means that $V_{H\nu} = I$ and the T -odd leptons have no effects on the flavor violating observable in the neutrino sector [14,19]. Certainly, this is a very limited scenario. However, in order to satisfy the constraints from $\mu \rightarrow 3e$ and $\mu \rightarrow e\gamma$, the mixing matrix V_{HI} must be rather

hierarchical or the first and second T -odd lepton masses are quasidegenerate. In this paper, we take $M_{l_H^1} = M_{l_H^2}$ and $V_{HI} = V_{\text{PMNS}}$ might be suitable. For the PMNS matrix V_{PMNS} , we take the standard parametrization form with parameters given by the neutrino experiments [20]. Since there are not constraints on the PMNS phases, we will take the Dirac phase to be equal to the CKM phase and set the two Majorana phases to zero in our numerical estimations.

Our numerical results are summarized in Fig. 2, in which we plot the production cross sections σ_{ij} as functions of the mass parameter M_3 for $M_2 = 400$ GeV and three values of the scale parameter f . Figures 2(a)–2(c) are corresponding to the LFV processes $e^+e^- \rightarrow e\bar{\tau}$, $e^+e^- \rightarrow \mu\bar{\tau}$ and $e^+e^- \rightarrow e\bar{\mu}$, respectively. In these figures, we have taken the c.m. energy $\sqrt{s} = 500$ GeV. One can see from Fig. 2 that the contributions of the LHT model to the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$ ($i \neq j$) increase as the third family T -odd lepton mass M_3 increases and the scale parameter f decreases. In most of the parameter space, the production cross sections for the states $e\bar{\tau}$, $\mu\bar{\tau}$ and $e\bar{\mu}$ are approximately at the same order of magnitude. For $\sqrt{s} = 500$ GeV, $M_2 = 400$ GeV, $450 \text{ GeV} \leq M_3 \leq 1000$ GeV and $500 \text{ GeV} \leq f \leq 1500$ GeV, there are $0.08 \text{ fb} \leq \sigma_{e\bar{\tau}} \leq 4.5 \text{ fb}$, $0.1 \text{ fb} \leq \sigma_{\mu\bar{\tau}} \leq 6.4 \text{ fb}$ and $0.09 \text{ fb} \leq \sigma_{e\bar{\mu}} \leq 5.3 \text{ fb}$, respectively. If we assume the integrated luminosity $\mathcal{L}_{\text{int}} = 500 \text{ fb}^{-1}$, there will be several tens and up to thousands of $l_i \bar{l}_j$ events to be generated in the future ILC experiments.

To see the effect of the first- and second-generation T -odd lepton masses on the production cross sections σ_{ij} , in Fig. 3, we plot σ_{ij} as functions of the mass parameter M_2 for $f = 1$ TeV and $M_3 = 1000$ GeV. From Fig. 3

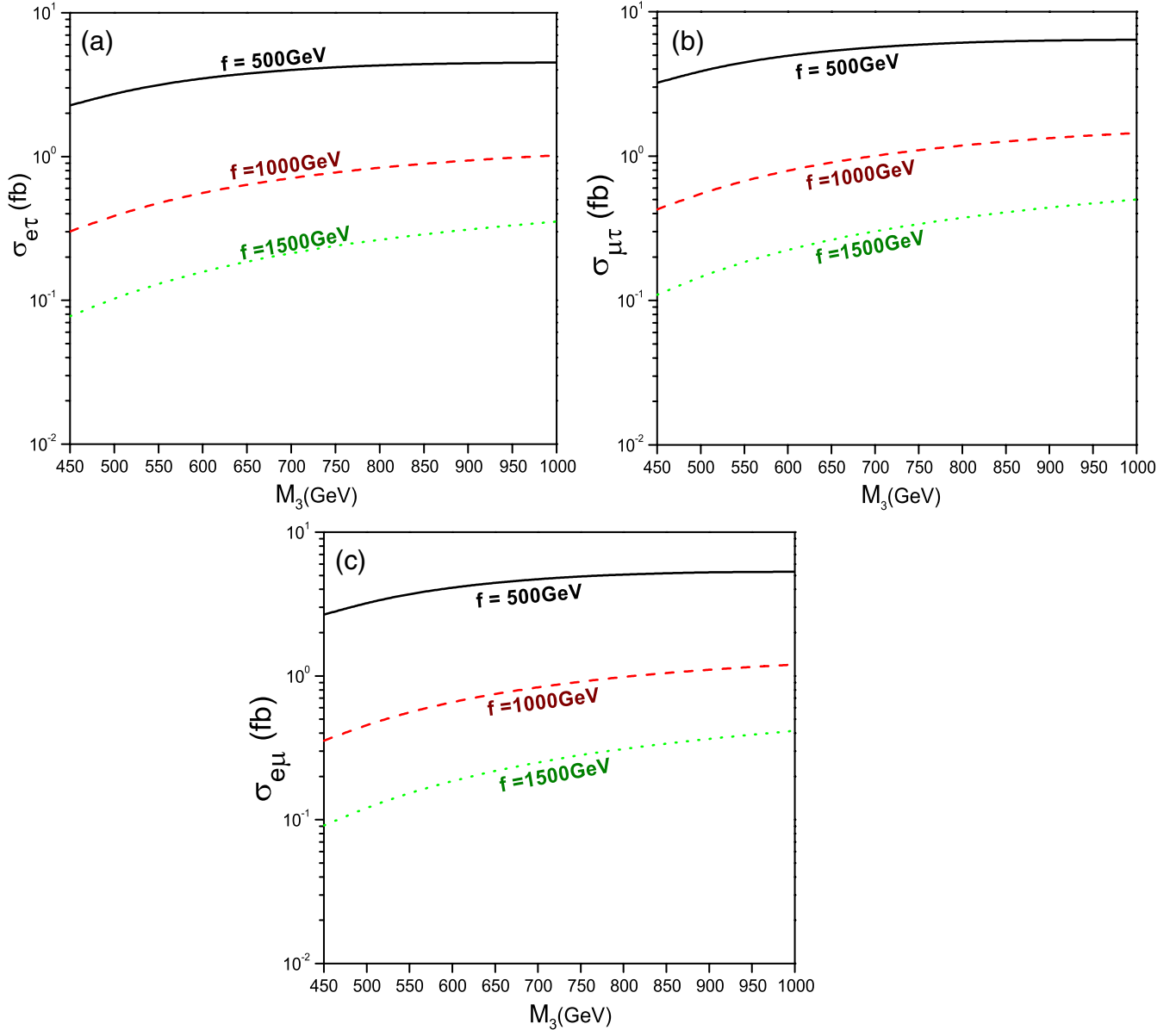


FIG. 2 (color online). The production cross sections σ_{ij} for the processes $e^+e^- \rightarrow l_i \bar{l}_j$ as function of the mass parameter M_3 for $M_2 = 400$ GeV and three values of the scale parameter f .

one can see that the contributions of the LHT model to the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$ also increase as the mass parameter M_2 . For $M_3 = 1000$ GeV, $f = 1$ TeV, $450 \text{ GeV} \leq M_2 \leq 900 \text{ GeV}$ and $\sqrt{s} = 500$ GeV, they are $1.1 \text{ fb} \leq \sigma_{e\bar{\tau}} \leq 3.3 \text{ fb}$, $1.5 \text{ fb} \leq \sigma_{\mu\bar{\tau}} \leq 4.7 \text{ fb}$ and $1.2 \text{ fb} \leq \sigma_{e\bar{\mu}} \leq 3.9 \text{ fb}$, respectively.

It is well known that, at the ILC, the LFV production processes can provide extremely clear signatures and are experimentally interesting. For the three LFV processes $e^+e^- \rightarrow e\bar{\tau}$, $e^+e^- \rightarrow \mu\bar{\tau}$ and $e^+e^- \rightarrow e\bar{\mu}$, the final leptons always emerge back to back and carrying a constant energy which is one-half of the c.m. energy \sqrt{s} . The last process is the best one and almost free of the SM backgrounds. For the first- and second- production processes,

we can assume the lepton tau decay $\bar{\tau} \rightarrow \bar{\mu}\nu_\mu\bar{\nu}_\tau$ and $\bar{\tau} \rightarrow \bar{e}\nu_e\bar{\nu}_\tau$, which give rise to the signal events with opposite-sign and different-flavor leptons and missing energy ($e\bar{\mu} + E$ and $\mu\bar{e} + E$). Although this kind of LFV signal is quite spectacular, it is not free of the SM backgrounds. For example, the leading backgrounds of the signal $e\bar{\mu} + E$ mainly come from the SM processes $e^+e^- \rightarrow e\bar{\mu}\nu_\mu\bar{\nu}_e$ and $e^+e^- \rightarrow \tau^+\tau^- \rightarrow e\bar{\mu}\nu_\mu\bar{\nu}_e\nu_\tau\bar{\nu}_\tau$, which have been discussed in Ref. [8]. They have shown that, with suitable cuts, the SM backgrounds can be largely suppressed. Thus, it is possible to observe the LFV signal of the LHT model in the future ILC experiments. Certainly, detailed study of the relevant backgrounds is needed, which is beyond the scope of this paper.

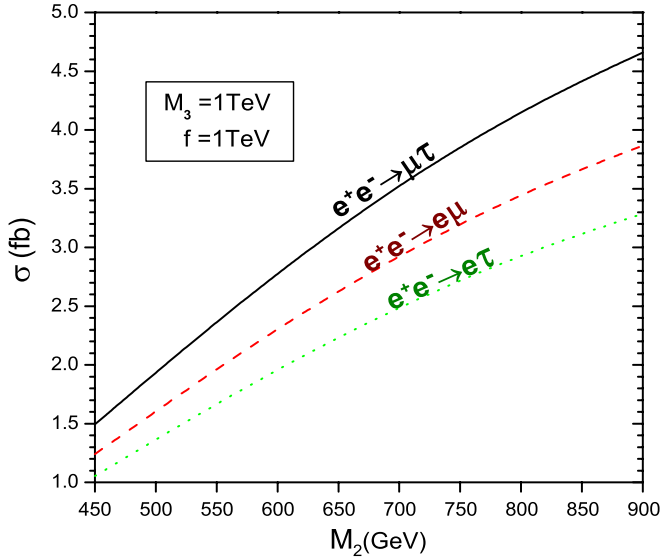


FIG. 3 (color online). The production cross sections σ_{ij} for the processes $e^+e^- \rightarrow l_i \bar{l}_j$ as function of the mass parameter M_2 for $f = 1$ TeV and $M_3 = 1000$ GeV.

IV. THE LFV PROCESSES $\gamma\gamma \rightarrow l_i \bar{l}_j$ IN THE LHT MODEL

It is well known that the ILC could offer the possibility of working in the $\gamma\gamma$ or $e\gamma$ collision thus realizing a very high energy photon collider [5]. For some production processes, their cross sections at the $\gamma\gamma$ collider might be larger than the corresponding e^+e^- ones; this collider will reveal crucial information about these production processes. To see whether the LHT model can give significant contributions to the LFV final states $l_i \bar{l}_j$ ($i \neq j$) via $\gamma\gamma$ collision, we will consider the LFV processes $\gamma\gamma \rightarrow l_i \bar{l}_j$ in this section.

From the discussions given in Sec. II, we can see that the LHT model can only induce the LFV processes $\gamma\gamma \rightarrow l_i \bar{l}_j$ at loop level. The relevant Feynman diagrams are shown in Fig. 4. The Feynman diagrams created by exchanging the initial photons, which are not shown in Fig. 4, are also involved in our calculations.

We use θ to denote the scattering angle between one of the photons and one of the final leptons. Then, in the center of mass (c.m.) system, we express all the four-momenta of the initial and final particles by means of the $\gamma\gamma$ c.m. energy $\sqrt{\hat{s}}$ and the scattering angle θ . The four-momentum components (E, p_x, p_y, p_z) of final particles l_i and \bar{l}_j can be written as

$$p_1 = (E_{l_i}, \sqrt{E_{l_i}^2 - m_{l_i}^2} \sin\theta, 0, \sqrt{E_{l_i}^2 - m_{l_i}^2} \cos\theta), \quad (4)$$

$$p_2 = (E_{\bar{l}_j}, -\sqrt{E_{\bar{l}_j}^2 - m_{\bar{l}_j}^2} \sin\theta, 0, -\sqrt{E_{\bar{l}_j}^2 - m_{\bar{l}_j}^2} \cos\theta), \quad (5)$$

where $E_{l_i} = E_{\bar{l}_j} = \sqrt{\hat{s}}/2$. The Mandelstam variables are defined as

$$\begin{aligned} \hat{s} &= (k_1 + k_2)^2 = (p_1 + p_2)^2, \\ \hat{t} &= (k_1 - p_1)^2 = (k_2 - p_2)^2, \\ \hat{u} &= (k_1 - p_2)^2 = (k_2 - p_1)^2, \end{aligned} \quad (6)$$

where k_1 and k_2 are the 4-momenta of the initial photons, which can be written as $k_1 = (\sqrt{\hat{s}}/2, 0, 0, \sqrt{\hat{s}}/2)$ and $k_2 = (\sqrt{\hat{s}}/2, 0, 0, -\sqrt{\hat{s}}/2)$. With the above definitions, we can give the renormalized amplitudes of the LFV processes $\gamma\gamma \rightarrow l_i \bar{l}_j$. To simplify our paper, we do not give their explicit expressions here. The cross section for the LFV process $\gamma\gamma \rightarrow l_i \bar{l}_j$ can be generally expressed in the form

$$\hat{\sigma}(\hat{s}) = \frac{1}{16\pi\hat{s}^2} \int_{\hat{t}^-}^{\hat{t}^+} d\hat{t} \sum_{\text{spin}} |\mathcal{M}|^2, \quad (7)$$

where $\hat{t}^\pm = \frac{1}{2}[(m_{l_i}^2 + m_{\bar{l}_j}^2 - \hat{s}) \pm \sqrt{(m_{l_i}^2 + m_{\bar{l}_j}^2 - \hat{s})^2 - 4m_{l_i}^2 m_{\bar{l}_j}^2}]$, and the bar over summation means to take the average over the initial polarizations of the photons.

At the ILC, the effective cross section of the subprocess $\gamma\gamma \rightarrow l_i \bar{l}_j$ can be written as

$$\sigma(s) = \int_{E_0/\sqrt{s}}^{x_{\max}} dz \frac{d\mathcal{L}_{\gamma\gamma}}{dz} \hat{\sigma}_{\gamma\gamma \rightarrow l_i \bar{l}_j}(\hat{s} = z^2 s), \quad (8)$$

where $E_0 = m_{l_i} + m_{\bar{l}_j}$, and $\sqrt{s}(\sqrt{\hat{s}})$ is the e^+e^- ($\gamma\gamma$) c.m. energy; $d\mathcal{L}_{\gamma\gamma}/dz$ is the photon-beam luminosity distribution, which is defined as

$$\frac{d\mathcal{L}_{\gamma\gamma}}{dz} = 2z \int_{z^2/x_{\max}}^{x_{\max}} \frac{dx}{x} F_{\gamma/e}(x) F_{\gamma/e}(z^2/x). \quad (9)$$

For the initial unpolarized electron and laser-photon beams, the energy spectrum of the backscattered photon is given by [21]

$$F_{\gamma/e}(x) = \frac{1}{D(\xi)} \left[1 - x + \frac{1}{1-x} - \frac{4x}{\xi(1-x)} + \frac{4x^2}{\xi^2(1-x)^2} \right] \quad (10)$$

with

$$D(\xi) = \left(1 - \frac{4}{\xi} - \frac{8}{\xi^2} \right) \ln(1 + \xi) + \frac{1}{2} + \frac{8}{\xi} - \frac{1}{2(1 + \xi)^2}, \quad (11)$$

where $\xi = 4E_0\omega_0/m_e^2$, m_e and E_0 are the incident electron mass and energy, respectively. ω_0 is the laser-photon energy; x is the fraction of the energy of the incident electron carried by the backscattered photon. In order to spoil the creation of e^+e^- pair by the interaction of the incident and backscattered photons, in our calculation, we require $\omega_0 x_{\max} \leq m_e^2/E_e$, which implies $\xi \leq 4.8$. For the choice $\xi = 4.8$, there are $x_{\max} \simeq 0.83$ and $D(\xi) = 1.8$.

Similarly with Sec. III, we also assume $V_{H\nu} = I$ and that the masses of the first-generation T -odd fermions are equal to those of the second-generation T -odd fermions. In this

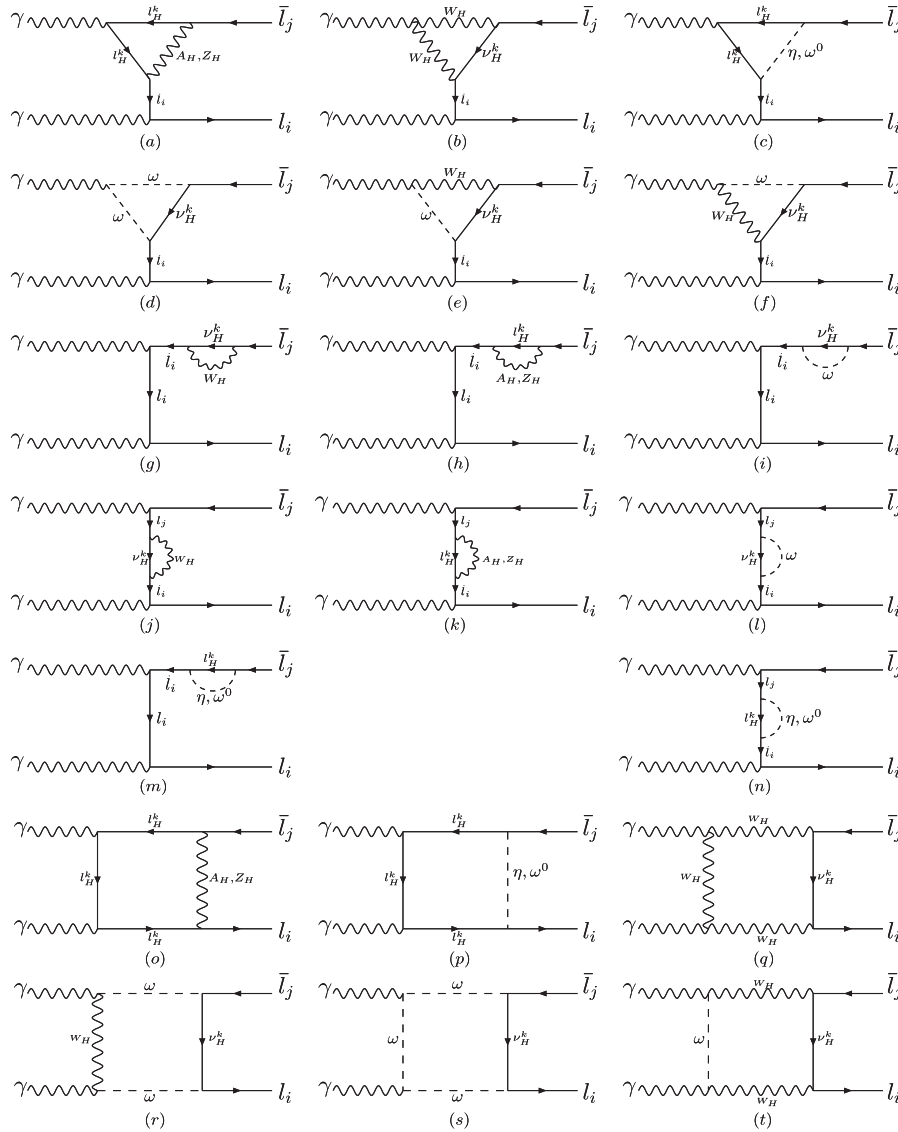


FIG. 4. The Feynman diagrams for the LFV processes $\gamma\gamma \rightarrow l_i \bar{l}_j$ in the LHT model. The diagrams obtained by exchanging the initial photons are not shown here.

case, our numerical results are summarized in Figs. 5 and 6. Figure 5 plots the effective production cross sections σ_{ij} as functions of the mass parameter M_3 for $M_2 = 400$ GeV and three values of the scale parameter f , while Fig. 6 plots the cross sections σ_{ij} as functions of M_2 for $f = 500$ GeV and $M_3 = 1000$ GeV. One can see from these figures that, in most of the parameter space, the contributions of the LHT model to the LFV processes $e^+e^- \rightarrow \gamma\gamma \rightarrow l_i \bar{l}_j$ are smaller than those for the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$. With the increasing of the mass parameter M_i and the decreasing of the scale parameter f , the effective cross sections of the LFV processes $\gamma\gamma \rightarrow l_i \bar{l}_j$ become larger. Such a behavior is similar to that for the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$. For $\sqrt{s} = 500$ GeV, $M_2 = 400$ GeV, $450 \text{ GeV} \leq M_3 \leq 1000$ GeV and $500 \text{ GeV} \leq f \leq 1500$ GeV, there are $6 \times 10^{-4} \text{ fb} \leq \sigma_{e\bar{\tau}} \leq 3 \times 10^{-2} \text{ fb}$,

$8 \times 10^{-4} \text{ fb} \leq \sigma_{\mu\bar{\tau}} \leq 5 \times 10^{-2} \text{ fb}$ and $7 \times 10^{-4} \text{ fb} \leq \sigma_{e\bar{\mu}} \leq 4 \times 10^{-2} \text{ fb}$, respectively. There will be several tens of $l_i \bar{l}_j$ events to be generated in the future ILC experiment with $\mathcal{L}_{\text{int}} = 500 \text{ fb}^{-1}$ and $\sqrt{s} = 500$ GeV.

Similar with the LFV process $e^+e^- \rightarrow e\bar{\mu}$, the LFV process $\gamma\gamma \rightarrow e\bar{\mu}$ is almost free of the SM background. For the LFV processes $\gamma\gamma \rightarrow e\bar{\tau}$ and $\gamma\gamma \rightarrow \mu\bar{\tau}$, the SM backgrounds mainly come from the processes $\gamma\gamma \rightarrow \tau^+\tau^-$ and $\gamma\gamma \rightarrow WW$ with the lepton τ and the electroweak gauge boson W leptonic decaying. It has been shown that, by applying appropriate kinematical cuts, the backgrounds can be significantly suppressed and the ratio of signal to background would be enhanced [6]. Thus, the LFV signatures of the LHT model might have a chance of being observed via the subprocesses $\gamma\gamma \rightarrow l_i \bar{l}_j$ in the future ILC experiments.

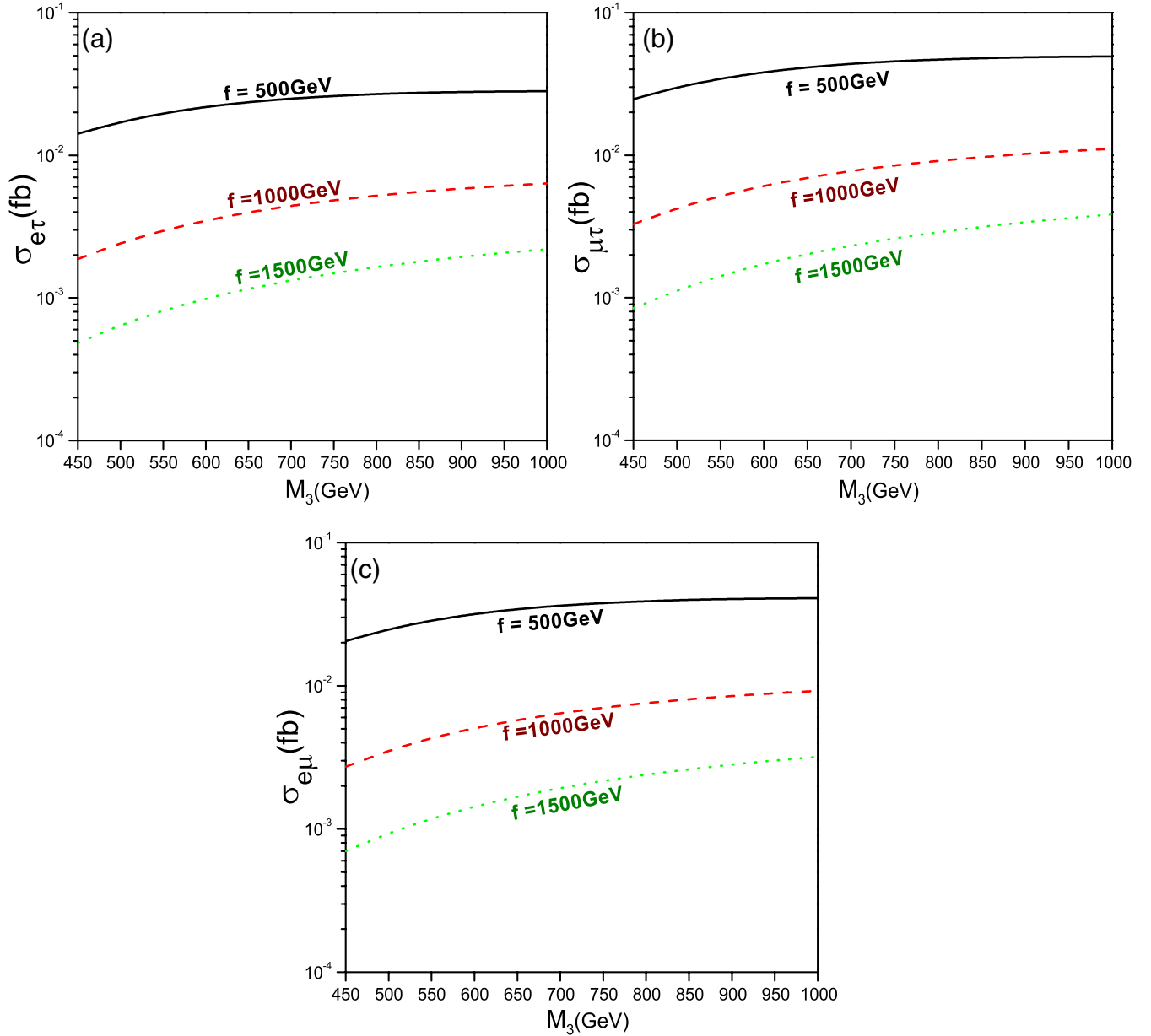


FIG. 5 (color online). The production cross sections σ_{ij} for the subprocesses $\gamma\gamma \rightarrow l_i \bar{l}_j$ as functions of the mass parameter M_3 for $M_2 = 400$ GeV and three values of the scale parameter f .

V. CONCLUSIONS AND DISCUSSIONS

The LHT model is one of the attractive little Higgs models. To simultaneously implement T parity, the LHT model introduces new mirror fermions (T -odd quarks and T -odd leptons). The flavor mixing in the mirror fermion sector gives rise to a new source of flavor violation, which might generate significant contributions to some flavor violation processes.

The evidence for the neutrino masses and flavor mixing, which can be seen as the first experimental clue of new physics beyond the SM, implies the nonconservation of the lepton flavor symmetry. Thus, the LFV processes related

charged leptons are expected, which are very sensitive to new physics beyond the SM. Taking into account the constraints on the free parameters of the LHT model from the rare decay processes $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$, in this paper, we have considered the contributions of the LHT model to the LFV processes $e^+e^- \rightarrow l_i \bar{l}_j$ and $\gamma\gamma \rightarrow l_i \bar{l}_j$ ($i \neq j$), which are induced at the one-loop level and will be of interest for the future ILC experiments. We find that, in wide range of the parameter space, the production cross section of the LFV process $e^+e^- \rightarrow l_i \bar{l}_j$ can reach several fb , and that of the LFV process $\gamma\gamma \rightarrow l_i \bar{l}_j$ can reach the order of magnitude of 10^{-2} fb . This means that there will

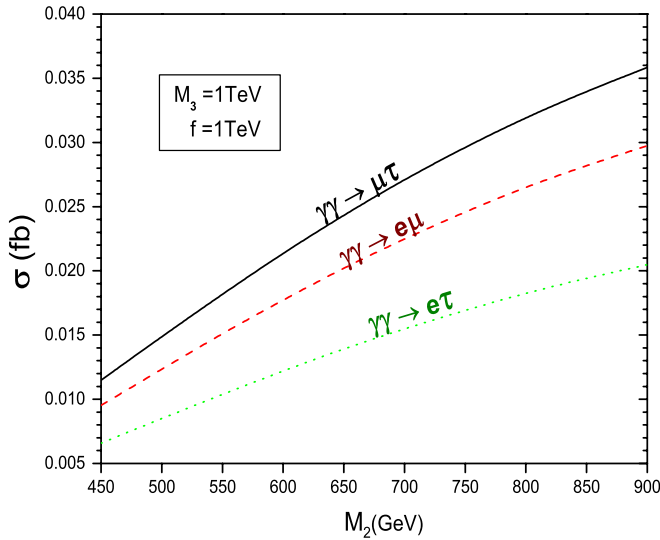


FIG. 6 (color online). The production cross sections σ_{ij} for the subprocesses $\gamma\gamma \rightarrow l_i \bar{l}_j$ as functions of the mass parameter M_2 for $f = 1$ TeV and $M_3 = 1000$ GeV.

be several and up to thousands of $l_i \bar{l}_j$ events to be generated each year for the designed luminosity of $\mathcal{L}_{\text{int}} = 500 \text{ fb}^{-1}$ at the ILC. Since the production rate of these LFV processes predicted by the SM is almost negligible, the observation of such $l_i \bar{l}_j$ events would be a robust evidence of the LHT model. Therefore, these LFV processes may serve as a sensitive probe of the LHT model.

An important tool of the ILC is the use of the polarized beams. One expects that a high polarization degree between 80% and 90% can be reached [3,4]. Beam polarization is not only useful for a possible reduction of the background, but might also serve as a possible tool to disentangle different contributions to the signal and to directly analyze the interaction structure of new physics models. In our calculation, we have not considered the polarization of the incident electron. Certainly, if we consider this case, our numerical results will be changed. Furthermore, we can also assume $V_{HI} = V_{\text{CKM}}$, which makes the values of the production cross sections of the LFV processes

$e^+ e^- \rightarrow l_i \bar{l}_j$ different from those for assuming $V_{HI} = V_{\text{PMNS}}$. However, our physical conclusions are not changed.

It is obvious that the production cross sections of the LFV processes $e^+ e^- \rightarrow l_i \bar{l}_j$ and $\gamma\gamma \rightarrow l_i \bar{l}_j$ are dependent on the factor $\sum_{\alpha} V_{HI}^{\alpha i*} V_{HI}^{\alpha j}$ with $\alpha = 1, 2, 3$. For assuming $V_{HI} = V_{\text{PMNS}}$, the PMNS phases should have effects on these cross sections. In our numerical estimations, we have fixed the values of the PMNS phases. If we vary these values, the numerical results for the cross sections $\sigma_{e\bar{\tau}}$, $\sigma_{\mu\bar{\tau}}$ and $\sigma_{e\bar{\mu}}$ are also changed. However, these variations are not significantly large. Our physical conclusions are not changed.

The T -odd quarks predicted by the LHT model also have new flavor violating interactions with the SM fermions mediated by the new gauge bosons and at higher order by the triplet scalar. These interactions are governed by the new mixing matrix V_{Hd} for down-quarks. So the T -odd quark sector can also generate significant contributions to some flavor changing processes [12,13,22,23]. For example, Ref. [24] has shown that it is possible to test the signatures of the LHT model at the ILC and LHC experiments via the flavor changing processes $e^+ e^- \rightarrow \bar{t}c$, $\gamma\gamma \rightarrow \bar{t}c$ and $pp \rightarrow \bar{t}c$.

The T -odd particles predicted by the LHT model can only be produced in pairs and give the direct signals at the LHC, which have close resemblance with those of supersymmetry with conserved R parity or universal extra dimensions with KK parity. The possibility of observing the T -odd leptons at the LHC has been studied in Ref. [25]. If the T -odd leptons are found at the LHC, we expect that our work will be helpful to determine their masses and couplings with high accuracy at the ILC.

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