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Less space for a new family of fermions

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We investigate the experimentally allowed parameter space of an extension of the standard model (SM3) by one additional family of fermions. Therefore we extend our previous study of the Cabibbo-Kobayashi-Maskawa (CKM)–like mixing constraints of a fourth generation of quarks. In addition to the bounds from tree-level determinations of the 3×3 CKM elements and flavor-changing neutral currents processes (K, D, B_d, B_s mixing and the decay $b \to s\gamma$) we also investigate the electroweak S, T, U parameters, the angle γ of the unitarity triangle, and the rare decay $B_s \to \mu^+\mu^-$. Moreover we improve our treatment of the QCD corrections compared to our previous analysis. We also take leptonic contributions into account, but we neglect the mixing among leptons. As a result we find that typically small mixing with the fourth family is favored, but still some sizeable deviations from the SM3 results are not yet excluded. The minimal possible value of V_{tb} is 0.93. Also very large CP-violating effects in B_s mixing seem to be impossible within an extension of the SM3 that consists of an additional fermion family alone. We find a delicate interplay of electroweak and flavor observables, which strongly suggests that a separate treatment of the two sectors is not feasible. In particular we show that the inclusion of the full CKM dependence of the S and T parameters in principle allows the existence of a degenerate fourth generation of quarks.

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I. INTRODUCTION

Increasing the number of fermion generations (see [1] for a review and [2] for an update) is probably the most obvious extension of the usual standard model with three generations (SM3). Although popular in the 1980s, such a possibility was discarded for a long time. Recently these models (SM4) celebrated a kind of resurrection. Partly, this was due to the fact that a fourth generation is not necessarily in conflict with electroweak precision observables [3–14].

Besides being a straightforward extension of the SM3, an increase of the number of fermion generations leads also to several desired effects:

(i) The authors of [7,9,11–13,15] have shown that a fourth generation softens the current low Higgs mass bounds from electroweak precision observables, see, e.g., [16], by allowing considerably higher values for the Higgs mass.

(ii) It might solve problems related to baryogenesis: An additional particle family could lead to a sizeable increase of the measure of CP violation, see [17,18]. Moreover, such an extension of the SM would increase the strength of the phase transition, see [19–21].

(iii) The gauge couplings can in principle be unified without invoking SUSY [22].

(iv) New heavy fermions lead to new interesting effects due to their large Yukawa couplings, see, e.g., [23,24]. Moreover dynamical electroweak symmetry breaking might be triggered by these heavy new fermions [25–35]. This mechanism can also be incorporated in models with warped extra dimensions, as done in [36,37].

There are also some modest experimental deviations that could be explained by the existence of a fourth generation:

(i) A new family might cure certain problems in flavor physics (CP violation in B_s mixing, $K - \pi$ puzzle, ϵ_k anomaly,...) see, e.g., [38–42] for some recent work and, e.g., [43,44] for some early work on 4th generation effects on flavor physics.

(ii) Investigations of lepton universality show a value of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) element $V_{e4} \neq 0$ at the 2.5σ level [45].

For more arguments in favor of a fourth generation see, e.g., [2] and also [46–49]. We conclude the list by repeating our statement from [50]: *In view of the (re)start of the LHC, it is important not to exclude any possibility for new physics scenarios simply due to prejudices*. Direct search strategies for heavy quarks at the LHC are worked out, e.g., in [31,51–57]. Signatures and consequences for collider physics, such as the modification of production rates, have been studied, e.g., in [31,58–61].

In this work we extend our analysis in [50], where we performed an exploratory study of the allowed parameter

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range for the Cabibbo-Kobayashi-Maskawa (CKM)–like mixing of hypothetical quarks of a fourth generation. Adding one generation of quarks results in several new parameters. In particular, we have the new masses $m_{b'}$ and $m_{t'}$, and nine parameters (six angles and three phases) in the 4×4 CKM matrix (compared to three angles and one phase in the SM3). Following our previous strategy, we consecutively add bounds on the CKM structure of the SM4 and perform a scan though the parameter space of the model to identify the allowed regions; while this treatment is insufficient to fit for the central values or standard deviation of the model parameters, it gives a very reasonable idea of the experimentally possible parameter space allowing for statements on the size of effects of the model on particularly interesting flavor observables. Apart from the unitarity of the 4×4 matrix and the direct bounds on the quark masses, the most important input comes from direct measurements of the absolute values of CKM matrix elements, e.g., from β decay. In [50] the next step was the inclusion of flavor observables sensitive to flavor-changing neutral currents (FCNC), mediated, i.e., by box or penguin diagrams. This led to some surprising results regarding the possible size of the quark mixing with the fourth generation quark, as rather large values for the mixing angle s_{34} could not be excluded.

However, in [62] Chanowitz found that the parameter sets, which we gave as an example for large mixing with the fourth generation, are excluded by electroweak precision constraints, in particular, by the oblique corrections [63]. Moreover, Chanowitz performed the whole electroweak fit for four different values of the mass of the t' quark. Here, some assumptions were used: (i) the lepton masses are fixed to $m_{l4} = 145$ GeV and $m_{\nu_4} = 100$ GeV, (ii) lepton mixing is not included, (iii) the mass difference of the heavy quark doublet is fixed to $m_{t'} - m_{b'} = 55$ GeV, (iv) only mixing between the third and fourth family was included. Assumptions (iii) and (iv) were also tested in [62].

Therefore, we supplement the analysis of the flavor sector by the S, T, and U parameters; also the lepton masses of the fourth generation have to be taken into account. For the present work we assume that the neutrinos have the Dirac character and neglect the possible mixing of the fourth neutrino in the lepton sector. Moreover, we extend the set of our FCNC observables to include also $B_s \to \mu^+\mu^-$ and we improve the simplified treatment of the decay $b \to s\gamma$ by using the full leading logarithmic result. Concerning the tree-level determination of the CKM elements we include now also the experimental results for the angle γ of the unitarity triangle, which gives a direct constraint on CKM phases. Similar studies have been recently performed, e.g., in [64–67]

In Sec. II we present all experimental constraints we use in our analysis. We start with the parametrization of $V_{\rm CKM4}$ in Sec. II A, next we discuss briefly tree-level determinations of CKM elements and direct mass limits. The electroweak parameters S, T, and U will be investigated before reviewing the FCNC constraints. We end Sec. II with the allowed regions for deviations of the SM4 results from the SM3 values.

In Sec. III we determine the bounds on the parameters of the model. After explaining our general strategy in III A, we present the results for the different mixing angles of $V_{\rm CKM4}$, the new results for V_{cx} and V_{tx} ($x = d, s, b$), and allowed effects of a fourth generation in neutral meson mixing.

In Sec. IV we give a Wolfenstein-like expansion of the 4×4 CKM matrix. With the additional information from the electroweak sector, tighter constraints on the fourth generation quark mixing can be utilized leading to a simplified expansion. We conclude with Sec V.

II. CONSTRAINTS ON $V_{\rm CKM4}$

A. Parametrization of $V_{\rm CKM4}$

In the SM3 the mixing between quarks is described by the unitary 3×3 CKM matrix [68,69], which can be parametrized by three angles, θ_{12}, θ_{13}, and θ_{23} (θ_{ij} describes the strength of the mixing between the i-th and j-th family) and the CP-violating phase δ_{13}. The so-called standard parametrization of $V_{\rm CKM3}$ reads

$$V_{\rm CKM3} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \tag{2.1}$$

with

$$s_{ij} := \sin(\theta_{ij}) \quad \text{and} \quad c_{ij} := \cos(\theta_{ij}). \tag{2.2}$$

Extending the minimal standard model to include a fourth family of fermions (SM4) introduces 3 additional angles in the CKM matrix θ_{14}, θ_{24}, and θ_{34} and 2 additional CP-violating phases δ_{14} and δ_{24}. To determine the allowed range for these new parameters we use an exact parametrization of the 4×4 CKM matrix. We have chosen the one suggested by Botella and Chau [70],[1] Fritzsch and Plankl [71],[2] and also by Harari and Leurer [72].

[1] In the published paper of Botella and Chau there is a typo in the element V_{td}: in the last term of V_{td} the factor s_y has to be replaced by c_y.

[2] In the published paper of Fritzsch and Plankl there is a typo in the element V_{cb}: the factor c_{23} has to be replaced by the factor s_{23}.

$$V_{\rm CKM4} = \begin{pmatrix} c_{12}c_{13}c_{14} & c_{13}c_{14}s_{12} & c_{14}s_{13}e^{-i\delta_{13}} & s_{14}e^{-i\delta_{14}} \\ -c_{23}c_{24}s_{12} - c_{12}c_{24}s_{13}s_{23}e^{i\delta_{13}} & c_{12}c_{23}c_{24} - c_{24}s_{12}s_{13}s_{23}e^{i\delta_{13}} & c_{13}c_{24}s_{23} & c_{14}s_{24}e^{-i\delta_{24}} \\ -c_{12}c_{13}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & -c_{13}s_{12}s_{14}s_{24}e^{i(\delta_{14}-\delta_{24})} & -s_{13}s_{14}s_{24}e^{-i(\delta_{13}+\delta_{24}-\delta_{14})} & \\ -c_{12}c_{23}c_{34}s_{13}e^{i\delta_{13}} + c_{34}s_{12}s_{23} & -c_{12}c_{34}s_{23} - c_{23}c_{34}s_{12}s_{13}e^{i\delta_{13}} & c_{13}c_{23}c_{34} & c_{14}c_{24}s_{34} \\ -c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}} & -c_{12}c_{23}s_{24}s_{34}e^{i\delta_{24}} & -c_{13}s_{23}s_{24}s_{34}e^{i\delta_{24}} & \\ +c_{23}s_{12}s_{24}s_{34}e^{i\delta_{24}} & -c_{13}c_{24}s_{12}s_{14}s_{34}e^{i\delta_{14}} & -c_{24}s_{13}s_{14}s_{34}e^{i(\delta_{14}-\delta_{13})} & \\ +c_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & +s_{12}s_{13}s_{23}s_{24}s_{34}e^{i(\delta_{13}+\delta_{24})} & & \\ -c_{12}c_{13}c_{24}c_{34}s_{14}e^{i\delta_{14}} & -c_{12}c_{23}c_{34}s_{24}e^{i\delta_{24}} + c_{12}s_{23}s_{34} & -c_{13}c_{23}s_{34} & c_{14}c_{24}c_{34} \\ +c_{12}c_{23}s_{13}s_{34}e^{i\delta_{13}} & -c_{13}c_{24}c_{34}s_{12}s_{14}e^{i\delta_{14}} & -c_{13}c_{34}s_{23}s_{24}e^{i\delta_{24}} & \\ +c_{23}c_{34}s_{12}s_{24}e^{i\delta_{24}} - s_{12}s_{23}s_{34} & +c_{23}s_{12}s_{13}s_{34}e^{i\delta_{13}} & -c_{24}c_{34}s_{13}s_{14}e^{i(\delta_{14}-\delta_{13})} & \\ +c_{12}c_{34}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & +c_{34}s_{12}s_{13}s_{23}s_{24}e^{i(\delta_{13}+\delta_{24})} & & \end{pmatrix}. \quad (2.3)$$

For our strategy the explicit form of $V_{\rm CKM4}$ does not matter, it is only important that the parametrization is exact. Besides the nine parameters of $V_{\rm CKM4}$ we have also the masses of the fourth generation particles, which we denote as $m_{b'}$, $m_{t'}$, m_{l_4}, and m_{ν_4}. We do not include leptonic mixing, yet.

B. Experimental bounds

In this section we summarize the experimental constraints that have to be fulfilled by the parameters of the fourth family.

The elements of the 3×3 CKM matrix have been studied intensely for many years and precision data on most of them is available. In principle there are two different ways to determine the CKM elements. On the one hand, they enter charged weak decays already at tree-level and a measurement of, e.g., the corresponding decay rate provides direct information on the CKM elements (see, e.g., [73] and references therein). We will refer to such constraints as *tree-level constraints*. On the other hand, processes involving FCNC are forbidden at tree-level and only come into play at loop level via the renowned penguin and box diagrams. These processes provide strong bounds—referred to as *FCNC constraints*—on the structure of the CKM matrix and its elements as well as on the masses of the heavy virtual particles appearing in the loops.

We will start with the tree-level constraints, since they only depend on the CKM elements and not on the fermion masses. Next we consider mass constraints on the fourth family members from direct searches at colliders. Since the oblique electroweak parameters are expected to reduce the allowed range of masses for a new fermion family notably, we consider them next and finally we discuss the FCNC constraints.

1. Tree-level constraints for the CKM parameters

Since the (absolute) value of only one CKM element enters the theoretical predictions for weak tree-level decays, no Glashow-Iliopoulos-Maiani mechanism or unitarity condition has to be assumed. By matching theory and experiment the matrix element can be extracted *independently* of the number of generations.[3] Therefore, all tree-level constraints have the same impact on the 4×4 matrix as they have on the 3×3 one.

We take the PDG values [74] for our analysis:

	Absolute value	Relative error	Direct measurement from
V_{ud}	$0.974\,18 \pm 0.000\,27$	0.028%	nuclear beta decay
V_{us}	0.2255 ± 0.0019	0.84%	semileptonic K decay
V_{ub}	$0.003\,93 \pm 0.000\,36$	9.2%	semileptonic B decay
V_{cd}	0.230 ± 0.011	4.8%	semileptonic D decay
V_{cs}	1.04 ± 0.06	5.8%	(semi-)leptonic D decay
V_{cb}	0.0412 ± 0.0011	2.7%	semileptonic B decay
V_{tb}	>0.74		(single) top-production

In the following, we denote the absolute values in the table above as $|V_i| \pm \Delta V_i$. In addition to the above tree-level constraints there exists a direct bound on the CKM angle

$$\gamma = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right).$$

It can be extracted via the decays $B \to DK, D\pi$ [75–78]. In principle the extraction of γ might be affected by the presence of a fourth generation of fermions [79], but it was shown in [80] that these effects are negligible. Therefore, γ gives direct information on the phases of the CKM matrix; with three CP violating phases present, this can provide a useful piece of information. We use the CKMfitter value from [81] (update of [82,83])

$$\gamma = 73^{\circ}\,{}^{-25^\circ}_{+22^\circ} \pm 2^\circ, \quad (2.4)$$

where the last error accounts for the tiny additional uncertainty due to the additional fermion generation.

[3]There is, however, one loop hole: In [45] the possibility of lepton mixing reducing the accuracy of the determination of, e.g., V_{ud} was discussed. Since we use the more conservative error estimate from the PDG, our relative error is similar to the final error of Lacker and Menzel, who started with a more ambitious error for V_{ud} in their analysis [45].

2. Direct mass limits for the fourth family

The PDG [74] gives from direct searches the following mass limits for a fourth family:

$$m_{\nu_4} > 80.5\ldots 101.5\ \text{GeV}, \tag{2.5}$$

$$m_{l_4} > 100.8\ \text{GeV}, \tag{2.6}$$

$$m_{b'} > 128\ldots 268\ \text{GeV}, \tag{2.7}$$

$$m_{t'} > 256\ \text{GeV}. \tag{2.8}$$

The mass bound on the heavy neutrino depends on the type of neutrino (Dirac or Majorana) and whether one considers a coupling of the heavy neutrino to e^-, μ^-, or τ^-. It is interesting to note that *LEP results in combination with* [84] *exclude a fourth stable neutrino with* $m < 2400$ GeV [74]. The quark mass bounds are obtained from direct searches at TeVatron [85,86], which were recently updated [87,88]

$$m_{b'} > 338\ \text{GeV}, \qquad m_{t'} > 335\ \text{GeV}. \tag{2.9}$$

In [89] it was pointed out that in deriving these bounds assumptions about the couplings of the fourth generation have been made (in [87] it is, e.g., explicitly assumed that the b' is short-lived and that it decays exclusively to tW^-, which corresponds to demanding $V_{ub'} \approx 0 \approx V_{cb'}$, $m_{b'} < m_{t'}$ and $V_{tb'}$ is not extremely small). Without these assumptions the mass bounds can be weaker, as the extraction of the masses has to be combined with the extraction of the CKM couplings. The inclusion of this dependence is beyond the scope of the current work. For some recent papers concerning the mass extraction of leptons and quarks, see [90,91].

In this work we investigate heavy quark masses in the range of 280 GeV to 650 GeV, heavy charged lepton masses in the range of 100 GeV to 650 GeV, and heavy neutrino masses in the range of 90 GeV to 650 GeV. Note that the triviality bound from unitarity of the $t't'$ S-wave scattering [92] indicates a maximal t' mass of around 504 GeV [93]. However, this estimate is based on tree-level expressions and while it seems prudent to treat too high quark masses with a grain of salt, one should not disregard higher masses based on this estimate alone. In this context it would be desirable to have, e.g., a lattice study of the effect of very heavy (fourth generation) quarks.

3. Electroweak constraints

We present here the expressions for the oblique electroweak S, T, and U parameters [63] in the presence of a fourth generation. They were originally defined as

$$\alpha S = 4e^2 \frac{d}{dq^2}[\Pi_{33}(q^2) - \Pi_{3Q}(q^2)]|_{q^2=0}, \tag{2.10}$$

$$\alpha T = \frac{e^2}{x_W \bar{x}_W M_Z^2}[\Pi_{11}(0) - \Pi_{33}(0)], \tag{2.11}$$

$$\alpha U = 4e^2 \frac{d}{dq^2}[\Pi_{11}(q^2) - \Pi_{33}(q^2)]|_{q^2=0}, \tag{2.12}$$

with the electric coupling α and e, Π_{xy} denotes the virtual self-energy contributions to the weak gauge bosons and with the Weinberg angle expressed as $x_W = \sin^2\theta_W$ and $\bar{x}_W = 1 - x_W$. In the first paper of Ref. [63] $\bar{x}_W M_Z^2$ was approximated by M_W^2. The T parameter is related to the famous ρ parameter [92,94,95]

$$\rho := \frac{M_W^2}{\bar{x}_W M_Z^2} := 1 + \Delta\rho, \tag{2.13}$$

$$= 1 + \alpha T. \tag{2.14}$$

In practice, it turns out to be considerably simpler to reexpress the derivatives in S and U as differences

$$\frac{d}{dq^2}\Pi_{XY}(q^2)|_{q^2=0} \approx \frac{\Pi_{XY}(M_Z^2) - \Pi_{XY}(0)}{M_Z^2}. \tag{2.15}$$

This approximation works very well for $m_{\text{new}} \gg M_Z$ and it is used by the PDG [74]. We will use however the original definitions given in Eqs. (2.10), (2.11), and (2.12) with $M_W^2 = \bar{x}_w M_Z^2$, because there are no correction terms and our expressions are exact.

Next, only the new physics contributions to the S, T, and U parameters will be considered, as the SM values of the oblique parameters are by definition set to zero. Fit results for the allowed regions of the S, T, and U parameters are obtained, e.g., by the PDG [74], EWWG [96], Gfitter [16], and most recent in [14]. Note that the more recent analyses [14,16] differ significantly from the old (November 2007) PDG version. Because of more refined experimental results and an improved theoretical understanding the best fit values shifted significantly toward higher values of S and T, see Fig. 1 for the Gfitter S-T ellipse [97]; this somewhat relaxes the previously observed tension with an additional fermion generation.

In the presence of a fourth generation the fermionic contribution to these parameters (before the necessary subtraction of the SM contribution) reads

$$S = \frac{N_c}{6\pi}\sum_{f=1}^{4}\left[1 - \frac{1}{3}\ln\frac{m_{u_f}^2}{m_{d_f}^2}\right] + \frac{1}{6\pi}\sum_{f=1}^{4}\left[1 + \ln\frac{m_{\nu_f}^2}{m_{l_f}^2}\right], \tag{2.16}$$

$$T = \frac{N_c}{16\pi x_W \bar{x}_W M_Z^2}\left[\sum_{q=u,d,s,\ldots,t',b'} m_q^2 - \sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{u_f d_{f'}}|^2 F_T(m_{u_f}^2, m_{d_{f'}}^2)\right] \tag{2.17}$$

$$+ \frac{1}{16\pi x_W \bar{x}_W M_Z^2}\left[\sum_{l=\nu_e,e^-,\ldots,\nu_4,l_4^-} m_l^2 - \sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{\nu_f l_{f'}}|^2 F_T(m_{\nu_f}^2, m_{l_{f'}}^2)\right], \tag{2.18}$$

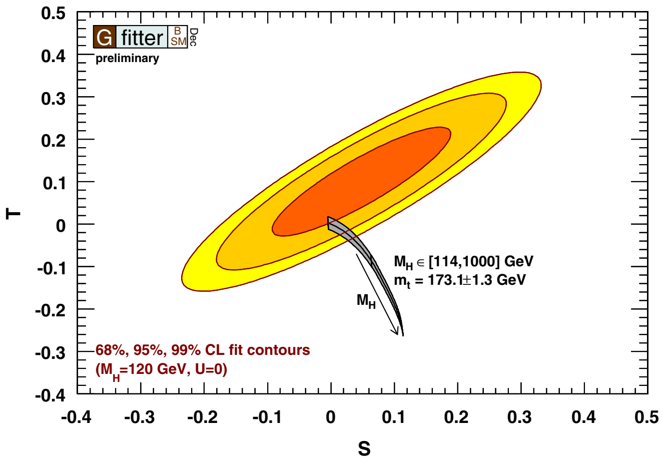


FIG. 1 (color online). Fit of the electroweak oblique parameters S and T. The plot is taken from [97].

$$U = \frac{N_c}{3\pi}\left[\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{u_f d_{f'}}|^2 F_U(m_{u_f}^2, m_{d_{f'}}^2) - \frac{5}{6}\sum_{f=1}^{4} 1\right] + \frac{1}{3\pi}\left[\sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{\nu_f l_{f'}}|^2 F_U(m_{\nu_f}^2, m_{l_{f'}}^2) - \frac{5}{6}\sum_{f=1}^{4} 1\right]. \tag{2.19}$$

u_f denotes the up-type quark of the f-th generation, d_f the down-type quark of the f-th generation, l_f the charged lepton of the f-th generation, and ν_f the neutrino of the f-th generation. We have used the following functions:

$$F_T(m_1^2, m_2^2) := 2\frac{m_1^2 m_2^2}{m_1^2 - m_2^2}\ln\frac{m_1^2}{m_2^2}, \tag{2.20}$$

$$F_U(m_1^2, m_2^2) := 2\frac{m_1^2 m_2^2}{(m_1^2 - m_2^2)^2} + \left(\frac{m_1^2 + m_2^2}{2(m_1^2 - m_2^2)} - \frac{m_1^2 m_2^2(m_1^2 + m_2^2)}{(m_1^2 - m_2^2)^3}\right)\ln\frac{m_1^2}{m_2^2}. \tag{2.21}$$

Both functions are symmetric in their arguments.

The formula for S is very well known—see, e.g., [5,11,62,63]. Using instead the PDG definition we would obtain the following corrections terms to S for heavy quark masses ($m_q^2 \gg M_Z^2$):

$$S_q^{\rm corr} = \frac{N_c}{6\pi}\left[\frac{M_Z^2}{3m_{b'}^2}\left(-\frac{1}{2} + \frac{2}{3}x_W - \frac{4}{9}x_W^2\right) + \frac{M_Z^2}{3m_{t'}^2}\left(-\frac{1}{2} + \frac{4}{3}x_W - \frac{16}{9}x_W^2\right)\right], \tag{2.22}$$

$$S_l^{\rm corr} = -\frac{1}{6\pi}\left[\frac{M_Z^2}{6m_{\nu_4}^2} + \frac{M_Z^2}{2m_{l_4}^2}\left(-\frac{1}{3} + \frac{1}{3}x_W - 3x_W^2\right)\right], \tag{2.23}$$

which are very small for the allowed mass ranges of the fourth family members. In the parameter T no mixing was usually assumed. We give here the full CKM and PMNS dependence. Our expression for T in Eq. (2.18) agrees with the one quoted in [62], if we make the same assumptions (only $4-3$ mixing or $4-3$ and $4-2$ mixing is considered).

By defining the SM3 values for S and T as zero, we only need to take the additional contributions due to the fourth generation into account. Keeping the full, previously neglected CKM dependences, we obtain

$$S_4 = \frac{1}{3\pi}\left[2 + \ln\frac{m_{b'}m_{\nu_4}}{m_{t'}m_{l_4}}\right], \tag{2.24}$$

$$\begin{aligned} T_4 = {} & \frac{N_c}{16\pi x_w \bar{x}_w M_Z^2}\left[m_{b'}^2 + m_{t'}^2 - \sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{u_f d'_f}|^2 F_T(m_{u_f}^2, m_{d_{f'}}^2) + F_T(m_t^2, m_b^2)\right] \\ & + \frac{1}{16\pi x_w \bar{x}_w M_Z^2}\left[m_{l_4}^2 + m_{\nu_4}^2 - \sum_{f=1}^{4}\sum_{f'=1}^{4}|V_{\nu_f l'_f}|^2 F_T(m_{\nu_f}^2, m_{l_{f'}}^2)\right], \end{aligned} \tag{2.25}$$

$$\begin{aligned} U_4 = {} & -\frac{N_c}{6\pi}\left[|V_{t'd}|^2\ln\frac{m_{t'}^2}{m_d^2} + |V_{t's}|^2\ln\frac{m_{t'}^2}{m_s^2} + |V_{t'b}|^2\ln\frac{m_{t'}^2}{m_b^2} + |V_{ub'}|^2\ln\frac{m_{b'}^2}{m_u^2} + |V_{cb'}|^2\ln\frac{m_{b'}^2}{m_c^2} \right. \\ & \left. - 2|V_{tb'}|^2F_U(m_t^2, m_{b'}^2) - 2|V_{t'b'}|^2F_U(m_{t'}^2, m_{b'}^2)\right] - \frac{1}{6\pi}\left[|V_{\nu_4 e}|^2\ln\frac{m_{\nu_4}^2}{m_e^2} + |V_{\nu_4\mu}|^2\ln\frac{m_{\nu_4}^2}{m_\mu^2} + |V_{\nu_4\tau}|^2\ln\frac{m_{\nu_4}^2}{m_\tau^2} \right. \\ & \left. + |V_{\nu_e l_4}|^2\ln\frac{m_{l_4}^2}{m_{\nu_e}^2} + |V_{\nu_\mu l_4}|^2\ln\frac{m_{l_4}^2}{m_{\nu_\mu}^2} + |V_{\mu_\tau l_4}|^2\ln\frac{m_{l_4}^2}{m_{\mu_\tau}^2} - 2|V_{\nu_4 l_4}|^2F_U(m_{\nu_4}^2, m_{l_4}^2)\right] - \frac{10}{9\pi} + U_{\rm SM3}. \end{aligned} \tag{2.26}$$

S_4 has a large positive contribution of about 0.21 which is independent of the parameters (masses and mixing) of the model. This value can, however, be diminished by the second logarithmic term that depends on the fermion masses.

In the SM3 the only significant contribution to T reads

$$T_4 = \frac{N_c}{16\pi x_w \bar{x}_w M_Z^2}[m_t^2 - F_T(m_t^2, m_b^2)]. \tag{2.27}$$

Here safely $V_{tb} = 1$ can be assumed. In the SM4, however, V_{tb} can in principle differ significantly from one, therefore we have the correction term in "$+F_T(m_t^2, m_b^2)$" in the formula for T_4. We also have included all previously neglected mixing terms within the SM3 particles. In principle we also should correct for the charm-strange contribution and for the up-down contribution with "$+F_T(m_u^2, m_d^2) + F_T(m_c^2, m_s^2)$," but their numerical effect is considerably below one per mille of the top-bottom contribution, so we do not show these two additional correction terms in the formula for T_4.

With the help of the S and T parameter Chanowitz [62] could exclude the three parameter sets, which we gave in [50] as an example for a very large mixing between the third and the fourth generation; these sets have passed all bounds set by precision flavor observables. We confirm the numbers from Table I in [62]. We also tested the approximation of taking only 3–4 mixing into account: Comparing with the full CKM dependence the differences are below 6% for these three parameter sets.

To simplify the expression for U we approximated

$$F_U(m_1^2, m_2^2) \approx -\frac{1}{2}\ln\frac{m_1^2}{m_2^2} \quad \text{for } m_1^2 \ll m_2^2. \tag{2.28}$$

Moreover we have only shown the contributions of the 4th family explicitly in Eq. (2.26), the previously neglected rest is denoted by U_{SM3}. It will be interesting to see in a future analysis, whether the large logarithms in the lepton sector will lead to strong constraints on the PMNS matrix. In the literature it is typically assumed that U is very small, see [5] for a notable exception. To our knowledge we incorporate for the first time the full CKM dependence in U. We find that arbitrary values for mixing and mass parameters could, in principle, generate values as large as 7.5 for U.[4] If one only takes into account mixing parameters that pass the tree-level flavor constraints still values of $\mathcal{O}(0.1)$ seem to be possible; however, in this case we observe a simultaneous blow up of the T parameter. For $T < 0.4$, U does not exceed 0.06. Note that, while still small, this value is larger than the 0.02 effect expected without flavor mixing [11].

At this point a few comments are appropriate: first, we would like to point out that our implementation of the S and T parameter is not "exact" from the SM4 point of view. In principle one would have to perform a full reanalysis of all electroweak data from the SM4 perspective to fit the new values of S and T, as advocated for in [62]. This is, of course, beyond the scope of the present work and it is generally accepted that a large deviation of the oblique parameters from their SM values cannot be accommodated in models that do not introduce new particles coupling to fermions [74]. Second, we will henceforth neglect the effect of lepton mixing due to a nontrivial modification of the PMNS matrix—the off diagonal elements including the fourth neutrino are, in any case, required to be small, see [45].

[4] We did not check whether this is the largest possible value.

As a first step in our analysis, we only take the tree-level constraints on V_{CKM4} into account and investigate the parameter ranges that pass the $S-T$ test at the 95% CL[5] The following values for the SM fit of the oblique parameters are used [14]:[6]

$$S_{\text{best fit}} = 0.03, \qquad \sigma_S = 0.09, \tag{2.29}$$

$$T_{\text{best fit}} = 0.07, \qquad \sigma_T = 0.08, \tag{2.30}$$

$$\rho_{\text{corr}} = 0.867, \tag{2.31}$$

where σ_x gives the standard deviation of x. The S and T parameters are not independent quantities; the strength of this correlation is given by ρ_{corr}.

As our work focuses on the flavor aspects of the 4th generation scenario, a short comment on the famous S-T ellipses seems to be in order. If the U parameter stays close to zero for some physics model it is feasible to set $U = 0$ and work with S and T alone. In this case the probability distribution of S and T reduces to a two-dimensional Gaussian distribution in the S-T plane centered on the best fit values. Because of the different σ_x and due to the strong correlation the distribution is essentially squeezed and rotated. Hence, the "equiprobability" lines are no longer circles but ellipses. In fact the two-dimensional case is somewhat special as the problem of finding the ellipse encircling an area corresponding to certain probability P can be solved analytically. The equation determining the contour for a given confidence level (CL) is then given by

$$\begin{pmatrix} S - S_{\text{best fit}} \\ T - T_{\text{best fit}} \end{pmatrix}^T \begin{pmatrix} \sigma_S\sigma_S & \sigma_S\sigma_T\rho \\ \sigma_S\sigma_T\rho & \sigma_T\sigma_T \end{pmatrix}^{-1} \begin{pmatrix} S - S_{\text{best fit}} \\ T - T_{\text{best fit}} \end{pmatrix} = -2\ln(1 - \text{CL}). \tag{2.32}$$

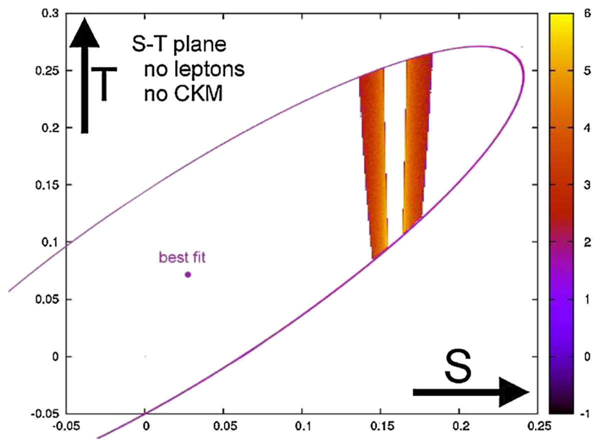
Already at that stage we find some interesting results:

(1) $S-T$ test with "no leptons" + "no V_{CKM}"
We do not take into account leptons as well as mixing of the quarks. By neglecting the leptonic contribution one can, of course, not make any conclusions as to how restrictive the oblique parameters are. However, we still find it instructive to consider the effect of the various contributions in the $S-T$ plane individually. The scatter plot shows the accessible region within the 95% CL ellipse of [14]. We find, as expected, that the masses of the fourth quark generation can not be degenerate if they fulfill the constraints from the oblique parameters. The

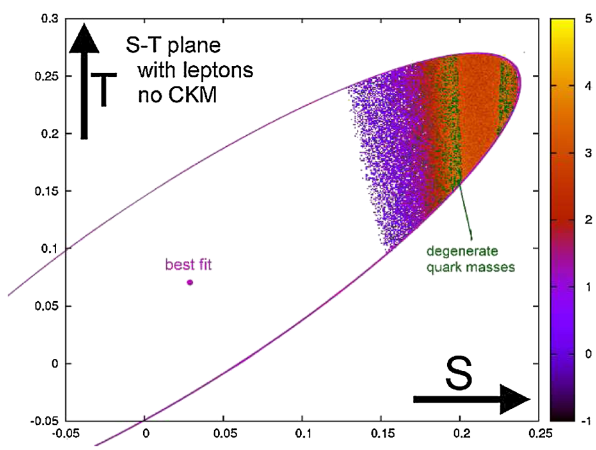
[5] For the final investigation of the allowed parameter range of a fourth fermion family we will use the 99% confidence level.

[6] Note that this fit is the most restrictive one currently available. Using instead the results from Gfitter a little more space is left for a fourth family. We simply decided to use the most recent numbers.

necessary mass difference is of the order of 50 GeV as stated in [11].

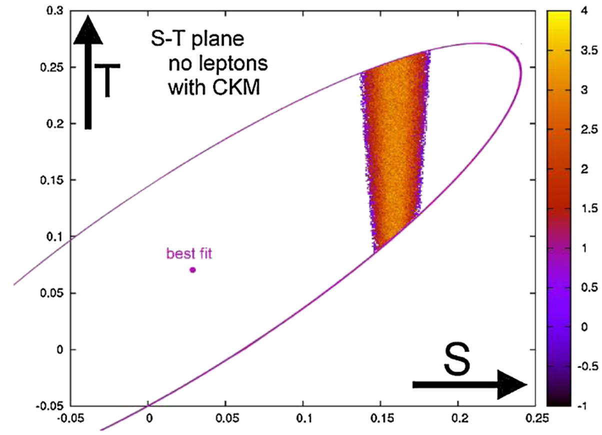


(2) $S - T$ test with "with leptons" + "no V_{CKM}"
Next, we include the leptonic contributions (without lepton mixing) and still neglect CKM mixing. In this case also degenerate values of the quark masses of the fourth generation are in principle not excluded; however, this would require a significant mass gap in the lepton doublet to increase T (and preferably reduce S).



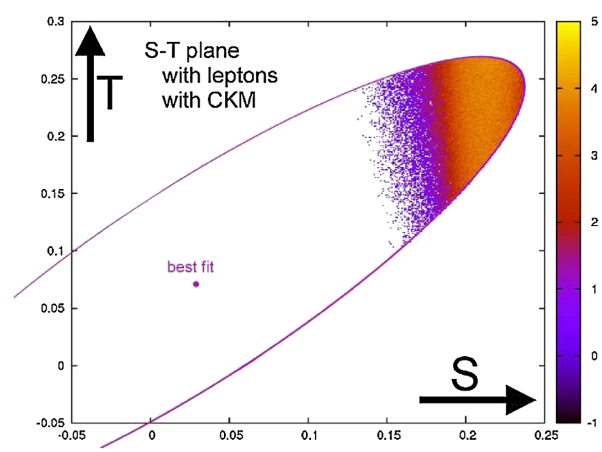
The dark green points indicate values not in conflict with a degeneracy of the quark masses of the fourth generation.

(3) $S - T$ test with "no leptons" + "with V_{CKM}"
To study the "unperturbed" effect of a nontrivial CKM structure of the fourth generation, let us discard again the leptons for the moment. The modified CKM structure results in an increase of the T parameter without changing S, cf. Equations (2.24) and (2.25).

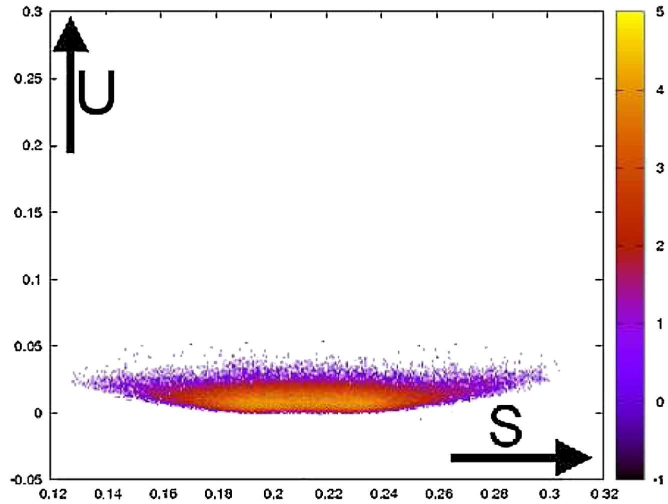


In this scenario an increase of T can originate either from a quark mass splitting or from nonzero mixing of the fourth generation with the SM quarks. However, a mass splitting must also induce a tiny (logarithmical) contribution to S, so the central area which could not be reached in scenario 1 corresponds to nonzero mixing and tiny mass splittings.

(4) $S - T$ test with "with leptons" + "with V_{CKM}"
Here, we use the full expressions for S and T including both leptons (without mixing) and CKM mixing. In this scenario we find a maximally allowed mass splitting of $|m_{t'} - m_{b'}| < 80$ GeV for quarks and $|m_{l_4} - m_{\nu_4}| < 140$ GeV for leptons.
Note that this splitting was also observed by the Gfitter group [97]; their fits also show a minimal required mass splitting as they do not take the possible effects of a nontrivial CKM structure into account. Because of the effects of quark mixing, we do not find a lower bound for the splitting. In fact, a simultaneous degeneracy of quark and lepton masses is *not* excluded, even though the S parameter favors larger t' and l_4 masses.



For completeness we also show the S-U plane using the exact expression for U and S.



Summarizing our investigation of the S, T, and U parameters we get the following results:

(i) U is not *a priori* small; only after the constraints on the quark mixing and the T parameter are used is the maximal value for U reduced below 0.06.

(ii) The quarks of a 4th generation can be degenerated without violating the 95% CL constraints from electroweak precision observables.[7] However, this requires taking into account the effects of the nontrivial flavor sector on T, i.e., mixing of the 4th generation fermions, or a sufficiently large mass splitting in the lepton sector. Note that, at first glance, this result seems to be in direct conflict with the standard statement that a degenerate fourth generation is excluded at the 6σ level [74] by virtue of the S parameter. However, this statement always tacitly assumed a trivial CKM structure. The CKM factors in Eq. (2.25) can lead to $T > 0$ even if both lepton and quark masses are degenerate. However, we did not investigate the effect of the $Z \rightarrow \bar{b}b$ vertex, which tends to favor small or no mixing. Still one can conclude that the situation for tiny mass splittings or even degenerate masses drastically improves once mixing is taken into account.

Finally, we also have to (re)consider the contribution of the Higgs particle, since in the presence of a fourth family higher values of the Higgs mass may be possible [63]. The correction terms to the S, T, and U parameters read

$$S_H = \frac{1}{12\pi} \ln \frac{M_H^2}{(117\ \text{GeV})^2}, \tag{2.33}$$

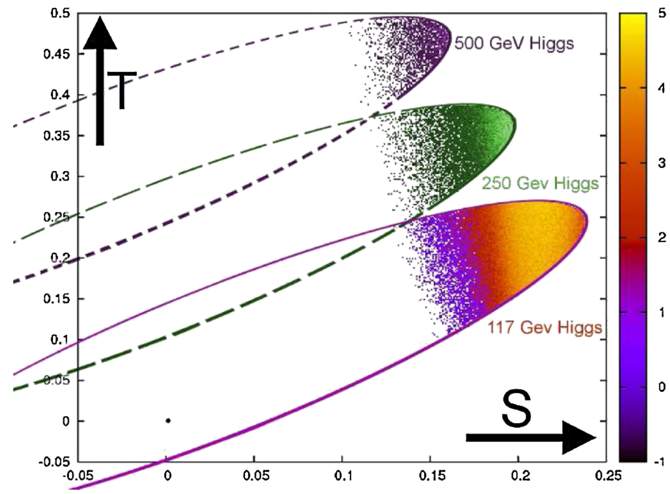
$$T_H = -\frac{3}{16\pi \bar{x}_W} \ln \frac{M_H^2}{(117\ \text{GeV})^2}, \tag{2.34}$$

$$U_H \approx 0. \tag{2.35}$$

[7] For the 99% CL this statement will even hold stronger.

Using that form for the Higgs contributions we implicitly subtract the used value for the Higgs mass in the fit (117 GeV) and add "our" value.

A heavier Higgs increases S and lowers T. Instead of adding S_H and T_H to our values for S and T, we subtracted the Higgs contributions from the fit values to make the diagram easier to understand. So we shift the ellipse and not our data sets. We investigate the S and T parameters for three values of the Higgs mass: 117 GeV, 250 GeV, and 500 GeV.



The decrease in T is welcome, as it allows even bigger mass splitting (or alternatively larger mixing); however, the simultaneous increase of S due to the heavy Higgs completely seems to neutralize or even reverse this effect. Hence, as stated recently in [14] very large values for the mass of a SM-like Higgs are clearly not favored. However, for a 250 GeV Higgs scenario the origin (SM3) is outside the ellipse, whereas some SM4 points are inside and thus more likely.

C. Flavor physics constraints—FCNC processes

After addressing the electroweak bounds, we turn to the constraints imposed by precision observables of flavor physics involving a FCNC. One can hope to impose severe constraints on the model by utilizing information from such processes as it is well known that the weak interaction bypasses the Appelquist-Carazzone decoupling theorem [98]; thus, FCNC processes are very sensitive to contributions of new physics.

However, the selection of flavor physics bounds on a hypothetical fourth family is a nontrivial issue. The reason for this is the fact that some processes known for being theoretically or experimentally very clean, may in fact specifically require the SM3 setup. Hence, it is always necessary to check if a specific feature (of the SM3) is crucial, e.g., for the data analysis is preserved in the fourth generation extension. If this is not the case, it may either be necessary to repeat the analysis without some SM3 simplification—much like the need to give up 3×3 unitarity—or the whole process may not even be feasible anymore.

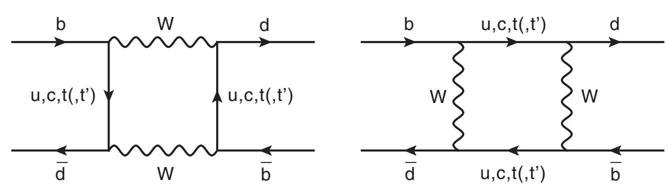
As an example for how unexpected complications may arise (see also [99] for a more detailed discussion), we discuss the so-called golden plated channel for the determination of the standard model CKM angle β: $B_d \to J/\Psi K_s$ [100]. This channel is renowned for being theoretically very clean (in the SM3). Since the decay process is tree-level dominated, it is usually taken for granted that the contribution of the fourth generation quarks to the decay is generally small. Therefore the SM4 could, in principle, be an explanation for discrepancies of the measurement of $\sin(2\beta)$ in the $B_d \to J/\Psi K_s$ and $B_d \to \Phi K_s$, as $B_d \to \Phi K_s$ is penguin dominated and as such more sensitive to new physics effects, see [101] for a more detailed version of this argument. However, it turns out that this elegant picture of the consequences of the fourth generation is, unfortunately, too simple. The reason for this is the following: $\sin(2\beta)$ is extracted via time-dependent CP asymmetries. The necessary ingredients are (i) B_d mixing, (ii) Kaon mixing, and the (iii) decay process itself, see Fig. 2 for a schematic picture of the relevant subprocesses. There are in fact two decay processes, the tree-level decay and the top mediated penguin decay (c and u penguin are expected to be tiny). However, the beauty of this process in the SM3 is that the tree-level decay and the t penguin have (to a fantastic accuracy) the same CKM phase. Hence, it is not necessary to take into account, e.g., different hadronization effects as they will only modify the overall amplitude but not the phase. Adding an additional generation has two new, separate effects. First of all, the expressions for the box diagram changes (see formulas below), so that additional CKM factors contribute; therefore, instead of the CKM angle β a different combination of CKM angles can be extracted from this process. This is, however, not a problem as one could still use $B_d \to J/\Psi K_s$ to constrain the SM4. The real problem is the simultaneous modification of the penguin diagram by the t' loop. As the t' will introduce some new virtually unconstrained phase, penguin and tree decay now have *different* CKM phases; the fourth generation introduces a mismatch between tree and penguin decay, which makes taking into account hadronization and QCD corrections mandatory. Therefore, it is not clear what quantity can be extracted from time-dependent CP asymmetries in $B_d \to J/\Psi K_s$ in the SM4 scenario. This, of course, limits the usefulness of this process for constraining the parameters of the model.

As the above example shows, not all processes can be used to obtain limits on the parameters of the SM4 and one has to be careful not to make use of a bound whose experimental input essentially requires the SM3 setup or some SM3 specific feature.

1. FCNC constraints with no sensitivity to lepton mixing

Another issue is the sensitivity of some processes to the properties of the leptons. Since we do not take mixing in the lepton sector into account and in fact assume lepton number conservation, quantities that are insensitive to the precise structure of the lepton sector are of course advantageous.

We first consider the mixing of the K, the D, the B_d, and the B_s system. For completeness we repeat the relevant formulas already given in [50]. The virtual part of the box diagram (here, e.g., for B_d mixing)



is encoded in M_{12}, which is very sensitive to new physics contributions. It is related to the mass difference of the heavy and light neutral mass eigenstate via

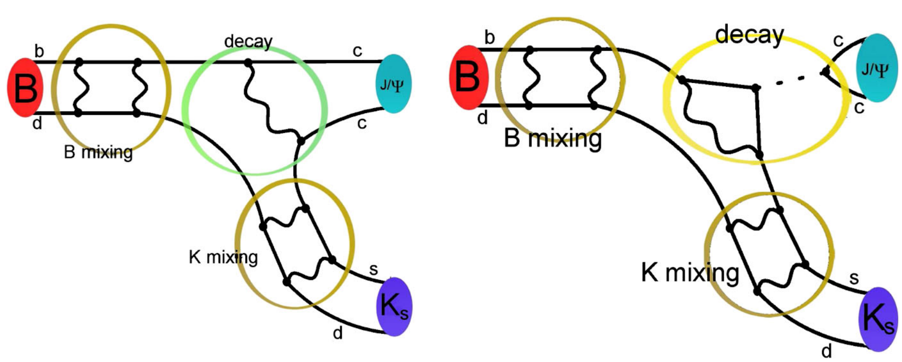


FIG. 2 (color online). Schematic diagrams for the necessary ingredients of the golden plated channel $B_d \to J/\Psi K_s$: B mixing, Kaon mixing, and the decay process itself. The left panel shows the tree level and the right panel the penguin mediated decay. The dashed line represents any current capable of creating a J/Ψ, e.g., two gluons.

$$\Delta M = M_{B_H} - M_{B_L} = 2|M_{12}|. \quad (2.36)$$

In the SM3 one obtains the following relations:

$$M_{12}^{K^0} \propto \eta_{cc}(\lambda_c^{K^0})^2 S_0(x_c) + 2\eta_{ct}\lambda_c^{K^0}\lambda_t^{K^0} S(x_c, x_t) + \eta_{tt}(\lambda_t^{K^0})^2 S_0(x_t), \quad (2.37)$$

$$M_{12}^{B_d} \propto \eta_{tt}(\lambda_t^{B_d})^2 S_0(x_t), \quad (2.38)$$

$$M_{12}^{B_s} \propto \eta_{tt}(\lambda_t^{B_s})^2 S_0(x_t), \quad (2.39)$$

with the Inami-Lim functions [102]

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln[x]}{2(1-x)^3}, \quad (2.40)$$

$$S(x,y) = xy\left[\frac{1}{y-x}\left(\frac{1}{4} + \frac{3}{2}\frac{1}{1-y} - \frac{3}{4}\frac{1}{(1-y)^2}\right)\ln[y] + \frac{1}{x-y}\left(\frac{1}{4} + \frac{3}{2}\frac{1}{1-x} - \frac{3}{4}\frac{1}{(1-x)^2}\right)\ln[x] - \frac{3}{4}\frac{1}{1-x}\frac{1}{1-y}\right], \quad (2.41)$$

where $x_{c,t} = \frac{m_{c,t}^2}{M_W^2}$, the CKM elements

$$\lambda_x^{K^0} = V_{xd}V_{xs}^*, \qquad \lambda_x^{B_d} = V_{xd}V_{xb}^*, \qquad \lambda_x^{B_s} = V_{xs}V_{xb}^*, \quad (2.42)$$

and the QCD corrections [103–105]

$$\eta_{cc} = 1.38 \pm 0.3, \qquad \eta_{ct} = 0.47 \pm 0.04, \qquad \eta_{tt} = 0.5765 \pm 0.0065. \quad (2.43)$$

The full expressions for M_{12} can be found, e.g., in [103,106]. In deriving these expressions the unitarity of the 3×3 matrix was explicitly used, i.e.,

$$\lambda_u^X + \lambda_c^X + \lambda_t^X = 0. \quad (2.44)$$

Moreover, in the B system the CKM elements of the different internal quark contributions are all roughly of the same size. Only the top contribution, which has by far the largest value of the Inami-Lim functions, survives. This is not the case in the K system. Here the top contribution is CKM suppressed, while the kinematically suppressed charm terms are CKM favored. Therefore, both have to be taken into account. More information about the mixing of neutral mesons can be found, e.g., in [106,107].

We define the parameter Δ as the ratio of the new physics model prediction (in our case SM4) for a generic observable to the SM3 theory value; thus, it quantifies the deviation from the standard model [106]. For M_{12} one would then define:

$$\Delta := \frac{M_{12}^{\rm SM4}}{M_{12}^{\rm SM3}} = |\Delta| e^{i\phi^\Delta}. \quad (2.45)$$

This representation is convenient as one can effectively map any observable to the complex Δ plane; this allows a straightforward comparison of the sensitivity of the various observables to the effect of the model. Experimental data can be mapped analogously by plotting $\tilde{\Delta} := O^{\rm Exp}/O^{\rm SM3}$ in the same complex plane. $O^{\rm Exp}$ denotes the experimental value of a generic observable and $O^{\rm SM3}$ the theory prediction within the SM3. In the SM4, we obtain

$$M_{12}^{K^0,\rm SM4} \propto \eta_{cc}(\lambda_c^{K^0})^2 S_0(x_c) + 2\eta_{ct}\lambda_c^{K^0}\lambda_t^{K^0} S(x_c, x_t) + \eta_{tt}(\lambda_t^{K^0})^2 S_0(x_t) + 2\eta_{ct'}\lambda_c^{K^0}\lambda_{t'}^{K^0} S(x_c, x_{t'}) + 2\eta_{tt'}\lambda_t^{K^0}\lambda_{t'}^{K^0} S(x_t, x_{t'}) + \eta_{t't'}(\lambda_{t'}^{K^0})^2 S_0(x_{t'}), \quad (2.46)$$

$$M_{12}^{B_d,\rm SM4} \propto \eta_{tt}(\lambda_t^{B_d})^2 S_0(x_t) + \eta_{t't'}(\lambda_{t'}^{B_d})^2 S_0(x_{t'}) + 2\eta_{tt'}\lambda_t^{B_d}\lambda_{t'}^{B_d} S(x_t, x_{t'}), \quad (2.47)$$

$$M_{12}^{B_s,\rm SM4} \propto \eta_{tt}(\lambda_t^{B_s})^2 S_0(x_t) + \eta_{t't'}(\lambda_{t'}^{B_s})^2 S_0(x_{t'}) + 2\eta_{tt'}\lambda_t^{B_s}\lambda_{t'}^{B_s} S(x_t, x_{t'}). \quad (2.48)$$

Note that CKM elements that describe the mixing within the first three families will now also change. For simplicity we take the new QCD corrections to be (see also [66,67])

$$\eta_{t't'} = \eta_{tt'} = \eta_{tt} \quad \text{and} \quad \eta_{ct'} = \eta_{ct}. \quad (2.49)$$

It is interesting to note here that the only information we have currently about the CKM elements V_{td} and V_{ts} comes from B and K mixing plus assuming the unitarity of $V_{\rm CKM3}$.

The mass difference in the neutral D system can be used to infer a very strong bound on $|V_{ub'}V_{cb'}|$, see [108].[8] We redid this analysis in [50] and softened the bound. The mass difference in the neutral D^0 system is typically expressed in terms of the parameter x_D:

[8] A similar strategy was recently used in [109].

$$x_D = \frac{\Delta M_D}{\Gamma_D} \leq \frac{2|M_{12}^{D^0}|}{\Gamma_D}. \tag{2.50}$$

For more information on the last inequality see, e.g., the discussion in [110,111]. HFAG [112] quotes for the experimental value of x_D

$$x_D = (0.811 \pm 0.334) \times 10^{-2}. \tag{2.51}$$

The main difference compared to the above discussed K- and B-mixing systems is that in the D system the theory prediction in the SM3 is theoretically not well under control, see, e.g., [110,111]. However, the pure contribution of a heavy fourth generation to M_{12} can be calculated reliably. Using the unitarity of the CKM matrix of the SM4 $\lambda_d^{D^0} + \lambda_s^{D^0} + \lambda_b^{D^0} + \lambda_{b'}^{D^0} = 0$ (with $\lambda_x^{D^0} = V_{cx}V_{ux}^*$), the full expression for M_{12} reads

$$\begin{aligned} M_{12}^{D^0} \propto\; & (\lambda_s^{D^0})^2 S_0(x_s) + 2\lambda_s^{D^0}\lambda_b^{D^0} S(x_s, x_b) + (\lambda_b^{D^0})^2 S_0(x_b) \\ & + \mathrm{LD} + 2\lambda_s^{D^0}\lambda_{b'}^{D^0} S(x_s, x_{b'}) + 2\lambda_b^{D^0}\lambda_{b'}^{D^0} S(x_b, x_{b'}) \\ & + \mathrm{LD} + (\lambda_{b'}^{D^0})^2 S_0(x_{b'}), \end{aligned} \tag{2.52}$$

where the proportionality constant is

$$\frac{G_F^2 M_W^2 M_D}{12\pi^2} f_D^2 B_D \eta(m_c, M_W). \tag{2.53}$$

We use the same numerical values as in [50]. The first line of (2.52) corresponds to the pure SM3 contribution, the third line is due to contributions of the heavy 4th generation, and the second line is a term arising when SM3 and b' contributions mix:

$$M_{12}^{D^0} = M_{12,\mathrm{SM3}}^{D^0} + M_{12,\mathrm{Mix}}^{D^0} + M_{12,b'}^{D^0}. \tag{2.54}$$

The idea of [108] was to neglect all terms in $M_{12}^{D^0}$, except $M_{12,b'}^{D^0}$, and to equate this term with the experimental number for x_D, since all perturbative short-distance contributions with light internal quarks are negligible. Since it is not completely excluded that there might be large nonperturbative contributions to both $M_{12,\mathrm{SM3}}^{D^0}$ and $M_{12,\mathrm{Mix}}^{D^0}$ (denoted by LD), each of the size of the experimental value of x_D, we get the following bound:

$$3M_{12}^{D^0,\mathrm{Exp}} \geq M_{12,b'}^{D^0}. \tag{2.55}$$

Allowing this possibility we obtain the following bounds on $|V_{ub'}V_{cb'}|$:

$$|V_{ub'}V_{cb'}| \leq \begin{cases} 0.003\,95 & \text{for } m_{b'} = 200 \text{ GeV}, \\ 0.002\,90 & \text{for } m_{b'} = 300 \text{ GeV}, \\ 0.001\,93 & \text{for } m_{b'} = 500 \text{ GeV}. \end{cases} \tag{2.56}$$

This bound is still by far the strongest direct constraint on $|V_{ub'}V_{cb'}|$.

Next, we consider the $b \to s\gamma$ transition. In [50] we approximated the treatment of the FCNC decay $b \to s\gamma$ by simply looking at the product of the CKM structure and the corresponding Inami-Lim function $D_0'(x_t)$ [102].[9]

$$\Delta_{b\to s\gamma} := \frac{|\lambda_t^{\mathrm{SM4}}|^2 D_0'(x_t)^2 + 2\mathrm{Re}(\lambda_t^{\mathrm{SM4}}\lambda_{t'}^{\mathrm{SM4}}) D_0'(x_t) D_0'(x_{t'}) + |\lambda_{t'}^{\mathrm{SM4}}|^2 D_0'(x_{t'})^2}{|\lambda_t^{\mathrm{SM3}}|^2 D_0'(x_t)^2}, \tag{2.57}$$

with

$$D_0'(x) = -\frac{-7x + 5x^2 + 8x^3}{12(1-x)^3} + \frac{x^2(2-3x)}{2(1-x)^4}\ln[x], \tag{2.58}$$

and $\lambda_x \equiv \lambda_x^{B_s}$. We assumed that parameters which give a value of $\Delta_{b\to s\gamma}$ close to 1 will also lead only to small deviations of $\Gamma(b\to s\gamma)^{\mathrm{SM4}}/\Gamma(b\to s\gamma)^{\mathrm{SM3}}$ from one. However, this crude treatment imposed a too strong bound on the 3–4 mixing.

In this work we will use the full leading logarithmic expression for $b \to s\gamma$, see also [66,67]. Following [113] we normalize the $b \to s\gamma$ decay rate to the semileptonic decay rate

$$R := \frac{\Gamma(b\to s\gamma)}{\Gamma(b\to ce\bar{\nu})} = \frac{|V_{ts}^* V_{tb}|^2}{|V_{cb}|^2}\frac{6\alpha}{\pi f(z)}|C_{7\gamma}^{(0)\mathrm{eff}}|^2, \tag{2.59}$$

$f(z = m_c^2/m_b^2)$ is a phase space factor, which we will not need later on. It is interesting to note that in deriving this formula the unitarity of the 3×3 CKM matrix was already used and the CKM combination $\lambda_u = V_{us}^* V_{ub}$ was neglected (in comparison to λ_c and λ_t). The effective Wilson coefficient $C_{7\gamma}^{(0)\mathrm{eff}}$ is a linear combination of the penguin Wilson coefficients C_7 and C_8, which are accompanied by the CKM structure λ_t and the current-current Wilson coefficient C_2 with the corresponding CKM structures λ_c and λ_u

$$C_{7\gamma}^{(0)\mathrm{eff}}(\mu) = C_7^{\mathrm{eff1}}(\mu) + C_7^{\mathrm{eff2}}(\mu) + C_7^{\mathrm{eff3}}(\mu), \tag{2.60}$$

$$C_7^{\mathrm{eff1}}(\mu) = \eta^{16/23} C_{7\gamma}^{(0)}(M_W), \tag{2.61}$$

$$C_7^{\mathrm{eff2}}(\mu) = \frac{8}{3}(\eta^{14/23} - \eta^{16/23}) C_{8g}^{(0)}(M_W), \tag{2.62}$$

[9]The Inami-Lim function $D_0'(x_t)$ is proportional to the Wilson coefficient $C_{7\gamma}(M_W)$.

$$C_7^{\text{eff3}}(\mu) = \sum_{i=1}^{8} h_i \eta^{a_i} C_2^{(0)}(M_W), \tag{2.63}$$

with

$$\eta(\mu) := \frac{\alpha_s(M_W)}{\alpha_s(\mu)}. \tag{2.64}$$

The values for h_i and a_i are given in Table XXVII of [113]. The initial conditions of the Wilson coefficients read

$$C_2^{(0)}(M_W) = 1, \tag{2.65}$$

$$C_{7\gamma}^{(0)}(M_W) = -\frac{1}{2} D_0'\left(x_t = \frac{m_t^2}{M_W^2}\right), \tag{2.66}$$

$$C_{8g}^{(0)}(M_W) = -\frac{1}{2} E_0'\left(x_t = \frac{m_t^2}{M_W^2}\right). \tag{2.67}$$

$D_0'(x)$ is given above in Eq. (2.58), $E_0'(x)$ reads

$$-\frac{1}{2}E_0'(x) = -\frac{2x + 5x^2 - x^3}{8(1-x)^3} - \frac{3x^2}{4(1-x)^4}\ln[x]. \tag{2.68}$$

Numerically it turns out that even for large values of m_t (up to 1000 GeV) C_7^{eff3} is the dominant contribution to $C_{7\gamma}^{(0)\text{eff}}(\mu)$. In [50] we have only taken C_7^{eff1} into account and therefore overestimated the effects of a fourth generation to the branching ratio (BR) of the decay $b \to s\gamma$. Putting everything together we get

$$\Delta_{b\to s\gamma} := \frac{R^{\text{SM3}}}{R^{\text{SM4}}} \tag{2.69}$$

$$\Delta_{b\to s\gamma} = \left|\frac{V_{cb}^{\text{SM3}}}{V_{cb}^{\text{SM4}}}\right|^2 \left|\frac{\lambda_c^{\text{SM4}} C_7^{\text{eff3}} - \lambda_t^{\text{SM4}}(C_7^{\text{eff1}} + C_7^{\text{eff2}}) - \lambda_{t'}^{\text{SM4}}(C_7^{\text{eff1}'} + C_7^{\text{eff2}'})}{\lambda_c^{\text{SM3}} C_7^{\text{eff3}} - \lambda_t^{\text{SM3}}(C_7^{\text{eff1}} + C_7^{\text{eff2}})}\right|^2. \tag{2.70}$$

In [67] it was suggested that we use the LO expression for $b \to s\gamma$ at a low scale of $\mu = 3.22$ GeV in order to reproduce the numerical value of the next-to-leading order (NLO) expression. We have checked that $\Delta_{b\to s\gamma}$ is quite insensitive to a variation of the scale between m_b and 3 GeV, so we use $\mu = m_b$.

2. FCNC constraints with sensitivity to lepton mixing

Next we also discuss FCNC processes that are sensitive to lepton mixing. In principle lepton mixing has to be investigated in the same manner as the quark mixing. For simplicity we have neglected lepton mixing in this paper. However, we will take into account the conservative bounds on $V_{e\nu_4}$, $V_{\mu\nu_4}$, and $V_{\tau\nu_4}$ given in [45] for the rare decay $B_s \to \mu^+\mu^-$. In the ratio of the SM4 and SM3 predictions for the branching ratios almost everything cancels out and one is left with the product of the CKM elements and Inami-Lim functions

$$\Delta_{B_s\to\mu\mu} := \frac{\text{BR}(B_s \to \mu^+\mu^-)^{\text{SM4}}}{\text{BR}(B_s \to \mu^+\mu^-)^{\text{SM3}}} = \frac{|\lambda_t^{\text{SM4}} Y_0(x_t) + \lambda_{t'}^{\text{SM4}} Y_0(x_{t'})|^2}{|\lambda_t^{\text{SM3}} Y_0(x_t)|^2}, \tag{2.71}$$

with the Inami-Lim function

$$Y_0[x] = \frac{x}{8}\left(\frac{x-4}{x-1} + 3\frac{x}{(x-1)^2}\ln[x]\right). \tag{2.72}$$

Including also the leptonic contributions we have to make the following substitutions [67] in Eq. (2.71):

$$Y_0(x_t) \to Y_0(x_t) - |U_{\mu 4}|^2 S(x_t, x_{\nu_4}), \tag{2.73}$$

$$Y_0(x_{t'}) \to Y_0(x_{t'}) - |U_{\mu 4}|^2 S(x_{t'}, x_{\nu_4}), \tag{2.74}$$

where S is the box function given in Eq. (2.41), x_{ν_4} is given by the mass of the fourth neutrino, and $U_{\mu 4}$ is the PMNS-matrix element describing the mixing between the μ and the fourth neutrino. In [45] the bound $U_{\mu 4} < 0.029$ was derived. Using this information we find that leptonic contributions give at most a relative correction of 0.5 per mille, so we can safely neglect them.

The branching ratio for $B_s \to \mu\mu$ is not measured yet, HFAG quotes [114] (for the current experimental bound from TeVatron, see also [115])

$$\text{Br}(B_s \to \mu^+\mu^-) < 3.6 \times 10^{-8}. \tag{2.75}$$

In the SM3 one expects a value of [67]

$$\text{Br}(B_s \to \mu^+\mu^-) = (3.2 \pm 0.2) \times 10^{-9}. \tag{2.76}$$

3. Allowed ranges for the Δ parameters

Now we come to a crucial point: the fixing of the allowed ranges for the values of the different Δ's. For our exploratory study—in comparison to a full fit that will be performed in the future—we fix reasonable ranges for the Δ's. Therefore we have to investigate theoretical and experimental errors. The FCNC quantities ΔM_s, ΔM_d, and $b \to s\gamma$ are dominated by theoretical uncertainties. Currently, in particular, the hadronic uncertainties are under intense discussion, see, e.g., [116]. Therefore, we use conservative estimates for the theoretical errors.

	Bound
$\lvert\Delta_{B_d}\rvert$	1 ± 0.3
$\phi^{\Delta}_{B_d}$	$0 \pm 10°$
$\lvert\Delta_{B_s}\rvert$	1 ± 0.3
$\phi^{\Delta}_{B_s}$	free
$\mathrm{Re}(\Delta_K)$	1 ± 0.5
$\mathrm{Im}(\Delta_K)$	0 ± 0.3
$\Delta_{b\to s\gamma}$	1 ± 0.15
$\Delta_{B_s\to\mu\mu}$	<15

Since we choose for the central values of our Δ's the value one, all resulting allowed parameter points for V_{CKM4} will be scattered around the SM3 values by definition. For a future fit we will use Δ's with the central value $\tilde{\Delta} = O^{\mathrm{Exp}}/O^{\mathrm{SM3}}$.

This means, in other words, that we do not take into account some current deviations in flavor physics in our current analysis, we simply include them in our error band for the Δ's.

III. PUTTING THINGS TOGETHER—CONSTRAINTS ON THE PARAMETER SPACE

In order to constrain the mixing with the fourth quark family we perform a scan through the parameter space of the model. To this end we use the exact parametrization of V_{CKM4} described in Sec. II A, Eq. (2.3). For the tree-level bounds we use the central values and standard deviations as given in II B 1; we allow for a variation at the 2σ level. The restrictive Peskin-Takeuchi parameters are allowed to vary at the 99% CL of [14], cf. Sec. II B 3. For the quark masses we use a hard lower limit of 280 GeV and allow for a maximal mass difference of 80 GeV as determined in Sec. II B 3. The lepton masses are chosen to be larger than 100 GeV with a maximal splitting of 140 GeV. For the FCNC we use the Δ's given in Sec II C 3.

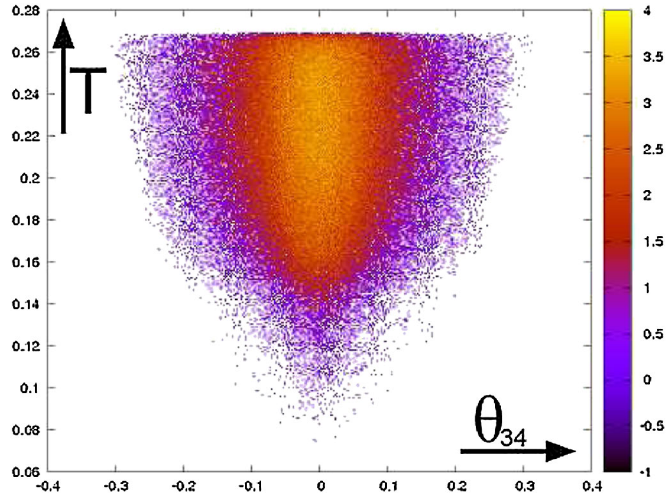
Then we generate a large number [$\mathcal{O}(10^{11})$] of randomly distributed points in the 13 dimensional parameter space.[10] For each point we determine the value of CKM matrix elements, flavor and electroweak observables, and check whether the various experimental bounds are passed (for details see [50]).

A. Result for the mixing angles

Let us for the moment ignore correlations among the various parameters and focus on the maximally allowed size of the mixing with the fourth generation. Table I shows the limits on the mixing angles θ_{14}, θ_{24}, and θ_{34}—without and with the electroweak bounds.

As already expected in Sec. II, the Peskin-Takeuchi parameters impose strong constraints on the mixing of standard model fermions with the fourth generation. These numbers are comparable with the ones quoted in [62,67]. The most dramatic effect is observed for the mixing angle θ_{34}. The maximal size is roughly halved by the virtue of the T parameter alone. So, already at this stage, one is able to conclude that a study of the flavor aspects of SM4 must not be decoupled from a simultaneous analysis of the electroweak sector.

To illustrate the dependence of T on θ_{34} we also show the scatter plot for T versus θ_{34}:



Next, the correlations between the different angles are examined. The results are depicted in Fig. 3. Obviously, the maximal mixing angles given in Table I cannot be simultaneously realized; especially θ_{14} and θ_{24} show a rather strong correlation and maximal θ_{24} is only possible for θ_{14} close to 0.018. Note, e.g., the $\theta_{14} - \theta_{34}$ correlation; simultaneous large mixing angles θ_{34} and θ_{14} are also disfavored. Indeed, this observation is rather natural, as V_{td} includes a term $c_{12}c_{13}c_{24}s_{14}s_{34}e^{i\delta_{14}}$. Hence, simultaneous large s_{14} and s_{34} would lead to a large modification of V_{td}; B_d mixing would be sensitive to this and indeed proves to be the most restrictive of the mixing observables. Because of this observation we disfavor a strategy based on starting from fixed bounds on the mixing angles without taking the correlations into account.

The large "voids" in the $\delta_{13} - \delta_{14}$ plane can be traced to the effect of the direct limit on the phases due to the CKM angle γ.

TABLE I. Maximal mixing of SM fermion generations with the fourth generation. The left column show the effect of tree-level bounds alone, for the center column FCNC bounds were added, and the right-most column gives the limits including electroweak parameters.

	Only tree-level	With FCNC bounds	With electroweak observables
θ_{14}	<0.07	<0.0535	<0.0535
θ_{24}	<0.19	<0.145	<0.121
θ_{34}	<0.8	<0.67	<0.35

[10] $m_{t'}$, $m_{b'}$, m_{l_4}, m_{ν_4}, θ_{12}, θ_{23}, θ_{13}, θ_{14}, θ_{24}, θ_{34}, δ_{13}, δ_{14}, δ_{24}.

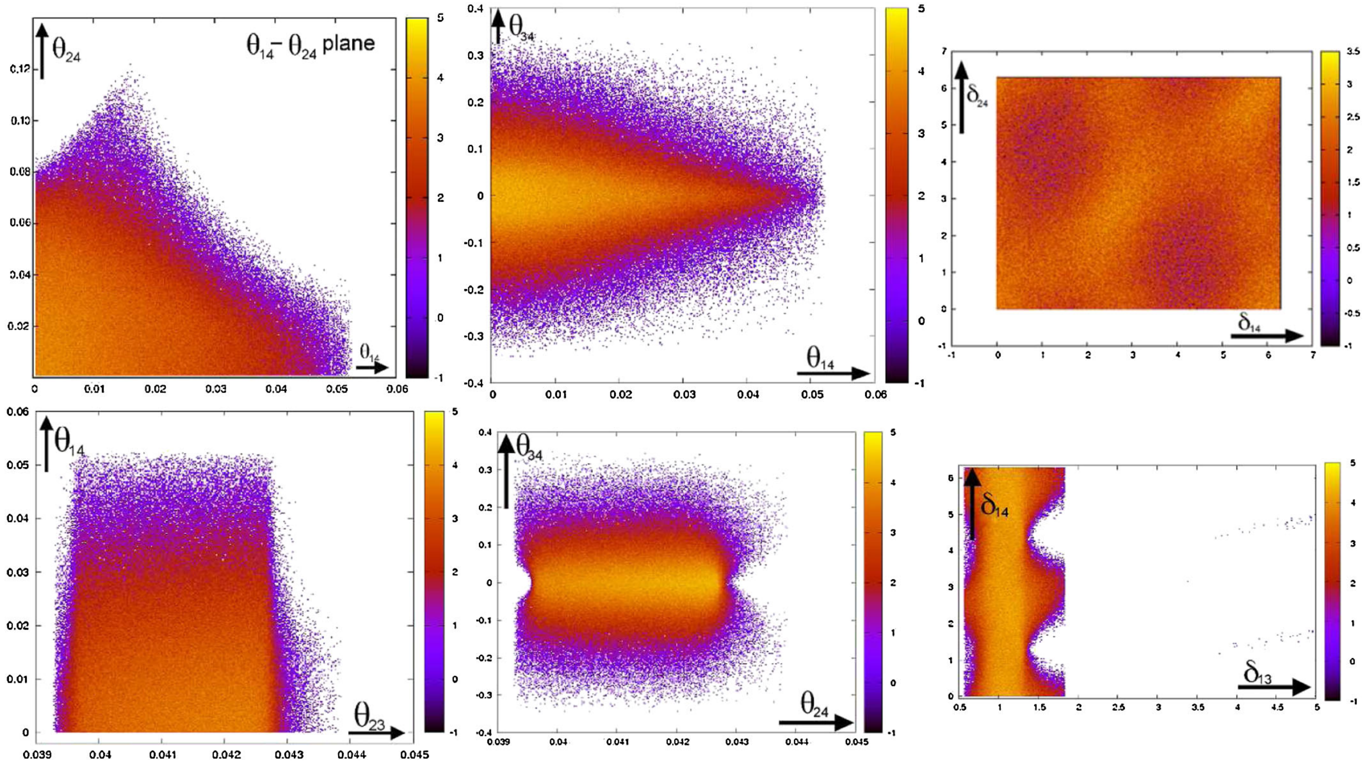


FIG. 3 (color online). Some correlations of the angles and phases.

B. Results for the CKM elements

Since the 3×3 unitarity fixes the values of the second and third row CKM elements rather precisely in the SM3, it is interesting to see the effect of the lifting of the unitarity constraint on V_{cd}, V_{cs}, V_{cb}, V_{td}, V_{ts}, and V_{tb}. In Fig. 4 we present the possible values for these CKM matrix elements in the complex plane. Note that a CKM matrix element itself is not a physical observable as it depends on phase conventions and CKM parametrization; one can, however, compare the values for the elements once the representation and phase convention is fixed. The plots correspond to the standard representation, cf. (2.1), which is the limit of the Botella-Chau representation for zero mixing with a fourth family.

The absolute value of the elements of the second row cannot change much with respect to the SM3; however, it is interesting to observe that the imaginary part of V_{cd} and V_{cs} can be increased by an order of magnitude. This might be potentially interesting for searches for CP violation in the charm sector. While V_{cb} does not have an imaginary part in the standard representation in SM3, a tiny imaginary part can be present in SM4.

The absolute value of both, V_{td} and V_{ts}, can be modified (with respect to their SM3 value) by approximately a factor of 2. More important, the imaginary part of V_{ts} can be an order of magnitude larger than in the SM. Therefore, one can expect that the weak phases of processes involving V_{ts}, e.g., B_s mixing, may experience large corrections.

The absolute value of V_{tb} can be as low as 0.93. Without the constraints coming from oblique parameters this limit would be much lower—around 0.8. This again shows that the electroweak sector imposes strong limits on the flavor structure of SM4.

This number is, in particular, interesting since it can be compared with direct determinations of V_{tb} from single top production from TeVatron [117–119]

$$V_{tb}^{\mathtt{TeVatron}} = 0.88 \pm 0.07. \tag{3.1}$$

C. New physics in B_s mixing

The results for the complex Δ_{B_s} plane is particularly interesting since there might be some hints on new physics effects in the CP-violating phase of B_s mixing, see [106,120] and the web updates of [82]. In [106] a visualization of the combination of the mixing quantities ΔM_s, $\Delta\Gamma_s$, a_{sl}^s, which are known to NLO-QCD [103,121–123] and of direct determinations of Φ_s in the complex Δ plane was suggested. Combining recent measurements [114,124] for the phase Φ_s one obtains a deviation from the tiny SM prediction [106] in the range of 2 to 3σ:

(i) HFAG: 2.2σ [114],
(ii) CKMfitter: $2.1 \ldots 2.5\sigma$ [125,126],
(iii) UTfit: 2.9σ [120].

The central values of these deviations cluster around

$$\Phi_s \approx -51^\circ. \tag{3.2}$$

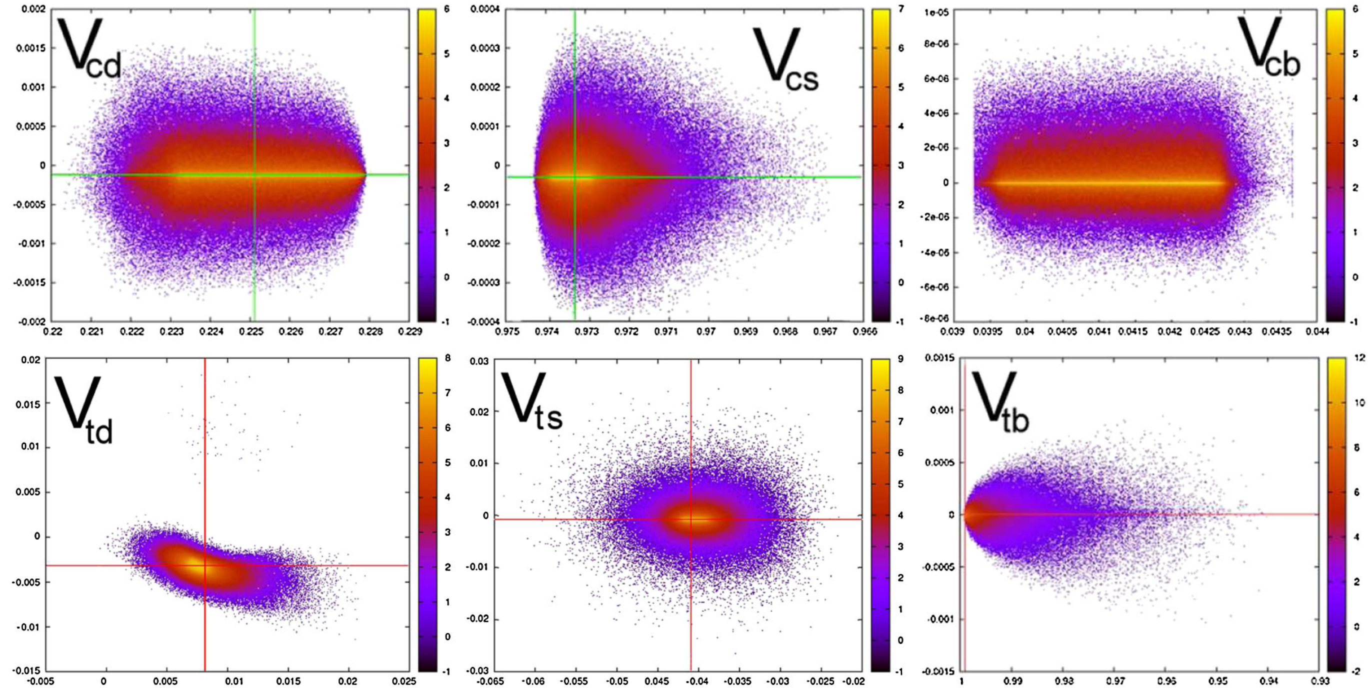


FIG. 4 (color online). Possible modifications of the SM3 CKM matrix elements V_{cd}, V_{cs}, V_{cb} V_{td}, V_{ts}, and V_{tb} in the SM4 scenario. Depicted is the real part versus the imaginary part of the CKM element (in the standard representation). The crossed lines show the SM3 value; V_{cb} is real by construction in the SM3.

Very recently the D0 Collaboration announced a 3.2σ deviation of a linear combination of a_{sl}^d and a_{sl}^s [127] from the standard model prediction in [106]. This deviation also indicates a large negative value of Φ_s.

As can be read off from the left picture of Fig. 5 sizeable values for Φ_s can also be obtained in scenarios with additional fermions. Such large values for Φ_s are not favored, but they are possible. An enhancement of Φ_s to large negative values by contributions of a fourth generation was first predicted in [40], by choosing the parameters of the fourth generations in such a way that other flavor problems like the $B \to K\pi$ puzzle are solved. Once the T parameter is implemented with full CKM dependence and used as an additional bound on the CKM elements, a large value of Φ_s seems to be very unlikely and requires a significant fine tuning of the parameters, see the right picture of Fig. 5. In that respect we differ slightly from the conclusion of, e.g., [40,66,67,128], where very large values for Φ_s are allowed.[11]

Here we expect new and considerably more precise data from TeVatron and LHC soon. If the central value stayed at the current position, the possibility would arise to find new physics that can not originate from an additional fermion family alone.

It is interesting to study the mass dependence of the phase Φ_s. Already in [128] it was noted that large phases clearly favor small t' (and b') masses and the largest phases require a value of $m_{t'}$ close to 300 GeV; this behavior is also present in our analysis and it seems that too large quark masses struggle to resolve the tension in the flavor sector should future experiments confirm, e.g., a phase of the order of 30°. In Fig. 6 we show the distribution of the scatter plot for ‘‘light’’ (< 440 GeV) and ‘‘heavy’’(> 440 GeV) t' masses.[12]

As a final remark, we would like to point out that the naively expected huge contributions to Δ_{B_s} due to the heavy t' can (still) be veiled by the mechanism described in [50]. The new contributions in the SM4 fall into two classes. The first class contains the ”direct” effects of the new heavy ferminons, i.e., the contributions of diagrams with at least one t' in the loop. The second class is more subtle as it includes the ‘‘indirect’’ effects of the SM4 scenario. These contributions arise from the breaking of the 3×3 unitarity; the standard model CKM elements have to be ‘‘thinned’’ out to accommodate for the nonzero values of the fourth row and column matrix elements: this results in a modification of the SM-like contributions.

[11]If we also described the problems in, e.g., $B \to K\pi$ by a fourth family, we also would exclude the points with Φ_s close to zero and we would predict a sizeable phase—around $-20°$—but no points with, e.g., $-50°$ survive in our analysis. Because of hadronic uncertainties we did not include $B \to K\pi$ in our analysis.

[12]Note that the points corresponding to heavy t' quarks are placed on top of the one corresponding to light t's. A light t' and a simultaneous small phase Φ_s are, of course, still possible.

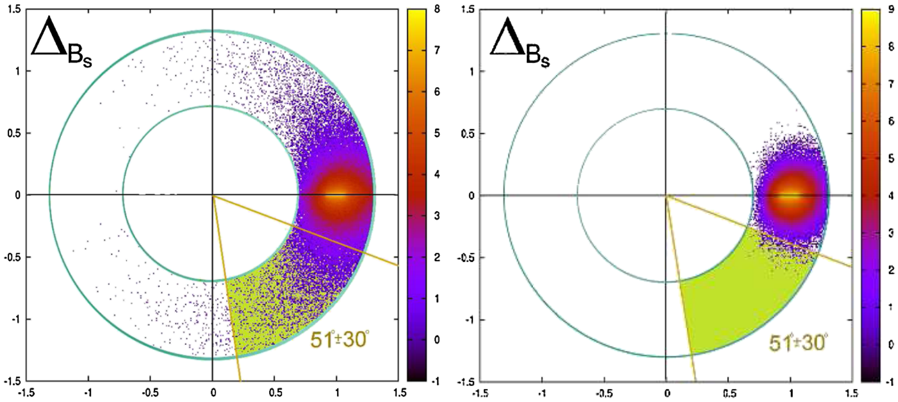


FIG. 5 (color online). Complex Δ plane for B_s mixing. The green (light grey) shaded area corresponds roughly to the experiment at the 1σ level [114]. The left panel shows the possible phase if one omits the T parameter, the right panel shows the impact of T on the allowed phase Φ_s.

These two sets of contributions are both sizeable (typically large enough to violate at least one experimental bound), but they can cancel to a very large extent. Here is one example for the case of Δ_{B_s} which survived all our constraints:

θ_{12}	θ_{13}	θ_{23}	θ_{14}	θ_{24}	θ_{34}	
0.2275	0.003 409	0.040 36	0.017 01	0.083 92	0.1457	
δ_{13}	δ_{14}	δ_{24}	$m_{t'}$	$m_{b'}$	m_{l4}	$m_{\nu 4}$
1.019	0.918 25	0.0787	385 GeV	378 GeV	602 GeV	636 GeV

This parameter set yields

$$\Delta_{B_s} = 1 + \underbrace{0.5379 - 0.9016i}_{\text{direct}} \underbrace{-0.4196 + 0.4262i}_{\text{indirect}} = 1.2e^{-i23^\circ}. \tag{3.3}$$

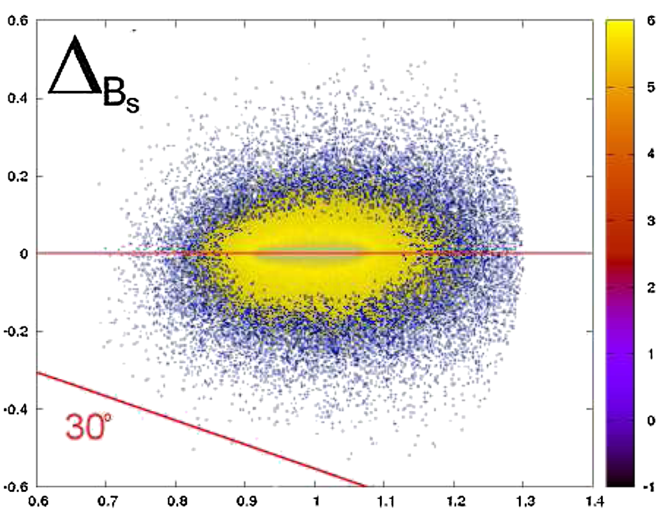


FIG. 6 (color online). Dependence of Δ_{B_s} on the t' mass. The dark blue points correspond to masses below 440 GeV, the yellow (light grey) points correspond to masses heavier than 440 GeV. The red line indicates a phase Φ_s of 30°.

One sees that each contribution is of the size of the standard model result; however, direct and indirect contribution have opposite signs and cancel to a large extent. Note that in this case the 3–4 mixing is strong enough to allow for almost degenerate quark masses.

IV. TAYLOR EXPANSION OF V_{CKM4}

In [50] we gave a Wolfenstein-like expansion of the CKM matrix for four fermion generations, which allows for a first estimate of possible effects of the fourth generation. Since the inclusion of the electroweak oblique parameters led to tighter constraints for the mixing, one can now further improve this expansion.

In the SM3 the hierarchy of the mixing between the three quark families can conveniently be visualized via a Taylor expansion in the small CKM element $V_{us} \approx 0.2255 = \lambda$: the Wolfenstein parametrization [129].

Following [130][13] we define

$$V_{ub} = s_{13}e^{-i\delta_{13}} =: A\lambda^4(\tilde{\rho} + i\tilde{\eta}), \tag{4.1}$$

$$V_{us} = s_{12}(1 + \mathcal{O}(\lambda^8)) =: \lambda, \tag{4.2}$$

[13]Note that due to historical reasons the element V_{ub} is typically defined to be of order λ^3, while it turned out that it is numerically of order λ^4.

$$V_{cb} = s_{23}(1 + \mathcal{O}(\lambda^8)) =: A\lambda^2. \tag{4.3}$$

For the case of 4 generations one also needs the possible size, i.e., the power in λ of the new CKM matrix elements. Using the results of the previous section we find:

$$|V_{ub'}| \leq 0.0535 \approx 1.05\lambda^2 \qquad (1.05\lambda^2),$$
$$|V_{cb'}| \leq 0.123 \approx 0.54\lambda^1 \approx 2.38\lambda^2 \qquad (2.8\lambda^2),$$
$$|V_{tb'}| \leq 0.35 \approx 1.55\lambda^1 \qquad (3.0\lambda^1).$$

In brackets we show the results of our previous analysis in [50]. The biggest effect of the inclusion of the electroweak precision constraints was the reduction of the allowed mixing between the third and fourth family. The mixing between the first and the fourth family can still be considerably larger than the mixing between the first and the third family. This bound is still dominated by D mixing.

(i) Defining the $V_{ub'}$ as

$$V_{ub'} = s_{14}e^{-i\delta_{14}} =: \lambda^2(x_{14} - iy_{14}),$$

one obtains

$$\Rightarrow s_{14} = \lambda^2\sqrt{x_{14}^2 + y_{14}^2},$$
$$\Rightarrow c_{14} = 1 - \lambda^4\frac{x_{14}^2 + y_{14}^2}{2} + O(\lambda^8). \tag{4.4}$$

The parameters x_{14} and y_{14} are effectively smaller than or equal to 1 for all cases.

(ii) Let us further define the matrix element $V_{cb'}$ via

$$V_{cb'} = c_{14}s_{24}e^{-i\delta_{24}} =: (x_{24} - iy_{24})\lambda^1. \tag{4.5}$$

A comparison with Eq. (2.3) then gives

$$\Rightarrow s_{24}e^{-i\delta_{24}} = (x_{24} - iy_{24})\lambda + O(\lambda^5),$$
$$\Rightarrow c_{24} = 1 + \tfrac{1}{2}(-x_{24}^2 - y_{24}^2)\lambda^2 + O(\lambda^5). \tag{4.6}$$

(iii) The last ingredient is the element $V_{tb'}$:

$$V_{tb'} = c_{14}c_{24}s_{34} =: B\lambda, \tag{4.7}$$

and therefore

$$\sin(\theta_{34}) = B\lambda + \frac{1}{2}\lambda^3(Bx_{24}^2 + By_{24}^2) + \mathcal{O}(\lambda^5),$$
$$\cos(\theta_{34}) = 1 - \frac{B^2\lambda^2}{2} + \frac{1}{8}\lambda^4(-B^4 - 4B^2x_{24}^2 - 4B^2y_{24}^2) + \mathcal{O}(\lambda^5). \tag{4.8}$$

Expanding the CKM4 matrix up to and including order λ^4, the matrix elements take the form:

$$V_{ud} = 1 - \tfrac{\lambda^2}{2} + \tfrac{1}{8}\lambda^4(-4x_{14}^2 - 4y_{14}^2 - 1) + O(\lambda^5),$$
$$V_{us} = \lambda, \qquad V_{ub} = A(\tilde{\rho} - i\tilde{\eta})\lambda^4,$$
$$V_{ub'} = (x_{14} - iy_{14})\lambda^2, \tag{4.9}$$

$$V_{cd} = -\lambda + \tfrac{1}{2}\lambda^3(x_{24}^2 + y_{24}^2) - (x_{14} + iy_{14})(x_{24} - iy_{24})\lambda^4 + O(\lambda^5),$$
$$V_{cs} = 1 + \tfrac{1}{2}\lambda^2(-x_{24}^2 - y_{24}^2 - 1) + \tfrac{1}{8}\lambda^4(-4A^2 - 2x_{24}^2(y_{24}^2 - 1) - x_{24}^4 - y_{24}^4 + 2y_{24}^2 - 1) + O(\lambda^5),$$
$$V_{cb} = A\lambda^2, \qquad V_{cb'} = \lambda^2(x_{24} - iy_{24}), \tag{4.10}$$

$$V_{td} = \lambda^3(A - Bx_{14} - iBy_{14}) + (-iA\eta - A\rho + Bx_{24} + iBy_{24})\lambda^4 + O(\lambda^5),$$
$$V_{ts} = -A\lambda^2 - B\lambda^3(x_{24} + iy_{24}) + \tfrac{1}{2}\lambda^4(AB^2 - Ax_{24}^2 - Ay_{24}^2 + A - 2Bx_{14} - 2iBy_{14}) + O(\lambda^5),$$
$$V_{tb} = 1 - \frac{B^2\lambda^2}{2} + \tfrac{1}{8}\lambda^4(-4A^2 - B^4 - 4B^2x_{24}^2 - 4B^2y_{24}^2) + O(\lambda^5),$$
$$V_{tb'} = B\lambda, \tag{4.11}$$

$$V_{t'd} = \lambda^2(-x_{14} - iy_{14}) + \lambda^3(x_{24} + iy_{24}) + \tfrac{1}{2}\lambda^4(-2AB + (x_{14} + iy_{14})(B^2 + x_{24}^2 + y_{24}^2) + x_{14} + iy_{14}) + O(\lambda^5),$$
$$V_{t's} = \lambda^2(-x_{24} - iy_{24}) + \lambda^3(AB - x_{14} - iy_{14}) + \tfrac{1}{2}(B^2 + 1)\lambda^4(x_{24} + iy_{24}) + O(\lambda^5),$$
$$V_{t'b} = -B\lambda - \tfrac{1}{2}\lambda^3B(x_{24}^2 + y_{24}^2) - A\lambda^4(x_{24} + iy_{24}) + O(\lambda^5),$$
$$V_{t'b'} = 1 + \tfrac{1}{2}\lambda^2(-B^2 - x_{24}^2 - y_{24}^2) + \tfrac{1}{8}\lambda^4(-B^4 - 2B^2x_{24}^2 - 2B^2y_{24}^2 - 2x_{24}^2y_{24}^2 - x_{24}^4 - 4x_{14}^2 - y_{24}^4 - 4y_{14}^2) + O(\lambda^5). \tag{4.12}$$

Naturally, this expansion cannot take into account correlations among the various CKM elements. However, the expansion is quite useful if one wants a rough estimate of the maximal size of a product of CKM elements determining the impact of the fourth generation on a certain process.

V. CONCLUSION

We have investigated the experimentally allowed parameter range for a hypothetical 4×4 quark mixing matrix. Therefore we extended our previous study [50] of the CKM-like mixing constraints of a fourth generation of quarks.

Besides the tree-level determinations of the 3×3 CKM elements we also included the angle γ of the unitarity triangle, which turned out to be a rather severe bound for the phases of V_{CKM4}. Next we included the electroweak S, T, U parameters. Here we reproduced some of the results from [62], in particular, we also excluded the three examples of very large mixing presented in [50]. In this paper we have included for the first time the full CKM dependence of the T and the U parameter. Doing so, we found that a mass degenerate fourth family of quarks is not excluded; in that respect we differ, e.g., from [11,74]. While degenerate quark masses can also arise if the lepton masses are adjusted accordingly, see Sec. II B 3 and [14], including the full CKM dependence allows for a greater ‘‘flexibility’’ in the parameter space: e.g., only then a simultaneous separate degeneracy of leptons and quarks of the fourth generation would not be excluded—this may be of interest if one wants to invoke a symmetry to motivate the tiny mass splittings. In addition we found also that large values of the parameter U are not excluded *a priori*; only after applying the T parameter constraint we are left with small values of U.

Concerning the FCNC constraints we studied K, D, B_d, B_s mixing, and the decay $b \to s\gamma$. In contrast to [50] we also included bounds to the rare decay $B_s \to \mu^+\mu^-$ and we improved our treatment of the QCD corrections to $b \to s\gamma$. It turned out that the naive bound for $b \to s\gamma$ used in [50] was too restrictive.

Performing a scan over the whole parameter space of the SM4 we found that typically small mixing with the fourth family is favored, but still some sizeable deviations from the SM3 results are not yet excluded. We demonstrated explicitly that, e.g., effects of $\mathcal{O}(100\%)$ in B_s mixing are not excluded, yet. Concerning CP violation in B_s mixing, we could have an almost arbitrarily large phase Φ_s without violating the tree-level constraints. After switching on the T constraint (99% CL of [14]) we could exclude values of the $\mathcal{O}(-50°)$ for the weak mixing phase Φ_s, while values of $\mathcal{O}(-20°)$ can easily be obtained—this is still about two orders of magnitude larger than the SM3 prediction $\Phi_s = 0.24° \pm 0.08°$ [106]. In that respect we differ slightly from [40,66,67,128]. If the real value of Φ_s was -50%, this could not be achieved by an extension of the SM3, which consists only of an additional fermion family.[14] Here new results for the B-mixing observables from TeVatron and LHCb are very desireable.

We found a minimal possible value for V_{tb} of 0.93 within the framework of the SM4, which can be compared with the result from single top production at the TeVatron [117–119]: $V_{tb}^{\mathtt{TeVatron}} = 0.88 \pm 0.07$. If in nature a central value of $V_{tb} = 0.88$ was realized, this could not be explained by the SM4 alone. Here also more precise experimental data are desirable. In general we found a delicate interplay of electroweak and flavor observables, which strongly suggests that a separate treatment of the two sectors is not feasible.

In our opinion the next steps to determine the allowed parameter space of the SM4 consist of (i) performing a combined electroweak and CKM fit, (ii) including lepton mixing, and (iii) including even more precision observables like, e.g., R_b or $B \to K^* ll$.

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