

Reconciling the $X(4630)$ with the $Y(4660)$

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The Belle Collaboration observed an enhancement called $X(4630)$ in the $\Lambda_c^+ \Lambda_c^-$ mass distribution using initial state radiation. We demonstrate that the enhancement could be consistent with the $\psi' f_0(980)$ molecular picture of the $Y(4660)$ taking into account the $\Lambda_c^+ \Lambda_c^-$ final state interaction. To test the hypothesis that the $X(4630)$ and $Y(4660)$ are the same molecular state, we give predictions for its spin partner, the $\eta'_c f_0(980)$ molecule. High statistic measurements of the B decays into the $K \Lambda_c^+ \Lambda_c^-$ and $K \eta'_c \pi^+ \pi^-$ are strongly recommended.

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The recently observed open and hidden charmed hadrons have stimulated many studies. They challenge our current knowledge of hadron spectroscopy, and provide us with an opportunity to understand nonperturbative QCD better. Among these hadrons, the $Y(4660)$ was observed by the Belle Collaboration in the $\psi' \pi^+ \pi^-$ mass distribution using the technique of initial state radiation (ISR) [1]. The mass and width were reported to be $4664 \pm 11 \pm 5$ MeV and $48 \pm 15 \pm 3$ MeV, respectively. This structure is very special because it was neither observed in $e^+ e^- \rightarrow \gamma_{\text{ISR}} \pi^+ \pi^- J/\psi$ [2], nor in the mass distributions of a charmed and anticharmed meson pair in the final states of electron-positron collisions [3,4]. Furthermore, the $\pi^+ \pi^-$ invariant mass spectrum shows a single peak at the high end, i.e. towards the mass region of the scalar meson $f_0(980)$. While these facts challenge other explanations [5–7], in Ref. [8] it was argued that they may be naturally explained in terms of a hadronic molecular picture, i.e. by $\psi' f_0(980)$ being bound together in an S -wave. This state is thus a candidate for a hadrocharmonium proposed in Ref. [9].

More recently, the Belle Collaboration reported another structure, called $X(4630)$, in the $\Lambda_c^+ \Lambda_c^-$ invariant mass distribution in $e^+ e^- \rightarrow \gamma_{\text{ISR}} \Lambda_c^+ \Lambda_c^-$ [10]. The reported mass is 4634_{-7-8}^{+8+5} MeV, and the width is 92_{-24-21}^{+40+10} MeV, consistent with the ones reported for the $Y(4660)$ within two sigma. Based on the tetraquark picture, both structures were proposed to be of the same origin in Ref. [7], however, there is no general consensus on this issue yet (see e.g. the discussion in the short review [11]). In this paper, we shall show that they could also be understood as the same state within the $\psi' f_0(980)$ hadronic molecular

picture, and discuss how this hypothesis can be tested in future experiments.

In the $\psi' f_0(980)$ hadronic molecular picture, one may expect naively that the bound state would decay mainly through the decays of the unstable $f_0(980)$, and hence into the $\psi' \pi \pi$, and the peak in the $\pi \pi$ invariant mass spectrum close to the $f_0(980)$ mass region appears naturally. While the latter statement is correct, the former one needs to be scrutinized. The mass of the $Y(4660)$ is higher than open charmed and anticharmed meson thresholds, and the $\Lambda_c^+ \Lambda_c^-$ threshold. If the binding energy $\varepsilon = M_{\psi'} + m_{f_0(980)} - M_{Y(4660)}$ is very small, the coupling of the bound state to its constituents determined by the equation [12,13]

$$\frac{g^2}{4\pi} = 4(M_{\psi'} + m_{f_0(980)})^2 \sqrt{\frac{2\varepsilon}{\mu}}, \quad (1)$$

with μ the reduced mass of the ψ' and $f_0(980)$ is small, and so is the partial width $\Gamma(Y(4660) \rightarrow \psi' \pi \pi)$. On the other hand, the open charm channels have larger phase space, and might have larger partial decay widths. In fact, there is a well-known example—the $f_0(980)$ decays mainly into two pions which have plenty of phase space although it can be understood as a $K\bar{K}$ bound state [13,14]. In this paper, we shall assume that the $\Lambda_c^+ \Lambda_c^-$ is the dominant open charm channel and study the implications of this assumption. This means we shall assume the total width of the $Y(4660)$ is given by the sum of the partial widths into the $\psi' \pi \pi$ and $\Lambda_c^+ \Lambda_c^-$, i.e.

$$\Gamma_Y^{\text{tot}} = \frac{3}{2} \Gamma_Y^{[\psi' \pi^+ \pi^-]} + \Gamma_Y^{[\Lambda_c^+ \Lambda_c^-]}, \quad (2)$$

where the factor 3/2 in front of $\Gamma_Y^{[\psi' \pi^+ \pi^-]}$ is from isospin symmetry.

The line shape of the $Y(4660)$ is given by its spectral function

$$\rho_Y(M) = \frac{M_Y \Gamma_Y^{\text{tot}}(M)}{|M^2 - M_Y^2 + \hat{\Pi}_Y(M)|^2}, \quad (3)$$

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convoluted with phase space, where M_Y is the mass, $\Gamma_Y^{\text{tot}}(M)$ is the energy-dependent total width, and $\hat{\Pi}_Y(M) = \Pi_Y(M) - \text{Re}[\Pi_Y(M_Y)]$ is defined as the self-energy with the real part subtracted at the mass [15]. The self-energy for arbitrary values of M is given by a dispersion integral (for further details, see Ref. [8])

$$\Pi_Y(M) = \frac{1}{\pi} \int_{M_{\text{thr}}}^{\infty} ds \frac{M_Y \Gamma_Y^{\text{tot}}(\sqrt{s})}{s - M^2 - i\epsilon}, \quad (4)$$

where M_{thr} denotes the relevant physical threshold. In Ref. [8], only the decays $Y \rightarrow \psi' \pi \pi (K \bar{K})$ were considered. In order to check whether or not the structure observed in the $\Lambda_c^+ \Lambda_c^-$ mass distribution is consistent with the $Y(4660)$ observed in the $\psi' \pi^+ \pi^-$, one needs to include the contribution of the $\Lambda_c^+ \Lambda_c^-$ in the total width Γ_Y^{tot} . For that, a simple Lagrangian for the $Y(4660) \Lambda_c^+ \Lambda_c^-$ coupling, which is assumed to be in an S wave, is used

$$\mathcal{L}_{Y\Lambda_c\Lambda_c} = -g_{Y\Lambda_c\Lambda_c} \bar{\Lambda}_c \gamma^\mu Y_\mu \Lambda_c, \quad (5)$$

with $g_{Y\Lambda_c\Lambda_c}$ a dimensionless coupling constant. Then the cross sections for $e^+e^- \rightarrow \gamma_{\text{ISR}} \psi' \pi^+ \pi^-$ and $e^+e^- \rightarrow \gamma_{\text{ISR}} \Lambda_c^+ \Lambda_c^-$ are simply given by the corresponding parts of the spectral function of the $Y(4660)$

$$\begin{aligned} \sigma(\psi' \pi^+ \pi^-) &= N \frac{M_Y \Gamma_Y^{[\psi' \pi^+ \pi^-]}(M)}{|M^2 - M_Y^2 + \hat{\Pi}_Y(M)|^2}, \\ \sigma(\Lambda_c^+ \Lambda_c^-) &= N \frac{M_Y \Gamma_Y^{[\Lambda_c^+ \Lambda_c^-]}(M)}{|M^2 - M_Y^2 + \hat{\Pi}_Y(M)|^2}, \end{aligned} \quad (6)$$

where $\Gamma_Y^{[\psi' \pi^+ \pi^-]}$ and $\Gamma_Y^{[\Lambda_c^+ \Lambda_c^-]}$ are the partial decay widths of the $Y(4660)$ into the $\psi' \pi^+ \pi^-$ and $\Lambda_c^+ \Lambda_c^-$ channels, respectively. The overall normalization constant N is the same for both processes since both structures were observed by the Belle Collaboration in the ISR processes.

Since the $Y(4660)$ has the quantum numbers $J^{\text{PC}} = 1^{--}$, it couples to the $\Lambda_c^+ \Lambda_c^-$ system in an S -wave, specifically to the 3S_1 , and, therefore, the impact of the final state interaction (FSI) is expected to be large. In principle, the

situation is comparable to J/ψ decays with the proton-antiproton channel in the final state where FSI effects are known to play a rather important role [16–22]. Unfortunately, there is no direct experimental information on the interaction between charmed and anticharmed baryons. Thus, we have to resort to a model of the $\Lambda_c^+ \Lambda_c^-$ interaction for taking into account FSI effects. Here we adopt the potential presented in Ref. [23], which was derived using SU(4) flavor-symmetry arguments, and compute the Jost function $\mathcal{J}(M)$ for this interaction. Multiplying the reaction amplitude with the inverse of the latter quantity, also known as enhancement factor, is practically equivalent to a treatment within a distorted-wave Born approximation [24,25]. The width of $Y(4660) \rightarrow \Lambda_c^+ \Lambda_c^-$ is then given by

$$\Gamma_Y^{[\Lambda_c^+ \Lambda_c^-]}(M) = \frac{g_{Y\Lambda_c\Lambda_c}^2}{|\mathcal{J}(M)|^2} \frac{p}{6\pi} \left(1 + 2 \frac{M_{\Lambda_c}^2}{M^2}\right) \theta(M - 2M_{\Lambda_c}), \quad (7)$$

where M_{Λ_c} is the mass of the Λ_c , $p = \sqrt{M^2/4 - M_{\Lambda_c}^2}$ is its three-momentum in the rest frame of the $Y(4660)$, and θ is the step function. According to the model of Ref. [23], the function $1/|\mathcal{J}(M)|^2$ for the 3S_1 channel decreases from about 2 at zero momentum to 0.3 at $p \approx 500$ MeV, and then slowly approaches unity only at very high momenta. In our calculations, we parametrize $1/|\mathcal{J}(M)|^2$ up to $p \approx 500$ MeV with the following function

$$\frac{1}{|\mathcal{J}(M)|^2} = d \frac{p^2 + b^2}{p^2 + cp + a^2}, \quad (8)$$

with the parameter values being $a = 247.7$ MeV, $b = 1390.4$ MeV, $c = 387.3$ MeV, and $d = 0.0677$. Then we set $d = 1$, which may always be done because such a normalization can be absorbed into a redefinition of the coupling constant $g_{Y\Lambda_c\Lambda_c}$, so that the remaining factor approaches unity asymptotically, and provides an enhancement to the amplitude close to the threshold. In Fig. 1(a), the FSI enhancement factor in the 3S_1 channel as well as

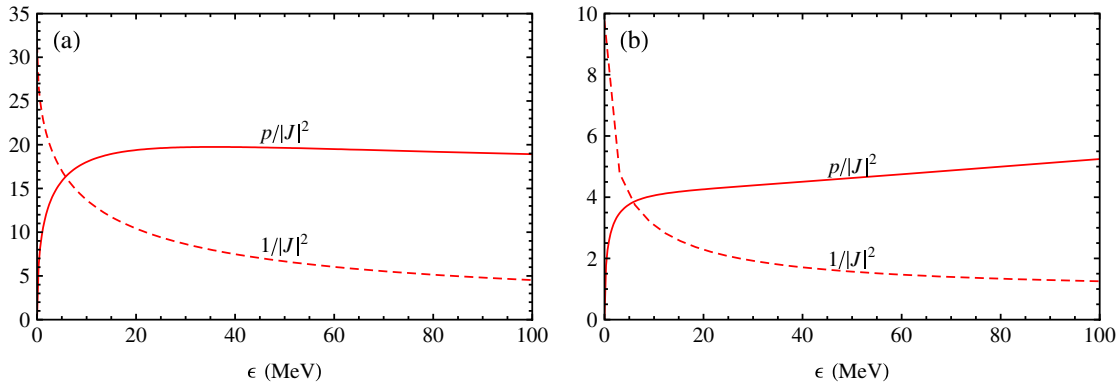


FIG. 1 (color online). The FSI enhancement factor $1/|\mathcal{J}(M)|^2$ (dashed line) and the quantity $p/|\mathcal{J}(M)|^2$ (solid line) as a function of the excess energy $\epsilon = M - 2M_{\Lambda_c}$. The latter curves are normalized arbitrarily. (a): the 3S_1 channel; (b): the 1S_0 channel.

this factor times the two-body phase space are shown as a function of the excess energy $\epsilon = M - 2M_{\Lambda_c}$. Note that the central value of the peak observed by the Belle Collaboration in the $\Lambda_c^+ \Lambda_c^-$ mass distribution is about 90 MeV above threshold, hence it cannot be due to the $\Lambda_c^+ \Lambda_c^-$ -FSI enhancement solely, as may be seen from the figure. An opposite claim was made recently in Ref. [26].

Using Eqs. (6), we perform a simultaneous fit to the cross sections of both processes. For simplicity, we assume that there is no background. Then there are three free parameters: the normalization constant N , the mass of the $Y(4660)$, M_Y , and the $Y(4660)\Lambda_c\Lambda_c$ coupling constant $g_{Y\Lambda_c\Lambda_c}$. The best fit gives

$$\begin{aligned} N &= 237_{-36}^{+40}, \\ M_Y &= 4662.5_{-0.2}^{+0.1} \text{ MeV}, \\ g_{Y\Lambda_c\Lambda_c} &= 0.7 \pm 0.1, \end{aligned} \quad (9)$$

with $\chi^2/\text{d.o.f.} = 1.4$. The uncertainties quoted above are only from the fit, and do not include an estimate of the systematic uncertainty of the procedure. In doing the above fit, we chose to use $M_{\psi'}$ as given by the PDG [27] and the central values of the parameters for the $f_0(980)$ measured recently by the KLOE Collaboration in the best fit K1 shown in Table 4 in Ref. [28], i.e. we used $m_{f_0} = 976.8$ MeV, $g_{f_0 K^+ K^-} = 3.76$ GeV and $g_{f_0 \pi^+ \pi^-} = -1.43$ GeV. The comparison of our best fit with the experimental data is presented in Fig. 2, cf. the solid lines. Also shown are the results for the case without the $\Lambda_c\Lambda_c$ FSI (dashed lines), which were obtained with the same parameters except for the coupling constant. We use $g_{Y\Lambda_c\Lambda_c}/|\mathcal{J}(M_Y)|$ as the coupling constant for the case without FSI such that it coincides with the FSI modified coupling at the mass of the $Y(4660)$. From the $\Lambda_c^+ \Lambda_c^-$ mass distribution, one immediately sees the enhancement effect of the FSI on the cross section close to the threshold. From the best fit, we obtain the partial widths of the $Y(4660)$

$$\begin{aligned} \Gamma(Y(4660) \rightarrow \psi' \pi^+ \pi^-) &= 8 \text{ MeV}, \\ \Gamma(Y(4660) \rightarrow \Lambda_c^+ \Lambda_c^-) &= 93 \text{ MeV}, \end{aligned} \quad (10)$$

and their ratio is

$$\frac{\Gamma(Y \rightarrow \Lambda_c^+ \Lambda_c^-)}{\Gamma(Y \rightarrow \psi' \pi^+ \pi^-)} = 11.5. \quad (11)$$

The ratio is smaller than the central value 24.8 extracted in Refs. [7,29] considering also an interference of the resonance with a polynomial background. In Ref. [7] the authors also treated the $X(4630)$ and the $Y(4660)$ as the same state, however, in this case as a compact tetraquark.

At this stage, we want to emphasize that the FSI obtained from the model of Ref. [23] is afflicted with sizeable uncertainties. However, it incorporates all essential features one expects from a realistic FSI, specifically it is generated by solving a scattering equation and it includes effects from the presence of annihilation channels. Therefore, it should be sufficient to give an illustration for the FSI effect in the problem at hand. The $\Lambda_c^+ \Lambda_c^-$ interaction of Ref. [23] contains two parts—an elastic part based on meson exchange and derived via SU(4) flavor symmetry, and an optical potential to simulate annihilation processes. In order to check in-how-far changes in the FSI influence our results we varied the strength of the optical potential by factors in the range from 1/2 to 2. It turned out that these variations only have a marginal effect on the resulting invariant mass distributions from the best fit. Moreover, note that we here only need the distortion of the spectral shape due to the FSI. Any normalization, which is certainly much more model dependent, can be absorbed into a redefinition of the coupling constant.

It should be clear that what we discussed above is only a possible scenario. The fact that one can obtain a combined fit of the $\Lambda_c^+ \Lambda_c^-$ and the $\pi\pi\psi'$ channels also in the molecular picture does not prove that the $X(4630)$ and the $Y(4660)$ are the same state. Observables should be found to further support or disprove this hypothesis. In

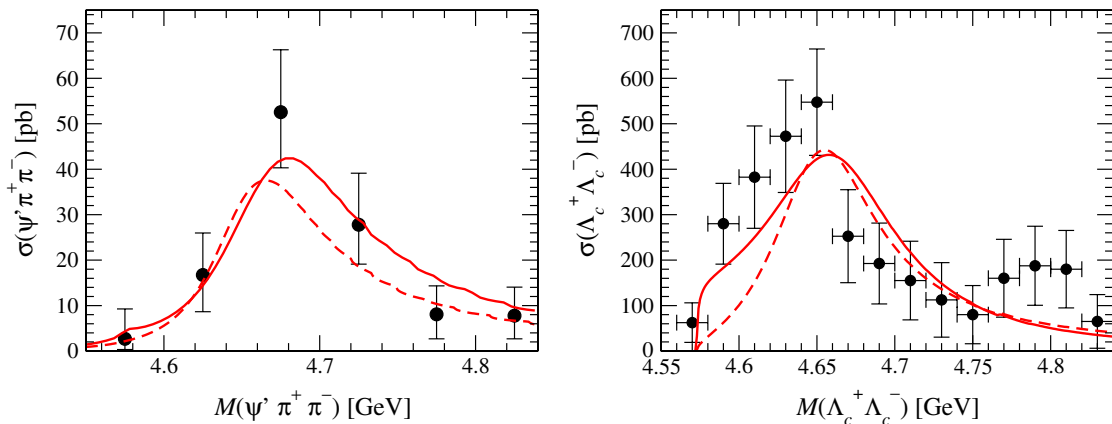


FIG. 2 (color online). The $\Lambda_c^+ \Lambda_c^-$ and $\psi' \pi^+ \pi^-$ invariant mass spectra. The data are taken from the Belle measurements. The solid curves are the results of the best fit, and the dashed curves are the results with FSI effects omitted.

this context, it is important to investigate the spin partner. Heavy quark spin symmetry in any case predicts the existence of a spin partner, however, the scenario outlined implies some very specific properties of that spin partner with respect to its mass and decay properties, as we will discuss now.

In Ref. [30], based on heavy quark spin symmetry, we predicted the presence of an $\eta'_c f_0(980)$ bound state, called Y_η , as the spin multiplet partner of the $\psi' f_0$ bound state. The mass of the Y_η should satisfy

$$M_{Y_\eta} = M_{Y(4660)} - (M_{\psi'} - M_{\eta'_c}) \quad (12)$$

to a high precision. Using the best fit value for the $Y(4660)$ mass given above and $M_{\eta'_c} = 3637 \pm 4$ MeV [27], one gets $M_{Y_\eta} = 4613 \pm 4$ MeV where the uncertainty is dominated by the one from the η'_c mass. Based on the same formalism as above, the line shape of the Y_η in the $\eta'_c \pi^+ \pi^-$ and the $\Lambda_c^+ \Lambda_c^-$ may be predicted. Heavy quark spin symmetry indicates that the coupling of the Y_η to the $\Lambda_c^+ \Lambda_c^-$ has the form, cf. Equation (5),

$$\mathcal{L}_{Y_\eta \Lambda_c \Lambda_c} = ig_{Y_\eta \Lambda_c \Lambda_c} \bar{\Lambda}_c \gamma^5 Y_\eta \Lambda_c, \quad (13)$$

with the same coupling constant as the $Y(4660)$.

In Fig. 3, the predictions for the Y_η line shapes in the $\eta'_c \pi^+ \pi^-$ and $\Lambda_c^+ \Lambda_c^-$ channels are shown in arbitrary units, however, with the relative normalization fixed. With the FSI, now in the 1S_0 partial wave and calculated again from the $\Lambda_c^+ \Lambda_c^-$ model of Ref. [23], shown in Fig. 1(b), the predicted line shapes are given by the solid curves, while the ones without FSI are given by the dashed curves. The Y_η mass is only about 40 MeV higher than the $\Lambda_c^+ \Lambda_c^-$ threshold, as a result the width of the Y_η is much smaller than that of the $Y(4660)$, and thus the line shapes are much narrower. The partial widths for decay into the $\eta'_c \pi^+ \pi^-$ and the $\Lambda_c^+ \Lambda_c^-$ channels are 8 MeV and 22 MeV, respectively. The ratio

$$\frac{\Gamma(Y_\eta \rightarrow \Lambda_c^+ \Lambda_c^-)}{\Gamma(Y_\eta \rightarrow \eta'_c \pi^+ \pi^-)} = 2.7 \quad (14)$$

is much smaller than the one for the $Y(4660)$ as a result of smaller phase spaces. Furthermore, the effect of the FSI is not so significant anymore. We expect that within other models for the spin partner of the $Y(4660)$ the discussed properties, especially the mass and the ratio of Eq. (14), will be very different.

In summary, taking into account the $\Lambda_c^+ \Lambda_c^-$ FSI, we found that the $X(4630)$ may be described as the same state as the $Y(4660)$ in the $\psi' f_0(980)$ bound state picture. One notices that there should be other open charm decay channels, such as decays into charmed and anticharmed mesons. We checked that an additional constant width from other possible decay channels of less than 30 MeV may still be accommodated. In principle, a polynomial background as in Ref. [7] allows one to improve the fit. Also possible interferences of the $X(4630)$ or $Y(4660)$ with other resonances, such as highly excited ψ resonances, could have an impact on the analysis. However, neither of these effects is under control quantitatively given the current quality of the experimental data. In addition, in the experimental data for the cross sections, the background is already subtracted. Hence, in our analysis, we refrain from considering them to reduce the number of parameters. Within the molecular picture for the $Y(4660)$, the presence of a Y_η with a mass given by Eq. (12) as the spin partner of the $\psi' f_0(980)$ bound state is almost unavoidable, since the spin-dependent interactions are highly suppressed by $1/m_c^2$, with m_c the charm quark mass [30]. Other models of the $Y(4660)$ should also provide a spin partner, but most probably with a different mass and different decay patterns. Thus, in order to test the molecular picture it is important to search for the Y_η experimentally, for instance in the decays $B^\pm \rightarrow \eta'_c K^\pm \pi^+ \pi^-$ which is expected to have a large branching fraction [30].

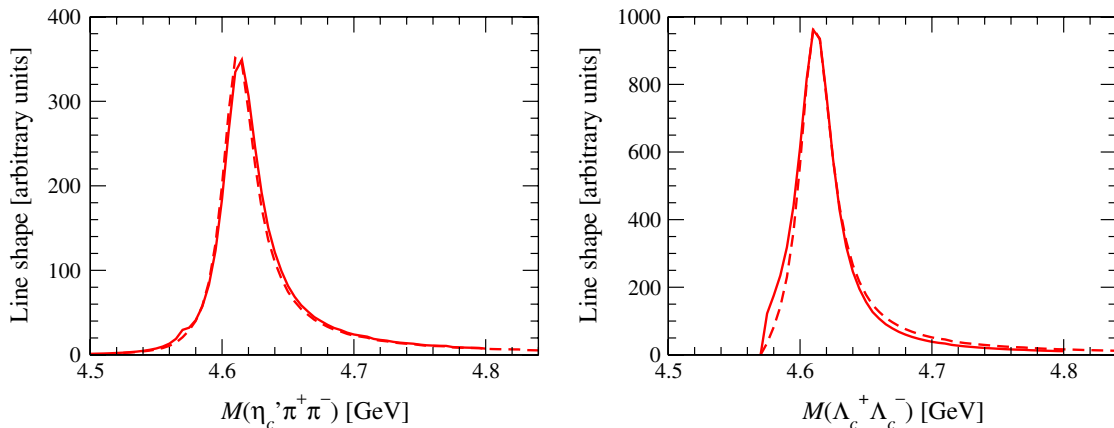


FIG. 3 (color online). Predictions of the Y_η line shapes in the $\eta'_c \pi^+ \pi^-$ and $\Lambda_c^+ \Lambda_c^-$ in arbitrary units. The solid and dashed curves represent results with and without FSI, respectively.

At last, we want to mention that a related observation was made by the *BABAR* Collaboration in the reaction $B^- \rightarrow \Lambda_c^+ \Lambda_c^- K^-$ [31]. They observed a structure at $2931 \pm 3 \pm 5$ MeV in the $\Lambda_c^+ K^-$ mass distribution. In the paper, the $\Lambda_c^+ \Lambda_c^-$ mass distribution is also provided, where one can see clearly two peaks. The measured branching ratio of the decay $B^- \rightarrow \Lambda_c^+ \Lambda_c^- K^-$ is of order 10^{-3} [31], which is several orders higher than the naive expectation 10^{-8} since this three-body decay is color-suppressed and with a small phase space [32]. In Ref. [32] Cheng *et al.* showed that the high suppression could be diminished, if there was a narrow hidden charm state with a mass of order 4.6–4.7 GeV or a charmed baryon, which was assumed to have $J^P = 1/2^+$, coupled to the $\Lambda_c^+ K^-$. We notice that the positions of the double peaks coincide with the masses of the $Y(4660)$ and the predicted Y_η . However, they could also be due to a charmed baryon Ξ_c with $J^P = 3/2^+$ —we found that a $J^P = 1/2^+$ Ξ_c baryon, as used in

Ref. [32], cannot describe the double peak structure in the $\Lambda_c^+ \Lambda_c^-$ mass distribution. Also some interference of a charmed baryon with the charmonia is possible. Better data with higher statistics, especially better Dalitz plots, would be very helpful in illuminating the situation.

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