PHYSICAL REVIEW D **82,** 094006 (2010)

$D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ rates on an unlikely impact of the littlest Higgs model with T parity

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(Received 23 August 2010; published 10 November 2010)

The decays $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ are highly suppressed in the standard model (SM) with the lion's share of the rate coming from long distance dynamics; $D^0 \to \mu^+\mu^-$ is driven predominantly by $D^0 \to \gamma\gamma \to \mu^+\mu^-$. Their present experimental bounds are small, yet much larger than SM predictions. New physics models like the littlest Higgs models with T parity (LHT) can induce even large indirect CP violation in D^0 transitions. One would guess that LHT has a "fighting chance" to affect these $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ rates in an observable way. We have found LHT contributions can be much larger than *short* distance SM amplitude by orders of magnitude. Yet those can barely compete with the *long* distance SM effects. *If* $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ modes are observed at greatly enhanced rates, LHT scenarios will *not* be candidates for generating such signals. LHT-like frameworks will not yield larger $D^0 \to \gamma\gamma/\mu^+\mu^-$ rates as they are constrained by B and K rare decays.

DOI: 10.1103/PhysRevD.82.094006 PACS numbers: 13.20.Fc, 11.15.Ex, 12.15.Ff

I. INTRODUCTION

Compelling evidence for $D^0 - \bar{D}^0$ oscillations has been presented [1–3]. The interpretation of the oscillation parameters x_D and y_D inferred from the data has not been settled: while they could contain sizable contributions from new physics (NP), they might still be compatible with what the SM can generate. Nevertheless it has sparked renewed interest in building NP models that can affect $\Delta C = 2$ dynamics significantly. This can be achieved even with models that had *not* been motivated by considerations of flavor dynamics. Littlest Higgs models with T parity provide an explicit class of examples that can generate sizable or even relatively large indirect CP violation in D^0 decays [4]. There are also other scenarios for such novel effects [5].

In littlest Higgs models with T parity (LHT) scenarios one gets new contributions also to $\Delta C = 1$ decays without hadrons in the final state, namely, $D^0 \to \gamma\gamma$, $\mu^+\mu^-$. Their rates are greatly suppressed both for fairly general reasons and those that are specific to SM dynamics. NP could then reveal its intervention through a significant enhancement in these rates. In this paper we present a rather detailed analysis of the possible impact of LHT scenarios: in contrast to the situation with indirect CP violation in D^0 transition we do not find any *significant* enhancements from LHT dynamics.

This paper is organized as follows: after discussing short distance (SD) and long distance (LD) SM contributions to $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ in Sec. II we sketch LHT models as an interesting class of NP scenarios and their potential impact on $D^0 - \bar{D}^0$ oscillations in Sec. III; then we analyze LHT contributions to $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ and present our quantitative findings on their potential impact in Sec. IV; after our general comments about flavor changing neutral currents (FCNCs) in LHT-like frameworks in Sec. V we give our conclusions in Sec. VI.

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II. SM CONTRIBUTIONS TO $D^0 \to \gamma\gamma$, $\mu^+\mu^-$

The rates for the modes $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ are highly suppressed, since they must be driven by charm changing neutral currents and also require the annihilation of the c and $\bar{u}$ quarks initially present in the D^0 meson; $D^0 \to \mu^+\mu^-$ is further reduced greatly by helicity suppression. The question is how much they are suppressed, which dynamics drive them, and whether they are short distance or long distance in nature.

A. $D^0 \to \gamma\gamma$

There is extensive literature both on $K_L \to \gamma\gamma$ and on the not yet observed $B^0 \to \gamma\gamma$. The former's reduced rate played an important role in the development of the SM, since it was one piece of evidence for nature's suppression of strangeness changing neutral currents. It was also realized that $K_L \to \gamma\gamma$ is driven mainly by long distance dynamics. $B^0 \to \gamma\gamma$ on the other hand should be shaped mainly by short distance contributions.

$D^0 \to \gamma\gamma$ can be treated in general analogy to $B^0 \to \gamma\gamma$ with the amplitude described by two form factors A and B:

$$T(M \to \gamma\gamma) = \epsilon_1^\mu \epsilon_2^\nu [(q_{1\mu} q_{2\nu} - q_1 . q_2 g_{\mu\nu}) A([Q\bar{q}]) + i\epsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma B([Q\bar{q}])]. \quad (1)$$

Those form factors receive contributions from two types of diagrams, the two-particle-reducible one (2PR) and the

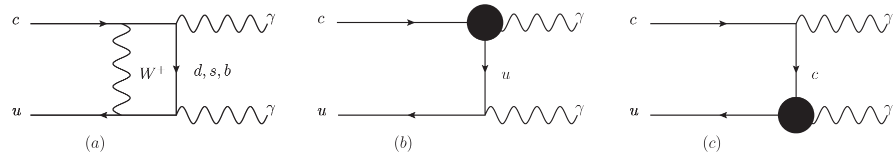

FIG. 1. The diagrams contributing to $D^0 \to \gamma\gamma$. (a) The 2PR (or 1PI) contribution. (a) and (c) The 1PR contribution. The vertices stand for $c \to u\gamma$ diagrams.

one-particle-reducible one (1PR) as shown in Fig. 1; the dark blob in the two diagrams on the right denote the effective $c \to u\gamma$ operator generated on the one-loop level.

Both types of diagrams had been evaluated for $K_L \to \gamma\gamma$ in Ref. [6] in terms of general quark masses. The pure electroweak contribution to the 1PR and 2PR amplitudes for $M^0 \equiv [Q\bar{q}] \to \gamma\gamma$ are given by

$$A^{\rm SD}([Q\bar{q}]) = i\frac{\sqrt{2}G_F\alpha}{\pi} f_M \sum_j V^*_{qj} V_{Qj} (A^{\rm 1PR}_j),$$
$$B^{\rm SD}([Q\bar{q}]) = \frac{\sqrt{2}G_F\alpha}{\pi} f_M \sum_j V^*_{qj} V_{Qj} (A^{\rm 2PR}_j + A^{\rm 1PR}_j),$$
$$A^{\rm 2PR}_j = (e_Q \pm 1)^2 \left[2 + \frac{4x_j}{x_M}\int_0^1 \frac{dy}{y} \ln\left[1 - y(1-y)\frac{x_M}{x_j}\right]\right],$$
$$A^{\rm 1PR}_j = \xi_M\{e_Q(e_Q \pm 1)F_{21}(x_j) + e_Q F_{22}(x_j)\},$$
$$F_{21}(x_j) = \frac{5}{3} + \frac{1 - 5x_j - 2x_j^2}{(1-x_j)^3} - \frac{6x_j^2}{(1-x_j)^4}\ln x_j,$$
$$F_{22}(x_j) = \frac{4}{3} + \frac{11x_j^2 - 7x_j + 2}{(1-x_j)^3} + \frac{6x_j^3}{(1-x_j)^4}\ln x_j,$$
$$\xi_M = \frac{m_M^2}{16}\left\langle M^0 \left| \frac{1}{p_1\cdot q_1} + \frac{1}{p_1\cdot q_2} + \frac{1}{p_2\cdot q_1} + \frac{1}{p_2\cdot q_2} \right| M^0 \right\rangle. \tag{2}$$

We have used the following notation: e_Q denotes the electric charge of the heavy quark Q, which is also carried by the lighter antiquark $\bar{q}$ inside the meson M^0; $x_M = m_M^2/m_W^2$ and $x_j = m_j^2/m_W^2$ where the nature of j depends on Q: For $Q = s$ or b, the internal summation j runs over the *up*-type quarks u, c, and t, while for $Q = c$—the case we will focus on—j runs over the *down*-type quarks d, s, and b. The $\pm$ in the 1PR and 2PR functions correspond to the cases of $Q = b$, s and $Q = c$ respectively. The functions $F_{21}(x_j)$ and $F_{22}(x_j)$ together correspond to the $Q\gamma\bar{q}$ effective vertex for an on-shell photon and only differ from [7] as they are valid for any arbitrary internal quark mass. ξ_M is a hadronic factor that can be safely taken as one for the D meson as it should be in the nonrelativistic limit, which is a pretty good approximation for the D meson. A and B correspond to the final state photons being in a state of parallel and perpendicular polarization, respectively. The branching fraction and the CP asymmetry parameter δ is then given by [8]

$$\mathrm{BR}_{\rm SD}(D^0 \to \gamma\gamma) = \frac{m_M^3}{64\pi}\Big(|A^{\rm SD}\big([Q\bar{q}]\big)|^2 + |B^{\rm SD}\big([Q\bar{q}]\big)|^2\Big), \tag{3}$$

$$\delta = \frac{|A^{\rm SD}([Q\bar{q}])|^2}{|A^{\rm SD}([Q\bar{q}])|^2 + |B^{\rm SD}([Q\bar{q}])|^2}. \tag{4}$$

Because of the very different mass hierarchies for the up- and down-type quarks and the very peculiar structure of the CKM parameters V_{ij}, one finds that the same algebraic expression yields very different results for these radiative K_L, D^0, and B^0 modes. For the $K_L \to \gamma\gamma$ decay, the 2PR dominates over the 1PR contribution by a few orders of magnitude. In $B^0 \to \gamma\gamma$ the 1PR contribution driven by $b \to s\gamma$ is comparable to the 2PR contribution [9]. Even if the $B^0 \to \gamma\gamma$ branching fraction is calculated solely from the $b \to s\gamma$ contribution, including the 2PR contribution raises the total branching fraction by about a factor of 2 and has been considered in quite a few works [9–12].

The situation is different for $D^0 \to \gamma\gamma$—and it is crafty in orders of QCD. The purely electroweak contributions from 1-loop without QCD are greatly dominated by 2PR over 1PR. Including QCD, leading logarithmic contributions of 1PR are significantly larger. Even more complete $\mathcal{O}(\alpha_S)$ corrections to the 1PR diagrams bring out the dominant contributions with amplitude $|A_{\rm SD}(D^0 \to \gamma\gamma)| \simeq (2.35 \pm 0.50) \times 10^4 \times |A^{\rm 1\text{-}loop}_{\rm SD}(D^0 \to \gamma\gamma)|$ [13,14]. From pure SD we get a branching fraction of

$$\mathrm{BR}^{\rm 2\text{-}loop}_{\rm SD}(D^0 \to \gamma\gamma) \simeq (3.6\text{–}8.1) \times 10^{-12}. \tag{5}$$

However, the $D^0 \to \gamma\gamma$ transition is dominated by long distance effects [14,15]:

$$\mathrm{BR}^{\rm LD}_{\rm SM}(D^0 \to \gamma\gamma) \sim (1\text{–}3) \times 10^{-8}. \tag{6}$$

This SM prediction is still substantially below the current experimental bound:

$$\mathrm{BR}_{\exp}(D^0 \to \gamma\gamma) \sim 2.7 \times 10^{-5}. \tag{7}$$

The LD contribution calculated in [14,15] is model dependent and even though they give similar contributions to the branching fraction, they have disagreement in the phases of the amplitude and the relative magnitude of the CP even and CP odd amplitudes. Taking into consideration this uncertainty in the LD estimates, our estimate for the CP asymmetry parameter δ (using the amplitudes calculated in [14,15]) stands at

$$\delta \sim (0.95)0.5. \tag{8}$$

B. $D^0 \to \mu^+\mu^-$

Realistically it seems one can improve the sensitivity for $D^0 \to \gamma\gamma$ only at an e^+e^- machine like a super-flavor or a super-tau-charm factory. The prospects for $D^0 \to \mu^+\mu^-$ are much better on the one hand, since one has a fighting chance to probe it in hadronic collisions, yet on the other hand the challenge is also much stiffer, since the rate for $D^0 \to \mu^+\mu^-$ suffers also from helicity suppression in the SM and most other NP scenarios.

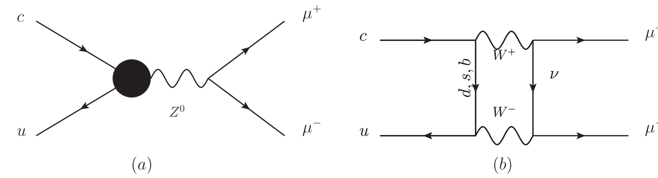

FIG. 2. The diagrams contributing to $D^0 \to \mu\mu$. (a) The $\bar{u}Z^0c$ effective vertex. (b) The W^+W^- contribution.

The SM short distance contributions are given by the diagrams shown in Fig. 2. Hence one obtains [16]

$$B_{\mathrm{SM}}^{\mathrm{SD}}(D^0 \to \mu^+\mu^-) = \frac{1}{\Gamma_D}\frac{G_F^2}{\pi}\left(\frac{\alpha}{4\pi \sin^2(\theta_W)}\right)^2 f_D^2 m_\mu^2 m_D \sqrt{1 - 4\frac{m_\mu^2}{m_D^2}} \sum_{j=d,s,b} |V_{uj}^* V_{cj}|^2 \left(Y_0(x_j) + \frac{\alpha_s}{4\pi} Y_1(x_j)\right)^2,$$
$$Y_0(x_j) = \frac{x_j}{8}\left[-\frac{2 + x_j}{1 - x_j} + \frac{3x_j - 6}{(1 - x_j)^2}\ln x_j\right],$$
$$Y_1(x_j) = \frac{4x_j + 16x_j^2 + 4x_j^3}{3(1 - x_j)^2} - \frac{4x_j - 10x_j^2 - x_j^3 - x_j^4}{(1 - x_j)^3}\ln x_j + \frac{2x_j - 14x_j^2 + x_j^3 - x_j^4}{2(1 - x_j)^3}\ln^2 x_j + \frac{2x_j + x_j^3}{(1 - x_j)^2} L_2(1 - x) + 8x_j \frac{\partial Y_0 x}{\partial x}\bigg|_{x=x_j} \ln x_\mu,$$
$$L_2(1 - x) = \int_1^x dt \frac{\ln t}{1 - t}, \tag{9}$$

where $x_\mu = \mu^2/m_W^2$. Numerically one finds

$$\mathrm{BR}_{\mathrm{SM}}^{\mathrm{SD}}(\mathrm{D}^0 \to \mu^+\mu^-) \sim 6 \times 10^{-19}, \tag{10}$$

i.e., a hopelessly tiny number.

However, a less than a tiny prediction in SM $D^0 \to \mu^+\mu^-$ can be made in analogy to $K_L \to \mu^+\mu^-$: a $\gamma\gamma$ intermediate state contributes from long distance dynamics [14]:

$$B_{\mathrm{SM}}^{LD}(D^0 \to \mu\mu) \sim 2.7 \times 10^{-5} B(D^0 \to \gamma\gamma). \tag{11}$$

Hence one arrives at

$$\mathrm{BR}_{\mathrm{SM}}(D^0 \to \mu\mu) \sim (2.7\text{–}8) \times 10^{-13} \tag{12}$$

using the SM estimate given above for $D^0 \to \gamma\gamma$.

III. ON LHT SCENARIOS

The SM predictions presented above leave a large range in rates for these rare transitions, where NP could *a priori* make its presence felt. So-called little Higgs models have been studied extensively over the past decade as a possible NP scenario [17,18]. Rather than attempting to solve the hierarchy problem, they "delay the day of reckoning" and address a maybe secondary, yet very relevant problem, namely, to reconcile the fact that the measured values of the electroweak parameters show no impact from NP even on the level of quantum corrections with the expectation that NP quanta exist with masses around the 1 TeV scale so that they could be produced at the LHC.

In this note we will analyze a subclass of little Higgs models, namely, LHT. In our view they possess several significant strong points:

(i) They contain several states with masses that can be below 1 TeV; i.e., those states should be produced and observed at the LHC.

(ii) Compared to supersymmetry (SUSY) models they introduce many fewer new entities and observable parameters.

(iii) Their motivation as sketched above lies outside of flavor dynamics. Thus, they have not been "cooked up" to induce striking effects in the decays of hadrons with strangeness, charm, or beauty.

(iv) Nevertheless they are not of the minimal flavor violating variety.

(v) Especially relevant for our study is the fact that they can have an observable impact on $D^0 - \bar{D}^0$ oscillations [4,19], as explained next.

A. Basics of LHT and impact on $D^0 - \bar{D}^0$ oscillations

Little Higgs models in general contain a large *global* symmetry that gets broken spontaneously to a lower subgroup leading to the emergence of a set of scalar particles as pseudo–Nambu-Goldstone Bosons of this broken symmetry that play the role of Higgs fields. These models push the hierarchy problem up to higher scales for its UV completion to deal with. To cancel out the radiative corrections to the SM Higgs mass one introduces a new set of gauge bosons and new fermions with judiciously arranged gauge couplings; the quadratic mass renormalization to the SM Higgs mass is achieved with the help of quanta of the *same statistics* unlike in SUSY extensions of the SM.

These models have to address a major challenge: Since nothing could prevent the tree-level coupling of the SM particles to these new, mostly heavier, particles, amongst other things, the ρ parameter gets shifted outside the allowed range for global symmetry breaking at the TeV scale. To address this, either the breaking scale had to be raised to above a few TeV or a new symmetry had to be incorporated into these models. Not surprisingly, preference has been given to keeping the breaking scale at a TeV so that new physics could be seen at the TeV scale.

Akin to what is generally done in SUSY, a discrete $\mathbb{Z}_2$ symmetry, T parity, has been postulated such that only pairs of the new quanta can couple to the SM states [20,21]. To accommodate this new symmetry into models that were already highly constrained, either an entire new set of scalars had to be brought into existence, or, as was done in the LHT, a set of mirror fermions had to be postulated [22].

The symmetry structure of the LHT (which it inherits from the littlest Higgs model [24]) is a global $SU(5)$ broken down to a global $SO(5)$ at the scale f. The T parity is implemented through the Callan-Coleman-Wess-Zumino formalism [21,25,26] using nonlinear representations of the symmetry group [27]. The Higgs sector (both T odd and even) is implemented as a nonlinear sigma model with a vacuum expectation value of f. The gauge group is a $[SU(2) \otimes U(1)] \otimes [SU(2) \otimes U(1)]$ broken down to $[SU(2) \otimes U(1)]_A$ which have the generators of the T-odd gauges and $[SU(2) \otimes U(1)]_V$ which become the SM electroweak gauge group.

The particle content of the LHT stands as follows: [29]

(i) T even

(a) All the SM particles.

(b) A heavy partner to the SM top.

(ii) T odd

(a) A set of T-odd $[SU(2) \otimes U(1)]$ heavy gauge bosons with the exact same couplings as the SM ones.

(b) A set of T-odd heavy mirror fermions which are familywise mass degenerate.

(c) A heavy Higgs triplet and a singlet.

While LHT have been crafted to deal with the nonobservation of NP in the electroweak parameters even on the quantum level, they generate non–minimal flavor violating dynamics. For imposing a $\mathbb{Z}_2$ symmetry in the LHT requires the introduction of the mirror fermions listed above. The two unitary 3×3 matrices V_{Hd} and V_{Hu} describing the mixing of the up- and down-type mirror quarks to the down- and up-type quarks, respectively, have no reason to exhibit the same pattern as the CKM matrix. However, since the mirror quark matrices can be diagonalized simultaneously, the two matrices are related to each other by the CKM matrix [30]:

$$V_{Hu}^\dagger V_{Hd} = V_{\rm CKM}. \tag{13}$$

Hence, assuming some form for V_{Hd} fixes V_{Hu} and vice versa. Since the CKM matrix does not differ too much from the identity matrix, one realizes that LHT contributions exhibit a clear correlation of the phases in the charm and strange sector.

The impact of LHT dynamics on K, B, and also D transitions has been explored in considerable detail, and potentially sizable effects have been identified [31–38]. Among other things it was found that sizable indirect CP violation can arise in D^0 decays [4,9] very close to the present experimental upper bounds [1–3]. This realization then naturally leads to the question whether they could affect the modes $D^0 \to \gamma\gamma$, $\mu^+\mu^-$ that are so highly suppressed in the SM in an observable way.

B. LHT contributions to $D^0 \to \gamma\gamma$

The LHT contributions to this decay channel will primarily come through the 1PR diagram where the W boson will be replaced by its T-odd partner, the W_H and the internal quarks will be replaced by their T-odd partner, the mirror quarks. These mirror quarks being very heavy ($\mathcal{O}(1\ \text{TeV})$), their contribution will be strictly short distance. The 2PR contribution benefits less from heavy fermion masses than the 1PR ones thus making it quite negligible. Redefining $x_{jH} = m_{jH}^2/m_{W_H}^2$, $x_H' = a x_H$ with $j = d_H,\ s_H,\ b_H$, $x_{DH} = m_D^2/m_{W_H}^2$, and V_{ij}^{Hu} as elements of V_{Hu}, where the subscript H refers to the T-odd sector, Eq. (2) will be modified as follows:

$$
\begin{aligned}
A^{\rm SD}_{\rm LHT}([Q\bar{q}]) &= i\frac{\sqrt{2}G_F\alpha}{\pi} f_M \sum_{j=d,s,b}\frac{v^2}{4f^2}[V^*_{uj_H}V_{cj_H}(A^{\rm 1PR}_{jH})],\\
B^{\rm SD}_{\rm LHT}([Q\bar{q}]) &= \frac{\sqrt{2}G_F\alpha}{\pi} f_M \sum_{j=d,s,b}\frac{v^2}{4f^2}[V^*_{uj_H}V_{cj_H}(A^{\rm 2PR}_{jH}+A^{\rm 1PR}_{jH})],\\
A^{\rm 2PR}_{H} &= (e_Q\pm 1)^2\left[2+\frac{4x_H}{x_M}\int_0^1\frac{dy}{y}\ln\left[1-y(1-y)\frac{x_M}{x_H}\right]\right],\\
A^{\rm 1PR}_{H} &= \xi_M e_Q\left[((e_Q\pm 1)F_{21}(x_H)+F_{22}(x_H))-\frac{1}{6}F_{22}(x_H)-\frac{1}{30}F_{22}(x'_H)\right],\\
F_{21}(x_H) &= \frac{5}{3}+\frac{1-5x_j-2x_j^2}{(1-x_j)^3}-\frac{6x_j^2}{(1-x_j)^4}\ln x_j, \qquad F_{22}(x_H)=\frac{4}{3}+\frac{11x_j^2-7x_j+2}{(1-x_j)^3}+\frac{6x_j^3}{(1-x_j)^4}\ln x_j,
\end{aligned}
\tag{14}
$$

where v is the vacuum expectation value of the SM Higgs and f is the vacuum expectation value which breaks the $SU(5)$ to $SO(5)$ in LHT. The two additional terms in the 1PR contribution which are proportional to $F_{22}(x_H)$ come from the effective $Q\gamma\bar{q}$ vertex with Z_H or A_H and heavy up-type quarks in the loop. In Sec. IV we will combine these amplitudes with the SM ones.

C. Impact on $D^0 \to \mu^+\mu^-$

The LHT contribution can come from three sources:

(i) Z_L penguins can contribute with the SM gauge boson in the loop being replaced by the corresponding heavy gauge boson and the internal SM quarks by their mirror partners. There can also be Z_L penguins with only neutral gauge bosons as the $\bar{u}_l Z_H u_H$ and $\bar{u}_l A_H u_H$ vertices are possible. Z_H or A_H penguins are forbidden by T parity.

(ii) There can be contributions from the box diagrams with the charged SM gauge bosons replaced by their T-odd partners and the same for the internal quarks. Box contributions can also come from the charged SM bosons being replaced by the neutral T-odd bosons and the internal quarks being replaced by the up-type mirror fermions [39].

(iii) Any of the aforementioned LHT contributions to $D^0 \to \gamma\gamma$ will affect also $D^0 \to \gamma\gamma \to \mu^+\mu^-$.

Full amplitudes have of course to be gauge invariant. The Z_L penguin contributions and those from the box diagrams have been calculated both in the unitary and 't Hooft-Feynman gauge [38]. The LHT contribution can be calculated by replacing the sum of $Y_0(x)$ and $Y_1(x)$ in Eq. (9) with $J^{\mu\bar{\mu}}(z, y)$ given by

$$
\begin{aligned}
J^{\mu\bar{\mu}}(z_i, y) = \frac{1}{64}\frac{v^2}{f^2}\Big[& z_i S(z_i) + F^{\mu\bar{\mu}}(z_i, y; W_H)\\
&+ 4[G(z_i, y; Z_H) + G_1(z'_i, y'; A_H)\\
&+ G_2(z_i, y; \eta)]\Big],
\end{aligned}
\tag{15}
$$

where

$$
\begin{aligned}
z_i &= \frac{m^2_{Hi}}{m^2_{W_H}} = \frac{m^2_{Hi}}{m^2_{Z_H}}, \qquad z'_i = a z_i, \qquad a = \frac{5}{\tan^2\theta_W},\\
y &= \frac{m^2_{HL}}{m^2_{W_H}} = \frac{m^2_{HL}}{m^2_{Z_H}}, \qquad y' = a y, \qquad \eta = \frac{1}{a},\\
S(z_i) &= \frac{z_i^2 - 2z_i + 4}{(1-z_i)^2}\ln(z_i) + \frac{7-z_i}{2(1-z_i)}.
\end{aligned}
\tag{16}
$$

$F^{\mu\bar{\mu}}(z_i, y; W_H)$, $G(z_i, y; Z_H)$, $G_1(z'_i, y'; A_H)$, $G_2(z_i, y; \eta)$ [30] correspond to the contributions from the $W_H W_H$, $Z_H Z_H$, $A_H A_H$, $Z_H A_H$ box diagrams, respectively. The function $S(z_i)$ is a contribution from the Z_L penguin diagram with internal mirror quarks. The replacement of a singularity [38] in $J^{\mu\bar{\mu}}(z_i, y)$ with the function $S(z_i)$ was first pointed out by [40] and subsequently by [41] and incorporated as an update to FCNC calculations in B and K physics cited earlier [37].

IV. NUMERICAL FINDINGS ON LHT CONTRIBUTIONS

Before we go into the details of the LHT contributions, let us clarify the parameter space that was probed and the value of the LHT parameters that were kept fixed in this study. The LHT has 20 new parameters of which the ones which will be relevant to us are as follows:

(i) The LHT breaking scale $f = 1$ TeV is fixed by choice.

(ii) The masses of the three T-odd mirror quarks, m_{dH}, m_{sH}, m_{bH} range from 300 to 1000 GeV.

(iii) There are three independent mixing angles in V_{Hu}, θ^{Hu}_{12}, θ^{Hu}_{13}, θ^{Hu}_{23} and

(iv) three irreducible phases in V_{Hu}, δ^{Hu}_{12}, δ^{Hu}_{13}, δ^{Hu}_{23}.

The parameter space used for these analyses is a set that satisfies all experimental constrains from B and K physics. A small parameter set was also used which did not follow such constraints to check whether constraints from B and K physics affect LHT contributions to D physics.

The mass spectrum for both the parameter sets is illustrated in Figs. 3. Using Eq. (13), the angles and phases of

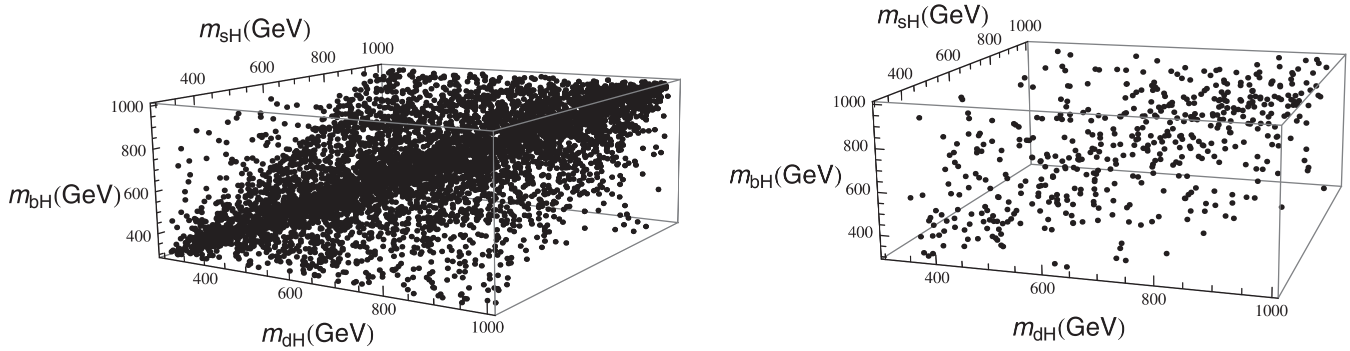

FIG. 3. Histogram of the parameter space of the mass of the mirror quarks.

V_{Hu} were calculated from those of V_{Hd} and hence were constrained by B and K physics too for the first parameter set and not so for the second. Histograms of the parameter space of the angles and phases are shown in Figs. 4. The angles and phases are familywise paired.

A. $D^0 \to \gamma\gamma$

The LHT contribution to the branching fraction amounts at most to $O(10\%)$ of the SM short distance contribution; for most of the LHT parameter space it reaches merely a few percent as seen from Fig. 5. The unconstrained

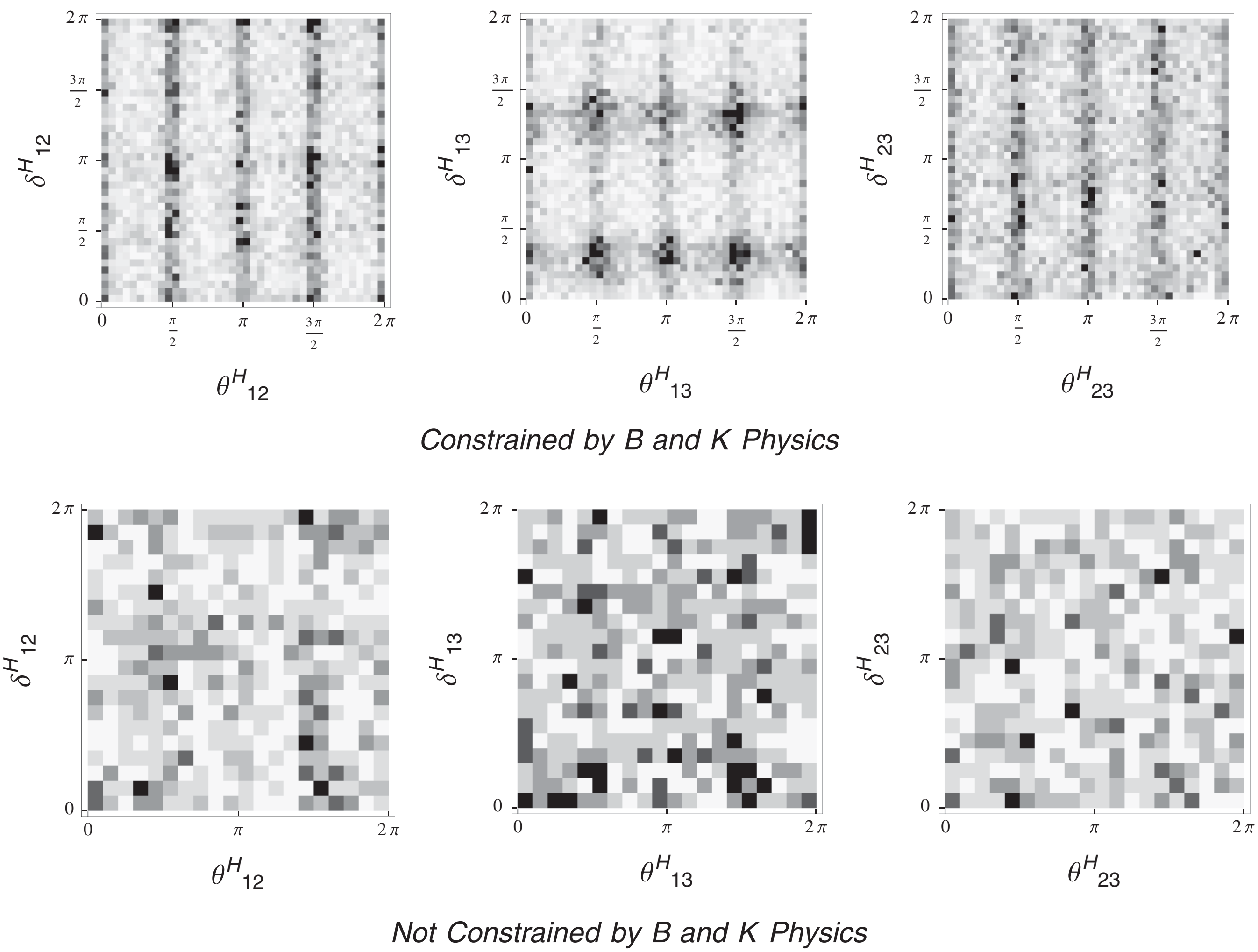

FIG. 4. Histogram of the parameter space of the angles and phases in V_{Hd}. Counts in any bin are represented in grayscale, darker representing higher density.

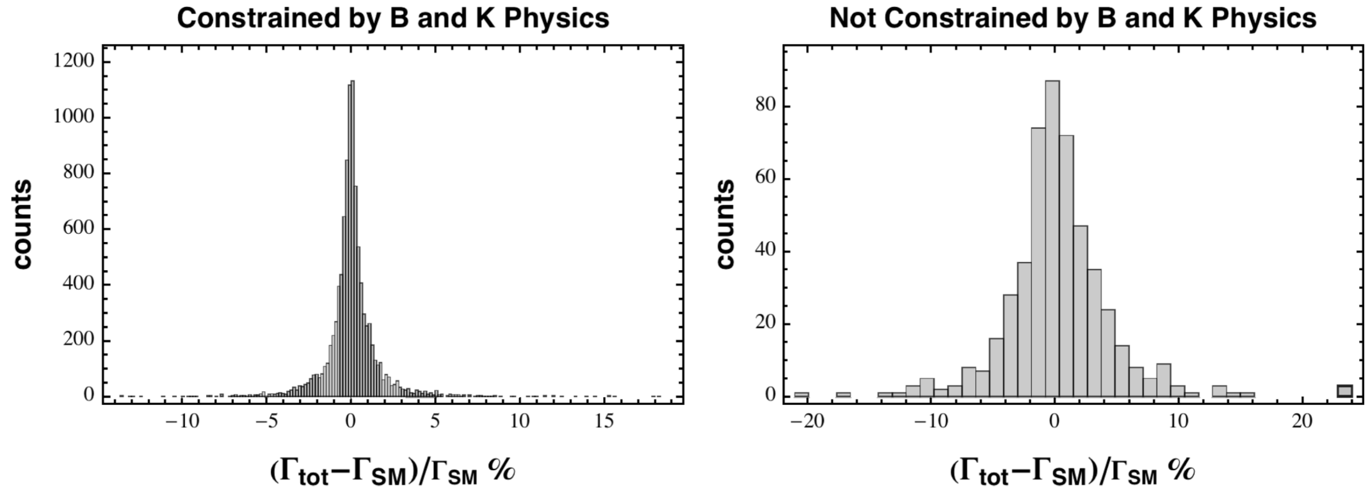

FIG. 5. Histogram of percentage enhancement to $\Gamma_{SD}(D^0 \to \gamma\gamma)$ due to LHT contributions.

parameter set gives us a very similar picture in Fig. 5. The LHT contributions hardly affect the *CP* asymmetry parameter δ. The dominance of LD contribution in the branching fraction and δ effectively swamps out any possible contribution that LHT can make to these. In view of the experimental challenges one can ''realistically'' hope to significantly improve the sensitivity for observing $D^0 \to \gamma\gamma$ only at a super-flavor factory. Yet even if one managed to measure this transition one could never claim a case for having found a LHT contribution considering the accuracy (or lack thereof) of the SM estimate given above.

B. $D^0 \to \mu^+\mu^-$

The potential impact of LHT dynamics has been analyzed for different lepton masses, yet only data for two of the mirror neutrino masses will be represented by the graphs below. The first choice for the mirror neutrino mass is 400 GeV so that it falls within the mirror quark mass spectrum used in our studies. The second choice is a mass of 1100 GeV so that it lies outside the mirror quark mass spectrum.

As explained above, the dominant SM contribution to $D^0 \to \mu^+\mu^-$ arises from $D^0 \to \gamma\gamma \to \mu^+\mu^-$, where it hardly matters, whether the intermediate transition $D^0 \to \gamma\gamma$ is generated by long or short distance effects.

We see that LHT contributions are orders of magnitude larger than the SM short distance contribution to the extent that LHT contributions alone can be comparable to the long distance contribution to this channel in some regions of the parameter space. A very small region of the LHT phase space brings about 6 orders of magnitude enhancement over the SM SD contribution. However the SM LD contribution is projected to be 6 to 7 orders of magnitude larger than the SM SD contribution and hence can easily be the dominant one. Enhancement to the total rate seems to be possible in rare cases, but only by a factor of 2 as seen in Fig. 6. The constraints set by B and K physics do not change much of the analyses as is evident from Fig. 6. However, it is almost impossible in this parameter space for LHT to provide the dominant contribution unless there is a larger mass splitting between the three generations of the mirror quark family.

V. FCNC(S) IN LHT-LIKE FRAMEWORKS

A careful analysis of the results of this study reveals that certain general conclusions can be drawn *beyond* the premises of LHT. In particular, the way with LHT affecting the rare decay channels depends purely on the *flavor* structure of LHT and not on the way this NP model is implemented as a *whole*. What defines the flavor structure of such models are

(i) A second sector of fermions that are an exact copy of the SM ones.
(ii) Mass mixing matrices which are unitary and loosely connected to V_{CKM} [Eq. (13)].
(iii) Possible large angles and phases in the mass mixing matrices.
(iv) Possible large hierarchies in the masses of the mirror quarks.
(v) A symmetry, like T parity, segregating the NP sector from the SM sector, hence forbidding tree-level FCNC.

We have seen that LHT can generate a significant effect in the $D^0 - \bar{D}^0$ oscillations [4]. However, we see that is not true for the $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ channels. The reasons are as follows: Both the $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ channels are dominated by SM LD contributions which are larger than the SM SD contributions by orders of magnitude. Since tree-level coupling to the heavier gauge bosons are forbidden by T parity, the only LHT contributions are through loops, which are essentially SM SD operators because of the flavor structure of LHT. It is true that LHT with its heavier gauge bosons and heavy

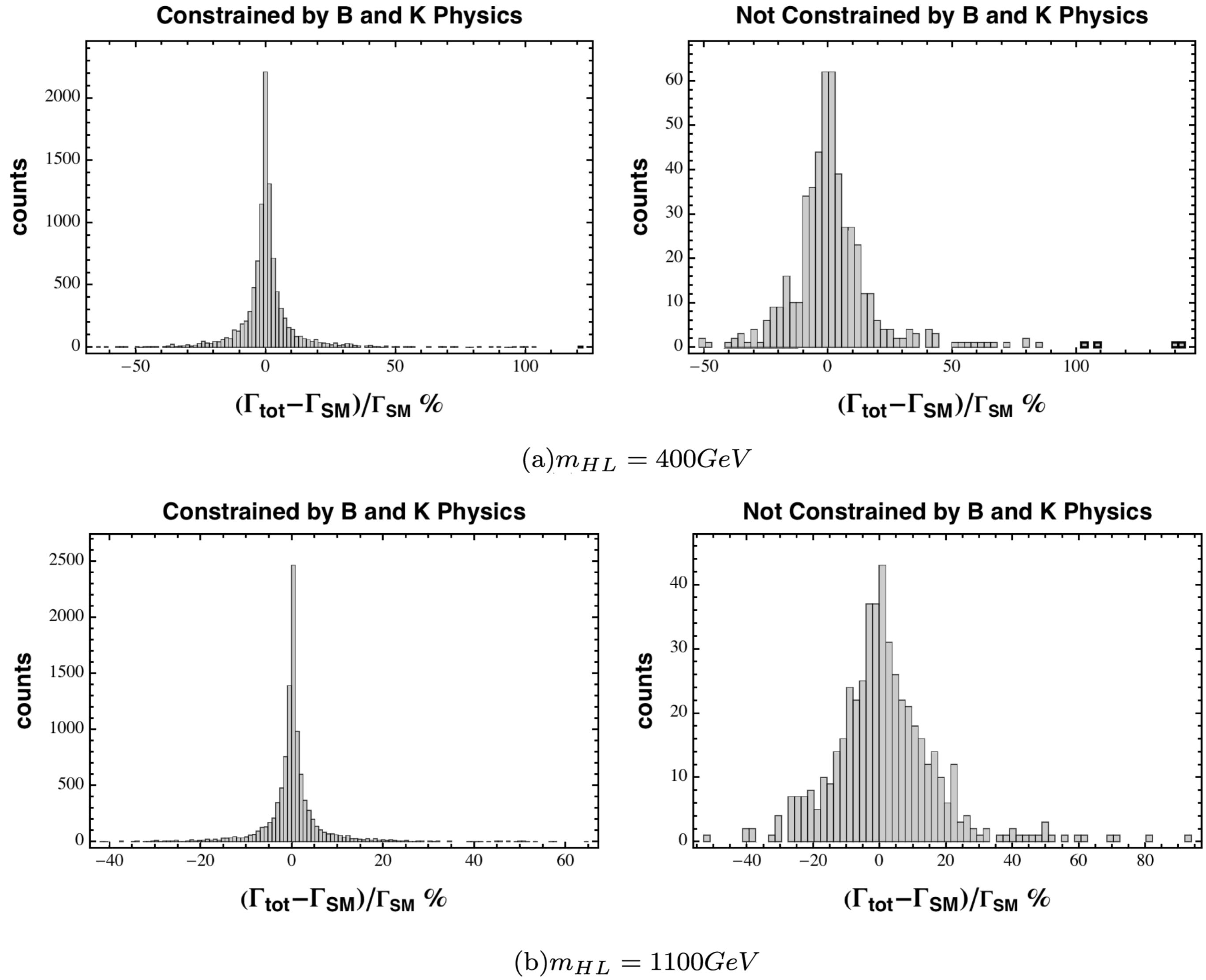

FIG. 6. Histograms of percentage enhancement to $\Gamma(D^0 \rightarrow \mu^+\mu^-)$ due to LHT contributions for mirror neutrino mass of (a) $m_{HL} = 400GeV$ and (b) $m_{HL} = 1100$ GeV.

quarks can produce orders of magnitude *enhancements* to SM SD contributions—but typically falls shy of or at best equals the LD contributions that these channels get from SM LD operators.

In [42] an interesting point has been shown that if NP can make a significant contribution to $D^0 - \bar{D}^0$ oscillations, then it can enhance the $D^0 \rightarrow \mu^+\mu^-$ channel well beyond the SM. We do not disagree in general. Yet our study shows that LHT and LHT-like [21,28] frameworks cannot produce a significant contribution to the $D^0 \rightarrow \mu^+\mu^-$ rate beyond the SM while a significant or even dominant signal can occur for $D^0 - \bar{D}^0$ oscillations. When a NP has ''construction plans'' not only for the charm sector, but also for strange and beauty sectors—what one has for a LHT-like framework—there are connections for charm, strange, and beauty hadrons. Weak experimental constraints are not very stringent in D, but are very impressive in B and K physics. Hence, these latter constraints can and have been used extensively to constrain the parameter space of any NP Models.

In our studies we have compared $D^0 \rightarrow \gamma\gamma$, $\mu^+\mu^-$ with $K_L \rightarrow \gamma\gamma$, $\mu^+\mu^-$, and $B^0 \rightarrow \gamma\gamma$, $\mu^+\mu^-$. From these numerical calculations, we can conclusively prove that given the constraints from B and K physics, significant effects over and above SM are possible in $D^0 - \bar{D}^0$ oscillations, but not in $D^0 \rightarrow \gamma\gamma$ and $D^0 \rightarrow \mu^+\mu^-$ decays.

The reason for this is as follows. LHT gives us the freedom of choosing large mixing angles and phases in the mixing matrices and also in the mass hierarchies of the quarks. Hence, large effects over SM SD, and possibly over SM LD, can be observed if we utilize both these freedoms. However, experimental constraints from B and K physics force us to choose between either large angles and phases or large mass hierarchies. In the current study we have chosen to live with large angles and phases rather than large mirror quark mass hierarchies. On the other hand, we could have made the mixing matrices (V_{Hd}, V_{Hu}) very diagonal and had large mass hierarchies but that would imply possibilities of the existence of quarks beyond the current experimental reach. In either case, LHT makes a significant contribution over SM SD rates but fails to overpower SM LD rates. Moreover, if any NP model is able to make a large impact on the $D^0 \to \gamma\gamma$ decay, it will also, indirectly, help in increasing the SM contribution to $D^0 \to \mu^+\mu^-$ as that depends primarily on the two photon unitary contribution [3]. This might well wash out any NP contribution to the $D^0 \to \mu^+\mu^-$ decay. Hence, in the absence of large mass hierarchies amongst the new set of quarks, it is not possible to generate large effects in the $\Delta F = 1$ processes although the same can be done in the $\Delta F = 2$ processes, even if we allow for large angles and phases. In the absence of large hierarchies in the new quark sector, unitarity of the mass mixing matrices result in very tiny FCNCs, something akin to what is seen in FCNCs in the SM. The possibility of such large mass hierarchies are limited by experimental limits on B and K decay branching fractions and CP violating parameters if we chose to use large angles and phases in the mixing matrices. Making the other choice does not help either. Hence, experimental observation of large effects in $\Delta F = 1$ processes will automatically lead to a loss of viability of models of this nature.

The littlest Higgs model with T parity has undergone extensive scrutiny in the past few years as a major candidate for the viable little Higgs class of theories. Many suggestions have been given for theoretical restructuring and for avoiding heavy constraints from experimental bounds [28,43,44]. The beauty of our analysis is that it is immune to changes in the way the specific flavor of the littlest Higgs model with T parity is implemented; it only depends on the final flavor structure of the fermionic sector, which remains unchanged in all these models. Hence, our conclusions are more general than the specific model that we have worked with.

Recently some light has been shed on what could possibly be UV completion of the effective littlest Higgs model [45–47]. The primary motivation for these models is the cancellation of anomalies [43,48] that arise from the Wess-Zumino-Witten [49,50] terms in the Lagrangian, which can save the lightest T-odd particle as a dark matter candidate. Furthermore, they address the obvious problem of the hierarchy between the 10 TeV and the Planck scale. Some of these models can possibly introduce new TeV scale particles into the low energy effective theory which, if they have the correct quantum numbers, can bring about new contributions to FCNCs. A more careful look at these brings us to some more general conclusions. UV completion models for the little Higgs models are usually constructed with the following constraints in mind which are interconnected amongst themselves:

(i) The breaking scale of the effective little Higgs models is preferred to be around 1 TeV.

(ii) FCNCs do not suffer from contributions significant enough to break the ρ parameter, i.e., the tree-level contribution to FCNC from NP is naturally suppressed at such low breaking scales.

(iii) Enhancement of FCNCs can appear only through loop contributions.

Under such conditions, any new contributions can at best be the size of the ones we have seen from LHT. Hence, we reemphasize, the conclusions that we have drawn shall hold good. On a final note we would also like to comment that since the lightest T-odd particle as a dark matter candidate need not be absolutely stable (à la proton), T parity (or any other discrete symmetry protecting it) need not be exact. How inexact the discrete symmetry can be is, of course, an open question and depends largely on how it is implemented.

VI. CONCLUSIONS

With the SM predicting tiny rates for $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ observing those modes would reveal the intervention of NP. That statement is still valid. What we have found in this paper is that LHT dynamics could not provide significantly enhanced rates even for those scenarios of LHT parameters that can generate observable indirect CP violation in D^0 transitions. To be sure LHT contributions can greatly enhance SM short distance rates even by orders of magnitude—in particular for $D^0 \to \mu^+\mu^-$—yet the SM short distance amplitude is so tiny relative to their long distance counterparts—at least as they are presently estimated—that the total rate is increased only very moderately. In that sense our findings are negative, though still significant: while LHT dynamics can generate striking effects in $D^0 - \bar{D}^0$ oscillations, they can barely enhance the rates for $D^0 \to \gamma\gamma$ and $D^0 \to \mu^+\mu^-$ beyond what one might conceivably predict for SM long distance contributions.

ACKNOWLEDGMENTS

We wish to thank Monika Blanke for her helpful comments on this work. This work was supported by the NSF under Grant No. PHY-0807959.

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