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Orbital structure of quarks inside the nucleon in the light-cone diquark model

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We study the orbital angular momentum structure of the quarks inside the proton. By employing the light-cone diquark model and the overlap representation formalism, we calculate the chiral-even generalized parton distribution functions $H_q(x, \xi, \Delta^2)$, $\tilde{H}_q(x, \xi, \Delta^2)$, and $E_q(x, \xi, \Delta^2)$ at zero skewedness for $q = u$ and d quarks. In our model, E_u and E_d have opposite sign with similar size. Those generalized parton distribution functions are applied to calculate the orbital angular momentum distributions, showing that $L_u(x)$ is positive, while $L_d(x)$ is consistent with zero compared with $L_u(x)$. We introduce the impact parameter dependence of the quark orbital angular momentum distribution. It describes the position space distribution of the quark orbital angular momentum at given x. We found that the impact parameter dependence of the quark orbital angular momentum distribution is axially symmetric in the light-cone diquark model.

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I. INTRODUCTION

Understanding the spin structure of the nucleon is one of the most important challenges in hadron physics. The naive picture that the nucleon spin is provided totally by the spin of its three valence quarks was proved to be wrong by the experimental measurements. The EMC result [1] indicates that a large fraction of the nucleon spin is carried by other sources of angular momentum. There have been many attempts to explain the EMC result from the fundamental theory. Besides the angular momentum of the gluon, the quark orbital angular momentum (OAM) [2] is believed to provide a substantial part of the nucleon spin. In the past two decades, the theoretical description of the quark OAM distribution has been established [3–7]. It has been shown by Ji that the quark angular momentum can be separated into [4] the usual quark helicity and a gauge-invariant orbital contribution L_q. One of the advantages of this decomposition is that L_q is related to generalized parton distributions (GPDs) [8–13], the experimental observables that enter the descriptions of hard exclusive processes, such as deeply virtual Compton processes [9,14] and meson exclusive production [15,16].

Moreover, recently it has been found that the quark OAM plays an essential role through spin-orbit correlations in some novel phenomena that appear in the physics of single spin asymmetries, among which a particular transverse momentum distribution [17–19]—the Sivers function [20,21]—has attracted a lot of interest, since it is an essential piece in our understanding of the single spin asymmetries observed in semi-inclusive deeply inelastic scattering. These single spin asymmetries have been measured recently by both the HERMES [22,23] and COMPASS [24–26] Collaborations. An interesting observation is that there is a quantitative relation [27–29] between the Sivers function $f_{1T}^{\perp q}$ and the GPD E^q, although it is obtained in a model-dependent way, suggesting that similar underlying physics plays a role for nonzero $f_{1T}^{\perp q}$ and E^q. Similar relations have been obtained between Boer-Muldes functions and chiral-odd quark GPDs [30,31]. A complete study on the relations between the GPDs and transverse momentum distributions has been presented in Ref. [32], which becomes more transparent through the conception of general parton correlation functions [33,34]. The relations between GPDs and transverse momentum distributions are more intuitive [35,36] if we interpret GPDs in the transverse position (impact parameter) space [37–40]. Of particular interest is the case of zero skewedness ($\xi = 0$), where a density interpretation of GPDs in impact parameter space may be obtained [37]. In particular, this interpretation allows one to study a three-dimensional picture of the nucleon.

In this paper, we study the orbital angular momentum structure of the quarks inside the proton in a light-cone diquark model. In this model the light-cone wave function of the proton can be obtained. It is then convenient to express the physical observables in the overlap representation formalism [41,42]. We calculate the chiral-even GPDs $H_q(x, \xi, \Delta^2)$, $\tilde{H}_q(x, \xi, \Delta^2)$, and $E_q(x, \xi, \Delta^2)$ at the zero skewedness for $q = u$ and d. We found that E_u and E_d have opposite sign with similar size in this model. The GPDs are applied to calculate the quark OAM distributions, showing that $L_u(x)$ is positive, while $L_d(x)$ is consistent with zero compared with $L_u(x)$, and the net OAM of the u and d quarks is positive. We also introduce the impact parameter dependence of quark OAM distribution. It describes the position space distribution of the quark OAM at given x. We found that the impact parameter dependence of quark OAM distribution is axially symmetric in the light-cone diquark model.

The paper is organized in the following way. In Sec. II, we review the GPDs and their connections with quark orbital angular momentum. In Sec. III, we present the calculation of chiral-even GPDs from the light-cone diquark model, by applying the overlap representation formalism. We also show the calculation of the quark OAM in the same approach. In Sec. IV, we introduce the impact parameter dependence of quark OAM distribution and present results of the position space distribution for an orbiting u quark, from the light-cone diquark model. We summarize our paper in Sec. V.

II. SYSTEMATICS OF GENERALIZED PARTON DISTRIBUTIONS AND THE ORBITAL ANGULAR MOMENTUM

GPDs are introduced to describe the exclusive process in which the momenta of the incoming and outgoing nucleon in the symmetric frame are given by

$$p = P + \tfrac{1}{2}\Delta, \qquad p' = P - \tfrac{1}{2}\Delta \tag{1}$$

and satisfy $p^2 = p'^2 = M^2$, with M denoting the nucleon mass. The GPDs depend on the following variables:

$$x = \frac{k^+}{P^+}, \qquad \xi = -\frac{\Delta^+}{2P^+}, \qquad t = \Delta^2, \tag{2}$$

where the light-cone coordinates are defined by

$$a^{\pm} = (a^0 \pm a^3), \qquad \vec{a}_T = (a^1, a^2) \tag{3}$$

for a generic 4-vector a. In a physical process the so-called skewness ξ and the momentum transfer t to the nucleon are fixed by the external kinematics, whereas x is typically an integration variable.

The chiral-even GPDs H_q, E_q and $\tilde{H}_q$, $\tilde{E}_q$ for quarks are defined through matrix elements of the bilinear vector and axial vector currents on the light cone:

$$\int \frac{dy^-}{8\pi} e^{ixP^+y^-/2}\langle p'|\bar{\psi}(0)\gamma^+\psi(y)|p\rangle|_{y^+=0,y_\perp=0} = \frac{1}{2P^+}\bar{U}(p')\bigg(\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M}E(x,\xi,t)\bigg)U(p), \tag{4}$$

$$\int \frac{dy^-}{8\pi} e^{ixP^+y^-/2}\langle p'|\bar{\psi}(0)\gamma^+\gamma_5\psi(y)|p\rangle|_{y^+=0,y_\perp=0} = \frac{1}{2P^+}\bar{U}(p')\bigg(\gamma^+\gamma_5\tilde{H}(x,\xi,t) + \frac{\Delta^+\gamma_5}{2M}\tilde{E}(x,\xi,t)\bigg)U(p). \tag{5}$$

An important implication of GPDs is that they are related to the OAM of the quark, which is expected to provide an essential contribution to the total spin of the nucleon. Here we follow the decomposition of the nucleon spin introduced by Ji [4]:

$$J^z = J_q^z + J_g^z = \frac{1}{2}\sum_q \Delta q + \sum_q L_q^z + J_g^z = \frac{1}{2}, \tag{6}$$

where Δq, L_q^z, and J_g^z denote the quark spin, quark OAM, and gluon angular momentum, respectively, which comes from the expectation value of the operator

$$M^{0xy} = \frac{1}{2}\sum_q \psi_q^\dagger \Sigma^z \psi_q + \sum_q \psi_q^\dagger(\vec{r}\times i\vec{D})^z\psi_q + [\vec{r}\times(\vec{E}\times\vec{B})]^z. \tag{7}$$

Note that in the literature [3,6,43] there are some other ways to decompose the nucleon spin. The advantage of the decomposition of J_q to Δq and L_q^z in (7) is that it ensures the gauge invariance of the operators. There has also been discussion about whether the gluon angular momentum can be further decomposed gauge-invariantly. In this work we will not consider the gluon contribution.

The quark OAM distribution $L_q(x)$ can then be defined as the expectation value of operator

$$\hat{O}_L = \int d\eta e^{-ixP^+\eta}\psi_q^\dagger(\vec{r}\times i\vec{D})^z\psi_q, \tag{8}$$

between the proton state $|PS\rangle$:

$$L_q^z(x) = \langle PS|\hat{O}_L|PS\rangle. \tag{9}$$

The quark OAM distribution can be obtained from [4,44]

$$L_q^z(x) = \tfrac{1}{2}\{x[H_q(x,0,0) + E_q(x,0,0)] - \tilde{H}_q(x,0,0)\}, \tag{10}$$

where $H_q(x,0,0)$, $\tilde{H}_q(x,0,0)$, and $E_q(x,0,0)$ are the forward limits of GPDs. Furthermore, the former two are the unpolarized and helicity distributions for the nucleon, respectively,

$$q(x) = H_q(x,0,0), \qquad \Delta q(x) = \tilde{H}_q(x,0,0), \tag{11}$$

and $E_q(x,0,0)$ is related to the anomalous magnetic momentum of the nucleon in the following way:

$$\int_0^1 dx E_q(x,0,0) = \kappa_q, \tag{12}$$

where κ_q is the contribution of quark flavor q to the nucleon anomalous magnetic momentum.

III. GPDS IN THE LIGHT-CONE DIQUARK MODEL FROM THE OVERLAP REPRESENTATION FORMALISM

In this section we present the calculation of the GPDs in the light-cone diquark model from the overlap representation formalism. The proton wave function with helicity $\Uparrow$, $\Downarrow$ in the SU(6) quark-diquark model [45–47] in the instant form is written as

$$\Psi^{\Uparrow,\Downarrow}(qD) = \frac{1}{\sqrt{2}}\varphi_V|qV\rangle^{\Uparrow,\Downarrow} + \frac{1}{\sqrt{2}}\varphi_S|qS\rangle^{\Uparrow,\Downarrow}, \tag{13}$$

where $D = V, S$ denotes the vector diquark and scalar diquark, respectively. Then

$$\begin{aligned}|qV\rangle^{\Uparrow,\Downarrow} &= \pm\tfrac{1}{3}[V_0(ud)u^{\uparrow,\downarrow} - \sqrt{2}V_{\pm1}(ud)u^{\downarrow,\uparrow} - \sqrt{2}V_0(uu)d^{\uparrow,\downarrow} \\ &\quad + 2V_{\pm1}(uu)d^{\downarrow,\uparrow}]; \\ |qS\rangle^{\Uparrow,\Downarrow} &= S(ud)u^{\uparrow,\downarrow}.\end{aligned} \tag{14}$$

The spin part of the light-cone wave function of the proton can be obtained from the instant form of the wave function by a Melosh rotation. For a spin-$\frac{1}{2}$ particle, the Melosh transformations are known to be [48]

$$\begin{aligned}\chi_T^{\uparrow} &= \omega[(k^+ + m_q)\chi_F^{\uparrow} - k^R\chi_F^{\downarrow}], \\ \chi_T^{\downarrow} &= \omega[(k^+ + m_q)\chi_F^{\downarrow} + k^L\chi_F^{\uparrow}],\end{aligned} \tag{15}$$

where χ_T and χ_F are instant and light-cone spinors, respectively, $\omega = [2k^+(k^0 + m_q)]^{-1/2}$, $k^{R,L} = k^1 \pm ik^2$, and m_q is the quark mass. In this work, for simplicity we treat the diquark as a point particle. The scalar diquark does not transform, since it has zero spin. For the spin-1 vector diquark, the Melosh transformations are given by [49]

$$\begin{aligned}V_T^1 &= \omega_V^2[(k_V^+ + \lambda_V)^2V_F^1 - \sqrt{2}(k_V^+ + \lambda_V)k_V^RV_F^0 + k_V^{R2}V_F^{-1}], \\ V_T^0 &= \omega_V^2[\sqrt{2}(k_V^+ + \lambda_V)k_V^LV_F^1 + 2((k_V^0 + \lambda_V)k_V^+ - k_V^Rk_V^L)V_F^0 \\ &\quad - \sqrt{2}(k_V^+ + \lambda_V)k_V^RV_F^{-1}], \\ V_T^{-1} &= \omega_V^2[k_V^{L2}V_F^1 + \sqrt{2}(k_V^+ + \lambda_V)k_V^LV_F^0 + (k_V^+ + \lambda_V)^2V_F^{-1}].\end{aligned} \tag{16}$$

Here, λ_V denotes the mass of the diquark, and V_T and V_F are the instant and light-cone spin-1 particles, respectively, which are constructed within the Weinberg-Soper formalism [50].

After some algebra we arrive at the two-body light-cone wave functions of the proton with

$$\Psi_F^{\uparrow,\downarrow} = \frac{1}{\sqrt{2}}|uS\rangle_F^{\Uparrow,\Downarrow} + \frac{1}{\sqrt{6}}|uV\rangle_F^{\Uparrow,\Downarrow} - \frac{1}{\sqrt{3}}|dV\rangle_F^{\Uparrow,\Downarrow}. \tag{17}$$

The scalar diquark component of the wave function for the proton has the form

$$|uS(P^+, \boldsymbol{k}_T)\rangle^{\Uparrow,\Downarrow} = \sum_{s_z=\pm(1/2)}\int\frac{d^2\boldsymbol{k}_Tdx}{\sqrt{x(1-x)}16\pi^3} \times \psi_S^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, s_z)|xP^+, \boldsymbol{k}_T, s_z\rangle, \tag{18}$$

while the vector diquark component is expressed as

$$|qV(P^+, \boldsymbol{k}_T)\rangle^{\Uparrow,\Downarrow} = \sum_{l_z=0,\pm1;s_z=\pm(1/2)}\int\frac{d^2\boldsymbol{k}_Tdx}{\sqrt{x(1-x)}16\pi^3} \times \psi_V^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, l_z, s_z)|xP^+, \boldsymbol{k}_T, l_z, s_z\rangle, \tag{19}$$

which is the same for $|uV\rangle_F$ and $|dV\rangle_F$. Here we denote s_z and l_z as the spin projections of the quark and the vector diquark, respectively. The forms of $\psi_S^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, s_z)$ and $\psi_V^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, l_z, s_z)$ are given in the Appendix.

Now we calculate the chiral-even GPDs in the zero skewedness ($\xi = 0$) where $t = -\boldsymbol{\Delta}_T^2$. In the overlap representation [41,42] H, E, and $\tilde{H}$ at $\xi = 0$ can be expressed in a symmetric frame as (in the domain $0 < x < 1$ and for $n \to n$ transition)

$$\begin{aligned}H(x, 0, -\boldsymbol{\Delta}_T^2) &= \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{dx_id^2\boldsymbol{k}_{Ti}}{16\pi^3}16\pi^3\delta\Big(1 - \sum_{j=1}^{n}x_j\Big) \\ &\quad\times \delta^{(2)}\Big(\sum_{j=1}^{n}\boldsymbol{k}_{Tj}\Big)\delta(x - x_1)\psi_n^{\uparrow\star}(x_i', \boldsymbol{k}_{Ti}', \lambda_i) \\ &\quad\times \psi_n^{\uparrow}(y_i, \boldsymbol{l}_{Ti}, \lambda_i),\end{aligned} \tag{20}$$

$$\begin{aligned}\frac{\boldsymbol{\Delta}_L}{2M}E(x, 0, -\boldsymbol{\Delta}_T^2) &= \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{dx_id^2\boldsymbol{k}_{Ti}}{16\pi^3}16\pi^3\delta\Big(1 - \sum_{j=1}^{n}x_j\Big) \\ &\quad\times \delta^{(2)}\Big(\sum_{j=1}^{n}\boldsymbol{k}_{Tj}\Big)\delta(x - x_1)\psi_n^{\uparrow\star}(x_i', \boldsymbol{k}_{Ti}', \lambda_i) \\ &\quad\times \psi_n^{\downarrow}(y_i, \boldsymbol{l}_{Ti}, \lambda_i),\end{aligned} \tag{21}$$

$$\begin{aligned}\tilde{H}(x, 0, -\boldsymbol{\Delta}_T^2) &= \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\mathrm{sgn}(\lambda_i)\frac{dx_id^2\boldsymbol{k}_{Ti}}{16\pi^3}16\pi^3\delta \\ &\quad\times \Big(1 - \sum_{j=1}^{n}x_j\Big)\delta^{(2)}\Big(\sum_{j=1}^{n}\boldsymbol{k}_{Tj}\Big)\delta(x - x_1) \\ &\quad\times \psi_n^{\uparrow\star}(x_i', \boldsymbol{k}_{Ti}', \lambda_i)\psi_n^{\uparrow}(y_i, \boldsymbol{l}_{Ti}, \lambda_i),\end{aligned} \tag{22}$$

with

$$x_1' = x_1, \qquad \boldsymbol{k}_{T1}' = \boldsymbol{k}_{T1} - (1 - x_1)\frac{\boldsymbol{\Delta}_T}{2}$$

for the final struck quark,

$$x_i' = x_i, \qquad \boldsymbol{k}_{Ti}' = \boldsymbol{k}_{Ti} + x_i\frac{\boldsymbol{\Delta}_T}{2}$$

for the final $(n - 1)$ spectators,

and

$$y_1 = x_1, \qquad \boldsymbol{l}_{T1}' = \boldsymbol{k}_{T1} + (1 - x_1)\frac{\boldsymbol{\Delta}_T}{2}$$

for the initial struck quark,

$$y_i = x_i, \qquad \boldsymbol{l}_{Ti} = \boldsymbol{k}_{Ti} - x_i\frac{\boldsymbol{\Delta}_T}{2}$$

for the initial $(n - 1)$ spectators.

We remind the reader that the light-cone wave functions for both a scalar and a vector diquark have already been studied in detail in Ref. [51], where $E_q(x, 0, 0)$ has been calculated. Also, the strategy of using the overlap representation to calculate the GPDs H and E in the scalar diquark model has been applied in Ref. [52]. Here we calculate three leading-twist chiral-even GPDs and study the combination which gives the quark OAM distribution through Eq. (10). In particular, we will study the impact parameter dependence of the quark OAM distributions in the next section.

From Eq. (21) we see that nonzero E_q needs a spin flip between the initial and final proton wave functions. The same kind of overlap integration of light-front wave functions (with $J_z = \pm 1/2$ in the initial and final states) also appears in the calculation [53] of Sivers functions, which indicates the presence of the quark OAM.

By employing the light-cone wave functions given in (17) and the overlap representation formalism, we calculate the GPDs $H_q(x, 0, -\mathbf{\Delta}_T^2)$, $\tilde{H}_q(x, 0, -\mathbf{\Delta}_T^2)$, and $E_q(x, 0, -\mathbf{\Delta}_T^2)$ at zero skewedness for $q = u$ and d quarks. The x dependences of these GPDs at different values of Δ_T are given in Figs. 1–3, respectively.

From Fig. 3, one can see that E_u and E_d have opposite sign (E_u is positive and E_d is negative) with similar size in our model. Since it has been shown that there is a quantitative relation [27,29,32] between the Sivers function $f_{1T}^{\perp q}$ and the GPD E^q, our result coincides with recent extractions [54–56] of the Sivers function from the semi-inclusive deeply inelastic scattering data, which show the Sivers functions of u and d have opposite sign with similar size.

Special attention should be paid to the limit of zero momentum transfer $\Delta_T^2 = 0$, since in this limit the GPDs H_q and $\tilde{H}_q$ are simplified to the forward distribution $q(x)$ and $\Delta_q(x)$. Also the quark OAMs are related in the way shown in (10), from which in principle one can calculate $L_q(x)$ from the known chiral-even GPDs.

By taking the GPDs in the forward limit, we calculate the OAM distributions of u and d quarks inside the proton, as shown in Fig. 4. It can be seen that in our model $L_u(x)$ is positive, while $L_d(x)$ is consistent with zero compared with $L_u(x)$, and the net OAM of the u and d quarks is positive. From Fig. 3 one can see that E_d is sizable. However, since E_d is negative, there is a cancellation between $d(x)$, $E_d(x)$, and $\Delta q(x)$. This leads to a small contribution of the d quark orbital angular momentum. We recall that there are lattice QCD [57–59], as well as phenomenological parametrizations and other model calculations of GPDs [60–64], which are used to estimate the OAM of the quarks.

We want to explain some details about the parameters used in our calculation. In our model there are five parameters to be determined: the quark mass m_q, the diquark masses $\lambda_{S/V}$, and the oscillation factors $\beta_{S/V}$ (see the Appendix for more details). The values of the parameters are adopted from Ref. [47]: $m_q = 0.22$ GeV, $\lambda_S = 0.5$ GeV, $\lambda_V = 0.7$ GeV, $\beta_S = 0.25$ GeV, and $\beta_V = 0.17$ GeV, which can describe the data of the nucleon form factors. It is also necessary to verify if our model predictions for $q(x)$ and $\Delta q(x)$ are comparable with well-known parametrizations. This is important since $q(x)$ and

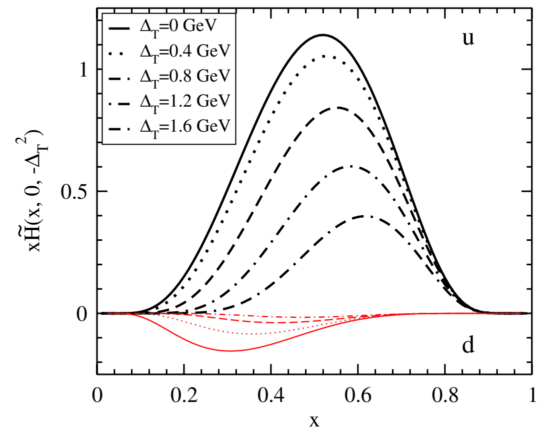

FIG. 2 (color online). The generalized parton distributions $\tilde{H}_u(x, 0, \Delta_T^2)$ and $\tilde{H}_d(x, 0, \Delta_T^2)$ for the proton in the light-cone diquark model as functions of x for different values of Δ_T.

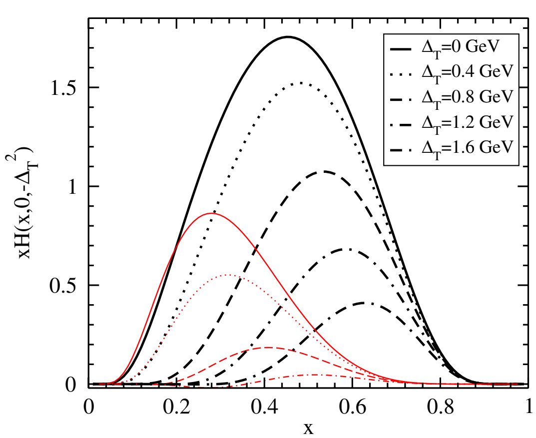

FIG. 1 (color online). The generalized parton distributions $H_u(x, 0, \Delta_T^2)$ and $H_d(x, 0, \Delta_T^2)$ for the proton in the light-cone diquark model as functions of x for different values of Δ_T.

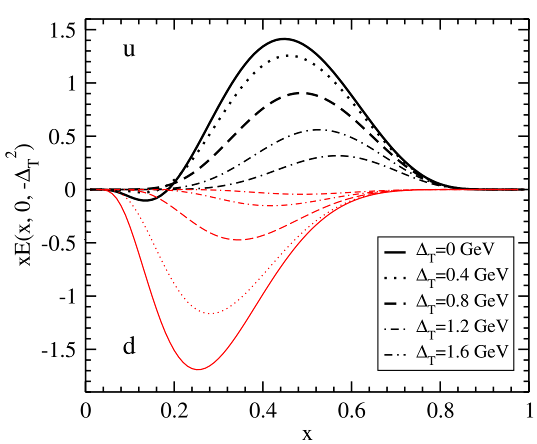

FIG. 3 (color online). The generalized parton distributions $E_u(x, 0, \Delta_T^2)$ and $E_d(x, 0, \Delta_T^2)$ for the proton in the light-cone diquark model as functions of x for different values of Δ_T.

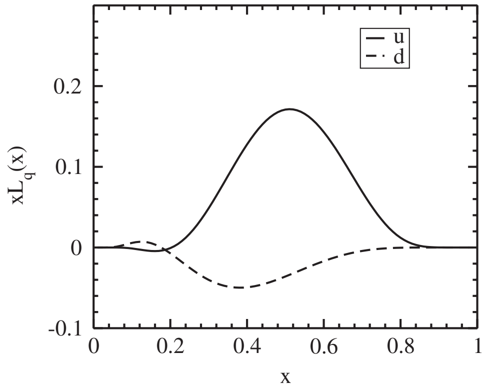

FIG. 4. The OAM distributions $L_q(x)$ of u and d quarks inside the proton in the light-cone diquark model as functions of x.

$\Delta q(x)$ are the forward limits of H_q and $\tilde{H}_q$, respectively, and, on the other hand, we have used the calculated $q(x)$ and $\Delta q(x)$ for our prediction of $L_q(x)$. In order to match the model and the parametrizations, we should figure out at which energy scale our model should be applied. Here we follow the strategy adopted in Refs. [65–67] to determine the scale at which our model is valid, by comparing the total momentum fraction carried by valence quarks and a well-known parametrization, for instance, CTEQ6L1 parametrization [68]. It turns out that this scale is $Q_0 = 0.25$ GeV. We then evolve our model results for $q(x)$ and $\Delta_q(x)$ by applying the leading order Dokshitzer-Gribov-Lipatov-Altarelli-Parisi equations to a higher scale ($Q = 1.0$ GeV) and compare them with CTEQ6L1 and DNS2005 leading order [69] parametrizations, respectively. We show our results in Fig. 5, which qualitatively agrees with these known parametrizations. Our aim is to obtain an estimate of the GPDs and the quark OAM distributions, which can be used to get the impact parameter dependence of the quark OAM distributions.

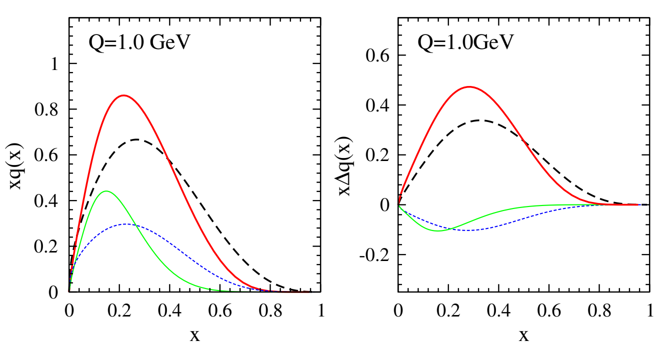

FIG. 5 (color online). Left panel: Our model calculation (solid line) of unpolarized parton distribution $xq(x)$ evolved to the scale $Q = 1.0$ GeV compared with CTEQ6L1 parametrization (dashed line). The thick and thin lines denote the curves for valence u and d quarks, respectively. Right panel: Our model calculation (solid line) of helicity distribution $x\Delta q(x)$ evolved to the scale $Q = 1.0$ GeV compared with de Florian-Navarro-Sassot leading order (set 3) parametrization (dashed line). The thick and thin lines denote the curves for valence u and d quarks, respectively.

As mentioned above, our model is applicable at a rather low energy scale $Q_0 = 0.25$ GeV. Therefore the results of the OAM distributions for u and d quarks (including their sign), shown in Fig. 4, are also for this model scale. It is important to know this scale in order to compare our results with other models and their phenomenology, which usually apply at different scales. A detailed study of the scale dependence of the quark OAM has been carried out in Ref. [70], showing that OAMs of u and d quarks can vary drastically with the scale and can even change sign. Here we refrain from evolving our model results to higher scales and keep that for a future study.

IV. IMPACT PARAMETER DEPENDENCE OF ORBITAL ANGULAR MOMENTUM

In this section we want to study the quark OAMs in transverse position (impact parameter) space. The GPDs in the impact parameter space have been studied in Refs. [37–39]. The most interesting case is the zero skewedness limit $\xi = 0$, in which a density interpretation of GPDs in the impact parameter space may be obtained [37]. Therefore studying GPDs in impact parameter space can provide a three-dimensional picture of the nucleon. In the following we restrict ourselves to the case $\xi = 0$.

The impact parameter parton distribution functions inside the nucleon can be obtained by sandwiching the parton correlator between nucleon states localized in transverse space

$$q(x, \boldsymbol{b}_T) = \langle P^+, \boldsymbol{0}_T; S|\hat{O}_q^{[\gamma^+]}(x, \boldsymbol{b}_T)|P^+, \boldsymbol{0}_T; S\rangle, \tag{23}$$

where

$$\hat{O}_q^{[\gamma^+]}(x, \boldsymbol{b}_T) = \int \frac{dy^-}{8\pi} e^{ixP^+y^-/2} \bar{\psi}\left(0, -\frac{y^-}{2}, \boldsymbol{b}_T\right) \times \gamma^+ \psi\left(0, \frac{y^-}{2}, \boldsymbol{b}_T\right), \tag{24}$$

and the initial and final states in the transverse space defined as [37,39,71]

$$|P^+, \boldsymbol{b}_T; S\rangle = \mathcal{N} \int \frac{d^2\boldsymbol{p}_T}{(2\pi)^2} e^{-i\boldsymbol{p}_T\cdot\boldsymbol{b}_T}|p; S\rangle, \tag{25}$$

$$\langle P^+, \boldsymbol{b}_T; S| = \mathcal{N}^* \int \frac{d^2\boldsymbol{p}'_T}{(2\pi)^2} e^{i\boldsymbol{p}'_T\cdot\boldsymbol{b}_T}\langle p'; S|, \tag{26}$$

which characterize a nucleon with momentum P^+ at a transverse position $\boldsymbol{b}_T$ and polarization specified by S.

One of the interesting features of impact-parameter-dependent parton distributions is that they are Fourier transformations of GPDs [37]. For instance, the impact parameter dependence of an unpolarized quark in the unpolarized nucleon can be obtained from

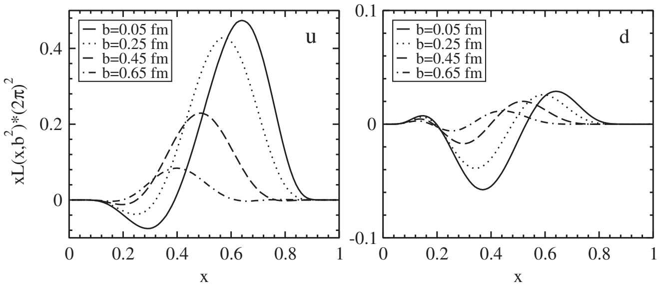

FIG. 6. The impact parameter distributions [scaled with a factor of $(2\pi)^2$] $xL_u(x, \boldsymbol{b}_T)$ (left) and $xL_d(x, \boldsymbol{b}_T)$ (right) for the proton in the light-cone diquark model as functions of x for different values of b.

$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} H_q(x, 0, -\boldsymbol{\Delta}_T^2), \tag{27}$$

where $\boldsymbol{b}_T$ and $\boldsymbol{\Delta}_T$ are two conjugated parameters.

Similarly, the impact parameter dependence of quark helicity distribution in the longitudinal polarized nucleon is defined as

$$\Delta q(x, \boldsymbol{b}_T) = \langle P^+, \boldsymbol{0}_T; S|\hat{\mathcal{O}}_q^{[\gamma^+\gamma_5]}(x, \boldsymbol{b}_T)|P^+, \boldsymbol{0}_T; S\rangle, \tag{28}$$

which is the Fourier transformation of $\tilde{H}_q$:

$$q(x, \boldsymbol{b}_T) = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} H_q(x, 0, -\boldsymbol{\Delta}_T^2). \tag{29}$$

We follow a similar approach by introducing the impact parameter dependence of quark OAM $\mathcal{L}(x, \boldsymbol{b}_T)$. It can be obtained from the expectation value of $\hat{O}_L$, given in Eq. (8), between the position state $|P^+, \boldsymbol{0}_T\rangle$:

$$L_q(x, \boldsymbol{b}_T) = \langle P, \boldsymbol{0}_T; S|\hat{O}_L|P, \boldsymbol{0}_T; S\rangle. \tag{30}$$

After a Fourier transformation on $L_q(x, \boldsymbol{b}_T)$, one can arrive at

$$\int d^2\boldsymbol{b}_T e^{i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} L_q^z(x, \boldsymbol{b}_T) = L_q(x, -\boldsymbol{\Delta}_T^2). \tag{31}$$

The function $L_q^z(x, -\boldsymbol{\Delta}_T^2)$ can be obtained by the GPDs at zero skewedness [4]:

$$L_q(x, -\boldsymbol{\Delta}_T^2) = \tfrac{1}{2}\{x[H(x, 0, -\boldsymbol{\Delta}_T^2) + E_q(x, 0, -\boldsymbol{\Delta}_T^2)] - \tilde{H}(x, 0, -\boldsymbol{\Delta}_T^2)\}, \tag{32}$$

and (10) is the forward limit of $L_q(x, -\boldsymbol{\Delta}_T^2)$.

Therefore, if we know the GPDs H_q, $\tilde{H}_q$, and E_q, from (10), we can calculate the impact parameter dependence of the quark OAM distribution by the Fourier transformation

$$L_q(x, \boldsymbol{b}_T) = \int \frac{d^2\boldsymbol{\Delta}_T}{(2\pi)^2} e^{-i\boldsymbol{b}_T\cdot\boldsymbol{\Delta}_T} L_q^z(x, \boldsymbol{\Delta}_T^2). \tag{33}$$

The integration over impact parameter dependence of the quark OAM leads to

$$\int d^2\boldsymbol{b}_T L_q(x, \boldsymbol{b}_T) = L_q(x). \tag{34}$$

In Fig. 6, we show the impact parameter distributions [scaled with a factor of $(2\pi)^2$] $L_u(x, \boldsymbol{b}_T)$ (left) and $L_d(x, \boldsymbol{b}_T)$ (right) for the proton in the light-cone diquark model, as functions of x, for different values of b. In Fig. 7, we show the profiles of the impact parameter distributions $L_u(x, \boldsymbol{b}_T)$ for the proton in the light-cone diquark model as functions of $\boldsymbol{b}_T$, for $x = 0.3$ and $x = 0.5$. It is shown that the impact parameter dependence of quark OAM is axially symmetric. Also at large x the impact parameter distribution is peaked at small b.

V. SUMMARY

In conclusion, we study the OAM structure of the quarks inside the proton in a light-cone diquark model. In this model the light-cone wave function of the proton is known. It is then convenient to express the physical observables in the overlap representation formalism. We calculate the

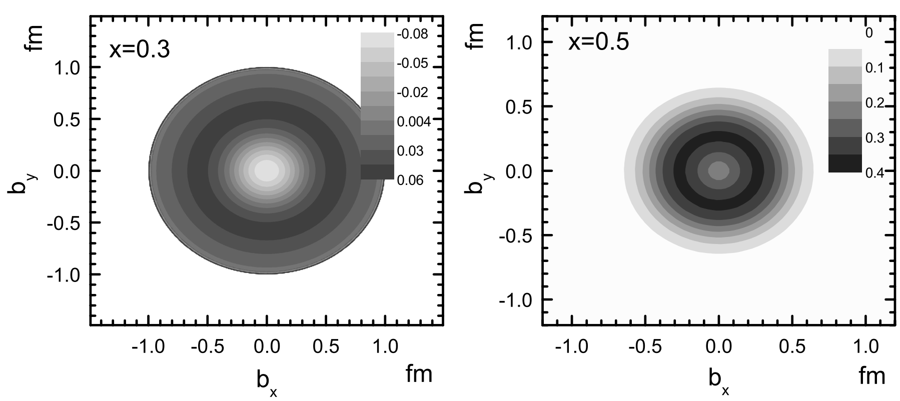

FIG. 7 (color online). The profiles of the impact parameter distribution [scaled by a factor of $(2\pi)^2$] $xL_u(x, \boldsymbol{b}_T)$ for the proton in the light-cone diquark model as functions of Δ_T for $x = 0.3$ (left) and $x = 0.5$ (right).

chiral-even GPDs $H_q(x, \xi, \Delta^2)$, $\tilde{H}_q(x, \xi, \Delta^2)$, and $E_q(x, \xi, \Delta^2)$ at zero skewedness for $q = u$ and d. We found that E_u and E_d have opposite sign, with similar size in our model. The GPDs are applied to calculate the OAM distributions, showing that $L_u(x)$ is positive, while $L_d(x)$ is consistent with zero compared with $L_u(x)$, and the net OAM of the u and d quarks is positive. We also introduce the impact parameter dependence of quark OAM distribution $L(x, \boldsymbol{b}_T)$. It describes the position space distribution of the quark OAM at given x. We found that the impact parameter dependence of quark OAM distribution is axially symmetric in the light-cone diquark model.

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APPENDIX A: LIGHT-CONE WAVE FUNCTIONS IN A DIQUARK MODEL

The expressions for $\psi_S^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, s_z)$ have the form

$$\begin{aligned} \psi_S^{\Uparrow}\left(x, \boldsymbol{k}_T, +\frac{1}{2}\right) &= \frac{(k^+ + m)}{\omega}\phi_S(x, k_T), \\ \psi_S^{\Uparrow}\left(x, \boldsymbol{k}_T, -\frac{1}{2}\right) &= -\frac{k_r}{\omega}\phi_S(x, k_T), \end{aligned} \tag{A1}$$

and

$$\begin{aligned} \psi_S^{\Downarrow}\left(x, \boldsymbol{k}_T, +\frac{1}{2}\right) &= \frac{k_l}{\omega}\phi_S(x, k_T), \\ \psi_S^{\Downarrow}\left(x, \boldsymbol{k}_T, -\frac{1}{2}\right) &= \frac{(k^+ + m)}{\omega}\phi_S(x, k_T), \end{aligned} \tag{A2}$$

respectively.

The expressions of $\psi_V^{\Uparrow,\Downarrow}(x, \boldsymbol{k}_T, l_z, s_z)$ can be expressed as

$$\begin{aligned}
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, +1, \uparrow) &= -\sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)(k^+ + m) + (k_V^+ + \lambda_V)^2]k^L, \\
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, +1, \downarrow) &= \sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)\boldsymbol{k}_T^2 - (k_V^+ + \lambda_V)^2(k^+ + m)], \\
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, 0, \uparrow) &= 2\frac{\phi_V(x, k_T)}{\omega\omega_V^2}\{[(k_V^0 + \lambda_V)k_V^+ - \boldsymbol{k}_T^2](k^+ + m) - (k_V^+ + \lambda_V)\boldsymbol{k}_T^2\}, \\
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, 0, \downarrow) &= \frac{\phi_V(x, k_T)}{\omega\omega_V^2}[-2((k_V^0 + \lambda_V)k_V^+ - \boldsymbol{k}_T^2) - 2(k_V^+ + \lambda_V)(k^+ + m)]k^R, \\
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, -1, \uparrow) &= \sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)(k^+ + m) - \boldsymbol{k}_T^2]k^R, \\
\psi_V^{\Uparrow}(x, \boldsymbol{k}_T, -1, \downarrow) &= -\sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[k_T^{R2}(k_V^+ + \lambda_V + k^+ + m)],
\end{aligned} \tag{A3}$$

and

$$\begin{aligned}
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, +1, \uparrow) &= -\sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[k_T^{L2}(k_V^+ + \lambda_V + k^+ + m)], \\
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, +1, \downarrow) &= -\sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)(k^+ + m) - \boldsymbol{k}_T^2]k^L, \\
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, 0, \uparrow) &= 2\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[((k_V^0 + \lambda_V)k_V^+ - \boldsymbol{k}_T^2) + (k_V^+ + \lambda_V)(k^+ + m)]k^L, \\
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, 0, \downarrow) &= 2\frac{\phi_V(x, k_T)}{\omega\omega_V^2}\{[(k_V^0 + \lambda_V)k_V^+ - \boldsymbol{k}_T^2](k^+ + m) - (k_V^+ + \lambda_V)\boldsymbol{k}_T^2\}, \\
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, -1, \uparrow) &= \sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)\boldsymbol{k}_T^2 - (k_V^+ + \lambda_V)^2(k^+ + m)], \\
\psi_V^{\Downarrow}(x, \boldsymbol{k}_T, -1, \downarrow) &= \sqrt{2}\frac{\phi_V(x, k_T)}{\omega\omega_V^2}[(k_V^+ + \lambda_V)(k^+ + m) + (k_V^+ + \lambda_V)^2]k^R.
\end{aligned} \tag{A4}$$

The momentum dependence of the wave functions in the above equations is described by $\phi_D(x, k_T^2)$ with the Gaussian form

$$\phi_D(x, k_T) = A_D \exp\left(-\frac{\mathcal{M}^2}{8\beta_D^2}\right), \tag{A5}$$

where

$$\mathcal{M}^2 = \frac{\boldsymbol{k}_T^2 + m_q^2}{x} + \frac{\boldsymbol{k}_T^2 + \lambda_V^2}{1-x}, \tag{A6}$$

A_D stands for the normalization constant, and β_D ($D = S, V$) is the oscillation factor. For the parameters we adopt the values from Ref. [47], which can describe the data of the nucleon form factors.

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