Transversity form factors of the pion in chiral quark models

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The transversity form factors of the pion, involving matrix elements of bilocal tensor currents, are evaluated in chiral quark models, both in the local Nambu-Jona-Lasinio with the Pauli-Villars regularization, as well as in nonlocal models involving momentum-dependent quark mass. After suitable QCD evolution, the agreement with recent lattice calculations is very good, in accordance to the fact that the spontaneously broken chiral symmetry governs the dynamics of the pion. Meson dominance of form factors with expected meson masses also works properly, conforming to the parton-hadron duality in the considered process.

The *transversity* form factors (TFFs) of the pion provide valuable insight into chirally-odd generalized parton distribution functions (GPDs) as well as into the nontrivial spin structure of the pion. These interesting quantities have been determined for the first time on the lattice [1]. Formally, the TFFs, denoted as $B_{Tni}^{\pi}(t)$, are defined as

$$\langle \pi^+(P')|\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}}|\pi^+(P)\rangle$$
$$= \mathcal{AS}\bar{P}^\mu\Delta^\nu\sum_{\substack{i=0\\ \text{even}}}^{n-1}\Delta^{\mu_1}\cdots\Delta^{\mu_i}\bar{P}^{\mu_{i+1}}\cdots\bar{P}^{\mu_{n-1}}\frac{B_{Tni}^{\pi,u}(t)}{m_\pi}, \quad (1)$$

where P' and P are the momenta of the pion, $\bar{P} = \frac{1}{2} \times (P' + P)$, $\Delta = P' - P$, and $t = \Delta^2$. The symbol $\mathcal{AS}$ denotes symmetrization in $\nu, \ldots, \mu_{n-1}$, followed by antisymmetrization in μ, ν, with the additional prescription that the traces in all index pairs are subtracted. The dividing factor of m_π is introduced by convention in order to have dimensionless form factors [1]. The tensor operators are given by

$$\mathcal{O}_T^{\mu\nu\mu_1\cdots\mu_{n-1}} = \mathcal{AS}\bar{u}(0)i\sigma^{\mu\nu}i\overleftrightarrow{D}^{\mu 1}\cdots i\overleftrightarrow{D}^{\mu_{n-1}}u(0), \quad (2)$$

where $\overleftrightarrow{D} = \frac{1}{2}(\overrightarrow{D} - \overleftarrow{D})$, with D denoting the QCD covariant derivative. As in [1], we use the positively charged pion and the up-quark density for definiteness.

The available full-QCD lattice results [1] are for $B_{10}^{\pi,u}$ and $B_{20}^{\pi,u}$ and for $-t$ reaching 2.5 GeV^2, with moderately low values of the pion mass, $m_\pi \sim 600$ MeV. The calculation uses the same set of QCDSF/UKQCD $N_f = 2$ ensembles with improved Wilson fermions and the Wilson gauge-action that were used in the determination of the pion charge form factor [2].

Form factors are related via sum rules to the GPDs (for extensive reviews see, e.g., [3–11], and references therein). Experimentally, the GPDs of the pion constitute rather elusive quantities which appear in rare exclusive processes, such as the deeply virtual Compton scattering or the hard electro-production of mesons. The high-Q^2 dependence of the transversity form factors has been addressed recently [12]; however, the comparison with the lattice was avoided. In the present paper we fill this gap and confront the lattice transversity form factors with the results of chiral quark models, where particular attention is paid to spontaneous chiral symmetry breaking and the Goldstone nature of the pion as a composite relativistic $\bar{q}q$ bound state. We apply the Nambu-Jona-Lasinio (NJL) model with the Pauli-Villars (PV) regularization, as well as nonlocal chiral quark models inspired by the nontrivial structure of the QCD vacuum [13,14]. These models provide the results at the quark-model scale [15]. After the necessary (multiplicative) QCD evolution [15], our model results are in a quite remarkable agreement with the lattice data. Lower values of the constituent quark mass, $\sim$250 MeV, are preferred. We use the techniques described in detail in [15,16].

Previously, chiral quark models have proved to correctly describe numerous features related to the pion GPDs. The parton distribution functions (PDF) have been evaluated in the NJL model in Refs. [17–19]. The extension to diagonal GPDs in the impact parameter space was carried out in [20]. Other analyses of the pionic GPDs and PDFs were performed in nonlocal chiral quark models [21–27], in the NJL model [15,22,28–30] and light-front constituent quark models [31,32]. The parton distribution amplitudes (PDAs), related to the GPD via a low-energy theorem [33], were evaluated in [34–41] (see [42] for a brief review of analyses of PDA). The gravitational form factors were computed in [43]. Finally, the pion-photon transition distribution amplitudes [44–47] were obtained in Refs. [48–52].

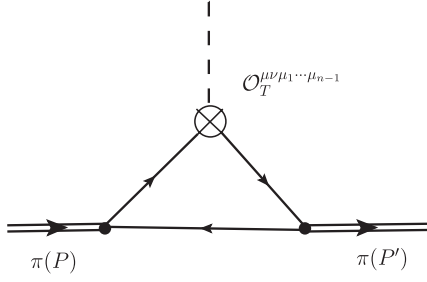*Wojciech.Broniowski@ifj.edu.pl
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FIG. 1.   (Color online) The one-quark-loop triangle diagram contribution to the form factors $B_{Tni}^{\pi}(t)$.

FIG. 2 (color online).   (Color online) The NJL model parameters, m and Λ, plotted as functions of the constituent quark mass, M.

In chiral quark models at the leading-N_c level the calculation of the form factors and GPDs proceeds according to the one-loop diagrams (Fig. 1), as explained in detail in [15,16]. The one-quark-loop action of the model is

$$\Gamma_{\rm NJL} = -iN_c {\rm Tr} \log(i\slashed{\partial} - MU^5 - m)|_{\rm reg}, \qquad (3)$$

where M is the constituent quark mass,

$$U^5 = \exp(i\gamma_5 \boldsymbol{\phi} \cdot \boldsymbol{\tau}), \qquad (4)$$

with $\boldsymbol{\phi}$ denoting the pion field, and m is the current quark mass. We apply the NJL with the PV regularization in the twice-subtracted version of Refs. [41,53,54]. Variants of chiral quark models differ in the way of performing the necessary regularization of the quark loop diagrams, which may to some extent influence the physical results.[1] Unlike many other studies, where one could work close to the chiral limit of $m = 0$, in the present case we need to tackle a situation with moderately large pion masses. This is because the lattice results for the transversity form factors are provided for $m_\pi = 600$ MeV. For that reason, we do the following. As usual, the three model parameters Λ, M, and m are traded for the constituent quark mass, M, f_π (the pion decay constant), and m_π. We assume that Λ depends on M only, and not on m. Constraining $f_\pi = 93$ MeV (the physical value) and using the given value of m_π leaves us with one free parameter only, M. The result of this procedure, with m for the two values of m_π of interest, is displayed in Fig. 2.

An explicit evaluation of the one-quark-loop diagram of Fig. 1, carried out along the standard lines explained, e.g., in [15], yields the simple result (holding at the quark-model scale):

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[1]We use the prescription where M^2 in the loop function is replaced with $M^2 + \Lambda^2$, and then the regularized observable is evaluated according to the formula $\mathcal{O}_{\rm reg} = \mathcal{O}(0) - \mathcal{O}(\Lambda^2) + \Lambda^2 d\mathcal{O}(\Lambda^2)/d\Lambda^2$. The premultiplying factor $g_\pi^2 = M^2/f_\pi^2$ is not regularized.
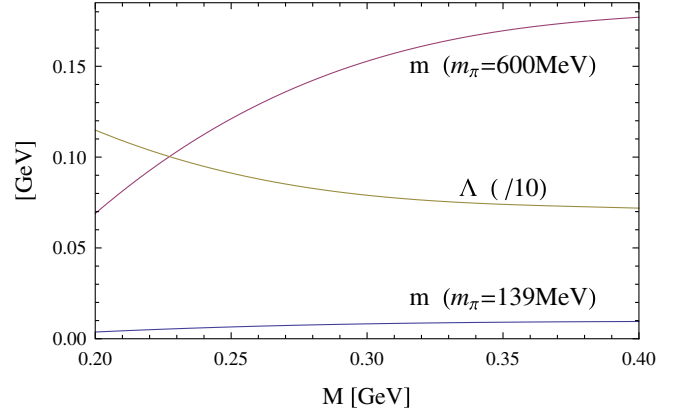
$$\frac{B_{T10}^{\pi,u}(t)}{m_\pi} = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta K,$$

$$\frac{B_{T20}^{\pi,u}(t)}{m_\pi} = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \alpha K,$$

$$K = \frac{N_c g_\pi^2 M}{2\pi^2(M^2 + m_\pi^2(\alpha - 1)\alpha + t\beta(\alpha + \beta - 1))}\bigg|_{\rm reg},$$
$$(5)$$

with $g_\pi = M/f_\pi$ and $N_c = 3$ denoting the number of colors. The variables α and β are the Feynman parameters.

Before comparing the results to the lattice data, we need to carry out the QCD evolution, as the transversity form factors, not corresponding to conserved quantities, evolve with the scale. The lattice data correspond to the scale of about $Q = 2$ GeV, while the quark-model calculation corresponds to a much lower scale,

$$\mu_0 = 320 \text{ MeV}. \qquad (6)$$

A detailed discussion of the evolution issue is presented in [15,55]. It turns out that $B_{T10}^{\pi,u}$ and $B_{T20}^{\pi,u}$ evolve multiplicatively as follows:

$$B_{Tn0}^{\pi,u}(t;\mu) = B_{Tn0}^{\pi,u}(t;\mu_0)\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{\gamma_{Tn}/(2\beta_0)}, \qquad (7)$$

with the anomalous dimensions $\gamma_{Tn} = \frac{32}{3}H_n - 8$, with $H_n = \sum_{k=1}^n 1/k$, which gives $\gamma_{T1} = \frac{8}{3}$ and $\gamma_{T2} = 8$. We use $\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_f$ and the running coupling constant $\alpha(\mu) = 4\pi/[\beta_0 \log(\mu^2/\Lambda_{\rm QCD}^2)]$, with $\Lambda_{\rm QCD} = 226$ MeV and $N_c = N_f = 3$. In particular, this gives

$$B_{T10}^{\pi,u}(t;2 \text{ GeV}) = 0.75 B_{T10}^{\pi,u}(t;\mu_0),$$
$$B_{T20}^{\pi,u}(t;2 \text{ GeV}) = 0.43 B_{T20}^{\pi,u}(t;\mu_0). \qquad (8)$$

Note a stronger reduction for B_{T20} compared to B_{T10} as the result of the evolution. In the chiral limit and at $t = 0$,

$$B_{T10}^{\pi,u}(t=0;\mu_0)/m_\pi = \frac{N_c M}{4\pi^2 f_\pi^2}, \qquad (9)$$

$$\frac{B_{T20}^{\pi,u}(t=0;\mu)}{B_{T10}^{\pi,u}(t=0;\mu)} = \frac{1}{3}\left(\frac{\alpha(\mu)}{\alpha(\mu_0)}\right)^{8/27}. \qquad (10)$$

In Fig. 3 we show the results from the NJL model, evolved to $\mu = 2$ GeV, confronted with the lattice data scanned from Fig. 1 of [1]. We have used $m_\pi = 600$ MeV and selected $M = 250$ MeV, which optimizes the comparison. As we see, the agreement is remarkable.

We have investigated the dependence of the values of the form factors at $t = 0$ on m_π, as studied in [1]. The result is displayed in Fig. 4. We note a fair agreement in the intermediate values of m_π, with a somewhat different character of the bent model curves and the flat data. Note, however, that the model, designed to work not too far from the chiral limit may need not be accurate at very large values of m_π. Also, the lattice data are extrapolated to $t = 0$ with a formula different from the NJL model, which may introduce some additional uncertainty.

We note that the results presented in Fig. 3 depend quite sensitively on the value of the constituent quark mass, M. Higher M yields lower values of the transversity form factors, for instance at $M = 275$ MeV we find the reduced value $B_{T10}^{\pi,u}(t = 0; \mu = 2 \text{ GeV}) = 0.67$. We recall that the optimum value of M used in chiral quark models depends on the particular observable used for the fit. While the application to the ρ meson suggests M somewhat above $m_\rho/2$ and the soliton models for the nucleon prefer $M \sim 300$–350 MeV [56], significantly lower values follow from other studies. The charge radius of the pion in the NJL model with the PV regulator suggests $M \sim 280$ MeV [41], however, the pion-loop corrections to this observable are significant. The analysis of the radii of the pion charged and transition form factors from quark tri-

FIG. 4 (color online).   (Color online) The values of the transversity form factors at $t = 0$ obtained in the NJL model (lines) for $M = 250$ MeV and evolved to the lattice scale of 2 GeV, plotted as functions of m_π^2 and compared to the lattice data of Fig. 4 of [1] (points).

angle diagrams yields $M = \sqrt{2/3}\pi f_\pi \sim 240$ MeV [57]. A strong restriction on the M follows from the Adler function and the corresponding vacuum polarization contribution to the gyroscopic factor $g - 2$ of the muon. In three different approaches—the simple loop [58], the analytic perturbation model [59], and the nonlocal chiral quark model [60]—the following values were obtained, correspondingly: $M = 180, 240$, and 250 MeV. Thus our optimum value of 250 MeV falls into this ball park. The mentioned models differ in details, thus one should not request exact agreement. Admittedly, a joint analysis of all observables within a uniform framework would be highly desirable in that regard.

We have also explored the nonlocal chiral quark models which incorporate the nontrivial structure of the QCD vacuum. In order to calculate the one-quark-loop diagram of Fig. 1, we use the nonperturbative quark propagator

$$S(k) = \frac{1}{k - m(k^2)} \qquad (11)$$

and the quark-pion vertex

$$\Gamma_\pi^a(k, q) = \frac{i}{f_\pi} \gamma_5 \tau^a F(k_+^2, k_-^2), \qquad (12)$$

where $p_\pm = k \pm q/2$. The quantity $m(k^2)$ is the dynamical quark mass normalized by $m(0) = M_0$, and the nonlocal vertex $F(k_+^2, k_-^2)$ is normalized by $F(k^2, k^2) = m(k^2)$. In the present study the nonlocal model calculations are performed in the chiral limit, which means that $m(k^2 \to \infty) = 0$.

Further, we will consider two variants of the quark-pion vertex (12),

$$F_I(k_+^2, k_-^2) = \sqrt{m(k_+^2)m(k_-^2)}, \qquad (13)$$

$$F_{\text{HTV}}(k_+^2, k_-^2) = \frac{1}{2}[m(k_+^2) + m(k_-^2)]. \qquad (14)$$
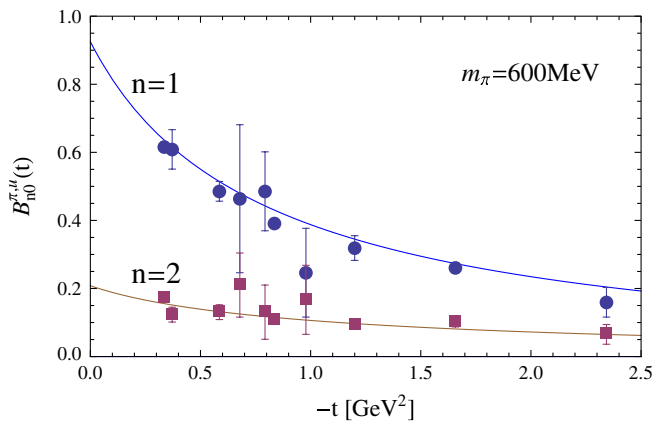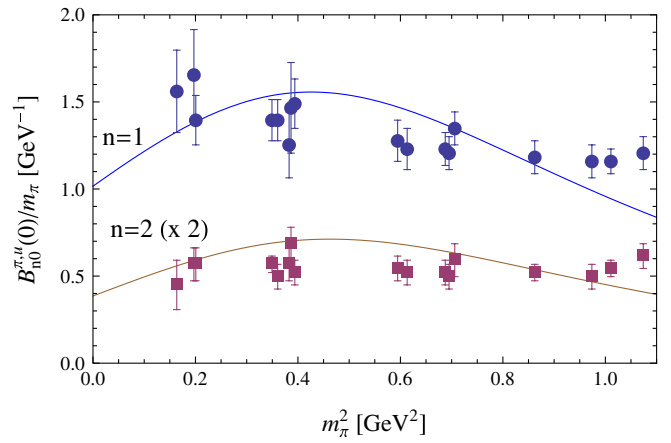

FIG. 3 (color online).   (Color online) The transversity form factors obtained in the NJL model (lines) for $M = 250$ MeV and $m_\pi = 600$ MeV, evolved to the lattice scale of 2 GeV and compared to the lattice data from Fig. 1 of [1] (points).

The form (12) is motivated by the instanton picture of the QCD vacuum [13], while (14), the Holdom-Terning-Verbeek (HTV) vertex, comes from the nonlocal chiral quark model of [14]. For $t = 0$, both models yield the normalization

$$B_{T10}^{\pi,u}(t = 0; \mu_0)/m_\pi$$

$$= \frac{N_c}{2\pi^2 f_\pi^2} \int_0^\infty du \frac{u m^2(u)}{(u + m^2(u))^3}(m(u) - u m'(u)), \tag{15}$$

$$B_{T20}^{\pi,u}(t = 0; \mu_0)/m_\pi$$

$$= \frac{N_c}{2\pi^2 f_\pi^2}\left\{\int_0^\infty du \frac{u m(u)}{(u + m^2(u))^3}\left(m^2(u) + \frac{1}{2}u m(u)m'(u)\right.\right.$$

$$\left.+ \frac{1}{6}u^2 m'^2(u)\right) - \int_0^\infty du \frac{u^2 m^2(u)}{(u + m^2(u))^4}(m(u)$$

$$+ 2m^2(u)m'(u))\bigg\}, \tag{16}$$

where $m'(u) = dm(u)/du$. In the local limit, where $m(k^2) \to$ const, one reproduces Eqs. (9) and (10).

The results for $B_{Tn0}^{\pi,u}(t)$, $n = 1, 2$, are shown in Fig. 5. In the present study, we have assumed that B_{Tn0}/m_π depends weakly on m_π, similarly to the local model (see Fig. 4). Hence, in order to compare to the lattice data for B_{Tn0} we simply multiply the results of calculations obtained in the chiral limit with $m_\pi = 600$ MeV. We have carried out the same QCD evolution procedure in the nonlocal models as given by Eq. (7). From Fig. 5, we note that the HTV model with the vertex function given by Eq. (14) (solid lines) and with $M_0 = 300$ MeV works best, describing accurately the data, while the instanton model, Eq. (13) (dashed lines), results in too steeply decreasing form factors. Also, we have found that lower values of M_0 spoil the agreement with the data.
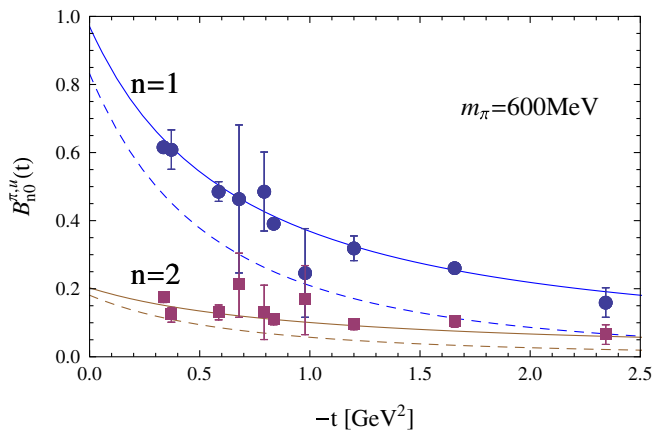

FIG. 5 (color online). (Color online) The transversity form factors in the HTV model (solid line) and in the instanton-motivated model (dashed line). The data as in Fig. 3.
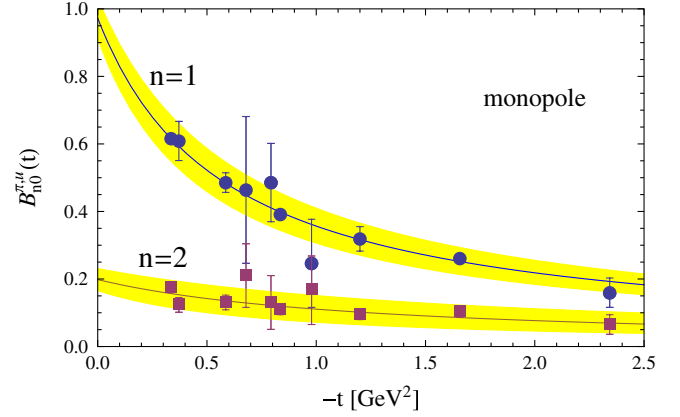

FIG. 6 (color online). (Color online) Monopole fits to the transversity form factors. The bands correspond to the uncertainties of the parameters of Eq. (18). The data as in Fig. 3.

In the large-N_c expansion all form factors are dominated by mesons with the proper quantum numbers (see, e.g., [61]). The well-known example is the experimentally measurable charge form factor, coupling to $\rho(770)$, $\rho'(1435)$, etc. (see, e.g., [62]), however meson dominance has also been checked in more elusive objects such as the spin-2 gravitational form factor [43] [coupling to $f_2(1270)$] and the trace-anomaly form factor [63] [coupling to $f_0(600)$]. We thus undertake a simple monopole χ^2-fit to the TFF lattice data of [1] for $B_{Tn0}^{\pi,u}(t)$ at $m_\pi = 600$ MeV, reading

$$B_{Tn0}^{\pi,u}(t) = A_n \frac{m_n^2}{m_n^2 - t}, \tag{17}$$

and obtain

$$\begin{aligned} A_1 &= 0.97(6), & m_1 &= 760(50) \text{ MeV}, \\ A_2 &= 0.20(3), & m_2 &= 1120(250) \text{ MeV}. \end{aligned} \tag{18}$$

The result of the fit is shown in Fig. 6. The ratio

$$B_{T20}^{\pi,u}(0)/B_{T10}^{\pi,u}(0) = A_2/A_1 = 0.20(4) \tag{19}$$

corresponds, according to Eq. (10), to the evolution ratio $\alpha(\mu)/\alpha(\mu_0) = 0.2(1)$, and hence to $\mu_0 = 350(80)$ MeV, in full agreement with the value (6) based on the PDF [17] and PDA [40] of the pion (see [15,55]).

The form factor B_{T10}^π couples to $I^G(J^{PC}) = 1^+(1^{--})$ states, while B_{T10}^π to $0^+(2^{++})$ and $1^+(1^{--})$ states. From Eq. (18) we note that indeed m_1 is compatible with the mass of $\rho(770)$, while m_2 with the mass of $f_2(1270)$, and within 2 standard deviations also with $\rho(770)$ or $\rho'(1435)$. These contributions cannot be disentangled with the current lattice accuracy. We note that the $n = 2$ case allows also the coupling to the $1^+(1^{+-})$ state, such as $b_1(1235)$, which, however, cannot decay into two pions (see, e.g., [64] for a discussion within chiral perturbation theory).

We conclude by presenting a comparison of the several considered chiral quark models in Fig. 7. We note the close
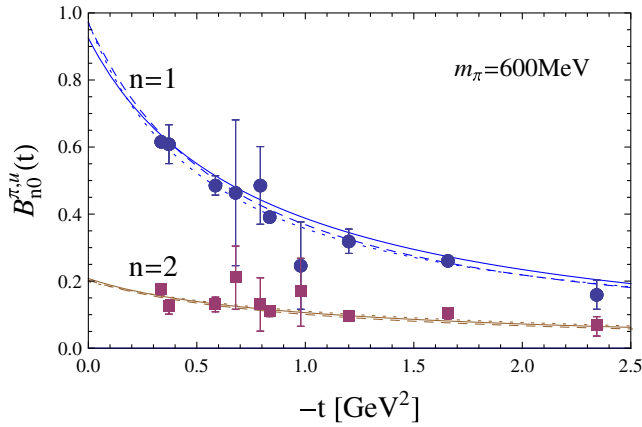
FIG. 7 (color online). (Color online) Comparison of the predictions of the NJL model (solid line), the HTV model [14] (dashed line), and the monopole fit (dotted line). The data as in Fig. 3.

proximity of all these model predictions. As we have shown, it is possible to describe the transversity form factors of the pion in chiral quark models. This is another manifestation of the fact that the spontaneously broken chiral symmetry is a key dynamical factor in the pion structure. Alternatively, one can describe the data with meson dominance, featuring parton-hadron duality for the TFFs. Indeed, appropriate meson masses govern the fall-off of form factors, an expectation which becomes exact in the large N_c limit. The considered form factors, being the matrix elements of nonconserved currents, undergo multiplicative QCD renormalization, thus their momentum dependence does not change as a function of the scale, although the absolute normalization is governed by anomalous dimensions and the corresponding evolution ratio from the actual scale to the model reference scale. Actually, we find that the ratio of the lowest transversity form factors at $t = 0$ is properly described when the QCD evolution is considered and the required model reference scale is fully compatible with other determinations.

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