Inverse seesaw mechanism, leptogenesis, observable proton decay, and $\Delta_R^{\pm\pm}$ in supersymmetric $SO(10)$ with heavy W_R

Mina K. Parida[1] and Amitava Raychaudhuri[2,3]

[1]*National Institute of Science Education and Research, Institute of Physics Campus, Sachivalaya Marg, Bhubaneswar 751005, India*
[2]*Harish-Chandra Research Institute, Jhunsi, Allahabad 211 019, India*
[3]*Department of Physics, University of Calcutta, Kolkata 700 009, India*

(Received 23 August 2010; published 29 November 2010)

We explore the prospects of low-scale leptogenesis in a class of supersymmetric $SO(10)$ models using extra singlet neutrinos (T_i, $i = 1, 2, 3$) and the Higgs representations $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ as well as $\mathbf{16_H} \oplus \overline{\mathbf{16}}_\mathbf{H}$. A singlet neutrino, which we show can be as light as 10^5–10^6 GeV, decays through its small mixings with right-handed (RH) neutrinos creating a lepton asymmetry which is explicitly shown to be flavor dependent. While the doublet vacuum expectation value in $\overline{\mathbf{16}}_\mathbf{H}$ triggers the generation of desired mixings, it also induces a large RH-triplet vacuum expectation value that breaks the left-right intermediate gauge symmetry and gives large right-handed neutrino masses. Manifest unification of gauge couplings and generation of heavy RH neutrino masses are achieved by purely renormalizable interactions. The canonical (Type-I) seesaw contributions to the light neutrino mass matrix cancel out while the Type-II seesaw contribution is negligible. Determining the parameters of the dominant inverse seesaw formula by using the underlying quark-lepton symmetry and neutrino oscillation data, we show how leptogenesis under the gravitino constraint is successfully implemented. New formulas for the decay rate and the asymmetry parameter are derived leading to baryon asymmetry within the observed range without invoking a resonant condition on RH neutrinos. The model is found to work for hierarchical as well as inverted hierarchical light neutrino masses. Testable predictions of the model are RH doubly charged Higgs bosons which may be leptophilic and accessible to the Tevatron, LHC or a linear collider. In a model-independent manner, the Drell-Yan pair production cross section at the Tevatron or LHC is shown to be bounded between 59%–79% of their left-handed counterparts with same mass. In contrast to single-step breaking supersymmetric grand unified theories, which predict a long proton lifetime for the decay $p \to e^+\pi^0$, here this lifetime is substantially reduced, bringing it within one order of the current experimental limit.

DOI: 10.1103/PhysRevD.82.093017 PACS numbers: 14.60.Pq, 12.10.Kt, 13.35.Hb

I. INTRODUCTION

$SO(10)$ [1] with supersymmetry (SUSY) has been at the center of attention for a number of attractive features. It contains just one right-handed (RH) neutrino per generation in its spinorial representaion **16**. With Pati-Salam [2] and left-right gauge symmetries [3] as its subgroups, in addition to unification of the three forces of Nature, it predicts high-scale unification of quark and lepton masses [4] and has the potential to explain the origin of parity ($\equiv$ P) and CP violations. Using the Higgs representations $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ and $\mathbf{10_H}$, it reproduces the small masses and large mixings of neutrinos through Type-I and Type-II seesaw mechanisms and their extensions [5–8]. It has been also shown that all the fermion masses can be fitted through SUSY $SO(10)$ by using suitable Higgs representations [8]. Another interesting aspect of the theory is that the observed tiny amount of matter-antimatter asymmetry of the Universe can be naturally explained through leptogenesis [9] and sphaleron effects [10–12].

In these theories neutrino masses indicated by oscillation data require the canonical seesaw scale of right-handed neutrinos to be in the range of $M_R \sim 10^{13}$–10^{15} GeV. This also sets the scale for the masses of associated Higgs triplets carrying $B - L = \pm 2$. This scale of neutrino mass generation is high in models with variants of the canonical seesaw [7,8,12] as well.

It is well known that the scale of leptogenesis through right-handed neutrino decays and canonical seesaw is constrained from below leading to the lower bound on the lightest RH neutrino mass $M_{N_1} \geq 10^9$ GeV [13]. This in turn requires the reheating temperature of the Universe after inflation to be at least $T_{\rm RH} \sim 10^9$ GeV. On the other hand, big-bang nucleosynthesis in SUSY theories sets a severe constraint on the gravitino mass and the reheating temperature leading to the upper bound $T_{\rm RH} \leq 10^7$ GeV [14]. While thermal leptogenesis in SUSY $SO(10)$ with a high seesaw scale easily satisfies the lower bound, the tension with the gravitino constraint is manifest.

Independent of quark-lepton unified theories, the question of baryogenesis via leptogenesis has been addressed in the context of the standard model (SM) and the minimal supersymmetric standard model (MSSM) [15] where freedom in the choice of Dirac neutrino Yukawa couplings permits fine-tuning them to very small values. In most of these models TeV scale resonant leptogenesis [16] is realized by degeneracy between right-handed neutrino masses. A major difficulty in having low-scale leptogenesis in

SUSY $SO(10)$ is the absence of such freedom because the underlying quark-lepton symmetry requires these Yukawa couplings to be of the same order as the corresponding up-quark Yukawa couplings. This latter difficulty persists even in some noncanonical seesaw models and several attempts have been made to bring down the scale of leptogenesis [17].

Another difficulty in renormalizable SUSY $SO(10)$ arises from the gauge coupling unification constraint and the need for an $SU(2)_R \times U(1)_{B-L}$ breaking intermediate scale that generates RH neutrino masses through renormalizable Majorana type interactions. It has been found that manifest unification of gauge couplings is spoiled in the presence of Higgs triplets of $\mathbf{126_H} \oplus \overline{\mathbf{126_H}}$ with intermediate symmetries such as $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_{3C}$ or $SU(2)_L \times SU(2)_R \times SU(4)_{4C}$ even at scales a few orders lower than the GUT-scale unless, in the first case, the LR gauge theory and $SO(10)$ are extended to include S_4 flavor symmetry [18] or additional light scalar degrees of freedom are introduced at lower scales [19,20]. On the other hand, there are a number of models with light right-handed gauge bosons [21,22] in which in place of the Higgs triplets with $B - L = \pm 2$ there are doublets carrying $B - L = \pm 1$. In contrast to the above scenarios, here we are interested in SUSY $SO(10)$ with both doublet and triplet scalars.

The Higgs triplets in $\mathbf{126_H}$ and $\overline{\mathbf{126_H}}$ representations include doubly charged bosons, $\Delta^{\pm\pm}$. Dedicated searches for such doubly charged scalars are being carried out at the Fermilab Tevatron [23]. Both the statistics and the energy reach are expected to be further enhanced at the CERN LHC. However, the high seesaw scale SUSY $SO(10)$ models will offer no prospects for these searches as the corresponding masses are large, $M_\Delta \geq 10^{11}$ GeV, while in the class of low intermediate scale SUSY $SO(10)$ models where only RH-doublets in $\mathbf{16_H} \oplus \overline{\mathbf{16_H}}$ are used near the TeV scale [21,22,24] no doubly charged Higgs bosons are present.

In this paper we address the issues of neutrino masses and mixings, low-scale leptogenesis consistent with the gravitino constraint, manifest unification of gauge couplings through renormalizable interactions, and testable experimental signatures of the proposed model at the Tevatron, LHC or ILC. We construct the desired SUSY $SO(10)$ model including the RH-triplets in $\mathbf{126_H} \oplus \overline{\mathbf{126_H}}$ as well as the RH-doublets in $\mathbf{16_H} \oplus \overline{\mathbf{16_H}}$, and three singlet fermions (T_i, $i = 1, 2, 3$) [25]. We find that a singlet fermion in the mass range $M_T = 10^5$–10^6 GeV can go out of equilibrium to generate lepton asymmetry; its decay is naturally suppressed by small mixing with heavy right-handed neutrinos (N_i). The vacuum expectation value of the RH-doublet in $\overline{\mathbf{16_H}}$ (or $\mathbf{16_H}$) responsible for this desired small mixing also induces a large vacuum expectation value (vev) of the RH-triplets in $\mathbf{126_H}$ (or $\overline{\mathbf{126_H}}$). This breaks $SU(2)_R \times U(1)_{B-L} \rightarrow U(1)_Y$ leading to large RH-neutrino masses through renormalizable interactions. We find that although heavy right-handed neutrinos are present in the model, the Type-I seesaw contributions to the neutrino mass cancel out as has been observed in the context of the standard model or its extension [26,27]. The Type-II contribution is also found to be negligible. The dominant contribution to light neutrino masses arises through an inverse seesaw which has attracted considerable attention over the recent years [22,28,29].

In an earlier work by us and S. K. Majee it was found that gauge coupling unification with threshold-like behavior would be possible through the presence of two nonrenormalizable dimension-5 operators [30]. Here, without using any dimension-5 operators, we obtain manifest unification of gauge couplings in the renormalizable theory with asymmetric left-right intermediate gauge symmetry ($g_{2L} \neq g_{2R}$) operative at any scale between 10^9 and 10^{15} GeV. Further, while lepton asymmetry was computed through solutions of Boltzmann equation [30] with an assumption about the asymmetry parameter, in this work we derive new analytic formulas for the decay rate and the CP-asymmetry parameter and find that they are explicitly flavor dependent. We then show analytically that when the model parameters estimated using the neutrino oscillation data are used in our new formula, the model yields desired values of the CP-asymmetry parameter leading to the observed baryon to photon density ratio. In addition, we demonstrate that the model is consistently successful for both hierarchical as well as invertedly hierarchical light neutrino masses. The model leaves its testable signature at the LHC, Tevatron and ILC [23,31,32] through doubly charged right-handed Higgs scalars $\Delta_R^{\pm\pm}$ in the mass range of 100 GeV to a few TeV. Since the decay mode $\Delta_R^{\pm\pm} \rightarrow W_R^\pm W_R^\pm$ is kinematically forbidden these Higgs bosons are leptophilic and predominantly result in like-sign charged bilepton pairs $\Delta_R^{\pm\pm} \rightarrow l_R^\pm l_R^\pm$. The absence of light $\Delta_L^{\pm\pm}$, $\Delta_L^\pm$ states and also the absence of left-handed bilepton pairs in the decays would provide signatures specific to this model which are different from other bilepton production modes.

In a model-independent manner without using any structure function data, we show analytically that the Drell-Yan hadronic pair production cross section for these RH Higgs bosons is bounded between 59%–79% of that for a left-handed boson of similar mass.

It is found that, triggered by low-mass RH doubly charged Higgs, at the unification scale the GUT coupling lies in the strong but perturbative regime and the gauge-boson mediated proton decay rate is enhanced. The lifetime $\tau_p(p \rightarrow e^+\pi^0)$ is shorter and remains within one order of the current experimental limit; this can be reached by the ongoing or planned proton decay searches [33–35].

This paper is organized in the following manner. In Sec. II we present the essence of the model. Unification of gauge couplings with left-right intermediate symmetry is examined in Sec. III where we also discuss proton lifetime predictions. Derivation of new formulas for the singlet-fermion decay rate and the CP-asymmetry parameter are in Sec. IV along with the predictions for the baryon asymmetry. In Sec. V we discuss testable predictions of the model at the Tevatron, LHC and ILC where we also provide an estimate of the upper and lower bounds on the Drell-Yan pair production cross section. A brief summary and conclusions are given in Sec. VI.

II. THE MODEL

In this section, we present the salient features of the model responsible for explaining neutrino masses, mixings, and leptogenesis with testable signature at accelerator energies. We consider the following pattern of spontaneous symmetry breaking originating from SUSY $SO(10)$,

$$\begin{aligned} SO(10) &\stackrel{(M_U)}{\rightarrow} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \\ &\quad \times D[\mathcal{G}_{2213P}] \\ &\stackrel{(M_P)}{\rightarrow} SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C[\mathcal{G}_{2213}] \\ &\stackrel{(M_R)}{\rightarrow} SU(2)_L \times U(1)_Y \times SU(3)_C[\mathcal{G}_{\rm std}] \\ &\stackrel{(M_Z)}{\rightarrow} SU(3)_C \times U(1)_Q. \end{aligned}$$

The first stage of spontaneous symmetry breaking (SSB) is carried out by assigning GUT-scale vacuum expectation values to the Φ_{54} of $SO(10)$ along the direction singlet[1] under the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv \mathcal{G}_{\rm PS}$ [2] as well as the singlet direction under the left-right gauge group $SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} \times SU(3)_C \equiv \mathcal{G}_{2213}$ in the $\mathcal{G}_{\rm PS}$ multiplet (1, 1, 15) contained in a $\Phi_{210}^{(1)}$ of $SO(10)$. At this stage D-parity remains intact and the gauge couplings of $SU(2)_L$ and $SU(2)_R$ are equal, $g_L = g_R$ [36]. The second stage of SSB takes place by assigning a vacuum expectation value to the D-Parity odd singlet also contained in $\Phi_{210}^{(2)}$ of $SO(10)$. By suitable fine tunings of the trilinear couplings between **210** and the $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ or $\mathbf{16_H} \oplus \overline{\mathbf{16}}_\mathbf{H}$ the right-handed triplets $\mathbf{\Delta_R} \oplus \bar{\mathbf{\Delta}}_\mathbf{R} \subset \mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ and the RH-doublets $\chi_\mathbf{R} \oplus \bar{\chi}_\mathbf{R} \subset \mathbf{16_H} \oplus \overline{\mathbf{16}}_\mathbf{H}$ are made much lighter compared to their left-handed counterparts. By adopting higher degree of fine-tuning for the RH-triplet compared to the RH-doublet, the components of the RH-triplet pairs can be assigned masses between 100 GeV to a few TeV while the RH-doublet pairs are kept heavier, but sufficiently lighter than the GUT scale. Although we do not ascribe any vev directly to the neutral components of the RH-triplets in $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$, we will find that once a vev is assigned to the neutral component of the RH-doublet in $\mathbf{16_H}$, the triplet vev is automatically induced. Smaller is the RH-triplet mass fixed by the D-parity breaking mechanism, larger is the induced triplet vev.

The reason behind such ordering of Higgs masses and vevs becomes transparent once we consider the Yukawa Lagrangian near the intermediate scale emerging from $SO(10)$,

$$\mathcal{L}_Y = Y\bar{\psi}_L\psi_R\Phi + f\psi_R^T\tau_2\psi_R\bar{\Delta}_R + F\bar{\psi}_R T\chi_R + \mu T^T T + \text{H.c.} \tag{1}$$

where $\psi_{L,R}$ are left- (right-) handed lepton doublets and T the three fermion singlet fields, one for each generation. The superscript T, of course, denotes transpose. In the (ν, N, T) basis this will lead to a 3×3 mass matrix[2] with vanishing 11, 13, and 31 blocks.

$$M_\nu = \begin{pmatrix} \nu & N^c & T \end{pmatrix}_L \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & M_N & M_X \\ 0 & M_X^T & \mu \end{pmatrix} \begin{pmatrix} \nu \\ N^c \\ T \end{pmatrix}_L. \tag{2}$$

Here the $N - T$ mixing matrix arises through the vev of the RH-doublet field with $M_X = Fv_\chi$, where $v_\chi = \langle \chi_R^0 \rangle$, and the RH-Majorana neutrino mass is generated by the induced vev of the RH-triplet with $M_N = fv_R$, with $v_R = \langle \bar{\Delta}_R^0 \rangle$. The vev of the weak bi-doublet $\Phi(2, 2, 0, 1) \subset 10_H$ of $SO(10)$ yields the Dirac mass matrix for neutrinos, $m_D = Y\langle \Phi^0 \rangle$.

While implementing leptogenesis in this model through T decays, the out-of equilibrium condition requires the mixing with RH neutrinos to be small. This will be naturally obtained if $M_N \gg M_X$ or if $v_R \gg v_\chi$.

Assuming $M_N \gg M_X \gg \mu, m_D$, which would be highly desirable for the present model, integrating out the heavy RH neutrinos leads to the effective Lagrangian [27],

$$\mathcal{L}_{(\rm mass)} = -(\mu - M_X^T M_N^{-1} M_X) T^T T - m_D M_N^{-1} m_D^T \nu^T \nu - M_X^T M_N^{-1} m_D^T \bar{T} \nu + \text{H.c.} \tag{3}$$

Interestingly, the block diagonalization of this mass matrix results in a cancellation among the Type-I seesaw contributions and the light neutrino mass m_ν is dominated by the inverse seesaw and one obtains,

$$m_\nu = -m_D[M_X^{-1}\mu(M_X^T)^{-1}]m_D^T, \tag{4}$$

$$M_T = \mu - M_X M_N^{-1} M_X^T, \tag{5}$$

$$M = M_N + M_X M_N^{-1} M_X^T. \tag{6}$$

It will be shown in the next section that the left-handed triplets are near the GUT scale while $v_R \sim 10^{10}$–10^{12} GeV leading to negligible Type-II contribution for light neutrino masses for suitable values of the model parameters.

To see how the induced vev is generated, consider the Higgs superpotential near the intermediate scale where all GUT-scale masses have decoupled,

[1]The rôle of this vev is discussed in [18].

[2]Each entry in this mass matrix is a (3×3) block.

$$W = M_{\Delta_R}\Delta_R\bar{\Delta}_R + M_{\chi_R}\chi_R\bar{\chi}_R + \lambda_1\bar{\Delta}_R\chi_R\chi_R + \lambda_2\Delta_R\bar{\chi}_R\bar{\chi}_R. \quad (7)$$

Using $\langle\chi_R^0\rangle = \langle\bar{\chi}_R^0\rangle = v_\chi$, $\langle\Delta_R^0\rangle = \langle\bar{\Delta}_R^0\rangle = v_R$ which requires $\lambda_1 = \lambda_2 \equiv \lambda$, the vanishing F-term conditions, $F_{\Delta_R^0} = F_{\bar{\Delta}_R^0} = F_{\chi_R^0} = F_{\bar{\chi}_R^0} = 0$ give

$$v_R = -\lambda\frac{v_\chi^2}{M_{\Delta_R}},\quad M_{\Delta_R}M_{\chi_R} = 2\lambda^2 v_\chi^2,\quad M_{\chi_R} = -2\lambda v_R. \quad (8)$$

The above equations imply that even though no direct vev is ascribed to Δ_R^0 or $\bar{\Delta}_R^0$, a large vev is induced once a direct vev is assigned to χ_R^0, the latter being essential to generate the desired $N - T$ mixings. With lighter RH-triplet masses $M_\Delta \simeq 100$ GeV–1 TeV, it is possible to have $v_R \simeq 10^{10}$–10^{12} GeV for $v_\chi = 10^6$–10^7 GeV. Since $v_R \gg v_\chi$, the spontaneous breaking $SU(2)_R \times U(1)_{B-L} \to U(1)_Y$ takes place at the higher scale generating large RH-Majorana neutrino masses $M_N \gg M_X$ leading to small $N_i - T_j$ mixings needed to establish the out-of equilibrium conditions for leptogenesis.

We assume the Majorana Yukawa coupling to be diagonal, $M_N = \mathrm{diag}(M_{N_1}, M_{N_2}, M_{N_3})$. This gives $N_i - T_j$ mixing angles,

$$\sin\xi_{ij} \simeq \frac{M_{X_{ij}}}{M_{N_i}}. \quad (9)$$

In the present model, the left-handed triplet pair in $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ acquires mass near the D-parity breaking scale $M_P \gg M_R$. In conventional models even with the left-handed triplet mass $\simeq 10^{13}$–10^{14} GeV, the Type-II seesaw contribution is comparable to the Type-I contribution. In this model the Type-II seesaw contribution to the light neutrino mass matrix is

$$m_{\mathrm{II}} = f\lambda'\frac{v_\chi^2 v_u^2}{M_P^2 M_\Delta}, \quad (10)$$

where M_P is the D-parity violation scale which is also the left-handed triplet mass. Now using $M_\Delta = 1$ TeV, $v_\chi = 10^6$–10^7 GeV, $v_u = 100$ GeV, and $M_P \simeq M_U = 10^{16.5}$ GeV, we obtain

$$m_{\mathrm{II}} = f\lambda'(10^{-20}\text{–}10^{-17})\ \mathrm{GeV}, \quad (11)$$

which is at least 7 orders of magnitude smaller than the highest value of hierarchical masses obtained from the neutrino oscillation data as proposed in [30].

Subject to small RG corrections, the underlying quark-lepton unification in $SO(10)$ approximates the Dirac neutrino mass matrix with the up-quark mass matrix. The light neutrino mass matrix is constructed using the available data on neutrino masses and mixings with a reasonable assumption on the leptonic phase of the PMNS matrix. Our strategy is to determine the mass eigenvalues and mixings of fermion singlets as well as their mixings with RH neutrinos to implement the leptogenesis scenario through their decays as will be discussed in Sec. IV.

Before addressing the leptogenesis issue we show in the next section that manifest gauge coupling unification occurs in SUSY $SO(10)$ with $\mathcal{G}_{\mathrm{LR}}$ intermediate gauge symmetry. No nonrenormalizable dim.5 operators are needed to support the unification idea.

III. UNIFICATION, HIGH W_R MASS, PROTON LIFETIME

Manifest unification of gauge couplings converging to a GUT-scale value in SUSY $SO(10)$ models having left-right intermediate symmetry has been found possible earlier by inclusion of additional scalar degrees of freedom beyond those needed for spontaneous symmetry breaking [19–21]. More recently this method has been evoked to fit masses of all charged fermions and for explaining small neutrino masses with W_R-boson mass even at the TeV scale [22]. In [30] unification of gauge couplings was accomplished by using threshold-like contributions of two nonrenormalizable dimension-5 operators at the GUT scale. Manifest unification has been also found to be possible when both the left-right intermediate gauge symmetry and SUSY $SO(10)$ are extended to contain S_4 flavor symmetry [18]. The left-right gauge symmetry in that case also has unbroken D-parity as well as unbroken R-parity down to the intermediate scale. In the present model there is no flavor symmetry. D-parity is broken at the GUT scale and R-parity is spontaneously broken at a lower scale by the vev of RH-doublets in 16_H. In addition the model has a testable novel feature of accessible doubly charged Higgs scalars.

In this section we show how manifest unification takes place with the gauge couplings of $\mathcal{G}_{2213}$ converging at the GUT scale without invoking the effect of any nonrenormalizable operators. We also show how the proton lifetime for the decay $p \to e^+\pi^0$ is brought closer to the current experimental limit [34].

A. Gauge coupling unification

We assume the superpartners of the SM particles to have masses of the order of a TeV. Using renormalization group equations (RGEs) for the gauge couplings up to one-loop [37]

$$\mu\frac{dg_i}{d\mu} = -\left(\frac{a_i}{16\pi^2}\right)g_i^3, \quad (12)$$

where i ranges over the set of gauge couplings. Below we list the particles which, with their superpartners, contribute to the a_i coefficients in different energy ranges.

(i) $M_Z \le \mu \le M_{\mathrm{SUSY}}$: Here the particle spectrum is the same as in the non-SUSY SM with three fermion generations,

$$a_Y = \frac{41}{10}, \qquad a_{2L} = -\frac{19}{6}, \qquad a_{3C} = -7. \quad (13)$$

(ii) $M_{\rm SUSY} \le \mu \le M_\sigma$: In this range, in addition to the MSSM particle spectrum, we have the doubly charged Higgs bosons left as unabsorbed components of RH Higgs triplets and these modify only the a_Y coefficients compared to the MSSM.

$$a_Y = \frac{57}{5}, \qquad a_{2L} = 1, \qquad a_{3C} = -3. \quad (14)$$

Because of relatively larger value of $a_Y = \frac{57}{5}$ ($a_Y = \frac{33}{5}$ for the MSSM), due to the $\Delta^{\pm\pm}$ near the TeV scale, the $U(1)_Y$ coupling grows faster, triggering a tendency of unification at substantially lower scales. This difficulty is bypassed by embedding G_{213} into the G_{2213} intermediate symmetry. At the boundary point, the $U(1)_{B-L}$ coupling starts from a lower value while the $SU(2)_R$ coupling is higher ensuring unification at the GUT scale. The exact unification of all four couplings of G_{2213} is achieved by introducing additional scalar submultiplets such as $\sigma_L(3, 0, 1)$ and $C_8(1, 0, 8)$ at scales M_σ and M_C, respectively. It has been noted earlier that such states in the adjoint representations of the standard model subgroups with $Y = 0$ could be naturally light and arise as continuous moduli states of string theory, playing a significant role to reconcile the discrepancy between the GUT scale and the string scale [38]. In our case these submultiplets are contained in the $SO(10)$ representations 210 and 45, whereas $C_8(1, 0, 8)$ is also contained in the Higgs representation $54 \subset SO(10)$. Alternatively, every pair of triplet σ_Ls can be replaced by a fermion triplet which has been noted to play the role of stable dark matter [39] if its mass is low. This fermionic state along with others may be present in nonminimal $SO(10)$ representations [40].

We will show below that one set of solutions of RGEs needs a pair of triplet scalars ($n_\sigma = 2$) or equivalently a fermionic triplet with mass $\sim$100 GeV. In that case, only the scalars $C_8(1, 0, 8)$ may be treated as naturally light continuous moduli states of string theory, or, purely from $SO(10)$ point of view, the mechanism of Refs. [18,22] can be utilized to make them light by exploiting the generalized superpotential [41].

Another pertinent question arises if one wishes to use a pair of moduli states $\sigma_L(3, 1, 0, 1)$ under $\mathcal{G}_{\rm LR}$. How is the lightness of these states ensured in the context of D-parity breaking at the GUT scale leading to lighter components of RH-triplets in $126_H \oplus \overline{126}_H$ and RH-doublets in $16_H \oplus \overline{16}_H$. This question is readily answered by examining the part of the superpotential,

$$\begin{aligned} W &= W_1 + W_2 + W_3 + \ldots. \\ W_1 &= M_{126}\overline{126}_H 126_H + \lambda_{126} 210_H \overline{126}_H 126_H, \\ W_2 &= M_{16}\overline{16}_H 16_H + \lambda_{16} 210_H \overline{16}_H 16_H, \\ W_3 &= M_{45} 45_H^2 + \lambda_{45} 210_H 45_H^2. \end{aligned} \quad (15)$$

Noting that the singlet under $\mathcal{G}_{\rm PS}$ contained in 210_H is D-odd, the RH-triplets are made light when the parameters M_{126} and $\lambda_{126}\langle 210_H\rangle$ are in the same phase. Similarly the RH-doublets are made lighter than the GUT scale when M_{16} and $\lambda_{16}\langle 210_H\rangle$ are in the same phase. Thus, it is clear that the same mechanism also renders $\sigma_L(3, 1, 0, 1) \subset 45_H$ substantially lighter than the GUT scale while keeping $\sigma_R(1, 3, 0, 1) \subset 45_H$ heavy when M_{45} and $\lambda_{45}\langle 210_H\rangle$ are in *opposite* phase.

Purely from SUSY $SO(10)$ considerations with standard three fermion generations, the method of keeping the relevant Higgs scalars substantially lighter than the GUT scale has been discussed in Refs. [18,22] by exploiting the minimization of the generalized superpotential of Ref. [41].

(iii) $M_\sigma \le \mu \le M_C$: In this range in addition to the contribution of the particles listed above we include those from n_σ members of Higgs scalar triplets $\sigma_L(3, 0, 1)$ leading to $a_{2L} = 1 + 2n_\sigma$, and $a_Y = \frac{57}{5}$, $a_{3C} = -3$ as before.

(iv) $M_C \le \mu \le M_R$: Over and above the contributions mentioned above, here we include the n_c color octets $C_8(1, 0, 8)$ resulting in $a_{3C} = -3 + 3n_C$ and $a_Y = \frac{57}{5}$, $a_{2L} = 1 + 2n_\sigma$, as before.

(v) $M_R \le \mu \le M_U$: In the presence of $\mathcal{G}_{\rm LR}$ gauge symmetry we have contributions of all the submultiplets discussed above. In addition, from the $126_H \oplus \overline{126}_H$ and $16_H \oplus \overline{16}_H$ the following submultiplets must now be included: $\Phi(2, 2, 0, 1) \oplus \chi_R(1, 2, -1, 1) \oplus \bar{\chi}_R(1, 2, +1, 1) \oplus \Delta_R(1, 3, -2, 1) \oplus \bar{\Delta}_R(1, 3, +2, 1) \oplus n_\sigma \sigma_L(3, 1, 0, 1) \oplus n_C C_8(1, 1, 0, 8)$.

(vi) $\mu \ge M_R$ with $n_\sigma = n_C = 3$ we have

$$\begin{aligned} a_{BL} &= 33/2, \qquad a_{2L} = 7, \\ a_{2R} &= 6, \qquad a_{3C} = 6. \end{aligned} \quad (16)$$

With the above particle content and keeping the possibilities of M_σ, M_C smaller or larger than the intermediate scale M_R, allowed solutions are realized with $M_R = 10^9$–$10^{12.5}$ GeV, $M_U = 10^{15.75}$–$10^{16.5}$ GeV and $\alpha_G^{-1} \simeq 5$–10. This covers the desired range $M_R = 10^{11}$–10^{12} GeV required to implement viable leptogenesis while satisfying the gravitino constraint.

For a typical example, the evolution of the gauge couplings and unification at the GUT scale are shown in Fig. 1 for which we have obtained

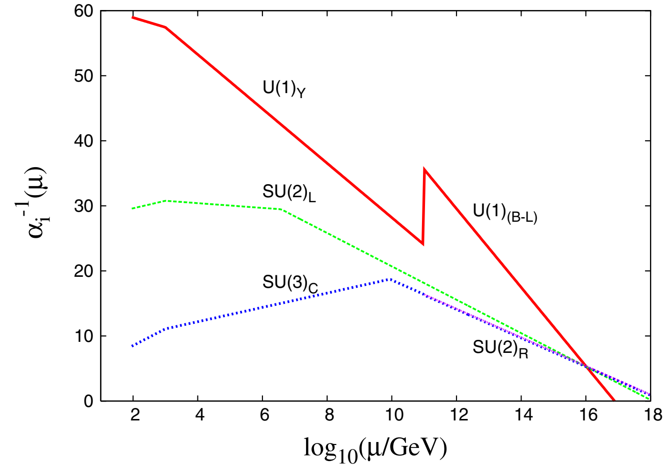

FIG. 1 (color online). Unification of gauge couplings with left-right symmetry breaking at $M_R = 10^{11}$ GeV with $n_\sigma = n_C = 3$ (see text). Below $M_U = M_P = 10^{16}$ GeV, $g_L \neq g_R$. The $SU(3)_C$ coupling (short-dashed line) and the $SU(2)_R$ coupling (dotted line) appear to merge for $\mu > M_R$ because fortuitously, in this example, the beta-function coefficients for both are nearly equal in this energy range as are the boundary values of the two couplings at M_R.

$$M_R = 10^{11}\ \text{GeV}, \qquad M_U = 10^{16}\ \text{GeV}, \tag{17}$$

with $\alpha_G^{-1} = 5.3$ which is well within the perturbative limit. In Fig. 1 the couplings for $SU(2)_R$ and $SU(3)_C$ are found to be almost overlapping above the scale M_R because of a fortuitous identity of their respective beta-function coefficients and near equality of the boundary values at M_R in this example. The change in slopes at M_σ and M_C is clearly noticeable.

In Table I we present several choices of n_σ and n_C and their respective mass scales for which the same values of $M_R = 10^{11}$ GeV and $M_U = 10^{16}$ GeV are obtained as in Fig. 1. The same results follow when a pair of σ_L's are replaced by a fermion triplet contained in additional $SO(10)$ representations. These fermions while driving type-III seesaw for their appropriate mass ranges, may also serve as stable dark matter candidates if their mass is low [39,40].

B. Observable gauge-boson mediated proton decay

There are elegant methods and models to suppress Higgsino mediated proton decay or allow both types of decays through dimension-5 or dimension-6 operators [42–44]. In most of the single-step breaking models, barring a few [44], neglecting threshold effects, the unification scale is $M_U^0 = 2 \times 10^{16}$ GeV with $\alpha_G^{-1} \simeq 25$ which imply large values of the lifetime for gauge-boson mediated proton decay, e.g. $p \to e^+\pi^0$, for which the current lower bound is [34] $(\tau_p)_{\text{expt}} \geq 1.01 \times 10^{34}$ yrs. Extensive estimations of the decay rate have been made in minimal GUTs and their extensions with or without SUSY [33,40,44,45]. Up to a good approximation, the decay width in the present model can be written as

$$\Gamma(p \to e^+\pi^0) = \frac{m_p}{64\pi f_\pi^2}\left(\frac{g_G^4}{M_U^4}\right)A_L^2\bar{\alpha}_H{}^2(1 + D + F)^2 \times [(A_{SR}^2 + A_{SL}^2)(1 + |V_{ud}|^2)^2]. \tag{18}$$

In the above formula $\bar{\alpha}_H$ is the hadronic matrix element, $m_p =$ proton mass $= 938.3$ MeV, $f_\pi =$ pion decay constant $= 139$ MeV, and the chiral Lagrangian parameters are $D = 0.81$, $F = 0.47$. The short distance renormalization for relevant dimension-6 operators evaluated with supersymmetry from $M_U \to M_{\text{SUSY}}$ and without supersymmetry from $M_{\text{SUSY}} = 1$ TeV $\to M_Z$ in the present model with appropriate anomalous dimensions [46] gives $A_{SL} \simeq A_{SR} \equiv A_{SD} = 2.38$. The long-distance renormalization factor is known to be $A_L = 1.25$. Noting that $A_R = A_L A_{SD} \simeq 2.98$, and $F_q = 2(1 + |V_{ud}|^2)^2 \simeq 7.6$, we then express the lifetime as

$$\Gamma^{-1}(p \to e^+\pi^0) = (1.0 \times 10^{34}\ \text{yrs})\left(\frac{0.012\ \text{GeV}^3}{\alpha_H}\right)^2\left(\frac{2.98}{A_R}\right)^2 \times \left(\frac{1/5}{\alpha_G}\right)^2 \times \left(\frac{7.6}{F_q}\right)\left(\frac{M_U}{1.3 \times 10^{16}\ \text{GeV}}\right)^4, \tag{19}$$

where we have used $\alpha_H = \bar{\alpha}_H(1 + D + F) \simeq 0.012\ \text{GeV}^3$ as per recent lattice estimations [47]. In a number of single-step breaking models or other intermediate breaking models with $M_U = M_U^0 = 2 \times 10^{16}$ GeV and $\alpha_G^{-1} \simeq 25$, the one-loop estimation gives large proton lifetime $\tau_p(p \to e + \pi^0) \sim O(10^{36})$ yrs, which is beyond the experimentally accessible limits of ongoing and planned proton decay searches for the $p \to e^+\pi^0$ mode.

In the present model some of our predictions at one-loop level using Eq. (19) and RGE solutions are given in Table II. Although two-loop and threshold corrections are likely to improve these results, at one-loop level itself our predictions on the lifetime are substantially less than a large number of single-step breaking models, with a few exceptions [44], and other intermediate breaking models in conventional SUSY $SO(10)$ GUTs. Our model predictions are found to remain within one order of the current experimental limit and are likely to be accessible to ongoing and planned experiments for proton decay searches [34,35]. Out of the two observable model predictions, namely, the

TABLE I. The number of $\sigma_L(3, 0, 1)$ and $C_8(1, 0, 8)$ submultiplets with their respective mass scales which lead to solutions with $M_U = 10^{16}$ GeV, $M_R = 10^{11}$ GeV, and $\alpha_G^{-1} = 5.3$.

n_σ	n_C	M_σ (GeV)	M_C (GeV)
2	2	100	7.16×10^6
3	3	3.8×10^6	8.67×10^9
4	3	8.63×10^8	8.67×10^9

TABLE II. Gauge-boson mediated decay lifetime for $p \to e^+\pi^0$ in SUSY $SO(10)$ with G_{2213} intermediate symmetry as described in the text.

M_R (GeV)	M_U (GeV)	α_G^{-1}	$\tau_p(p \to e^+\pi^0)$ (yrs.)
10^{11}	1.4×10^{16}	5.3	1.5×10^{34}
10^{11}	2×10^{16}	4.2	4×10^{34}
10^{12}	2×10^{16}	4.1	3.8×10^{34}
10^{13}	2×10^{16}	3.3	2.5×10^{34}
10^{9}	1.4×10^{16}	6.2	8.6×10^{34}

low-mass RH doubly charged Higgs (discussed later) and proton decay, if any one is first observed experimentally, the observation on the other should follow.

IV. LEPTOGENESIS THROUGH SINGLET FERMION DECAY

A. Leptogenesis and canonical seesaw

In the standard formulation of leptogenesis the lightest right-handed neutrino decays into either $l^-\phi^+$ and $\nu\phi^0$ or into their conjugate channels $l^+\phi^-$ and $\bar{\nu}\bar{\phi}^0$ and the desired CP-asymmetry is generated by the interference of the tree-level amplitude with one-loop amplitudes (vertex and self-energy corrections). Denoting the mass eigenvalue of the ith RH neutrino as M_{N_i}, using the canonical Type-I seesaw formula the decay rate of N_1 is

$$\Gamma_1 = \frac{1}{8\pi} M_{N_1}(Y_D^\dagger Y_D)_{11} = \frac{1}{8\pi v_u^2}\tilde{m}_1 M_{N_1}^2, \qquad (20)$$

where $\tilde{m}_1$ is roughly the lightest left-handed Majorana neutrino mass, v_u the vev of the up-type Higgs, and Y_D the Dirac-type neutrino Yukawa matrix which, up to RG corrections, is the same as the up-quark Yukawa matrix. A net lepton asymmetry is generated when the decay process goes out of equilibrium at temperature $\sim M_{N_1}$ satisfying the condition,

$$\Gamma_1 < H(T = M_{N_1}), \qquad H(T) = 1.66 g_*^{1/2}\frac{T^2}{M_{Pl}}, \qquad (21)$$

where H is the Hubble expansion rate. In the normal hierarchical case the generated CP-asymmetry is expressed as

$$\epsilon_1 = -\frac{3}{8\pi v_u^2}\frac{M_{N_1}}{M_{N_2}}\frac{\mathrm{Im}[(m_D^\dagger m_D)_{12}]^2}{(m_D^\dagger m_D)_{11}}. \qquad (22)$$

The canonical seesaw mechanism gives rise to the lower bound $M_{N_1} \geq 2.9 \times 10^9$ GeV [13]. Since this exceeds the upper bound on the gravitino mass by several orders, the tension between standard leptogenesis and the gravitino constraint in SUSY theories is explicit.

In a large class of solutions of the present model, examples of which are considered in the following subsections, the decay of two of the three singlet neutrinos to $l^-\phi^+$ and $\nu\phi^0$ (and the charge conjugate states) is kinematically forbidden. Also, the mass of the remaining singlet neutrino is $\simeq 10^5$–10^6 GeV, determined by the neutrino oscillation data, and is substantially smaller than that of the RH neutrinos. Thus, its decay can reconcile with the gravitino constraint provided it goes out of equilibrium at temperatures $\sim M_{T_1}$ and it generates the required lepton asymmetry. The out-of-equilibrium condition is found to be naturally achieved due to the small mixings of T_1 with heavy right-handed neutrinos dictated by the model.

B. Model parameters for the inverse seesaw

In this subsection we show how the parameters needed for leptogenesis are obtained in this model using the inverse seesaw formula, neutrino oscillation data, up-quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. Apart from the RH-neutrino mass matrix M_N assumed to be diagonal with the largest element of order v_R, Eqs. (4)–(6) contain four mass matrices: the light neutrino mass matrix m_ν, the neutrino Dirac mass matrix m_D, the $N - T$ mixing matrix M_X, and the singlet-fermion mass matrix μ.

We construct m_ν from the mass eigenvalues ($m_{1,2,3}$) via the PMNS matrix, U_{PMNS}, for which we use $\theta_{12} = 32°$, $\theta_{23} = 45°$, $\theta_{13} = 7°$ and take the leptonic Dirac phase $\delta_{\mathrm{PMNS}} = 1.0$ radian.

$$m_\nu = U_{\mathrm{PMNS}}^T \mathrm{diag}(m_1, m_2, m_3) U_{\mathrm{PMNS}}. \qquad (23)$$

The Dirac neutrino mass matrix is fixed by the underlying quark-lepton symmetry of $SO(10)$. Neglecting small RG corrections, it is taken to be approximately equal to the up-quark mass matrix. Using the PDG values [48] of the CKM matrix elements, its Dirac phase, and the running masses of the three up-type quarks, namely, $m_u = 2$ MeV, $m_c = 1.5$ GeV, $m_t = 171$ GeV, we have,

$$m_D \simeq M_U = V_{\mathrm{CKM}}^\dagger \mathrm{diag}(m_u, m_c, m_t) V_{\mathrm{CKM}}, \qquad (24)$$

where we have used the CKM phase $\delta_{\mathrm{CKM}} = 1.0$ radian, and the quark mixing angles $\sin\theta_{12}^q = 0.2243$, $\sin\theta_{23}^q = 0.0413$, and $\sin\theta_{13}^q = 0.0037$.

The mass matrices M_X and μ are not constrained by experimental data. To minimize the unknown parameters we assume a simple form for the $N - T$ mixing matrix M_X,

$$\mathbf{M_X} = \begin{pmatrix} M_{X_{11}} & 0 & 0 \\ 0 & 0 & M_{X_{23}} \\ 0 & M_{X_{32}} & 0 \end{pmatrix}, \qquad (25)$$

where using Eqs. (1) and (2) we have defined

$$M_{X_{ij}} = F_{ij} v_\chi. \qquad (26)$$

With the knowledge of m_ν, m_D and M_X we then use the inverse seesaw mass formula—Eq. (4)—to obtain elements of the matrix μ for hierarchical as well as invertedly hierarchical light neutrino masses. The μ and M_X matrices are used in Eq. (5) to compute the mass eigenvalues of the singlet fermions and their mixings by the diagonalization procedure with

$$\hat{T}_i = \sum_j \tilde{U}_{ij} T_j, \tag{27}$$

where we denote the mass eigenstates by $\hat{T}_i$ and the corresponding mixing matrix by $\tilde{U}$. Thus the two inputs matrices M_N (chosen diagonal) and M_X—Eq. (25)—completely determine the singlet neutrino, T_i, masses and mixings consistent with the data on the light neutrino mass spectrum, mixing, and grand unification.

C. Analytic formulas for decay rate and asymmetry parameter

The physical processes responsible for leptogenesis are shown in Fig. 2 where crosses denote appropriate $N - T$ mixings. The flavor-dependent decay rate for the singlet fermion T_i of mass M_{T_i} through its mixing with N_i—recall Eq. (9)—can now be expressed as

$$\Gamma_{T_i} = \frac{1}{8\pi v_u^2} M_{T_i} \sum_{jk} |\tilde{U}_{ij}|^2 \sin^2\xi_{jk} (m_D^\dagger m_D)_{kk}. \tag{28}$$

We find that, depending on the choices of M_N and M_X, there is a wide possibility for the singlet neutrino mass eigenvalues. In particular, the model permits a class of solutions where only one state has mass above the $l\phi$ threshold (~ 100 GeV) while two others have masses substantially below. Denoting this eigenstate as $\hat{T}_1$ we discuss leptogenesis through its decay in the rest of this paper. Because of the simple assumption on the M_X matrix given in Eq. (25) the decay rate and the asymmetry parameter are reduced to the forms

$$\begin{aligned} \Gamma_{T_1} = \frac{1}{8\pi} M_{T_1} \frac{K_1}{K_2} [(|\tilde{U}_{11}|)^2 \sin^2\xi_{11} (Y_D^\dagger Y_D)_{11} \\ + (|\tilde{U}_{12}|)^2 \sin^2\xi_{32} (Y_D^\dagger Y_D)_{33} \\ + (|\tilde{U}_{13}|)^2 \sin^2\xi_{23} (Y_D^\dagger Y_D)_{22}], \end{aligned} \tag{29}$$

where K_1, K_2 are modified Bessel functions. Even though Y_D is of the same order as the up-quark Yukawa matrix, the smallness of Γ_{T_1} compared to the Type-I seesaw case—Eq. (20)—originates from two sources: (i) Allowed values of $M_{T_1} \ll M_{N_i} (i = 1, 2, 3)$, (ii) $\sin^2\xi_{jk} \ll 1 (j, k = 1, 2, 3)$. These two features achieve the out-of-equilibrium condition at temperature $\sim M_{T_1}$ satisfying the gravitino constraint. The asymmetry parameter can be expressed as

$$\begin{aligned} \epsilon_1 &= \frac{-3M_{T_i}}{8\pi} \frac{P}{Q}, \\ P &= [(|\tilde{U}_{11}|)^2 \sin^2\xi_{11}/M_{N_3} - (|\tilde{U}_{12}|)^2 \sin^2\xi_{32}/M_{N_1}] \mathrm{Im}[Y_{1i} Y_{3i}^*]^2 + [(|\tilde{U}_{13}|)^2 \sin^2\xi_{23}/M_{N_3} \\ &\quad - (|\tilde{U}_{12}|)^2 \sin^2\xi_{32}/M_{N_2}] \mathrm{Im}[Y_{2i} Y_{3i}^*]^2 + [(|\tilde{U}_{11}|)^2 \sin^2\xi_{11}/M_{N_2} - (|\tilde{U}_{13}|)^2 \sin^2\xi_{23}/M_{N_1}] \mathrm{Im}[Y_{1i} Y_{2i}^*]^2, \\ Q &= |\tilde{U}_{11}|^2 \sin^2\xi_{11} (Y_D^\dagger Y_D)_{11} + |\tilde{U}_{12}|^2 \sin^2\xi_{32} (Y_D^\dagger Y_D)_{33} + |\tilde{U}_{13}|^2 \sin^2\xi_{23} (Y_D^\dagger Y_D)_{22}. \end{aligned} \tag{30}$$

In Table III we present for two typical solutions the right-handed Majorana neutrino masses, matrix elements of M_X, the mixing matrix $\tilde{U}$, and the T-particle masses when the light neutrino masses are hierarchical or invertedly hierarchical. ϵ_1 is obtained through Eq. (30).

D. The baryon asymmetry

For a large departure from equilibrium in the T_1 decay, the lepton asymmetry per unit entropy at temperature $T > M_{T_1}$ is [49]

$$\frac{n_L}{s} \simeq \frac{\kappa\epsilon_1}{s} \frac{g_{T_1} T^3}{\pi^2} = \frac{45}{2\pi^4} \frac{g_{T_1}}{g_*} \kappa\epsilon_1 = 4.33 \times 10^{-3} \kappa\epsilon_1, \tag{31}$$

where κ is the efficiency factor and $g_{T_1} = 2$ the number of degrees of freedom of T_1. The entropy density $s = (2/45) g_* \pi^2 T^3$ where $g_* = 106.75$, the effective number of relativistic degrees of freedom contributing to entropy in the standard model. Denoting by N_H the number of Higgs doublets ($N_H = 1$ in this model), the baryon to entropy ratio is

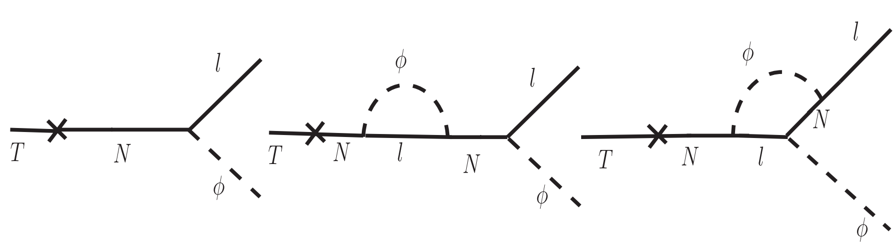

FIG. 2. The tree- and one-loop level contribution to the decay of T_1 that generate the lepton asymmetry.

TABLE III. Sample solutions with one singlet, T_1, at the right mass scale for leptogenesis. For both normal (NH) and inverted (IH) hierarchies the masses of the singlet neutrinos and the light neutrinos are displayed. $M_N = \mathrm{diag}(2 \times 10^7, 5 \times 10^{10}, 9 \times 10^{10})$ GeV has been chosen for both cases. $\kappa = 0.40$–0.55 for agreement with the observed baryon asymmetry.

Hier.	$M_{X_{11}}$ (10^5 GeV)	$M_{X_{23}}$ (10^5 GeV)	$M_{X_{32}}$ (10^5 GeV)	M_{T_i} (GeV)	m_i (10^{-2} eV)	$\tilde{U}_{11}$	$\tilde{U}_{12}$	$\tilde{U}_{13}$	ϵ_1 (10^{-7})
NH	1.2	0.18	18	1.11×10^5	1.1	−0.992	−0.125	0.035	1.11
				92	1.4				
				7.7	5.2				
IH	1.2	60.0	1.8	1.83×10^6	5.3	−0.085	−0.001	0.996	1.40
				65	5.4				
				9	2.0				

$$\frac{n_B}{s} = -\frac{24 + 4N_H}{66 + 13N_H}\frac{n_L}{s} = -\frac{28}{79}\frac{n_L}{s} = -1.53 \times 10^{-3}\kappa\epsilon. \tag{32}$$

Noting that $s = 7.04 n_\gamma$, where n_γ is the photon density, the observed baryon asymmetry is

$$\eta_B \equiv \frac{n_B}{n_\gamma} \simeq 10^{-2}\kappa\epsilon_1. \tag{33}$$

This is to be compared with [50]:

$$(\eta_B)_{\mathrm{expt}} = (6.15 \pm 0.25) \times 10^{-10}. \tag{34}$$

We find that for both the cases (NH as well as IH) the predictions are in agreement with the observed value. In Table III we have exhibited only two out of a large number of allowed solutions with efficiency factors $\kappa = 0.4$–0.5.

For comparison, in Refs. [49,51] maximal efficiency, $\kappa \simeq 1$, has been considered to obtain the requisite baryon asymmetry. In Ref. [51] constraints on the Dirac Yukawa coupling of the RH neutrino have been examined and it turns out to be small. In our model the effective Yukawa coupling of the decaying particle T_1 (instead of N_1) to the $l\phi$ pair is essentially modified by the product of two extra factors each of which is a mixing substantially smaller than unity. Thus, the effective Yukawa coupling of the decaying singlet neutrino remains small.

The present model permits a variety of solutions with $\epsilon_1 \simeq 10^{-6}$–$10^{-8}$ which match the observed baryon asymmetry when the efficiency factors $\kappa \simeq \mathcal{O}(10^{-2}) - \mathcal{O}(1)$.

V. RIGHT-HANDED DOUBLY CHARGED HIGGS AT COLLIDERS

In this section we briefly discuss how this model can be experimentally tested at high energy colliders such as the Tevatron, LHC or ILC. The light doubly-charged Higgs boson provides a clear scope for this. We relate the production cross section of the $\Delta_R^{\pm\pm}$ with that of a $\Delta_L^{\pm\pm}$ of the same mass.[3] This relationship permits the setting of upper and lower bounds on the pair production cross sections at the Tevatron or LHC in a model-independent manner.

As explained in Sec. II, the RH-triplets in $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ carrying $B - L = \pm 2$ are made light in this model through the D-parity breaking mechanism with the component masses $M_{\Delta_R} \simeq 100$ GeV to a few TeV. This enhances the induced vev, v_R, resulting in high-scale LR gauge symmetry breaking with large $W_R^\pm$, Z_R gauge-boson masses and heavy RH-Majorana neutrinos. The largeness of the RH-Majorana masses in the model lead to naturally small $T - N$ mixings essential to satisfy the out-of-equilibrium condition for the T-decay to drive leptogenesis.

Because of large masses of the RH gauge bosons, $m_{W_R^\pm}$, $m_{Z_R} \sim M_R = 10^9$–10^{15} GeV, the decays $\Delta_R^{\pm\pm} \to W_R^\pm W_R^\pm$ are kinematically forbidden and so only leptonic decays are possible. The $SO(10)$ invariant Yukawa interaction with fermions given in Eq. (1), $f_{ij}.16_i.16_j.\overline{126}_H$, makes the doubly charged Higgs boson leptophilic, its only decay modes being $\Delta_R^{\pm\pm} \to l_{R_i}^\pm l_{R_j}^\pm$ $(i, j = 1, 2, 3)$.

Because of their heaviness, W_R and Z_R also play no role in $\Delta^{\pm\pm}$ production at the Tevatron or LHC. The production will dominantly be through the electromagnetic or Z^*-exchange Drell-Yan mechanism.

The couplings f_{ij} determine the branching ratios of the $\Delta_R^{\pm\pm}$ to the different leptonic final states. Although for the sake of economy and simplicity we have chosen f_{ij} to be diagonal in the previous section, our choice has been guided by negligibly small values of the nondiagonal elements suggested by current limits on lepton flavor violating decays such as $\mu \to 3e$ and $\tau \to 3e$. Out of a large number of possible solutions, the two sets given in Table III have $M_{N_1} = 2 \times 10^7$ GeV, $M_{N_2} = 5 \times 10^{10}$ GeV, $M_{N_3} = 9 \times 10^{10}$ GeV, which for $v_R \simeq 10^{11}$ GeV corresponds to $f_1 \equiv f_{ee} = 0.0002$, $f_2 \equiv f_{\mu\mu} = 0.5$, $f_3 \equiv f_{\tau\tau} = 0.9$. Keeping in mind the wide classes of solutions permitted in this model, we will discuss possible implications for $f_1 = 0.0002 - 0.001$, and $f_2 \leq f_3$.

The mass ordering of the $W_R^\pm$, Z_R, and $\Delta_R^{\pm\pm}$ discussed above is specific to this model. Interestingly, low-mass doubly charged Higgs bosons with similar interactions have been shown to be generic in a class of SUSYLR models with $M_{W_R} \geq 10^9$ GeV which require nonrenormalizable terms in the superpotential [52–54]. Within the non-SUSY left-right model prospects of Drell-Yan pair

[3] In our model the $\Delta_L^{\pm\pm}$ is very heavy due to D-parity breaking.

production of LH doubly charged Higgs bosons at high energy colliders and their detection [55] and the impact of QCD corrections thereon [56] have also been investigated.

A. Bounds from muonium-antimuonium conversion

Muonium (M)-anti-muonium ($\bar{M}$) conversion, $\mu^+ e^- \to \mu^- e^+$ can be mediated by $\Delta_R^{\pm\pm}$ giving rise to an effective coupling [57],

$$G_{M-\bar{M}} \simeq \frac{f_1 f_2}{4\sqrt{2}M_\Delta^2}. \tag{35}$$

Experimental searches for this transition yields the upper bound [58]

$$G_{M-\bar{M}} \leq 3 \times 10^{-3} G_F, \tag{36}$$

where $G_F = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$ is the Fermi coupling. Combining Eqs. (35) and (36) gives

$$M_\Delta \geq \left(\frac{f_1 f_2}{12 \times 10^{-3}\sqrt{2}G_F}\right)^{1/2}. \tag{37}$$

For example, choosing $f_1 \sim f_2 \sim 1$, a condition applicable to quasidegenerate RH neutrinos but which is outside the presently allowed solutions, we obtain $M_\Delta \geq 2.24$ TeV which is beyond the Tevatron limit, but within the LHC range. But when we use the class of solutions which permit $f_1 \simeq 0.0002$ and $f_2 \simeq 0.5$, we obtain $M_\Delta \geq 22.4$ GeV. This is not inconsistent with the experimental search limit reached by DO and CDF Collaborations at the Fermilab Tevatron with $M_\Delta \geq 112\ \mathrm{GeV}/c^2$ and $M_\Delta \geq 127\ \mathrm{GeV}/c^2$, respectively [59,60].

B. Drell-Yan pair production at the LHC or Tevatron

Previous searches at LEP have already excluded $\Delta_R^{\pm\pm}$ below $97\ \mathrm{GeV}/c^2$ [61]. At hadron colliders the doubly charged Higgs boson will be dominantly created through pair production via the basic Drell-Yan process $q\bar{q} \to \gamma^*$, $Z^* \to \Delta_i^{++}\Delta_i^{--}$, ($i = L, R$). At the quark level, the cross section depends only on the quantum numbers and mass of the doubly charged scalars. In the present model with supersymmetry and purely renormalizable interactions, the only allowed decay mode is

$$\Delta_R^{++} \to l_R^+ l_R^+ \tag{38}$$

and its conjugate. At the Fermilab Tevatron or the LHC, the production of the doubly charged boson will be through

$$\mathrm{p}\,\bar{\mathrm{p}}(\mathrm{p}) \to (\gamma^*, Z^*)X \to \Delta_R^{++}\Delta_R^{--}X \to l_R^+ l_R^+ l_R'^- l_R'^- X \tag{39}$$

where for the dominant modes in our model l, $l' = \tau$, μ since $f_1 \ll f_2 \simeq f_3 \simeq 1$.

The parton level $\Delta_i^{\pm\pm}$ pair production cross section through γ^* and Z^* exchange is expressed as [54,56]

$$\hat{\sigma}_i = \frac{\pi\alpha^2\beta_i^3}{9\hat{s}}\Sigma_i^q,$$
$$\Sigma_i^q = \left[Q_q^2 Q_{\Delta_i}^2 + \frac{\hat{s}^2(g_{qA}^2 + g_{qv}^2)g_{\Delta_i v}^2}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2} + \frac{2\hat{s}Q_q Q_{\Delta_i}(\hat{s} - M_Z^2)g_{qv}g_{\Delta_i v}}{(\hat{s} - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}\right],$$
$$i = L, R. \tag{40}$$

Here $\hat{s} \equiv Q^2 = \tau s$ is the square of the c.m. energy of the colliding quark-antiquark pair and τ the product of momentum fractions carried by them. α is the fine-structure constant at the relevant energy scale and $\beta_i = \sqrt{(1 - 4M_{\Delta_i}^2/\hat{s})}$ is the velocity of the doubly charged Higgs, $\Delta_i^{\pm\pm}$, produced in the collision ($i = L$, R). $g_{qv} = (I_{3q} - 2Q_q \sin^2\theta_W)/(2\sin\theta_W\cos\theta_W)$, $g_{qA} = I_{3q}/(2\sin\theta_W\cos\theta_W)$ for the quark q and $g_{\Delta_i v} = (I_{3\Delta_i} - Q_{\Delta_i}s_W^2)/(2s_w c_w)$. $Q_q(Q_{\Delta_i})$ is the electric charge number of the quark q (Higgs Δ_i) and $I_{3q}(I_{3\Delta_i})$ the third component of $SU(2)_L$ isospin for q (Δ_i).

It is clear from Eq. (40) that for

$$\hat{s} > 4M_{\Delta_i}^2 \gg M_Z^2, \tag{41}$$

the Σ_i^q become independent of $\hat{s}$ and the momentum fractions carried by the quarks leading to

$$\Sigma_i^q \to \bar{\Sigma}_i^q = [Q_q^2 Q_{\Delta_i}^2 + (g_{qA}^2 + g_{qv}^2)g_{\Delta_i v}^2 + 2Q_q Q_{\Delta_i} g_{qv} g_{\Delta_i v}], \qquad (i = L, R). \tag{42}$$

Noting that $I_{3\Delta_i} = 1(0)$ for $i = L(R)$ we obtain from Eq. (42)

$$\bar{\Sigma}_R^u = 0.59\bar{\Sigma}_L^u, \qquad \bar{\Sigma}_R^d = 0.79\bar{\Sigma}_L^d. \tag{43}$$

These relations translate to upper and lower bounds on the production cross section of the $\Delta_R^{\pm\pm}$ which do not depend on the proton structure functions or the model-origin of these Higgs bosons as long as the gauge symmetry at the electroweak scale is the standard model. Thus,

$$0.59\sigma_L \leq \sigma_R \leq 0.79\sigma_L. \tag{44}$$

The above bounds for Drell-Yan pair production relate the cross section for RH doubly charged Higgs at the LHC or Tevatron with that for their LH counterparts with the same mass provided the mass is $\gtrsim 150$ GeV.

Using these relations and the published results for $\Delta_L^{\pm\pm}$ production one can estimate the number of events in this model. For example,[4] at the LHC, for $m_{\Delta_L^{\pm\pm}} = 200$ GeV (1 TeV) the Drell-Yan production cross section in fb is 49.4, 96.4, 169.5 (0.004, 0.04, 0.14) for $\sqrt{s} = 7$, 10, or

[4]We thank Anindya Datta for providing these numbers.

14 TeV, respectively. Using Eq. (44) and assuming an integrated luminosity of 30 fb^{-1} for $m_{\Delta_R^{\pm\pm}} = 200$ GeV one would expect 1022, 1995, 3509 events for the three cases. If $m_{\Delta_L^{\pm\pm}} = 1$ TeV then for $\sqrt{s} = 10$ TeV one would require about 200 fb^{-1} integrated luminosity for a 5-event signal.

At the Tevatron with $\sqrt{s} = 2$ TeV, $\sigma_R \simeq 12$–16 fb for $M_{\Delta_R} = 150$ GeV and with an integrated luminosity of 350 pb^{-1} the predicted number of events is nearly 4-6. With an acquired integrated luminosity of 10 fb^{-1}, the mass reach of up to $M_{\Delta_{R_2}} = 300$ GeV can be achieved where $\sigma_R \simeq (5 - 7) \times 10^{-2}$ fb.

In our model since $f_1 \ll f_2 \simeq f_3$, the dominant decay modes of the produced pair would be through the following four lepton channels, every one of which would be almost equally likely: $\Delta_R^{++}\Delta_R^{--} \rightarrow \tau_R^+\tau_R^+\tau_R^-\tau_R^-$, $\mu_R^+\mu_R^+\mu_R^-\mu_R^-$, $\tau_R^+\tau_R^+\mu_R^-\mu_R^-$, $\tau_R^-\tau_R^-\mu_R^+\mu_R^+$.

The standard model backgrounds for such production processes have been discussed in [55]. The signal event should have negligible missing p_T. Moreover, the two pairs of like-charged leptons are constrained to each have the invariant mass equal to m_{Δ_R}. These criteria and a judicious cut on the l^+l^- pair invariant mass to remove ZZ contributions effectively removes the entire background.

Unlike the Tevatron and the LHC where the doubly charged bosons are pair-produced, at proposed muon colliders resonant production of these bosons could take place if $\mu^-\mu^-$ colliders are arranged. The singly produced Δ^{--} would decay via 2μ or 2τ channels providing the cleanest signals for these bosons. In contrast to a large class of asymmetric left-right models where the decay of the right-handed doubly charged bosons could proceed via kinematically allowed channels such as $\Delta_R^{++} \rightarrow W_R^+W_R^+$, $W_R^+\Delta_R^+$, $\Delta_R^+\Delta_R^+$, this model allows decay only in the bilepton channel providing a signature of its genuine leptophilic property.

VI. SUMMARY AND CONCLUSIONS

In summary, we have implemented flavor-dependent leptogenesis through the decay of singlet fermions with masses $M_{T_1} = 10^5$–10^6 GeV in SUSY $SO(10)$ while satisfying the gravitino constraint. The left-right intermediate symmetry is at a high scale corresponding to W_R and Z_R masses larger than 10^{11} GeV. This has been made possible by using the RH-triplets in $\mathbf{126_H} \oplus \overline{\mathbf{126}}_\mathbf{H}$ as well as the RH-doublets in $\mathbf{16_H} \oplus \overline{\mathbf{16}}_\mathbf{H}$. Not only is the singlet fermion-RH-neutrino mixing generated by the vev of $\overline{\mathbf{16}}_\mathbf{H}$ but also this mixing becomes naturally small through the large vev of the RH-triplet induced by the doublet vev. In addition to obtaining renormalizable mass for the RH neutrino through this mechanism, manifest unification of gauge couplings is also achieved purely by renormalizable interactions and, thus, nonrenormalizable dimension-5 operators used earlier are dispensed with. In contrast to the earlier attempts where an assumed value of the CP-asymmetry parameter was shown to yield the lepton asymmetry numerically, in this work we have derived and suggested new analytic formulas leading to the correct asymmetry parameter and the observed baryon to photon density ratio. We have found that both the decay rate and the CP-asymmetry are explicitly flavor dependent. Whereas our previous work [30] required a normal hierarchy, here we have also found successful implementation in the case of inverted hierarchical neutrino masses. Unlike a host of low-scale leptogenesis models, this model works with hierarchical heavy RH-neutrino masses and no resonant condition with extreme degeneracy among them is needed. A decisive test of the present model would be through the detection of doubly charged Higgs bosons $\Delta_R^{\pm\pm} \rightarrow l_R^\pm l_R^\pm$ at the Tevatron, LHC, or a future muon collider. The model provides an example of truly leptophilic doubly charged Higgs bosons. Without using parton-density distribution functions we have also shown in a model-independent manner that the pair production cross sections for RH doubly charged Higgs at Tevatron or LHC energies are bounded between 59%–79% of their LH counterparts with same masses.

As the unification, triggered by low-mass doubly charged bosons, occurs with a large (but perturbative) unified gauge coupling, the decay lifetime $\tau_p(p \rightarrow e^+\pi^0)$ is substantially reduced compared to conventional SUSY GUTs and remains within one order of the current experimental limit. This is likely to be accessible to the ongoing and planned proton decay searches. The model appears to be rich in dark matter candidates which will be investigated elsewhere.

ACKNOWLEDGMENTS

The authors acknowledge useful discussions with Professor R. N. Mohapatra. They also thank Dr. Anindya Datta for help with some of the material in Sec. V B. M. K. P. thanks the Harish-Chandra Research Institute (HRI), Allahabad, for hospitality. The work of A. R. is supported by the XI Plan "Neutrino Physics" and RECAPP projects at HRI.

[1] H. Georgi, *Particles and Fields*, edited by C. Carlson, AIP Conf. Proc. (1974) Vol. 23, pp. 575; H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).
[2] J. C. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973); Phys. Rev. D **10**, 275 (1974).
[3] R. N. Mohapatra and J. C. Pati, Phys. Rev. D **11**, 566 (1975); G. Senjanović and R. N. Mohapatra, Phys. Rev. D **12**, 1502 (1975).
[4] K. S. Babu, J. C. Pati, and F. Wilczek, Nucl. Phys. B **566**, 33 (2000).
[5] P. Minkowski, Phys. Lett. B **67**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, *Supergravity*, edited by D. Freedman *et al.* (North-Holland, Amsterdam, 1980); T. Yanagida, in Proceedings of the KEK Workshop (unpublished); S. L. Glashow, NATO Adv. Stud. Inst. Ser. B Phys. Sci. **59**, 687 (1979); R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
[6] R. N. Mohapatra and G. Senjanović, Phys. Rev. D **23**, 165 (1981); G. Lazaridis, Q. Shafi, and C. Wetterich, Nucl. Phys. **B181**, 287 (1981); J. Schechter and J. W. F. Valle, Phys. Rev. D **22**, 2227 (1980).
[7] B. Bajc, G. Senjanović, and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003).
[8] H. S. Goh, R. N. Mohapatra, and S. Nasri, Phys. Rev. D **70**, 075022 (2004); R. N. Mohapatra, N. Okada, and H.-B. Yu, *ibid.* **76**, 015013 (2007); S. Bertolini, M. Frigerio, and M. Malinsky, *ibid.* **70**, 095002 (2004); S. Bertolini and M. Malinsky, *ibid.* **72**, 055021 (2005); B. Dutta, Y. Mimura, and R. N. Mohapatra, *ibid.* **72**, 075009 (2005); **80**, 095021 (2009); A. S. Joshipura, B. P. Kodrani, and K. M. Patel, *ibid.* **79**, 115017 (2009).
[9] S. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).
[10] J. C. Pati, Phys. Rev. D **68**, 072002 (2003).
[11] X. Ji, Y. Li, R. N. Mohapatra, S. Nasri, and Y. Zhang, Phys. Lett. B **651**, 195 (2007).
[12] S. M. Barr, Phys. Rev. Lett. **92**, 101601 (2004); C. H. Albright and S. M. Barr, Phys. Rev. D **69**, 073010 (2004); **70**, 033013 (2004); T. Fukuyama, T. Kikuchi, and T. Osaka, J. Cosmol. Astropart. Phys. 06 (2005) 005.
[13] S. Davidson and A. Ibarra, Phys. Lett. B **535**, 25 (2002); W. Buchmuller, P. Di Bari, and M. Plumacher, Nucl. Phys. **B643**, 367 (2002); T. Hambye and G. Senjanović, Phys. Lett. B **582**, 73 (2004); S. Blanchet and P. Di Bari, J. Cosmol. Astropart. Phys. 03 (2007) 018.
[14] M. Y. Khlopov and A. D. Linde, Phys. Lett. B **138**, 265 (1984); J. R. Ellis, D. V. Nanopoulos, and S. Sarkar, Nucl. Phys. **B259**, 175 (1985); J. R. Ellis, D. V. Nanopoulos, K. A. Olive, and S. J. Rey, Astropart. Phys. **4**, 371 (1996); M. Kawasaki and T. Moroi, Prog. Theor. Phys. **93**, 879 (1995); V. S. Rychkov and A. Strumia, Phys. Rev. D **75**, 075011 (2007).
[15] W. Buchmuller, P. Di Bari, and M. Plumacher, Ann. Phys. (N.Y.) **315**, 305 (2005).
[16] M. Flanz, E. A. Paschos, and U. Sarkar, Phys. Lett. B **345**, 248 (1995); **389**, 693 (1996); A. Pilaftsis, Nucl. Phys. **B692**, 303 (2004); W. Buchmuller and M. Plumacher, Phys. Lett. B **431**, 354 (1998); W. Buchmuller, P. Di Bari, and M. Plumacher, New J. Phys. **6**, 105 (2004).
[17] J. C. Romão, M. A. Tortola, M. Hirsch, and J. W. F. Valle, Phys. Rev. D **77**, 055002 (2008).
[18] M. K. Parida, Phys. Rev. D **78**, 053004 (2008).
[19] D. G. Lee and R. N. Mohapatra, Phys. Rev. D **52**, 4125 (1995); M. Bando, J. Sato, and T. Takahashi, *ibid.* **52**, 3076 (1995); M. K. Parida, *ibid.* **57**, 2736 (1998); E. Ma, *ibid.* **51**, 236 (1995); E. Ma, Phys. Lett. B **344**, 164 (1995); **625**, 76 (2005).
[20] S. K. Majee, M. K. Parida, A. Raychaudhuri, and U. Sarkar, Phys. Rev. D **75**, 075003 (2007).
[21] N. G. Deshpande, E. Keith, and T. G. Rizzo, Phys. Rev. Lett. **70**, 3189 (1993); M. Malinsky, J. C. Romão, and J. W. F. Valle, *ibid.* **95**, 161801 (2005).
[22] P. S. Bhupal Dev and R. N. Mohapatra, Phys. Rev. D **81**, 013001 (2010); P. S. B. Dev and R. N. Mohapatra, Phys. Rev. D **82**, 035014 (2010).
[23] D. Acosta *et al.* (CDF Collaboration), Phys. Rev. Lett. **93**, 221802 (2004); T. Aaltonen *et al.* (CDF Collaboration), *ibid.* **101**, 121801 (2008); V. M. Abazov *et al.* (D0 Collaboration), *ibid.* **101**, 071803 (2008).
[24] M. Malinsky, T. Ohlsson, and H. Zhang, Phys. Rev. D **79**, 073009 (2009).
[25] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986); R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986).
[26] J. Ellis, J. L. Lopez, and D. V. Nanopoulos, Phys. Lett. B **292**, 189 (1992); J. Ellis, D. V. Nanopoulos, and K. Olive, Phys. Lett. B **300**, 121 (1993).
[27] S. K. Kang and C. S. Kim, Phys. Lett. B **646**, 248 (2007).
[28] M. Lindner, M. A. Schmidt, and A. Yu. Smirnov, J. High Energy Phys. 07 (2005) 048.
[29] F. Deppisch, T. S. Kosmas, and J. W. F. Valle, Nucl. Phys. **B752**, 80 (2006); J. Garayoa, M. C. Gonzalez-Gartia, and N. Rius, J. High Energy Phys. 02 (2007) 021; C. Arina, F. Bazzocchi, N. Fornengo, J. C. Romão, and J. W. F. Valle, Phys. Rev. Lett. **101**, 161802 (2008); M. Malinsky, T. Ohlsson, Z.-z. Xing, and H. Zhang, Phys. Lett. B **679**, 242 (2009); E. Ma, Mod. Phys. Lett. A **24**, 2491 (2009); M. B. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, J. High Energy Phys. 09 (2009) 038; M. Hirsch, T. Kernreiter, J. Romão, and A. Villanova del Moral, arXiv:0910.2435.
[30] S. K. Majee, M. K. Parida, and A. Raychaudhuri, Phys. Lett. B **668**, 299 (2008).
[31] F. del Aguila and J. Aguilar-Saavedra, Nucl. Phys. **B813**, 22 (2009).
[32] O. Cakir, New J. Phys. **8**, 145 (2006); C.-S. Chen, C.-Q. Geng, and D. V. Zhuridov, Phys. Lett. B **666**, 340 (2008); Eur. Phys. J. C **60**, 119 (2009).
[33] Pran Nath and P. F. Pérez, Phys. Rep. **441**, 191 (2007).
[34] H. Nishino *et al.* (Super-Kamiokande Collaboration), Phys. Rev. Lett. **102**, 141801 (2009); arXiv:0903.0676.
[35] S. Raby *et al.*, arXiv:0810.4551.
[36] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); Phys. Rev. D **30**, 1052 (1984); D. Chang, R. N. Mohapatra, J. M. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D **31**, 1718 (1985).
[37] H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
[38] C. Bachas, C. Fabre, and T. Yanagida, Phys. Lett. B **370**, 49 (1996).
[39] M. Cirelli, N. Fornengo, and A. Strumia, Nucl. Phys. **B753**, 178 (2006); P. F. Pérez, H. Iminniyaz, and G. Rodrigo, Phys. Rev. D **78**, 015013 (2008); M. Kadastik,

K. Kannike, and M. Raidal, Phys. Rev. D **80**, 085020 (2009).
[40] D.-G. Lee, R. N. Mohapatra, M. K. Parida, and M. Rani, Phys. Rev. D **51**, 229 (1995); M. Frigerio and T. Hambye, Phys. Rev. D **81**, 075002 (2010); I. Dorsner and I. Mocioiu, Nucl. Phys. **B796**, 123 (2008).
[41] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac, and N. Okada, J. Math. Phys. (N.Y.) **46**, 033505 (2005); T. Fukuyama, A. Ilakovac, T. Kikuchi, and K. Matsuda, J. High Energy Phys. 06 (2005) 016.
[42] S. Dimopoulos and F. Wilczek, *The Unity of the Fundamental Interactions*, edited by A. Zichichi (Plenum, New York, 1983); K. S. Babu and S. M. Barr, Phys. Rev. D **48**, 5354 (1993); **50**, 3529 (1994); Z. Chacko and R. N. Mohapatra, Phys. Rev. D **59**, 011702 (1998).
[43] B. Dutta, Y. Mimura, and R. N. Mohapatra, Phys. Rev. Lett. **94**, 091804 (2005); **100**, 181801 (2008); A. T. Azatov and R. N. Mohapatra, Phys. Rev. D **78**, 015002 (2008); R. N. Mohapatra, Phys. Lett. B **679**, 382 (2009).
[44] K. S. Babu, J. C. Pati, and Z. Tavartkiladze, J. High Energy Phys. 06 (2010) 84.
[45] B. Bajc, I. Dorsner, and M. Nemevsek, J. High Energy Phys. 11 (2008) 007.
[46] A. J. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978); T. Goldman and D. A. Ross, Nucl. Phys. **B171**, 273 (1980); J. Ellis, D. V. Nanopoulos, and S. Rudaz, Nucl. Phys. **B202**, 43 (1982); L. E. Ibanez and C. Munoz, Nucl. Phys. **B245**, 425 (1984).
[47] Y. Aoki, C. Dawson, J. Noaki, and A. Soni, Phys. Rev. D **75**, 014507 (2007); Y. Aoki *et al.* (RBC-UKQCD Collaboration), Phys. Rev. D **78**, 054505 (2008).
[48] C. Amsler *et al.*, Phys. Lett. B **667**, 1 (2008).
[49] See, for example, L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B **384**, 169 (1996).
[50] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003); E. Komatsu *et al.* (WMAP Collaboration), Astrophys. J. Suppl. Ser. **180**, 330 (2009).
[51] M. Frigerio, T. Hambye, and E. Ma, J. Cosmol. Astropart. Phys. 09 (2006) 009.
[52] C. S. Aulakh, A. Melfo, and G. Senjanović, Phys. Rev. D **57**, 4174 (1998); C. S. Aulakh, K. Benakli, and G. Senjanović, Phys. Rev. Lett. **79**, 2188 (1997); R. N. Mohapatra, A. Rašin, and G. Senjanović, Phys. Rev. Lett. **79**, 4744 (1997).
[53] Z. Chacko and R. N. Mohapatra, Phys. Rev. D **58**, 015003 (1998).
[54] B. Dutta and R. N. Mohapatra, Phys. Rev. D **59**, 015018 (1998); S. Dar, Q. Shafi, and A. Sil, Phys. Lett. B **632**, 517 (2006).
[55] A. Datta and A. Raychaudhuri, Phys. Rev. D **62**, 055002 (2000).
[56] M. Muhlleitner and M. Spira, Phys. Rev. D **68**, 117701 (2003).
[57] P. Herczeg and R. N. Mohapatra, Phys. Rev. Lett. **69**, 2475 (1992); A. Halprin, Phys. Rev. Lett. **48**, 1313 (1982); J. Gunion, J. Grifols, A. Mendez, B. Kayser, and F. Olness, Phys. Rev. D **40**, 1546 (1989).
[58] L. Willmann *et al.*, Phys. Rev. Lett. **82**, 49 (1999).
[59] D. Acosta *et al.*, Ref. [23]; T. Aaltonen *et al.*, Ref. [23].
[60] V. M. Abazov *et al.*, Ref. [23].
[61] G. Abbiendi *et al.* (OPAL Collaboration), Phys. Lett. B **526**, 221 (2002); P. Achard *et al.* (L3 Collaboration), Phys. Lett. B **576**, 18 (2003); J. Abdallah *et al.* (DELPHI Collaboration), Phys. Lett. B **552**, 127 (2003).