Duality-symmetric supersymmetric Yang-Mills theory in three dimensions

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We formulate a duality-symmetric $N = 1$ supersymmetric Yang-Mills theory in three dimensions. Our field content is $(A_\mu{}^I, \lambda^I, \varphi^I)$, where the index I is for the adjoint representation of an arbitrary gauge group G. Our Hodge duality symmetry is $F_{\mu\nu}{}^I = +\epsilon_{\mu\nu}{}^\rho D_\rho \varphi^I$. Because of this relationship, the presence of two physical fields $A_\mu{}^I$ and φ^I within the same $N = 1$ supermultiplet poses no problem. We can couple this multiplet to another vector multiplet $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$ with $1 + 1$ physical degrees of freedom modulo dim G. Thanks to peculiar couplings and supersymmetry, the usual problem with an extra vector field in a nontrivial representation does *not* arise in our system.

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I. INTRODUCTION

The so-called "duality-symmetric theory" or "self-dual theory" has drawn much attention [1–3]. However, there is a caveat about these theories, associated with Lagrangian constructions. The problem can be most easily seen by the following example in two dimensions (2D). Suppose we require the "self-duality" (heterotic) condition $\partial_\mu \varphi = \epsilon_\mu{}^\nu \partial_\nu \varphi$ on the "field strength" $\partial_\mu \varphi$. If we use a Lagrange multiplier Ω^μ, such as

$$\mathcal{L} = \Omega^\mu(\partial_\mu \varphi - \epsilon_\mu{}^\nu \partial_\nu \varphi), \tag{1.1}$$

then not only the original field φ, but also the multiplier field Ω^μ starts propagating [1,2]. This is because the field equation of φ gives[1]

$$\begin{aligned} &\partial_\mu \Omega^\mu + \epsilon^{\mu\nu}\partial_\mu \Omega_\nu \doteq 0 \\ &\quad \Rightarrow \partial_\mu \Omega_{(+)}{}^\mu \doteq 0, \qquad \partial_\mu \Omega_\nu^{(+)} - \partial_\nu \Omega_\mu^{(+)} \doteq 0, \end{aligned} \tag{1.2}$$

where $\Omega_\mu^{(+)} \equiv (1/2)[\Omega_\mu + (1/2)\epsilon_\mu{}^\nu \Omega_\nu]$. Applying ∂_μ to the last equation yields

$$\partial_\mu^2 \Omega_\nu^{(+)} - \partial_\nu(\partial_\mu \Omega_{(+)}^\mu) \doteq \partial_\mu^2 \Omega_\nu^{(+)} \doteq 0, \tag{1.3}$$

meaning that $\Omega_\nu^{(+)}$ is a propagating component.

Propagating multiplier fields is sometimes problematic. For example in 11D, introducing extra propagating fields other than the conventional 128 + 128 components [4] is not allowed. There are several nontrivial ways to solve this sort of problem, such as using nonlinear constraint terms [1,5], introducing infinitely many auxiliary fields [6], harmonic space [2], or harmonic unit vector fields [3,6].

In the present paper, we deal with a duality-symmetric Yang-Mills (DSYM) multiplet in 3D with $N = 1$ supersymmetry. However, we do not give a Lagrangian formulation, in order to avoid the usage of a multiplier field, caused by the above-mentioned problem with multiplier fields. Instead, we give the set of field equations consistent with $N = 1$ supersymmetry. This is similar to the type IIB supergravity in 10D [7].

In order to show the nontrivial nature of our duality-symmetric system, we further couple the DSYM multiplet to another vector multiplet in a nontrivial way. The extra vector multiplet has the fields $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$ with the adjoint index I. We show that the coupling to this extra vector multiplet is consistent, despite the nontrivial duality symmetry within the YM multiplet.

It is usually problematic to introduce an extra vector field in nontrivial representation of a gauge group G, in addition to the ordinary YM gauge field. This can be seen as follows: Suppose we have an additional vector field $C_\mu{}^I$ in addition to the YM gauge field $A_\mu{}^I$ in the adjoint representation of the group G. Their field strengths are $F_{\mu\nu}{}^I \equiv 2\partial_{[\mu}A_{\nu]}{}^I + mf^{IJK}A_\mu{}^J A_\nu{}^K$ and $H_{\mu\nu}{}^I \equiv 2D_{[\mu}C_{\nu]}{}^I$, where m is a coupling constant. The field equation for the C-field is generally given by

$$D_\nu H^{\mu\nu I} \doteq J^{\mu I}. \tag{1.4}$$

Now the problem is the application of another D_μ to this equation yielding

$$\begin{aligned} D_\mu D_\nu H^{\mu\nu I} &= \frac{1}{2}[D_\mu, D_\nu]H^{\mu\nu I} \\ &= \frac{1}{2} mf^{IJK}F_{\mu\nu}{}^J H^{\mu\nu K} \doteq D_\mu J^{\mu I}. \end{aligned} \tag{1.5}$$

If the current J^μ is "conserved," the last side of (1.5) should vanish. However, the trouble is that its penultimate side is a product of two field strengths F and H, which does *not* simply vanish! A similar problem had been known [8] for the case of the spin 3/2 field, before supergravity was discovered [9]

In our formulation below, the fields in the YM multiplet $(A_\mu{}^I, \lambda^I, \varphi^I)$ have the respective degrees of freedom (DOF): $(1/2, 1, 1/2)$ modulo dim G, where 1/2 come from the duality condition $F_{\mu\nu}{}^I \doteq \epsilon_{\mu\nu}{}^\rho D_\rho \varphi^I$, reducing each of the original DOF 1 of $A_\mu{}^I$ and φ^I into 1/2, while the sum of these will be unity. Relevantly, the extra vector multiplet $(C_\mu{}^I, \chi^i; B_{\mu\nu}{}^I)$ has the expected DOF (1, 1, 0).

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[1] We use the symbol $\doteq$ for a field equation distinguished from an algebraic identity.

In a sense, the formulation below is similar to the Stueckelberg formulation for self-duality in 3D [10], because the latter also leads to the duality symmetry $D_\mu \varphi^I = (1/2)\epsilon_\mu{}^{\rho\sigma} F_{\rho\sigma}{}^I$. However, the difference is that our present formulation is *not* with such a compensator generating a mass to a vector field, but all the fields are massless. Another difference is that we interact two vector multiplets $(A_\mu{}^I, \lambda^I, \varphi^I)$ and $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$, while the work in [10] uses a YM multiplet and a scalar multiplet combined together.

A similar formulation was also given in [11], where we have shown in 10D the duality symmetry between an Abelian vector multiplet with A_μ and its Hodge dual multiplet with $B_{\mu_1\cdots\mu_8}$. However, in [11] the duality symmetry was established only for an *Abelian* vector multiplet and its dual multiplet. In the formulation below, we will establish a duality-symmetric non-Abelian system with nontrivial interactions.

The problem in (1.5) can be solved, if the current $J^{\mu I}$ has a very sophisticated structure, e.g., a "Chern-Simons term" present in the field strength $H_{\mu\nu}{}^I$. In the present paper, we establish such a formulation, i.e., our field strength $H_{\mu\nu}{}^I$ is *not* just a covariant curl $2D_{[\mu} C_{\nu]}{}^I$, but it contains also the potential $B_{\mu\nu}{}^I$ within the same vector multiplet.

II. DUALITY SYMMETRY RELATIONS

Our YM multiplet is $(A_\mu{}^I, \lambda^I, \varphi^I)$, where $A_\mu{}^I$ is the usual YM gauge field with the adjoint index I, λ^I is the gaugino as its superpartner Majorana field, and φ^I is a scalar field in the adjoint representation of the gauge group G.

Our basic duality symmetry between $A_\mu{}^I$ and φ^I is expressed as $F_{\mu\nu}{}^I \doteq \epsilon_{\mu\nu}{}^\rho D_\rho \varphi^I$,[2] or equivalently

$$F_{\mu\nu}{}^I - \epsilon_{\mu\nu}{}^\rho D_\rho \varphi^I \equiv \mathcal{F}_{\mu\nu}{}^I \doteq 0, \quad (2.1a)$$

$$D_\mu \varphi^I + \frac{1}{2}\epsilon_\mu{}^{\rho\sigma} F_{\rho\sigma}{}^I \equiv D_\mu \varphi^I + \tilde{F}_\mu{}^I \equiv \tilde{\mathcal{F}}_\mu{}^I \doteq 0. \quad (2.1b)$$

The field strengths are defined by

$$F_{\mu\nu}{}^I \equiv +2\partial_{[\mu} A_{\nu]}{}^I + m f^{IJK} A_\mu{}^J A_\nu{}^K, \quad (2.2a)$$

$$D_\mu \varphi^I \equiv +\partial_\mu \varphi^I + m f^{IJK} A_\mu{}^J \varphi^K \quad (\text{idem for } \lambda^I), \quad (2.2b)$$

where m is a constant with the dimension of a mass.

The consistency of the duality (2.1) with $N = 1$ supersymmetry fixes the superpartner gaugino λ^I-field equation to be

$$\not{D}\lambda^I + m f^{IJK} \lambda^J \varphi^K \equiv \Lambda^I \doteq 0. \quad (2.3)$$

To be more specific, (2.2) and (2.3) are consistent with $N = 1$ supersymmetry

$$\delta_Q A_\mu{}^I = +(\bar{\epsilon}\gamma_\mu \lambda^I), \quad (2.4a)$$

$$\delta_Q \lambda^I = -\frac{1}{4}(\gamma^{\mu\nu}\epsilon) F_{\mu\nu}{}^I + \frac{1}{2}(\gamma^\mu \epsilon) D_\mu \varphi^I = -\frac{1}{2}(\gamma^\mu \epsilon)(\tilde{F}_\mu{}^I - D_\mu \varphi^I), \quad (2.4b)$$

$$\delta_Q \varphi^I = +(\bar{\epsilon}\lambda^I). \quad (2.4c)$$

The explicit consistency is summarized as

$$\delta_Q \mathcal{F}_\mu{}^I = +(\bar{\epsilon}\gamma_\mu \Lambda^I) \doteq 0, \quad (2.5a)$$

$$\delta_Q \Lambda^I = +\frac{1}{2}\epsilon D_\mu \tilde{\mathcal{F}}^{\mu I} + \frac{1}{2}(\gamma^\mu \epsilon)(D_\nu \mathcal{F}_\mu{}^{\nu I} - m f^{IJK} \varphi^J \tilde{\mathcal{F}}_\mu{}^K) \doteq 0, \quad (2.5b)$$

for $\mathcal{F}_\mu{}^I$ and Λ^I defined in (2.1) and (2.3). In (2.5), field equations have been used only at the last equalities.

The closure of supersymmetry (2.4) is up to field Eqs. (2.1) and (2.3):

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] \doteq \delta_\xi + \delta_\alpha, \quad (2.6)$$

where δ_ξ is the translation, and δ_α is the YM-gauge transformation with the respective parameters

$$\xi^\mu \equiv +2(\bar{\epsilon}_2 \gamma^\mu \epsilon_1), \qquad \alpha^I \equiv -\xi^\mu A_\mu{}^I. \quad (2.7)$$

The actions of δ_ξ and δ_α on the fundamental fields are

$$\delta_\xi(A_\mu{}^I, \lambda^I, \varphi^I) = +\xi^\nu \partial_\nu (A_\mu{}^I, \lambda^I, \varphi^I), \quad (2.8a)$$

$$\delta_\alpha A_\mu{}^I = +D_\mu \alpha^I, \quad \delta_\alpha(\lambda^I, \varphi^I) = -f^{IJK}\alpha^J(\lambda^K, \varphi^K). \quad (2.8b)$$

The covariances of (2.1) and (2.3) under δ_ξ and δ_α are easily confirmed, once we see that all the field strengths are "covariant" under these symmetries.

III. COUPLING TO EXTRA VECTOR MULTIPLET

As has been mentioned, we can couple the DSYM multiplet above to an extra vector multiplet $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$ with the common adjoint index I.

The field equations for the extra vector multiplet are highly nontrivial:

$$+D_\nu H^{\mu\nu I} - m\epsilon^{\mu\nu\rho} f^{IJK} \tilde{H}^{\mu J} \varphi^K - m f^{IJK} (\bar{\chi}^J \gamma_\mu \lambda^K) \equiv \mathcal{H}_\mu{}^I \doteq 0, \quad (3.1a)$$

$$+\not{D}\chi^I - m f^{IJK} \chi^J \varphi^K \equiv \mathcal{Y}^I \doteq 0, \quad (3.1b)$$

$$+\frac{1}{6}\epsilon^{\mu\nu\rho} G_{\mu\nu\rho}{}^I + m f^{IJK} (\bar{\chi}^J \lambda^K) \equiv \mathcal{G}^I \doteq 0. \quad (3.1c)$$

[2]We use the Greek indices $\mu, \nu, \cdots = 0, 1, 2$ for 3D coordinates. We also use the symbol $\doteq$ for a field equation distinguished from an algebraic identity.

Here the field strengths H and G have nontrivial structures

$$H_{\mu\nu}{}^{I} \equiv +2\partial_{[\mu}C_{\nu]}{}^{I} + 2mf^{IJK}A_{[\mu}{}^{J}C_{\nu]}{}^{K} + B_{\mu\nu}{}^{I} \equiv +2D_{[\mu}C_{\nu]}{}^{I} + B_{\mu\nu}{}^{I}, \tag{3.2a}$$

$$G_{\mu\nu\rho}{}^{I} \equiv +3\partial_{[\mu}B_{\nu\rho]}{}^{I} + 3mf^{IJK}A_{[\mu}{}^{J}B_{\rho\sigma]}{}^{K} + 3mf^{IJK}F_{[\mu\nu}{}^{J}C_{\rho]}{}^{K} \equiv +3D_{[\mu}B_{\nu\rho]}{}^{I} + 3mf^{IJK}F_{[\mu\nu}{}^{J}C_{\rho]}{}^{K}, \tag{3.2b}$$

$$\tilde{H}_{\mu}{}^{I} \equiv +\frac{1}{2}\epsilon_{\mu}{}^{\rho\sigma}H_{\rho\sigma}{}^{I}. \tag{3.2c}$$

The covariant derivative $D_\mu\chi^I$ is defined in the same way as (2.2b). Note the nontrivial Chern-Simons-like terms added in the H and G-field strengths. In particular, the B-linear term in the former plays a crucial role, avoiding the usual problem with an extra vector field in a nontrivial representation of the YM group G.

The field equations in (3.1) are fixed by requiring the consistency of them with $N = 1$ supersymmetry:

$$\delta_Q C_\mu{}^I = +(\bar{\epsilon}\gamma_\mu\chi^I), \tag{3.3a}$$

$$\delta_Q \chi^I = -\frac{1}{2}(\gamma^{\mu\nu}\epsilon)H_{\mu\nu}{}^I = -(\gamma^\mu\epsilon)\tilde{H}_\mu{}^I, \tag{3.3b}$$

$$\delta_Q B_{\mu\nu}{}^I = +\frac{1}{2}[\bar{\epsilon}\gamma_{\mu\nu}(\not{D}\chi^I - mf^{IJK}\chi^J\varphi^K)] - 2mf^{IJK}(\delta_Q A_{[\mu|}{}^J)C_{|\nu]}{}^K, \tag{3.3c}$$

in addition to the transformation rule (2.4). The first term in the right-hand side of (3.3c) contains the factor $\mathcal{Y}^I$.

The field Eqs. (3.1) are consistent with supersymmetry (2.4) and (3.3):

$$\delta_Q \mathcal{H}^{\mu I} = -\frac{1}{2}(\bar{\epsilon}\gamma^{\mu\nu}D_\nu\mathcal{Y}^I) - \frac{1}{2}mf^{IJK}(\bar{\epsilon}\gamma^\mu Y^I)\varphi^K - \frac{1}{2}mf^{IJK}(\bar{\epsilon}\gamma^\mu\gamma^\nu\chi^J)\tilde{\mathcal{F}}_\nu{}^K \doteq 0, \tag{3.4a}$$

$$\delta_Q \mathcal{Y}^I = -\epsilon\mathcal{G}^I + (\gamma^\mu\epsilon)\mathcal{H}_\mu{}^I + mf^{IJK}(\gamma^{\mu\nu}\epsilon)C_\mu{}^J\tilde{\mathcal{F}}_\nu{}^K \doteq 0, \tag{3.4b}$$

$$\delta_Q \mathcal{G}^I = -\frac{1}{2}(\bar{\epsilon}\not{D}\mathcal{Y}^I) - \frac{1}{2}mf^{IJK}(\bar{\epsilon}\gamma^\mu\chi^J)\tilde{\mathcal{F}}_\mu{}^K \doteq 0, \tag{3.4c}$$

where $\mathcal{H}$, $\mathcal{Y}$ and $\mathcal{G}$ are defined in (3.1). To elucidate the on-shell vanishing structure explicitly, field equations have been used only at the last equalities.

Our field equations in (3.1) are also consistent with δ_ξ, δ_α, δ_β and δ_γ-symmetries defined by

$$\delta_\alpha(C_\mu{}^I, \chi^I, B_{\mu\nu}{}^I) = -f^{IJK}\alpha^J(C_\mu{}^K, \chi^K, B_{\mu\nu}{}^K),$$

$$\delta_\beta B_{\mu\nu}{}^I = +2D_{[\mu}\beta_{\nu]}{}^I, \qquad \delta_\beta C_\mu{}^I = -\beta_\mu{}^I,$$

$$\delta_\beta(A_\mu{}^I, \lambda^I, \varphi^I, \chi^I) = 0, \tag{3.5a}$$

$$\delta_\gamma C_\mu{}^I = +D_\mu\gamma^I,$$

$$\delta_\gamma B_{\mu\nu}{}^I = -mf^{IJK}F_{\mu\nu}{}^J\gamma^K,$$

$$\delta_\gamma(A_\mu{}^I, \lambda^I, \varphi^I, \chi^I) = 0, \tag{3.5b}$$

combined with the already-given δ_ξ and δ_α in (2.8). The δ_γ is associated with the gauge symmetry of $C_\mu{}^I$, while δ_β is the tensorial gauge symmetry for $B_{\mu\nu}{}^I$. The consistency of our field equations in (3.1) with these symmetries is easily confirmed, if we use the covariance of the relevant field strengths under these transformations.

The closure of supersymmetries on the two multiplets $(A_\mu{}^I, \lambda^I, \varphi^I)$ and $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$ is also consistent with these transformations:

$$[\delta_Q(\epsilon_1), \delta_Q(\epsilon_2)] = \delta_\xi + \delta_\alpha + \delta_\beta + \delta_\gamma, \tag{3.6}$$

where the ξ and α-parameters are the same as (2.7), while

$$\beta^I \equiv -\xi^\nu B_{\nu\mu}{}^I, \qquad \gamma^I \equiv -\xi^\mu C_\mu{}^I. \tag{3.7}$$

Note that the field Eq. (3.1a) has the free vectorial index μ. As has been mentioned in the Introduction, its divergence usually causes a problem in naïvely constructed theories. In our system, this problem is avoided in a highly nontrivial way:

$$0 \stackrel{?}{=} D_\mu\mathcal{H}^{\mu I} = +m\epsilon^{\mu\nu\rho}f^{IJK}\tilde{H}^{\mu J}\tilde{\mathcal{F}}_\mu{}^K + mf^{IJK}\mathcal{G}^J\varphi^K - mf^{IJK}(\bar{\chi}^J\Lambda^K) - mf^{IJK}(\bar{\lambda}^J\mathcal{Y}^K) - \epsilon^{\mu\nu\rho}f^{IJK}D_\mu(C_\nu{}^J\tilde{\mathcal{F}}_\rho{}^K) \doteq 0. \tag{3.8}$$

As before, field equations have been used only at the last equality. The usual problem has been avoided in our theory, because the divergence $D_\mu J^{\mu I}$ has absorbed the unwanted term $mfFH$ in (1.5) into the first term $mf\tilde{\mathcal{F}}\tilde{H}$ in (3.8) that vanishes on-shell thanks to the duality relationship (2.1)

The original DOF of $B_{\mu\nu}{}^I$ is $(3-2)(3-3)/2 = 0$, due to the δ_β-symmetry (3.5a), and the two longitudinal components for each index are to be deleted. This tells that the field $B_{\mu\nu}{}^I$ is a nonpropagating field, as it is also a well-known fact in 3D. From this viewpoint, the B-field is *not* physical, while the $C_\mu{}^I$-field propagates, as the field Eq. (3.1a) has its conventional kinetic term $D_\nu H^{\mu\nu I}$. So in this way of counting, the DOF of each field in the extra vector multiplet is $C_\mu{}^I(1)$, $\chi^I(1)$, $B_{\mu\nu}{}^I(0)$.

However, there is an alternative counting method for the B and C-fields. In the field strength $H_{\mu\nu}{}^I$ in (3.2a), the tensorial δ_β-gauge symmetry $\delta_\beta B_{\mu\nu}{}^I$ in (3.5a) absorbs the gradient term $2D_{[\mu}B_{\nu]}{}^I$. In other words, the C-field is absorbed into the B-field. Now, the counting of physical DOF works as follows: The usual counting of $(3-2)\times(3-1)/2 = 0$ is *no longer* valid, because the δ_β-symmetry has been used up to absorb the C-field. It is now modified to $(3-1)(3-2)/2 = 1$, i.e., the B-field after the absorption carries one DOF which is a propagating physical freedom. So in this second way of counting, the DOF are $C_\mu{}^I(0)$, $\chi^I(1)$, $B_{\mu\nu}{}^I(1)$, again consistently with $N = 1$ supersymmetry.

We mention one subtlety related to a possible Lagrangian for the extra vector multiplet $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$.

When the non-Abelian coupling is turned off by $m = 0$, the field equations in (3.1) are possibly obtained from the tentative Lagrangian

$$\mathcal{L}_{m=0} = -\frac{1}{4}(H_{\mu\nu}{}^I)^2 - \frac{1}{2}(\bar{\chi}^I \not{\partial} \chi^I), \tag{3.9}$$

where the index I here means just copies of fields, but without any non-Abelian couplings due to $m = 0$. The invariance $\delta_Q \int d^3x \mathcal{L}_{m=0} = 0$ is confirmed with (3.3) for $m = 0$. Now the subtlety is about the absence of $C_{\mu\nu\rho}{}^I$-containing term in (3.9), because (3.1c) is *not* obtained from (3.9), unless there is a term involving G in the latter. On the other hand, the field strength G of the C-field is vanishing on-shell: $G_{\mu\nu\rho}{}^I \doteq 0$, so that the presence or absence of G in the transformation rule (3.3) is not essential, because it will not affect the closure of supersymmetry. Once such a G-dependent term is added to the Lagrangian (3.9), then the transformation rule (3.3c) should be modified with G-dependent terms accordingly, because the field equation $G_{\mu\nu\rho}{}^I \doteq 0$ should *not* be used for action invariance. However, since we do not present Lagrangian formulation in this paper, this point is not essential, although we mention at least the existence of such subtlety.

IV. CONCLUDING REMARKS

In this paper, we have presented an apparently simple but simultaneously sophisticated YM multiplet $(A_\mu{}^I, \lambda^I, \varphi^I)$ that possesses the peculiar duality-symmetric relationship (2.1). We showed that this YM multiplet can be consistently coupled to the extra vector multiplet $(C_\mu{}^I, \chi^I; B_{\mu\nu}{}^I)$. Here the tensor field $B_{\mu\nu}{}^I$ is a nonpropagating auxiliary field, while the extra vector field $C_\mu{}^I$ has a physical propagating DOF. Our system has in total 2 bosonic and 2 fermionic DOF consistent with supersymmetry, because the DOF of each field is $A_\mu{}^I(1/2)$, $\lambda^I(1)$, $\varphi^I(1/2)$, $C_\mu{}^I(1)$, $\chi^I(1)$, $B_{\mu\nu}{}^I(0)$.

We have also mentioned the alternative DOF counting, such that the C-field is absorbed into the B-field by the δ_β-symmetry. From this viewpoint, the C-field carries *no* physical DOF. In other words, the $B_{\mu\nu}{}^I$-field now propagates, instead of being an auxiliary multiplier field, whereas the C-field lost its physical degree of freedom. Accordingly, the DOF of each field is $A_\mu{}^I(1/2)$, $\lambda^I(1)$, $\varphi^I(1/2)$, $C_\mu{}^I(0)$, $\chi^I(1)$, $B_{\mu\nu}{}^I(1)$, in total 2 + 2 DOF, again in agreement with supersymmetry.

We have seen that all of our field equations in (2.1), (2.3), and (3.1), are consistent with supersymmetry (2.4) and (3.3), as shown in (2.5) and (3.4). In particular, there arises no problem for the divergence of the field equation $\mathcal{H}^{\mu I} \doteq 0$. In other words, the extra vector field $C_\mu{}^I$ in the adjoint representation poses *no* problem for consistent interactions in our system.

To our knowledge, there has been no formulation in 3D, in which a DSYM multiplet is coupled to an extra vector multiplet with the 2nd-rank tensor, either propagating or not propagating, depending whether absorbing the vector field $C_\mu{}^I$ or not, respectively. This peculiar feature of the extra vector multiplet, especially the role played by the fields $B_{\mu\nu}{}^I$ and $C_\mu{}^I$, avoiding the usual problem of an extra vector field does not seem to have any example in the past, especially in the presence of a DSYM multiplet.

Our results above have opened a completely new avenue for duality-symmetric theory in 3D, and possibly in higher dimensions. For example, the application of similar mechanisms to extended supersymmetries is the next natural subject to be explored in the near future.

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