Bubbles of nothing in flux compactifications

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(Received 10 June 2010; published 29 October 2010)

We construct a simple $AdS_4 \times S^1$ flux compactification stabilized by a complex scalar field winding the single extra dimension and demonstrate an instability to nucleation of a bubble of nothing. This occurs when the Kaluza-Klein dimension degenerates to a point, defining the bubble surface. Because the extra dimension is stabilized by a flux, the bubble surface must be charged, in this case under the axionic part of the complex scalar. This smooth geometry can be seen as a de Sitter topological defect with asymptotic behavior identical to the pure compactification. We discuss how a similar construction can be implemented in more general Freund-Rubin compactifications.

DOI: 10.1103/PhysRevD.82.086015 PACS numbers: 11.25.Mj, 04.50.−h, 98.80.Qc, 11.27.+d

I. INTRODUCTION

Some time ago, Witten showed that the Kaluza-Klein vacuum suffers from a nonperturbative instability due to the semiclassical nucleation of so-called *bubbles of nothing*. These smooth ''boundaries'' of space expand and soon engulf the whole of spacetime [1]. Such instabilities may be cause for concern regarding the viability of certain higher dimensional spacetimes as acceptable vacua. It is therefore necessary to consider the existence of analogous decay processes for perturbatively stable compactifications.

Flux compactifications [2] provide an elegant solution to the moduli problem in higher dimensional field theories [3–6] as well as string theory [7,8]. By stabilizing the extra dimensions with sufficiently high moduli masses, we can construct a model realistic enough to accommodate the low energy physics as well as a cosmological framework compatible with observations. These masses are induced by a flux potential, which depends on a discrete set of flux winding numbers. The diverse set of fluxes and associated winding numbers gives rise to the multitude of (meta)stable vacua known as the string landscape [9]. Although supersymmetric flux compactifications are known to be stable [10], the more phenomenologically interesting compactifications may enjoy/suffer from several instabilities, including decompactification [11–13] or more general transdimensional tunneling [13,14], flux transitions [12,15–18] and, as we will demonstrate here, nucleation of bubbles of nothing.[1] The bubble of nothing geometry asymptotic to a given flux compactification is a gravitational instanton which represents both the decay mediator and subsequent classical evolution of the metastable spacetime. This is the case for Witten's bubble of nothing in the minimal five-dimensional Kaluza-Klein (KK) model [1].

The outline of the paper is as follows. In Sec. II we review the original bubble of nothing. In Sec. III we discuss how these bubble solutions can be obtained as the spacetimes of de Sitter topological defects in a simple five-dimensional flux compactification, where the extra dimension is stabilized by the presence of a winding complex scalar field. In Sec. IV we obtain numerical solutions describing these bubbles of nothing. We conclude in Sec. V and speculate on similar constructions in more general flux compactifications.

[1]Bubbles of nothing in flux compactifications have been recently discussed in [18] by matching different spacetime geometries across a codimension *one* brane. This is different from our present construction describing a geometry which is smooth everywhere, obviating the need to postulate the existence of domain walls.

II. THE ORIGINAL BUBBLE OF NOTHING

The bubble of nothing geometry introduced by Witten [1] is easily obtained from the five-dimensional Schwarzschild black hole,

$$ds^2 = -\left(1 - \frac{\ell^2}{\rho^2}\right)dt^2 + \left(1 - \frac{\ell^2}{\rho^2}\right)^{-1} d\rho^2 + \rho^2(d\psi^2 + \sin^2\psi\, d\Omega_2^2). \quad (1)$$

It will be convenient for us to express this metric in terms of a new radial coordinate, $r = \sqrt{\rho^2 - \ell^2}$, as

$$ds^2 = -\frac{r^2}{r^2 + \ell^2} dt^2 + dr^2 + (r^2 + \ell^2)(d\psi^2 + \sin^2\psi\, d\Omega_2^2). \quad (2)$$

We can now Wick rotate two of the coordinates via

$$t \to i\ell y \qquad \psi \to it + \frac{\pi}{2} \quad (3)$$

to yield

$$ds^2 = \frac{r^2}{1 + r^2/\ell^2} dy^2 + dr^2 + (r^2 + \ell^2)(-dt^2 + \cosh^2 t\, d\Omega_2^2). \quad (4)$$

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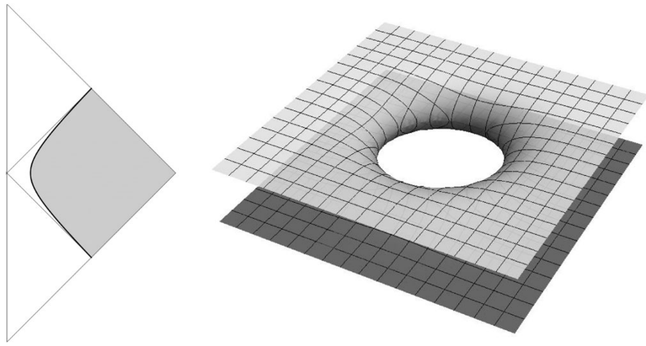

FIG. 1. Left: A four-dimensional conformal diagram for Witten's bubble of nothing. The spacetime only exists outside of the bubble (shaded region). Right: The bubble surface is smooth, and located where the compactification volume, shown as the apparent vertical separation, degenerates to zero size.

This is the bubble of nothing metric written in a somewhat unfamiliar gauge. (See the discussion in [19].) In the limit $r \longrightarrow 0$, it becomes

$$ds^2 \approx r^2 dy^2 + dr^2 + \ell^2(-dt^2 + \cosh^2 t d\Omega_2^2), \quad (5)$$

which is devoid of a conical singularity if we impose periodicity with $0 \leq y < 2\pi$. In this limit the r slice degenerates onto a $2 + 1$ dimensional de Sitter space of size ℓ, representing the induced metric on the bubble surface.

In the limit of $r \gg \ell$, the metric asymptotes to

$$ds^2 \approx \ell^2 dy^2 + dr^2 + r^2(-dt^2 + \cosh^2 t d\Omega_2^2), \quad (6)$$

which represents the Cartesian product of four-dimensional Rindler space and a circle of circumference $2\pi\ell$. The bubble of nothing geometry can therefore be regarded as a deformation of Rindler space (times a circle), whereby the horizon region near $r \longrightarrow 0$ is replaced with a smooth tip consisting of $dS_3 \times \mathcal{B}_2$, where $\mathcal{B}_2$ is a cigar-shaped disk. This is illustrated in Fig. 1 below.

In order for this geometry to represent an instability of the Kaluza-Klein vacuum, it must have the same asymptotics as the pure compactification, in particular, the same (zero) value of Schwarzschild mass. This can be seen to be the case in a number of ways. The KK compactification possesses the 11 isometeries of Poincaré $\times\, U(1)$, which are broken down to the seven in $O(3, 1) \times U(1)$ by the bubble of nothing (four translations are lost). This remaining symmetry is nevertheless larger than the five isometeries found in a generic Schwarzschild-Kaluza-Klein spacetime: $O(3) \times \mathbb{R} \times U(1)$, and so the mass of the bubble must be zero. Hence, barring any symmetry in place to prevent bubble nucleation, the KK vacuum is unstable.

III. BUBBLE OF NOTHING IN A FIVE-DIMENSIONAL FLUX COMPACTIFICATION

In this section we will discuss the construction of a bubble of nothing geometry similar to that presented above, but with one crucial difference: we will stabilize the Kaluza-Klein radion. This is accomplished by introducing an axionic flux winding around the extra dimension. The simplest example of this type of flux compactification was described in [12] using the action for a complex scalar field given by

$$S = \int d^5x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_M\Phi\partial^M\bar{\Phi} - \frac{\lambda}{4}(\Phi\bar{\Phi} - \eta^2)^2 - \Lambda\Big), \quad (7)$$

with $M, N, = 0, \ldots 4$ and $\kappa^2 = M_P^{-3}$, where M_P is the five-dimensional reduced Planck mass, and Λ denotes the five-dimensional cosmological constant. We now review the results of [12].

A. The flux vacua

To begin, we shall constrain the magnitude of the scalar field to lie at $|\Phi| = \eta$, leading to the effective action

$$S = \int d^5x\sqrt{-g}\Big(\frac{1}{2\kappa^2}R - \frac{1}{2}\eta^2\partial_M\theta\partial^M\theta - \Lambda\Big), \quad (8)$$

where θ is the phase of Φ. The equations of motion for this model are

$$\partial_M(\sqrt{-g}\partial^M\theta) = 0, \quad (9)$$

$$R_{AB} - \tfrac{1}{2}g_{AB}R = \kappa^2 T_{AB}, \quad (10)$$

where

$$T_{AB} = \eta^2(\partial_A\theta\partial_B\theta - \tfrac{1}{2}g_{AB}\partial_M\theta\partial^M\theta) - g_{AB}\Lambda \quad (11)$$

is the energy momentum tensor. We will look for a solution of the form

$$ds^2 = g_{MN}dx^M dx^N = g_{\mu\nu}dx^\mu dx^\nu + g_{yy}(x^\mu)dy^2, \quad (12)$$

where $\mu, \nu = 0, 1, 2, 3$ denote the four-dimensional coordinates, and the compact extra dimension is parameterized by the coordinate $0 \leq y < 2\pi$. We are interested in the case

$$g_{yy}(x^\mu) = L^2 = \text{const}, \quad (13)$$

i.e., in solutions with the extra dimension stabilized at a constant circumference $2\pi L$. We shall also require that the four-dimensional slices are described by a spacetime of maximal symmetry whose scalar curvature is given by $R^{(4)} = 12H^2$, with H^2 negative for the anti-de Sitter (AdS) case.

The solutions to the equations of motion for the scalar field Eq. (9) which are compatible with maximal spacetime symmetry are given by

$$\theta(x^M) = ny. \quad (14)$$

The change of axion phase θ around the compact dimension must be an integer multiple of 2π, and hence the various flux vacua are parameterized by the integer n.

With these assumptions we arrive at the Einstein equations

$$3H^2 = \kappa^2\left(\frac{n^2\eta^2}{2L^2} + \Lambda\right), \tag{15}$$

$$6H^2 = -\kappa^2\left(\frac{n^2\eta^2}{2L^2} - \Lambda\right), \tag{16}$$

which fix the values of H and L in terms of n and the parameters of the original Lagrangian according to

$$L^2 = -\frac{3n^2\eta^2}{2\Lambda}, \tag{17}$$

$$H^2 = \frac{2\kappa^2\Lambda}{9}. \tag{18}$$

We thus conclude from Eq. (17) that flux vacua exist for S^1 compactifications provided the five-dimensional cosmological constant is negative ($\Lambda < 0$). Equation (18) then indicates that these compactifications always yield a four-dimensional anti-de Sitter spacetime, enumerated by the integer $n \neq 0$. Furthermore, it can be easily shown by studying the four-dimensional effective action associated with these models [12], that the above solutions are perturbatively stable. We will now investigate the possibility that there exist nonperturbative instabilities of this geometry,[2] similar to the pure Kaluza-Klein bubble of nothing presented in the previous section.

B. The bubble of nothing as a de Sitter brane

A particularly useful description of our $AdS_4 \times S^1$ compactification is given by the following metric which covers a Rindler-like portion of the four-dimensional spacetime shown in Fig. 2 below:

$$ds^2 = dr^2 + H^{-2}\sinh^2(Hr)(-dt^2 + \cosh^2 t d\Omega_2^2) + L^2 dy^2. \tag{19}$$

Comparing with the right-hand illustration in Fig. 2, consider a metric of the form

$$ds^2 = dr^2 + B^2(r)(-dt^2 + \cosh^2 t d\Omega_2^2) + r^2 C(r)^2 dy^2, \tag{20}$$

with boundary conditions

$$B(r) \to \ell \qquad C(r) \to 1 \tag{21}$$

for $r \to 0$, and

$$\frac{\partial_r B(r)}{B(r)} \to H\coth(Hr) \qquad rC(r) \to L \tag{22}$$

in the limit $r \to \infty$.

[2]Flux-changing instantons [12] have previously been shown to exist. Their relation to the bubble of nothing presented here will be discussed below.

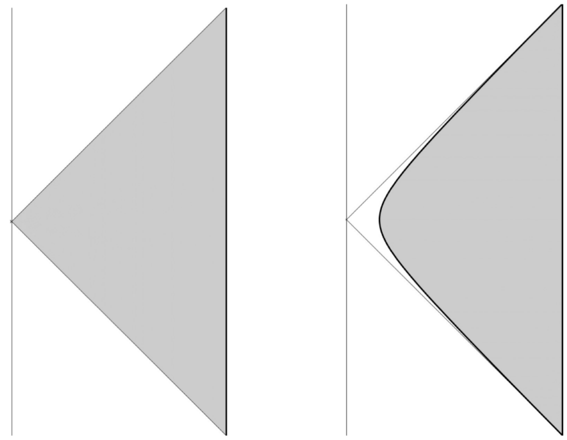

FIG. 2. Left: A conformal diagram for AdS_4. An S^2 is suppressed at each point except the left vertical line. The coordinates used in Eq. (19) cover the shaded region. Right: The spacetime only exists outside (shaded region) the bubble wall, which respects the de Sitter slicing of AdS_4.

It is clear from this description that such a solution, if it exists, would have the appropriate asymptotics as $r \to \infty$ to match to the flux compactification solution Eq. (19) at the conformal boundary, where we must additionally impose that the axion θ approach the form Eq. (14). Looking at the boundary conditions imposed above, one can see that the metric in Eq. (20) has the same structure at $r \to 0$ as Witten's solution discussed previously in Eq. (4); we have a compelling ansatz for a bubble of nothing in this flux compactification.

Because the compact dimension in this solution closes smoothly at $r = 0$, one must introduce some dynamical object that is able to resolve the flux singularity on the surface of the bubble. We can accomplish this simply by examining our original Lagrangian for the complex scalar field in Eq. (7). The problem arises only if one insists on keeping the modulus of the scalar field finite near the surface of the bubble. The divergence of gradient energy is cured by allowing the scalar modulus to relax along the radial direction in such a way that it vanishes at the tip of the cigarlike geometry. This is the same regularization found on a global string (i.e., codimension-two) solitonic solution associated with a complex scalar field.

Singular bubbles of nothing in $AdS_5 \times S^5/\mathbb{Z}_k$ were constructed in [20], yielding a bubble surface charged with respect to the stabilizing flux. The bubble, located where an S^1 fiber degenerates, is singular due to the required de Sitter D3 branes smeared on the $\mathbb{C}P^2$ base. Our aim is to find nonsingular solutions using a solitonic rather than smeared pointlike sources.

As pointed out in [12], our Lagrangian admits such solitonic solutions describing 2-branes *charged* with respect to the axion θ. We therefore conjecture that one should identify the solution described above, the bubble of nothing within our $AdS_4 \times S^1$ flux compactification, as

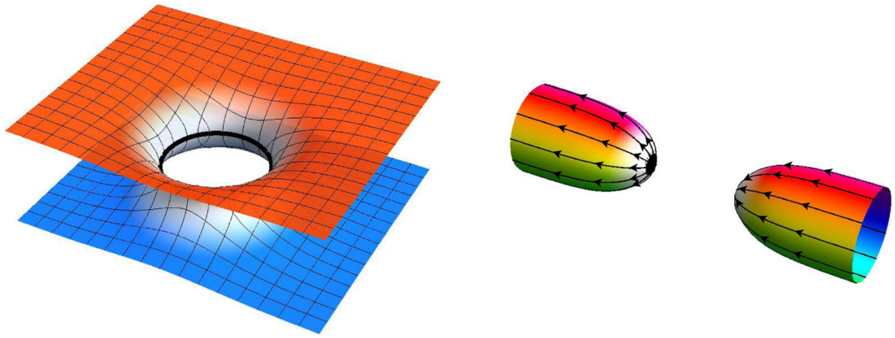

FIG. 3 (color online). An axionic bubble of nothing. As in Witten's solution, the bubble wall lies where the S^1 extra dimension degenerates. In this case, the wall is charged under an axionic phase. Hue and saturation represent the scalar phase θ, and modulus f, respectively. The core of the solitonic 2-brane has an ill-defined phase, and is shown in black. The left-hand picture shows two large dimensions, with the apparent vertical separation representative of the KK radion. The right-hand picture shows the full KK extra dimension, but only one of the four large dimensions.

a de Sitter solitonic 2-brane in a five-dimensional AdS spacetime. We prove in the next section that one can indeed find such smooth bubble geometries in our model by numerically solving the equations of motion using the ansatz described above.

Solutions for de Sitter branes as topological defects have been previously discussed in [21], whose numerical solutions introduced many characteristics found in our bubble of nothing. Here we give a different interpretation for these spacetimes in the context of flux compactifications. We illustrate such a bubble of nothing in Fig. 3 below.

IV. THE BUBBLE SOLUTION

We begin by considering the action

$$S = \int d^5x\sqrt{-g}\left(\frac{1}{2\kappa^2}R - \frac{1}{2}\partial_M\Phi\partial^M\bar{\Phi} - \frac{\lambda}{4}(\Phi\bar{\Phi} - \eta^2)^2 - \Lambda\right), \tag{23}$$

with metric

$$ds^2 = dr^2 + B^2(r)(-dt^2 + \cosh^2(t)d\Omega_2^2) + r^2C(r)^2dy^2, \tag{24}$$

and scalar field

$$\Phi(x^M) = f(r)e^{i\theta(y)} = f(r)e^{iny}. \tag{25}$$

The $O(3,1)\times U(1)$ symmetry of the bubble of nothing is easily seen within this ansatz. We arrive at the equations

$$R_{MN} - \tfrac{1}{2}g_{MN}R = \kappa^2 T_{MN}, \tag{26}$$

and

$$\partial_M(\sqrt{-g}\partial^M\Phi) - \sqrt{-g}\lambda\Phi(|\Phi|^2 - \eta^2) = 0, \tag{27}$$

having denoted

$$T_{MN} = \partial_M\Phi\partial_N\bar{\Phi} + g_{MN}\left(-\frac{1}{2}\partial_P\Phi\partial^P\bar{\Phi} - \frac{\lambda}{4}(\Phi\bar{\Phi} - \eta^2)^2 - \Lambda\right). \tag{28}$$

The Einstein tensor then becomes

$$\begin{aligned} G_t^t &= -\frac{1}{B^2} + \frac{2B'}{rB} + \frac{B'^2}{B^2} + \frac{2C'}{rC} + \frac{2B'C'}{BC} + \frac{2B''}{B} + \frac{C''}{C},\\ G_r^r &= -\frac{3}{B^2} + \frac{3B'}{rB} + \frac{3B'^2}{B^2} + \frac{3B'C'}{BC},\\ G_y^y &= -\frac{3}{B^2} + \frac{3B'^2}{B^2} + \frac{3B''}{B}, \end{aligned} \tag{29}$$

and the energy momentum tensor is given by

$$\begin{aligned} T_t^t &= -\frac{1}{2}f'^2 - \frac{n^2f^2}{2r^2C^2} - \frac{\lambda}{4}(f^2 - \eta^2)^2 - \Lambda,\\ T_r^r &= \frac{1}{2}f'^2 - \frac{n^2f^2}{2r^2C^2} - \frac{\lambda}{4}(f^2 - \eta^2)^2 - \Lambda,\\ T_y^y &= -\frac{1}{2}f'^2 + \frac{n^2f^2}{2r^2C^2} - \frac{\lambda}{4}(f^2 - \eta^2)^2 - \Lambda. \end{aligned}$$

The equations of motion are then any three of the four equations

$$-\frac{1}{B^2}+\frac{2B'}{rB}+\frac{B'^2}{B^2}+\frac{2C'}{rC}+\frac{2B'C'}{BC}+\frac{2B''}{B}+\frac{C''}{C}=\kappa^2\left(-\frac{1}{2}f'^2-\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right),$$

$$-\frac{3}{B^2}+\frac{3B'}{rB}+\frac{3B'^2}{B^2}+\frac{3B'C'}{BC}=\kappa^2\left(\frac{1}{2}f'^2-\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right),$$

$$-\frac{3}{B^2}+\frac{3B'^2}{B^2}+\frac{3B''}{B}=\kappa^2\left(-\frac{1}{2}f'^2+\frac{n^2f^2}{2r^2C^2}-\frac{\lambda}{4}(f^2-\eta^2)^2-\Lambda\right),$$

$$f''+\left(3\frac{B'}{B}+\frac{C'}{C}+\frac{1}{r}\right)f'=\frac{n^2f}{C^2r^2}+\lambda f(f^2-\eta^2).$$

Because our ansatz is explicitly time-independent, the Lorentzian and Euclidean solutions are trivially related.

A. Asymptotic solution of the full equations

Before numerically solving the near-bubble region, we determine the asymptotic values of all functions, which can be done exactly. The solution below exhibits the expected backreaction on the scalar modulus, the KK radion, and the vacuum energy density.

$$\Phi(x)=f_\infty e^{iny}, \tag{30}$$

$$ds^2=dr^2+\frac{1}{H^2}\sinh^2(Hr)(-dt^2+\cosh^2(t)d\Omega_2^2)+L^2dy^2, \tag{31}$$

where

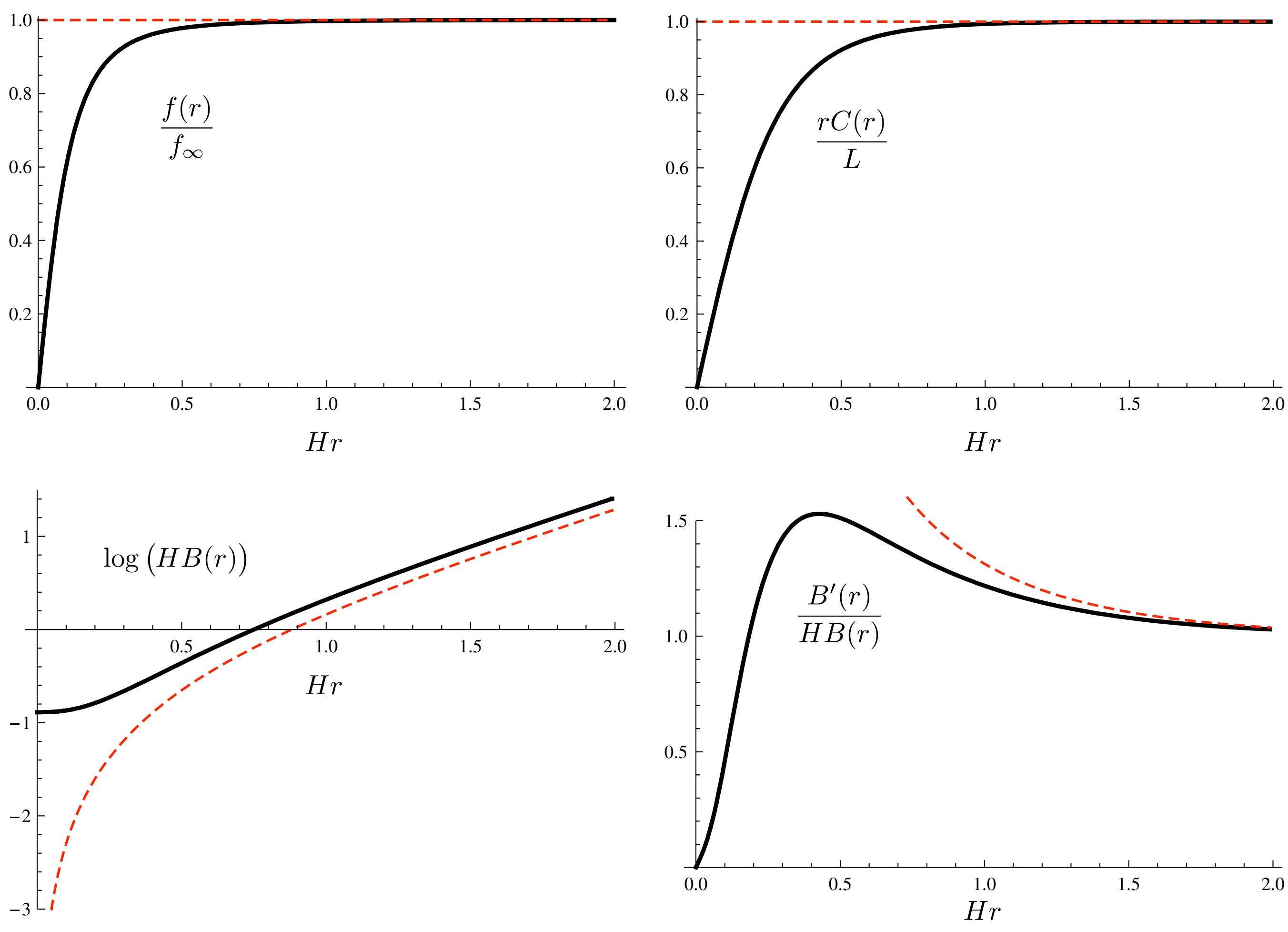

FIG. 4 (color online). The numerical solutions for the scalar modulus (top left), the KK radion modulus (top right), the de Sitter slice radius (bottom left), and its derivative (bottom right). We express r in units of the AdS radius $1/H$. Dashed red lines represent the pure compactification solution of Eqs. (30) and (31), which shares the conformal boundary with the bubble solution.

$$f_\infty^2 = \eta^2 - \frac{n^2}{\lambda L^2} = \frac{2\eta^2}{5}\left(1 + \frac{3}{2}\Delta\right), \tag{32}$$

$$L^2 = -\frac{3n^2\eta^2}{4\Lambda}(1 + \Delta), \tag{33}$$

$$H^2 = -\frac{4\kappa^2\Lambda}{15}\left(\frac{\frac{2}{3} + \Delta}{1 + \Delta}\right), \tag{34}$$

and we have introduced the parameter Δ,

$$\Delta = \sqrt{1 + \frac{20\Lambda}{9\eta^4\lambda}}. \tag{35}$$

Note that $\Delta \to 1$ in the limit $\lambda \to \infty$, so we recover the pure flux compactification geometry described in previous section [See Eqs. (17) and (18)].

B. Near core expansion

We can Taylor expand the equations of motion about the tip of the cigar. This leaves two unknown boundary conditions, which we will use as shooting parameters. For $n = 1$ they are ℓ and f_0'. The remaining terms are then completely specified:

$$B(r) = \ell + \left(\frac{1}{2\ell} - \frac{\kappa^2\ell(\eta^4\lambda + 4\Lambda)}{24}\right)r^2 + \ldots, \tag{36}$$

$$C(r) = 1 + \left(-\frac{1}{2\ell^2} + \frac{\kappa^2(\eta^4\lambda + 4\Lambda - 12f_0'^2)}{72}\right)r^2 + \ldots, \tag{37}$$

$$f(r) = f_0' r + \ldots. \tag{38}$$

We then numerically integrate the equations of motion outward from $r = 0$ to obtain the functions pictured in Fig. 4. The numerical values for the parameters used are

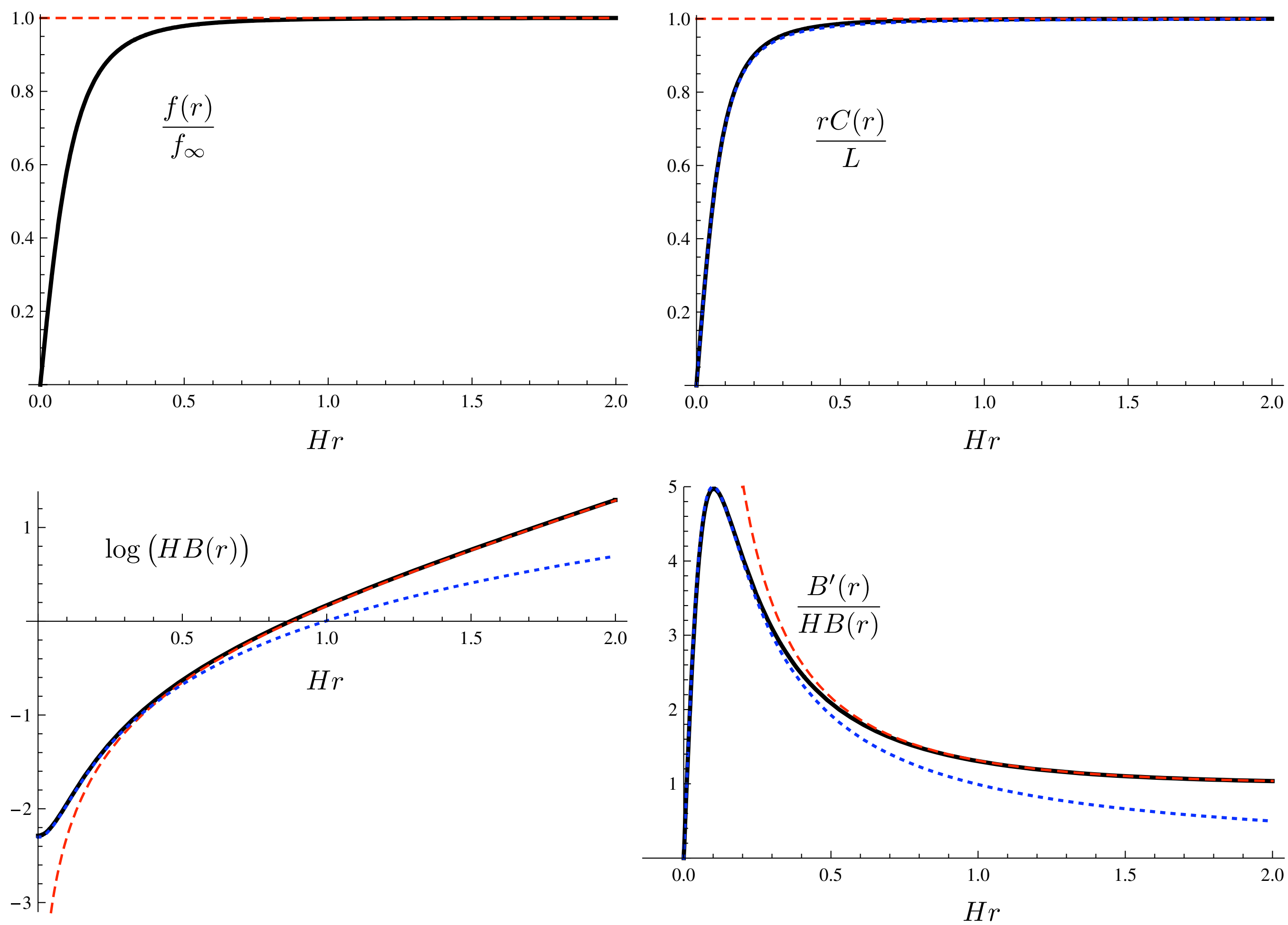

FIG. 5 (color online). Parameters were chosen to demonstrate the smooth embedding of Witten's bubble in a mildly AdS flux compactification. Dotted blue lines represent Witten's solution of identical KK circumference $2\pi L$. Dashed red lines represent the pure flux compactification solution, as before. Near the bubble, the solution resembles that of Witten, while on scales larger than H^{-1}, the solution approaches the AdS compactification.

$$n = 1, \qquad \Lambda = -(0.347 M_P)^5, \qquad \eta = (0.630 M_P)^{3/2}, \qquad \lambda = (0.995 M_P)^{-1}, \tag{39}$$

which give the solution

$$H = 0.0332 M_P, \qquad L = (0.118 M_P)^{-1}, \qquad \ell = (0.0806 M_P)^{-1}, \qquad f'_0 = (0.456 M_P)^{5/2}, \tag{40}$$

where $M_P = \kappa^{-2/3}$ is the five-dimensional reduced Planck mass. The Euclidean action of this solution is given by

$$S_B = S_E[\text{bubble}] - S_E[\text{compactification}] \approx 7.4 \times 10^4, \tag{41}$$

where the numerical solution was matched to the background in a box of circumference $25.566 H^{-1}$. Unlike Witten's solution, the size ℓ of our bubble depends not only on the compactification radius L, but on both the tension of the vortex and the AdS curvature scale as well. This taxonomy will be explored in a future publication. Our chosen parameter values are rather generic, but we expect a large range of solutions to exist. In particular, we can smoothly deform our parameters and corresponding solution to match the bubble of Witten.

This is shown in the solution Fig. 5, where the chosen parameters are

$$n = 1, \qquad \Lambda = -(0.224 M_P)^5, \qquad \eta = (0.422 M_P)^{3/2}, \qquad \lambda = (4.50 M_P)^{-1}, \tag{42}$$

which give the solution

$$H = 0.010 M_P, \qquad L = (0.100 M_P)^{-1}, \qquad \ell = (0.0985 M_P)^{-1}, \qquad f'_0 = (0.179 M_P)^{5/2}. \tag{43}$$

The Euclidean action of this solution is given by

$$S_B = S_E[\text{bubble}] - S_E[\text{compactification}] \approx 6.4 \times 10^4, \tag{44}$$

which is similar to the analogous action for Witten: $S_B = 2\pi^3 L^3/\kappa^2 \approx 6.2 \times 10^4$. As seen in Fig. 5, the solution resembles that of Witten on scales smaller than H^{-1}, but then asymptotes to the AdS compactification far from the bubble.

V. CONCLUSIONS

We have shown that a simple $AdS_4 \times S^1$ flux compactification exhibits an instability to the nucleation of bubbles of nothing. The key feature of this geometry is the flux through the compactification cycle at the conformal boundary, which demands a source wherever the cycle degenerates. Appropriately, the field theoretic model considered has solitonic 2-brane solutions which are charged with respect to the axionic flux stabilizing the extra dimension. One can construct a bubble of nothing in this context as the spacetime sourced by an "inflating" solitonic 2-brane whose worldvolume is given by a codimension-two de Sitter space (the surface of the bubble). We have obtained numerical examples of these bubbles which asymptotically match the pure compactification geometry. Although our numerical work only considered the decay of $n = 1$ vacua, analogous decays should exist involving higher n vortex solutions. Because the relevant instanton is probably less symmetric [22], $|n| > 1$ bubbles of nothing require numerical analysis beyond the scope of ordinary differential equations.

We have presented our construction using the simplest field theory that possesses the minimal ingredients for the scenario to work, but we believe this kind of instability should be a generic feature of other nonsupersymmetric flux compactifications. In particular, we have recently shown how to implement these ideas [23] in a model based on a six-dimensional Einstein-Maxwell theory, where one finds a sizable landscape [5,12] of dS_4, M_4 and AdS_4 vacua. As explained in [12], this theory possesses 2-brane solutions which could be used to construct instantons interpolating between vacua of differing flux numbers. One can then extend this model to an Einstein-Yang-Mills-Higgs theory [3]. The new degrees of freedom incorporated in this model allow for smooth codimension-three solitonic 2-branes. Such magnetically charged de Sitter 2-branes have the correct asymptotics, similar to those in [24], to be bubbles of nothing [23]. The difficulty of finding higher n solutions is exacerbated in the codimension-three case: there are no finite energy configurations with both $|n| > 1$ and spherical symmetry [25].

An alternate perspective of the solutions presented here is as the critical case of a flux-changing instanton whose final vacuum has zero flux. Typical flux-changing transitions do not exhibit large backreaction on the spacetime geometry, provided one considers the nucleation of a single-charge brane, i.e., small relative changes in flux. On the other hand, the bubble of nothing corresponds to the extreme case, where the brane acts as a sink for *all* the flux found in the asymptotic compactification. It is not surprising that the effect on the geometry is drastic, since a zero-flux vacuum is unstable to collapse. One can see this in our model by examining the effective four-dimensional theory for the KK radion, which in the absence of flux is parameterized solely by the contribution from the negative bulk cosmological constant. The resulting tachyonic potential is divergent as one approaches zero-size compact dimension (See [12]). This suggests that one should be able to understand the bubble of nothing from a purely four-dimensional point of view by including the field f as a new scalar degree of freedom in the four-dimensional effective theory. Such a solution may appear singular from the four-dimensional perspective, but this is just an artifact of the dimensional reduction, much like the case of Witten's bubble [19].

The bubbles of nothing considered here affect many otherwise stable flux vacua. One should consider this instability as a new decay channel whose end result is a type of terminal vacuum, a vacuum without a classical spacetime. This is clearly relevant for any program undertaking the assigning of probabilities to different vacua in the landscape, the so-called measure problem. (See, for example, [26] and references therein.)

ACKNOWLEDGMENTS

We would like to thank J. Garriga, O. Pujolas, and A. Vilenkin for very helpful conversations and discussions. J. J. B.-P. would like to thank the Theory Division at CERN for its hospitality and support in the early stages of this work. J. J. B.-P. is supported in part by the National Science Foundation under Grant No. 06533561. B. S. is supported in part by the Foundational Questions Institute under Grant No. RFP2-08-26A.

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