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Remarks on decay of defects with internal degrees of freedom

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We consider the decay of metastable walls and strings populated by additional degrees of freedom. The examples involve the decay of an axion wall in an external magnetic field, pionic walls, and metastable walls in dense QCD. It is shown that the induced fermions escape from the wall during the decay process, providing an example of the chiral magnetic effect. An absolute stabilization of metastable and unstable walls in a large magnetic field is found. A possible higher-dimensional generalization of the chiral magnetic effect is mentioned.

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I. INTRODUCTION

Metastable brane-type extended objects such as walls and strings are an essential part of the nonperturbative spectrum in gauge theories, and are also frequently discussed in geometrical engineering of supersymmetric (SUSY) gauge theories and in cosmology. The metastability of such extended defects is ensured by the well-known semiclassical exponential suppression factor in their decay rate. A somewhat less-studied problem is that of the decay of these defects while being populated by additional degrees of freedom, including the degrees of freedom associated with defects of lower dimensions. In this situation an interesting question arises concerning the fate of such additional degrees of freedom and the effect of their presence on the decay rate.

In this paper we discuss three such types of problems: the decay of a metastable axion wall in an external magnetic field, the decay of walls in dense matter, and the decay of a non-Abelian string with the excitation. It is known that due to the one-loop Goldstone-Wilczek current [1], an axion wall in an external magnetic field develops the homogeneous density of the charged fermions [2–5]. The question we investigate here is whether the fermions stay localized on the remaining part of the domain wall during its decay or escape into the bulk? We find that in the leading approximation the fermions leave the domain wall and provide a very clear pattern of the chiral magnetic effect (CME) which currently attracts considerable attention related to the observed charge separation effect at the Relativistic Heavy Ion Collider [6]. We shall thus argue that the decay of the domain wall provides an effective chiral chemical potential responsible for the chiral magnetic effect.

The second problem is related to the decay of mesonic walls in QCD. The nontrivial density of the Skyrmions is generated in the external magnetic field on π^0 walls, and the field prevents the wall from nonperturbative decay above some B_{crit} [7]. There is a new point in the mesonic wall decay in dense matter. It is known [8] that in this case, due to the anomalous term in the action, the current is generated along the axionlike string at the boundary of the wall. In the decay problem the axion string is the boundary of the hole, and therefore there is a circular boundary current during the decay. We analyze the impact of the created current on the magnetic field.

The third problem in our discussion concerns the decay process of a non-Abelian string with excited internal CP_N degrees of freedom. The Lagrangian of the world-sheet theory supports a kink excitation [9] which can exist in isolation by itself in SUSY theories and as kink-antikink bound states in a generic non-SUSY case. We shall argue that in the non-Abelian string decay the kink excitations on the string decrease the decay rate.

The paper is organized as follows. In Sec. II we consider the process of the axion domain wall decay. The decays of mesonic walls in QCD are considered in Sec. III. In Sec. IV we comment on the decay of non-Abelian strings with an emphasis on the role of kink excitations. The analogue of the CME in higher dimensions is briefly discussed in Sec. V.

II. DECAY OF AXIONLIKE DOMAIN WALLS IN D = 3 + 1 THEORIES

In this section we shall discuss the decay of the Abelian domain walls in the magnetic field. Let us discuss the decay of the axion wall in the theory with the Lagrangian for the axion field $a(x)$ given by

$$L = f_a^{-2}[\tfrac{1}{2}(\partial a)^2 + m_a^2 \cos a(x)]. \tag{1}$$

The model also contains charged fermions interacting with the axion as

$$L_f = \bar{\psi}[i(\partial_\nu - iA_\nu)\gamma^\nu - m_f e^{ia(x)\gamma^5}]\psi \tag{2}$$

with A_ν being the potential of the electromagnetic field. (The coupling e is absorbed into the normalization of A.) An integration over the fermionic field gives rise to the anomalous interaction between the axion and the electromagnetic field [1] described by the Lagrangian

$$L_{\rm anom} = \frac{1}{16\pi^2}\epsilon_{\mu\nu\lambda\sigma}A_\mu F_{\lambda\sigma}\partial_\nu a. \tag{3}$$

A derivation of this term through the analysis of the fermionic modes in the external field in the simplified model for the fermions with the Lagrangian

$$L = \bar{\psi}[i(\partial_\nu - iA_\nu)\gamma^\nu - \mu_1(z) - i\mu_2(z)\gamma^5]\psi \tag{4}$$

can be found in [4]. The z dependence of the mass term $\mu_1(z) - i\mu_2(z)\gamma^5$ is driven by a varying spinless field, and the variation of μ_2 breaks the CP parity similarly to the axion field.

If the axion model admits a wall solution, the anomalous term yields the density of the electric charge at the domain wall [2]. The details of the domain wall solution are not important, and the surface density of the induced charge in the background magnetic field is equal to

$$q = \frac{B\Delta a}{4\pi^2}, \tag{5}$$

where B is the magnetic field perpendicular to the axion wall and Δa is the total variation of the axion field across the wall. Therefore a constant external magnetic field creates a homogeneous density of the induced electric charge on the wall. In the simplified model and for a slowly varying ratio $\mu_2(z)/\mu_1(z)$, the distribution of the induced charge density in the domain wall background equals

$$\rho(z) = \frac{B}{4\pi^2}\frac{d}{dz}\arctan\frac{\mu_2(z)}{\mu_1(z)}. \tag{6}$$

For any realistic magnetic field, created by physical sources, its total flux through an infinite wall is zero, so that the total charge of such a wall is zero as well. This behavior, however, is a result of the exact cancellation between areas with positive and negative surface charges.

An axion wall is metastable and decays through nucleation and subsequent expansion of a hole bounded by an axion string. When considering this process in a constant magnetic field, a natural question arises concerning the fate of the induced electric charge during the decay as well as the backreaction of the decay on the background magnetic field. In order to analyze this issue we consider different components of the Goldstone-Wilczek (GW) current, $J_\mu = \delta L_{\rm anom}/\delta A_\mu$, which are generated during the decay process. Let us assume for definiteness that the hole is created at the origin in the (x, y) plane.

During the decay the axion field can be approximated as

$$a(z, r, t) = f(z)\theta(r - r(t)) \tag{7}$$

with some profile function $f(z)$, and $r(t)$ is the time-dependent radius of the expanding hole, which is assumed to be large, so that any edge effects near the boundary of the hole that violate the form of the field in Eq. (7) can be neglected. Fermions are bounded by the domain wall and therefore there is no reason for them to stay at the same point when the hole is created. There are two logical possibilities: the fermions fly away from the domain wall plane or they are captured by the axion string at the boundary of the hole. Using the GW expression and the form (3) of the anomalous Lagrangian, we find that as the hole expands there arises a current perpendicular to the wall plane

$$J_z \propto (\partial_t a)B_z \propto f(z)\dot{r}\delta(r - r(t))B_z \tag{8}$$

which is clearly localized near the boundary of the hole.

The current J_z is directed along the external magnetic field and provides an explicit example of the chiral magnetic effect resulting in the charge separation effect [6,10] which was recently the subject of intensive theoretical and experimental studies. The domain wall decay process amounts to the effective time-dependent chiral chemical potential

$$\mu_{5,\rm eff} = \partial_t a.$$

The effective chiral chemical potential is localized at the boundary of the hole—the axion string. This is not a surprise since the axion string in the magnetic field is chiral; i.e. there is an asymmetry between left and right modes of the fermions on the string.

We thus conclude that the fermions, initially localized at the wall, do fly away to the bulk in the decay process. One can perform a cross-check of this conclusion by comparing the current "to the bulk" J_z with the rate of the disappearance of the area of the wall that initially carried the fermions. Indeed, the rate at which the surface charge disappears through the growth of the hole is given by

$$dN = 2\pi q r\dot{r}dt, \tag{9}$$

where q is the surface charge density on the wall, $r(t)$ is the radius of the hole, and the speed $\dot{r}$ is fixed by the bounce solution. On the other hand, if all fermions fly away we must have, from the continuity equation,

$$\frac{dN}{dt} = -\int d^3x\frac{dJ_z}{dz}. \tag{10}$$

Substituting the expression for the axion field corresponding to the undeformed $O(4)$ symmetric bounce solution into the anomalous current, we obtain that the continuity equation is fulfilled. This implies that all fermions fly away from the wall plane during the decay process and there is no need for accumulating any fermions on the boundary of the hole.

It should be mentioned that in the considered case, where the area of the wall supporting the fermion charge changes and the fermions flow into the bulk, the charge conservation is implemented differently from the case of a fixed patch of the wall and varying magnetic field. In the latter situation the surface term of the GW current is not conserved by itself [11] and its divergence is localized at the string

$$\partial_\mu j_\mu \propto B\delta(r). \tag{11}$$

The apparent nonconservation of the current is compensated by the accumulation of the charge on the axionic string at the boundary of the patch.

The escape of the fermions to the bulk produces an effect on the probability of the wall decay in a magnetic field. Indeed, the tunneling process is described by a spherical Euclidean bounce configuration which is determined from the effective action

$$S_{\rm eff} = 4\pi R^2 T_{\rm string} - 4/3\pi R^3 T_{\rm wall}, \tag{12}$$

where R is the radius of the bounce. At the extremum of this action the surface of the bounce is described by

$$R_{\rm crit} = \frac{2T_{\rm string}}{T_{\rm wall}}, \qquad x^2 + y^2 + t_E^2 = R_{\rm crit}^2, \tag{13}$$

with t_E being the Euclidean time, so that the coordinate z transverse to the wall is not essential in the bounce solution.

The transverse direction, however, becomes of relevance when the magnetic field is switched on. In the leading approximation the effect of the magnetic field can be taken into account as follows. While the fermions are localized at the wall they are effectively massless and there is no energy associated with them. Once they escape in the bulk each fermion costs energy equal to its mass m_f. That is, energetically the localization of the fermions on the wall suppresses the decay probability. From the $(2 + 1)$-dimensional viewpoint the decay proceeds with the energy loss since fermions escape from the wall. The fermion charge in a uniform magnetic field is proportional to the area of the wall [Eq. (5)]. Therefore the effect of the energy loss due to the emission of fermions can be described by replacing the wall tension by an effective one,

$$T_{\rm wall} \rightarrow T_{\rm wall,eff} = T_{\rm wall} - \frac{B}{2\pi} m_f, \tag{14}$$

resulting in a suppression of the wall decay. Notably the decay is entirely suppressed; i.e. the wall is stabilized at the magnetic field exceeding the critical value

$$B_{\rm crit} = 2\pi \frac{T_{\rm wall}}{m_f}. \tag{15}$$

Let us remark that for the case of an induced wall decay at a nonvanishing energy, the solution necessarily involves a Minkowski part in the time evolution. An example of such a two-step process involving a resonance behavior at particular values of the energy has been discussed in detail in [12]. In the current problem this would happen if in a process, e.g. in a collision, an excited state of the axionic string is produced, which then tunnels through the Euclidean region and eventually reaches the classical expansion regime.

It can be noted also that in four dimensions it is possible to trade the pseudoscalar field for a rank-two field $B_{\lambda\sigma}$ via the duality relation

$$\partial_\mu a = \epsilon_{\mu\nu\lambda\sigma} \partial_\nu B_{\lambda\sigma}. \tag{16}$$

In the presence of an external gauge field this relation gets modified,

$$\partial_\mu a = \epsilon_{\mu\nu\lambda\sigma}(\partial_\mu B_{\lambda\sigma} - A_\mu F_{\lambda\sigma}), \tag{17}$$

where the second Chern-Simons term emerges at one loop. Then the kinetic term of the axion at one loop yields the term in the Lagrangian responsible for the GW current. In terms of such a dual description, the domain wall implies a nontrivial "electric" curvature of the rank-two field $H = dB$ localized along the domain wall. The axionic string is charged with respect to $B_{\lambda\sigma}$; that is, in the dual representation the decay of the axion domain wall corresponds to a Schwinger-type production of axion strings in the external field.

III. DECAYS OF MESONIC WALLS

A. Decay of π^0 domain walls

In this section we consider the decay of walls in conventional QCD. One example of such an object is provided by a wall built from π^0 mesons. A π^0 wall is not topological and can be "unwound" inside the $SU(2)$ flavor group. Furthermore, such walls are absolutely unstable in the absence of the magnetic field. However, at $B > B_0 = 3m_\pi^2$ the wall becomes locally stable and at $B > B_1 = 16\pi f_\pi^2 m_\pi / m_N$ a patch of such a wall carrying a baryon number becomes the lowest energy state with baryon number [7].

The tension of the domain wall calculated at the explicit solution [7] reads as

$$T_{\rm pwall} = 8 f_\pi^2 m_\pi. \tag{18}$$

A magnetic field B applied perpendicularly to the wall generates a surface density of the baryon charge

$$q_B = \frac{B}{2\pi}, \tag{19}$$

which can also be viewed as a liquid of the Skyrmions on the surface [7].

The decay of the pionic wall implies a nontrivial baryonic current

$$J_\mu = \frac{1}{4\pi^2} \epsilon_{\mu\nu\lambda\sigma} \partial_\nu \pi^0 F_{\lambda\sigma} \tag{20}$$

flowing into the bulk similar to the electric current in the axion example. While escaping from the wall the Skyrmions have mass m_N. Therefore the effective wall tension can be found as

$$T_{\rm eff} = T_{\rm pwall} - q_B m_N = 8 f_\pi^2 m_\pi - \frac{B}{2\pi} m_N. \qquad (21)$$

One readily concludes from this expression that at $B > B_1$ the effective tension of the wall is negative, so that the decay is energetically impossible and the wall is absolutely stable. It can be noted that a somewhat similar behavior has been observed for the decay of electric strings in the magnetic field [13].

B. Wall decay in QCD at high density

At high baryon density the ground state of QCD is in a color-flavor locking (CFL) phase and the system develops color superconductivity (see [14] for a review). The theory at large baryon chemical potential μ is in the weak coupling regime and the dynamics of the low-energy degrees of freedom can be calculated perturbatively. In particular, the existence of a ϕ domain wall can be justified [15] from the effective Lagrangian for the Goldstone mode ϕ of $U(1)_A$ symmetry which is spontaneously broken by the condensate in the color-superconducting vacuum state.

The explicit form of the Lagrangian in the two-flavor case reads as follows:

$$L_{\rm dense} = f^2[(\partial_0 \phi)^2 - u^2(\partial_i \phi)^2] - a\mu^2\Delta^2 \cos\phi, \qquad (22)$$

where a is dimensionless and vanishes in the limit $\mu \to \infty$, and u is the speed of sound: $u^2 = 1/3$. The parameters of the Lagrangian are

$$f^2 = \frac{\mu^2}{8\pi u^2}, \qquad (23)$$

and Δ is the value of the gap. The tension of the wall can be derived immediately from the effective Lagrangian,

$$T_{\rm wall} = 8\sqrt{2a} u f \mu \Delta. \qquad (24)$$

In the CFL phase the $U(1)_A$ symmetry is spontaneously broken and the lightest degree of freedom is the η' meson. The kinetic term for this degree of freedom is the same as in the two-flavor superconducting phase, while the instanton induced potential term of the lightest meson gets modified as

$$V_{\rm CFL} = -\tilde{a}\left(\frac{m_s}{\mu}\right)\mu^2\Delta^2 \cos\phi, \qquad (25)$$

where m_s is the mass of the strange quark. Thus the tension of the wall in the CFL phase acquires an additional m_s-dependent factor.

The decay of the domain walls in the dense QCD matter has some peculiarities. In particular, one can notice that in the dense matter there is an anomalous Chern-Simons term in the Lagrangian of the pseudoscalar meson proportional to the chemical potential μ [7,8]:

$$\delta L = \frac{e}{24\pi^2} \mu \epsilon_{0\nu\lambda\sigma} \partial_\nu \phi F_{\lambda\sigma}. \qquad (26)$$

An immediate consequence of this term is that there is an electric current circulating along the axial string [8,16],

$$J = \frac{\mu e}{12\pi}. \qquad (27)$$

It can be noted that the current does not depend on the value of the external field. This current can be derived by the summation over the fermion modes [16], similar to the calculation of the induced charge in the magnetic field [4].

The main difference from the wall decay discussed in the previous section is that in dense matter there necessarily is a current along the hole boundary identified with the axial ϕ string. The current plays a twofold role. First, its existence implies that during the decay process not all of the fermions populating the wall fly away from the plane. Some of them are captured by the axial string at the boundary instead. Second, the circular current induces a magnetic field inside the hole, and the direction of the field depends on the sign of the chemical potential. One thus concludes that in dense matter magnetic effects in the decay of the wall are necessarily essential since the field is generated by the induced current circulating along the hole boundary. The tunneling starts at zero energy, so that there is no cusp at the bottom of the bounce configuration. However, contrary to the axion and π^0-mesonic walls in the vacuum, the current along the boundary makes it impossible to describe the outflow of the fermion energy by an appropriately modified effective wall tension as in Eq. (21).

One more issue is related to the angular momentum conservation during the domain wall decay. It was argued in [8] that the domain wall in dense matter carries an induced constant angular momentum per unit area,

$$\mathcal{M} = \frac{\mu}{6\pi}, \qquad (28)$$

so that it may appear that during the decay the angular momentum of the part of the wall turning into a hole is "lost." However, the current along the hole boundary also yields an induced angular momentum and can in fact ensure the conservation.

IV. NON-ABELIAN STRING DECAY

The strings are quite common objects corresponding to effective solutions to the equations of motion in various models. The problem of their decays can be formulated, and a detailed analysis of the decay of an Abrikosov-Nielsen-Olesen string in the Abelian Higgs model can be found in [17]. As of yet, only the decay of such Abrikosov-Nielsen-Olesen Abelian effective strings has been considered. However, more general stringy solutions exist both in the SUSY and in the non-SUSY theories in the color-flavor locking phase. Their key feature is the existence of additional degrees of freedom due to the nontrivial embedding of the non-Abelian string into the gauge group, which amounts to the orientational moduli providing CP_N degrees of freedom on the world sheet [9]. Thus the problem that parallels the discussion in the previous section is that of the fate of the CP_N degrees of freedom living on the non-Abelian string during the decay. There is, however, an essential difference between non-Abelian strings in SUSY and non-SUSY cases.

The world-sheet theory is built from an N-component complex field n^i subject to the constraint

$$n_i^* n^i = 1. \tag{29}$$

The Lagrangian has the form

$$L = \frac{2}{g^2}[(\partial_\mu - iA_\mu)n_i^*(\partial_\mu + iA_\mu)n^i - \lambda(n_i^* n^i - 1)], \tag{30}$$

where λ is the Lagrange multiplier enforcing the condition (29). At the quantum level this constraint is effectively eliminated and λ becomes dynamical. Moreover, A_μ is an auxiliary field which, at the classical level, enters the Lagrangian with no kinetic term. A kinetic term is generated, however, at the quantum level, so that the field A_μ becomes dynamical too.

One can also add to the Lagrangian a θ term of the form

$$L_\theta = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu A^\nu = \frac{\theta}{2\pi}\varepsilon_{\mu\nu}\partial^\mu(n_i^*\partial^\nu n^i). \tag{31}$$

In the SUSY case the world-sheet theory is supplemented by fermionic terms. There are $(N-1)$ vacuum states in the model and, respectively, kinks interpolating between those states. In a sense, the bounce configuration provides the decay of the CP_N model on a plane into a pair of CP_N models on a semiplane. If the string is in one of the ground states, the decay corresponds to the creation of a heavy monopole-antimonopole pair located at the ends of two semi-infinite strings.

We consider here the decay of a string in the presence of kinks. We start by discussing the case where only one kink is present, and we question whether this kink induces the string decay or suppresses it instead. Clearly, the kink, being a monopole in the Higgs phase, cannot escape into the bulk, contrary to the previously considered situations, since the kink does not exist in the bulk at all. Hence it can only be at the end of one semi-infinite string or located somewhere on the string world sheet. The location at the end is impossible since it would yield an additional non-Abelian magnetic flux in the emerging hole, which cannot be the case due to the Meissner effect. One therefore concludes that the kink has to be outside the decay region and that it thus effectively suppresses the decay.

In non-SUSY D = 4 gauge theories there is only a single vacuum state, so that a single kink solution is impossible. However, the spectrum involves a kink-antikink pair, which corresponds, from the four-dimensional viewpoint, to a monopole-antimonopole pair localized at the non-Abelian string [18]. Such a pair cannot exist in the bulk theory as well, so that similarly to the SUSY case, the string excitations suppress the decay rate. In the non-SUSY case one can also introduce the θ term in the bulk theory, which makes its way to the world-sheet theory of the non-Abelian string as well [18]. In the world-sheet theory the θ term induces a constant Abelian electric field of the auxiliary gauge field $A(x)$ along the string. In the string decay the electric field is completely screened in the emerging hole. One therefore concludes that the dyons have to be created at the ends of the string in this case.

Note that the explicit calculation of the decay probability would require detailed knowledge of the quantum boundary CP_N model.

V. AN ANALOGUE OF CME IN HIGHER DIMENSIONS

In this section we shall make some remarks concerning possible higher-dimensional generalization of the chiral magnetic effect. To this aim let us consider a 3-brane providing a four-dimensional world volume. One could think of a brane world setup where the 3-brane is embedded in higher-dimensional, e.g. D = 5, space-time with an additional two-form field. The problem we would like to comment on is the decay of the 3-brane world volume due to the creation of the hole surrounded by a domain wall.

Let us consider the specific situation where the 3-brane is embedded into the D = 5 space-time with the transverse coordinate z. The question we address here is that of the anomalous flow from the 3 + 1-brane world volume into the bulk during the decay process. In the two-form field background there is the current

$$J_\mu = \epsilon_{\mu\nu\lambda\rho\sigma}\partial_\nu\phi H_{\lambda\rho\sigma} \tag{32}$$

emerging from the anomalous one-loop term in the Lagrangian

$$\delta L = \int d^5x \phi F_{\mu\nu} * H_{\mu\nu}, \tag{33}$$

where ϕ is a pseudoscalar field similar to the axion and $H = dB$ is the curvature of the two-form field.

In a constant curvature H a homogeneous electric charge is generated in the 3d space volume with the density

$$\rho \propto \epsilon_{zijk}H_{ilk}, \tag{34}$$

where it is assumed that the 3-brane interpolates between different values of the field ϕ similar to an axion wall. One thus may wonder what happens to the charge during the decay. The corresponding current providing the flow from the brane reads as

$$J_z \propto \partial_0\phi\epsilon_{zijk}H_{ilk}. \tag{35}$$

For a 3 + 1-dimensional observer the charge is seemingly not conserved. However, having in mind a holographical interpretation of the fifth coordinate, one could interpret this flow as a renormalization-group one.

VI. SUMMARY

In this paper we have discussed the decay of defects in an external field. It was shown that in a magnetic field the axion domain wall evaporates all induced electric charges into the bulk. Such a decay of the axion wall provides an explicit example of the chiral magnetic effect, where the axion strings are responsible for the chiral chemical potential.

The decay of the mesonic walls in the magnetic field has some peculiarities. Namely, the decay probability of the pionic walls is suppressed by a magnetic field, and above a critical value of the field, the wall is nonperturbatively stable. In the CFL phase at high density the current along the boundary of the hole in the η-meson walls is generated, decreasing the initial magnetic field. An example of similar phenomena in a higher-dimensional case has been discussed. This can potentially be interesting for a holographic description of gauge theories.

Our consideration here was limited to the leading approximation in the electromagnetic coupling constant. It would be interesting to take into account the emerging currents on the field distribution around the hole in the next order in the electromagnetic interaction. One more potentially interesting effect is related to the possible evaporation of closed axion or mesonic strings from the decaying wall, rather than the evaporation of particles. This subleading effect can potentially be of relevance in an analysis of the stability of mesonic walls.

The processes discussed in this paper can be of interest for astrophysical applications. In particular, the unstable cosmic domain walls could be stabilized in the space regions with large magnetic fields.

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