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Cosmological intersecting brane solutions

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The recent discovery of an explicit dynamical description of p-branes makes it possible to investigate the existence of intersection of such objects. We generalize the solutions depending on the overall transverse space coordinates and time to those which depend also on the relative transverse space and satisfy new intersection rules. We give classification of these dynamical intersecting brane solutions involving two branes, and discuss the application of these solutions to cosmology and show that these give Friedmann-Lemaitre-Robertson-Walker cosmological solutions. Finally, we construct the brane world models, using the (cut-)copy-paste method after compactifying the trivial spatial dimensions. We then find that interesting brane world models can be obtained from codimension-one branes and several static branes with higher codimensions. We also classify the behaviors of the brane world near the future/past singularity.

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I. INTRODUCTION

The dynamics of the brane world model in five or six dimensions have been much explored (see [1–4] and references therein) because of the possible cosmological and phenomenological interests. Although some results have recently emerged on the applications of the solutions in higher-dimensional supergravity to the brane world cosmology, e.g., in [5–15], the construction of the brane world model in string theory is much less extensive. One motivation for the present work is to improve this situation. For this purpose, it is first necessary to construct dynamical brane solutions depending on the time as well as space coordinates.

It has already been known [16,17] that dynamical brane solutions arise when the gravity is coupled not only to a single gauge field but also to several combinations of scalars and forms, as generalization of the static intersecting brane solutions in the supergravity [18–23]. Similar solutions which have only time dependence have been obtained in [24] and other related solutions in [25–32]. Here we construct dynamical brane solutions by generalizing these static solutions to a dynamical one. The first class of dynamical solutions we study in this paper has the dependence on the time as well as overall transverse space coordinates in the metric and obeys the well-known intersection rules. However, it has also been known for some time that some static intersecting brane solutions may not follow these intersection rules [33]. These intersecting brane solutions are derived for the case when the branes depend on the relative transverse directions of the intersecting branes.

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Our goal in the present paper is to exhaust and classify all two-intersecting-brane solutions which depend on the time and (relative) transverse dimensions and to study their applications to the cosmological evolutions and the brane world models, in particular in the ten-dimensional string theory and eleven-dimensional supergravity theory. We first find cosmological solutions for possible intersections including the above exceptional cases for two intersecting branes by extending the similar solutions obeying the usual intersection rules [16,17]. Our results on the dynamical branes are given for general cases of arbitrary dimensions and forms, but in their applications to cosmology and brane world models, we mainly focus on the dynamical branes in ten- and eleven-dimensional supergravities because these are the most important low-energy effective theories of superstrings. We show that they exhibit physical phenomena of general interest, including the evolution of the four-dimensional universe in the brane world cosmology and dynamics of the internal space via compactification.

The paper is organized as follows. In Sec. II, we show that the dynamical intersecting brane solutions of two p-branes exist as an almost immediate generalization of the static p-brane solution. We then go on in Sec. III to apply these solutions to cosmology. In Sec. IV, we discuss construction of brane world models from these solutions and identify physically relevant solutions among these. Section V is devoted to discussions.

II. INTERSECTING BRANE SOLUTIONS IN D-DIMENSIONAL THEORY

In this section, we consider a D-dimensional theory composed of the metric g_{MN}, dilaton ϕ, and the antisymmetric tensor fields of rank $(p_r + 2)$ and $(p_s + 2)$:

$$S = \frac{1}{2\kappa^2}\int\Big[R*\mathbf{1} - \frac{1}{2}d\phi\wedge *d\phi - \frac{1}{2}\frac{1}{(p_r+2)!}\mathrm{e}^{\epsilon_r c_r\phi}F_{(p_r+2)}\wedge *F_{(p_r+2)} - \frac{1}{2}\frac{1}{(p_s+2)!}\mathrm{e}^{\epsilon_s c_s\phi}F_{(p_s+2)}\wedge *F_{(p_s+2)}\Big], \qquad (1)$$

where κ^2 is the D-dimensional gravitational constant, $*$ is the Hodge operator in the D-dimensional spacetime, $F_{(n)}$ is an n-form field strength, and c_I, ϵ_I ($I = r, s$) are constants given by

$$c_I^2 = 4 - \frac{2(p_I+1)(D-p_I-3)}{D-2}, \qquad (2a)$$

$$\epsilon_I = \begin{cases} + & \text{if } p_I\text{-brane is electric} \\ - & \text{if } p_I\text{-brane is magnetic.}\end{cases} \qquad (2b)$$

After variations with respect to the metric, the dilaton, and the forms, we obtain the field equations:

$$R_{MN} = \frac{1}{2}\partial_M\phi\partial_N\phi + \frac{1}{2}\frac{\mathrm{e}^{\epsilon_r c_r\phi}}{(p_r+2)!}\Big[(p_r+2)F_{MA_2\cdots A_{(p_r+2)}}F_N{}^{A_2\cdots A_{(p_r+2)}} - \frac{p_r+1}{D-2}g_{MN}F^2_{(p_r+2)}\Big] + \frac{1}{2}\frac{\mathrm{e}^{\epsilon_s c_s\phi}}{(p_s+2)!}\Big[(p_s+2)F_{MA_2\cdots A_{(p_s+2)}}F_N{}^{A_2\cdots A_{(p_s+2)}} - \frac{p_s+1}{D-2}g_{MN}F^2_{(p_s+2)}\Big], \qquad (3a)$$

$$d*d\phi - \frac{1}{2}\frac{\epsilon_r c_r}{(p_r+2)!}\mathrm{e}^{\epsilon_r c_r\phi}F_{(p_r+2)}\wedge *F_{(p_r+2)} - \frac{1}{2}\frac{\epsilon_s c_s}{(p_s+2)!}\mathrm{e}^{\epsilon_s c_s\phi}F_{(p_s+2)}\wedge *F_{(p_s+2)} = 0, \qquad (3b)$$

$$d[\mathrm{e}^{\epsilon_r c_r\phi}*F_{(p_r+2)}] = 0, \qquad (3c)$$

$$d[\mathrm{e}^{\epsilon_s c_s\phi}*F_{(p_s+2)}] = 0. \qquad (3d)$$

To solve these field equations, we assume that the D-dimensional metric takes the form

$$ds^2 = h_r^\alpha h_s^\beta[h_r^{-1}h_s^{-1}q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h_s^{-1}\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_r^{-1}w_{mn}(\mathrm{Y}_2)dv^m dv^n + u_{ab}(\mathrm{Z})dz^a dz^b], \qquad (4)$$

where $q_{\mu\nu}$, γ_{ij}, w_{mn}, and u_{ab} are the metrics depending only on x^μ, y^i, v^m, and z^a coordinates of dimensions $(p+1)$, (p_s-p), (p_r-p), and $(D+p-p_r-p_s-1)$, respectively. The parameters α and β in the metric (4) are given as

$$\alpha = \frac{p_r+1}{D-2}, \qquad \beta = \frac{p_s+1}{D-2}. \qquad (5)$$

Here we suppose that $p_s(p_r)$-brane extends along X and Y_1 (Y_2) spaces.

The D-dimensional metric (4) implies that the p-brane solutions are characterized by a function which depends on the coordinates transverse to the brane as well as the worldvolume coordinate. For the configurations of two branes, we should sort the coordinates in three sets and the powers of harmonic functions are different for each set of coordinates according to the intersection rules. One set of the coordinates is the overall worldvolume coordinates, which are common to the two branes. The others are overall transverse coordinates and the last are the relative transverse coordinates, which are transverse to only one of the two branes.

The field equations of intersecting branes allow for the following three kinds of possibilities on p_r- and p_s-branes in D dimensions [34,35]:

(I) Both h_r and h_s depend on the overall transverse coordinates: $h_r = h_r(x,z)$, $h_s = h_s(x,z)$.

(II) Only h_s depends on the overall transverse coordinates, but the other h_r does on the corresponding relative coordinates: $h_r = h_r(x,y)$, $h_s = h_s(x,z)$.

(III) Each of h_r and h_s depends on the corresponding relative coordinates: $h_r = h_r(x,y)$, $h_s = h_s(x,v)$.

In the following, we consider intersections where each participating brane corresponds to an independent harmonic function in the solution and derive the dynamical intersecting brane solution in D dimensions satisfying the above conditions.

A. Case (I)

For completeness, let us first consider case (I) though this has been already discussed in [17]. For this class, the D-dimensional metric (4) becomes

$$ds^2 = h_r^\alpha(x,z)h_s^\beta(x,z)[h_r^{-1}(x,z)h_s^{-1}(x,z)q_{\mu\nu}(\mathrm{X})dx^\mu dx^\nu + h_s^{-1}(x,z)\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_r^{-1}(x,z)w_{mn}(\mathrm{Y}_2)dv^m dv^n + u_{ab}(\mathrm{Z})dz^a dz^b]. \qquad (6)$$

We also assume that the scalar field ϕ and the gauge field strengths are given as

$$e^\phi = h_r^{\epsilon_r c_r/2}h_s^{\epsilon_s c_s/2}, \qquad (7a)$$

$$F_{(p_r+2)} = d[h_r^{-1}(x,z)]\wedge\Omega(\mathrm{X})\wedge\Omega(\mathrm{Y}_2), \qquad (7b)$$

$$F_{(p_s+2)} = d[h_s^{-1}(x,z)]\wedge\Omega(\mathrm{X})\wedge\Omega(\mathrm{Y}_1), \qquad (7c)$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ denote the volume forms of dimensions $(p+1)$, (p_s-p), and (p_r-p), respectively:

$$\Omega(\mathrm{X}) = \sqrt{-q}dx^0 \wedge dx^1 \wedge \cdots \wedge dx^p, \tag{8a}$$
$$\Omega(\mathrm{Y}_1) = \sqrt{\gamma}dy^1 \wedge dy^2 \wedge \cdots \wedge dy^{p_s - p}, \tag{8b}$$
$$\Omega(\mathrm{Y}_2) = \sqrt{w}dv^1 \wedge dv^2 \wedge \cdots \wedge dv^{p_r - p}. \tag{8c}$$

Here, q, γ, and w are the determinant of the metric $q_{\mu\nu}$, γ_{ij}, and w_{mn}, respectively.

Let us now consider gauge field equations (3c) and (3d). Under the assumptions (7b) and (7c), we find

$$d[\mathrm{e}^{\epsilon_r c_r \phi} * F_{(p_r+2)}] = -d[h_s^\chi \partial_a h_r (*_{\mathrm{Z}} dz^a) \wedge \Omega(\mathrm{Y}_1)] = 0, \tag{9a}$$
$$d[\mathrm{e}^{\epsilon_s c_s \phi} * F_{(p_s+2)}] = -d[h_r^\chi \partial_a h_s (*_{\mathrm{Z}} dz^a) \wedge \Omega(\mathrm{Y}_2)] = 0, \tag{9b}$$

where $*_{\mathrm{Z}}$ denotes the Hodge operator on Z, and χ is defined by

$$\chi = p + 1 - \frac{(p_r + 1)(p_s + 1)}{D - 2} + \frac{1}{2}\epsilon_r \epsilon_s c_r c_s. \tag{10}$$

The vanishing condition of χ is the intersection rule [17,20,21]. Then, Eq. (9a) leads to

$$h_s^\chi \triangle_{\mathrm{Z}} h_r = 0, \qquad \partial_\mu h_s^\chi \partial_a h_r + h_s^\chi \partial_\mu \partial_a h_r = 0, \tag{11}$$

where $\triangle_{\mathrm{Z}}$ is the Laplace operators on the space of Z. On the other hand, it follows from (9b) that

$$h_r^\chi \triangle_{\mathrm{Z}} h_s = 0, \qquad \partial_\mu h_r^\chi \partial_a h_s + h_r^\chi \partial_\mu \partial_a h_s = 0. \tag{12}$$

When the intersection rule $\chi = 0$ is obeyed, Eq. (11) gives

$$\triangle_{\mathrm{Z}} h_r = 0, \qquad \partial_\mu \partial_a h_r = 0, \tag{13}$$

and Eq. (12) reduces to

$$\triangle_{\mathrm{Z}} h_s = 0, \qquad \partial_\mu \partial_a h_s = 0. \tag{14}$$

We note that, in this case, the functions h_r and h_s can be written by linear combinations of terms depending on both x^μ and z^a. We are now going to see that the Einstein equations also hold if we use this result and the intersection rule $\chi = 0$.

Next we consider the Einstein equations (3a). With the assumptions (6) and (7), they reduce to

$$\begin{aligned} &R_{\mu\nu}(\mathrm{X}) - h_r^{-1} D_\mu D_\nu h_r - h_s^{-1} D_\mu D_\nu h_s - \tfrac{1}{2}(h_r h_s)^{-1}(\partial_\mu h_r \partial_\nu h_s + \partial_\mu h_s \partial_\nu h_r) \\ &\quad - \tfrac{1}{2}(\alpha + \beta - 2) q_{\mu\nu} q^{\rho\sigma} \partial_\sigma \ln h_r \partial_\sigma \ln h_s - \tfrac{1}{2} q_{\mu\nu} h_r^{-1}(\alpha - 1)[\triangle_{\mathrm{X}} h_r + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_r] \\ &\quad - \tfrac{1}{2} q_{\mu\nu} h_s^{-1}(\beta - 1)[\triangle_{\mathrm{X}} h_s + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_s] = 0, \end{aligned} \tag{15a}$$
$$h_r^{-1} \partial_\mu \partial_a h_r = 0, \tag{15b}$$
$$h_s^{-1} \partial_\mu \partial_a h_s = 0, \tag{15c}$$
$$\begin{aligned} &R_{ij}(\mathrm{Y}_1) - \tfrac{1}{2}(\alpha + \beta - 1)\gamma_{ij} h_r q^{\rho\sigma} \partial_\rho \ln h_r \partial_\sigma \ln h_s - \tfrac{1}{2}\gamma_{ij}\alpha[\triangle_{\mathrm{X}} h_r + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_r] \\ &\quad - \tfrac{1}{2}\gamma_{ij}(\beta - 1) h_r h_s^{-1}[\triangle_{\mathrm{X}} h_s + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_s] = 0, \end{aligned} \tag{15d}$$
$$\begin{aligned} &R_{mn}(\mathrm{Y}_2) - \tfrac{1}{2}(\alpha + \beta - 1) w_{mn} h_s q^{\rho\sigma} \partial_\rho \ln h_r \partial_\sigma \ln h_s - \tfrac{1}{2}\beta w_{mn}[\triangle_{\mathrm{X}} h_s + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_s] \\ &\quad - \tfrac{1}{2}(\alpha - 1) w_{mn} h_r^{-1} h_s[\triangle_{\mathrm{X}} h_r + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_r] = 0, \end{aligned} \tag{15e}$$
$$\begin{aligned} &R_{ab}(\mathrm{Z}) - \tfrac{1}{2}(\alpha + \beta) u_{ab} h_r h_s q^{\rho\sigma} \partial_\rho \ln h_r \partial_\sigma \ln h_s - \tfrac{1}{2}\alpha u_{ab} h_s[\triangle_{\mathrm{X}} h_r + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_r] \\ &\quad - \tfrac{1}{2}\beta u_{ab} h_r[\triangle_{\mathrm{X}} h_s + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_s] = 0, \end{aligned} \tag{15f}$$

where we have used the intersection rule $\chi = 0$, and D_μ is the covariant derivative with respect to the metric $q_{\mu\nu}$, $\triangle_{\mathrm{X}}$ is the Laplace operators on the space of X, and $R_{\mu\nu}(\mathrm{X})$, $R_{ij}(\mathrm{Y}_1)$, $R_{mn}(\mathrm{Y}_2)$, and $R_{ab}(\mathrm{Z})$ are the Ricci tensors of the metrics $q_{\mu\nu}(\mathrm{X})$, $\gamma_{ij}(\mathrm{Y}_1)$, $w_{mn}(\mathrm{Y}_2)$, and $u_{ab}(\mathrm{Z})$, respectively.

We see from Eqs. (15b) and (15c) that the warp factors h_r and h_s must be of the form

$$h_r(x, z) = h_0(x) + h_1(z), \qquad h_s(x, z) = k_0(x) + k_1(z). \tag{16}$$

With this form of h_r and h_s, the other components of the Einstein equations (15) are rewritten as

$$R_{\mu\nu}(\mathrm{X}) - h_r^{-1} D_\mu D_\nu h_0 - h_s^{-1} D_\mu D_\nu k_0 - \frac{1}{2}(h_r h_s)^{-1}(\partial_\mu h_0 \partial_\nu k_0 + \partial_\mu k_0 \partial_\nu h_0) - \frac{1}{2}(\alpha + \beta - 2)(h_r h_s)^{-1} q_{\mu\nu} q^{\rho\sigma} \partial_\sigma h_0 \partial_\sigma k_0 - \frac{1}{2} q_{\mu\nu} h_r^{-1}(\alpha - 1)[\triangle_{\mathrm{X}} h_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_1] - \frac{1}{2} q_{\mu\nu} h_s^{-1}(\beta - 1)[\triangle_{\mathrm{X}} k_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} k_1] = 0, \tag{17a}$$

$$R_{ij}(\mathrm{Y}_1) - \frac{1}{2}(\alpha + \beta - 1)\gamma_{ij} h_s^{-1} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma k_0 - \frac{1}{2}\gamma_{ij}\alpha[\triangle_{\mathrm{X}} h_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_1] - \frac{1}{2}\gamma_{ij}(\beta - 1) h_r h_s^{-1}[\triangle_{\mathrm{X}} k_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} k_1] = 0, \tag{17b}$$

$$R_{mn}(\mathrm{Y}_2) - \frac{1}{2}(\alpha + \beta - 1) w_{mn} h_r^{-1} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma k_0 - \frac{1}{2}\beta w_{mn}[\triangle_{\mathrm{X}} k_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} k_1] - \frac{1}{2}(\alpha - 1) w_{mn} h_r^{-1} h_s[\triangle_{\mathrm{X}} h_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_1] = 0, \tag{17c}$$

$$R_{ab}(\mathrm{Z}) - \frac{1}{2}(\alpha + \beta) u_{ab} q^{\rho\sigma} \partial_\rho h_0 \partial_\sigma k_0 - \frac{1}{2}\alpha u_{ab} h_s[\triangle_{\mathrm{X}} h_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} h_1] - \frac{1}{2}\beta u_{ab} h_r[\triangle_{\mathrm{X}} k_0 + (h_r h_s)^{-1} \triangle_{\mathrm{Z}} k_1] = 0. \tag{17d}$$

Finally, we should consider the scalar field equation. Substituting Eqs. (7) and (16) and the intersection rule $\chi = 0$ into Eq. (3b), we obtain

$$h_r^{-\alpha} h_s^{-\beta}[\epsilon_r c_r\{h_s \triangle_{\mathrm{X}} h_0 + q^{\rho\sigma}\partial_\rho h_0 \partial_\sigma k_0 + h_r^{-1} \triangle_{\mathrm{Z}} h_1\} + \epsilon_s c_s\{h_r \triangle_{\mathrm{X}} k_0 + q^{\rho\sigma}\partial_\rho h_0 \partial_\sigma k_0 + h_s^{-1} \triangle_{\mathrm{Z}} k_1\}] = 0. \tag{18}$$

Thus, the warp factors h_r and h_s should satisfy the equations

$$\triangle_{\mathrm{X}} h_0 = 0, \quad \triangle_{\mathrm{Z}} h_1 = 0, \quad \triangle_{\mathrm{Z}} h_s = 0, \quad \text{for } \partial_\mu k_0 = 0, \tag{19a}$$

$$\triangle_{\mathrm{X}} k_0 = 0, \quad \triangle_{\mathrm{Z}} k_1 = 0, \quad \triangle_{\mathrm{Z}} h_r = 0, \quad \text{for } \partial_\mu h_0 = 0. \tag{19b}$$

Combining these, we find that these field equations lead to [17]

$$R_{\mu\nu}(\mathrm{X}) = 0, \quad R_{ij}(\mathrm{Y}_1) = 0, \quad R_{mn}(\mathrm{Y}_2) = 0, \quad R_{ab}(\mathrm{Z}) = 0, \tag{20a}$$

$$h_r = h_0(x) + h_1(z), \quad h_s = k_0(x) + k_1(z), \tag{20b}$$

$$D_\mu D_\nu h_0 = 0, \quad \triangle_{\mathrm{Z}} h_1 = 0, \quad \triangle_{\mathrm{Z}} h_s = 0, \quad \text{for } \partial_\mu h_s = 0, \tag{20c}$$

$$D_\mu D_\nu k_0 = 0, \quad \triangle_{\mathrm{Z}} k_1 = 0, \quad \triangle_{\mathrm{Z}} h_r = 0, \quad \text{for } \partial_\mu h_r = 0. \tag{20d}$$

If $F_{(p_r+2)} = 0$ and $F_{(p_s+2)} = 0$, the functions h_1 and k_1 become trivial. Namely, the D-dimensional spacetime is no longer warped [36,37].

As a special example, let us consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \quad \gamma_{ij} = \delta_{ij}, \quad w_{mn} = \delta_{mn}, \quad u_{ab} = \delta_{ab}, \tag{21}$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$-, and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x, z) = A_\mu x^\mu + B + \sum_l \frac{M_l}{|z - z_l|^{D+p-p_r-p_s-3}}, \tag{22a}$$

$$h_s(z) = C + \sum_c \frac{M_c}{|z - z_c|^{D+p-p_r-p_s-3}}, \tag{22b}$$

where A_μ, B, C, M_l, and M_c are constant parameters, and z_l and z_c are constant vectors representing the positions of the branes. We can also choose the solution in which the p_s-brane part depends on both t and z. Then, we have

$$h_r(z) = B + \sum_l \frac{M_l}{|z - z_l|^{D+p-p_r-p_s-3}}, \tag{23a}$$

$$h_s(x, z) = A_\mu x^\mu + C + \sum_c \frac{M_c}{|z - z_c|^{D+p-p_r-p_s-3}}. \tag{23b}$$

Now we discuss the intersecting brane solutions in eleven-dimensional supergravity and in ten-dimensional string theories. For the M-branes in eleven-dimensional supergravity, there is 4-form field strength without dilaton, so the intersection rule $\chi = 0$ gives

$$p = \frac{(p_r + 1)(p_s + 1)}{9} - 1, \tag{24}$$

where p denotes the number of overlapping dimensions of the p_r and p_s branes. Then we get the intersections involving the M2 and M5-branes [20,21]

$$\mathrm{M2} \cap \mathrm{M2} = 0, \quad \mathrm{M2} \cap \mathrm{M5} = 1, \quad \mathrm{M5} \cap \mathrm{M5} = 3. \tag{25}$$

For the ten-dimensional string theories, the couplings to dilaton for the RR-charged D-branes are given by

$$\epsilon_r c_r = \frac{1}{2}(3 - p_r), \quad \epsilon_s c_s = \frac{1}{2}(3 - p_s). \tag{26}$$

The condition $\chi = 0$ then gives

$$p = \frac{1}{2}(p_r + p_s - 4). \tag{27}$$

The intersections for the D-branes are thus given by [20,21]

$$\mathrm{D}\,p_r \cap \mathrm{D}p_s = \frac{1}{2}(p_r + p_s) - 2. \tag{28}$$

We finally consider the intersections for NS-branes. The parameters c_r for fundamental string (F1) and solitonic 5-brane are $\epsilon_1 c_1 = -1$ (for F1) and $\epsilon_5 c_5 = 1$ (for NS5), respectively. Then the intersections involving the F1 and NS5-branes are [20,21]

$$\mathrm{F1} \cap \mathrm{NS5} = 1, \qquad \mathrm{NS5} \cap \mathrm{NS5} = 3, \tag{29a}$$

$$\mathrm{F1} \cap \mathrm{D}\bar{p} = 0, \tag{29b}$$

$$\mathrm{D}\bar{p} \cap \mathrm{NS5} = \bar{p} - 1, \qquad 1 \leq \bar{p} \leq 6. \tag{29c}$$

There is no solution for the F1-F1 and D0-NS5 intersecting brane systems because the numbers of space dimensions for each pairwise overlap are negative by the intersection rule.

B. Case (II)

We next consider the case (II). For this class, the D-dimensional metric ansatz (4) gives

$$\begin{aligned} ds^2 = {} & h_r^{\alpha}(x, y) h_s^{\beta}(x, z)[h_r^{-1}(x, y) h_s^{-1}(x, z) q_{\mu\nu}(\mathrm{X}) dx^{\mu} dx^{\nu} \\ & + h_s^{-1}(x, z)\gamma_{ij}(\mathrm{Y}_1) dy^i dy^j \\ & + h_r^{-1}(x, y) w_{mn}(\mathrm{Y}_2) d\boldsymbol{v}^m d\boldsymbol{v}^n + u_{ab}(\mathrm{Z}) dz^a dz^b]. \end{aligned} \tag{30}$$

We also take the following ansatz for the scalar field ϕ and the gauge field strengths:

$$e^{\phi} = h_r^{\epsilon_r c_r/2} h_s^{\epsilon_s c_s/2}, \tag{31a}$$

$$F_{(p_r+2)} = d[h_r^{-1}(x, y)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{31b}$$

$$F_{(p_s+2)} = d[h_s^{-1}(x, z)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \tag{31c}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ are defined in (8). Since we use the same procedure as in Sec. II A, we can derive the intersection rule $\chi = 0$ from the field equations. For $\chi = 0$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{mn}(\mathrm{Y}_2) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \tag{32a}$$

$$h_r = h_0(x) + h_1(y), \qquad h_s = h_s(z), \tag{32b}$$

$$D_{\mu} D_{\nu} h_0 = 0, \qquad \triangle_{\mathrm{Y}_1} h_1 = 0, \qquad \triangle_{\mathrm{Z}} h_s = 0, \qquad \partial_{\mu} h_s = 0, \tag{32c}$$

where $\triangle_{\mathrm{Y}_1}$ is the Laplace operators on the space of Y_1. If $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$, the functions h_1 and k_1 are nontrivial.

Let us consider the following case in more detail:

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \tag{33}$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$-, and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_{\mu} h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x, y) = A_{\mu} x^{\mu} + B + \sum_l \frac{M_l}{|\boldsymbol{y} - \boldsymbol{y}_l|^{p_s - p - 2}}, \tag{34a}$$

$$h_s(z) = C + \sum_c \frac{M_c}{|\boldsymbol{z} - \boldsymbol{z}_c|^{D + p - p_r - p_s - 3}}, \tag{34b}$$

where A_{μ}, B, C, $\boldsymbol{y}_l$, $\boldsymbol{z}_c$, M_l, and M_c are constant parameters.

On can easily get the solution for $\partial_{\mu} h_r = 0$ and $\partial_{\mu} h_s \neq 0$ if the roles of Y_1 and Y_2 are exchanged. The solution of field equations is thus expressed as

$$h_r(z) = B + \sum_l \frac{M_l}{|\boldsymbol{z} - \boldsymbol{z}_l|^{D + p - p_s - p_r - 3}}, \tag{35a}$$

$$h_s(x, \boldsymbol{v}) = A_{\mu} x^{\mu} + C + \sum_c \frac{M_c}{|\boldsymbol{v} - \boldsymbol{v}_c|^{p_r - p - 2}}. \tag{35b}$$

Since the dynamical solution (34) obeys the same intersection rule $\chi = 0$, the intersections of M-branes in eleven-dimensional supergravity and D-branes in ten-dimensional string theories are given as (25), (28), and (29).

C. Case (III)

Finally, we consider the case (III). For this class, the D-dimensional metric ansatz (4) reduces to

$$\begin{aligned} ds^2 = {} & h_r^{\alpha}(x, y) h_s^{\beta}(x, \boldsymbol{v})[h_r^{-1}(x, y) h_s^{-1}(x, \boldsymbol{v}) q_{\mu\nu}(\mathrm{X}) dx^{\mu} dx^{\nu} \\ & + h_s^{-1}(x, \boldsymbol{v})\gamma_{ij}(\mathrm{Y}_1) dy^i dy^j \\ & + h_r^{-1}(x, y) w_{mn}(\mathrm{Y}_2) d\boldsymbol{v}^m d\boldsymbol{v}^n + u_{ab}(\mathrm{Z}) dz^a dz^b]. \end{aligned} \tag{36}$$

We also assume that the scalar field ϕ and the gauge field strengths are given as

$$e^{\phi} = h_r^{\epsilon_r c_r/2} h_s^{\epsilon_s c_s/2}, \tag{37a}$$

$$F_{(p_r+2)} = h_s d[h_r^{-1}(x, y)] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_2), \tag{37b}$$

$$F_{(p_s+2)} = h_r d[h_s^{-1}(x, \boldsymbol{v})] \wedge \Omega(\mathrm{X}) \wedge \Omega(\mathrm{Y}_1), \tag{37c}$$

where $\Omega(\mathrm{X})$, $\Omega(\mathrm{Y}_1)$, and $\Omega(\mathrm{Y}_2)$ denote the volume $(p+1)$-, $(p_s - p)$-, and $(p_r - p)$-forms, respectively.

Under the assumption, the field equations give the intersection rule $\chi = -2$. This is different from the usual rule applicable to the cases (I) and (II). Upon using the intersection rule $\chi = -2$, it is easy to show that the field equations reduce to

$$R_{\mu\nu}(\mathrm{X}) = 0, \qquad R_{ij}(\mathrm{Y}_1) = 0, \qquad R_{mn}(\mathrm{Y}_2) = 0, \qquad R_{ab}(\mathrm{Z}) = 0, \tag{38a}$$

$$h_r = h_0(x) + h_1(y), \qquad h_s = k_0(x) + k_1(v), \tag{38b}$$

$$D_\mu D_\nu h_0 = 0, \qquad \triangle_{\mathrm{Y}_1} h_1 = 0, \qquad \triangle_{\mathrm{Y}_2} h_s = 0, \quad \text{for } \partial_\mu h_s = 0, \tag{38c}$$

$$D_\mu D_\nu k_0 = 0, \qquad \triangle_{\mathrm{Y}_1} h_r = 0, \qquad \triangle_{\mathrm{Y}_2} k_1 = 0, \quad \text{for } \partial_\mu h_r = 0. \tag{38d}$$

where $\triangle_{\mathrm{Y}_1}$ and $\triangle_{\mathrm{Y}_2}$ are the Laplace operators on the spaces of Y_1 and Y_2, respectively. The functions h_1 and k_1 are nontrivial for $F_{(p_r+2)} \neq 0$ and $F_{(p_s+2)} \neq 0$.

Now we consider the case

$$q_{\mu\nu} = \eta_{\mu\nu}, \qquad \gamma_{ij} = \delta_{ij}, \qquad w_{mn} = \delta_{mn}, \qquad u_{ab} = \delta_{ab}, \tag{39}$$

where $\eta_{\mu\nu}$ is the $(p+1)$-dimensional Minkowski metric and δ_{ij}, δ_{mn}, δ_{ab} are the $(p_s - p)$-, $(p_r - p)$-, and $(D + p - p_r - p_s - 1)$-dimensional Euclidean metrics, respectively. For $\partial_\mu h_s = 0$, the solution for h_r and h_s can be obtained explicitly as

$$h_r(x, y) = A_\mu x^\mu + B + \sum_l \frac{M_l}{|\boldsymbol{y} - \boldsymbol{y}_l|^{p_s - p - 2}}, \tag{40a}$$

$$h_s(v) = C + \sum_c \frac{M_c}{|\boldsymbol{v} - \boldsymbol{v}_c|^{p_r - p - 2}}, \tag{40b}$$

where A_μ, B, C, $\boldsymbol{y}_l$, $\boldsymbol{v}_c$, M_l, and M_c are constant parameters. We can also get the solution in which the function h_s depends on both t and v. The solution (40) is replaced by

$$h_r(y) = B + \sum_l \frac{M_l}{|\boldsymbol{y} - \boldsymbol{y}_l|^{p_s - p - 2}}, \tag{41a}$$

$$h_s(x, v) = A_\mu x^\mu + C + \sum_c \frac{M_c}{|\boldsymbol{v} - \boldsymbol{v}_c|^{p_r - p - 2}}. \tag{41b}$$

Let us consider the intersecting brane solutions in eleven-dimensional supergravity and in ten-dimensional string theories. We first discuss the intersections of M-branes in eleven-dimensional supergravity. The intersection rule $\chi = -2$ leads to

$$p = \frac{(p_r + 1)(p_s + 1)}{9} - 3. \tag{42}$$

Then we get the intersection involving the M5-brane

$$\mathrm{M5} \cap \mathrm{M5} = 1. \tag{43}$$

Equation (42) tells us that the numbers of intersection for M2-M2 and M2-M5 branes are negative, which means that there is no intersecting solution of these brane systems.

Next we consider the intersection in the ten-dimensional string theory. The couplings to dilaton for the RR-charged D-branes are

$$\epsilon_r c_r = \tfrac{1}{2}(3 - p_r), \qquad \epsilon_s c_s = \tfrac{1}{2}(3 - p_s), \tag{44}$$

and the condition $\chi = -2$ is expressed as

$$p = \tfrac{1}{2}(p_r + p_s - 8). \tag{45}$$

The intersections for the RR-charged D-branes are thus given by

$$\mathrm{D}p_r \cap \mathrm{D}p_s = \tfrac{1}{2}(p_r + p_s) - 4. \tag{46}$$

We finally consider the intersections for NS-branes. The parameters c_r for fundamental string (F1) and solitonic 5-brane are $\epsilon_1 c_1 = -1$ for F1 and $\epsilon_5 c_5 = 1$ for NS5, respectively. Then the intersection with F1-brane is forbidden by the intersection rule. The intersections involving the NS5-branes are

$$\mathrm{NS5} \cap \mathrm{NS5} = 1, \tag{47a}$$

$$\mathrm{D}\bar{p} \cap \mathrm{NS5} = \bar{p} - 3, \qquad 3 \leq \bar{p} \leq 8. \tag{47b}$$

There is no brane solution involving other intersections because the numbers of space dimensions for each pairwise overlap become negative by the intersection rule.

III. COSMOLOGY

In this section, we discuss the application of the above solutions to four-dimensional cosmology. We assume an isotropic and homogeneous three-space in the four-dimensional spacetime known as Friedmann-Lemaitre-Robertson-Walker (FLRW) universe, and do not discuss solutions which break these properties after compactification. In what follows, we concentrate on the $(p+1)$-dimensional Minkowski spacetime with $q_{\mu\nu}(\mathrm{X}) = \eta_{\mu\nu}(\mathrm{X})$, and drop the coordinate dependence on X space except for the time.

The D-dimensional metric (4) can be expressed as

$$ds^2 = -h dt^2 + ds^2(\tilde{\mathrm{X}}) + ds^2(\mathrm{Y}_1) + ds^2(\mathrm{Y}_2) + ds^2(\mathrm{Z}), \tag{48}$$

where we have defined

$$ds^2(\tilde{\mathrm{X}}) \equiv h \delta_{PQ}(\tilde{\mathrm{X}}) d\theta^P d\theta^Q, \tag{49a}$$

$$ds^2(\mathrm{Y}_1) \equiv h_r^\alpha h_s^{\beta - 1} \gamma_{ij}(\mathrm{Y}_1) dy^i dy^j, \tag{49b}$$

$$ds^2(\mathrm{Y}_2) \equiv h_r^{\alpha - 1} h_s^\beta w_{mn}(\mathrm{Y}_2) dv^m dv^n, \tag{49c}$$

$$ds^2(\mathrm{Z}) \equiv h_r^\alpha h_s^\beta u_{ab}(\mathrm{Z}) dz^a dz^b, \tag{49d}$$

$$h \equiv h_r^{\alpha - 1} h_s^{\beta - 1}. \tag{49e}$$

Here $\delta_{PQ}(\tilde{\mathrm{X}})$ is the p-dimensional Euclidean metric, and θ^P denotes the coordinate of the p-dimensional Euclid space $\tilde{\mathrm{X}}$. In the following, we assume $\partial_\mu h_s = 0$ and set $h_r = At + h_1$. The D-dimensional metric (49) can be written as

$$ds^2 = h_s^{\beta-1}\left[1+\left(\frac{\tau}{\tau_0}\right)^{-2/(\alpha+1)}h_1\right]^{\alpha-1}\left[-d\tau^2+\left(\frac{\tau}{\tau_0}\right)^{2(\alpha-1)/(\alpha+1)}\delta_{PQ}(\tilde{\mathrm{X}})d\theta^P d\theta^Q + \left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(\alpha+1)}h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2\alpha/(\alpha+1)}\gamma_{ij}(\mathrm{Y}_1)dy^i dy^j + h_s\left(\frac{\tau}{\tau_0}\right)^{2(\alpha-1)/(\alpha+1)}w_{mn}(\mathrm{Y}_2)dv^m dv^n + h_s\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(\alpha+1)}h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2\alpha/(\alpha+1)}u_{ab}(\mathrm{Z})dz^a dz^b\right], \quad (50)$$

where we have introduced the cosmic time τ defined by

$$\frac{\tau}{\tau_0} = (At)^{(\alpha+1)/2}, \qquad \tau_0 = \frac{2}{(\alpha+1)A}. \quad (51)$$

On the other hand, for $h_s(t, v) = At + k_1(v)$, the metric (49) is given by replacing α and h_1 with β and k_1.

Now we apply these solutions to lower-dimensional effective theory. We compactify d ($\equiv d_1 + d_2 + d_3 + d_4$) dimensions to fit our universe, where d_1, d_2, d_3, and d_4 denote the compactified dimensions with respect to the $\tilde{\mathrm{X}}$, Y_1, Y_2, and Z spaces. The metric (48) is then described by

$$ds^2 = ds^2(\mathrm{M}) + ds^2(\mathrm{N}), \quad (52)$$

where $ds^2(\mathrm{M})$ is the $(D-d)$-dimensional metric and $ds^2(\mathrm{N})$ is the metric of compactified dimensions.

By the conformal transformation

$$ds^2(\mathrm{M}) = h_r^B h_s^C ds^2(\bar{\mathrm{M}}), \quad (53)$$

we can rewrite the $(D-d)$-dimensional metric in the Einstein frame. Here B and C are

$$B = \frac{-\alpha d + d_1 + d_3}{D-d-2}, \qquad C = \frac{-\beta d + d_2 + d_4}{D-d-2}. \quad (54)$$

Hence, the $(D-d)$-dimensional metric in the Einstein frame is

$$ds^2(\bar{\mathrm{M}}) = h_r^{B'}h_s^{C'}[-d\tau^2 + \delta_{P'Q'}(\tilde{\mathrm{X}}')d\theta^{P'}d\theta^{Q'} + h_r\gamma_{k'l'}(\mathrm{Y}_1{}')dy^{k'}dy^{l'} + h_s w_{m'n'}(\mathrm{Y}_2{}')dv^{m'}dv^{n'} + h_r h_s u_{a'b'}(\mathrm{Z}')dz^{a'}dz^{b'}], \quad (55)$$

where B' and C' are defined by $B' = -B + \alpha - 1$ and $C' = -C + \beta - 1$, and $\tilde{\mathrm{X}}'$, $\mathrm{Y}_1{}'$, $\mathrm{Y}_2{}'$, and Z' denote the $(p - d_1)$-, $(p_s - p - d_2)$-, $(p_r - p - d_3)$-, and $(D + p - p_r - p_s - d_4)$-dimensional spaces, respectively.

For $h_r = At + h_1$, the metric (55) is thus rewritten as

$$ds^2(\bar{\mathrm{M}}) = h_s^{C'}\left[1+\left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)}h_1\right]^{B'}\left[-d\tau^2+\left(\frac{\tau}{\tau_0}\right)^{2B'/(B'+2)}\delta_{P'Q'}(\tilde{\mathrm{X}}')d\theta^{P'}d\theta^{Q'} + \left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)}h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2(B'+1)/(B'+2)}\gamma_{k'l'}(\mathrm{Y}_1{}')dy^{k'}dy^{l'} + h_s\left(\frac{\tau}{\tau_0}\right)^{2B'/(B'+2)}w_{m'n'}(\mathrm{Y}_2{}')dv^{m'}dv^{n'} + h_s\left\{1+\left(\frac{\tau}{\tau_0}\right)^{-2/(B'+2)}h_1\right\}\left(\frac{\tau}{\tau_0}\right)^{2(B'+1)/(B'+2)}u_{a'b'}(\mathrm{Z}')dz^{a'}dz^{b'}\right], \quad (56)$$

where the cosmic time τ is defined by

$$\frac{\tau}{\tau_0} = (At)^{(B'+2)/2}, \qquad \tau_0 = \frac{2}{(B'+2)A}. \quad (57)$$

For $h_s = At + k_1$ and $\partial_\mu h_r = 0$, we can also get results similar to (50) and (56).

We list the FLRW cosmological solutions with an isotropic and homogeneous three-space for the solutions (56) in Table I for M-branes, Tables II, III, IV, V, VI, VII, VIII, and IX for D-branes, and X, XI, XII, XIII, and XIV for F1 and NS5-branes. The power exponents of the scale factor of possible four-dimensional cosmological models are given by $a(\tilde{\mathrm{M}}) \propto \tau^{\lambda(\tilde{\mathrm{M}})}$, where τ is the cosmic time, and $a(\tilde{\mathrm{M}})$ and $a_{\mathrm{E}}(\tilde{\mathrm{M}})$ denote the scale factors of the space $\tilde{\mathrm{M}}$ in Jordan and Einstein frames with the exponents carrying the same suffices, respectively. Here $\tilde{\mathrm{M}}$ denotes the spatial part of spacetime M and includes our four-dimensional universe besides the time coordinate. The mark $\surd$ in the tables shows which brane is time dependent.

Since the time dependence in the metric comes from only one brane in the intersections, the obtained expansion law is simple. In order to find an expanding universe, one may have to compactify the vacuum bulk space as well as the brane worldvolume. Unfortunately, we find that the fastest expanding case in the Jordan frame has the power $a \propto \tau^{7/15}$, which is too small to give a realistic expansion law like that in the matter dominated era ($a \propto \tau^{2/3}$) or that in the radiation dominated era ($a \propto \tau^{1/2}$). Note that all these cases correspond to the solutions involving D6-branes, given in Tables IV, V, VI, VII, VIII, IX, XI, XII, and XIII.

When we compactify the extra dimensions and go to the four-dimensional Einstein frame, the power exponents are different depending on how we compactify the extra dimensions even within one solution. We give the power exponent of the fastest expansion of our four-dimensional universe in the Einstein frame in Tables XIV, XV, XVI, XVII, XVIII, XIX, XX, XXI, XXII, XXIII, XXIV, and XXV. We again see that the expansion is too small. Hence, we have to conclude that in order to find a realistic expansion of the universe in this type of models, one has to include additional "matter" fields on the brane.

IV. BRANE WORLD APPROACH

A. Construction of the brane world model

In this section, we discuss the applications of our solutions to construct the brane world models. Starting from a given ten- or eleven-dimensional solution, we compactify the trivial extra dimensions. After the reduction, we further move to the Einstein frame. Then, there are the ordinary four-dimensional spacetime and extra dimensions. The following procedure depends on the number of codimensions n.

For $n > 2$, the brane is so singular that one cannot put the ordinary matter and we employ the cut-copy-paste method as a way of regularization, which is explained later. As a result, the original brane with n codimensions is replaced with a codimension-one object including the internal angular dimensions. For $n = 2$, there is a curvature singularity at the infinity due to the logarithmic spatial dependence of the metric and we do not discuss this case any more. For $n = 1$, the spacetime is regular. In all cases, due to the presence of the time dependence, there can be the other singularity in the future or past, where $h_r = 0$ in brane solutions. For the moment, we focus on the prescription for the brane and will discuss the behavior of the brane world near the time dependent singularity in the next subsection. In both cases, the boundary of the bulk spacetime is a codimension-one hypersurface, which is called "brane world" in the rest. By construction, the extra space is Z_2-symmetric with respect to the brane world.

We consider the time dependent Einstein-frame metric with n conformally flat extra dimensions,

$$ds^2 = -d(t,\xi)^2 dt^2 + a(t,\xi)^2 \delta_{ij} d\mathcal{X}^i d\mathcal{X}^j + f(t,\xi)^2 (d\xi^2 + \xi^2 \mathcal{G}_{ab} d\theta^a d\theta^b), \tag{58}$$

where metric $\mathcal{G}_{ab}$ $[a = 1, 2, \ldots, (n-1)]$ represents the unit $(n-1)$ sphere. The coordinates (ξ, θ^a) and $\mathcal{X}^i$ denote the radial and angular directions of the extra space, and the ordinary three-space, respectively. We assume that in the original intersecting brane solution, the brane is at $\xi = 0$. For $n > 2$, we need a regularization to put the matter on the brane and we employ the cut-copy-paste method:

(a) The region including the brane $0 \leq \xi < \xi_0$ is removed.
(b) The remaining piece is glued to its identical copy at $\xi = \xi_0$.

In this way, the brane is replaced with a codimension-one brane world including the extra angular dimensions. For $n = 1$, we assume that the brane world is at the place where the original brane source exists.

The brane world moves along the trajectory $(t(\tau), \xi(\tau))$, where τ satisfies

$$-d(t(\tau), \xi(\tau))^2 \dot{t}^2 + f(t(\tau), \xi(\tau))^2 \dot{\xi}^2 = -1. \tag{59}$$

The "dot" denotes the derivative with respect to τ. Thus, the induced metric becomes cosmological

$$ds^2_{\rm ind} = -d\tau^2 + a(t(\tau), \xi(\tau))^2 \delta_{ij} d\mathcal{X}^i d\mathcal{X}^j + \xi(\tau)^2 f(t(\tau), \xi(\tau))^2 \mathcal{G}_{ab} d\theta^a d\theta^b. \tag{60}$$

In the case of $n > 2$, since $\mathcal{G}_{ab}$ denotes the $(n-1)$-dimensional sphere, the corresponding dimensions in the brane worldvolume are automatically compact as long as the scale factor in these directions ξf is finite. To construct the brane world by our cut-copy-paste method, further simplifications should be required: We first assume that each function h_r and h_s in Eqs. (22), (34), and (40) contains only a contribution from a single brane, with charge M_r and M_s, respectively. We then impose additional restrictions in each of the cases (I)–(III).

1. Case (I)

Both functions h_r and h_s depend on z:

$$h_r(z,t) = A_0 t + B + \frac{M_r}{|z - z_r|^{n-2}}, \quad h_s(z) = C + \frac{M_s}{|z - z_s|^{n-2}} \tag{61}$$

[see Eq. (22)]. The positions of branes, z_r and z_s, are different in general. Our cut-copy-paste procedure works when the extra space is spherically symmetric. To realize the spherical symmetry, we put the branes at the same place $z_r = z_s = 0$ and then set $\xi = |z|$. Alternatively, we may choose $z_s = 0$ for $M_r = 0$ and $z_r = 0$ for $M_s = 0$, with $\xi = |z|$, but we focus on the former general case with $M_s M_r \neq 0$ here. For $n = 1$, we may take $z_r \neq z_s$.

2. Case (II)

Here h_r and h_s depend on $\boldsymbol{y}$ and z. We write

$$h_r(\boldsymbol{y},t) = A_0 t + B + \frac{M_r}{|\boldsymbol{y} - \boldsymbol{y}_r|^{n_r-2}}, \quad h_s(z) = C + \frac{M_s}{|z - z_s|^{n_s-2}} \tag{62}$$

[see Eq. (34)]. We need to assume either $M_r = 0$ or $M_s = 0$, since in our construction the position of the brane world is specified by a single coordinate $|\boldsymbol{y}|$ or $|z|$. For $M_r = 0$, we set $\xi = |z|$ and $n_s = n$ with $z_s = 0$, while for $M_s = 0$ $\xi = |\boldsymbol{y}|$ and $n_r = n$ with $\boldsymbol{y}_r = 0$.

3. Case (III)

Here h_r and h_s depend on $\boldsymbol{y}$ and $\boldsymbol{v}$. We again write

$$h_r(\boldsymbol{y}, t) = A_0 t + B + \frac{M_r}{|\boldsymbol{y} - \boldsymbol{y}_r|^{n_r - 2}},\qquad h_s(\boldsymbol{v}) = C + \frac{M_s}{|\boldsymbol{v} - \boldsymbol{v}_s|^{n_s - 2}} \tag{63}$$

[see Eq. (40)]. Similarly we need to set either $M_r = 0$ or $M_s = 0$. For $M_r = 0$, we set $\xi = |\boldsymbol{v}|$ and $n_s = n$ with $\boldsymbol{v}_s = 0$, while for $M_s = 0$, $\xi = |\boldsymbol{y}|$ and $n_r = n$ with $\boldsymbol{y}_r = 0$.

Let us illustrate a D3-D1 solution of the case (I), where the D3 brane is time dependent. Under our assumptions, after compactifying the trivial Y_1 directions, the Einstein-frame metric is given by

$$ds^2 = h_r^{-3/7}(z, t) h_s^{-6/7}(z)(-dt^2 + h_s(z)\delta_{mn} dv^m dv^n + h_r(z, t) h_s(z)\delta_{ab} dz^a dz^b), \tag{64}$$

where v^m and z^a are the coordinates of three- and five-dimensional Euclidean spaces. Now, we identify $\xi = |z|$, $\{\theta\}$ = angular part of $\{z\}$, and $\{\mathcal{X}\} = \{v\}$. Then, functions in (58) read $a = h_r^{-3/14} h_s^{1/14}$, $d = h_r^{-3/14} h_s^{-3/7}$, and $f = h_r^{2/7} h_s^{1/14}$, respectively.

B. Properties of brane world near the singularity

We discuss the properties of singularity at $h_r = 0$, which arises because of the time dependence. We assume that $B > 0$ and $M_r \geq 0$. We also assume that $C > 0$ and $M_s \geq 0$ in h_s, so that no singularity other than the brane at $\xi = 0$ appears from h_s. In the case $n > 2$, the dynamics of the spacetime is changed at the critical time $t = -\frac{B}{A_0}$. For $M_r > 0$ and $A_0 > 0$, in the infinite past $t \to -\infty$ the regular spatial region is small. The spatial region gradually expands and spreads to the infinity at $t = -B/A_0$. Subsequently, the spacetime is regular except at $\xi = 0$. For $A_0 < 0$, initially spacetime is regular except at the brane. But at $t = \frac{B}{|A_0|}$, a singularity appears at the spatial infinity, and then the spatial domain shrinks as the time evolves. See Fig. 1. In the case of $n = 1$, for $M_r > 0$ and $A_0 > 0$, the spacetime is defined for $\xi > -\frac{1}{M_r}(A_0 t + B)$, while for $A_0 < 0$ it can be done for $\xi < \frac{1}{M_r}(|A_0| t - B)$. For $M_r = 0$, a spacelike singularity appears at $t = -\frac{B}{A_0}$. For $A_0 > 0$, spacetime can be defined for $t > -\frac{B}{A_0}$ while for $A_0 < 0$ it can be done for $t < \frac{B}{|A_0|}$.

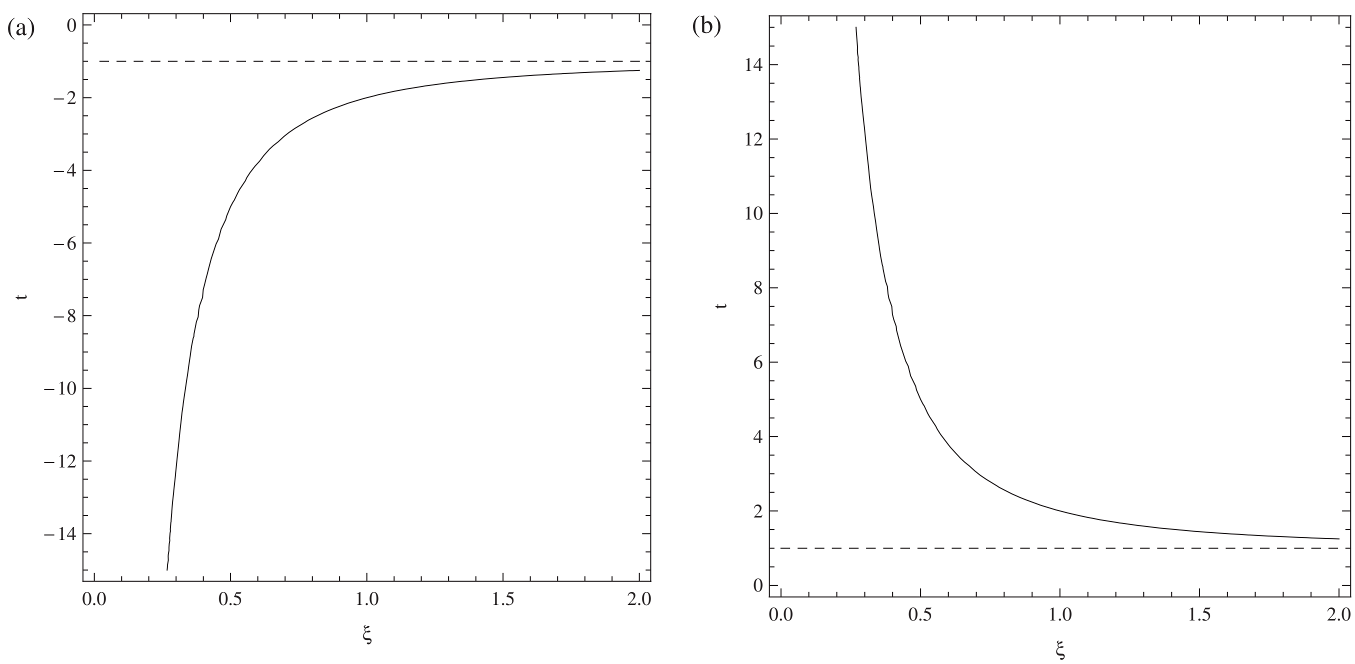

FIG. 1. Curves in the figures show the level of $h_r = 0$ (solid curves) in the case of $n = 3$. We set $|A_0| = 1$. The horizontal and vertical axes show ξ and t, respectively. The dashed lines denote the critical times. (a) For $A_0 > 0$, the size of the spacetime domain (the left side of the solid curve) expands and diverges at $t = -1$. (b) For $A_0 < 0$, at $t = +1$ singularity appears at the infinity and then the spacetime domain (the left side of the solid curve) shrinks as the time progresses.

We embed the brane world and discuss the dynamics near the singularity. In Tables XXVI, XXVII, and XXVIII, we have classified the future singularities of brane worlds for $A_0 < 0$. The behavior of past singularities can be discussed for $A_0 > 0$. In the tables, for example, "Dm-Dn" denotes the intersecting D-branes, where the first Dm-brane has the time dependence. Also "(A, B)" represents the directions of the ordinary three-space and the bulk, respectively. These rules are also applied to the subsequent tables.

For $n > 2$, in the metric (58) $d^2 \propto h_r^{q-1}$ ($f^2 \propto h_r^q$ or $f^2 \propto h_r^{q-1}$), where $0 < q < 1$. From (59), $\frac{dt}{d\tau} \sim h_r^{(1-q)/2} \to 0$ as $h_r \to 0$, which implies that for $A_0 < 0$ the brane world cannot reach the future singularity within a finite proper time. Note that for $f^2 \propto h_r^q$ the brane world trajectory is always timelike as $h_r \to 0$ while for $f^2 \propto h_r^{q-1}$ it becomes timelike or null in approaching singularity. Conversely, for $A_0 > 0$, the universe is born in the infinite past in terms of the proper time. On the other hand, the solutions listed in Table XXIX give $q > 1$, and hence $\frac{dt}{d\tau} \sim h_r^{(1-q)/2} \to \infty$ as $h_r \to 0$. Thus, the brane world can reach the singularity within a finite proper time. Some of the $n = 1$ solutions are in this class.

C. Cosmological equations

Now we derive the cosmological equations in the brane world through the junction conditions. Under the Z_2 symmetry, they are given by

$$K^{\hat{A}}{}_{\hat{B}} - \delta^{\hat{A}}{}_{\hat{B}} K^{\hat{C}}{}_{\hat{C}} = -\frac{1}{2}\kappa^2 S^{\hat{A}}{}_{\hat{B}}, \tag{65}$$

where $K_{\hat{A}\hat{B}} := q^A_{\hat{A}} q^B_{\hat{B}} \nabla_A n_B$ is the extrinsic curvature tensor, and $S^{\hat{A}}{}_{\hat{B}}$ is the energy-momentum tensor of the brane world matter. n_A and $q^A_{\hat{A}}$ are the unit normal vector and projection tensor to the brane world, respectively, The hatted indices $\{\hat{A}\}$ run over the brane world directions. The nonvanishing (i, j), (a, b), and (τ, τ) components of the left-hand side of Eq. (65) are related to the pressure in the ordinary 3-spatial direction, that in the $(n-1)$-sphere and the energy density on the brane world.

From now on, we focus on the (square of the) (τ, τ) component, which is given by

$$1 + \frac{\dot{a}^2}{\alpha_\xi^2} - \frac{\alpha_t^2(\alpha_\xi^4 - \alpha_t^2\dot{a}^2 - \alpha_\xi^2(\alpha_t^2 - 3\dot{a}^2))}{\alpha_\xi^2(\alpha_\xi^2 - \alpha_t^2)^2} \pm \frac{2\alpha_\xi\alpha_t\dot{a}\sqrt{\alpha_\xi^2 - \alpha_t^2 + \dot{a}^2}}{(\alpha_\xi^2 - \alpha_t^2)^2} = \frac{1}{(3\frac{\alpha_\xi}{a} + (n-1)\frac{\gamma_\xi}{f\xi})^2}\left\{\frac{1}{2}\epsilon\kappa^2\rho - \left(3\frac{\alpha_t}{f} + (n-1)\frac{\gamma_t}{d}\right) \times \frac{\alpha_\xi\dot{a} \pm \alpha_t\sqrt{\alpha_\xi^2 - \alpha_t^2 + \dot{a}^2}}{\alpha_\xi^2 - \alpha_t^2}\right\}^2, \tag{66}$$

where ρ is the energy density in the brane world. For convenience, we have introduced

$$\alpha_\xi := \frac{a_{,\xi}}{f}, \qquad \alpha_t := \frac{a_{,t}}{d}, \qquad \gamma_\xi := 1 + \frac{\xi f_{,\xi}}{f} \qquad \gamma_t := \frac{\xi f_{,t}}{d}. \tag{67}$$

$\epsilon = +1\ (-1)$ denotes the normal vector pointing the increasing (decreasing) ξ direction. When the spacetime is approximately static, i.e., $|\alpha_\xi| \gg |\alpha_t|$ and $|\gamma_\xi| \gg |\gamma_t|$, Eq. (66) reduces to

$$\frac{\dot{a}^2}{a^2} \approx \frac{\kappa^4\rho^2}{4(3 + (n-1)\frac{\gamma_\xi a}{\alpha_\xi f\xi})^2} - \frac{\alpha_\xi^2}{a^2}, \tag{68}$$

where "≈" becomes "=" when $\alpha_t = \gamma_t = 0$. When the time dependence dominates the system, i.e., $|\alpha_\xi| \ll |\alpha_t|$ and $|\gamma_\xi| \ll |\gamma_t|$, Eq. (66) reduces to

$$\frac{\dot{a}^2}{a^2} \approx \frac{\kappa^4\rho^2}{4(3 + (n-1)\frac{\gamma_t a}{\alpha_t f\xi})^2} + \frac{\alpha_t^2}{a^2}, \tag{69}$$

where "≈" becomes "=" when $\alpha_\xi = 0$ and $\gamma_\xi = 1$.

Furthermore, we integrate the energy density over the $(n-1)$-sphere in the brane world $\rho_{4D} = \xi^{n-1} f^{n-1} V_{n-1}\rho$, where $V_{n-1} := \frac{2\pi^{n/2}}{\Gamma(n/2)}$ is the surface area of the unit $(n-1)$-sphere. Then, we decompose the energy density into the constant part σ and the time dependent one $\delta\rho$: $\rho_{4D} := \sigma + \delta\rho$. In both of the above limiting cases, at the low-energy scale ($\delta\rho \ll \sigma$), Eq. (68) reduces to

$$\frac{\dot{a}^2}{a^2} \approx \begin{cases} \frac{8\pi G_{\rm eff}}{3}\delta\rho + \frac{1}{3}\Lambda_{\rm eff}^{(\xi)}(\tau) + O(\delta\rho^2) & \text{(static limit)} \\ \frac{8\pi G_{\rm eff}}{3}\delta\rho + \frac{1}{3}\Lambda_{\rm eff}^{(t)}(\tau) + O(\delta\rho^2) & \text{(time-dependent limit),} \end{cases} \tag{70}$$

where the effective gravitational constant is given by

$$\frac{8\pi G_{\rm eff}}{3} := \begin{cases} \frac{\kappa^4\sigma}{2V_{n-1}^2(\xi f)^{2n-2}(3 + (n-1)(\frac{1}{\xi} + \frac{f_{,\xi}}{f})\frac{a}{a_{,\xi}})^2} & \text{(static limit)} \\ \frac{\kappa^4\sigma}{2V_{n-1}^2(\xi f)^{2n-2}(3 + (n-1)\frac{f_{,t}a}{f a_{,t}})^2} & \text{(time-dependent limit),} \end{cases} \tag{71}$$

respectively. $\Lambda_{\rm eff}^{(\xi)}(\tau)$ and $\Lambda_{\rm eff}^{(t)}(\tau)$ give the nonstandard contributions. The necessary condition to obtain a realistic low-energy cosmology is that the effective gravitational coupling in Eq. (71) remains (almost) constant during the cosmological evolution. It is clear that in the definition of $G_{\rm eff}$, Eq. (71), the motion of the $(n-1)$ compact dimensions on the brane world is taken in account. Its effect makes it difficult to obtain a constant $G_{\rm eff}$. Nevertheless, as we will discuss later, we will find a few examples in which the constant $G_{\rm eff}$ is obtained.

Before closing this subsection, we briefly mention the other requirement for the recovery of the four-dimensional gravity, i.e., the localizability of the graviton zero mode. This issue is independent of the number of extra dimensions n. Although to clarify it we have to investigate the perturbations, at least we need to require the finiteness of the bulk volume off the brane world. In our solutions, in general it is not ensured. Then, it is necessary to add, for example, the cut-off second brane world. In this case, the gravity in the brane world may not coincide with the ordinary general relativity, due to the degree of freedom associated with the interbrane distance. For now, we leave this issue for a future study and focus on the minimal requirement that $G_{\rm eff}$ must be constant.

D. Effective gravitational coupling

Finally, we discuss the behavior of the effective gravitational coupling (71) and list the realistic brane world models.

1. $n = 1$

The requirement of a constant $G_{\rm eff}$ is trivially satisfied for $n = 1$, with $G_{\rm eff} = \frac{\kappa^4\sigma}{48\pi}$. The solutions which give $n = 1$ brane world models are listed in Table XXX. We classify the solutions with $h_r = A_0 t + B + M_r|\xi|$ and $h_r = A_0 t + B$ in (a) and (b) in the table, respectively. Also in the right column of Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, XIII, XIV, XV, XVI, XVII, and XVIII, the original ten- or eleven-dimensional solutions which give these brane worlds are listed.

In Table XXXI we further classify $n = 1$ brane worlds in terms of the behavior near the future singularity for $A_0 < 0$. We find that no brane world model whose scale factor diverges within a finite time, i.e., a big-rip singularity, is obtained. Solutions in the category of ($a = 0$,"Finite") provide the brane worlds which collapse within a finite time. But ever expanding brane worlds are also obtained from the intersecting D-branes of the cases (I) and (II). For $A_0 > 0$, from Table XXXI, the behavior of the brane near the past singularity can be discussed. In particular, the solutions of ($a = 0$,Finite) in Table XXXI provides the brane worlds with initial big bang singularities in the finite past.

Before closing this part, it is important to summarize the property of brane world models of $n = 1$ in each case. First, it is clear that in our treatment M-branes do not provide such a brane world model in all the cases (I)–(III). Concerning the ten-dimensional solutions, we summarize their properties below:

Case (I) A remarkable property of this case is that all the $n = 1$ brane world models are obtained only from the intersecting D-branes. By definition, the bulk direction is always Z. In addition, all of models are of type (a) (see Table XXX). In approaching the future $h_r = 0$ singularity, the model of D5-D7($\rm Y_2$, Z) gives an ever expanding universe, while the model of D5-D7($\rm Y_1$, Z) provides an ever contracting universe. All the rest give universes collapsing within the finite time.

Case (II) All classes of solutions can provide the $n = 1$ brane world models. This is in part because some of the case (II) solutions correspond to particular cases of case (I). The bulk direction can be either $\rm Y_1$ or Z. For the models of type (a) $\rm Y_1$ is the bulk direction, while for those of type (b) Z is the bulk direction. In approaching the future $h_r = 0$ singularity, the model of D5-D7($\tilde{\rm X}$, Z) gives an ever expanding universe, while the models of D5-D7($\rm Y_1$, Z), D3-D1(Z, $\rm Y_1$), and NS5-D4(Z, $\rm Y_1$) provide ever contracting universes. All the rest give universes collapsing within the finite time.

Case (III) The $n = 1$ brane world models are obtained only from the solutions including an NS5-brane. The bulk direction can be either $\rm Y_1$ or $\rm Y_2$. For the models of type (a) $\rm Y_1$ is the bulk direction, while for those of type (b) $\rm Y_2$ is the bulk direction. In approaching the future $h_r = 0$ singularity, the model of NS5-D7($\rm Y_1$, $\rm Y_2$) provides an ever contracting universe. All the rest give universes collapsing within the finite time and an ever expanding universe is not realized.

2. $n > 2$

We consider the case of $n > 2$. First, for simplicity, we discuss the case of the completely static solutions $\alpha_t = \gamma_t = 0$. In this case, there is no future or past singularity. We do not find examples that the effective gravitational coupling approaches a constant. In the opposite limit $\xi \to \infty$, we find that there are examples with constant $G_{\rm eff}$ in cases (I) and (III), which are listed in Table XXXII. From the table, we see that for case (I) the four models of $n = 3$, i.e., D5-D3 ($\rm Y_2$, Z), NS5-D3($\rm Y_2$, Z), D3-D5 ($\rm Y_1$, Z), D3-NS5($\rm Y_1$, Z), and for the case (III) D3-D7(($\tilde{\rm X}$, $\rm Y_2$), $\rm Y_1$) model of $n = 6$ can be realistic. In case (I), we have to assume both $M_r \neq 0$ and $M_s \neq 0$. In the right column of Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, and XIII, the original ten-dimensional solutions which give these models are indicated. Note that the static M-brane solutions do not give realistic models. Although in our model there are many solutions and ways of compactification, the number of realistic solutions is rather small. This is because the compact $(n-1)$ dimensions in the brane world evolve with time and make $G_{\rm eff}$ in Eq. (71) time dependent even in the static solutions.

By allowing the time dependence $\alpha_t \neq 0$ and $\gamma_t \neq 0$, in the near-brane limit $\xi \to 0$, the static approximation is still valid, namely $|\alpha_t| \ll |\alpha_\xi|$ and $|\gamma_t| \ll |\gamma_\xi|$. But in the $\xi \to \infty$ limit, the time dependence now dominates the system since $|\alpha_t| \gg |\alpha_\xi|$ and $|\gamma_t| \gg |\gamma_\xi|$. Then, in both the limits of $\xi \to 0$ and $\xi \to \infty$, there is no example of the constant gravitational coupling. Therefore, for $n > 2$, it is impossible to construct the realistic brane world models from our solutions.

V. DISCUSSIONS

In this paper, we have derived intersecting dynamical brane solutions and discussed their dynamics in the ten- and eleven-dimensional supergravity models. These solutions are obtained by replacing a constant A in the warp factor $h = A + h_1(y)$ of a supersymmetric solution by a function $h_0(x)$ of the coordinates x^μ [16,17,36,38]. Our solutions can contain only one function depending on both time as well as overall or relative transverse space coordinates. In particular, the solutions in Sec. II B tell us that the brane which depends on overall transverse coordinate can be extended to the time dependent case. It is possible to get the dynamical intersecting brane solutions which obey the intersection rule $\chi = -2$ different from the usual one, as we have discussed in Sec. II C.

We have used the intersection rules to find the cosmological solution because it is not so easy to find it analytically without their rules. The intersection rules have led to the functions h_r and h_s which can be written by linear combinations of terms depending on both coordinates of worldvolume and transverse space. This feature is expected to be shared by a wide class of supersymmetric solutions beyond the examples considered in the present paper, because the result has been obtained by analyzing the general structure of solutions for warped compactification with field strength of the ten- or eleven-dimensional supergravities under ansatz that is natural to include supersymmetric solutions as a special case. We have showed that these solutions give a FLRW universe if we regard the homogeneous and isotropic part of the brane worldvolumes as our spacetime. Unfortunately, the power of the scale factor is so small that the solutions of field equations cannot give a realistic expansion law. This means that we have to consider additional matter on the brane in order to get a realistic expanding universe. As the number p_r or p_s increases, the power of the scale factor becomes large. We find that the intersection with D6-brane in ten-dimensional theory gives the fastest expansion of our universe because the three-dimensional spatial space of our universe stays in the transverse space to the D6-brane. Though the power of the scale factor for the transverse space in solutions with the D7- or D8-branes is larger than those with the D6-brane, the number of the transverse space to these branes is less than three. Hence, these solutions cannot provide the isotropic universe if we assume that the transverse space to the brane is the part of our universe.

The solutions we have obtained may give some moduli instabilities because of the flat direction of the moduli potential in the lower-dimensional effective theories after compactifications [15,17,36,37]. Such instability will grow unless the global or local minimum of the potential can be produced by some correction in the effective theory.

The dynamical solutions contain only one function depending on both time and transverse space coordinates. One possible reason for this is that the ansatz concerning the structure of the D-dimensional metric is too restrictive. However, a recent study of similar systems shows that it is possible to obtain solutions with each function depending on both time and transverse space coordinates (see [31] for recent discussion). It is interesting to examine if our solutions can be extended to more general solutions by relaxing the assumptions of the field ansatz.

Finally, we have constructed the brane world models from our solutions. This approach makes it clear how the ordinary four-dimensional matter contributes to the cosmology. In our approach, we first compactify the trivial spatial directions in a given ten- or eleven-dimensional spacetime and then move to the Einstein frame. This approach gives a way of regularization of the brane source to put matter there. For a brane with higher codimensions we have applied the cut-copy-paste method. We then need to integrate over the angular dimensions in the brane worldvolume to define the effective four-dimensional quantities. For a codimension-one brane, we need just the copy and paste. For our prescription to work, we have restricted our solutions. In particular, we have chosen the parameters of ten- or eleven-dimensional solutions that after compactifying the trivial spatial dimensions the extra dimensions become spherically symmetric with respect to a single brane. We have also classified the singularity in the time dependent solutions, and discussed the behaviors of the brane world universe around it.

Then, we have derived the effective gravitational equations via the junction condition. The necessary condition to obtain a realistic cosmological model is that the effective gravitational constant must approach constant. A brane world model obtained from a codimension-one brane can automatically realize a constant gravitational coupling. The intersecting M-branes could not provide such models for all types of brane intersection. The existence of some ten-dimensional solutions which provide such brane world models crucially depend on the types of the brane intersection. In terms of the behavior around the singularity, an ever expanding universe cannot be obtained from the solutions where the metric does not depend on the overall transverse space. Concerning the models constructed from higher-codimensional branes, for the purely static case, we have found a few solutions where the effective gravitational coupling approaches constant in the far brane limit. In contrast, however, for the generic time dependent solutions, we did not find such examples.

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TABLES

TABLE I. Intersections of two M-branes in the metric (50) and (56). Whichever of the two M2's or M5's is time dependent does not make any difference. In the right column of Tables I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII, and XIII, "BW" denotes the solutions which provide realistic brane world models. From the solutions in the list, no realistic brane world model is obtained.

Case		0	1	2	3	4	5	6	7	8	9	10		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
(I), (II)	M2	∘			∘	∘							√	Y_1 & Z	$\lambda(Y_1)=1/4$	$\lambda_E(Y_1)=\frac{-3+d_3}{-12+d_2+2d_3+d_4}$	
M2-M2	M2	∘	∘	∘											$\lambda(Z)=1/4$	$\lambda_E(Z)=\frac{-3+d_3}{-12+d_2+2d_3+d_4}$	
	x^N	t	y^1	y^2	v^1	v^2	z^1	z^2	z^3	z^4	z^5	z^6					
(I), (II)	M2	∘	∘					∘					√	Y_1 & Z	$\lambda(Y_1)=1/4$	$\lambda_E(Y_1)=\frac{-3+d_1+d_3}{-12+2d_1+d_2+2d_3+d_4}$	
M2-M5	M5	∘	∘	∘	∘	∘	∘								$\lambda(Z)=1/4$	$\lambda_E(Z)=\frac{-3+d_1+d_3}{-12+2d_1+d_2+2d_3+d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v	z^1	z^2	z^3	z^4					
(I), (II)	M2	∘	∘	∘										$\tilde{X}$ & Y_2	$\lambda(Y_2)=-1/5$	$\lambda_E(Y_2)=\frac{3-d_2-d_4}{-15+2d_1+d_2+2d_3+d_4}$	
M2-M5	M5	∘	∘		∘	∘	∘	∘					√	or	$\lambda(Y_1)=2/5$	$\lambda_E(Y_1)=\frac{-6+d_1+d_3}{-15+2d_1+d_2+2d_3+d_4}$	
	x^N	t	x	y	v^1	v^2	v^3	v^4	z^1	z^2	z^3	z^4		Y_1 & Z	$\lambda(Z)=2/5$	$\lambda_E(Z)=\frac{-6+d_1+d_3}{-15+2d_1+d_2+2d_3+d_4}$	
(I), (II)	M5	∘	∘	∘	∘			∘	∘				√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X})=-1/5$	$\lambda_E(\tilde{X})=\frac{3-d_2-d_4}{-15+2d_1+d_2+2d_3+d_4}$	
M5-M5	M5	∘	∘	∘	∘	∘	∘							or	$\lambda(Y_2)=-1/5$	$\lambda_E(Y_2)=\frac{3-d_2-d_4}{-15+2d_1+d_2+2d_3+d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v^1	v^2	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1)=\lambda(Z)=\frac{2}{5}$	$\lambda_E(Y_1)=\lambda_E(Z)=\frac{-6+d_1+d_3}{-15+2d_1+d_2+2d_3+d_4}$	
(III)	M5	∘	∘					∘	∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X})=-1/5$	$\lambda_E(\tilde{X})=\frac{3-d_2-d_4}{-15+2d_1+d_2+2d_3+d_4}$	
M5-M5	M5	∘	∘	∘	∘	∘	∘							or	$\lambda(Y_2)=-1/5$	$\lambda_E(Y_2)=\frac{3-d_2-d_4}{-15+2d_1+d_2+2d_3+d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v^1	v^2	v^3	v^4	z		Y_1 & Z	$\lambda(Y_1)=\lambda(Z)=\frac{2}{5}$	$\lambda_E(Y_1)=\lambda_E(Z)=\frac{-6+d_1+d_3}{-15+2d_1+d_2+2d_3+d_4}$	

TABLE II. Intersections of two D-branes with $p = 0$ intersection in cases (I) and (II). Whichever of the two D2's is time dependent does not make any difference. In the right column, the notation (A, B) corresponds to the directions which give the ordinary 3-space $\mathcal{X}^i$ and the bulk (ξ, θ^a) of the brane world, respectively. The D3-D1 solution can give a brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D0	∘										√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/9$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{1}{9-d_2-d_4}$	
D0-D4	D4	∘	∘	∘	∘	∘								$\lambda(\mathrm{Z}) = 1/9$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{1}{9-d_2-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	z^1	z^2	z^3	z^4	z^5					
	D0	∘											Y_2 or Z	$\lambda(\mathrm{Y}_2) = -3/13$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{-3+d_4}{13-2d_3-d_4}$	
D0-D4	D4	∘	∘	∘	∘	∘						√		$\lambda(\mathrm{Z}) = 5/13$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_3}{13-2d_3-d_4}$	
	x^N	t	v^1	v^2	v^3	v^4	z^1	z^2	z^3	z^4	z^5					
	D1	∘				∘						√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
D1-D3	D3	∘	∘	∘	∘									$\lambda(\mathrm{Z}) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	v	z^1	z^2	z^3	z^4	z^5					
	D1	∘	∘										Y_2 or Z	$\lambda(\mathrm{Y}_2) = -1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-4}{12-d_2-2d_3-d_4}$	$(\mathrm{Y}_2, \mathrm{Y}_1)$, II
D1-D3	D3	∘		∘	∘	∘						√		$\lambda(\mathrm{Z}) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	$(\mathrm{Z}, \mathrm{Y}_1)$, II
	x^N	t	y	v^1	v^2	v^3	z^1	z^2	z^3	z^4	z^5					
	D2	∘			∘	∘						√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{3-d_3}{11-d_2-2d_3-d_4}$	
D2-D2	D2	∘	∘	∘										$\lambda(\mathrm{Z}) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{3-d_3}{11-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	v^1	v^2	z^1	z^1	z^3	z^4	z^5					

TABLE III. Intersections of two D-branes with $p = 1$ intersection in cases (I) and (II). Whichever of the two D3's is time dependent does not make any difference. No solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D1	∘	∘									√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{2-d_1}{10-2d_1-d_2-d_4}$	
D1-D5	D5	∘	∘	∘	∘	∘	∘							$\lambda(\mathrm{Z}) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{2-d_1}{10-2d_1-d_2-d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	z^1	z^2	z^3	z^4					
	D1	∘	∘										$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/7$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{-2+d_4}{14-2d_1-2d_3-d_4}$	
D1-D5	D5	∘	∘	∘	∘	∘	∘					√	or Z	$\lambda(\mathrm{Y}_2) = -1/7$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{-2+d_4}{14-2d_1-2d_3-d_4}$	
	x^N	t	x	v^1	v^2	v^3	v^4	z^1	z^2	z^3	z^4			$\lambda(\mathrm{Z}) = 3/7$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_1-d_3}{14-2d_1-2d_3-d_4}$	
	D2	∘	∘				∘					√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{3-d_1-d_3}{11-2d_1-d_2-2d_3-d_4}$	
D2-D4	D4	∘	∘	∘	∘	∘								$\lambda(\mathrm{Z}) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{3-d_1-d_3}{11-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y^1	y^2	y^3	v	z^1	z^2	z^3	z^4					
	D2	∘	∘	∘									$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -3/13$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D2-D4	D4	∘	∘		∘	∘	∘					√	or	$\lambda(\mathrm{Y}_2) = -3/13$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y	v^1	v^2	v^3	z^1	z^2	z^3	z^4		Y_1 & Z	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{5}{13}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	D3	∘	∘			∘	∘					√	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/3$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	
D3-D3	D3	∘	∘	∘	∘								or	$\lambda(\mathrm{Y}_2) = -1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y^1	y^2	v^1	v^2	z^1	z^2	z^3	z^4		Y_1 & Z	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{1}{3}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_1-d_3}{12-2d_1-d_2-2d_3-d_4}$	

TABLE IV. Intersections of two D-branes with $p = 2$ intersection in cases (I) and (II). Whichever of the two D4's is time dependent does not make any difference. The static D3-D5 solution of case (I) provides realistic brane world models.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D2	$\circ$	$\circ$	$\circ$								$\surd$	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{3-d_1}{11-2d_1-d_2-d_4}$	
D2-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						$\lambda(\mathrm{Z}) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{3-d_1}{11-2d_1-d_2-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	y^3	y^4	z^1	z^2	z^3					
	D2	$\circ$	$\circ$	$\circ$									$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/15$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_4-1}{15-2d_1-2d_3-d_4}$	
D2-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\surd$	or Z	$\lambda(\mathrm{Y}_2) = -1/15$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_4-1}{15-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	v^1	v^2	v^3	v^4	z^1	z^2	z^3			$\lambda(\mathrm{Z}) = 7/15$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{7-d_1-d_3}{15-2d_1-2d_3-d_4}$	
	D3	$\circ$	$\circ$	$\circ$				$\circ$				$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/3$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	(Y_1, Z), I(s)
D3-D5	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(\mathrm{Y}_2) = -1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	y^3	v	z^1	z^2	z^3		Y_1 & Z	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{1}{3}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_1-d_3}{12-2d_1-d_2-2d_3-d_4}$	
	D3	$\circ$	$\circ$	$\circ$	$\circ$								$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/7$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	(Y_2, Z), I(s)
D3-D5	D5	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$				$\surd$	or	$\lambda(\mathrm{Y}_2) = -1/7$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y	v^1	v^2	v^3	z^1	z^2	z^3		Y_1 & Z	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{3}{7}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	D4	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$				$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = -\frac{3}{13}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D4-D4	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{5}{13}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	v^1	v^2	z^1	z^2	z^3		Y_1 & Z			

TABLE V. Intersections of two D-branes with $p = 3$ intersection in cases (I) and (II). Whichever of the two D5's is time dependent does not make any difference. No solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D3	$\circ$	$\circ$	$\circ$	$\circ$							$\surd$	$\tilde{\mathrm{X}}$	$\lambda(\tilde{\mathrm{X}}) = -1/3$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-4}{12-2d_1-d_2-d_4}$	
D3-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				or	$\lambda(\mathrm{Y}_1) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{4-d_1}{12-2d_1-d_2-d_4}$	
	x^N	t	x^1	x^1	x^3	y^1	y^2	y^3	y^4	z^1	z^2		Y_1 & Z	$\lambda(\mathrm{Z}) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_1}{12-2d_1-d_2-d_4}$	
	D3	$\circ$	$\circ$	$\circ$	$\circ$								$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 0$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_4}{16-2d_1-2d_3-d_4}$	
D3-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			$\surd$		$\lambda(\mathrm{Y}_2) = 0$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_4}{16-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	v^1	v^2	v^3	v^4	z^1	z^2					
	D4	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$			$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-3}{13}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D4-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{5}{13}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	x^2	x^3	y^1	y^2	y^3	v	z^1	z^2		Y_1 & Z			
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{15}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-1}{15-2d_1-d_2-2d_3-d_4}$	
D4-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$			$\surd$	or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{7}{15}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y	v^1	v^2	v^3	z^1	z^2		Y_1 & Z			
	D5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$			$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{7}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-D5	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{3}{7}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v^1	v^2	z^1	z^2		Y_1 & Z			

TABLE VI. Intersections of two D-branes with $p = 4$ intersection in cases (I) and (II). Whichever of the two D6's is time dependent does not make any difference. D5-D7 solutions of the cases (I) and (II) provide realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						$\surd$	$\tilde{\mathrm{X}}$	$\lambda(\tilde{\mathrm{X}}) = -3/13$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4-3}{13-2d_1-d_2-d_4}$	
D4-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			Y_1 & Z	$\lambda(\mathrm{Y}_1) = 5/13$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{5-d_1}{13-2d_1-d_2-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	y^4	z			$\lambda(\mathrm{Z}) = 5/13$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_1}{13-2d_1-d_2-d_4}$	
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 1/17$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_4+1}{17-2d_1-2d_3-d_4}$	
D4-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\surd$		$\lambda(\mathrm{Y}_2) = 1/17$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_4+1}{17-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	v^1	v^2	v^3	v^4	z					
	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$		$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{7}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	$(\mathrm{Y}_1, \mathrm{Z})$, I, II
D5-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{3}{7}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	$(\tilde{\mathrm{X}}, \mathrm{Z})$, I, II
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	v	z		Y_1 & Z			
	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 0$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+d_4}{16-2d_1-d_2-2d_3-d_4}$	$(\mathrm{Y}_2, \mathrm{Z})$, I
D5-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$		$\surd$		$\lambda(\mathrm{Y}_2) = 0$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4}{16-2d_1-d_2-2d_3-d_4}$	$(\tilde{\mathrm{X}}, \mathrm{Z})$, I, II
	x^N	t	x^1	x^2	x^3	x^4	y	v^1	v^2	v^3	z					
	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$		$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = \lambda(\mathrm{Y}_2) = \frac{-1}{15}$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-1}{15-2d_1-d_2-2d_3-d_4}$	
D6-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					or	$\lambda(\mathrm{Y}_1) = \lambda(\mathrm{Z}) = \frac{7}{15}$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	v^1	v^2	z		Y_1 & Z			

TABLE VII. Intersections of two D-branes with $p = 5$ intersection in cases (I) and (II). Whichever of the two D7's is time dependent does not make any difference. No solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\circ$	$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = -1/15$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2-1}{15-2d_1-d_2-2d_3}$	
D6-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			or Y_1	$\lambda(\mathrm{Y}_2) = -1/15$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2-1}{15-2d_1-d_2-2d_3}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	v			$\lambda(\mathrm{Y}_1) = 7/15$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3}$	
	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 1/17$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2+1}{17-2d_1-d_2-2d_3}$	
D6-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\surd$		$\lambda(\mathrm{Y}_2) = 1/17$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+1}{17-2d_1-d_2-2d_3}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	y	v^1	v^2	v^3					
	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$	$\surd$	$\tilde{\mathrm{X}}$ & Y_2	$\lambda(\tilde{\mathrm{X}}) = 0$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{X}}) = \frac{d_2}{16-2d_1-d_2-2d_3}$	
D7-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\lambda(\mathrm{Y}_2) = 0$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2}{16-2d_1-d_2-2d_3}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	y^1	y^2	v^1	v^2					

TABLE VIII. Intersections of two D-branes with $p = 0$ intersection in case (III). Whichever of the two D4's is time dependent does not make any difference. No solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{\mathrm{M}}$	$\lambda(\tilde{\mathrm{M}})$	$\lambda_{\mathrm{E}}(\tilde{\mathrm{M}})$	BW
	D0	∘										√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/9$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{1}{9-d_2-d_4}$	
D0-D8	D8	∘	∘	∘	∘	∘	∘	∘	∘	∘				$\lambda(\mathrm{Z}) = 1/9$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{1}{9-d_2-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	y^7	y^8	z					
	D0	∘											Y_2	$\lambda(\mathrm{Y}_2) = 1/17$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{1+d_4}{17-2d_3-d_4}$	
D0-D8	D8	∘	∘	∘	∘	∘	∘	∘	∘	∘		√				
	x^N	t	v^1	v^2	v^3	v^4	v^5	v^6	v^7	v^8	z					
	D1	∘								∘		√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
D1-D7	D7	∘	∘	∘	∘	∘	∘	∘	∘					$\lambda(\mathrm{Z}) = 1/5$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	y^7	v	z					
	D1	∘	∘										Y_2	$\lambda(\mathrm{Y}_2) = 0$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4}{16-d_2-2d_3-d_4}$	
D1-D7	D7	∘		∘	∘	∘	∘	∘	∘	∘		√				
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	v^6	v^7	z					
	D2	∘							∘	∘		√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{3-d_3}{11-d_2-2d_3-d_4}$	
D2-D6	D6	∘	∘	∘	∘	∘	∘	∘						$\lambda(\mathrm{Z}) = 3/11$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{3-d_3}{11-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	v^1	v^2	z					
	D2	∘	∘	∘									Y_2 or	$\lambda(\mathrm{Y}_2) = -1/15$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-1}{15-d_2-2d_3-d_4}$	
D2-D6	D6	∘			∘	∘	∘	∘	∘	∘		√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 7/15$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{7-d_3}{15-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	v^1	v^2	v^3	v^4	v^5	v^6	z			$\lambda(\mathrm{Z}) = 7/15$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{7-d_3}{15-d_2-2d_3-d_4}$	
	D3	∘						∘	∘	∘		√	Y_2 or	$\lambda(\mathrm{Y}_2) = -1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-4}{12-d_2-2d_3-d_4}$	
D3-D5	D5	∘	∘	∘	∘	∘	∘						Y_1 & Z	$\lambda(\mathrm{Y}_1) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	v^1	v^2	v^3	z			$\lambda(\mathrm{Z}) = 1/3$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	
	D3	∘	∘	∘	∘								Y_2 or	$\lambda(\mathrm{Y}_2) = -1/7$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-2}{14-d_2-2d_3-d_4}$	
D3-D5	D5	∘				∘	∘	∘	∘	∘		√	Y_1 & Z	$\lambda(\mathrm{Y}_1) = 3/7$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	v^1	v^2	v^3	v^4	v^5	z			$\lambda(\mathrm{Z}) = 3/7$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	D4	∘					∘	∘	∘	∘		√	Y_2 or	$\lambda(\mathrm{Y}_2) = -3/13$	$\lambda_{\mathrm{E}}(\mathrm{Y}_2) = \frac{d_2+d_4-3}{13-d_2-2d_3-d_4}$	
D4-D4	D4	∘	∘	∘	∘	∘							Y_1 & Z	$\lambda(\mathrm{Y}_1) = 5/13$	$\lambda_{\mathrm{E}}(\mathrm{Y}_1) = \frac{5-d_3}{13-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	v^1	v^2	v^3	v^4	z			$\lambda(\mathrm{Z}) = 5/13$	$\lambda_{\mathrm{E}}(\mathrm{Z}) = \frac{5-d_3}{13-d_2-2d_3-d_4}$	

TABLE IX. Intersections of two D-branes with $p = 1$ intersection in case (III). Whichever of the two D5's is time dependent does not make any difference. The static D3-D7 solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
	D2	$\circ$	$\circ$								$\circ$	$\surd$	Y_1	$\lambda(Y_1) = 3/11$	$\lambda_E(Y_1) = \frac{-3+d_1+d_3}{-11+2d_1+d_2+2d_3}$	
D2-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						
	x^N	t	x	y^1	y^2	y^3	y^4	y^5	y^6	y^7	v					
	D2	$\circ$	$\circ$	$\circ$									Y_2	$\lambda(Y_2) = 1/17$	$\lambda_E(Y_2) = \frac{-1-d_2}{-17+2d_1+d_2+2d_3}$	
D2-D8	D8	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$				
	x^N	t	x	y	v^1	v^2	v^3	v^4	v^5	v^6	v^7					
	D3	$\circ$	$\circ$							$\circ$	$\circ$	$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/3$	$\lambda_E(\tilde{X}) = \frac{d_2-4}{12-2d_1-d_2-2d_3}$	$((\mathcal{X}, Y_2), Y_1)$, I(s)
D3-D7	D7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				or Y_1	$\lambda(Y_2) = -1/3$	$\lambda_E(Y_2) = \frac{d_2-4}{12-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	y^3	y^4	y^5	y^6	v^1	v^2			$\lambda(Y_1) = 1/3$	$\lambda_E(Y_1) = \frac{4-d_1-d_3}{12-2d_1-d_2-2d_3}$	
	D3	$\circ$	$\circ$	$\circ$	$\circ$								$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = 0$	$\lambda_E(\tilde{X}) = \frac{d_2}{16-2d_1-d_2-2d_3}$	
D3-D7	D7	$\circ$	$\circ$			$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$		$\lambda(Y_2) = 0$	$\lambda_E(Y_2) = \frac{d_2}{16-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	v^1	v^2	v^3	v^4	v^5	v^6					
	D4	$\circ$	$\circ$						$\circ$	$\circ$	$\circ$	$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -3/13$	$\lambda_E(\tilde{X}) = \frac{d_2-3}{13-2d_1-d_2-2d_3}$	
D4-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					or Y_1	$\lambda(Y_2) = -3/13$	$\lambda_E(Y_2) = \frac{d_2-3}{13-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	y^3	y^4	y^5	v^1	v^2	v^3			$\lambda(Y_1) = 5/13$	$\lambda_E(Y_1) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3}$	
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/15$	$\lambda_E(\tilde{X}) = \frac{d_2-1}{15-2d_1-d_2-2d_3}$	
D4-D6	D6	$\circ$	$\circ$				$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$	or Y_1	$\lambda(Y_2) = -1/15$	$\lambda_E(Y_2) = \frac{d_2-1}{15-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	y^3	v^1	v^2	v^3	v^4	v^5			$\lambda(Y_1) = 7/15$	$\lambda_E(Y_1) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3}$	
	D5	$\circ$	$\circ$					$\circ$	$\circ$	$\circ$	$\circ$	$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2-2}{14-2d_1-d_2-2d_3}$	
D5-D5	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or Y_1	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2-2}{14-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v^1	v^2	v^3	v^4			$\lambda(Y_1) = 3/7$	$\lambda_E(Y_1) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3}$	

TABLE X. Intersections of F1-branes in cases (I) and (II). The F1-D3 solution can provide a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
	F1	$\circ$	$\circ$									$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_1}{10-2d_1-d_2-d_4}$	
F1-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_1}{10-2d_1-d_2-d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	z^1	z^2	z^3	z^4					
	F1	$\circ$	$\circ$										$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{-2+d_4}{14-2d_1-2d_3-d_4}$	
F1-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\surd$	or Z	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{-2+d_4}{14-2d_1-2d_3-d_4}$	
	x^N	t	x	v^1	v^2	v^3	v^4	z^1	z^2	z^3	z^4			$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-2d_3-d_4}$	
	F1	$\circ$		$\circ$								$\surd$	Z	$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D1	D1	$\circ$	$\circ$													
	x^N	t	y	v	z^1	z^2	z^3	z^4	z^5	z^6	z^7					
	F1	$\circ$	$\circ$										Z	$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D1	D1	$\circ$		$\circ$								$\surd$				
	x^N	t	y	v	z^1	z^2	z^3	z^4	z^5	z^6	z^7					
	F1	$\circ$			$\circ$							$\surd$	Z	$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D2	D2	$\circ$	$\circ$	$\circ$												
	x^N	t	y^1	y^2	v	z^1	z^2	z^3	z^3	z^5	z^6					
	F1	$\circ$	$\circ$										Z	$\lambda(Z) = 3/11$	$\lambda_E(Z) = \frac{3-d_3}{11-d_2-2d_3-d_4}$	
F1-D2	D2	$\circ$		$\circ$	$\circ$							$\surd$				
	x^N	t	y	v^1	v^2	z^1	z^2	z^3	z^4	z^5	z^6					
	F1	$\circ$				$\circ$						$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D3	D3	$\circ$	$\circ$	$\circ$	$\circ$									$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	v	z^1	z^2	z^3	z^4	z^5					
	F1	$\circ$	$\circ$										Y_2 or Z	$\lambda(Y_2) = -1/3$	$\lambda_E(Y_2) = \frac{d_2+d_4-4}{12-d_2-2d_3-d_4}$	(Y_2, Y_1), II
F1-D3	D3	$\circ$		$\circ$	$\circ$	$\circ$						$\surd$		$\lambda(Z) = 1/3$	$\lambda_E(Z) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	(Z, Y_1), II
	x^N	t	y	v^1	v^2	v^3	z^1	z^2	z^3	z^4	z^5					
	F1	$\circ$					$\circ$					$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D4	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$								$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	v	z^1	z^2	z^3	z^4					
	F1	$\circ$	$\circ$										Y_2 or Z	$\lambda(Y_2) = -3/13$	$\lambda_E(Y_2) = \frac{d_2+d_4-3}{13-d_2-2d_3-d_4}$	
F1-D4	D4	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$					$\surd$		$\lambda(Z) = 5/13$	$\lambda_E(Z) = \frac{5-d_3}{13-d_2-2d_3-d_4}$	
	x^N	t	y	v^1	v^2	v^3	v^4	z^1	z^2	z^3	z^4					
	F1	$\circ$						$\circ$				$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D5	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	v	z^1	z^2	z^3					
	F1	$\circ$	$\circ$										Y_2 or Z	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-d_2-2d_3-d_4}$	
F1-D5	D5	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\surd$		$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	z^1	z^2	z^3					

TABLE XI. Intersections of F1-branes in cases (I) and (II). No solution provides a realistic brane world model.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
	F1	$\circ$							$\circ$			$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D6	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	v	z^1	z^2					
	F1	$\circ$	$\circ$										Y_2 or	$\lambda(Y_2) = -1/15$	$\lambda_E(Y_2) = \frac{d_2+d_4-1}{15-d_2-2d_3-d_4}$	
F1-D6	D6	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$			$\surd$	Y_1 & Z	$\lambda(Y_1) = 7/15$	$\lambda_E(Y_1) = \frac{7-d_3}{15-d_2-2d_3-d_4}$	
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	v^6	z^1	z^2			$\lambda(Z) = 7/15$	$\lambda_E(Z) = \frac{7-d_3}{15-d_2-2d_3-d_4}$	
	F1	$\circ$								$\circ$		$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
F1-D7	F7	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	y^7	v	z					
	F1	$\circ$	$\circ$										Y_2	$\lambda(Y_2) = 0$	$\lambda_E(Y_2) = \frac{d_2+d_4}{16-d_2-2d_3-d_4}$	
F1-D7	D7	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\surd$				
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	v^6	v^7	z					
	F1	$\circ$									$\circ$	$\surd$	Y_1	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3}$	
F1-D8	D8	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						
	x^N	t	y^1	y^2	y^3	y^4	y^5	y^6	y^7	y^8	v					
	F1	$\circ$	$\circ$										Y_2	$\lambda(Y_2) = 1/17$	$\lambda_E(Y_2) = \frac{d_2+1}{17-d_2-2d_3}$	
F1-D8	D8	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\surd$				
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	v^6	v^7	v^8					

TABLE XII. Intersections of NS-branes in cases (I) and (II). Whichever of the two NS5's is time dependent does not make any difference. D3-NS5 and D4-NS5 solutions can provide realistic brane world models.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
	NS5	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$	$\circ$			$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = \lambda(Y_2) = \frac{-1}{7}$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v^1	v^2	z^1	z^2		Y_1 & Z			
	D1	$\circ$						$\circ$				$\surd$	Y_1 & Z	$\lambda(Y_1) = 1/5$	$\lambda_E(Y_1) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
D1-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\lambda(Z) = 1/5$	$\lambda_E(Z) = \frac{2-d_3}{10-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	v	z^1	z^2	z^3					
	D1	$\circ$	$\circ$										Y_2	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-d_2-2d_3-d_4}$	
D1-NS5	NS5	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\surd$	or	$\lambda(Y_1) = 3/7$	$\lambda_E(Y_1) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	x^N	t	y	v^1	v^2	v^3	v^4	v^5	z^1	z^2	z^3		Y_1 & Z	$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	D2	$\circ$	$\circ$					$\circ$				$\surd$	Y_1 & Z	$\lambda(Y_1) = 3/11$	$\lambda_E(Y_1) = \frac{3-d_1-d_3}{11-2d_1-d_2-2d_3-d_4}$	
D2-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\lambda(Z) = 3/11$	$\lambda_E(Z) = \frac{3-d_1-d_3}{11-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v	z^1	z^2	z^3					
	D2	$\circ$	$\circ$	$\circ$									$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D2-NS5	NS5	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$	$\circ$				$\surd$	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y	v^1	v^2	v^3	v^4	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	D3	$\circ$	$\circ$	$\circ$				$\circ$				$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/3$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	(Y_2, Z), I(s)
D3-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(Y_2) = -1/3$	$\lambda_E(Y_2) = \frac{d_2+d_4-4}{12-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	y^3	v	z^1	z^2	z^3		Y_2 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{1}{3}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{4-d_1-d_3}{12-2d_1-d_2-2d_3-d_4}$	
	D3	$\circ$	$\circ$	$\circ$	$\circ$								$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	(Y_2, Y_1), II
D3-NS5	NS5	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$	$\circ$				$\surd$	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	(Y_1, Z), I(s)
	x^N	t	x^1	x^2	y	v^1	v^2	v^3	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	D4	$\circ$	$\circ$	$\circ$	$\circ$			$\circ$				$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -3/13$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D4-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(Y_2) = -3/13$	$\lambda_E(Y_2) = \frac{d_3+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{5}{13}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	D4	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$							$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	(Z, Y_1), II
D4-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$	$\circ$				$\surd$	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	$(\tilde{X}, Y_1)$, II
	x^N	t	x^1	x^2	x^3	y	v^2	v^2	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-2d_3-d_4}$	
	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$				$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	y	v	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	D5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		$\circ$				$\surd$	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	y	v	z^1	z^2	z^3		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$				$\surd$	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/15$	$\lambda_E(\tilde{X}) = \frac{d_4-1}{15-2d_1-2d_3-d_4}$	
D6-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$						or Z	$\lambda(Y_2) = -1/15$	$\lambda_E(Y_2) = \frac{d_4-1}{15-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	v	z^1	z^2	z^3			$\lambda(Z) = \frac{7}{15}$	$\lambda_E(Z) = \frac{7-d_1-d_3}{15-2d_1-2d_3-d_4}$	
	D6	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\tilde{X}$ & Y_1	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4-2}{14-2d_1-d_2-d_4}$	
D6-NS5	NS5	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$					$\surd$	or Z	$\lambda(Y_1) = 3/7$	$\lambda_E(Y_1) = \frac{6-d_1}{14-2d_1-d_2-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	y	z^1	z^2	z^3			$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6-d_1}{14-2d_1-d_2-d_4}$	

TABLE XIII. Intersections of NS-branes in case (III). Whichever of the two NS5's is time dependent does not make any difference. D7-NS5 solutions can provide realistic brane world models.

Branes		0	1	2	3	4	5	6	7	8	9		$\tilde{M}$	$\lambda(\tilde{M})$	$\lambda_E(\tilde{M})$	BW
	NS5	∘	∘					∘	∘	∘	∘	√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_2-2}{14-2d_1-d_2-2d_3}$	
NS5	NS5	∘	∘	∘	∘	∘	∘						or Y_1	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2-2}{14-2d_1-d_2-2d_3}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v^1	v^2	v^3	v^4			$\lambda(Y_1) = 3/7$	$\lambda_E(Y_1) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3}$	
	D3	∘						∘	∘	∘		√	Y_2 or	$\lambda(Y_2) = -1/3$	$\lambda_E(Y_2) = \frac{d_2+d_4-4}{12-d_2-2d_3-d_4}$	
D3-NS5	NS5	∘	∘	∘	∘	∘	∘						Y_1 & Z	$\lambda(Y_1) = 1/3$	$\lambda_E(Y_1) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	y^4	y^5	v^1	v^2	v^3	z			$\lambda(Z) = 1/3$	$\lambda_E(Z) = \frac{4-d_3}{12-d_2-2d_3-d_4}$	
	D3	∘	∘	∘	∘								Y_2 or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_2) = \frac{d_2+d_4-2}{14-d_2-2d_3-d_4}$	
D3-NS5	NS5	∘				∘	∘	∘	∘	∘		√	Y_1 & Z	$\lambda(Y_1) = 3/7$	$\lambda_E(Y_1) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	x^N	t	y^1	y^2	y^3	v^1	v^2	v^3	v^4	v^5	z			$\lambda(Z) = 3/7$	$\lambda_E(Z) = \frac{6-d_3}{14-d_2-2d_3-d_4}$	
	D4	∘	∘					∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -3/13$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-3}{13-2d_1-d_2-2d_3-d_4}$	
D4-NS5	NS5	∘	∘	∘	∘	∘	∘						or	$\lambda(Y_2) = -3/13$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{5-d_1-d_3}{13-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y^1	y^2	y^3	y^4	v^1	v^2	v^3	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{5}{13}$		
	D4	∘	∘	∘	∘	∘							$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D4-NS5	NS5	∘	∘				∘	∘	∘	∘		√	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x	y^1	y^2	y^3	v^1	v^2	v^3	v^4	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$		
	D5	∘	∘	∘				∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-NS5	NS5	∘	∘	∘	∘	∘	∘						or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	y^3	v^1	v^2	v^3	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$		
	D5	∘	∘	∘	∘	∘	∘						$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D5-NS5	NS5	∘	∘	∘				∘	∘	∘		√	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	y^1	y^2	y^3	v^1	v^2	v^3	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$		
	D6	∘	∘	∘	∘			∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/15$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-1}{15-2d_1-d_2-2d_3-d_4}$	
D6-NS5	NS5	∘	∘	∘	∘	∘	∘						or	$\lambda(Y_2) = -1/15$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{7-d_1-d_3}{15-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	v^1	v^2	v^3	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{7}{15}$		
	D6	∘	∘	∘	∘	∘	∘	∘					$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	
D6-NS5	NS5	∘	∘	∘	∘				∘	∘		√	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	y^1	y^2	y^3	v^1	v^2	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$		
	D7	∘	∘	∘	∘	∘		∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = 0$	$\lambda_E(\tilde{X}) = \frac{d_2+d_4}{16-2d_1-d_2-2d_3-d_4}$	(Y_2, Y_1)
D7-NS5	NS5	∘	∘	∘	∘	∘	∘							$\lambda(Y_2) = 0$	$\lambda_E(Y_2) = \frac{d_2+d_4}{16-2d_1-d_2-2d_3-d_4}$	$(\tilde{X}, Y_1)$
	x^N	t	x^1	x^2	x^3	x^4	y	v^1	v^2	v^3	z					
	D7	∘	∘	∘	∘	∘	∘	∘	∘				$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \lambda_E(Y_2) = \frac{d_2+d_4-2}{14-2d_1-d_2-2d_3-d_4}$	(Y_1, Y_2)
D7-NS5	NS5	∘	∘	∘	∘	∘				∘		√	or	$\lambda(Y_2) = -1/7$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-d_2-2d_3-d_4}$	$(\tilde{X}, Y_2)$
	x^N	t	x^1	x^2	x^3	x^4	y^1	y^2	y^3	v	z		Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$		
	D8	∘	∘	∘	∘	∘	∘	∘	∘	∘		√	$\tilde{X}$ & Y_2	$\lambda(\tilde{X}) = 1/17$	$\lambda_E(\tilde{X}) = \frac{d_4+1}{17-2d_1-2d_3-d_4}$	
D8-NS5	NS5	∘	∘	∘	∘	∘	∘							$\lambda(Y_2) = 1/17$	$\lambda_E(Y_2) = \frac{d_4+1}{17-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	v^1	v^2	v^3	z					
	D8	∘	∘	∘	∘	∘	∘	∘	∘	∘			$\tilde{X}$ or	$\lambda(\tilde{X}) = -1/7$	$\lambda_E(\tilde{X}) = \frac{d_4-2}{14-2d_1-2d_3-d_4}$	
D8-NS5	NS5	∘	∘	∘	∘	∘	∘					√	Y_1 & Z	$\lambda(Y_1) = \lambda(Z) = \frac{3}{7}$	$\lambda_E(Y_1) = \lambda_E(Z) = \frac{6-d_1-d_3}{14-2d_1-2d_3-d_4}$	
	x^N	t	x^1	x^2	x^3	x^4	x^5	y^1	y^2	y^3	z					

TABLE XIV. The power exponent of the fastest expansion in the Einstein frame for M-brane. "TD" in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
M2-M2	M2	9	Y_1 & Z	(0, 2, 0, 0)	3/10	I
	M2	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	1/5	II
M2-M5	M2	7	$\tilde{X}$ & Y_2 & Z	(0, 4, 0, 0)	3/8	I
	M2	10	Y_1 & Y_2 & Z	(1, 0, 0, 0)	1/5	II
	M2	10	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	1/5	II
	M5	10	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	3/7	I
	M5	10	Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/13	II
	M5	10	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/13	II
M5-M5	M5	9	$\tilde{X}$ & Y_2 & Z	(0, 2, 0, 0)	6/13	I
	M5	10	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/13	II
	M5	10	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/13	II
	M5	10	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	3/7	III

TABLE XV. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 0$). "TD" in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D0-D4	D0	6	Z	(0, 4, 0, 0)	1/5	I
	D0	10	Y_1 & Z	(0, 0, 0, 0)	1/9	II
	D4	9	Y_2 & Z	(0, 0, 1, 0)	4/11	I & II
D1-D3	D1	7	Y_2 & Z	(0, 3, 0, 0)	2/7	I
	D1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D3	9	Y_2 & Z	(0, 1, 0, 0)	4/11	I
	D3	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	3/10	II
D2-D2	D2	8	Y_2 & Z	(0, 2, 0, 0)	1/3	I
	D2	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	2/9	II

TABLE XVI. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 1$). "TD" in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D1-D5	D1	6	$\tilde{X}$ & Z	(0, 4, 0, 0)	1/3	I
	D1	9	Y_1 & Z	(1, 0, 0, 0)	1/8	II
	D5	9	Y_2 & Z	(1, 0, 0, 0)	5/12	I & II
	D5	9	$\tilde{X}$ & Y_2 & Z	(0, 0, 1, 0)	5/12	I & II
D2-D4	D2	7	$\tilde{X}$ & Y_2 & Z	(0, 3, 0, 0)	3/8	I
	D2	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	2/9	II
	D2	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	2/9	II
	D4	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	5/12	I
	D4	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	4/11	II
	D4	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	4/11	II
D3-D3	D3	8	$\tilde{X}$ & Y_2 & Z	(0, 2, 0, 0)	2/5	I
	D3	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	3/10	II
	D3	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	3/10	II

TABLE XVII. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 2$). "TD" in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D2-D6	D2	6	$\tilde{X}$ & Z	(0, 4, 0, 0)	3/7	I
	D2	9	$\tilde{X}$ & Y_1 & Z	(1, 0, 0, 0)	2/9	II
	D6	9	$\tilde{X}$ & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
	D6	9	$\tilde{X}$ & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II
D3-D5	D3	7	$\tilde{X}$ & Y_2 & Z	(0, 3, 0, 0)	4/9	I
	D3	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	3/10	II
	D3	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	3/10	II
	D5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
D4-D4	D4	8	$\tilde{X}$ & Y_2 & Z	(0, 2, 0, 0)	5/11	I
	D4	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	4/11	II
	D4	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	4/11	II

TABLE XVIII. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 3$). "TD" in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D3-D7	D3	7	$\tilde{X}$ & Y_1 & Z	(0, 3, 0, 0)	4/9	I
	D3	9	$\tilde{X}$ & Y_1 & Z	(1, 0, 0, 0)	3/10	II
	D7	$10 - d$	$\tilde{X}$ & Y_2 & Z or $\tilde{X}$ & Z or Y_2 & Z	$(d_1, 0, d_3, 0)$	0	I & II
D4-D6	D4	8	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 2, 0, 0)	5/11	I
	D4	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	4/11	II
	D4	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	4/11	II
	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II
D5-D5	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II

TABLE XIX. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 4$). “TD” in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D4-D8	D4	8	$\tilde{X}$ & Y_1 & Z	(0, 2, 0, 0)	5/11	I
	D4	9	$\tilde{X}$ & Y_1 & Z	(1, 0, 0, 0)	4/11	II
	D8	5	Y_2 & Z	(4, 0, 1, 0)	1/7	I & II
	D8	5	$\tilde{X}$ & Z	(1, 0, 4, 0)	1/7	I & II
	D8	5	$\tilde{X}$ & Y_2 & Z	(3, 0, 2, 0)	1/7	I & II
	D8	5	$\tilde{X}$ & Y_2 & Z	(2, 0, 3, 0)	1/7	I & II
D5-D7	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	D5	9	$\tilde{X}$ & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D7	5	Y_2 & Z	(4, 1, 0, 0)	1/6	I
	D7	5	$\tilde{X}$ & Y_2 & Z	(3, 1, 1, 0)	1/6	I
	D7	5	$\tilde{X}$ & Y_2 & Z	(2, 1, 2, 0)	1/6	I
	D7	5	$\tilde{X}$ & Z	(1, 1, 3, 0)	1/6	I
	D7	$10 - d$	$\tilde{X}$ & Y_1 & Y_2 & Z or $\tilde{X}$ & Y_1 & Z or Y_1 & Y_2 & Z	$(d_1, 0, d_3, 0)$	0	II
D6-D6	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II

TABLE XX. The power exponent of the fastest expansion in the Einstein frame for D-brane of cases (I) and (II) ($p = 5$). “TD” in the table shows which brane is time dependent.

Branes	TD	dim(M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D6-D8	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II
	D8	5	Y_1 & Y_2	(5, 1, 0, 0)	1/3	I
	D8	5	$\tilde{X}$ & Y_2	(4, 1, 1, 0)	1/3	I
	D8	5	$\tilde{X}$ & Y_2	(3, 1, 2, 0)	1/3	I
	D8	5	$\tilde{X}$	(2, 1, 3, 0)	1/3	I
	D8	6	Y_1 & Y_2	(5, 0, 0, 0)	1/7	II
	D8	6	$\tilde{X}$ & Y_1 & Y_2	(4, 0, 1, 0)	1/7	II
	D8	6	$\tilde{X}$ & Y_1 & Y_2	(3, 0, 2, 0)	1/7	II
	D8	6	$\tilde{X}$ & Y_1 & Y_2	(2, 0, 3, 0)	1/7	II
D7-D7	D7	4	$\tilde{X}$ & Y_2	(4, 2, 0, 0)	1/3	I
	D7	4	$\tilde{X}$ & Y_2	(3, 2, 1, 0)	1/3	I
	D7	4	$\tilde{X}$	(2, 2, 2, 0)	1/3	I
	D7	$10 - d$	$\tilde{X}$ & Y_1 & Y_2 or $\tilde{X}$ & Y_1	$(d_1, 0, d_3, 0)$	0	II

TABLE XXI. The power exponent of the fastest expansion in the Einstein frame for D-brane of case (III) ($p = 0$). “TD” in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D0-D8	D0	9	Y_1	(0, 0, 0, 1)	1/8	III
	D8	9	Y_2	(0, 0, 0, 1)	1/8	III
D1-D7	D1	9	Y_1 & Y_2	(0, 0, 0, 1)	2/9	III
	D7	9	Y_1 & Y_2	(0, 0, 0, 1)	1/15	III
D2-D6	D2	9	Y_1 & Y_2	(0, 0, 0, 1)	3/10	III
	D6	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	III
D3-D5	D3	9	Y_1 & Y_2	(0, 0, 0, 1)	4/11	III
	D5	9	Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D4-D4	D4	9	Y_1 & Y_2	(0, 0, 0, 1)	5/12	III

TABLE XXII. The power exponent of the fastest expansion in the Einstein frame for D-brane of case (III) ($p = 1$). "TD" in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
D2-D8	D2	9	Y_1 & Y_2	(1, 0, 0, 0)	2/9	III
	D8	9	Y_1 & Y_2	(1, 0, 0, 0)	1/15	III
	D3	9	Y_1 & Y_2	(1, 0, 0, 0)	3/10	III
D3-D7	D7	10	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 0)	0	III
	D7	9	Y_1 & Y_2	(1, 0, 0, 0)	0	III
D4-D6	D4	9	Y_1 & Y_2	(1, 0, 0, 0)	4/11	III
	D6	9	Y_1 & Y_2	(1, 0, 0, 0)	6/13	III
D5-D5	D5	9	Y_1 & Y_2	(1, 0, 0, 0)	5/12	III

TABLE XXIII. The power exponent of the fastest expansion in the Einstein frame for F1-string of cases (I) and (II). "TD" in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
	F1	6	$\tilde{X}$ & Z	(0, 4, 0, 0)	1/3	I
F1-NS5	F1	9	Y_1 & Z	(1, 0, 0, 0)	1/8	II
	NS5	9	Y_1 & Z	(1, 0, 0, 0)	5/12	I & II
	NS5	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	5/12	I & II
	F1	9	Y_2 & Z	(0, 1, 0, 0)	2/9	I
F1-D1	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D1	9	Y_2 & Z	(0, 1, 0, 0)	2/9	I
	D1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	F1	8	Y_2 & Z	(0, 2, 0, 0)	1/4	I
F1-D2	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D2	9	Y_2 & Z	(0, 1, 0, 0)	3/10	I
	D2	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	2/9	II
	F1	7	Y_2 & Z	(0, 3, 0, 0)	2/7	I
F1-D3	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D3	9	Y_2 & Z	(0, 1, 0, 0)	4/11	I
	D3	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	3/10	II
	F1	6	Y_2 & Z	(0, 4, 0, 0)	1/3	I
F1-D4	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D4	9	Y_2 & Z	(0, 1, 0, 0)	5/12	I
	D4	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	4/11	II
	F1	5	Y_2 & Z	(0, 5, 0, 0)	2/5	I
F1-D5	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D5	9	Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	F1	5	Y_1 & Y_2 & Z	(0, 5, 0, 0)	2/5	I
F1-D6	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D6	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	6/13	I & II
	F1	5	Y_1 & Y_2 & Z	(0, 5, 0, 0)	2/5	I
F1-D7	F1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	D7	5	Y_2 & Z	(0, 1, 4, 0)	1/7	I
	D7	$10 - d_3$	Y_1 & Y_2 & Z	$(0, 0, d_3, 0)$	0	II
	F1	5	Y_1 & Y_2	(0, 5, 0, 0)	2/5	I
F1-D8	F1	9	Y_1	(0, 0, 1, 0)	1/8	II
	D8	4	Y_2	(0, 1, 5, 0)	1/3	I
	D8	5	Y_1 & Y_2	(0, 0, 5, 0)	1/7	II

TABLE XXIV. The power exponent of the fastest expansion in the Einstein frame for NS-brane of cases (I) and (II). "TD" in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 1, 0, 0)	6/13	I
NS5-NS5	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D1	5	Y_2 & Z	(0, 5, 0, 0)	2/5	I
D1-NS5	D1	9	Y_1 & Z	(0, 0, 1, 0)	1/8	II
	NS5	9	Y_2 & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D2	6	$\tilde{X}$ & Y_2 & Z	(0, 4, 0, 0)	3/7	I
	D2	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	2/9	II
D2-NS5	D2	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	2/9	II
	NS5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D3	7	$\tilde{X}$ & Y_2 & Z	(0, 3, 0, 0)	4/9	I
	D3	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	3/10	II
D3-NS5	D3	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	3/10	II
	NS5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D4	8	$\tilde{X}$ & Y_2 & Z	(0, 2, 0, 0)	5/11	I
	D4	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	4/11	II
D4-NS5	D4	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	4/11	II
	NS5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(0, 0, 1, 0)	5/12	II
	D5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	D5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
D5-NS5	D5	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_2 & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	5/12	II
	NS5	9	$\tilde{X}$ & Y_1 & Z	(0, 0, 1, 0)	5/12	II
	D6	9	$\tilde{X}$ & Y_2 & Z	(1, 0, 0, 0)	6/13	I & II
D6-NS5	D6	9	$\tilde{X}$ & Z	(0, 0, 1, 0)	6/13	I & II
	NS5	9	$\tilde{X}$ & Z	(0, 1, 0, 0)	6/13	I
	NS5	9	$\tilde{X}$ & Y_1 & Z	(1, 0, 0, 0)	5/12	II

TABLE XXV. The power exponent of the fastest expansion in the Einstein frame for NS-brane of case (III). "TD" in the table shows which brane is time dependent.

Branes	TD	dim (M)	$\tilde{M}$	(d_1, d_2, d_3, d_4)	$\lambda_E(\tilde{M})$	Case
NS5-NS5	NS5	9	Y_1 & Y_2	(1, 0, 0, 0)	5/12	III
D3-NS5	D3	9	Y_1 & Y_2	(0, 0, 0, 1)	4/11	III
	NS5	9	Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D4-NS5	D4	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	5/12	III
	NS5	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D5-NS5	D5	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
	NS5	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D6-NS5	D6	9	$\tilde{X}$ & Y_1 & Y_2 & Z	(1, 0, 0, 0)	6/13	III
	NS5	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D7-NS5	D7	5	Y_1 & Y_2	(4, 0, 0, 1)	1/7	III
	NS5	9	$\tilde{X}$ & Y_1 & Y_2	(0, 0, 0, 1)	6/13	III
D8-NS5	D8	4	Y_2	(5, 0, 0, 1)	1/3	III
	NS5	9	$\tilde{X}$ & Y_1	(0, 0, 0, 1)	6/13	III

TABLE XXVI. Future singularity of brane worlds in case (I) with $A_0 < 0$.

Future singularity	Type	Intersecting branes
$a = 0$	D	D1-D3(Y_1, Z), D7-D5(Y_1, Z), D2-D4(Y_1, Z), D3-D5(Y_1, Z), D5-D7(Y_1, Z), D7-D5($\tilde{X}$, Z)
	M	M2-M5(Y_1, Z)
	F1	F1-D3(Y_1, Z)
	NS5	D3-NS5(Y_1, Z)
$a = \infty$	D	D3-D1(Y_2, Z), D4-D2(Y_2, Z), D5-D3(Y_2, Z), D5-D7($\tilde{X}$, Z)
	M	M5-M2(Y_2, Z), M5-M5($\tilde{X}$, Z)
	F1	D3-F1(Y_2, Z)
	NS5	NS5-D3(Y_2, Z), D4-NS5($\tilde{X}$, Z), NS5-D4($\tilde{X}$, Z)

TABLE XXVII. Future singularity of brane worlds in case (II) with $A_0 < 0$.

Future singularity	Type	Intersecting branes
$a = 0$	D	D3-D1(Z, Y_1), D3-D1(Y_2, Y_1), D2-D4(Y_1, Z), D3-D5(Y_1, Z), D5-D7(Y_1, Z), D7-D5($\tilde{X}$, Z)
	M	M2-M5(Y_1, Z), M2-M5(Z, Y_1), M5-M2(Y_2, Z)
	F1	F1-D3(Y_1, Z), F1-D3(Z, Y_1), D3-F1(Y_2, Y_1), D3-F1(Z, Y_1)
	NS5	D3-NS5(Y_1, Z), D3-NS5(Z, Y_1), NS5-D3(Y_1, Z), NS5-D3(Y_2, Y_1), D4-NS5(Z, Y_1), NS5-D4($\tilde{X}$, Y_1), NS5-D4(Z, Y_1)
$a = \infty$	D	D5-D7($\tilde{X}$, Z)
	M	M5-M5($\tilde{X}$, Z)
	F1	D3-F1(Y_2, Z)
	NS5	NS5-D4($\tilde{X}$, Z)

TABLE XXVIII. Future singularity of brane worlds in case (III) with $A_0 < 0$.

Future singularity	Type	Intersecting branes
$a = 0$	D	D5-D3(Y_1, Y_2), D6-D4(Y_1, Y_2)
	NS5	D6-NS5($\tilde{X}$, Z), D5-NS5(Y_1, Y_2), D7-NS5(Y_2, Y_1), D7-NS5($\tilde{X}$, Y_1) NS5-D3(Y_1, Y_2), NS5-D4(Y_1, Y_2), NS5-D5(Y_1, Y_2), NS5-D7(Y_1, Y_2), NS5-D7($\tilde{X}$, Y_2)
$a = \infty$	D	D3-D5(Y_2, Y_1), D4-D6(Y_2, Y_1), D3-D7(($\tilde{X}$, Y_2), Y_1)
	NS5	D3-NS5(Y_2, Y_1), D4-NS5(Y_2, Y_1), D5-NS5(Y_2, Y_1), NS5-D3(Y_2, Y_1), NS5-D4(Y_2, Y_1), NS5-D5(Y_2, Y_1), NS5-D7(Y_2, Y_1)

TABLE XXIX. Solutions with $d^2 \propto h_r^{q-1}$ with $q > 1$.

Case	Intersecting branes
I	D7-D5(Y_2, Z), D7-D5($\tilde{X}$, Z)
II	D3-D1(Y_2, Y_1), D7-D5($\tilde{X}$, Z), D3-F1(Y_2, Y_1), D3-F1(Z, Y_1), NS5-D3(Y_2, Y_1), NS5-D4($\tilde{X}$, Y_1)
III	D6-NS5($\tilde{X}$, Y_1), D7-NS5(Y_2, Y_1), D7-NS5($\tilde{X}$, Y_1), NS5-D7($\tilde{X}$, Y_2)

TABLE XXX. Solutions which give codimension-one brane world models.

Case	Type	Intersecting branes
I	(a)	D7-D5(Y_2, Z), D5-D7(Y_1, Z), D7-D5($\tilde{X}$, Z), D5-D7($\tilde{X}$, Z)
	(b)	No solution
II	(a)	D3-D1(Z, Y_1), D3-D1(Y_2, Y_1), D3-F1(Y_2, Y_1), D3-F1(Z, Y_1), NS5-D3(Y_2, Y_1), NS5-D4($\tilde{X}$, Y_1), NS5-D4(Z, Y_1)
	(b)	D5-D7(Y_1, Z), D7-D5($\tilde{X}$, Z), D5-D7($\tilde{X}$, Z)
III	(a)	D7-NS5(Y_2, Y_1), D7-NS5($\tilde{X}$, Y_1)
	(b)	NS5-D7(Y_1, Y_2), NS5-D7($\tilde{X}$, Y_2)

TABLE XXXI. The possible future singularities for the codimension-one brane world for $A_0 < 0$. "(In)finite" means that the brane world reaches the singularity within a (in)finite proper time.

	Case	$a = 0$	$a = \infty$
	I	D7-D5(Y_2, Z), D7-D5($\tilde{X}$, Z)	
Finite	II	D3-D1(Y_2, Y_1), D7-D5($\tilde{X}$, Z), D3-F1(Y_2, Y_1), D3-F1(Z, Y_1) NS5-D3(Y_2, Y_1), NS5-D4($\tilde{X}$, Y_1)	No solution
	III	NS5-D7($\tilde{X}$, Y_2), D7-NS5(Y_2, Y_1), D7-NS5($\tilde{X}$, Y_1)	
	I	D5-D7(Y_1, Z)	D5-D7(Y_2, Z)
Infinite	II	D5-D7(Y_1, Z), D3-D1(Z, Y_1), NS5-D4(Z, Y_1)	D5-D7($\tilde{X}$, Z)
	III	NS5-D7(Y_1, Y_2)	

TABLE XXXII. Static solutions which give a constant effective gravitational coupling in the limit of $\xi \to \infty$.

Case	n	Intersecting branes	$\frac{8\pi G_{\text{eff}}}{3}$
I	3	D5-D3(Y_2, Z), NS5-D3(Y_2, Z)	$\frac{\kappa^4 \sigma (M_r C - M_s B)^2}{3200\pi^2 M_r^{18/5} M_s^{12/5}}$
I	3	D3-D5(Y_1, Z), D3-NS5(Y_1, Z)	$\frac{\kappa^4 \sigma (M_r C - M_s B)^2}{3200\pi^2 M_r^{12/5} M_s^{18/5}}$
III	6	D3-D7(($\tilde{X}$, Y_2), Y_1)	$\frac{\kappa^4 \sigma}{18\pi^6 M_r^{5/2}}$

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