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Superconductivity of QCD vacuum in strong magnetic field

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We show that in a sufficiently strong magnetic field the QCD vacuum may undergo a transition to a new phase where charged $\rho^{\pm}$ mesons are condensed. In this phase the vacuum behaves as an anisotropic inhomogeneous superconductor which supports superconductivity along the axis of the magnetic field. In the directions transverse to the magnetic field the superconductivity is absent. The magnetic-field-induced anisotropic superconductivity—which is realized in the cold vacuum, i.e. at zero temperature and density—is a consequence of a nonminimal coupling of the ρ mesons to the electromagnetic field. The onset of the superconductivity of the charged $\rho^{\pm}$ mesons should also induce an inhomogeneous superfluidity of the neutral ρ^0 mesons. We also argue that due to simple kinematical reasons a strong enough magnetic field makes the lifetime of the ρ mesons longer by closing the main channels of the strong decays of the ρ mesons into charged pions.

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I. INTRODUCTION

Properties of QCD matter subjected to very strong magnetic fields have recently attracted increasing interest of the community. The interest is motivated by the possibility to create strong magnetic fields in the heavy-ion collisions at RHIC and LHC. The strength of the magnetic field is estimated to be of the hadronic scale [1], $eB \sim (1 \ldots 15)m_\pi^2$, or even higher (here $m_\pi \approx 140$ MeV is the pion mass). The duration of the magnetic field "flashes" is expected to be rather short (a few fm/c).

Both analytical calculations [2–4] and lattice simulations [5] indicate that the QCD phase diagram is affected by the strong magnetic field. In particular, the external magnetic field splits the chiral and deconfinement transitions [3,4]. In a constant magnetic field of the typical LHC magnitude, $eB \sim 15m_\pi^2$ [1], the splitting between the critical temperatures of these transitions reaches 10 MeV [3].

In the quark-gluon plasma the strong magnetic field may also lead to the chiral magnetic effect [6]. This effect generates an electric current of quarks along the magnetic field axis provided the densities of left- and right-handed quarks are not equal.

In the cold matter the external magnetic field may create spatially inhomogeneous structures which are made of quark condensates [7].

A recent lattice simulation has revealed that in the cold confinement phase the external magnetic field induces nonzero electric conductivity along the direction of the field, thus transforming the QCD vacuum from an insulator into an anisotropic conductor [8]. In our paper we argue that there is a chance that a stronger magnetic field may be able to make the QCD vacuum unstable toward creation of a superconducting state. We would like to stress that we discuss here the electromagnetic superconductivity which should be distinguished from the color superconductivity in the dense matter [9]. We discuss a superconducting state which may presumably be formed in the cold vacuum, i.e. at zero temperature and density.

Basically, we follow the works of Ambjørn, Nielsen, and Olesen on two subjects: (i) on the condensate of color magnetic flux tubes ("spaghetti states") [10] created by an unstable gluonic mode in the QCD vacuum [11]; and (ii) on the condensation of the W bosons in the standard electroweak model due to sufficiently strong external magnetic field [12,13]. The key idea of Refs. [10–13] is that the vacuum of charged vector particles is unstable in the background of a sufficiently strong magnetic field provided these particles have anomalously large gyromagnetic ratio $g = 2$. The large value of g guarantees that the magnetic moment of such particles is too large to withstand a spontaneous condensation at sufficiently strong external magnetic fields.

As we have mentioned, there are at least two examples of such instabilities. A strong enough chromomagnetic field leads to the instability of the gluonic QCD vacuum since the gluon is the vector particle with the (color) gyromagnetic ratio $g = 2$ [11]. As a result of the instability, a spaghetti of the chromomagnetic flux tubes is formed. These flux tubes tend to arrange themselves into a lattice structure similar to the Abrikosov lattice which is realized in a mixed state of a type-II superconductor subjected to a near-critical external magnetic field [10].

The second example is suggested to be realized in the standard electroweak model. The gyromagnetic ratio of the W boson is also large, $g = 2$, so that in the strong magnetic field the vacuum of the electroweak theory is unstable toward formation of the condensate of the W bosons. The

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W condensate is accompanied by a similar lattice vortex state [12,13]. Note that in the second example the external field is the electromagnetic field and not the color (gluon) one.

Our work is based on the fact that the ρ meson is the charged vector particle with the gyromagnetic ratio $g = 2$ so that this particle may condense in a background of strong enough magnetic field. It is important to stress that in all discussed cases of the spontaneous condensation—we mentioned the gluons in QCD [11], the W bosons in the electroweak theory [12,13], and the ρ mesons in QCD (this paper)—the condensation takes place in the *vacuum* at zero temperature (as opposed to dense and/or hot environment).

The structure of the paper is as follows. In Sec. II we outline the basic idea of the ρ-meson condensation. In the same section we argue that the ρ mesons are (at least, partially) stabilized by the strong magnetic field background. This is an important property which should make the ρ condensate "intrinsically" stable against decays of the ρ mesons (the ρ mesons have a very short lifetime in the absence of the external fields). In Sec. III we describe the quantum electrodynamics of the ρ mesons. Section IV is devoted to a short overview of basic features of the Ginzburg-Landau model of the superconductivity (homogeneity, isotropy, effects of the magnetic field, the Abrikosov vortices, the Meissner effect, the London equations). In Sec. V we discuss the same features in the superconducting state of condensed ρ mesons in QCD and find a few similarities and many surprising dissimilarities with the ordinary superconductivity. The last section is devoted to our conclusions.

II. ρ MESONS IN STRONG MAGNETIC FIELD: CONDENSATION AND LONGER LIFE

A. Condensation of charged ρ mesons

The basic idea of our work is as follows. Consider a charged relativistic spin-s particle moving in a background of an external magnetic field. Without loss of generality we assume that the magnetic field $\vec{B}_{\rm ext} = (0, 0, B_{\rm ext})$ is directed along the z axis, $B_{\rm ext} \geq 0$ and we consider spatially uniform and time-independent external fields only. The energy levels ε of the free particle of the mass m in the magnetic field are characterized by three parameters: the nonnegative integer $n \geq 0$, the spin projection on the field's axis $s_z = -s, \ldots, s$, and the particle momentum along the field's axis, p_z:

$$\varepsilon^2_{n,s_z}(p_z) = p_z^2 + (2n - 2s_z + 1)eB_{\rm ext} + m^2. \qquad (1)$$

In this work we consider the charged particles, pions ($s = 0$) and the vector particles, ρ mesons ($s = 1$), for reasons that will be clear later. For a moment, we assume that these particles are free, so that their (squared) minimal effective masses, corresponding to lowest energy states (1) with $p_z = 0$, are respectively,

$$m^2_{\pi^\pm}(B_{\rm ext}) = m^2_{\pi^\pm} + eB_{\rm ext}, \qquad (2)$$

$$m^2_{\rho^\pm}(B_{\rm ext}) = m^2_{\rho^\pm} - eB_{\rm ext}. \qquad (3)$$

The zero-field vacuum masses of the $\pi^\pm$ and $\rho^\pm$ mesons are, respectively [14],

$$m_\pi = 139.6\ \text{MeV}, \qquad m_\rho = 775.5\ \text{MeV}. \qquad (4)$$

Equation (3) implies that the lowest energy of the charged ρ meson in the external magnetic field may become purely imaginary if the magnetic field exceeds the following critical value:

$$B_c = \frac{m_\rho^2}{e} \approx 10^{16}\ \text{Tesla}. \qquad (5)$$

This observation indicates that the strong magnetic field ($B_{\rm ext} > B_c$) makes the QCD vacuum unstable toward condensation of the charged ρ mesons. This new QCD effect is very similar to the magnetic-field-induced condensation of the W bosons which was predicted by Ambjørn and Olesen [12,13]. The behavior of the lowest mass (3) of the charged $\rho^\pm$ meson in the region $0 \leq B_{\rm ext} \leq B_c$ is shown in Fig. 1 by the solid line.

The subtle point of Eq. (3) [and of Eq. (1) for $s = 1$ as well] is that the gyromagnetic ratio of the vector $\rho^\pm$ meson is set to be $g = 2$. In fact, this g-factor is "anomalously" large compared to the standard gyromagnetic ratio $g_{\rm min} = 1$ of a charged vector particle which is minimally coupled to the electromagnetism. Notice, that it is the anomalous gyromagnetic ratio $g_W = 2$ which drives the condensation of the W bosons in the strong magnetic field [12,13]. The

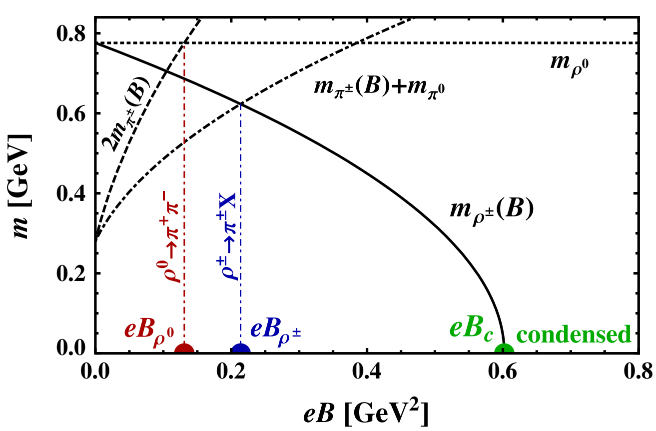

FIG. 1 (color online). Masses of the lowest ρ-meson eigenstates and of the products of their dominant decay modes as functions of the external magnetic field $B \equiv B_{\rm ext}$. The left (red) point and the middle (blue) point mark the onsets of the $\pi^\pm$-stability regions for the neutral (11) and charged (9) ρ mesons, respectively. The right (green) point marks the critical field B_c which corresponds to the onset of the $\rho^\pm$ condensation (5).

large g-factor for the W boson is a direct consequence of the non-Abelian nature of the electroweak gauge group.

As for the ρ mesons, the electrodynamics of these particles has also elements of a non-Abelian structure which is visible in phenomenological Lagrangians [15–17]. The vector dominance hypothesis [18] as well as the QCD sum rules [19] point out that the g-factor of the ρ mesons is 2. We discuss the quantum electrodynamics for these vector mesons in more detail in Sec. III.

B. Larger lifetime of charged and neutral ρ mesons

In the absence of the external magnetic field both the charged and neutral ρ mesons are very unstable particles characterized by the mean lifetime $\tau_\rho \approx 4.5 \times 10^{-24}$ s ≈ 1.35 fm$/c$ which corresponds to the full width [14]

$$\Gamma_{\rho\to\mathrm{all}} = 149.1 \pm 0.8\ \mathrm{MeV}. \tag{6}$$

Thus, one may incorrectly conclude that if even the ρ-meson condensate is formed at the strong magnetic fields, then it will be unstable due to the intrinsic instability of the ρ mesons themselves. Below we show that this statement is incorrect.

1. Charged vector mesons

Consider first the charged vector mesons. All known decays of the $\rho^\pm$ mesons are going via the modes [14]:

$$\rho^\pm \to \pi^\pm X, \qquad X = \pi^0, \eta, \gamma, \pi\pi\pi. \tag{7}$$

The fraction of the primary decay mode, $X = \pi^0$, is greater than 99%.

As the strength of the background magnetic field increases, the product of the decay, the charged pion [which is always created in the known decay modes of the $\rho^\pm$ mesons (7)] becomes heavier (2) while the decaying particle, the lowest state of the $\rho^\pm$ meson, becomes lighter (3). Obviously, at a certain magnetic field $B_{\rho^\pm}$ the masses of the initial and final states in the dominant channel, $\rho^\pm \to \pi^\pm\pi^0$, should become equal,

$$m_{\rho^\pm}(B_{\rho^\pm}) = m_{\pi^\pm}(B_{\rho^\pm}) + m_{\pi^0}, \tag{8}$$

and the fast strong decays (7) of the charged ρ mesons should eventually become impossible due to obvious kinematical reasons. The strength of this "$\pi^\pm$-stabilizing" field is approximately 3 times weaker[1] compared to the critical field of the ρ condensation (5),

$$B_{\rho^\pm} = \frac{1}{2e}[m_\rho^2 - m_\pi^2 - m_\pi(m_\pi^2 + 2m_\rho^2)^{1/2}] \simeq 0.36 B_c. \tag{9}$$

[1]Here and below we always neglect the difference between the masses of the charged $\pi^\pm$ and $\rho^\pm$ mesons, and their neutral counterparts, π^0 and ρ^0, respectively.

The left- and right-hand sides of Eq. (8) are shown by the solid and dot-dashed lines in Fig. 1. The point of the intersection of these lines gives us the critical field (9).

At $B > B_{\rho^\pm}$ the charged ρ mesons may in principle decay via other slower (and undetected so far) channels that avoid fast gluon-mediated $\pi^\pm$ production. On the other side, the QCD string (which confines the quarks and antiquarks into mesons and baryons) is partially stabilized by the external magnetic field [20]. Thus, in the sufficiently strong magnetic field the allowed modes of the decays of the charged ρ mesons should be much slower. As a result, the lifetime of the $\rho^\pm$ mesons should be much longer compared to the lifetime of these particles in the absence of the external magnetic field.

One can also make a qualitative prediction for the behavior of the spectral function of the charged ρ meson in the strong magnetic field. Expected behavior of the lowest-mass peak is plotted in Fig. 2 as a function of an invariant mass. At zero magnetic field the $\rho^\pm$ meson is seen as a broad resonance (the right peak in Fig. 2). As we switch on the background magnetic field, the single peak should split into multiple peaks corresponding to different levels of the charged vector particle ($s = 1$) in the external magnetic field (1). The increase of the strength of the background magnetic field leads to the kinematical suppression of the ρ meson decay modes and, consequently, to a narrower lowest-mass peak in the corresponding spectral function (the peak in the middle of Fig. 2). At $B \geq B_c$, the onset of the condensation of the ρ mesons occurs. This effect can be seen as the appearance of a singularity of the δ-function–type located at the zero invariant mass (the left peak in Fig. 2).

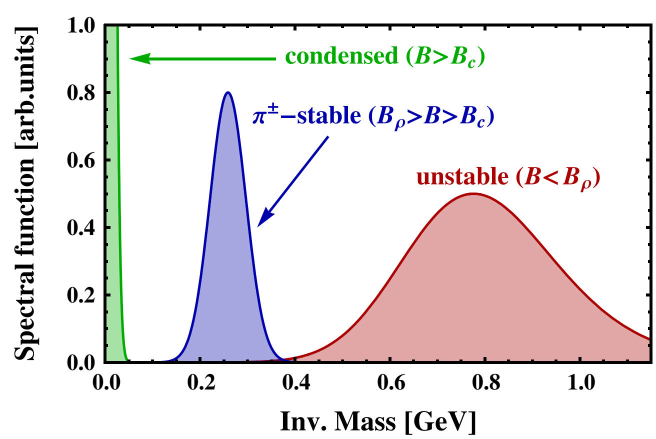

FIG. 2 (color online). Prediction: a qualitative effect of the external magnetic field on the lowest-mass peak in the spectral function of the $\rho^\pm$ mesons vs the invariant mass. As the magnetic field strength $B \equiv B_{\mathrm{ext}}$ increases, the broad peak in the unstable low-B phase (right) turns into a much narrower peak in the $\pi^\pm$-stable phase (middle). At the onset of the condensation of the ρ mesons the peak transforms into the δ-function–like singularity located at the vanishing invariant mass. Features at higher invariant masses are not shown.

2. Neutral vector mesons

Similarly to its charged counterpart, the neutral ρ^0 meson should also be $\pi^\pm$-stabilized in a sufficiently strong magnetic field background. The primary channel of the ρ^0 decay $\rho^0 \to \pi^+\pi^-$ (it corresponds to more than 99% of the decays) becomes inoperative due to the same kinematical reasons provided $B_{\rm ext} \geq B_{\rho^0}$, where

$$m_{\rho^0}(B_{\rho^0}) = 2m_{\pi^\pm}(B_{\rho^0}). \tag{10}$$

The ρ^0 mass is expected to be practically independent of the magnetic field,[2] $m_{\rho^0}(B) \simeq m_{\rho^0}(B=0)$, so that

$$B_{\rho^0} = \frac{m_\rho^2 - 4m_\pi^2}{4e} \simeq 0.22 B_c. \tag{11}$$

The left- and right-hand sides of Eq. (10) are shown by the dashed and dotted lines in Fig. 1. The intersection of these lines occurs at the critical field (11).

In the absence of the external magnetic field the neutral ρ^0 meson has also other decay channels which do not involve the production of the charged $\pi^\pm$ pions. Such decay modes, however, are much slower compared to the primary decay $\pi^\pm$ modes like $\rho^0 \to \pi^+\pi^-$. For example, the most effective $\pi^\pm$-less decay of ρ^0 is $\rho^0 \to \pi^0\gamma$, with the width

$$\Gamma_{\rho^0\to\pi^0\gamma} = 0.089 \pm 0.012\ \text{MeV}, \tag{12}$$

which is more than 3 orders of magnitude narrower compared to the full width (6). In this paper we are not discussing how the $\pi^\pm$-less decays are affected by the strong magnetic field. However, it is clear that the electromagnetically driven decay channels should be slower compared to the strongly mediated ones. Thus, there are good kinematical reasons to believe that the prolongation of the ρ-meson life—induced by the strong magnetic field background—should be substantial.

As for the evolution of the ρ^0 peak in the spectral function, we expect that the background magnetic field makes it narrower, while its position is largely unaffected by the external field. As will be clear from the results reported below, at $B_{\rm ext} > B_c$ we may expect an appearance of a singular peak at zero ρ^0-meson mass due to (quite weak, though) condensation of the neutral ρ^0 mesons.

3. Reversed decays and effects of chiral condensates

The estimations of the values of the critical fields (5), (9), and (11) are obviously approximate, as one may expect systematic corrections coming from other effects of the strong magnetic field on the mass spectrum of the mesons. For example, in our qualitative considerations we do not take into account effects of mixing of the ρ^0 meson with the neutral ω and φ mesons. We also neglect influence of the magnetic field on the ρ mesons and pions at the quark level. However, the latter effect may be estimated, at least partially. Indeed, the background magnetic field enhances the chiral symmetry breaking [21]. According to a leading order of the chiral perturbation theory [22] (confirmed by the results of the recent lattice simulations [23]) the chiral condensate Σ is a linearly increasing function of the strength of the external magnetic field $B_{\rm ext}$:

$$\Sigma(B_{\rm ext}) = \Sigma(0)\left(1 + \frac{\ln 2}{32\pi^2 f_\pi^2} e B_{\rm ext}\right), \tag{13}$$

where $f_\pi = 92.4$ MeV is the pion decay constant. At the critical fields (5), (9), and (11) the corrections (13) to the chiral condensate are 16%, 6%, and 3%, respectively. We expect that uncertainties in our estimations of the critical values (11), (9), and (5) may be of the same scale at least.

One should also note that our considerations imply that at $B > B_{\rho^\pm}$ ($B > B_{\rho^0}$) the charged pions may decay into the charged (neutral) ρ mesons. The statement, that the presence of the strong enough magnetic field interchanges the decaying and created particles, should not be disappointing. For example, it is known that the magnetic field may reverse the β decay of the neutron because at the background magnetic fields with the strength greater than $5 \cdot 10^{14}\ \text{T} \approx 0.1 m_e^2/e$ the proton becomes heavier than the neutron. As a consequence, the proton may decay into the neutron by positron emission [24].

Summarizing this section, the charged and neutral ρ mesons are very unstable particles provided the magnetic field is weaker than the critical values $B_{\rho^\pm}$, Eq. (9), and B_{ρ^0}, Eq. (11), respectively (Fig. 1). We expect, however, that as the external field becomes stronger than these critical values, the corresponding ρ mesons get stabilized with respect to the vast majority of the strong decays which are going via the production of the $\pi^\pm$ mesons (we call these regions of the magnetic field intensities the "$\pi^\pm$-stable" phases both for the charged and neutral ρ mesons). If the background field surpasses the critical value (5), $B_{\rm ext} > B_c$, the condensation of the charged $\rho^\pm$ mesons should occur. Below we show that at the same point $B_{\rm ext} = B_c$ the neutral ρ^0 mesons may simultaneously form an inhomogeneous superfluid.

III. ELECTRODYNAMICS OF ρ MESONS

A. The DSGS Lagrangian

The self-consistent quantum electrodynamics for the ρ mesons was recently constructed by Djukanovic, Schindler, Gegelia, and Scherer (DSGS) in Ref. [16] starting from an effective Lagrangian for vector mesons developed by Weinberg [15] long ago. The chiral, Lorentz and

[2] Here we ignore a weak coupling of the magnetic field to the magnetic dipole moment of the ρ^0 meson. This coupling makes the critical field (11) slightly stronger.

discrete symmetries of the Weinberg Lagrangian were extended to the Maxwellian $U(1)$ sector by adding all allowed interactions with electromagnetic fields. In terms of the renormalized fields the bosonic part of the DSGS Lagrangian reads as follows [16]:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\rho^\dagger_{\mu\nu}\rho^{\mu\nu} + m_\rho^2\rho^\dagger_\mu\rho^\mu - \frac{1}{4}\rho^{(0)}_{\mu\nu}\rho^{(0)\mu\nu} + \frac{m_\rho^2}{2}\rho^{(0)}_\mu\rho^{(0)\mu} + \frac{e}{2g_s}F^{\mu\nu}\rho^{(0)}_{\mu\nu}, \quad (14)$$

where A_μ is the photon field, $\rho_\mu = (\rho^{(1)}_\mu - i\rho^{(2)}_\mu)/\sqrt{2}$ and $\rho^{(0)}_\mu \equiv \rho^{(3)}_\mu$ are, respectively, the fields of the (negatively) charged and neutral vector mesons[3] characterized by the mass m_ρ. The DSGS Lagrangian possesses the $U(1)$ gauge invariance

$$U(1)_{\rm em}: \begin{cases} \rho^{(0)}_\mu(x) & \to \rho^{(0)}_\mu(x), \\ \rho_\mu(x) & \to e^{i\omega(x)}\rho_\mu(x), \\ A_\mu(x) & \to A_\mu(x) + \partial_\mu\omega(x), \end{cases} \quad (15)$$

where "em" represents "electromagnetic."

The tensor quantities in (14) are

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \quad (16a)$$

$$f^{(0)}_{\mu\nu} = \partial_\mu\rho^{(0)}_\nu - \partial_\nu\rho^{(0)}_\mu, \quad (16b)$$

$$\rho^{(0)}_{\mu\nu} = f^{(0)}_{\mu\nu} - ig_s(\rho^\dagger_\mu\rho_\nu - \rho_\mu\rho^\dagger_\nu), \quad (16c)$$

$$\rho_{\mu\nu} = D_\mu\rho_\nu - D_\nu\rho_\mu, \quad (16d)$$

and the covariant derivative is

$$D_\mu = \partial_\mu + ig_s\rho^{(0)}_\mu - ieA_\mu. \quad (17)$$

Equation (16) indicates that $\rho^-_\mu \equiv \rho_\mu$ and $\rho^+_\mu \equiv \rho^\dagger_\mu$ meson fields carry the electric charges $-e$ and $+e$ respectively (here $e = |e|$ is the elementary electric charge). The coupling constant g_s can be estimated [16,17] from the Kawarabayashi-Suzuki-Riadzuddin-Fayyazuddin relation [25]:

$$g_s \equiv g_{\rho\pi\pi} = \frac{m_\rho}{\sqrt{2}f_\pi} = 5.88, \quad (18)$$

so that $g_s \gg e \equiv \sqrt{4\pi\alpha_{\rm em}} \approx 0.303$.

The most important fact for us is that the last term of the DSGS Lagrangian (14) describes a nonminimal coupling of the ρ mesons to the electromagnetic field. This term has two parts,

$$\delta\mathcal{L} = \delta\mathcal{L}^{(0)} + \delta\mathcal{L}^{\rm ch}, \quad (19)$$

$$\delta\mathcal{L}^{(0)} = \frac{e}{2g_s}(\partial_\mu\rho^{(0)}_\nu - \partial_\nu\rho^{(0)}_\mu)F^{\mu\nu}, \quad (20)$$

$$\delta\mathcal{L}^{\rm ch} = ie\rho_\mu\rho^\dagger_\nu F^{\mu\nu}. \quad (21)$$

[3]We denote the field of the neutral meson as $\rho^{(0)}(x)$ in order to discriminate it from the timelike component $\rho^0(x)$ of the charged $\rho^\pm$-meson field.

where the first part $\delta\mathcal{L}^{(0)}$ corresponds to the coupling of the electromagnetic field to the magnetic dipole moment of the ρ^0 meson, while the second part $\delta\mathcal{L}^{\rm ch}$ describes the nonminimal coupling of the charged $\rho^\pm$ mesons to the electromagnetic field. The presence of the former may lead to an instability of the vacuum of the neutral vector particles ($\rho^{(0)}$ mesons in our case) [26], while the latter implies the anomalous gyromagnetic ratio ($g = 2$) of the charged $\rho^\pm$ mesons, so that the magnetic dipole moment of the $\rho^\pm$ mesons is

$$\vec{\mu}_{\rho^\pm} = \pm 2\cdot\frac{e}{2m_\rho}\vec{s} \quad (22)$$

(here $\vec{s}$ is the meson's spin). It is the coupling (21) that plays a dominant effect in our paper while the interaction (20) makes a subleading contribution.

As we have already discussed in Sec. II A, spin-one particles with the gyromagnetic ratio $g = 2$ in strong enough external magnetic field should experience a tachyonic instability toward development of a Bose-Einstein condensate. Since the condensed particles are charged, the condensate should be superconducting, and this fact is our central observation which is discussed in detail below.

B. Equations of motion

A variation of the DSGS Lagrangian (14) with respect to the electromagnetic potential A_μ provides us with the Maxwell-type equation of motion,

$$\partial^\nu F_{\nu\mu} = -J_\mu, \quad (23)$$

where the electric current J_μ contains two contributions,

$$J_\mu = J^{\rm ch}_\mu + J^{(0)}_\mu, \quad (24)$$

coming from the charged and neutral mesons,

$$J^{\rm ch}_\mu = ie[\rho^{\nu\dagger}\rho_{\nu\mu} - \rho^\nu\rho^\dagger_{\nu\mu} + \partial^\nu(\rho^\dagger_\nu\rho_\mu - \rho^\dagger_\mu\rho_\nu)] \equiv ie[(D_\mu\rho^\nu)^\dagger\rho_\nu - \rho^{\nu\dagger}D_\mu\rho_\nu + \partial^\nu(\rho^\dagger_\nu\rho_\mu - \rho^\dagger_\mu\rho_\nu) + \rho^\dagger_\nu D^\nu\rho_\mu - (D^\nu\rho_\mu)^\dagger\rho_\nu], \quad (25a)$$

$$J^{(0)}_\mu = -\frac{e}{g_s}\partial^\nu f^{(0)}_{\nu\mu}, \quad (25b)$$

respectively. The currents (25) are separately conserved,

$$\partial^\mu J_\mu = \partial^\mu J^{\rm ch}_\mu = \partial^\mu J^{(0)}_\mu = 0. \quad (26)$$

A variation of the DSGS Lagrangian (14) with respect to the field $\rho^{(0)}_\mu$ gives us the second equation of motion,

$$\partial^\nu\rho^{(0)}_{\nu\mu} + m_\rho^2\rho^{(0)}_\mu - \frac{e}{g_s}\partial^\nu F_{\nu\mu} - ig_s(\rho^\dagger_{\mu\nu}\rho^\nu - \rho_{\mu\nu}\rho^{\nu\dagger}) = 0. \quad (27)$$

It can be rewritten as follows [we used (23)–(25)]:

$$(\partial^\nu\partial_\nu + m^2_{\rho^{(0)}})\rho^{(0)}_\mu - \partial_\mu\partial^\nu\rho^{(0)}_\nu - \frac{g_s}{e}J^{\rm ch}_\mu = 0, \quad (28)$$

so that Eq. (26) gives us

$$\partial^{\mu}\rho^{(0)}_{\mu} = 0. \tag{29}$$

Equation (28) provides us with the mass of the neutral $\rho^{(0)}$ meson,

$$m_0 \equiv m_{\rho^{(0)}} = m_{\rho}\left(1 - \frac{e^2}{g_s^2}\right)^{-(1/2)}. \tag{30}$$

Using Eqs. (24), (25), and (28) one can get a well-known relation (emerged originally in the scope of vector dominance models long time ago [18]) between the electromagnetic current J_μ and the neutral meson field $\rho^{(0)}_{\mu}$,

$$J_\mu = \frac{e m_0^2}{g_s}\rho^{(0)}_{\mu} \tag{31}$$

(notice that in our notations $e = |e| > 0$).

The third equation of motion is

$$D^{\nu}\rho_{\nu\mu} + m_\rho^2\rho_\mu + i(g_s\rho^{(0)}_{\mu\nu} - eF_{\mu\nu})\rho^{\nu} = 0. \tag{32}$$

Using the identity $[D_\mu, D_\nu] = i(g_s f^{(0)}_{\mu\nu} - eF_{\mu\nu})$, one gets

$$[(D^{\alpha}D_{\alpha} + m_\rho^2)g_{\mu\nu} - D_\mu D_\nu + i(g_s\rho^{(0)}_{\mu\nu} + g_s f^{(0)}_{\mu\nu} - 2eF_{\mu\nu})]\rho^{\nu} = 0. \tag{33}$$

Equations (31) and (32) imply that

$$(\partial_\mu - ieA_\mu)\rho^{\mu} \equiv \left[D_\mu - \frac{i g_s^2}{e m^2_{\rho^{(0)}}} J_\mu\right]\rho^{\mu} = 0. \tag{34}$$

The linear part of Eq. (33) gives us the mass of the charged $\rho^{\pm}$ meson,

$$m_{\rho^{\pm}} = m_\rho. \tag{35}$$

The neutral vector $\rho^{(0)}$ meson is heavier compared to its charged counterpart $\rho^{\pm}$. According to Eqs. (18), (30), and (35), the difference in the masses is very small [16],

$$\delta m_\rho \equiv m_0 - m_{\rho^{\pm}} \simeq \frac{4\pi\alpha_{\rm em} f_\pi^2}{m_\rho} \approx 1\ \text{MeV}. \tag{36}$$

This mass difference is consistent with the available experimental bounds [14].

IV. EXAMPLE: GINZBURG-LANDAU MODEL

In Sec. V we analyze the condensation of the ρ mesons in the strong magnetic field, starting from the phenomenological field-theoretical DSGS Lagrangian (14). However, before going into the details of the ρ condensation in QCD, it is very useful to discuss a few basic properties of conventional superconductivity in the condensed matter physics. Below we concentrate on the Ginzburg-Landau (GL) model which provides us with a simplest phenomenological description of the superconductivity.

A. The Ginzburg-Landau Lagrangian

The relativistic version of the GL Lagrangian for a superconductor is

$$\mathcal{L}_{\rm GL} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\mathfrak{D}_\mu\Phi)^*\mathfrak{D}^{\mu}\Phi - \lambda(|\Phi|^2 - \eta^2)^2, \tag{37}$$

where $\mathfrak{D}_\mu = \partial_\mu - ieA_\mu$ is the covariant derivative and Φ is the complex scalar field carrying a unit[4] electric charge e.

The ground state of the model (37) is characterized by the homogeneous condensate of the scalar field, $\Phi_0 = \langle\Phi\rangle$ with $|\Phi_0| = \eta$. In the condensed state the mass of the scalar excitation, $\delta\Phi = \Phi - \Phi_0$, and the mass of the photon field A_μ are, respectively, as follows:

$$m_\Phi = \sqrt{4\lambda}\,\eta, \qquad m_A = \sqrt{2}e\eta. \tag{38}$$

The classical equations of motion of the GL model are

$$\mathfrak{D}_\mu\mathfrak{D}^{\mu}\Phi + 2\lambda(|\Phi|^2 - \eta^2)\Phi = 0, \tag{39}$$

$$\partial_\nu F^{\nu\mu} + J^{\mu}_{\rm GL} = 0, \tag{40}$$

where the electric current is

$$J^{\mu}_{\rm GL} = -ie[\Phi^*\mathfrak{D}^{\mu}\Phi - (\mathfrak{D}^{\mu}\Phi)^*\Phi]. \tag{41}$$

B. Destructive role of magnetic field

The superconducting state in the GL model is completely destroyed ($\Phi = 0$) in a background of the strong magnetic field $B_{\rm ext}$, if the strength of the field exceeds the critical value

$$B_c^{\rm GL} = \frac{m_\Phi^2}{2e} \equiv \frac{2\lambda}{e}\eta^2. \tag{42}$$

Let us assume that $B_{\rm ext} = F_{12}$ is the only nonvanishing component of the field-strength tensor. Consider the case when the uniform time-independent magnetic field $B_{\rm ext}$ is slightly smaller than the critical value (42), $B < B_c^{\rm GL}$, so that

$$1 - \frac{B_{\rm ext}}{B_c^{\rm GL}} \ll 1. \tag{43}$$

Then the condensate is very small

$$|\Phi_0(B)| \ll \eta \tag{44}$$

and Eq. (39) can be linearized,

$$\{(\mathfrak{D}_1 - i\mathfrak{D}_2)(\mathfrak{D}_1 + i\mathfrak{D}_2) + e[B_c - B(x)]\}\Phi = 0, \tag{45}$$

where $B(x)$ is the field inside the superconductor (here we consider static and z-independent solutions which

[4]Without loss of generality, it is convenient to consider the singly charged bosons Φ instead of the usual doubly charged bosons.

correspond to a lowest energy of the system). In the vicinity of the critical field $B \simeq B_{\rm ext} \simeq B_c$, so that Eq. (45) reduces to the following equation for the condensate Φ:

$$\mathfrak{D}\,\Phi \simeq 0 \quad \text{with} \quad \mathfrak{D} = \mathfrak{D}_1 + i\mathfrak{D}_2. \tag{46}$$

The magnetic field *destroys* the superconductivity in the ordinary superconductor. On the contrary, we show below that a strong enough magnetic field should *induce* the superconductivity of the charged ρ mesons in the QCD vacuum.

C. Abrikosov lattice of vortices in mixed state

The GL model (37) admits a topological stringlike solution to the classical equations of motion (39) and (40), which is known as the Abrikosov vortex [27]. The Abrikosov vortices are formed when the superconductors are subjected to external magnetic fields.

A single Abrikosov vortex carries the quantized magnetic flux (remember that we consider the condensed bosons Φ which carry the electric charge e and not $2e$):

$$\int d^2 x_\perp B(x_\perp) = \frac{2\pi}{e}, \tag{47}$$

where the integral of the vortex magnetic field B is taken over the two-dimensional coordinates $x_\perp = (x_1, x_2)$ of the plane which is transverse to the infinitely long, straight, and static vortex. In the original solution, the scalar field of the unit-flux vortex is singular at the vortex center,

$$\Phi(x_\perp) \propto |x_\perp| e^{i\varphi} \equiv x_1 + i x_2, \tag{48}$$

where φ is the azimuthal angle in the transverse plane, and $|x_\perp|$ is the distance from the vortex center. Equation (48) corresponds to small $|x_\perp|$: $m_\Phi |x_\perp| \ll 1$ and $m_A |x_\perp| \ll 1$.

In a type-II superconductor [in which $m_\Phi > m_A$ or, according to (38), $2\lambda > e^2$] the Abrikosov vortices repel each other. If the external field is strong enough [but lower than the critical value (42)] then multiple Abrikosov vortices are created. Because of the mutual repulsion, the vortices arrange themselves in a regular structure known as the Abrikosov lattice [28,29]. Since the normal (nonsuperconducting) phase is restored inside the vortices, the Abrikosov lattice corresponds to a "mixed state" of the superconductor, in which both normal and superconducting states of matter are present.

There are various types of the Abrikosov lattices which are characterized by different energies [29]. The stable lattice corresponds to a minimal energy of the system. If the magnetic field $B_{\rm ext}$ approaches the critical magnetic field (42) from below, then the simplest lattice type is given by the square lattice solution of Eq. (46),

$$\Phi(x_1, x_2) = C_0 \exp\left\{-\frac{(eB_{\rm ext})^2}{2} x_1^2\right\} \cdot \sum_{n=-\infty}^{+\infty} \exp\left\{-\pi n^2 + 2\pi n \frac{x_1 + i x_2}{L_B}\right\}. \tag{49}$$

In this equation the parameter C_0 is independent of the transverse coordinates $x_\perp$. The intervortex distance L_B is expressed via the magnetic length ℓ_B,

$$L_B = \sqrt{2\pi}\ell_B, \qquad \ell_B = \frac{1}{\sqrt{eB_{\rm ext}}}. \tag{50}$$

The area of the elementary square cell (i.e., of a cell which contains one Abrikosov vortex) is $L_B^2 \equiv 2\pi \ell_B$. The absolute value of the condensate, $|\Phi(x_\perp)|$, has a square symmetry in the solution (49) and the vortices are located at the sites of the square lattice,

$$\frac{x_i}{L_B} = n_i + \frac{1}{2}, \qquad n_i \in \mathbb{Z}, \qquad i = 1, 2. \tag{51}$$

In this case the distance between the vortex centers is L_B. At the points (51) the condensate $\Phi(x_1, x_2)$ vanishes exactly and in the vicinity of these points the scalar field (49) follows the behavior of Eq. (48).

As we will see below, the pure superconducting state cannot be formed in the ρ meson superconductor contrary to the ordinary superconductor. Instead, the Abrikosov lattice state is created.

D. Homogeneous isotropic superconductivity

Let us now apply a very weak external electromagnetic field to the superconductor. Neglecting the effect of the external field on the condensate Φ_0, one gets from (41)

$$\partial^\mu J^\nu_{\rm GL} - \partial^\nu J^\mu_{\rm GL} = -m_A^2 F^{\mu\nu}, \tag{52}$$

where m_A is given in Eq. (38). Setting $\mu = 0$ and $\nu = i$ in Eq. (52) one gets the first London relation for an electrically neutral ($J_0 = 0$) superconductor

$$\frac{\partial \vec{J}_{\rm GL}}{\partial t} = m_A^2 \vec{E}, \tag{53}$$

where $E^i \equiv -F^{0i}$ is the time-independent and uniform electric field. Equation (53) implies a linear growth of the electric current in the external electric field, thus indicating a vanishing electric resistance of the superconducting state.

In the long-wavelength limit, $|\vec{q}| \to 0$, the weak electric field $\vec{E}(\vec{x}, t) = \vec{E}_0 e^{i(\vec{x}\cdot\vec{q} - \omega t)}$ induces the local current

$$J_k(\vec{x}, t; \omega) = \sum_{k=1}^{3} \sigma_{kl}(\omega) E_l(\vec{x}, t), \tag{54}$$

where $\sigma_{kl} = {\rm Re}\sigma_{kl} + i{\rm Im}\sigma_{kl}$ is the complex electric conductivity. The London equation (53) indicates that

$$\sigma_{kl}(\omega) = \sigma_{kl}^{\rm sing}(\omega) + \sigma_{kl}^{\rm reg}(\omega), \tag{55}$$

where the first contribution is a singular isotropic part associated with the superconducting state,

$$\sigma_{kl}^{\rm sing}(\omega) = \frac{\pi m_A^2}{2}\left[\delta(\omega) + \frac{2i}{\pi\omega}\right]\delta_{kl}. \tag{56}$$

The regular part $\sigma^{\rm reg}$ accounts for all other contributions to the conductivity.

It is clear that the superconductivity described by Eq. (53) is homogeneous (it is independent of the spatial coordinate) and isotropic (it is independent of the direction). On the contrary, we will see below that a strong enough magnetic field induces *inhomogeneous* and *anisotropic* superconductivity of the charged ρ mesons in the QCD vacuum.

E. Meissner effect

The spatial components of Eq. (52) give us the second London relation,

$$\vec{\partial} \times \vec{J}_{\rm GL} = -m_A^2 \vec{B}, \tag{57}$$

so that in the absence of the external electric field ($\vec{E} = 0$) one of the Maxwell equations (40), $\vec{J}_{\rm GL} = \vec{\partial} \times \vec{B}$, implies

$$(-\Delta + m_A^2)\vec{B} = 0. \tag{58}$$

This equation indicates that the photon inside the superconductor acquires the mass m_A, Eq. (38). Consequently, the superconductor tends to expel the external magnetic field (''the Meissner effect''). Physically, the Meissner effect is realized because the external magnetic field induces the circulating superconducting currents (57) inside the superconductor. These currents, in turn, screen the external magnetic field since they induce their own magnetic field which is opposite to the external one (here we always assume that $B_{\rm ext} < B_c$).

A weak magnetic field which is parallel to the boundary of the superconductor is always screened inside the bulk of the superconductor. The perpendicular magnetic field may penetrate the superconductor and create a mixed phase of the Abrikosov vortices.

As we will see below, the second London equation (57) is not realized in the superconducting phase of the QCD vacuum contrary to the conventional superconductor. Consequently, the Meissner effect cannot be formulated in a self-consistent way in the suggested superconducting phase of QCD.

V. CONDENSATION OF ρ MESONS

A. Homogeneous approximation

The energy density of the DSGS model (14) is

$$\begin{aligned}\epsilon \equiv T_{00} &= \frac{1}{2}F_{0i}^2 + \frac{1}{4}F_{ij}^2 + \frac{1}{2}(\rho_{0i}^{(0)})^2 \\ &\quad + \frac{1}{4}(\rho_{ij}^{(0)})^2 + \frac{m_\rho^2}{2}[(\rho_0^{(0)})^2 + (\rho_i^{(0)})^2] + \rho_{0i}^\dagger \rho_{0i} \\ &\quad + \frac{1}{2}\rho_{ij}^\dagger \rho_{ij} + m_\rho^2(\rho_0^\dagger \rho_0 + \rho_i^\dagger \rho_i) - \frac{e}{g_s}F_{0i}\rho_{0i}^{(0)} \\ &\quad - \frac{e}{2g_s}F_{ij}\rho_{ij}^{(0)},\end{aligned} \tag{59}$$

where $T_{\mu\nu}$ is the energy-momentum tensor,

$$T_{\mu\nu} = 2\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} - \mathcal{L} g_{\mu\nu}. \tag{60}$$

In order to understand the phase structure of the ρ mesons in the background magnetic field, it is useful to study first the homogeneous field approximation. To this end we ignore the kinetic terms $\partial_\mu \rho_\nu^{(0)} = 0$ and $D_\mu \rho_\nu = 0$ in Eq. (59). The remaining (potential) part of the energy density in the external uniform magnetic field $B_{\rm ext}$ is

$$\begin{aligned}\epsilon_0(\rho_\mu, \rho_\nu^{(0)}) &= \frac{1}{2}B_{\rm ext}^2 + \frac{g_s^2}{4}[i(\rho_\mu^\dagger \rho_\nu - \rho_\nu^\dagger \rho_\mu)]^2 \\ &\quad + ieB_{\rm ext}(\rho_1^\dagger \rho_2 - \rho_2^\dagger \rho_1) + \frac{m_\rho^2}{2}(\rho_\mu^{(0)})^2 \\ &\quad + m_\rho^2 \rho_\mu^\dagger \rho_\mu.\end{aligned} \tag{61}$$

where the sums over silent indices are written in the Euclidean metric, $O_\mu^2 \equiv \sum_{\mu=0}^3 O_\mu O_\mu$. We remind that we always take $B_{\rm ext} \equiv F_{12} > 0$, and in this section $F_{0i} = F_{3i} = 0$.

The ground state of the model can be found via the minimization of the potential energy (61) with respect to the meson fields. To this end we notice that the field of the neutral meson is vanishing at the energy minimum, $\rho_\mu^{(0)} = 0$. Then, the quadratic part of Eq. (61) becomes as follows:

$$\begin{aligned}\epsilon_0^{(2)}(\rho_\mu) &= ieB_{\rm ext}(\rho_1^\dagger \rho_2 - \rho_2^\dagger \rho_1) + m_\rho^2 \rho_\mu^\dagger \rho_\mu \\ &= \sum_{a,b=1}^{2} \rho_a^\dagger \mathcal{M}_{ab} \rho_b + m_\rho^2(\rho_0^\dagger \rho_0 + \rho_3^\dagger \rho_3).\end{aligned} \tag{62}$$

The Lorentz components ρ_1 and ρ_2 possess the nondiagonal mass matrix

$$\mathcal{M} = \begin{pmatrix} m_\rho^2 & ieB_{\rm ext} \\ -ieB_{\rm ext} & m_\rho^2 \end{pmatrix}. \tag{63}$$

The eigenvalues $\mu_\pm$ and the corresponding eigenvectors $\rho_\pm$ of the mass matrix (63) are, respectively, as follows:

$$\mu_\pm^2 = m_\rho^2 \pm eB_{\rm ext}, \qquad \rho_\pm = \frac{1}{\sqrt{2}}(\rho_1 + i\rho_2). \tag{64}$$

The mass terms for ρ_0 and ρ_3 components are diagonal in (62) and their prefactors m_ρ^2 are unaltered by the external magnetic field.

It is clear from Eq. (62) that in terms of the ''longitudinal'' components ρ_0 and ρ_3, the ground state of the model corresponds to $\rho_0 = \rho_3 = 0$ at any value of the magnetic field. We express the transverse components $\rho_{1,2}$ via the eigenvalues and eigenvectors (64) of the mass matrix (63), and then we get for (the potential part of) the energy density (59) the following expression:

$$\epsilon_0(\rho_+, \rho_-) = \frac{1}{2}B_{\rm ext}^2 + \frac{g_s^2}{2}(|\rho_+|^2 - |\rho_-|^2)^2 + \mu_+^2|\rho_+|^2 + \mu_-^2|\rho_-|^2. \quad (65)$$

Since $\mu_+^2 > 0$ regardless of the value of the magnetic field $B_{\rm ext}$, the ground state corresponds to $\rho_+ = 0$. In turn, this means that $\rho_- \equiv \sqrt{2}\rho$ and

$$\rho_1 = -i\rho_2 = \rho, \qquad \rho_0 = \rho_3 = 0, \quad (66)$$

where ρ is a scalar complex field. In terms of the new field ρ the energy density (65) takes the simple form

$$\epsilon_0(\rho) = \frac{1}{2}B_{\rm ext}^2 + 2(m_\rho^2 - eB_{\rm ext})|\rho|^2 + 2g_s^2|\rho|^4. \quad (67)$$

Thus, we get the familiar Mexican-hat potential which describes various spontaneously broken systems. In particular, the same potential appears in the GL model of superconductivity (37).

The ground state of the model (67) depends on the value of the external magnetic field: if the field strength is weaker than the critical value $B_c = m_\rho^2/e$, Eq. (5), then the potential is trivial, while if $B_{\rm ext} > B_c$ then we get a nontrivial ground state

$$|\rho|_0 = \begin{cases} \sqrt{\frac{e(B_{\rm ext}-B_c)}{2g_s^2}}, & B_{\rm ext} \geq B_c, \\ 0, & B_{\rm ext} < B_c, \end{cases} \quad (68)$$

(the subscript "0" in $|\rho|_0$ indicates that we consider the homogeneous-field approximation). In Fig. 3 we plot the behavior of the condensate (68) as the function of the external magnetic field $B_{\rm ext}$. The value of the condensate follows a typical behavior of an order parameter for a second-order phase transition at $B_{\rm ext} = B_c$.

In Fig. 3 the subscript "AS" in $|\rho_{\rm AS}|$ stands for the "anisotropic superconductor." Indeed, the scalar field $\rho(x)$ enjoys the gauge symmetry (15) of its vector predecessor $\rho_\mu(x)$,

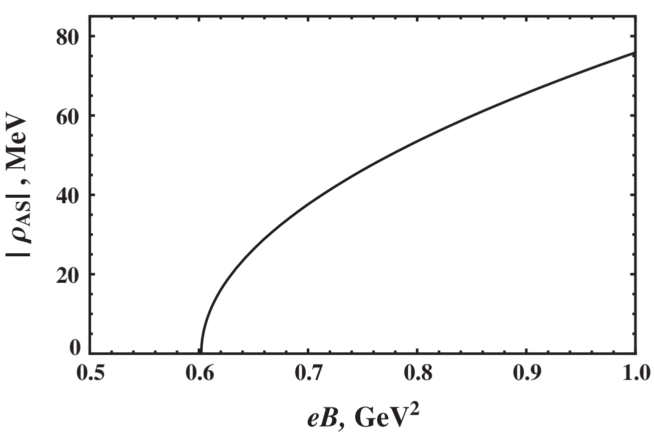

FIG. 3. The condensate $|\rho_{\rm AS}|$ of the charged $\rho^\pm$ mesons as a function of the external magnetic field $B \equiv B_{\rm ext}$ at the ground state. This single curve describes both the uniform condensate $|\rho_{\rm AS}| \equiv |\rho|_0$ in the homogeneous approximation (68) and the mean-cell value $|\rho_{\rm AS}| \equiv |\rho_{\rm AS}|_{\mathcal{A}}$ of the inhomogeneous condensate (96) in the weak-amplitude approximation.

$$U(1)_{\rm em}\colon\ \rho(x) \to e^{i\omega(x)}\rho(x). \quad (69)$$

The formation of the nontrivial ground state ρ in the strong external magnetic field $B_{\rm ext} \geq B_c$ breaks spontaneously the gauge symmetry (15) and forms, consequently, a superconducting state. The superconductor should exhibit spatially anisotropic properties due to spatially anisotropic condensate (66). This issue will be discussed in detail later.

Note that in the presence of the background magnetic field $\vec{B}_{\rm ext}$ the rotational group $SO(3)_{\rm rot}$ is explicitly broken to its $O(2)_{\rm rot}$ subgroup generated by rotations around the axis of the magnetic field. In the homogeneous approximation, the ground state (66) is transformed under the global $O(2)_{\rm rot}$ rotations as follows:

$$O(2)_{\rm rot}\colon\ \rho(x) \to e^{i\varphi}\rho(x), \quad (70)$$

where φ is the azimuthal angle of the rotation in the transverse plane. Thus, the ground state (66) is invariant under a combination of the global transformation from the gauge group (69) and the global rotation around the field axis (70) provided the parameters of these transformations are related ("locked") to each other as follows: $\omega(x) = -\varphi$. In analogy with the color superconductivity [9] one can say that the ground state "locks" the residual rotational symmetry with the electromagnetic gauge symmetry,

$$U(1)_{\rm em} \times O(2)_{\rm rot} \to U(1)_{\rm locked}. \quad (71)$$

Below we will see that the inhomogeneities of the condensate break the locked group (71) further to the group of discrete rotations of the vortex lattice.

In the ground state (68) the potential energy (62) has the form

$$\frac{\varepsilon_0(|\rho| = |\rho|_0)}{\varepsilon_0(|\rho| = 0)} = \begin{cases} 1 - \frac{e^2}{g_s^2}\left(1 - \frac{B_c}{B_{\rm ext}}\right)^2, & B_{\rm ext} \geq B_c, \\ 1, & B_{\rm ext} < B_c. \end{cases} \quad (72)$$

Obviously, for a strong magnetic field $B_{\rm ext} \geq B_c$, the condensed state has lower energy compared to the energy $\varepsilon_0(|\rho| = 0) = B_{\rm ext}^2/2$ of the normal (noncondensed) state.

Thus, we observed that the condensation of the $\rho^\pm$ mesons in the QCD vacuum should be very different from the condensation of the Cooper pairs Φ in the standard superconductor which is described by the phenomenological GL model (37). Indeed, in Sec. IV B we have illustrated the destructive role of the strong magnetic field on the conventional superconductivity. On the contrary, in this section we have found that the strong-enough magnetic field enforces the ρ-meson superconductivity.

B. Two-dimensional equations of motion

In order to study the properties of the emerged superconductor in more detail we should definitely go beyond the homogeneous approximation. The inhomogeneous state can be treated with the full system of the 3 + 1 dimensional equations of motion for the ρ-meson fields which were discussed in Sec. III B. We notice, however, that a wave function of the lowest energy state of a free particle in a uniform static magnetic field is independent on the coordinate x_3 which is longitudinal to the magnetic field. The dependence on the time coordinate x_0 comes as a trivial phase factor only. The Abrikosov lattice solution in the type-II superconductors is also known to be independent of x_0 and x_3 coordinates (Sec. IV C). These well-known properties suggest that we concentrate on x_0- and x_3-independent solutions to the classical equation of motions for the ρ mesons. To this end we choose the complex coordinate $z = x_1 + ix_2$ where $x_\perp = (x_1, x_2)$ are the coordinates in the spatial plane which is transverse of the magnetic field axis. We define the complex variables

$$\mathcal{O} = \mathcal{O}_1 + i\mathcal{O}_2, \qquad \bar{\mathcal{O}} = \mathcal{O}_1 - i\mathcal{O}_2 \tag{73}$$

for the fields $\mathcal{O} = J^{(0)}$, J, $\rho^{(0)}$, A, and for the derivative $\mathcal{O} = \partial$. It is also convenient to introduce two covariant derivatives,

$$D \equiv D_1 + iD_2 = \mathfrak{D} + ig_s\rho^{(0)}, \qquad \mathfrak{D} = \partial - ieA. \tag{74}$$

For the sake of convenience we use below both $x_\perp$ and z notations interchangeably, so that the two-dimensional Laplacian, for example, can be written in the three different ways: $\partial\bar{\partial} \equiv \partial_\perp^2 \equiv \partial_1^2 + \partial_2^2$.

Our homogeneous field analysis (Sec. V A) suggests that the charged currents should be chosen in the form

$$\rho_0 = \rho_3 = 0, \qquad \rho_1 = -i\rho_2 = \rho(z), \tag{75}$$

where ρ is a complex field.[5]

The magnetic field (16a) and the field strength of the neutral vector bosons (16b) are as follows:

$$F_{12} \equiv \mathrm{Im}(\bar{\partial}A) = B(z), \tag{76}$$

$$f_{12}^{(0)} \equiv \mathrm{Im}(\bar{\partial}\rho^{(0)}) = C(z). \tag{77}$$

Notice that, despite the assumption that the external magnetic field B_{ext} is uniform, the magnetic field (76) of the classical solution may be (and, in fact, will be) inhomogeneous. The tensor quantities (16c) and (16d) take, respectively, the following form (we omit the argument z hereafter):

$$\rho_{12}^{(0)} = C + 2g_s|\rho|^2, \qquad \rho_{12} = iD\rho. \tag{78}$$

The charged and neutral components of the current (25) become simple expressions, respectively,

$$J^{\mathrm{ch}} = 2ie(\rho^\dagger D\rho + \partial|\rho|^2), \qquad J^{(0)} = i\frac{e}{g_s}\partial C. \tag{79}$$

[5]In a strong field limit one can show that due to presence of inhomogeneities the ansatz (75) may be generalized : $\rho_1 = \rho(z) + \xi(z)$, $\rho_2 = i[\rho(z) - \xi(z)]$. In our analysis we ignore the subleading field ξ because its amplitude is suppressed by the factor $e/g_s \ll 1$.

The conservation law for the charged current (26), $\mathrm{Im}\{\bar{\partial}[\rho^\dagger\mathfrak{D}\rho - \rho(\bar{\mathfrak{D}}\rho)^\dagger]\} = 0$, is satisfied automatically due to relation (34),

$$\mathfrak{D}\rho = 0 \tag{80}$$

[we also used the identity $\partial|\rho|^2 \equiv \rho^\dagger D\rho + (\bar{D}\rho)^\dagger\rho$].

Equations (23), (28), and (32) reduce, respectively, to

$$g_s\partial B + iem_0^2\rho^{(0)} = 0, \tag{81}$$

$$(-\bar{\partial}\partial + m_0^2 + 2g_s^2|\rho|^2)\rho^{(0)} - 2ig_s\partial|\rho|^2 = 0, \tag{82}$$

$$[-\bar{D}D + 2(g_sC - eB + 2g_s^2|\rho|^2 + m_\rho^2)]\rho = 0. \tag{83}$$

Equation (77) along with the conservation law (29), $\mathrm{Re}(\bar{\partial}\rho^{(0)}) = 0$, lead to a simple expression for the transverse component of the field tensor (16b) of the neutral mesons,

$$C = -i\bar{\partial}\rho^{(0)}. \tag{84}$$

C. Inhomogeneous condensate of small amplitude

1. Linearized equations of motion

The classical equations of motion (80)–(84) comprise a complicated system of equations which is difficult to solve analytically due to the nonlinearities. However, following our discussion for the GL model (Sec. IV B), let us assume that the amplitude of the condensate ρ is very small. Then the equations of motion can be linearized and a leading analytical solution can be obtained. The condensate ρ should be small if the background magnetic field B_{ext} exceeds slightly the critical value B_c, Eq. (5). Concretely, for $B_{\mathrm{ext}} \geq B_c$ we consider the condition

$$2g_s^2|\rho|^2 \ll m_0^2, \quad \text{or} \quad \frac{B_{\mathrm{ext}}}{B_c} - 1 \ll 1. \tag{85}$$

These relations are analogous to, respectively, weak-condensate conditions (44) and (43) in the GL model of superconductivity. We show below that the first and the second relations in (85) are, in fact, equivalent.

Notice that Eq. (80) coincides with Eq. (46) for the order parameter of the ordinary superconductivity in the GL model (37) provided that the external magnetic field is close to the critical field (42) of this model. Therefore we should expect emergence of an analogue of the vortex lattice (49) in the ρ system (14) similarly to the appearance of the Abrikosov lattice (49) in the GL model. Thus, the condensate of the ρ mesons in the external magnetic field should definitely be inhomogeneous. Following the classic example [29], we consider below the simplest case of the square lattice with the elementary length (50).

In the weak-condensate regime (85) we can work in the leading order in terms of the condensate ρ_{AS} (higher order corrections are always omitted below). Then the equation of motion (82) gives the following relation:

$$\rho^{(0)}_{\rm AS}(x_\perp) = \frac{2ig_s}{-\partial_\perp^2 + m_0^2}\partial|\rho_{\rm AS}|^2, \tag{86}$$

where

$$\frac{1}{-\partial_\perp^2 + m_0^2}(x_\perp) = \frac{1}{2\pi}K_0(m|x_\perp|) \tag{87}$$

is the two-dimensional Euclidean propagator of a scalar particle with the mass m_0 and K_0 is a modified Bessel function (remember that the subscript AS stands for the anisotropic superconductor solution).

It is very important to notice that Eq. (86) relates the condensate of the neutral ρ^0 mesons with the condensate of the charged $\rho^\pm$ mesons. Thus, if the have an inhomogeneous condensate of the charged $\rho^\pm$ mesons, then we automatically get the inhomogeneous condensate (86) of the neutral ρ^0 mesons as well! This fact may indicate that the superconductivity of the $\rho^\pm$ mesons may induce the superfluidity of the ρ^0 mesons. Notice that a relation between the superfluidity of the neutral $\rho^{(0)}$ mesons and the superconductivity of the charged $\rho^\pm$ mesons may be guessed from the fact of the vector dominance, Eq. (31).

We interpret the nonzero condensate (86) as a "superfluid" because of the complex nature of the field $\rho^{(0)}$. Moreover, if for a moment we assume that this field is homogeneous (i.e., coordinate-independent) then rotations of the system around the magnetic field axis (70) would transform it as a usual complex field in simplest bosonic theories of superfluidity [28], $\rho^{(0)} \to e^{i\varphi}\rho^{(0)}$. The inhomogeneities of the condensate (86) break spontaneously this global group down to a discrete group of the rotations of the vortex lattice.

We would like also to note an important role of the inhomogeneities in the charged $\rho^\pm$ condensate for the superfluidity. In Sec. VA we have seen that the homogeneous condensate of the charged $\rho^\pm$ mesons alone is unable to induce the superfluidity of the $\rho^{(0)}$ mesons: a uniform nonzero expectation value of $\rho^\pm$ does not imply $\rho^{(0)} \neq 0$. However, the inhomogeneous charged condensate of $\rho^\pm$ automatically induces the inhomogeneous neutral condensate of $\rho^{(0)}$ as one can see from the presence of the derivative ∂ in the numerator in the right-hand side of Eq. (86).

The transverse component of the strength tensor (77) of the neutral ρ^0 mesons is given by Eq. (84),

$$C_{\rm AS}(x_\perp) = -i\bar\partial\rho^{(0)}_{\rm AS} = 2g_s\frac{\partial_\perp^2}{-\partial_\perp^2 + m_0^2}|\rho_{\rm AS}|^2. \tag{88}$$

Because of the identity

$$\int d^2x_\perp \frac{\partial_\perp^2}{-\partial_\perp^2 + m_0^2}(x_\perp - y_\perp) = 0, \tag{89}$$

the total "flux" of the neutral ρ^0 mesons through the transverse plane is always zero,

$$\int_{\mathcal{A}} d^2x_\perp (f^{(0)}_{12})_{\rm AS}(x_\perp) \equiv \int_{\mathcal{A}} d^2x_\perp C_{\rm AS}(x_\perp) = 0, \tag{90}$$

where the integral is taken over a unit cell $\mathcal{A}$ of the periodic structure of the "ρ vortices."

Next, Eq. (81) gets simplified,

$$\partial\left(B - \frac{2em_0^2}{-\partial_\perp^2 + m_0^2}|\rho_{\rm AS}|^2\right) = 0, \tag{91}$$

and its solution becomes as follows:

$$B_{\rm AS}(x_\perp) = B_{\rm ext} + \frac{2em_0^2}{-\partial_\perp^2 + m_0^2}|\rho_{\rm AS}|^2 - 2e(\overline{|\rho_{\rm AS}|^2})_{\mathcal{A}}. \tag{92}$$

Here the last term

$$(\overline{|\rho_{\rm AS}|^2})_{\mathcal{A}} = \frac{1}{L_B^2}\int_{\mathcal{A}} d^2y_\perp |\rho_{\rm AS}(y_\perp)|^2 \tag{93}$$

is "the mean-cell value" of the condensate squared $|\rho_{\rm AS}|^2$. Because of the identity

$$\int d^2x_\perp \frac{m_0^2}{-\partial_\perp^2 + m_0^2}(x_\perp - y_\perp) = 1, \tag{94}$$

the last term in Eq. (92) guarantees the conservation of the net magnetic flux through each elementary cell $\mathcal{A}$,

$$\int_{\mathcal{A}} d^2x_\perp B_{\rm AS}(x_\perp) = \int_{\mathcal{A}} d^2x_\perp B^{\rm ext} \equiv L_B^2 B^{\rm ext} = \frac{2\pi}{e} \tag{95}$$

[here we have used Eq. (50)]. The quantization of the magnetic flux (95) is similar to the quantization of the flux of the Abrikosov vortex (47).

Finally, Eqs. (83), (90), (92), and (95) give us

$$|\rho_{\rm AS}|_{\mathcal{A}} \equiv (\overline{|\rho_{\rm AS}|^2})^{1/2}_{\mathcal{A}} = \begin{cases} \sqrt{\frac{e(B_{\rm ext}-B_c)}{2g_s^2}}, & B_{\rm ext} \geq B_c, \\ 0, & B_{\rm ext} < B_c, \end{cases} \tag{96}$$

for the mean value (93) of the condensate. The mean-cell value of the condensate (96) is shown in Fig. 3. Notice that the mean-cell value of the condensate (96) coincides with the value of the uniform condensate (68) obtained in the homogeneous-field approximation.

Equation (96) has a few interesting properties. First, this equation represents a typical behavior of an order parameter. Second, Eq. (96) suggests that the phase transition, which separates the superconducting and the nonsuperconducting phases at $B_{\rm ext} = B_c$, is of a second order (as is shown clearly in Fig. 3). And third, Eq. (96) proves the equivalence between the first and the second conditions of the weak-condensate regime, Eq. (85).

Concluding this section we would like to stress that here we have introduced the new topological object, the ρ vortex, which is the vortex made of the superconducting $\rho^\pm$ mesons and superfluid ρ^0 mesons. This unit vortex cell carries the nonzero quantized flux of the magnetic field (95) and zero ρ^0 flux (90). The lattice of such vortices is a ground state of the superconductivity of the QCD vacuum at strong magnetic field. We discuss this lattice state in detail in the next section.

2. Inhomogeneous condensate: ρ-vortex lattice

In the regime (85) the degree of the inhomogeneity of the magnetic field $\delta B(x_\perp) = B_{\rm AS}(x_\perp) - B_{\rm ext}$ in the superconducting state is extremely small. Indeed, according to Eq. (92),

$$|\delta B| \sim 2e|\rho|^2 \ll \frac{em_0^2}{g_s^2} \approx \frac{e^2}{g_s^2} B_{\rm ext} \ll B_{\rm ext}. \qquad (97)$$

Thus, the inhomogeneity of the magnetic field δB is suppressed both by the small amplitude of the condensate (85) and by the very small factor $e^2/g_s^2 = 8.8 \times 10^{-3}$. From Eqs. (88) and (92) one also finds that the stress tensor of the neutral bosons (77) is small compared to the magnetic field (76), $|C| \ll (e/g_s)B_{\rm ext}$. Therefore we can set $B(x) \simeq B_{\rm ext}$ with very good accuracy. Then,

$$\mathfrak{D} \simeq \mathfrak{D}_{\rm ext} = \partial - eA_{\rm ext} = \partial + \frac{eB}{2} z \qquad (98)$$

so that the solution of Eq. (80) is

$$\rho_{\rm AS}(z) = e^{-(eB/4)|z|^2} H_{\rm AS}(z/L_B), \qquad (99)$$

where $H_{\rm AS}(z)$ is an arbitrary analytic function of the argument z and the intervortex distance L_B is given in Eq. (50).

Following the known solution (49) in the conventional superconductivity [29], we choose the square form of the lattice cells. For such periodic structure, one gets

$$H_{\rm AS}(z) = \sqrt{\frac{e(B_{\rm ext} - B_c)}{\sqrt{2} g_s^2}} e^{-(\pi/2)z^2} \sum_{n=-\infty}^{+\infty} e^{-\pi n^2 + 2\pi n z}, \qquad (100)$$

where the prefactor was determined with the help of the normalization relation (96) supplemented by the explicit expressions (99) and (100).

We already know that the homogeneous condensate locks the rotational and gauge degrees of freedom (71). The inhomogeneities in the condensate break the locked subgroup (71) further down to a discrete subgroup of the lattice rotations $G^{\rm lat}_{\rm locked}$,

$$U(1)_{\rm em} \times O(2)_{\rm rot} \to U(1)_{\rm locked} \to G^{\rm lat}_{\rm locked}. \qquad (101)$$

The discrete group $G^{\rm lat}_{\rm locked}$ depends on the lattice structure formed by the vortices.

Similarly to the mixed state of the ordinary type-II superconductivity, the ρ-vortex centers are located at the points (51), where the condensate $\rho_{\rm AS}$ vanishes. In the vicinity of the ρ-vortex centers the condensate (99) follows the typical Abrikosov-vortex behavior (48). However, there are many essential dissimilarities between the vortex systems in the GL model and in the system of the condensed ρ mesons.

In Fig. 4 we visualize four elementary lattice cells of the ρ-vortex lattice in the transverse plane. We take the external magnetic field with the strength $eB_{\rm ext} = (800\ {\rm MeV})^2 > eB_c$, so that the system is already in the superconducting state. The strength of the field satisfies the weak-condensate condition (85). The magnetic length and the elementary distance between the vortices in the square vortex lattice are, respectively,[6] (50), $\ell_B = 0.25$ fm and $L_B = 0.63$ fm. The mean value of the condensate (96) of the $\rho^\pm$ mesons is $|\phi_{\rm AS}| \simeq 23$ MeV. In Fig. 4 we plot various quantities that characterize the vortex: the amplitudes of the superconducting and superfluid condensates, the excess of the magnetic field with respect to the external magnetic field, and the field strength of the neutral meson field C. One can clearly see that:

(1) The superconducting condensate ρ of the charged vortices $\rho^\pm$, Eqs. (99) and (100), vanishes at the centers of the vortices (51), Fig. 4(a). In the vortex core the amplitude of the condensate $|\rho|$ is a linear function of the distance from the vortex center. This feature is similar to the behavior of the condensate near a typical Abrikosov vortex with a unit vorticity (48).

(2) The superfluid condensate $\rho^{(0)}$, Eq. (86), has a toothlike structure, Fig. 4(b). It vanishes at the locations of all local extrema of the superconducting condensate including the centers of the vortices. The amplitude of the superfluid condensate is maximal at the points of steepest behavior of the superconducting condensate, Fig. 4(a).

(3) The magnetic field strength B, Eq. (92), takes its minimal values at the centers of the vortices, Fig. 4(c). The maxima of B are located outside the vortex cores. This feature contradicts our intuition: in the ordinary superconductivity the strength of the magnetic field is maximal at the center of the Abrikosov vortex. In fact, the $\rho^\pm$ condensate has its own magnetic dipole moment due to the large, $g = 2$, gyromagnetic ratio of the ρ vortex. This dipole moment contributes only to the magnetic field outside the vortex cores, where the condensate of the $\rho^\pm$ condensate is large, Fig. 4(a). The electric current J, Eq. (24), is visualized in Fig. 5.

(4) The strength of the neutral meson field C of the superfluid [Eq. (88)] takes its maxima at the locations of the ρ vortices, Fig. 4(d). Thus, the ρ vortices

[6] Note that the magnetic length ℓ_B is of the order of the size of the ρ meson itself, $r_\rho \sim m_\rho \simeq 0.25$ fm. Thus, at these magnetic fields the ρ mesons should mutually overlap similarly to the overlapping Cooper pairs in the conventional superconductivity. However, regarding the success of the phenomenological GL model of the superconductivity we do not question the applicability of the phenomenological DSGS model (14) in the strong-field regime.

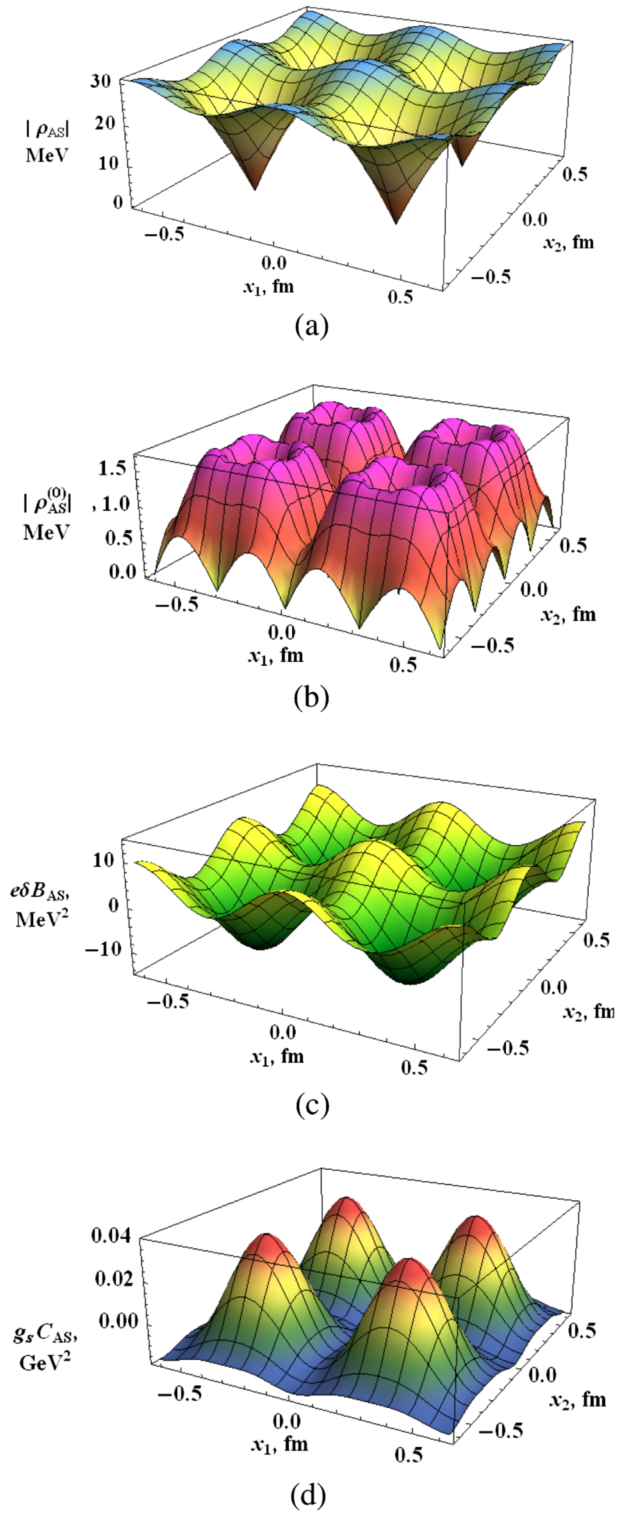

FIG. 4 (color online). Four elementary cells of the ρ-vortex lattice in the plane $x_\perp = (x_1, x_2)$, which is perpendicular to the external magnetic field with $eB_{\rm ext} = (800~{\rm MeV})^2$. From top to bottom: (a) the amplitude of the superconducting condensate ρ [Eqs. (99) and (100)]; (b) the amplitude of the superfluid condensate $\rho^{(0)}$ [Eq. (86)]; (c) the excess of the magnetic field $\delta B(x_\perp) \equiv B(x_\perp) - B_{\rm ext}$ [Eq. (92)]; and (d) the field strength C of the superfluid condensate $\rho^{(0)}$ [Eq. (88)].

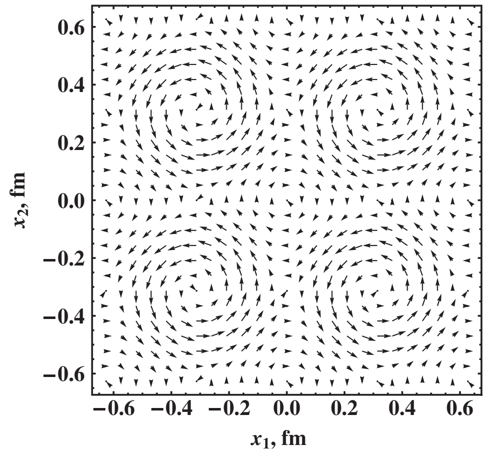

FIG. 5. The transverse components J_1 and J_2 of the electric current J, Eq. (24), in the transverse plane $x_\perp = (x_1, x_2)$. Four elementary cells of the ρ-vortex lattice at the external magnetic field with $eB_{\rm ext} = (800~{\rm MeV})^2$ are shown.

share this important property of the ordinary superfluid vortices as well.

Summarizing, the vortex core expels both superconducting and superfluid condensates of the charged and neutral ρ mesons, respectively. The magnetic field takes its maxima outside the vortices, while the strength of the superfluid (electrically neutral) field is peaked at the vortex centers.

3. Anisotropic superconductivity

The basic property of a superconductor is the absence of the resistivity. This feature is reflected, in particular, in the first London equation (53) in the GL model.

There is a simple way to derive analogues of the London equations for the condensed state of ρ mesons in the external magnetic field. First, we notice that Eqs. (28) and (31) imply

$$(\partial^\alpha \partial_\alpha + m_0^2)\partial_{[\mu} J_{\nu]} = m_0^2 \partial_{[\mu} J^{\rm ch}_{\nu]}. \qquad (102)$$

Then, we take $\mu = 0$ and $\nu = 3$ in Eq. (102) and use Eq. (25a) to get expressions for the $\mu = 0, 3$ components of the charged currents,

$$J^{\rm ch}_a = 2ie[\rho^* D_a \rho - (D_a \rho)^* \rho], \qquad a = 0, 3. \qquad (103)$$

Following the logic of the derivation of the London equation (53) in the Ginzburg-Landau approach (Sec. IV D), one gets from Eqs. (102) and (103)

$$\frac{\partial J_3(x_0, x_\perp)}{\partial x_0} = -4e^2 h^2_{\rm AS}(x_\perp) E_3, \qquad (104)$$

where $x_\perp = (x_1, x_2)$. The inhomogeneous quantity

$$h^2_{\rm AS} = \frac{m_0^2}{-\partial_\perp^2 + m_0^2} |\rho_{\rm AS}|^2 \qquad (105)$$

plays the role of the $|\Phi_0|^2$ condensate or m_A^2/e^2 in the conventional London relation (53).

Equation (104) implies that the ρ-meson condensate exhibits the superconductivity phenomenon along the direction of the external magnetic field $\vec{B}$: the electric current is growing linearly as a function of time once a weak external electric field is applied.

Notice that due to the periodicity of the inhomogeneous condensed state the mean-cell values of the squares of the effective condensate (105) and of the real condensate (93) coincide identically,

$$(\overline{h_{\rm AS}^2})_{\mathcal{A}} \equiv (\overline{|\rho_{\rm AS}|^2})_{\mathcal{A}}. \tag{106}$$

Averaging Eq. (104) over an elementary square cell in transverse directions and using Eq. (96) we get the cell-averaged value of the electric current $(\bar{J}_3)_{\mathcal{A}}$,

$$\frac{\partial}{\partial t}(\bar{J}_3)_{\mathcal{A}} = -\frac{2e^3}{g_s^2}(B_{\rm ext} - B_c)E_3, \tag{107}$$

where $B_{\rm ext} > B_c$ and we assumed, as usual, that the external electric field E_3 is a spacetime-independent quantity. The longitudinal (i.e., directed along $\vec{B}_{\rm ext}$) superconductivity sets in as the external field $B_{\rm ext}$ exceeds the critical value B_c, Eq. (5).

It is easy to prove that the superconductivity phenomenon has an anisotropic nature: in the transverse (i.e., perpendicular to $\vec{B}_{\rm ext}$) directions the superconductivity is absent. In order to prove this fact let us apply a weak spacetime-independent electric field $\vec{E}_{\rm ext} = (E_{{\rm ext},1}, E_{{\rm ext},2}, 0)$ perpendicularly to the strong magnetic field background $\vec{B}_{\rm ext} = (0, 0, B_{\rm ext})$. This electric field should test a possible transverse superconductivity of the $\rho^\pm$-meson condensate which could also be created by the strong magnetic field.

In order to show that the $\vec{B}_{\rm ext}$-transverse electric field does not create an accelerating electric current, we notice that an appropriate Lorentz boost may transform this system of the nonparallel $\vec{E}_{\rm ext}$ and $\vec{B}_{\rm ext}$ fields into the frame where the electric field is zero, $\vec{E}'_{\rm ext} = 0$. Obviously, in the new frame there are no linearly growing electric currents, so that in the initial frame such runaway currents are absent as well and

$$\frac{\partial J_i(x_0, x_\perp)}{\partial t} = 0, \qquad \frac{\partial}{\partial t}(\bar{J}_i)_{\mathcal{A}} = 0, \qquad i = 1, 2. \tag{108}$$

This argument does not work for the parallel electric and magnetic fields which were used to prove the longitudinal superconductivity (104). Indeed, in this case the scalar product $(\vec{E}_{\rm ext} \cdot \vec{B}_{\rm ext}) \propto \varepsilon_{\mu\nu\alpha\beta} F^{\mu\nu}_{\rm ext} F^{\alpha\beta}_{\rm ext}$ is a Lorentz-invariant quantity which is insensitive to boosts and rotations. Thus, if $\vec{E}_{\rm ext} \parallel \vec{B}_{\rm ext}$ then there is no frame where the external electric field $\vec{E}'_{\rm ext}$ is zero.

Equations (107) and (108) imply that the (cell-averaged) electric conductivity (54) contains an anisotropic complex contribution (55) which is singular at $\omega = 0$:

$$\sigma^{\rm sing}_{kl}(\omega) = \frac{\pi e^3}{g_s^2}(B_{\rm ext} - B_c)\left[\delta(\omega) + \frac{2i}{\pi\omega}\right]\delta_{k3}\delta_{l3}, \tag{109}$$

where the $i = 3$ is the direction of the external magnetic field $\vec{B}_{\rm ext}$.

The anisotropy of the superconductivity is quite similar to the anisotropy of the "usual" conductivity of the QCD vacuum which was found in lattice simulation in Ref. [8] for much weaker magnetic fields. An explanation of the anisotropy could be as follows: in a background of a uniform magnetic field the electric charges may move along the axis of the magnetic field while the motion in the transverse direction is limited to the spatial size ℓ_B of the low Landau orbits (50). In a sufficiently strong magnetic field, and in the absence of scattering of the charge carriers (we are working in a vacuum), the net transverse motion of the charges is suppressed contrary to the motion in the longitudinal direction.

4. Absence of a longitudinal Meissner effect

We have a very unusual situation In our paper we suggest that in the QCD vacuum the strong magnetic field induces the superconductivity of ρ mesons, while all our experience in the condensed matter systems tells us that we should expect the opposite phenomenon [28,29]: the external magnetic field should destroy the superconductivity due to the Meissner effect (Sec. IV E). In order to find a reason for this would-be inconsistency between the usual superconductor and the ρ-meson system, let us apply the considerations of Sec. IV E to the ρ mesons.

According to the Maxwell equations the electric currents that could screen the external magnetic field $\vec{B}_{\rm ext} = (0, 0, B_{\rm ext})$ should circulate in the transverse $x_\perp$ plane. In turn, the superconducting current in the transverse plane, $J_{\rm AS} \equiv J_{{\rm AS},1} + iJ_{{\rm AS},2}$, can be related to the neutral meson current (86) via the vector dominance relation (31),

$$J_{\rm AS}(x_\perp) = \frac{e m_0^2}{g_s}\rho^{(0)}_{\rm AS}(x_\perp) = \frac{2ie m_0^2}{-\partial_\perp^2 + m_0^2}\partial|\rho_{\rm AS}|^2. \tag{110}$$

Then in the system of the condensed ρ mesons, the analogue of the second London equation (57) for the longitudinal magnetic field can be written as follows (here we use the relation $\bar\partial\partial = \partial_\perp^2$):

$$(\vec{\partial} \times \vec{J}_{\rm AS})_3 \equiv {\rm Im}(\bar\partial J_{\rm AS}) = 2e m_0^2 \frac{\partial_\perp^2}{-\partial_\perp^2 + m_0^2}|\rho_{\rm AS}|^2. \tag{111}$$

The right-hand side of this equation depends on the external magnetic field $B_{\rm ext}$ via the superconducting density $\rho_{\rm AS}$, Eq. (99).

Equations (99), (100), and (111) provide us with an implicit expression for the curl of the screening currents. However, even without knowledge of the explicit form of these solutions one can show that these transverse currents both *screen and enhance* the external magnetic field in such a way that the net effect in one elementary vortex cell is precisely zero. Indeed, let us integrate left- and right-hand sides of Eq. (111) over an elementary unit cell, take into account the periodicity of the solution (99), and use the following property:

$$\int d^2x_\perp \frac{\partial_\perp^2}{-\partial_\perp^2 + m_0^2}(x_\perp - y_\perp) = 0. \qquad (112)$$

Thus, the cell-averaged right-hand side of the second London equation (111) for ρ mesons is zero,

$$\int_{\mathcal{A}} d^2x_\perp (\vec{\partial} \times \vec{J}_{\rm AS})_3 = 0, \quad [\text{condensed } \rho^\pm \text{ mesons}], \qquad (113)$$

while in the GL model the same procedure would give us the constant quantity in the right-hand side of (57),

$$\int_{\mathcal{A}} d^2x_\perp (\vec{\partial} \times \vec{J}_{\rm GL})_3 = -m_A^2 B_{\rm ext}, \qquad [\text{GL model}]. \qquad (114)$$

This fact simply means that, in the state of the condensed ρ mesons, the external magnetic field induces the transverse superconducting currents which are circulating both clockwise and counterclockwise. Consequently, the external magnetic field is enhanced in some regions of the transverse plane and it is suppressed in the other regions. Contrary to the ordinary superconductor, the net current circulation of the superconducting ρ currents per a unit lattice cell is exactly zero (113), while in the ordinary superconductor the net circulation is a linearly growing function of the external magnetic field.

Thus, we have found that the external magnetic field of any strength $B_{\rm ext} > B_c$ does not experience the screening inside the ρ superconductor: the magnetic flux propagates freely inside the superconductor. The same statement is not true for the ordinary superconductor in the purely superconducting state: the magnetic field tries to avoid the superconductor (the Meissner effect). Thus, in a loose sense one can interpret the absence of the net circulating currents (113) as the absence of the longitudinal Meissner effect.

On the other hand, our system is very similar to the ordinary Abrikosov lattice in the mixed state of the type-II superconductor, Sec. IV C: in the mixed state the magnetic field forms an inhomogeneous state and propagates through the superconductor, basically, in the cores of the Abrikosov vortices. In this case, however, the external magnetic field must be bounded both from above and from below, contrary to our ρ superconductivity in the QCD vacuum.

One can try to address the question about the existence of the Meissner effect in the ρ superconductor in a different way. In the ordinary superconductivity the Meissner effect is usually formulated as follows: if we apply a weak "test" magnetic field, say $\vec{B}'_{\rm ext} = (B'_{\rm ext}, 0, 0)$, along the boundary of a superconductor then this field will be screened inside the superconductor according to Eq. (58), i.e. $\vec{B}(x_3) = (e^{-m_A x_3} B'_{\rm ext}, 0, 0)$. This experiment, however, is senseless in the case of the ρ condensation because this condensation is induced in the rotationally invariant vacuum by the magnetic field itself. Indeed, assume that we have a combination of the two external magnetic fields: the strong field $\vec{B}'_{\rm ext}$, which induces the conductivity, and the additional weak field $\vec{B}''_{\rm ext}$, which is superimposed onto $\vec{B}'_{\rm ext}$ transversely $(\vec{B}'_{\rm ext} \cdot \vec{B}''_{\rm ext}) = 0$, in order to check the Meissner effect. Because of the vacuum environment, it is clear that the sole role of the additional field $\vec{B}''_{\rm ext}$ is to rotate the primary field $\vec{B}'_{\rm ext}$. After simple rotation of our coordinate around its origin we get a new field $\vec{B}_{\rm ext} = \vec{B}'_{\rm ext} + \vec{B}''_{\rm ext}$ so that the role of the additional test field is to rotate the directions of the ρ vortices in the condensed state. Thus, the question of the (non)existence of the transverse Meissner effect cannot be formulated in a self-consistent way.

VI. CONCLUSIONS

We argue that in a sufficiently strong background magnetic field the QCD vacuum may undergo a spontaneous transition to a superconducting state via condensation of the charged $\rho^\pm$ mesons. The critical strength of the magnetic field is given in Eq. (5). The superconductivity is understood in the usual electromagnetic sense. Moreover, unlike the color superconductivity, the superconducting QCD state is suggested to be formed in the cold vacuum, i.e. at zero temperature and at zero chemical potentials. Our vision of the phase diagram of the cold QCD vacuum in terms of the ρ-meson degrees of freedom is illustrated is Fig. 6.

We have found the following basic properties of the superconducting state:

(1) The effect occurs because of the nonminimal coupling of the charged ρ mesons to the electromagnetic field. The strong magnetic field enhances the superconductivity instead of destroying it.

(2) Because of simple kinematical reasons a strong-enough magnetic field makes the lifetime of the ρ mesons much longer by closing the dominant decay channels ($\rho^\pm \to \pi^\pm \pi^0$ and $\rho^0 \to \pi^+ \pi^-$) of the ρ mesons into the charged pions. The estimations of

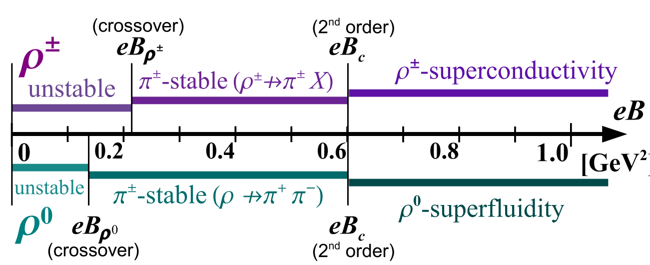

FIG. 6 (color online). The expected phase diagram: the impact of strong external magnetic field $B \equiv B_{\rm ext}$ on the ρ-meson degrees of freedom in the QCD vacuum.

the corresponding critical field strengths for the charged and neutral ρ mesons are given in Eqs. (9) and (11). Since these critical strengths are smaller than the condensation point (5), the condensate should be intrinsically stable, at least at the scale of the strong interactions.

(3) The transitions between the unstable and stable regions of the ρ mesons are expected to be smooth crossovers while the onset of the superconductivity is expected to be a second-order phase transition.

(4) The superconducting state is anisotropic: the electric resistance is zero only along the axis of the magnetic field.

(5) The superconducting state is inhomogeneous: the condensate shares similarity with the Abrikosov vortex lattice in the mixed state of a type-II superconductor.

(6) The pure homogeneous superconducting state is not formed.

(7) The onset of the superconductivity of the charged $\rho^{\pm}$ mesons leads to emergence of an inhomogeneous superfluidity of the neutral ρ^0 mesons. The superfluidity is induced by the inhomogeneities of the superconducting condensate.

(8) The inhomogeneous superconducting state is given by the ρ-vortex lattice. Locally, the ρ-vortex core expels both superconducting and superfluid condensates of the charged and neutral ρ mesons, respectively. The magnetic field takes its maxima outside the vortices, while the strength of the superfluid (electrically neutral) field is peaked at the vortex centers. However, the unit ρ-vortex cell carries one unit of the quantized magnetic flux of the magnetic field and no net ρ^0 flux.

(9) The spontaneous emergence of the superconducting condensate locks the rotations of the system around the magnetic field axis with a global subgroup of the gauge transformations. The inhomogeneities of the condensate break the locked group further to the group of discrete rotations of the vortex lattice.

(10) The Meissner effect (understood in the usual sense) cannot be realized in the superconducting QCD state due to the Lorenz invariance of the vacuum.

Our results also imply that the inhomogeneous Ambjørn-Olesen state [12,13] of the vacuum of the electroweak model is, in fact, a superconducting state. This state may in principle be realized in the first moments of the Universe if strong-enough magnetic fields are created in the primordial era [30]. The superconducting nature of the Ambjørn-Olesen state may have imprints in the large-scale structure of the magnetic fields in the present-day Universe.

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