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From 10 + 1 to 3 + 1 dimensions in an early universe with mutually BPS intersecting branes

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We assume that the early universe is homogeneous, anisotropic, and is dominated by the mutually Bogomol'nyi-Prasad-Sommerfeld $22'55'$ intersecting branes of M theory. The spatial directions are all taken to be toroidal. Using analytical and numerical methods, we study the evolution of such an universe. We find that, asymptotically, three spatial directions expand to infinity and the remaining spatial directions reach stabilized values. Any stabilized values can be obtained by a fine-tuning of initial brane densities. We give a physical description of the stabilization mechanism. Also, from the perspective of four-dimensional spacetime, the effective four-dimensional Newton's constant G_4 is now time varying. Its time dependence will follow from explicit solutions. We find in the present case that, asymptotically, G_4 exhibits characteristic log periodic oscillations.

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I. INTRODUCTION

In the early universe, temperatures and densities reach Planckian scales. Its description then requires a quantum theory of gravity. A promising candidate for such a theory is string/M theory. When the temperatures and densities reach string/M theory scales, the appropriate description is expected to be given in terms of highly energetic and highly interacting string/M theory excitations [1–8].[1] In this context, one of us has proposed in an earlier work an entropic principle according to which the final spacetime configuration that emerges from such high temperature string/M theory phase is the one that has maximum entropy for a given energy. This principle implies, under certain assumptions, that the number of large spacetime dimensions is 3 + 1 [8].

High densities and high temperatures also arise near black hole singularities. Therefore, it is reasonable to expect that the string/M theory configurations which describe such regions of black holes will describe the early universe also.

Consider the case of black holes. Various properties of a class of black holes have been successfully described using mutually Bogomol'nyi-Prasad-Sommerfeld (BPS) intersecting configurations of string/M theory branes.[2] Black hole entropies are calculated from counting excitations of such configurations, and Hawking radiation is calculated from interactions between them.

In the extremal limit, such brane configurations consist of only branes and no antibranes. In the near extremal limit, they consist of a small number of antibranes also. It is the interaction between branes and antibranes which gives rise to Hawking radiation. String theory calculations are tractable and match those of Bekenstein and Hawking in the extremal and near extremal limits. But they are not tractable in the far extremal limit where the numbers of branes and antibranes are comparable. However, even in the far extremal limit, black hole dynamics is expected to be described by mutually BPS intersecting brane configurations where they now consist of branes, antibranes, and other excitations living on them, all at nonzero temperature and in dynamical equilibrium with each other [10–19]. For the sake of brevity, we will refer to such far extremal configurations also as brane configurations even though they may now consist of branes and antibranes, left moving and right moving waves, and other excitations.

The entropy S of N stacks of mutually BPS intersecting brane configurations, in the limit where $S \gg 1$, is expected to be given by

$$S \sim \prod_I \sqrt{n_I + \bar{n}_I} \sim \mathcal{E}^{N/2} \tag{1}$$

where n_I and $\bar{n}_I$, $I = 1, \cdots, N$, denote the numbers of branes and antibranes of Ith type, $\mathcal{E}$ is the total energy, and the second expression applies for the charge neutral case where $n_I = \bar{n}_I$ for all I. The proof for this expression is given by comparing it in various limits with the entropy of the corresponding black holes [10,11], see also [12–21]. For $N \le 4$ and when other calculable factors omitted here are restored, this expression matches that for the corresponding black holes in the extremal and near extremal limit and, in the models based on that of Danielsson *et al* [12], matches up to a numerical factor in the far extremal limit [10–21] also. However, no such proof exists for

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[1] Only string theory is considered in these references. But their arguments can be extended for M theory also leading to similar conclusions.

[2] Mutually BPS intersecting configurations means, for example, that in M theory two stacks of 2 branes intersect at a point; two stacks of 5 branes intersect along three common spatial directions; a stack of 2 branes intersect a stack of 5 branes along one common spatial direction; waves, if present, will be along a common intersection direction; and each stack of branes is smeared uniformly along the other brane directions. See [9] for more details and for other such string/M theory configurations.

$N > 4$ since no analogous object, black hole or otherwise, is known whose entropy is $\propto \mathcal{E}^*$ with $* > 2$.

Note that, in the limit of large $\mathcal{E}$, the entropy $S(\mathcal{E})$ is $\ll \mathcal{E}$ for radiation in a finite volume and is $\sim\mathcal{E}$ for strings in the Hagedorn regime. In comparison, the entropy given in (1) is much larger when $N > 2$. This is because the branes in the mutually BPS intersecting brane configurations form bound states, become fractional, and support very low energy excitations which lead to a large entropy. Thus, for a given energy, such brane configurations are highly entropic.

Another novel consequence of fractional branes is the following. According to the "fuzz ball" picture for black holes [22], the fractional branes arising from the bound states formed by intersecting brane configurations have nontrivial transverse spatial extensions due to quantum dynamics. The size of their transverse extent is of the order of the Schwarzschild radius of the black holes. Therefore, essentially, the region inside the "horizon" of the black hole is not empty but is filled with a fuzz ball whose fuzz arises from the quantum dynamics of fractional strings/branes.

Chowdhury and Mathur have recently extended the fuzz ball picture to the early universe [20,21]. They have argued that the early universe is filled with fractional branes arising from the bound states of the intersecting brane configurations, and that the brane configurations with highest N are entropically favorable, see Eq. (1).

However, as mentioned below Eq. (1) and noted also in [20,21], the entropy expression in (1) is proved in various limits for $N \leq 4$ only and no proof exists for $N > 4$. Also, we are not certain of the existence of any system whose entropy $S(\mathcal{E})$ is parametrically larger than $\mathcal{E}^2$ for large $\mathcal{E}$. See related discussions in [23,24]. Therefore, in the following we will assume that $N \leq 4$. Then, a homogeneous early universe in string/M theory may be taken to be dominated by the maximum entropic $N = 4$ brane configurations distributed uniformly in the common transverse space.

Such $N = 4$ mutually BPS intersecting brane configurations in the early universe may then provide a concrete realization of the entropic principle proposed earlier by one of us to determine the number $(3 + 1)$ of large spacetime dimensions [8]. Indeed, in further works [23–25], using M theory symmetries and certain natural assumptions, we have shown that these configurations lead to three spatial directions expanding and the remaining seven spatial directions stabilizing to constant sizes.

In this paper, we assume that the early universe in M theory is homogeneous and anisotropic and that it is dominated by $N = 4$ mutually BPS intersecting brane configurations.[3] In this context, it is natural to assume that all spatial directions are on equal footing to begin with. Therefore we assume that the ten-dimensional space is toroidal. We then present a thorough analysis of the evolution of such a universe.

The corresponding energy momentum tensor T_{AB} has been calculated in [20] under certain assumptions. However, general relations among the components of T_{AB} may be obtained [24] using U duality symmetries of M theory which are, therefore, valid more generally.[4] We show in this paper that these U duality relations alone imply, under a technical assumption, that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes will asymptotically lead to an effective $(3 + 1)$—dimensional expanding universe.

In order to proceed further, and to obtain the details of the evolution, we make further assumptions about T_{AB}. We then analyze the evolution equations in D dimensions in general, and then specialize to the 11-dimensional case of interest here.

We are unable to solve explicitly the relevant equations. However, applying the general analysis mentioned above, we describe the qualitative features of the evolution of the $N = 4$ brane configuration. In the asymptotic limit, three spatial directions expand as in the standard FRW universe and the remaining seven spatial directions reach constant, stabilized values. These values depend on the initial conditions and can be obtained numerically. Also, we find that any stabilized values may be obtained, but this requires a fine-tuning of the initial brane densities.

Using the analysis given here, we present a physical description of the mechanism of stabilization of the seven brane directions. The stabilization is due, in essence, to the relations among the components of T_{AB} which follow from U duality symmetries, and to each of the brane directions in the $N = 4$ configuration being wrapped by, and being transverse to, just the right number and kind of branes. This mechanism is very different from the ones proposed in string theory or in brane gas models [28–31] to obtain large $3 + 1$—dimensional spacetime. (See Sec. 1A below also.)

In the asymptotic limit, the 11-dimensional universe being studied here can also be considered from the perspective of four-dimensional spacetime. One then obtains an effective four-dimensional Newton's constant G_4 which is now time varying. Its precise time dependence will follow from explicit solutions of the 11-dimensional evolution equations.

We find that, in the case of $N = 4$ brane configuration, G_4 has a characteristic asymptotic time dependence: the fractional deviation δG_4 of G_4 from its asymptotic value exhibits log periodic oscillations given by

$$\delta G_4 \propto \frac{1}{t^\alpha} \mathrm{Sin}(\omega \ln t + \phi). \tag{2}$$

The proportionality constant and the phase angle ϕ depend on initial conditions and matching details of the

[3]There is an enormous amount of work on the study of early universe in string/M theory. For a small, nonexhaustive, sample of such works, see [26–38].

[4]Such U duality relations are present in the case of black holes also. We point them out in Appendix A.

asymptotics, but the exponents α and ω depend only on the configuration parameters. Such log periodic oscillations seem to be ubiquitous and occur in a variety of physical systems [39–41]. But, to our knowledge, this is the first time it appears in a cosmological context. One expects such a behavior to leave some novel imprint in the late time universe, but its implications are not clear to us.

Since we are unable to solve the evolution equations explicitly, we analyze them using numerical methods. We present the results of the numerical analysis of the evolution. We illustrate the typical evolution of the scale factors showing stabilization and the log periodic oscillations mentioned above. By way of illustration, we choose a few sets of initial values and present the resulting values for the sizes of the stabilized directions and ratios of the string/M theory scales to the effective four-dimensional scale.

We also discuss critically the implications of our assumptions. As we will explain, many important dynamical questions must be answered before one understands how our known $(3 + 1)$—dimensional universe may emerge from M theory. Until these questions are answered and our assumptions justified, our assumptions are to be regarded conservatively as amounting to a choice of initial conditions which are fine tuned and may not arise naturally.

The organization of this paper is as follows. In Sec. II, we describe the U duality symmetries of M theory and their consequences, and present our ansatzes for the energy momentum tensor T_{AB} and for the equations of state. In Sec. III, we present a general analysis of D-dimensional evolution equations. In Sec. IV, we specialize to the 11-dimensional case of $N = 4$ intersecting brane configurations and describe the various results mentioned above. In Sec. V, we discuss the stabilized values of the brane directions, their ranges, and the necessity of fine-tuning. In Sec. VI, we present the four-dimensional perspective and the time variations of G_4. In Sec. VII, we present the results of numerical analysis. In Sec. VIII, we conclude by presenting a brief summary, a few comments on the assumptions made, and by mentioning a few issues which may be studied further. In Appendix A, we highlight the points related to U duality symmetries in the black hole case. In Appendices B, C, and D, we present certain results required in the text of the paper.

A. Intersecting brane vs brane gas models

In this subsection, we note that the branes and antibranes in the mutually BPS intersecting brane configurations considered here and in [20–25] are different from those in the string/brane gas models [28–31] in many important aspects. These differences are explained in detail in Sec. 2.6 of [20] and Sec. 6 of [21]. Briefly, the differences are the following.

(1) In brane gas models, the branes can intersect each other arbitrarily. In the brane configurations here, the intersections must follow specific rules. Consequently, U duality symmetries of M theory imply certain relations among the components of the energy momentum tensor T_{AB} which turn out to be crucial elements in our case [23,24].

(2) The branes in brane gas models support excitations on their surfaces and, at high energies, have $S \sim \mathcal{E}$ where S is the entropy and $\mathcal{E}$ the energy. Here, the intersecting branes form bound states, become fractional, support very low energy excitations and, hence, are highly entropic. At high energies, $S \sim \mathcal{E}^{N/2}$ which, for $N > 2$, vastly exceeds the entropy in brane gas models. Such intersecting brane configurations are, therefore, the entropically favorable ones.

(3) In brane gas models, the branes are assumed to annihilate if they intersect each other. Here, the intersections are necessary for formation of bound states, and thereby of fractional branes leading to high entropic excitations. Also, the intersecting configurations here consist of branes, antibranes, and the large number of low-energy excitations living on them. All these constituents are at nonzero temperature and in dynamical equilibrium with each other [10–19]. The excitations are long-lived and noninteracting to the leading order, hence the branes and antibranes here are metastable and do not immediately annihilate.

Note that, in string/M theory description, the Hawking evaporation of black holes is due to the annihilation of branes and antibranes. Also, large black holes consist of stacks of large numbers of branes and antibranes, and have a long lifetime. This implies that such stacks of branes and antibranes do not immediately annihilate and have a long lifetime. Hence, we assume that the mutually BPS intersecting brane configurations describing the early universe also consists of stacks of large numbers of branes and antibranes having a long lifetime and, in particular, that the brane antibrane annihilation effects are negligible during the evolution of the universe at least until the brane directions are stabilized resulting in an effective $(3 + 1)$—dimensional universe.

II. U DUALITY SYMMETRIES AND EQUATIONS OF STATE

In this paper, we assume that the early universe in M theory is homogeneous and anisotropic and that it is dominated by $N = 4$ mutually BPS intersecting brane configurations. To be specific, we consider $22'55'$ configurations.[5]

[5] In our notation, $22'55'$ denotes two stacks each of 2 branes and 5 branes, all intersecting each other in a mutually BPS configuration. Similarly for other configurations, e.g. $55'5''W$ denotes three stacks of 5 branes intersecting in a mutually BPS configuration with a wave along the common intersection direction.

We study the consequent evolution of such an universe dictated by a $(10+1)$—dimensional effective action given, in the standard notation, by

$$S_{11} = \frac{1}{16\pi G_{11}} \int d^{11}x\sqrt{-g}R + S_{br} \qquad (3)$$

where S_{br} is the action for the fields corresponding to the branes. The corresponding equations of motion are given, in the standard notation and in units where $8\pi G_{11} = 1$, by[6]

$$R_{AB} - \frac{1}{2}g_{AB}R = T_{AB}, \qquad \sum_A \nabla_A T^A{}_B = 0 \qquad (4)$$

where $A = (0, i)$ with $i = 1, 2, \cdots, 10$ and T_{AB} is the energy momentum tensor corresponding to the action S_{br}.

For black hole case, T_{AB} is obtained from the action for higher form gauge fields. With a suitable ansatz for the metric, equations of motion (4) are solved to obtain black hole solutions. For cosmological case, T_{AB} is often determined using equations of state of the dominant constituent of the universe. Such equations of state may be obtained if the underlying physics is known; or, one may assume a general ansatz for them and proceed.[7]

T_{AB} for intersecting branes in the early universe has been calculated in [20] assuming that the branes and antibranes in the intersecting brane configurations are noninteracting and that their numbers are all equal, i.e. $n_I = \bar{n}_I$ for $I = 1, 2, \cdots, N$ and $n_1 = \cdots = n_N$. However, general relations among the components of T_{AB} may be obtained [24] using U duality symmetries of M theory, involving chains of dimensional reduction and uplifting and T and S dualities of string theory, using which 2 branes and 5 branes or $22'55'$ and $55'5''W$ configurations can be interchanged. Such relations are valid more generally, for example, even when n_I and $\bar{n}_I$ are all different.

These general relations on the equations of state are sufficient to show, under a technical assumption, that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes, i.e. with $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, will asymptotically lead to an effective $(3+1)$—dimensional expanding universe. To obtain the details of the evolution, however, we need further assumptions and an ansatz of the type $p = w\rho$ [24,25].

We now present the details. Let the line element ds be given by

$$ds^2 = -dt^2 + \sum_i e^{2\lambda^i}(dx^i)^2 \qquad (5)$$

where e^{λ^i} are scale factors and, due to homogeneity, λ^i are functions of the physical time t only. (Parametrizing the scale factors as e^{λ^i} turns out to be convenient for our purposes.) It then follows that T_{AB} depends on t only and that it is of the form

$$T^A{}_B = \mathrm{diag}(-\rho, p_i). \qquad (6)$$

We assume that $\rho > 0$. From Eqs. (4) one now obtains

$$\Lambda_t^2 - \sum_i (\lambda_t^i)^2 = 2\rho \qquad (7)$$

$$\lambda_{tt}^i + \Lambda_t \lambda_t^i = p_i + \frac{1}{9}\Big(\rho - \sum_j p_j\Big) \qquad (8)$$

$$\rho_t + \rho\Lambda_t + \sum_i p_i \lambda_t^i = 0 \qquad (9)$$

where $\Lambda = \sum_i \lambda^i$ and the subscripts t denote time derivatives. Note, from Eq. (7), that Λ_t cannot vanish. Hence, if $\Lambda_t > 0$ at an initial time t_0 then it follows that e^Λ increases monotonically for $t > t_0$. We assume the evolution to be such that $e^\Lambda \to \infty$ eventually.

In the context of early universe in M theory, it is natural to assume that all spatial directions are on equal footing to begin with. Therefore we assume that the ten-dimensional space is toroidal. Further, we assume that the early universe is homogeneous and is dominated by the $22'55'$ configuration where, with no loss of generality, we take two stacks each of 2 branes and 5 branes to be along (x^1, x^2), (x^3, x^4), $(x^1, x^3, x^5, x^6, x^7)$, and $(x^2, x^4, x^5, x^6, x^7)$ directions, respectively, and take these intersecting branes to be distributed uniformly along the common transverse space directions (x^8, x^9, x^{10}). Note that the total brane charges must vanish, i.e. $n_I = \bar{n}_I$ for all I, since the common transverse space is compact. We denote this $22'55'$ configuration as (12, 34, 13567, 24567). The meaning of this notation is clear and, below, such a notation will be used to denote other configurations also.

A. U duality relations

We now describe the relations which follow from U duality symmetries, involving chains of dimensional reduction and uplifting and T and S dualities of string theory. See [24] for more details. Let $\downarrow_k$ and $\uparrow_k$ denote dimensional reduction and uplifting along kth direction between M theory and type IIA string theory, T_i denote T duality along ith direction in type IIA and IIB string theories, and S denote S duality in type IIB string theory. Then U dualities of the type $\uparrow_j T_i S T_i \downarrow_j$ interchange i and j directions, and U dualities of the type $\uparrow_k T_i T_j \downarrow_k$ transform one mutually BPS N intersecting brane configuration to another.

[6] In the following, the convention of summing over repeated indices is not always applicable. Hence, we will always write the summation indices explicitly. Unless indicated otherwise, the indices $A, B, \cdots$ run from 0 to 10, the indices $i, j, \cdots$ from 1 to 10, and the indices $I, J, \cdots$ from 1 to N.

[7] This is similar to the FRW case. Equation of state $p = \frac{\rho}{3}$ for radiation, or $p = 0$ for pressureless dust, may be obtained from the physics of radiation or of massive particles; or, one may assume a general ansatz $p = w\rho$ and proceed.

For example, the U duality $\uparrow_5 T_3T_4 \downarrow_5$ transforms a 2 brane configuration (12) to the 5 brane configuration (12345). Similarly, the U duality $\uparrow_5 T_1T_2 \downarrow_5$ transforms the $22'55'$ configuration (12,34, 13567, 24567) to the $W55'5''$ configuration (5, 12345, 23567, 14567); whereas $\uparrow_6 T_4T_5 \downarrow_6$ transforms it to the $\tilde{5}'2'5\tilde{2}$ configuration (12456, 35, 13467, 27).

The U dualities transform the corresponding gravitational fields also. In the case of metric of the form given in Eq. (5), the U duality $\uparrow_k T_iT_j \downarrow_k$ transforms the λ^is in the scale factors to λ'^is given by

$$\lambda'^i = \lambda^j - 2\lambda, \quad \lambda'^j = \lambda^i - 2\lambda, \quad \lambda'^k = \lambda^k - 2\lambda$$
$$\lambda'^l = \lambda^l + \lambda, \quad l \neq i, j, k, \quad \lambda = \frac{\lambda^i + \lambda^j + \lambda^k}{3}. \tag{10}$$

Consider the 2 brane configuration (12) with scale factors e^{λ^i} and the 5 brane configuration (12345) with scale factors $e^{\lambda'^i}$. By inspection, or by using U dualities $\uparrow_j T_iST_i \downarrow_j$ for appropriate (i, j), these scale factors may be expected to obey the "obvious" symmetries:

$$2\colon \lambda^1 = \lambda^2, \qquad \lambda^3 = \cdots = \lambda^{10} \tag{11}$$

$$5\colon \lambda'^1 = \cdots = \lambda'^5, \qquad \lambda'^6 = \cdots = \lambda'^{10}. \tag{12}$$

Now, these two configurations are related by the U duality $\uparrow_5 T_3T_4 \downarrow_5$. Hence the corresponding λ^is and λ'^is must obey the relations of the type given in (10). Combined with the obvious symmetry relations above, it is straightforward to show that

$$\lambda^{\parallel} + 2\lambda^{\perp} = 2\lambda'^{\parallel} + \lambda'^{\perp} = 0 \tag{13}$$

where the superscripts $\parallel$ and $\perp$ denote spatial directions along and transverse to the branes, respectively.

Similarly, the obvious symmetry relations for the $22'55'$ configuration (12, 34, 13567, 24567) are

$$22'55'\colon \lambda^5 = \lambda^6 = \lambda^7, \qquad \lambda^8 = \lambda^9 = \lambda^{10}. \tag{14}$$

Proceeding as in the case of 2 and 5 branes above, and using the U duality $\uparrow_5 T_1T_2 \downarrow_5$ which relates the $22'55'$ and $W55'5''$ configurations, one obtains two more relations given by [24]

$$\lambda^1 + \lambda^4 + \lambda^5 = \lambda^2 + \lambda^3 + \lambda^5 = 0. \tag{15}$$

In general, for an N intersecting brane configuration, the U duality symmetries will lead to $10 - N$ relations among the λ^is, leaving only N of them independent. These relations are of the form $\sum_i c_i\lambda^i = 0$ where c_i are constants. Clearly, such a relation can be violated by constant scaling of x^i coordinates. Hence, we interpret it as implying a relation among the components of T_{AB}. In view of Eq. (8), we interpret a U duality relation $\sum_i c_i\lambda^i = 0$ as implying that

$$\sum_i c_i f^i = 0, \qquad f^i = p_i + \frac{1}{9}\Big(\rho - \sum_j p_j\Big). \tag{16}$$

Substituting Eq. (16) in Eq. (8), it follows upon an integration that

$$\sum_i c_i\lambda^i_t = ce^{-\Lambda} \tag{17}$$

where c is an integration constant. If $\sum_i c_i\lambda^i_t = 0$ initially at $t = t_0$ then $c = 0$. In such cases then $\sum_i c_i\lambda^i_t = 0$ for all t and, hence, $\sum_i c_i\lambda^i = \upsilon$ where υ is another integration constant.

In general $\sum_i c_i\lambda^i_t \neq 0$ initially at $t = t_0$ and, hence, $c \neq 0$. Let the evolution be such that $e^\Lambda \sim t^\beta \to \infty$ in the limit $t \to \infty$. Then it follows from Eq. (17) that $\sum_i c_i\lambda^i_t \to 0$ in this limit. If, furthermore, $\beta > 1$ then Eq. (17) also gives

$$\sum_i c_i\lambda^i = \upsilon + \mathcal{O}(t^{1-\beta}) \to \upsilon \tag{18}$$

where υ is an integration constant. If $\beta \leq 1$ then $\sum_i c_i\lambda^i$ is a function of t. We will see later that, in the solutions we obtain with further assumptions, β turns out to be >1 for $N > 1$.

Note that, as can be seen from the above steps, the U duality relations follow as long as the directions involved in the U duality operations are isometry directions. Since none of the common transverse directions are involved in obtaining the relations above, it follows that they are valid even if the common transverse directions are not compact. Thus the U duality relations are applicable in such cases also.

Similarly, the time dependence of λ^is played no role in obtaining the U duality relations here. Hence, these relations may be expected to arise for the black hole case also. They indeed arise as we point out in Appendix A.

B. A general result

We now consider a general result for the $22'55'$ configuration that follows from the U duality relations alone [24]. The λ^is for this configuration obey the relations given in Eqs. (14) and (15). Note that a suitable U duality, for example $\uparrow_6 T_4T_5 \downarrow_6$, can transform 2 branes and 5 branes into each other. Hence, we will refer to two types of branes as being identical if they have identical numbers of branes and antibranes, i.e. Ith type is identical to Jth type if $n_I = n_J$ and $\bar{n}_I = \bar{n}_J$.

Consider the case when 2 and $2'$ branes in the $22'55'$ configurations are identical. This will enhance the obvious symmetry relations. It is easy to see that we now have one more independent relation $\lambda^1 = \lambda^3$. If, instead, 5 and $5'$ branes are identical, then the extra independent relation is $\lambda^1 = \lambda^2$. Similarly, if 2 and $5'$ branes are identical then, after a few steps involving U duality $\uparrow_6 T_4T_5 \downarrow_6$ which interchanges 2 and $5'$ branes, it follows that the extra independent relation is $\lambda^2 = \lambda^5$.

Now if all the four types of branes in the $22'55'$ configuration are identical, i.e. if $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, then, we have three extra independent relations

$$\lambda^1 = \lambda^2 = \lambda^3 = \lambda^5. \tag{19}$$

Combined with Eqs. (14) and (15), we get $\lambda^1 = \cdots = \lambda^7 = 0$ which is to be interpreted as $f^1 = \cdots = f^7 = 0$, see Eq. (16). Hence, as described earlier, it follows for $i = 1, \cdots, 7$ that if $\lambda^i_t = 0$ initially at $t = t_0$ then $\lambda^i_t = 0$ and $\lambda^i = v^i$ for all t where v^i are constants. Or, if $e^\Lambda \sim t^\beta \to \infty$ in the limit $t \to \infty$ with $\beta > 1$, it then follows for $i = 1, \cdots, 7$ that $\lambda^i_t \to 0$ and $\lambda^i \to v^i$ in this limit. Obtaining the values of the asymptotic constants v^i, however, requires knowing the details of evolution. It also follows similarly that $e^{\lambda^i} \sim e^{\Lambda/3} \to \infty$ for $i = 8, 9, 10$. It is straightforward to show that same results are obtained for the equivalent $55'5''W$ configuration also.

Thus, assuming either that $\lambda^1_t = \cdots = \lambda^7_t = 0$ initially at $t = t_0$ or that $e^\Lambda \sim t^\beta \to \infty$ in the limit $t \to \infty$ with $\beta > 1$, we obtain that the $N = 4$ mutually BPS intersecting brane configurations with identical numbers of branes and antibranes, i.e. with $n_1 = \cdots = n_4$ and $\bar{n}_1 = \cdots = \bar{n}_4$, will asymptotically lead to an effective $(3 + 1)$—dimensional expanding universe with the remaining seven spatial directions reaching constant sizes. This result follows as a consequence of U duality symmetries alone, which imply relations of the type given in Eq. (16) among the components of the energy momentum tensor T_{AB}. This result is otherwise independent of the details of the equations of state, and also of the ansatzes for T_{AB} we make in the following in order to proceed further.

C. Ansatz for T_{AB}

The dynamics underlying the general result given above may be understood in more detail, and the asymptotic constants v^i can be obtained, if an explicit solution for the evolution is available. In the following, we will make a few assumptions which enable us to obtain such details.

Consider now the case of 2 branes or 5 branes only. From the U duality relations given by Eqs. (11)–(13) and (16), it follows easily that $p_\parallel = -\rho + 2p_\perp$ where $p_\parallel$ is the pressure along the brane directions and $p_\perp$ is the pressure along the transverse directions. For the case of waves, one obtains $p_\parallel = \rho$. We write these U duality relations for the $N = 1$ configurations in the form

$$p_\parallel = z(\rho - p_\perp) + p_\perp \tag{20}$$

where $z = -1$ for 2 and 5 branes and $= +1$ for waves. (A similar relation may be obtained in the black hole case also, see Appendix A.) In general, ρ, $p_\parallel$, and $p_\perp$ are functions of the numbers n and $\bar{n}$ of branes and antibranes, satisfying the U duality relations (20). If $n = \bar{n}$ then $p_\parallel$ and $p_\perp$ may be thought of as functions of ρ satisfying Eq. (20) [24].

Consider now the mutually BPS N intersecting brane configuration. In the black hole case, it turns out that the energy momentum tensors ${T^A}_{B(I)}$ of the Ith type of branes are mutually noninteracting and separately conserved [42–51]. That is,

$${T^A}_B = \sum_I {T^A}_{B(I)}, \qquad \sum_A \nabla_A {T^A}_{B(I)} = 0. \tag{21}$$

We assume that this is the case in the context of the early universe also where ${T^A}_B = \mathrm{diag}(-\rho, p_i)$, ${T^A}_{B(I)} = \mathrm{diag}(-\rho_I, p_{iI})$, $\rho_I > 0$, and (ρ_I, p_{iI}) satisfy the U duality relations in (20) for all I. Eqs. (21) now give

$$\rho = \sum_I \rho_I, \qquad p_i = \sum_I p_{iI} \tag{22}$$

$$(\rho_I)_t + \rho_I \Lambda_t + \sum_i p_{iI} \lambda^i_t = 0. \tag{23}$$

We have verified explicitly for a variety of mutually BPS N intersecting brane configurations that Eqs. (20) and (22) are sufficient to satisfy all the relations of the type $\sum_i c_i f^i = 0$ implied by U duality symmetries. See [24] for more details.

To solve the evolution Eqs. (7), (8), (22), and (23), we need the functions ρ_I, $p_{\parallel I}$, and $p_{\perp I}$. To proceed further, we assume that $n_I = \bar{n}_I$ for all I. This is necessary if, as is the case here, the common transverse directions are compact and hence the net charges must vanish. Then $p_{\parallel I}$ and $p_{\perp I}$ may be thought of as functions of ρ_I satisfying Eq. (20).

It is natural to expect that $p_{\perp I}(\rho_I)$ is the same function for waves, 2 branes, and 5 branes since they can all be transformed into each other by U duality operations which do not involve the transverse directions. We assume that this is the case. We further assume that this function $p_\perp(\rho)$ is given by

$$p_\perp = (1 - u)\rho \tag{24}$$

where u is a constant. Such a parametrization of the equation of state, instead of the usual one $p = w\rho$, leads to elegant expressions as will become clear in the following, see [32,33] also. The results of [20] correspond to the case where $u = 1$. Here, we assume only that $0 < u < 2$. The constant u is arbitrary otherwise.

It now follows that p_{iI} in Eq. (22) are of the form $p_{iI} = (1 - u^I_i)\rho_I$ and that the constants u^I_i can be obtained in terms of u using Eqs. (20) and (24). Thus, for 2 branes, 5 branes, and waves, we have $u_\perp = u$, $u_\parallel = (1 - z)u$, and hence

$$\begin{aligned} 2&: u_i = (2, 2, 1, 1, 1, 1, 1, 1, 1, 1)u \\ 5&: u_i = (2, 2, 2, 2, 2, 1, 1, 1, 1, 1)u \\ W&: u_i = (0, 1, 1, 1, 1, 1, 1, 1, 1, 1)u \end{aligned} \tag{25}$$

where the I superscripts have been omitted since $N = 1$. Similarly, u^I_i for the $22'55'$ configuration are given by

$$\begin{aligned} 2&: u_i^1 = (2, 2, 1, 1, 1, 1, 1, 1, 1, 1)u \\ 2'&: u_i^2 = (1, 1, 2, 2, 1, 1, 1, 1, 1, 1)u \\ 5&: u_i^3 = (2, 1, 2, 1, 2, 2, 2, 1, 1, 1)u \\ 5'&: u_i^4 = (1, 2, 1, 2, 2, 2, 2, 1, 1, 1)u. \end{aligned} \tag{26}$$

This completes our ansatz for the energy momentum tensor T_{AB} for the intersecting brane configurations in the early universe.

III. GENERAL ANALYSIS: EVOLUTION EQUATIONS

The evolution of the universe can now be analyzed. In this section, we first present the analysis in a general form which is applicable to a D-dimensional homogeneous, anisotropic universe. We specialize to the intersecting brane configurations in the next section.

The D-dimensional line element ds is given by Eq. (5), now with $i = 1, 2, \cdots, D-1$. The total energy momentum tensor T_{AB} of the dominant constituents of the universe is given by Eq. (6). The equations of motion for the evolution of the universe is given, in units where $8\pi G_D = 1$, by Eqs. (7)–(9) with 9 in Eq. (8) now replaced by $D-2$. Defining

$$G_{ij} = 1 - \delta_{ij}, \qquad G^{ij} = \frac{1}{D-2} - \delta_{ij}, \tag{27}$$

the Eqs. (7) and (8), with 9 replaced by $D-2$, may be written compactly as

$$\sum_{i,j} G_{ij}\lambda_t^i\lambda_t^j = 2\rho \tag{28}$$

$$\lambda_{tt}^i + \Lambda_t\lambda_t^i = \sum_j G^{ij}(\rho - p_j) \tag{29}$$

where $i, j, \cdots$ run from 1 to $D-1$.

Let the universe be dominated by N types of mutually noninteracting and separately conserved matter labeled by $I = 1, \cdots, N$. Then the corresponding energy momentum tensors $T_{AB(I)}$ and their components ρ_I and p_{iI} satisfy Eqs. (21)–(23).

Further, let the equations of state be given by $p_{iI} = (1 - u_i^I)\rho_I$ where u_i^I are constants. Eqs. (23), (28), and (29) may now be simplified and cast in various useful forms as follow.

Using $p_{iI} = (1 - u_i^I)\rho_I$, Eq. (23) can be integrated to give

$$\rho_I = e^{l^I - 2\Lambda}, \qquad l^I = \sum_i u_i^I\lambda^i + l_0^I \tag{30}$$

where l_0^I are integration constants. Further using Eqs. (22) and (30), Eqs. (28) and (29) become

$$\sum_{i,j} G_{ij}\lambda_t^i\lambda_t^j = 2\sum_J e^{l^J - 2\Lambda} \tag{31}$$

$$\lambda_{tt}^i + \Lambda_t\lambda_t^i = \sum_J u^{iJ}e^{l^J - 2\Lambda} \tag{32}$$

where $u^{iJ} = \sum_j G^{ij}u_j^J$. Let the initial conditions at an initial time t_0 be given, with no loss of generality, by

$$(\lambda^i, \lambda_t^i, l^I, l_t^I, \rho_I)_{t=t_0} = (0, k^i, l_0^I, K^I, \rho_{I0}) \tag{33}$$

where

$$\rho_{I0} = e^{l_0^I}, \qquad K^I = \sum_i u_i^I k^i, \qquad \sum_{i,j} G_{ij}k^ik^j = 2\sum_J e^{l_0^J}. \tag{34}$$

Eqs. (31) and (32) may now be solved for the $D-1$ variables λ^i with the above initial conditions. Or, instead, these equations may be manipulated so that one needs to solve for N variables l^I only, see Eqs. (35), (39), (42), and (44), below. We now perform these manipulations.

First define a variable $\tau(t)$ as follows:

$$d\tau = e^{-\Lambda}dt, \qquad \tau(t_0) = 0. \tag{35}$$

Then, for $\lambda^i(t)$ or equivalently $\lambda^i(\tau(t))$, we have

$$\lambda_t^i = e^{-\Lambda}\lambda_\tau^i, \qquad \lambda_{tt}^i + \Lambda_t\lambda_t^i = e^{-2\Lambda}\lambda_{\tau\tau}^i \tag{36}$$

where the subscripts τ denote τ-derivatives. Note that the initial values at $\tau(t_0) = 0$ remain unchanged since $\Lambda = 0$, and hence $\lambda_t^i = \lambda_\tau^i$ at $t = t_0$. Eqs. (31) and (32) now become

$$\sum_{i,j} G_{ij}\lambda_\tau^i\lambda_\tau^j = 2\sum_J e^{l^J} \tag{37}$$

$$\lambda_{\tau\tau}^i = \sum_J u^{iJ}e^{l^J}. \tag{38}$$

Also, from $l^I = \sum_i u_i^I\lambda^i + l_0^I$, it follows that

$$l_{\tau\tau}^I = \sum_J \mathcal{G}^{IJ}e^{l^J} \tag{39}$$

where

$$\mathcal{G}^{IJ} = \sum_i u_i^I u^{iJ} = \sum_{i,j} G^{ij}u_i^I u_j^J. \tag{40}$$

We assume that $\mathcal{G}_{IJ}$ exists such that $\sum_J \mathcal{G}_{IJ}\mathcal{G}^{JK} = \delta_I^K$, i.e. that the matrix $\mathcal{G}$ formed by $\mathcal{G}^{IJ}$ is invertible.[8] Then, from Eq. (39), we have

$$\sum_J \mathcal{G}_{IJ}l_{\tau\tau}^J = e^{l^I}. \tag{41}$$

Substituting this expression for e^{l^I} into Eq. (38), then integrating it twice and incorporating the initial conditions given in Eq. (33), we get

$$\lambda^i = \sum_I u_I^i(l^I - l_0^I) + L^i\tau, \qquad u_I^i = \sum_{j,J}\mathcal{G}_{IJ}G^{ij}u_j^J \tag{42}$$

[8] This is not always the case. For example, $u_i^I = u^I$ for all i in the isotropic case. Then $\mathcal{G}^{IJ} \propto u^I u^J$ and $\mathcal{G}$ is not invertible. This is not a problem: it just means that the set of variables l^I can be reduced to a smaller independent set; one then proceeds with the smaller set.

where L^i are integration constants. It follows from $\lambda^i_\tau(0)$ that L^i are related to initial values k^i and K^I by $k^i = \sum_I u^i_I K^I + L^i$. Using this expression for k^i in the relation $K^I = \sum_i u^I_i k^i$, or substituting the expression for λ^i given in Eq. (42) into the Eq. (30) for l^I, leads to the following N constraints on L^i:

$$\sum_i u^I_i L^i = 0, \qquad I = 1, 2, \cdots, N. \tag{43}$$

Now, using Eqs. (42) and (43), Eq. (37) may be written in terms of l^I as follows:

$$\sum_{I,J} \mathcal{G}_{IJ} l^I_\tau l^J_\tau = 2\Big(E + \sum_I e^{l^I}\Big), \quad 2E = -\sum_{i,j} G_{ij} L^i L^j. \tag{44}$$

One may now solve Eqs. (39) and (44) for N variables $l^I(\tau)$. Then $\lambda^i(\tau)$ are obtained from Eq. (42) and $t(\tau)$ from Eq. (35). Inverting $t(\tau)$ then gives $\tau(t)$, and thereby $\lambda^i(t)$.

A. $N = 1$ case

Consider the $N = 1$ case. Note that we are still considering the general D-dimensional universe, not the 11-dimensional one. We assume here that $\mathcal{G}^{11} = \mathcal{G} > 0$. Now, as shown in Appendix B, it follows in general that if $\sum_i u_i L^i = 0$ and $\sum_{i,j} G^{ij} u_i u_j > 0$ then $E \geq 0$ and E vanishes if and only if L^i all vanish. Since $\sum_i u^1_i L^i = 0$, see Eq. (43), and we assume that $\mathcal{G}^{11} = \sum_{i,j} G^{ij} u^1_i u^1_j > 0$, we have $E \geq 0$. We further assume that $E > 0$, equivalently that L^is do not all vanish.

Omitting the I labels, Eqs. (39) and (44) for $l(\tau)$ become

$$l_{\tau\tau} = \mathcal{G} e^l, \qquad (l_\tau)^2 = 2\mathcal{G}(E + e^l). \tag{45}$$

The initial values are $l_0 = l(0)$ and $K = l_\tau(0)$ obeying $K^2 = 2\mathcal{G}(E + e^{l_0})$. We take $K > 0$ with no loss of generality. Then the solution for $l(\tau)$ is given by

$$e^l = \frac{E}{\mathrm{Sinh}^2 \alpha(\tau_\infty - \tau)} \tag{46}$$

where

$$2\alpha^2 = \mathcal{G}E, \quad \mathrm{Sinh}^2 \alpha\tau_\infty = E e^{-l_0}, \quad K = 2\alpha \mathrm{Coth} \alpha\tau_\infty. \tag{47}$$

The sign of α is immaterial but, just to be definite, we take it to be positive. The sign of τ_∞ is same as that of K, hence $\tau_\infty > 0$. Also, $\lambda^i(\tau)$ and $t(\tau)$ may now be obtained but are not needed here for our purposes.

Note that $e^l \to 4E e^{2\alpha(\tau - \tau_\infty)}$ and vanishes in the limit $\tau \to -\infty$, whereas $e^l \to \frac{2}{\mathcal{G}}(\tau_\infty - \tau)^{-2}$ and diverges in the limit $\tau \to \tau_\infty$. The value of τ_∞ depends on the initial values l_0 and K, or equivalently E, as given in Eqs. (47). It is finite and can be evaluated exactly. However, if $e^{l_0} \ll E$ then τ_∞ may be approximated in a way that will be useful later on.

From the exact solution given above, we have $\mathrm{Sinh}^2 \alpha\tau_\infty = E e^{-l_0}$ and $K = 2\alpha \mathrm{Coth} \alpha\tau_\infty$. In the limit $e^{l_0} \ll E$, we then have $e^{2\alpha\tau_\infty} \simeq 4E e^{-l_0}$ and $K \simeq 2\alpha$. It, therefore, follows that

$$\tau_\infty \simeq \frac{1}{K}(lnE - l_0 + ln4). \tag{48}$$

In the limit $e^{l_0} \ll E$, the evolution of $l(\tau)$ may also be thought of as follows. Consider E to be fixed and e^{l_0} to be very small so that $e^{l_0} \ll E$. It then follows from Eqs. (45) that, at initial times, $l_{\tau\tau}$ is very small and that $l_\tau \simeq \sqrt{2\mathcal{G}E} = 2\alpha$ is independent of e^l. Hence, $l(\tau)$ evolves as if there is no "force," i.e. $l(\tau) \simeq l_0 + K\tau$ where $K = l_\tau(0) > 0$ is the initial "velocity." Once e^l becomes of $\mathcal{O}(E)$ then it affects l_τ. But, from then on, e^l evolves quickly and diverges soon after.

This suggests that one may well approximate τ_∞ by the time τ_a required for l, which starts from l_0 with a velocity K and evolves freely with no force, to reach lnE—namely, to reach a value where $e^l = e^{l_0 + K\tau_a} = E$. In other words, if $e^{l_0} \ll E$ then

$$\tau_\infty \simeq \tau_a = \frac{1}{K}(lnE - l_0). \tag{49}$$

A comparison with Eq. (48) shows that the exact τ_∞ which follows from solving the evolution equations is indeed well approximated by τ_a in Eq. (49) in the limit $e^{l_0} \ll E$. Note that τ_a is calculated using only the initial values, requiring no knowledge of the exact solution.

B. $N > 1$ case

When $N > 1$, the equations of motion can be solved if $\mathcal{G}^{IJ}$ are of certain form [32–38]. For example, if $\mathcal{G}^{IJ} \propto \delta^{IJ}$ then the solutions are similar to those in the $N = 1$ case described above. For general forms of $\mathcal{G}^{IJ}$, we are unable to obtain explicit solutions. Nevertheless, the general evolution can still be analyzed if one assumes suitable asymptotic forms for the scale factors e^{λ^i}.

It follows from Eqs. (27) and (28) that Λ_t cannot vanish. With no loss of generality, let $\Lambda_t > 0$ initially at $t = t_0$. Then e^Λ decreases monotonically for $t < t_0$, equivalently $\tau < 0$, and increases monotonically for $t > t_0$, equivalently $\tau > 0$. Further features of the evolution depend on the structure of $\mathcal{G}^{IJ}$ and u^I_i. In the cases of interest here, it turns out that e^Λ and also all e^{l^I} vanish in the limit $\tau \to -\infty$, and diverge in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. We assume such a behavior and analyze the asymptotic solutions.

1. Asymptotic evolution: $e^\Lambda \to 0$

We assume that $(e^\Lambda, e^{l^I}) \to 0$ in the limit $\tau \to -\infty$. Then, Eqs. (38) and (39) can be solved since their right-hand sides depend only on e^{l^I}s which now vanish. Hence, in the limit $\tau \to -\infty$, we write

$$e^{l^I} = e^{\tilde{c}^I \tau} = t^{\tilde{b}^I}, \qquad e^{\lambda^i} = e^{\tilde{c}^i \tau} = t^{\tilde{b}^i} \tag{50}$$

which are valid up to multiplicative constants and where $(\tilde{c}^I, \tilde{c}^i, \tilde{b}^I, \tilde{b}^i)$ are constants. Also, the equalities in the asymptotic expressions here and in the following are valid only up to the leading order. Equation (42) now implies that $\tilde{c}^i = \sum_I u_I^i \tilde{c}^I + L^i$. Also, $e^\Lambda = e^{\tilde{c}\tau}$ where $\tilde{c} = \sum_i \tilde{c}^i$. Then it follows from Eq. (35) that $t \sim e^{\tilde{c}\tau}$. Hence,

$$\tilde{b}^I = \frac{\tilde{c}^I}{\tilde{c}}, \qquad \tilde{b}^i = \frac{\tilde{c}^i}{\tilde{c}}, \qquad \sum_i \tilde{b}^i = 1. \tag{51}$$

Furthermore, Eq. (37) implies that $(\sum_i \tilde{b}^i)^2 - \sum_i (\tilde{b}^i)^2 = 0$. Thus the evolution is of Kasner type in the limit $\tau \to -\infty$. The constants $\tilde{c}^I$s in Eqs. (50) must be such that the resulting $\sum_i \tilde{b}^i = \sum_i (\tilde{b}^i)^2 = 1$, but are otherwise arbitrary. In an actual evolution, however, $\tilde{c}^I$s can be determined in terms of the initial values l_0^I and K^I with no arbitrariness, but this requires complete solution for $l^I(\tau)$.

2. *Asymptotic evolution:* $e^\Lambda \to \infty$

We assume that $e^\Lambda \to \infty$ in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. Whether this limit is reached at a finite or infinite physical time t depends on the values of u_i^I, see below. $\Lambda(\tau)$ may be obtained in terms of $l^I(\tau)$ using Eq. (42). Hence, in the limit $e^\Lambda \to \infty$, some or all of the e^{l^I}s diverge. Consider the following ansatz in the limit $\tau \to \tau_\infty$:

$$e^{l^I} = e^{c^I}(\tau_\infty - \tau)^{-2\gamma^I}, \qquad e^{\lambda^i} = e^{c^i}(\tau_\infty - \tau)^{-2\gamma^i}, \tag{52}$$

where c^I and γ^I are constants, and some or all of the γ^Is must be >0 so that some or all of the e^{l^I}s diverge. Equation (42) now implies that

$$\gamma^i = \sum_I u_I^i \gamma^I, \qquad c^i = \sum_I u_I^i (c^I - l_0^I) + L^i \tau_\infty. \tag{53}$$

Also, $e^\Lambda = e^c(\tau_\infty - \tau)^{-2\gamma}$ where $c = \sum_i c^i$ and $\gamma = \sum_i \gamma^i$. For the ansatz in Eqs. (52) to be consistent, it is necessary that $\gamma > 0$ so that $e^\Lambda \to \infty$ in the limit $\tau \to \tau_\infty$. Now $t(\tau)$ follows from Eq. (35) and is given by

$$t - t_s = \frac{1}{2\gamma - 1} e^c (\tau_\infty - \tau)^{-(2\gamma - 1)}, \qquad \gamma = \sum_{i,I} u_I^i \gamma^I \tag{54}$$

where t_s is a finite constant. If $2\gamma < 1$ then $t \to t_s$ which means that $e^\Lambda \to \infty$ at a finite physical time t_s. If $2\gamma > 1$ then $t \to \infty$ in the limit $e^\Lambda \to \infty$. Which case is realized, i.e. whether $2\gamma < 1$ or >1, depends on the values of u_i^I.

Using Eq. (54), the asymptotic behavior of e^{l^I} and e^{λ^i} can be obtained in terms of t. For example, let $2\gamma > 1$ and

$$e^{l^I} = e^{b^I + 2b} t^{\beta^I}, \qquad e^{\lambda^i} = e^{b^i} t^{\beta^i}, \qquad e^\Lambda = e^b t^\beta \tag{55}$$

in the limit $t \to \infty$. It then follows that

$$\beta^I = \frac{2\gamma^I}{2\gamma - 1}, \qquad \beta^i = \frac{2\gamma^i}{2\gamma - 1}, \qquad \beta = \frac{2\gamma}{2\gamma - 1}. \tag{56}$$

Note that, in this case, we have $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. See the discussion below Eq. (16) for the relevance of this feature.

To obtain the values of γ^I, and thereby γ^i, in Eq. (52), consider Eq. (41) from which it follows that

$$2\sum_J \mathcal{G}_{IJ} \gamma^J = e^{c^I} (\tau_\infty - \tau)^{2(1-\gamma^I)}. \tag{57}$$

The left-hand side in the above equation is a constant but the right-hand side depends on τ. This is consistent if $\gamma^I = 1$ in which case the right-hand side becomes a positive constant, or if $\gamma^I < 1$ in which case the right-hand side vanishes in the limit $\tau \to \tau_\infty$. Thus, there are two possibilities:

$$\text{(i) } \gamma^I = 1 \Rightarrow 2\sum_J \mathcal{G}_{IJ}\gamma^J = e^{c^I} > 0 \tag{58}$$

$$\text{(ii) } \gamma^I \neq 1 \Rightarrow \sum_J \mathcal{G}_{IJ}\gamma^J = 0, \qquad \gamma^I < 1. \tag{59}$$

For a given $\mathcal{G}_{IJ}$, the possible consistent solutions for (γ^I, e^{c^I}) are to be obtained as follows. Assume that some I's are of type (i) and the remaining ones are of type (ii). Then solve Eqs. (58) and (59) for e^{c^I} in type (i) and for γ^I in type (ii). Such a solution is consistent if the resulting (e^{c^I}, γ^I) satisfy $e^{c^I} > 0$ for Is in type (i) and $\gamma^I < 1$ for Is in type (ii). Also, some or all of the γ^Is must be >0 as required in Eq. (52). (It is further necessary that the resulting $\gamma > 0$ so that $e^\Lambda \to \infty$, but calculating γ requires u_i^I.)

Consider an example, which will be useful later, where $\mathcal{G}^{IJ}$ and $\mathcal{G}_{IJ}$ are given by

$$\mathcal{G}^{IJ} = a(b - \delta^{IJ}), \qquad \mathcal{G}_{IJ} = \frac{1}{a}\left(\frac{b}{Nb - 1} - \delta_{IJ}\right) \tag{60}$$

with $a > 0$ and $Nb > 1$. It is then easy to show that the only possibility is the one given in (i). Also $\sum_J \mathcal{G}_{IJ} = \frac{1}{a(Nb-1)} > 0$, and thus $\gamma^I = 1$ for all I is a consistent solution as required by Eq. (58). In the $N = 1$ case, we get $\mathcal{G}^{11} = \mathcal{G} = a(b - 1) > 0$, and e^l in the limit $\tau \to \tau_\infty$ obtained as described above agrees with that obtained from the explicit solution, see below Eq. (47).

Thus e^{c^I} and γ^I, and thereby $\gamma^i = \sum_I u_I^i \gamma^I$ and $\gamma = \sum_{i,I} u_I^i \gamma^I$, are all determined ultimately by u_i^I. The constants c^i are given by Eq. (53) and they depend on u_i^I, on the initial values l_0^I and L^i, and also on τ_∞. But determining τ_∞, and hence determining c^i when L^i do not all vanish, requires complete solution for $l^I(\tau)$.

C. Deviations from $e^{l^I} \to \infty$ asymptotics

We consider the deviations of $l^I(\tau)$ from its asymptotic behavior given in Eq. (52), which will turn out to be of interest. Let the deviations $s^I(\tau)$ for $I = 1, 2, \cdots, N$ be defined, in the limit $\tau \to \tau_\infty$, by

$$e^{l^I} = e^{c^I}(\tau_\infty - \tau)^{-2\gamma^I} e^{s^I(\tau)} \tag{61}$$

where c^I and γ^I are determined as described earlier. For the purpose of illustration, and also for later use, we now assume that all the Is are of type (i), namely, that $\gamma^I = 1$ and $e^{c^I} = 2\sum_J \mathcal{G}_{IJ} > 0$ for all I. It then follows straightforwardly from Eq. (39) that

$$(\tau_\infty - \tau)^2 s^I_{\tau\tau} = 2\sum_{K,L} \mathcal{G}^{IK}\mathcal{G}_{KL}(e^{s^K} - 1). \tag{62}$$

Consider the example of $\mathcal{G}^{IJ}$ given in Eq. (60). Then $\sum_J \mathcal{G}_{IJ} = \frac{1}{a(Nb-1)}$ and, for any σ^K, one has

$$\sum_{K,L} \mathcal{G}^{IK}\mathcal{G}_{KL}\sigma^K = -\frac{1}{Nb-1}\left(\sigma^I - b\sum_K \sigma^K\right). \tag{63}$$

In Eq. (62), $\sigma^K = 2(e^{s^K} - 1)$. It now follows easily that, up to the leading order in $\{s^K\}$, the difference $s^I - s^J$ obeys the equation

$$(\tau_\infty - \tau)^2(s^I - s^J)_{\tau\tau} + \frac{2}{Nb-1}(s^I - s^J) = 0. \tag{64}$$

The solutions to these equations are of the form

$$(s^I - s^J) \sim (\tau_\infty - \tau)^{(1/2)(1\pm\sqrt{\Delta})}, \quad \Delta = 1 - \frac{8}{Nb-1}. \tag{65}$$

Note that $s^I - s^J = l^I - l^J$ since γ^I and c^I are the same for all I, see Eq. (61). Hence, Eqs. (64) and (65) can be used to understand in more detail the behavior of l^Is as they all diverge in the limit $\tau \to \tau_\infty$ as given in Eq. (52). We will discuss these features in Secs. VI and VII.

IV. INTERSECTING BRANES

We now analyze the evolution of the universe dominated by mutually BPS N intersecting brane configurations of M theory. The number of spacetime dimensions $D = 11$. The equations of state are assumed to be given by $p_{iI} = (1 - u^I_i)\rho_I$ where, as a consequence of U duality symmetries, u^I_i are parametrized in terms of one constant u. The indices $i, j, \cdots$ run from 1 to 10 and the indices $I, J, \cdots$ from 1 to N. For 2 branes, 5 branes, and waves, $N = 1$ and the corresponding u^I_i are given in Eqs. (25). For $22'55'$ configuration, $N = 4$ and the corresponding u^I_i are given in Eqs. (26).

A. EVOLUTION EQUATIONS

The evolution of λ^i describing the scale factors is given by the equations described earlier which, for ease of reference, we summarize below:

$$\lambda^i_{\tau\tau} = \sum_J u^{iJ} e^{l^J} \tag{66}$$

$$l^I_{\tau\tau} = \sum_J \mathcal{G}^{IJ} e^{l^J} \tag{67}$$

$$\lambda^i = \sum_J u^i_J (l^J - l^J_0) + L^i \tau \tag{68}$$

where

$$u^{iI} = \sum_j G^{ij} u^I_j, \quad \mathcal{G}^{IJ} = \sum_{i,j} G^{ij} u^I_i u^J_j, \quad u^i_I = \sum_{j,J} \mathcal{G}_{IJ} G^{ij} u^J_j \tag{69}$$

with G^{ij} and $\mathcal{G}_{IJ}$ as defined earlier, and L^i are arbitrary constants satisfying the constraints $\sum_i u^I_i L^i = 0$ for all I. Also, l^I_τ obey the constraint

$$\sum_{I,J} \mathcal{G}_{IJ} l^I_\tau l^J_\tau = 2\left(E + \sum_J e^{l^J}\right) \tag{70}$$

where $2E = -\sum_{i,j} G_{ij} L^i L^j$. Eqs. (67) and (70) are to be solved for $l^I(\tau)$ with initial conditions $l^I(0) = l^I_0 = ln\rho_{I0}$ and $l^I_\tau(0) = K^I$ where ρ_{I0} are initial densities and

$$\sum_{I,J} \mathcal{G}_{IJ} K^I K^J = 2\left(E + \sum_J e^{l^J_0}\right). \tag{71}$$

Then $\lambda^i(\tau)$ follow from Eq. (68) and the physical time $t(\tau)$ from $dt = e^\Lambda d\tau$. Inverting $t(\tau)$ then gives $\tau(t)$, and thereby $\lambda^i(t)$.

We can now calculate $\mathcal{G}^{IJ}$ for the mutually BPS intersecting brane configurations. As explained in footnote 2, in the BPS configurations two stacks of 2 branes intersect at a point; two stacks of 5 branes intersect along three common spatial directions; a stack of 2 branes intersects a stack of 5 branes along one common spatial direction; and, waves, if present, will be along a common intersection direction. With these rules given, it is now straightforward to calculate $\mathcal{G}^{IJ}$ using Eqs. (25) and (69). It turns out because of the BPS intersection rules that the resulting $\mathcal{G}^{IJ}$ are given by

$$\mathcal{G}^{IJ} = 2u^2(1 - \delta^{IJ}). \tag{72}$$

The corresponding $\mathcal{G}_{IJ}$ exists for $N > 1$, and is given by

$$\mathcal{G}_{IJ} = \frac{1}{2u^2}\left(\frac{1}{N-1} - \delta_{IJ}\right). \tag{73}$$

Note that, for $N > 1$, the above $\mathcal{G}^{IJ}$ is a special case of the example considered earlier in Eq. (60), now with $a = 2u^2$ and $b = 1$,

It is also straightforward to calculate u^{iI} and u^i_I for the $22'55'$ configuration using the definitions in Eq. (69) and the u^I_i in Eq. (26). They are given by

$$\begin{aligned} &2:\ u^{i1} \propto (-2,-2,1,1,1,1,1,1,1,1) \\ &2':\ u^{i2} \propto (1,1,-2,-2,1,1,1,1,1,1) \\ &5:\ u^{i3} \propto (-1,2,-1,2,-1,-1,-1,2,2,2) \\ &5':\ u^{i4} \propto (2,-1,2,-1,-1,-1,-1,2,2,2) \end{aligned} \tag{74}$$

where the proportionality constant is $\frac{u}{3}$, and by

$$\begin{aligned} &2:\ u_1^i \propto (2,2,-1,-1,-1,-1,-1,1,1,1) \\ &2':\ u_2^i \propto (-1,-1,2,2,-1,-1,-1,1,1,1) \\ &5:\ u_3^i \propto (1,-2,1,-2,1,1,1,0,0,0) \\ &5':\ u_4^i \propto (-2,1,-2,1,1,1,1,0,0,0) \end{aligned} \tag{75}$$

where the proportionality constant is $\frac{1}{6u}$.

We are unable to solve Eqs. (67), (70), and (72), for $N > 1$.[9] However, applying the general analysis described in Sec. III and making further use of the explicit forms of u_i^I and $\mathcal{G}^{IJ}$ given in Eqs. (26) and (72), one can understand the qualitative features of the evolution of the $22'55'$ configuration.

We first make several remarks which will lead to an immediate understanding of the evolution of this configuration.

(1) Let $u_i = \sum_I u_i^I$. It can then be checked that $\sum_{i,j} G^{ij} u_i u_j > 0$. Also, $\sum_i u_i L^i = 0$ since $\sum_i u_i^I L^i = 0$ for all I. Hence, as shown in Appendix B, it follows that E given in Eq. (70) is ≥ 0 and that it vanishes if and only if L^i all vanish.

(2) The constraints $\sum_i u_i^I L^i = 0$ imply that

$$\begin{aligned} &L^1 - L^4 = L^2 - L^3 = L^5 + L^6 + L^7 = 0 \\ &L^8 + L^9 + L^{10} = -3(L^1 + L^2). \end{aligned} \tag{76}$$

Thus, for example, we may take $(L^1, L^2, L^6, L^7, L^8, L^9)$ to be independent. The remaining L^is are then determined by the above equations. Also, we have

$$L \equiv \sum_i L^i = -(L^1 + L^2). \tag{77}$$

Using Eqs. (76) and (77), and the Schwarz inequality (B1) in Appendix B, we write E as

$$\begin{aligned} 2E &= \sum_i (L^i)^2 - \Big(\sum_i L^i\Big)^2 \\ &= 3(L)^2 + \sum_{i=5}^{7}(L^i)^2 + 2\sigma_2^2 + \sigma_3^2 \\ &= 3(L^1)^2 + (L^1 + 2L^2)^2 + \sum_{i=5}^{7}(L^i)^2 + \sigma_3^2 \end{aligned} \tag{78}$$

where the first line is the definition of E, $\sigma_2 = 0$ if and only if $L^1 = L^2$, and $\sigma_3 = 0$ if and only if $L^8 = L^9 = L^{10}$. See the Schwarz inequality given in Eq. (B1). It is easy to show that the above expressions for E imply that $(L^i)^2$ for all i are bounded above by E as follows: $E \geq c_i (L^i)^2 \geq 0$ where c_i are constants of $\mathcal{O}(1)$. In particular, note the inequality $2E \geq 3(L)^2$ which is required in Appendix C.

(3) It follows from Eqs. (68), (75), and (77), that

$$\Lambda_\tau = \sum_i \lambda_\tau^i = \frac{1}{6u}(2l_\tau^1 + 2l_\tau^2 + l_\tau^3 + l_\tau^4) + L. \tag{79}$$

Using the explicit form of $\mathcal{G}_{IJ}$ given in Eq. (73) with $N = 4$, Eq. (70) becomes

$$\Big(\sum_I l_\tau^I\Big)^2 - 3\sum_I (l_\tau^I)^2 = 12u^2\Big(E + \sum_I e^{l^I}\Big) > 0 \tag{80}$$

where the inequality follows since $E \geq 0$ and $e^{l^I} > 0$. We show in Appendix C that this inequality implies that none of (Λ_τ, l_τ^I) may vanish, and that they must all have same sign. Hence, for all τ throughout the evolution, (Λ_τ, l_τ^I) must all be nonvanishing, and be all positive or all negative.

B. Asymptotic evolution

With no loss of generality, let $\Lambda_t > 0$ initially at $t = t_0$. Then it follows from the above result that (Λ_τ, l_τ^I) must all be positive and nonvanishing for all τ. Hence, (Λ, l^I) are all monotonically increasing functions for all τ throughout the evolution.

Equation (67) may be written, using Eq. (72), as

$$l_{\tau\tau}^I = 2u^2 \sum_{J \neq I} e^{l^J}. \tag{81}$$

In the past, τ and all l^I decrease continuously. Hence, the right-hand side in Eq. (81) becomes more and more negligible. The asymptotic solution in the limit $\tau \to -\infty$ is then given by $l^I = \tilde{c}^I \tau + \tilde{d}^I$ where $\tilde{c}^I > 0$. Thus $e^{l^I} \to 0$ in this limit.

Similarly, in the future, τ and all l^I increase continuously. However, the right-hand side in Eq. (81) increases exponentially now. It is then obvious that all $e^{l^I} \to \infty$ within a finite interval of τ, i.e. at a finite value τ_∞ of τ. In this context, see Eqs. (45) and (46), and the general analysis given in Sec. III B 2.

[9] In the case of black holes, the equations of motion for the corresponding harmonic functions $H^I = 1 + \frac{Q^I}{r} \equiv e^{\tilde{h}_I}$ can also be written in a form similar to that of Eq. (67). The main steps are indicated in Appendix A. The analogous $\mathcal{G}^{IJ}$ in the black hole case turns out to be $\propto \delta^{IJ}$, and the equations can then be solved.
Also, note that if $L^i = 0$ for all i then λ^i in Eq. (68) here may be written as in Eq. (A4) in Appendix A The role of $\tilde{h}_I$ there is played by the functions $2uh_I = 2u\sum_J \mathcal{G}_{IJ}(l^J - l_0^J)$ here. Such a similarity is present for other intersecting brane configurations also.

We now analyze the corresponding asymptotic solutions.

1. Asymptotic evolution: $e^{\Lambda} \to 0$

It follows from the above discussion that $e^{\Lambda} \to 0$ in the limit $\tau \to -\infty$. Also, in this limit, we have

$$e^{l^I} = e^{\tilde{c}^I \tau} = t^{\tilde{b}^I}, \qquad e^{\lambda^i} = e^{\tilde{c}^i \tau} = t^{\tilde{b}^i} \tag{82}$$

up to multiplicative constants where $(\tilde{c}^I, \tilde{c}^i, \tilde{b}^I, \tilde{b}^i)$ are constants. The evolution is then of Kasner type and is similar to that described in Sec. III B 1. The constants $\tilde{c}^I$s are determined by the initial values l_0^I and K^I, but obtaining the exact dependence in the general case requires complete solution for $l^I(\tau)$. However, if the initial values l_0^I are large and negative then we have $e^{l^I} \ll 1$ for all $\tau < 0$ and, hence, $\tilde{c}^I = K^I$ to a good approximation.

2. Asymptotic evolution: $e^{\Lambda} \to \infty$

It follows from the above discussion that $e^{\Lambda} \to \infty$ in the limit $\tau \to \tau_\infty$ where τ_∞ is finite. Also, $e^{l^I} \to \infty$ in this limit and τ_∞ depends on the initial values l_0^I and K^I.

Although solutions for $l^I(\tau)$ are not known, their asymptotic forms in the limit $\tau \to \tau_\infty$, and hence those of $\lambda^i(\tau)$, may be obtained following the analysis given in Sec. III B 2. $\mathcal{G}^{IJ}$ in Eq. (72) is a special case of the example (60) where, now, $N = 4$, $a = 2u^2$, and $b = 1$. Hence, it can be shown to correspond to the possibility (i) given in Eq. (58). Therefore, we have $\gamma^I = 1$ and $e^{c^I} = 2\sum_J \mathcal{G}_{IJ} = \frac{1}{3u^2}$.

It then follows from Eq. (52) that e^{l^I} and e^{λ^i} are given in the limit $\tau \to \tau_\infty$ by

$$e^{l^I} = \frac{1}{3u^2} \frac{1}{(\tau_\infty - \tau)^2} \tag{83}$$

$$e^{\lambda^i} = e^{v^i} \left(\frac{1}{3u^2} \frac{1}{(\tau_\infty - \tau)^2} \right)^{\sum_I u_I^i} \tag{84}$$

where, since $\rho_{I0} = e^{l_0^I}$, we have

$$v^i = -\sum_J u_J^i l_0^J + L^i \tau_\infty, \qquad e^{v^i} = e^{L^i \tau_\infty} \prod_J (\rho_{J0})^{-u_J^i}. \tag{85}$$

Also, since $\gamma = \sum_{i,I} u_I^i = \frac{1}{u}$, we have from Eq. (54) that the physical time t is given in this limit by

$$t - t_s = A(\tau_\infty - \tau)^{-(2-u)/u} \tag{86}$$

where t_s and A are finite constants. Clearly, $t \to \infty$ in the limit $\tau \to \tau_\infty$ since it is assumed that $0 < u < 2$. In this limit, the scale factors e^{λ^i} may be written in terms of t as

$$e^{\lambda^i} = e^{v^i} (Bt)^{\beta^i} \tag{87}$$

where B is a constant and $\beta^i = \frac{2u}{2-u} \sum_J u_J^i$. Using Eq. (75) for u_I^i, the exponents β^i are given by

$$\beta^i \propto (0, 0, 0, 0, 0, 0, 0, 1, 1, 1) \tag{88}$$

where the proportionality constant is $\frac{2}{3(2-u)}$. Note that $\beta = \sum_i \beta^i = \frac{2}{2-u} > 1$. Hence, we have $e^{\Lambda} \sim t^{\beta}$ in the limit $t \to \infty$ with $\beta > 1$. See the discussion below Eq. (16) for the relevance of this feature.

Thus, asymptotically in the limit $t \to \infty$, we obtain that $e^{\lambda^i} \to t^{2/3(2-u)}$ for the common transverse directions $i = 8$, 9, 10. Hence, these directions continue to expand, their expansion being precisely that of a (3 + 1)—dimensional homogeneous, isotropic universe containing a perfect fluid whose equation of state is $p = (1 - u)\rho$. Also, $e^{\lambda^i} \to e^{v^i}$ for the brane directions $i = 1, \cdots, 7$. Hence, these directions cease to expand or contract. Their sizes are thus stabilized and are given by e^{v^i}. Note that this result is in accord with the general result described in Sec. II B since, in the limit $\tau \to \tau_\infty$, the brane densities $\rho_I \propto e^{l^I}$ all become equal and hence the four types of branes all become identical; and, $t \to \infty$ and $e^{\Lambda} \sim t^{\beta} \to \infty$ with $\beta > 1$.

C. Mechanism of stabilization

Using the asymptotic solutions, we can now give a physical interpretation of the dynamical mechanism underlying the stabilization of the brane directions seen above for the $22'55'$ configuration.

We first study the stabilization process. Consider Eq. (66) for $\lambda^1_{\tau\tau}$, for example. Using the values of u^{iI} given in Eq. (74), we have

$$\lambda^1_{\tau\tau} \propto (-2e^{l^1} + e^{l^2} - e^{l^3} + 2e^{l^4}). \tag{89}$$

In the $22'55'$ configuration, x^1 direction is wrapped by 2 branes and 5 branes and is transverse to $2'$ branes and $5'$ branes. Thus, from the above equation for λ^1 and from similar equations for $\lambda^2, \cdots, \lambda^7$, we see that 2 brane and 5 brane directions "contract with a force" proportional to $2\rho_{(2)}$ and $\rho_{(5)}$ respectively, whereas the directions transverse to them "expand with a force" proportional to $\rho_{(2)}$ and $2\rho_{(5)}$ respectively, where $\rho_{(*)} \propto e^{l^{(*)}}$ are the time dependent densities of the corresponding branes.

When $\rho_I \propto e^{l^I}$ all become equal, the forces of expansion cancel the forces of contraction resulting in vanishing net force for the x^1 direction. Then, using Eq. (36), one has

$$\lambda^1_{\tau\tau} = e^{2\Lambda}(\lambda^1_{tt} + \Lambda_t \lambda^1_t) = 0. \tag{90}$$

Now, as described earlier in the context of Eqs. (17) and (18), the transient "velocity" λ^1_t is damped and λ^1 reaches a constant value in the expanding universe here since we have $e^{\Lambda} \sim t^{\beta}$ in the limit $t \to \infty$ with $\beta > 1$. The result is the stabilization of the x^1 direction.

The stabilized size e^{v^1} of x^1 direction is given by

$$e^{v^1} = e^{L^1 \tau_\infty} \left(\frac{\rho_{20} \rho_{40}^2}{\rho_{30} \rho_{10}^2} \right)^{1/6u}, \tag{91}$$

see Eq. (85). Note that e^{v^1} can be interpreted as arising from the imbalance among the initial brane densities ρ_{I0}, and from the parts L^1 of $\lambda_t^1(0)$ which indicate the transients. The above analysis can be similarly applied to the stabilization of other brane directions $(x^2, \cdots, x^7)$ in the $22'55'$ configuration.

Thus, three conditions need to be satisfied for stabilization: (1) the time dependent brane densities $\rho_I \propto e^{l^I}$ all become equal; (2) the forces of expansion and contraction for each of the brane directions be just right so that the net force vanishes; (3) the universe be expanding as $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$ so that the transient velocities are damped and the corresponding scale factors reach constant values.

For any mutually BPS $N > 1$ intersecting brane configurations with the equations of state as assumed here, it is straightforward to show using the earlier analysis that the evolution equations ensure that e^{l^I} all become equal asymptotically even if they were unequal initially, and that $e^\Lambda \sim t^\beta$ in the limit $t \to \infty$ with $\beta > 1$. Thus conditions (1) and (3) are satisfied. Condition (2) requires the brane configuration to be such that each of the brane directions is wrapped by, and is transverse to, just the right number and kind of branes. This condition is satisfied for the $N = 4$ configurations $22'55'$ and $55'5''W$, both of which result in the stabilization of seven brane directions and the expansion of the remaining three spatial directions. To our knowledge, the only other configurations which satisfy the condition (2) are the $N = 3$ configurations $22'2''$ and $25W$, both of which result in the stabilization of six brane directions and the expansion of the remaining four spatial directions [24]. However it is the $N = 4$ configurations that are entropically favorable, see Eq. (1).

Note that, as described in Sec. II B and up to certain technical assumptions regarding the equality of brane densities and the asymptotic behavior of e^Λ, the stabilization here follows essentially as a consequence of U duality symmetries. In particular, it is independent of the ansatz for energy momentum tensors, or of the assumptions about equations of state, as long as the components of the energy momentum tensors obey the U duality constraints of the type given in Eq. (16). Obtaining the details of the stabilization, however, requires further assumptions, e.g., of the type made here.

Note also that the present mechanism of stabilization of seven brane directions, and the consequent emergence of three large spatial directions, is very different from the ones proposed in string theory or in brane gas models [28–31].

V. STABILIZED SIZES OF BRANE DIRECTIONS

We thus see for the $22'55'$ configuration that, asymptotically in the limit $e^\Lambda \to \infty$, the initial $(10 + 1)$—dimensional universe effectively becomes $(3 + 1)$—dimensional. Also, if $v^s = \min\{v^1, \cdots, v^7\}$ then a dimensional reduction of the $(10 + 1)$—dimensional M theory along the corresponding x^s direction gives type IIA string theory with its dilaton now stabilized. Using the standard relations, one can obtain the string coupling constant g_s, the string scale M_s, and the four-dimensional Planck scale M_4 in terms of the M theory scale M_{11} and the stabilized values e^{v^i}. Defining $v^c = \sum_{i=1}^7 v^i$ and assuming, with no loss of generality, that the coordinate sizes of all spatial directions are of $\mathcal{O}(M_{11}^{-1})$, we obtain

$$g_s^2 = e^{3v^s}, \qquad M_4^2 = e^{v^c - v^s} M_s^2 = e^{v^c} M_{11}^2 \tag{92}$$

where the equalities are valid up to numerical factors of $\mathcal{O}(1)$ only and

$$e^{v^c} = e^{L^c \tau_\infty} \left(\frac{\rho_{10}\rho_{20}}{\rho_{30}\rho_{40}} \right)^{1/6u}, \qquad L^c = \sum_{i=1}^7 L^i \tag{93}$$

as follows from Eqs. (75) and (85), and $\rho_{I0} = e^{l_0^I}$. Also, note that $g_s = (\frac{M_s}{M_{11}})^3$.

Since we have an asymptotically $(3 + 1)$—dimensional universe evolving from a $(10 + 1)$—dimensional one, it is of interest to study the resulting ratios $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$, and study their dependence on the initial values (l_0^I, K^I, L^i). In particular, one may like to know the generic values of these ratios and to know whether arbitrarily small values are possible. Setting $M_4 = 10^{19}$ GeV, one then knows the generic scales of M_{11} and M_s and, for example, whether $M_{11} = 10^{-15} M_4 = 10$ TeV is possible.

In view of the relations between (M_{11}, M_s, M_4) given in Eq. (92), this requires studying the stabilized values e^{v^c} and $e^{v^c - v^s}$, their dependence on (l_0^I, K^I, L^i), and knowing whether they can be arbitrarily large. Note that if $L^i = 0$ for all i then v^i are all determined in terms of l_0^I only, see Eq. (85). It is then obvious from Eqs. (85) and (93) that any values for e^{v^c} and $e^{v^c - v^s}$, no matter how large, may be obtained by fine-tuning ρ_{I0} correspondingly.[10]

This statement remains true even when L^is do not all vanish. In this case, however, one may question the necessity of fine-tuning since, for example, the relation $e^{v^c} \propto e^{L^c \tau_\infty}$ suggests that large values such as $10^{30} \sim e^{70}$ may be obtained by tuning L^is, or τ_∞, or both to within a couple of orders of magnitude only. It turns out, as we explain below, that fine-tuning is still necessary to obtain such large values.

Consider first the possibility of tuning L^i. Note that Eqs. (67) and (70) are invariant under the scaling

$$(E, e^{l^I}, \tau) \Rightarrow \left(\sigma^2 E, \sigma^2 e^{l^I}, \frac{\tau}{\sigma} \right) \tag{94}$$

[10]It follows from Eq. (71) and the definition of E that the generic ranges of the initial values may be taken to be given by $|L^i| \simeq K^I \simeq \sqrt{E} \simeq \sqrt{\rho_{I0}}$ within a couple of orders of magnitude. If the initial values lie way beyond such a range then we consider it as fine-tuning.

where σ is a positive constant. The initial values scale correspondingly as

$$(e^{l_0^I}, K^I, L^i) \rightarrow (\sigma^2 e^{l_0^I}, \sigma K^I, \sigma L^i). \qquad (95)$$

It then follows from Eq. (68) that λ^i, and hence e^{v^i}, remain invariant.[11] This scaling property merely reflects the choice of a scale for time. For example, using this scaling, one may set $\sum_J e^{l_0^J} = 1$ or, when $E > 0$ as is the case here, set $E = 1$. The corresponding σ then provides a natural time scale for evolution. We set $E = 1$ using the above scaling.

With $E = 1$, the value of τ_∞ now depends only on l_0^I and K^I. Since $2E = \sum_i (L^i)^2 - (\sum_i L^i)^2$, it is still plausible to have a range of nonzero measure where L^i are large and $E = 1$, and thereby obtain large values for e^{v^c} and $e^{v^c - v^s}$. However, L^is are further constrained by $\sum_i u_i^I L^i = 0$, $I = 1, \cdots, 4$, and consequently their magnitudes are all bounded from above. For example, with $E = 1$, we obtain $(L^c)^2 \le \frac{8}{3}$. See remark (2) in Sec. IV A. Thus, large values of e^{v^i} cannot be obtained by tuning L^i alone.

Consider now the possibility of tuning τ_∞. Obtaining the dependence of τ_∞ on (l_0^I, K^I) requires explicit solutions which are not available. Hence, we obtain τ_∞ numerically. We will present the numerical results in the next section. Here we point out that an approximate expression for τ_∞ can be given in the limit when $e^{l_0^I} \ll E$ for all I. The reasoning involved is analogous to that used in obtaining τ_a in Eq. (49). Using similar reasoning and setting $E = 1$ now, we have that if $e^{l_0^I} \ll 1$ for all I then

$$\tau_\infty \simeq \tau_a = \min\{\tau_I\}, \qquad \tau_I = -\frac{l_0^I}{K^I}. \qquad (96)$$

Note that τ_a can be calculated easily and requires no knowledge of explicit solutions. Our numerical results show that τ_a given above indeed provides a good approximation to τ_∞ when $e^{l_0^I} \ll 1$ for all I.

Note also that K^I must satisfy Eq. (71) with $E = 1$. It then follows from an analysis similar to that given in Appendix C that K^I are all positive, cannot be too small, and are of $\mathcal{O}(1)$ generically. Hence, in the limit $e^{l_0^I} \ll 1$ for all I, τ_a in Eq. (96) are of $\mathcal{O}(\min\{-l_0^I\})$. This indicates that large values of τ_∞, and hence of e^{v^i}, cannot be obtained by tuning K^I alone; a tuning of l_0^I, which translates to fine-tuning of $\rho_{I0} = e^{l_0^I}$, is required. Our numerical analysis also supports this conclusion.

We thus find that, even when L^is do not all vanish, a fine-tuning of $\rho_{I0} = e^{l_0^I}$ is necessary to obtain large values for $e^{v^c - v^s}$ and e^{v^c}.

VI. TIME VARYING NEWTON'S CONSTANT

The evolution of the 11-dimensional early universe which is dominated by the $22'55'$ configuration described here can also be considered from the perspective of four-dimensional spacetime. Indeed, in general, let the 11-dimensional line element ds be given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \sum_{i=1}^{7} e^{2\lambda^i} (dx^i)^2 \qquad (97)$$

where $x^\mu = (x^0, x^8, x^9, x^{10})$, with $x^0 = t$, describes the four-dimensional spacetime, and the fields $g_{\mu\nu}$ and λ^i, $i = 1, \cdots, 7$, depend on x^μ only. Also, let $\Lambda^c = \sum_{i=1}^7 \lambda^i$. It is then straightforward to show that the gravitational part of the 11-dimensional action S_{11} given in Eq. (3) becomes

$$S_4 = \frac{V_7}{16\pi G_{11}} \int d^4x \sqrt{-g_{(4)}} e^{\Lambda^c} \Big\{ R_{(4)} + (\nabla_{(4)} \Lambda^c)^2 - \sum_{i=1}^{7} (\nabla_{(4)} \lambda^i)^2 \Big\} \qquad (98)$$

where V_7 is the coordinate volume of the seven-dimensional space and the subscripts (4) indicate that the corresponding quantities are with respect to the four-dimensional metric $g_{\mu\nu}$. The action S_4 describes four-dimensional spacetime in which the effective Newton's constant G_4 is spacetime dependent and is given by

$$G_4(x^\mu) = e^{-\Lambda^c(x^\mu)} \frac{G_{11}}{V_7}. \qquad (99)$$

In the case of early universe, the fields $g_{\mu\nu}$ and λ^i depend on t only. Then G_4 is time dependent and we have, for G_4 and its fractional time derivative,

$$G_4(t) = e^{-\Lambda^c(t)} \frac{G_{11}}{V_7}, \qquad \frac{(G_4)_t}{G_4} = -\Lambda_t^c. \qquad (100)$$

For the four-dimensional spacetime arising from the $22'55'$ configuration, the $g_{\mu\nu}$ fields are just the scale factors $(e^{\lambda^8}, e^{\lambda^9}, e^{\lambda^{10}})$ for (x^8, x^9, x^{10}) directions, and all $\lambda^i(\tau)$ are given in Eq. (68) in terms of $l^I(\tau)$, $I = 1, \cdots, 4$. Then, using Eq. (75) and the definitions of Λ^c, L^c, and v^c, we have

$$\Lambda^c = -\frac{1}{6u}(l^1 + l^2 - l^3 - l^4) - L^c(\tau_\infty - \tau) + v^c. \qquad (101)$$

In the limit $t \rightarrow \infty$, we have from the results given earlier that $\tau \rightarrow \tau_\infty$ and the fields l^I all become equal. Then $\Lambda^c \rightarrow v^c$ where e^{v^c} is given in Eq. (93), and the three-dimensional scale factors evolve as in the standard FRW case, namely $e^{\lambda^8} = e^{\lambda^9} = e^{\lambda^{10}} \sim t^{2/3(2-u)}$ as given in Eqs. (87) and (88).

It thus follows that the effective Newton's constant G_4 varies with time in the early universe and, in the case of $22'55'$ configuration, approaches a constant value $= e^{-v^c} \frac{G_{11}}{V_7}$ as the four-dimensional universe expands to large size. The precise time dependence of G_4 will follow from

[11] This invariance is equivalent to that of Eqs. (28) and (29) under the scaling $(\lambda^i, \rho, p_i, t) \rightarrow (\lambda^i, \sigma^2\rho, \sigma^2 p_i, \frac{t}{\sigma})$.

explicit solutions to Eqs. (67) and (70). The consequences of a such a time dependent G_4 are clearly interesting, and are likely to be important too. But their study is beyond the scope of the present paper.

However, we like to point out here a characteristic feature of the time dependence of G_4 which arises in the case of $22'55'$ configuration. Consider the behavior of the differences $l^I - l^J$ in the limit $\tau \rightarrow \tau_\infty$ which, in our case, vanish to the leading order. These quantities have been analyzed in Sec. III C and, for the example of the $\mathcal{G}^{IJ}$ given in Eq. (60), they are given by Eqs. (64) and (65) to the nontrivial leading order. The case of the $22'55'$ configuration corresponds to $N = 4$, $a = 2u^2$, and $b = 1$. Noting that $s^I - s^J = l^I - l^J$ and that $\Delta < 0$ in our case, Eq. (65) now gives

$$(l^I - l^J) \sim (\tau_\infty - \tau)^{(1/2)(1 \pm i\sqrt{5/3})} \tag{102}$$

to the leading order. Clearly, $\Lambda^c(\tau)$ given in Eq. (101) will also have the same form as above to the nontrivial leading order. Thus, taking the real part and writing in terms of t using Eq. (86), we have

$$\Lambda^c = v^c + \frac{b}{t^\alpha}\mathrm{Sin}(\omega lnt + \phi) \tag{103}$$

to the nontrivial leading order in the limit $t \rightarrow \infty$ where b and ϕ are constants, $\alpha = \frac{u}{2(2-u)}$, and $\omega = \sqrt{\frac{5}{3}}\frac{u}{2(2-u)}$. Correspondingly, the time varying Newton's constant is given by

$$G_4 \propto e^{-\Lambda^c} = e^{-v^c}\left(1 - \frac{b}{t^\alpha}\mathrm{Sin}(\omega lnt + \phi)\right) \tag{104}$$

to the leading order in the limit $t \rightarrow \infty$. Note that the constants b and ϕ depend on the details of matching. The constants α and ω arise as real and imaginary parts of an exponent on time variable, see Eq. (102). They do not depend on the initial values (l_0^I, K^I, L^i) and thus are independent of the details of evolution, but depend only on the configuration parameters N and u.

The amplitude of time variation of G_4 is dictated by α, and it vanishes in the limit $t \rightarrow \infty$. Hence, the time variation of G_4 in Eq. (104) is unlikely to contradict any late time observations. The time variation of G_4 has log periodic oscillations also: G_4 has an oscillatory behavior where the nth and $(n+1)$th nodes occur at times t_n and t_{n+1} which are related by $lnt_{n+1} = \frac{\pi}{\omega} + lnt_n$, i.e. by $t_{n+1} = e^{\pi/\omega}t_n$. The characteristic signatures and observational consequences of such log periodic variations of G_4 are not clear to us.

Log periodic behavior occurs in many physical systems with "discrete self similarity" or "discrete scale symmetry": for example, in quantum mechanical systems with strongly attractive $\frac{1}{r^2}$ potentials near zero energy [39]; in Choptuik scaling and brane–black hole merger transitions [40]; and in a variety of dynamical systems [41]. Algebraically, the log periodicity arises when an exponent on an independent variable becomes complex for certain values of system parameters. The relevant equations and solutions can often be cast in a form given in Eqs. (64) and (65). But we are not aware of a physical reason which explains the ubiquity of the log periodicity.

To our knowledge, this is the first time a log periodic behavior appears in a cosmological context. One expects such a behavior to leave some novel imprint in the universe. But it is not clear to us which effects to look for, or which observables are sensitive to the log periodic variations of G_4.

VII. NUMERICAL RESULTS

We are unable to solve explicitly the Eqs. (67)–(70) describing the early universe evolution. Hence, we have analyzed these equations numerically. In this section, we briefly describe our procedure and present a few illustrative results. We have analyzed both the $u = \frac{2}{3}$ and $u = 1$ cases which would correspond to a four-dimensional universe dominated by radiation and pressureless dust, respectively. The results are qualitatively the same and, hence, we take $u = \frac{2}{3}$ in the following. Note that ω in Eq. (104) is then determined and, for $u = \frac{2}{3}$, the nth and $(n+1)$th nodes in the log periodic oscillations occur at times t_n and t_{n+1} related by $ln(\frac{t_{n+1}}{t_n}) = 4\pi\sqrt{\frac{3}{5}} \simeq 9.734$.

We proceed as follows. We start at an initial time $\tau = 0$ and choose a set of initial values $l_0^I = ln\rho_{I0}$. For each set of l_0^I, we further choose numerous arbitrary sets of (K^I, L^i) such that $K^I > 0$, $E = 1$, and Eqs. (71) and (76) are satisfied.[12] For each set of initial values (l_0^I, K^I, L^i), we then numerically analyze the evolution for $\tau > 0$ and obtain the value of τ_∞; the evolution of l^I, $(\lambda^1, \cdots, \lambda^{10})$, and t; the stabilized values $(v^1, \cdots, v^7)$; and the resulting values for $(g_s, \frac{M_{11}}{M_4}, \frac{M_s}{M_4})$. For a few sets of initial values, we have analyzed the evolution for $\tau < 0$ also.

We find that the numerical results we have obtained confirm the asymptotic features described in this paper:

(1) e^{λ^i} and l^I all vanish in the limit $\tau \rightarrow -\infty$. In this limit, the evolution of the scale factors e^{λ^i} is of Kasner type.

(2) l^I and the physical time t all diverge in the limit $\tau \rightarrow \tau_\infty$ where τ_∞ is finite. In this limit, the scale factors $(e^{\lambda^8}, e^{\lambda^9}, e^{\lambda^{10}})$ evolve as in the standard FRW case and $(e^{\lambda^1}, \cdots, e^{\lambda^7})$ reach constant values.

(3) τ_a given in Eq. (96) provides a good approximation to τ_∞ when $e^{l_0^I} \ll 1$ for all I.

[12]There are two special choices for the set of K^I. One is where $K^1 = \cdots = K^4$ and another is the one which maximizes the approximation τ_a given in Eq. (96). The later set may be determined by the algorithm given in Appendix D.

TABLE I. The initial values $-(l_0^1, l_0^2, l_0^3, l_0^4)$ and the resulting values of τ_a, τ_∞, $\frac{M_{11}}{M_4}$, and $\frac{M_s}{M_4}$. The values in the last four columns have been rounded off to two decimal places.

	$-(l_0^1, l_0^2, l_0^3, l_0^4)$	τ_a	τ_∞	$\frac{M_{11}}{M_4}$	$\frac{M_s}{M_4}$
(1)	(2, 5, 8, 8)	1.88	3.21	5.77×10^{-2}	2.86×10^{-2}
(2)	(5, 4, 6, 9)	2.96	4.16	4.56×10^{-2}	1.93×10^{-2}
(3)	(15, 12, 10, 16)	4.88	6.61	5.95×10^{-2}	1.96×10^{-2}
(4)	(25, 26, 27, 28)	22.00	22.59	1.99×10^{-7}	7.30×10^{-10}
(5)	(41, 30, 50, 43)	25.80	28.30	1.87×10^{-10}	2.92×10^{-11}
(6)	(44.5, 34, 49, 49.5)	34.82	36.20	2.59×10^{-14}	3.80×10^{-15}

TABLE II. The initial values of (K^I, L^i) for the data shown in Table I, tabulated here up to overall positive constants. These constants and the remaining L^is are to be fixed as explained in the text.

	$(K^1, K^2, K^3 K^4) \propto$	$(L^1, L^2, L^6, L^7, L^8, L^9) \propto$
(1)	(4.65, 9.14, 4.57, 6.87)	(0.60, 0.62, 0.76, 0.72, −0.94, −0.26)
(2)	(8.86, 8.26, 6.01, 6.62)	−(0.08, −0.93, 0.08, −0.72, 0.54, 0.63)
(3)	(1.61, 2.65, 0.69, 2.1)	−(0.2, −0.68, −0.14, 0.3, 0.08, 0.19)
(4)	(1.03, 1.18, 1.17, 1.27)	(0.08, 0.58, 0.27, 0.27, −0.66, −0.66)
(5)	(5.24, 4.83, 4.30, 4.96)	(0.74, 0.02, 0.24, −0.22, −0.61, −0.75)
(6)	(33.79, 24.23, 35.4, 32.29)	(11.72, 9.31, 4.59, −6.46, −21.02, −21.02)

(4) Any values for the ratios $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ can be obtained, but a corresponding fine-tuning of $\rho_{I0} = e^{l_0^I}$ is necessary.

(5) The log periodic oscillations of $l^I - l^J$, equivalently of $(\lambda^1, \cdots, \lambda^7)$, can also be seen in the limit $\tau \to \tau_\infty$. They can be matched to solutions of the type given in Eq. (102).

To illustrate the values of τ_∞ and the ratios $(\frac{M_{11}}{M_4}, \frac{M_s}{M_4})$ one obtains, and to give an idea of their dependence on the initial values l_0^I, we tabulate these quantities in Table I for a few sets of initial values (l_0^I, K^I, L^i). We have also tabulated the values of τ_a as given by Eq. (96). The value of g_s follows from $g_s = (\frac{M_s}{M_{11}})^3$ and, hence, is not tabulated.

In Table II, the corresponding initial values (K^I, L^i), $i = 1, 2, 6, 7, 8, 9$, are tabulated up to overall positive constants. The remaining L^is are given by Eqs. (76) and the overall positive constants are determined by $E = 1$ and Eq. (71). All the sets of initial values (l_0^I, K^I, L^i) are chosen arbitrarily with no particular pattern and are presented here to give an idea of the typical results.

We find, by analyzing numerous sets of initial values, that changing the values of (K^I, L^i) for a given set of l_0^I changes the values of $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ only up to about 4 orders of magnitude. Any bigger change requires changing $e^{l_0^I}$ to a similar order, confirming that any values for $\frac{M_{11}}{M_4}$ and $\frac{M_s}{M_4}$ can be obtained but only by fine-tuning $\rho_{I0} = e^{l_0^I}$.

We illustrate the evolution of the universe for the data set (3) given in Tables I and II where many features can be seen clearly. The evolution with respect to τ of l^I is shown in Figs. 1, 2(a), and 2(b). For negative values of τ not shown in Fig. 1, all l^I evolve along straight lines with no further crossings and their evolution is of Kasner type. Also, all l^I diverge at a finite value $\tau_\infty \simeq 6.612$ of τ. The magnified plots in Figs. 2(a) and 2(b) for $\tau > 6.40$ and for $\tau > 6.55$ respectively show the continually criss-crossing evolution of l^I which, near τ_∞, represent the log periodic oscillations and are well described by Eq. (102).

The evolution with respect to lnt of $(\lambda^1, \cdots, \lambda^7)$ is shown in Fig. 3. It can be seen that $(\lambda^1, \cdots, \lambda^7)$, and hence the scale factors $(e^{\lambda^1}, \cdots, e^{\lambda^7})$ of the brane directions, all stabilize to constant values as $t \to \infty$.

The evolution with respect to lnt of $(\lambda^8, \lambda^9, \lambda^{10})$ and $\Lambda^c = \sum_{i=1}^7 \lambda^i$ is shown in Fig. 4. Note that the seven-dimensional volume of the brane directions $\propto e^{\Lambda^c}$ and

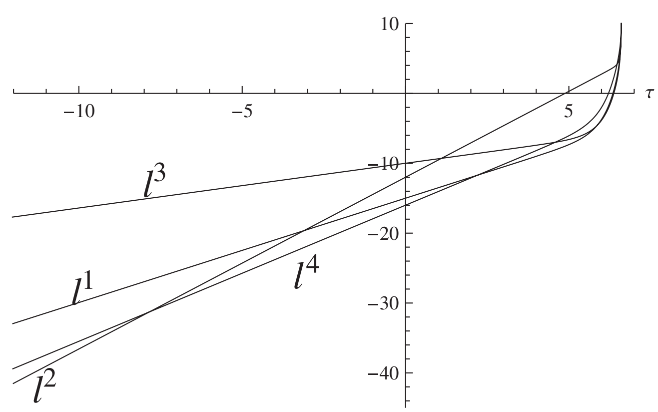

FIG. 1. The plots of l^I with respect to τ. The lines continue with no further crossings for negative values of τ not shown in the figure. All l^I diverge at $\tau_\infty \simeq 6.612$. All figures in this paper are for the data set (3) given in Tables I and II.

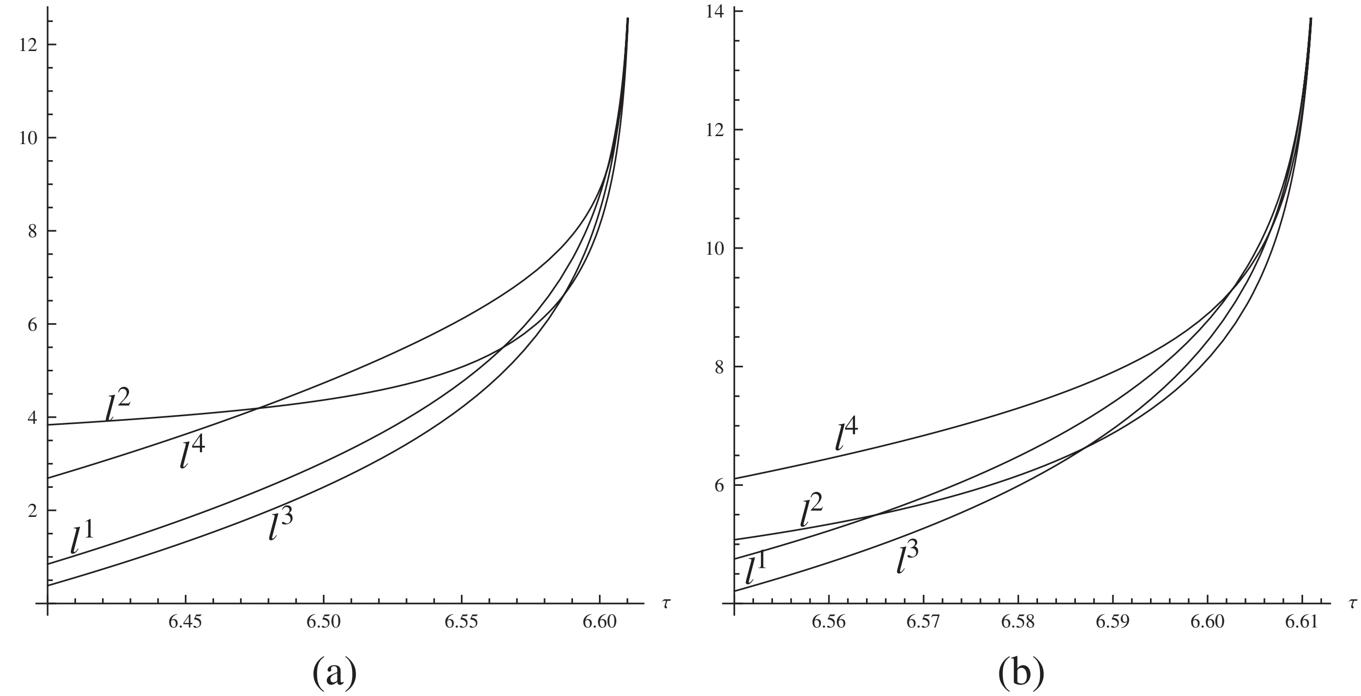

FIG. 2. (a), (b) The magnified plots of l^I with respect to τ for $\tau > 6.40$ and for $\tau > 6.55$ showing the continually criss-crossing evolution of l^I. Near $\tau_\infty \simeq 6.612$, these crossings are well described by Eq. (102).

that it stabilizes to a constant value e^{v^c} as $t \longrightarrow \infty$. We have also verified that the evolution of $(\lambda^8, \lambda^9, \lambda^{10})$ as $t \longrightarrow \infty$ is the same as that of the corresponding ones in a four-dimensional radiation-dominated FRW universe.

The log periodic oscillations of Λ^c are illustrated in Figs. 5(a) and 5(b) by magnifying the plots of $(\Lambda^c - v^c)$ with respect to lnt for $lnt > 20$ and for $lnt > 30$. The internode separations can be seen in these figures, and they match the value $\simeq 9.734$ obtained in Eq. (103) from the asymptotic analysis.

In all the cases we have analyzed, the evolutions of (l^I, λ^i) are qualitatively similar to the ones shown in the

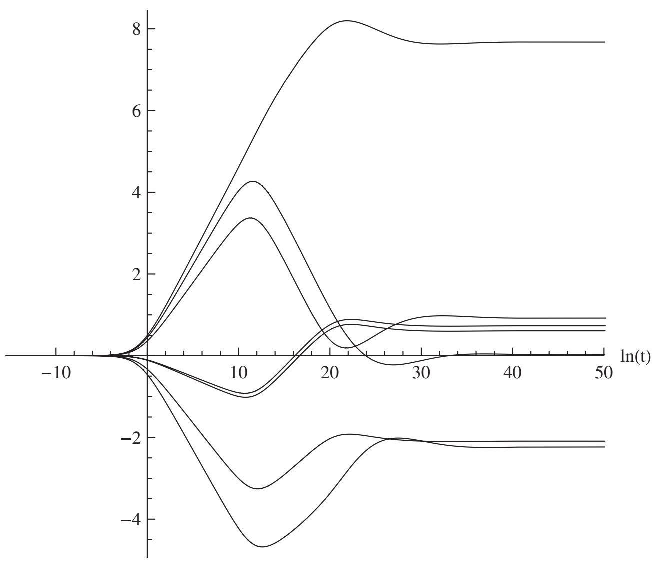

FIG. 3. The plots of $(\lambda^1, \cdots, \lambda^7)$ with respect to lnt. The lines, from top to bottom at the rightmost end, correspond to $(\lambda^2, \lambda^3, \lambda^5, \lambda^6, \lambda^4, \lambda^7, \lambda^1)$. $(\lambda^1, \cdots, \lambda^7)$ all stabilize to constant values as $t \longrightarrow \infty$.

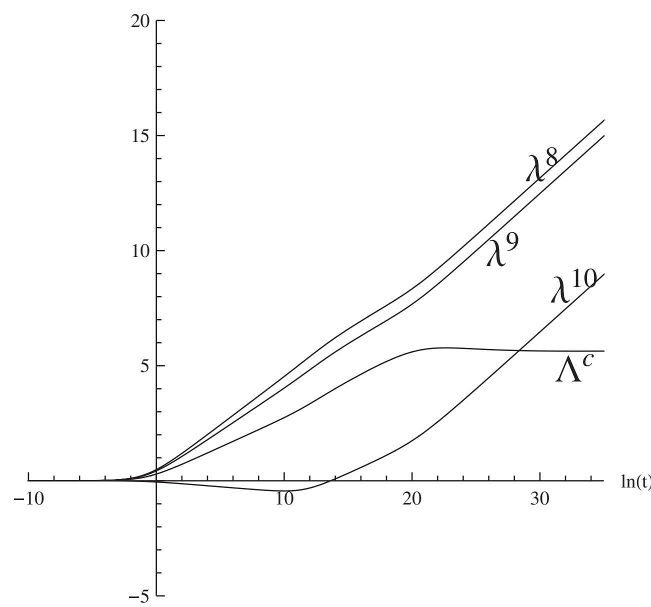

FIG. 4. The plots of $(\lambda^8, \lambda^9, \lambda^{10}, \Lambda^c)$ with respect to lnt. The seven-dimensional volume of the brane directions $\propto e^{\Lambda^c}$. The evolution of $(\lambda^8, \lambda^9, \lambda^{10})$ as $t \longrightarrow \infty$ is the same as that of the corresponding ones in a four-dimensional radiation-dominated FRW universe.

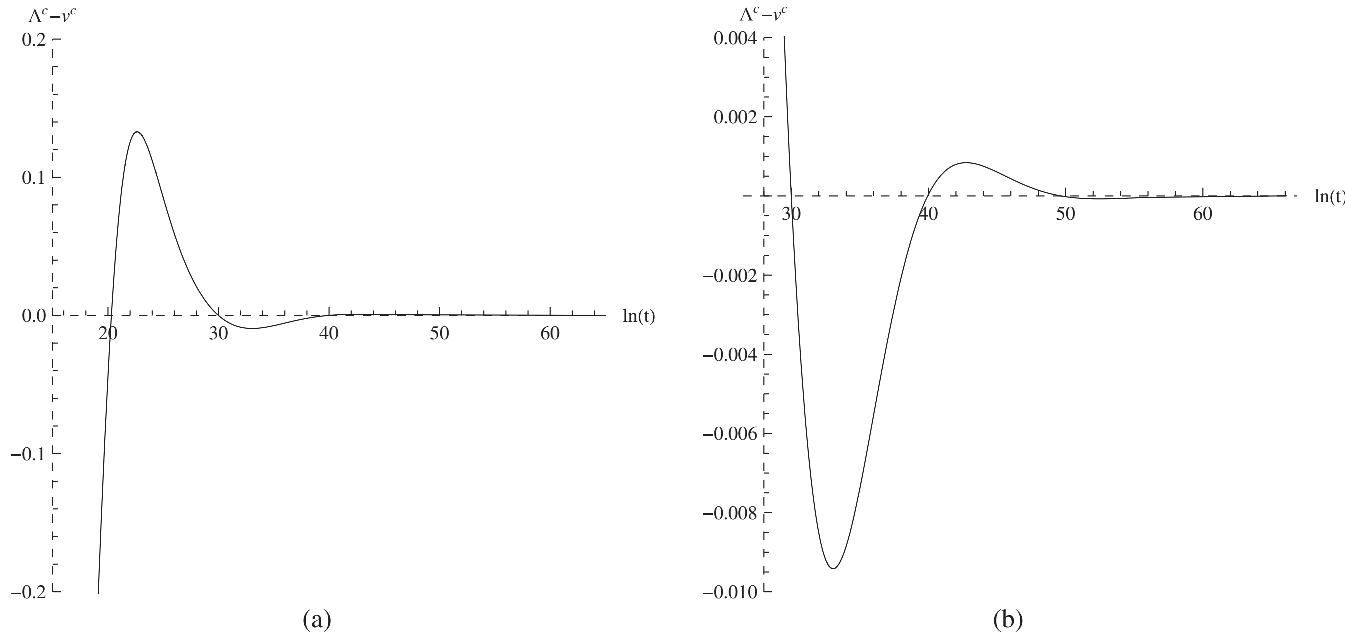

FIG. 5. (a), (b) The magnified plots of $(\Lambda^c - \upsilon^c)$ with respect to lnt for $lnt > 20$ and for $lnt > 30$ showing log periodic oscillations and the internode separation which is $\simeq 9.734$.

figures above. The details, such as the rise and fall of λ^i in the initial times or the value of τ_∞ or the stabilized values of the brane directions, depend on the initial values but the asymptotic features described in the beginning of this section are all the same. Hence, we have presented the plots for one illustrative set of initial values only.

VIII. SUMMARY AND CONCLUSIONS

We summarize the main results of the paper. We assume that the early universe in M theory is homogeneous, anisotropic, and is dominated by $N = 4$ mutually BPS $22'55'$ intersecting brane configurations which are assumed to be the most entropic ones. Also, the ten-dimensional space is assumed to be toroidal. We further assumed that the brane antibrane annihilation effects are negligible during the evolution of the universe at least until the brane directions are stabilized resulting in an effective (3 + 1)—dimensional universe.

We then present a thorough analysis of the evolution of such an universe. We obtain general relations among the components of the energy momentum tensor T_{AB} using U duality symmetries of M theory and show that these relations alone imply, under a technical assumption, that the $N = 4$ mutually BPS $22'55'$ intersecting brane configurations with identical numbers of branes and antibranes will asymptotically lead to an effective (3 + 1)—dimensional expanding universe.

To obtain further details of the evolution, we make further assumptions about T_{AB}. We then analyze the evolution equations in D dimensions in general, and then specialize to the 11-dimensional case of interest here. Since explicit solutions are not available, we apply the general analysis and describe the qualitative features of the evolution of the $N = 4$ brane configuration: In the asymptotic limit, three spatial directions expand as in the standard FRW universe and the remaining seven spatial directions reach constant, stabilized values. These values depend on the initial conditions and can be obtained numerically. Also, any stabilized values may be obtained but it requires a fine-tuning of the initial brane densities.

We also present a physical description of the mechanism of stabilization of the seven brane directions. The stabilization is due, in essence, to the relations among the components of T_{AB} which follow from U duality symmetries, and to each of the brane directions in the $N = 4$ configuration being wrapped by, and being transverse to, just the right number and kind of branes. This mechanism is very different from the ones proposed in string theory or in brane gas models.

In the asymptotic limit, from the perspective of four-dimensional spacetime, we obtain an effective four-dimensional Newton's constant G_4 which is now time varying. Its precise time dependence will follow from explicit solutions of the 11-dimensional evolution equations. We find that, in the case of $N = 4$ brane configuration, G_4 has characteristic log periodic oscillations. The oscillation "period" depends only on the configuration parameters.

Using numerical analysis, we have confirmed the qualitative features mentioned above.

We now make a few comments on the assumptions made in this paper. Note that the assumptions mentioned above in the first paragraph of this section pull a rug over many important dynamical questions that must be answered in a final analysis. Some of these questions,[13] in the context of M theory, are:

(i) Starting from the highly energetic and highly interacting M theory excitations, which are expected to describe the high temperature state of the universe, how does a 11-dimensional spacetime emerge?

(ii) What determines the topology of the ten-dimensional space? Here, we assumed it to be toroidal. How does the universe evolve if its spatial topology is not toroidal?

(iii) From what stage onwards, does the 11-dimensional "low-energy" effective action provide a good description of further evolution?

(iv) What are the relevant "low-energy" configurations of M theory? Here, based on the black hole studies, we have assumed that the $N = 4$ mutually BPS $22'55'$ intersecting brane configurations are the most entropic ones and, hence, that they are the dominant configurations in the early universe studied here.

This raises further questions: Are the $22'55'$, and not some other mutually BPS $N \geq 4$ or some other non-BPS, configurations really the most entropic and the dominant ones? Even assuming that mutually BPS $N = 4$ is the answer, are there other $N = 4$ configurations beside the $22'55'$ ones and, if so, how do they affect the evolution described here? What are the effects of the subdominant configurations? In particular, will the effects of other brane configurations mentioned above undo the stabilization of seven directions presented here?

Note that unless these questions are answered and, furthermore, it is shown that other brane configurations mentioned above do not undo the stabilization presented here, our assumption that the evolution of the universe is dictated by the $22'55'$ configuration amounts to a fine-tuning: The $22'55'$ configuration assumed here, where the sets of 2 branes and 5 branes wrap the directions $(x^1, \cdots, x^7)$ homogeneously everywhere in the mutually transverse three-dimensional space, may not arise generically. Also, the implicitly required absence of other brane configurations is not natural in the context of the early universe. Then the problem of the emergence of an effective $(3 + 1)$—dimensional universe, a solution for which is presented here, gets shifted to answering how the required, finely tuned, initial conditions may arise naturally from M theory.

(v) What is the time scale of brane antibrane annihilations in the $22'55'$ configuration studied here? Is it long enough for the brane directions to be stabilized as described in this paper? Here, based on the black hole studies, we have assumed it to be long enough.

(vi) A related question, but applicable after stabilization of brane directions, is the following: If all the branes and antibranes will eventually decay, as seems natural, then what are the decay products? How can one obtain the known constituents of our present universe?

Although one of us has presented a principle in [8] that may be of help, the fact is that we do not know even where to begin in answering these questions quantitatively, much less know the answers. Nevertheless we present the above list of questions, unlikely to be complete, in order to emphasize the further work required to understand how our known $(3 + 1)$—dimensional universe may emerge from M theory.

In the present work, with many attendant assumptions, we considered the $22'55'$ configurations and explained a mechanism by which seven directions stabilize and an effective $(3 + 1)$—dimensional universe results. Clearly, it is important to answer the questions listed above and thereby determine the relevance of this mechanism.

Within the present framework, there are many other issues that may be studied further. We conclude by mentioning a sample of them. We have shown here that a large stabilized seven-dimensional volume can be obtained but it requires a corresponding fine-tuning of initial brane densities. This is within the context of our ansatzes for T_{AB} and the equations of state. It will be of interest to prove or disprove the necessity of such a fine-tuning in more general contexts.

The $N = 4$ intersecting brane configuration studied here is the entropically favorable one and, as proposed in [8], may be thought of as emerging from the high temperature phase of M theory in the early universe. Such an emergence suggests that there may be novel solutions to the horizon problem and to the primordial density fluctuations, perhaps similar to those explored recently in the Hagedorn phase of string theory by Nayeri *et al* [52]. Note that this involves answering many of the questions listed above.

It may be of interest to study further the consequences of time varying Newton's constant which appears here, in particular, possible imprints of its asymptotic log periodic oscillations.

In the case of a class of black holes, the brane configurations describe well their entropy and Hawking radiation. In the present description of a four-dimensional early universe in terms of $N = 4$ intersecting branes, it is not clear which quantities to calculate which, analogously to entropy or Hawking radiation in the black hole case, may provide further validation. It is important to study this further.

ACKNOWLEDGMENTS

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[13]Many of the questions listed below have been raised by the referee also.

APPENDIX A: U DUALITY RELATIONS IN BLACK HOLE CASE

Consider black holes in $m + 2$ dimensional spacetime described by mutually BPS intersecting brane configurations in M theory. The brane action S_{br} in Eq. (3) is the standard one for higher form gauge fields. The corresponding black hole solutions and their properties are well known, so here we only highlight the points related to U duality symmetries. Also, for illustration, we consider only 2 branes and 5 branes.

As mentioned in Sec. II, the method of U duality symmetries applies here also and leads to the same relations between λ^i. They are best seen in the extremal case. (The nonextremal case requires further analysis and is more involved.) The 11-dimensional line element ds for the extremal brane configurations are of the form

$$ds^2 = -e^{2\lambda^0} dt^2 + \sum_i e^{2\lambda^i} (dx^i)^2 \tag{A1}$$

where (λ^0, λ^i) depend on r, the radial coordinate of the $m + 1$ dimensional transverse space. For 2 branes and 5 branes, the λ^is may be written as

$$\lambda^1 = \lambda^2 = -\frac{2\tilde{h}}{6}, \quad \lambda^3 = \cdots = \lambda^{10} = \frac{\tilde{h}}{6} \tag{A2}$$

$$\lambda^1 = \cdots = \lambda^5 = -\frac{\tilde{h}}{6}, \quad \lambda^6 = \cdots = \lambda^{10} = \frac{2\tilde{h}}{6} \tag{A3}$$

where $e^{\tilde{h}} = H = 1 + \frac{Q}{r^{m-1}}$ is the corresponding harmonic function and Q is the charge. See, for example, [53] for more details. Clearly, the U duality relations in Eqs. (11)–(13) are valid here also.

For the extremal $22'55'$ configuration (12, 34, 13567, 24567), the transverse space is three-dimensional and the λ^is may be written as [53]

$$\begin{aligned}
\lambda^1 &= \frac{1}{6}(-2\tilde{h}_1 + \tilde{h}_2 - \tilde{h}_3 + 2\tilde{h}_4) \\
\lambda^2 &= \frac{1}{6}(-2\tilde{h}_1 + \tilde{h}_2 + 2\tilde{h}_3 - \tilde{h}_4) \\
\lambda^3 &= \frac{1}{6}(\tilde{h}_1 - 2\tilde{h}_2 - \tilde{h}_3 + 2\tilde{h}_4) \\
\lambda^4 &= \frac{1}{6}(\tilde{h}_1 - 2\tilde{h}_2 + 2\tilde{h}_3 - \tilde{h}_4) \\
\lambda^5 = \lambda^6 = \lambda^7 &= \frac{1}{6}(\tilde{h}_1 + \tilde{h}_2 - \tilde{h}_3 - \tilde{h}_4) \\
\lambda^8 = \lambda^9 = \lambda^{10} &= \frac{1}{6}(\tilde{h}_1 + \tilde{h}_2 + 2\tilde{h}_3 + 2\tilde{h}_4)
\end{aligned} \tag{A4}$$

where $e^{\tilde{h}_I} = H^I = 1 + \frac{Q^I}{r}$ are the corresponding harmonic functions and Q^Is are the charges. Clearly, the U duality relations in Eqs. (14) and (15) are valid here also. Furthermore, if 2 and $2'$ branes are identical then $\tilde{h}_1 = \tilde{h}_2$ and we get $\lambda^1 = \lambda^3$, and similarly other relations when different sets of branes are identical.

We further illustrate the U duality methods by interpreting a U duality relation $\sum_i c_i \lambda^i = 0$ as implying a relation among the components of the energy momentum tensor T_{AB}. The relations thus obtained are indeed obeyed by the components of T_{AB} calculated explicitly using the corresponding higher form gauge field action S_{br}.

For this purpose, let the spacetime coordinates be $x^A = (r, x^\alpha)$ where $x^\alpha = (x^0, x^i, \theta^a)$ with $x^0 = t$, $i = 1, \cdots, q$, $a = 1, \cdots, m$, and $q + m = 9$. The x^i directions may be taken to be toroidal, some or all of which are wrapped by branes, and θ^a are coordinates for an m-dimensional space of constant curvature given by $\epsilon = \pm 1$ or 0. The metric and brane fields depend only on r coordinate. We write the line element ds, in an obvious notation, as

$$ds^2 = -e^{2\lambda_0} dt^2 + \sum_i e^{2\lambda^i} (dx^i)^2 + e^{2\lambda} dr^2 + e^{2\sigma} d\Omega^2_{m,\epsilon}. \tag{A5}$$

The independent nonvanishing components of $T^A{}_B$ are given by $T^r{}_r = f$ and $T^\alpha{}_\alpha = p_\alpha$ where $\alpha = (0, i, a)$. These components can be calculated explicitly using the action S_{br}. For example, for an electric p-brane along $(x^1, \cdots, x^p)$ directions, they are given by

$$p_0 = p_\parallel = -p_\perp = -p_a = f = \frac{1}{4} F_{01\cdots pr} F^{01\cdots pr} \tag{A6}$$

where $p_\parallel = p_i$ for $i = 1, \cdots, p$, $p_\perp = p_i$ for $i = p + 1, \cdots, q$, and note that f is negative. For mutually BPS N intersecting brane configurations, it turns out [42–51] that the respective energy momentum tensors $T^A{}_B$ and $T^A{}_{B(I)}$ obey equations analogous to those given in (21)–(23).

Equations of motion may now be obtained from Eqs. (4) and (21). After some manipulations, they may be written in a form similar to those given in (7)–(9) as follows:

$$\Lambda_r^2 - \sum_\alpha (\lambda_r^\alpha)^2 = 2f + \epsilon m(m-1) e^{-2\sigma} \tag{A7}$$

$$\lambda_{rr}^\alpha + \Lambda_r \lambda_r^\alpha = -p_\alpha + \frac{1}{9}\Big(f - \sum_\beta p_\beta\Big) + \epsilon(m-1) e^{-2\sigma} \delta^{\alpha a} \tag{A8}$$

$$f_r + f\Lambda_r - \sum_\alpha p_\alpha \lambda_r^i = 0 \tag{A9}$$

where $\Lambda = \sum_\alpha \lambda^\alpha = \lambda^0 + \sum_i \lambda^i + m\sigma$ and the subscripts r denote r-derivatives. See [50] particulary, whose set up and the equations of motions are closest to the present ones.

Consider now the case of 2 branes or 5 branes. We assume that $p_a = p_\perp$ which is natural since θ^a directions are transverse to the branes. Applying the U duality relations in Eq. (13) then implies, for both 2 branes and 5 branes, the relation

$$p_{\parallel} = p_0 + p_{\perp} + f \tag{A10}$$

among the components of their energy momentum tensors. Note that it is also natural to take $p_0 = p_{\parallel}$ since $x^0 = t$ is one of the world volume coordinates and may naturally be taken to be on the same footing as the other ones $(x^1, \cdots, x^p)$. Equation (A10) then implies that $p_{\perp} = -f$. The relation between $p_{\parallel}$ and f is to be specified by an equation of state which, in the black hole case, is that given in Eq. (A6).

For now, however, we take p_0 and $p_{\parallel}$ to be different. Keeping in mind that f is negative, we assume the equations of state to be of the form $p_{\alpha I} = -(1 - u^I_\alpha) f_I$ where $\alpha = (0, i, a)$, $I = 1, \cdots, N$, and u^I_α are constants. Then for 2 branes and 5 branes, we have

$$\begin{aligned} &2\colon u_\alpha = (u_0, u_{\parallel}, u_{\parallel}, u_{\perp}, u_{\perp}, u_{\perp}, u_{\perp}, u_{\perp}, u_{\perp}, u_{\perp}) \\ &5\colon u_\alpha = (u_0, u_{\parallel}, u_{\parallel}, u_{\parallel}, u_{\parallel}, u_{\parallel}, u_{\perp}, u_{\perp}, u_{\perp}, u_{\perp}) \end{aligned} \tag{A11}$$

where the I superscripts have been omitted here and $u_{\parallel} = u_0 + u_{\perp}$ which follows from Eq. (A10). Note that $u_{\perp} = 0$ and $u_0 = u_{\parallel} = 2$ in the black hole case given in Eq. (A6).

Let

$$f_I = -e^{l^I - 2\Lambda}, \qquad l^I = \sum_\alpha u^I_\alpha \lambda^\alpha + l^I_0, \qquad d\tau = e^{-\Lambda} dr$$
$$G_{\alpha\beta} = 1 - \delta_{\alpha\beta}, \qquad \mathcal{G}^{IJ} = \sum_{\alpha,\beta} G^{\alpha\beta} u^I_\alpha u^J_\beta. \tag{A12}$$

Then, after a straightforward algebra, one obtains

$$l^I_{\tau\tau} = -\sum_J \mathcal{G}^{IJ} e^{l^J} + u_{\perp} \epsilon m(m-1) e^{2(\Lambda - \sigma)}, \tag{A13}$$

which are similar to Eqs. (39). The remaining equations are not needed and, hence, not given here. Using Eqs. (A11) and (A12), it is now straightforward to calculate $\mathcal{G}^{IJ}$ for N intersecting brane configurations. It turns out because of the BPS intersection rules that $\mathcal{G}^{IJ}$ may be written as

$$\mathcal{G}^{IJ} = 2u_0(u_{\perp} - u_0 \delta^{IJ}). \tag{A14}$$

The corresponding $\mathcal{G}_{IJ}$ is given by

$$\mathcal{G}_{IJ} = \frac{1}{2u_0^2}\left(\frac{u_{\perp}}{N u_{\perp} - u_0} - \delta_{IJ}\right). \tag{A15}$$

Now take $p_0 = p_{\parallel}$. Then Eq. (A10) gives $p_{\perp} + f = 0$. In terms of u_α, we now have $u_0 = u_{\parallel}$ and $u_{\perp} = 0$. Clearly, then $\mathcal{G}^{IJ} \propto \delta^{IJ}$ and Eqs. (A13) can be solved for $l^I(\tau)$. See [50] for such solutions, with $u_0 = 2$ as follows from Eq. (A6), and their analysis.

Tracing through the steps in the above derivation, it can also be seen that for the homogeneous early universe case, (r, f) here get replaced by $(t, -\rho)$, and (t, p_0, u_0) here gets replaced by $(r, p_{\perp}, u_{\perp} = u)$. Then, Eqs. (A10) and (A14) become Eqs. (20) with $z = -1$ and (72) respectively.

APPENDIX B: TO SHOW $E \geq 0$

Let $\vec{1} = (1, 1, \cdots, 1)$ and $\vec{v} = (v_1, v_2, \cdots, v_n)$ be the standard n–component vectors with the standard vector product. Let θ_n be the angle between them. Then $\vec{1} \cdot \vec{1} = n$, $\vec{v} \cdot \vec{v} = \sum_a v_a^2$, $(\vec{1} \cdot \vec{v})^2 = (\sum_a v_a)^2 = n\cos^2\theta_n \sum_a v_a^2$, and we have the Schwarz inequality in the form

$$n \sum_{a=1}^n v_a^2 - \left(\sum_{a=1}^n v_a\right)^2 = n\sigma_n^2 \geq 0 \tag{B1}$$

where $\sigma_n^2 = \sin^2\theta_n \sum_{a=1}^n v_a^2$. The equality is valid, i.e. $\sigma_n = 0$, if and only if $\sin\theta_n = 0$, equivalently $v_1 = \cdots = v_n$.

We now show the following:

Let G^{ij} and G_{ij} be given by Eq. (27). If u_i and L^i satisfy the relations $\sum_i u_i L^i = 0$ and $\sum_{ij} G^{ij} u_i u_j > 0$ then $2E = -\sum_{ij} G_{ij} L^i L^j \geq 0$. E vanishes if and only if L^i all vanish.

Proof: It is clear that E vanishes if L^i all vanish. Now, let $\vec{1} = (1, 1, \cdots, 1)$, $\vec{u} = (u_1, \cdots, u_{D-1})$, and θ be the angle between them. Then $(\sum_i u_i)^2 = (D-1)\cos^2\theta \sum_i u_i^2$. Hence, $\sum_{ij} G^{ij} u_i u_j = \frac{1}{D-2}(\sum_i u_i)^2 - \sum_i u_i^2 > 0$ implies that

$$1 - (D-1)\sin^2\theta > 0. \tag{B2}$$

The vector $\vec{L} = (L^1, \cdots, L^{D-1})$ is perpendicular to $\vec{u}$ since $\sum_i u_i L^i = 0$. Let $\vec{L} = \vec{L}_{\perp} + \vec{L}_{\parallel}$ where $\vec{L}_{\perp}$ is perpendicular to the plane defined by $\vec{1}$ and $\vec{u}$, and $\vec{L}_{\parallel}$ lies in it. Then $\sum_i (L^i)^2 = L_{\perp}^2 + L_{\parallel}^2$ where $L_{\perp}^2 = \vec{L}_{\perp} \cdot \vec{L}_{\perp}$ and $L_{\parallel}^2 = \vec{L}_{\parallel} \cdot \vec{L}_{\parallel}$. Since $\vec{L}$ and $\vec{u}$ are perpendicular and $\vec{L}_{\parallel}$ lies in the plane defined by $\vec{1}$ and $\vec{u}$, it follows that $\vec{L}_{\parallel}$ is perpendicular to $\vec{u}$, and that the angle between the vectors $\vec{1}$ and $\vec{L}_{\parallel}$ is $\frac{\pi}{2} \pm \theta$. We then have

$$\begin{aligned} 2E &= -\sum_{ij} G_{ij} L^i L^j = \sum_i (L^i)^2 - \left(\sum_i L^i\right)^2 \\ &= L_{\perp}^2 + L_{\parallel}^2 - (D-1) L_{\parallel}^2 \sin^2\theta \geq 0 \end{aligned}$$

where the inequality follows from Eq. (B2). The equality holds, and hence E vanishes, only when $L_{\perp}^2 = L_{\parallel}^2 = 0$, i.e. only when L^i all vanish. This completes the proof.

APPENDIX C: SIGNS AND NONVANISHING OF (Λ_τ, l^I_τ)

Here, we show that the inequality in Eq. (80) implies that none of (Λ_τ, l^I_τ) may vanish, and that they must all have the same sign.

Setting $x_I = l^I_\tau$, Eq. (80) becomes $X = 12u^2(E + \sum_I e^{l^I}) > 0$ where the polynomial $X = (x_1 + x_2 + x_3 + x_4)^2 - 3(x_1^2 + x_2^2 + x_3^3 + x_4^2)$. Now, if any of the x_I vanishes then $X \leq 0$, see the Schwarz inequality given in Eq. (B1). Hence, none of the x_I may vanish. Rewrite X as

$$\begin{aligned} X &= \{(x_1 + x_2 + x_3)^2 - 3(x_1^2 + x_2^2 + x_3^3)\} - 2x_4^2 \\ &\quad + 2x_4(x_1 + x_2 + x_3) \\ &= \{(x_1 + x_2)^2 - 2(x_1^2 + x_2^2)\} + \{(x_3 + x_4)^2 - 2(x_3^2 + x_4^2)\} \\ &\quad - (x_1^2 + x_2^2 + x_3^3 + x_4^2) + 2(x_1 + x_2)(x_3 + x_4) \end{aligned}$$

and note that $\{\cdots\} \leq 0$ for each curly bracket, see Eq. (B1). Hence, the necessary conditions for $X > 0$ are

$$x_4(x_1 + x_2 + x_3) > 0, \qquad (x_1 + x_2)(x_3 + x_4) > 0.$$

Let one of the x_I, e.g. x_4, be negative and the other three positive. This violates the first inequality above and, hence, is not possible. Let two of the x_I, e.g. x_3 and x_4, be negative and the other two positive. This violates the second inequality above and, hence, is not possible. Similarly, three of the x_I being negative and one positive is also not possible. Thus, the only possibility is that all x_I have same sign. Thus we have that none of the l^I_τ may vanish, and that they must all have the same sign.

With l^I_τ denoted as x_I, Eq. (79) for Λ_τ becomes

$$6u\Lambda_\tau = 2x_1 + 2x_2 + x_3 + x_4 + 6uL.$$

Note that $u > 0$. If $L = 0$ then it follows that Λ_τ does not vanish and has the same sign as x_I. Consider now the case where $L \neq 0$. Using Eq. (B1) to eliminate $\sum_I x_I^2$ in the polynomial X, we obtain

$$X = \frac{1}{4}(x_1 + x_2 + x_3 + x_4)^2 - 3\sigma_4^2 = 12u^2\Big(E + \sum_I e^{l^I}\Big).$$

Using the inequality $2E > 3(L)^2$, see Eq. (78), it follows that $(x_1 + x_2 + x_3 + x_4)^2 > 72u^2(L)^2$. Combined with the earlier result on l^I_τ, this inequality implies that $(x_1 + x_2 + x_3 + x_4 + 6uL)$, and hence Λ_τ given above, may not vanish and must have the same sign as $x_I = l^I_\tau$, irrespective of whether L is positive or negative. This completes the proof.

APPENDIX D: SET OF K^I WHICH MAXIMIZES τ_a

With no loss of generality, let $0 < -l_0^1 \leq \cdots \leq -l_0^4$. The corresponding set of K^I which satisfies Eq. (71), with $E = 1$, and which maximizes $\tau_a = \min\{\tau_I\}$, where $\tau_I = -\frac{l_0^I}{K^I}$, may be obtained by the following algorithm. The required analysis is straightforward but a little tedious and, hence, is omitted.

(i) Let $K^1 = -l_0^1 K$. It will turn out that $\tau_a = \tau_1 = \frac{1}{K}$.
(ii) Choose $K^2 = -l_0^2 K$. Then $\tau_2 = \tau_1$.
(iii) If $-l_0^1 - l_0^2 \leq -l_0^3$ then choose $K^3 = K^4 = -(l_0^1 + l_0^2)K$. Then $\tau_4 \geq \tau_3 \geq \tau_2 = \tau_1$.
(iv) If $-l_0^1 - l_0^2 > -l_0^3$ then choose $K^3 = -l_0^3 K$. Then $\tau_3 = \tau_2 = \tau_1$.
(v) If $-l_0^1 - l_0^2 > -l_0^3$ and if $-l_0^1 - l_0^2 - l_0^3 \leq -2l_0^4$ then choose $K^4 = -\frac{1}{2}(l_0^1 + l_0^2 + l_0^3)K$. Then $\tau_4 \geq \tau_3 = \tau_2 = \tau_1$.
(vi) If $-l_0^1 - l_0^2 > -l_0^3$ and if $-l_0^1 - l_0^2 - l_0^3 > -2l_0^4$ then choose $K^4 = -l_0^4 K$. Then $\tau_4 = \cdots = \tau_1$.
(vii) K^I are all thus determined in terms of K. Equation (71), with $E = 1$, will now determine K.

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