

Big bang nucleosynthesis with a stiff fluid

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Models that lead to a cosmological stiff fluid component, with a density ρ_S that scales as a^{-6} , where a is the scale factor, have been proposed recently in a variety of contexts. We calculate numerically the effect of such a stiff fluid on the primordial element abundances. Because the stiff fluid energy density decreases with the scale factor more rapidly than radiation, it produces a relatively larger change in the primordial helium-4 abundance than in the other element abundances, relative to the changes produced by an additional radiation component. We show that the helium-4 abundance varies linearly with the density of the stiff fluid at a fixed fiducial temperature. Taking ρ_{S10} and ρ_{R10} to be the stiff fluid energy density and the standard density in relativistic particles, respectively, at $T = 10$ MeV, we find that the change in the primordial helium abundance is well-fit by $\Delta Y_p = 0.00024(\rho_{S10}/\rho_{R10})$. The changes in the helium-4 abundance produced by additional radiation or by a stiff fluid are identical when these two components have equal density at a “pivot temperature,” T_* , where we find $T_* = 0.55$ MeV. Current estimates of the primordial ${}^4\text{He}$ abundance give the constraint on a stiff fluid energy density of $\rho_{S10}/\rho_{R10} < 30$.

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I. INTRODUCTION

In recent years, a host of cosmological observations have provided an increasingly precise picture of the constituents of the Universe. The baryon density has long been known to provide roughly 5% of the critical density; earlier estimates from big bang nucleosynthesis [1] have been confirmed by CMB observations from WMAP [2]. Cosmological data from a wide range of sources including type Ia supernovae [3], the cosmic microwave background [2], baryon acoustic oscillations [4], cluster gas fractions [5,6], and gamma ray bursts [7,8] seem to indicate that about 72% of the energy density of the Universe is in the form of an exotic, negative-pressure component called dark energy. In addition, a slew of evidence from modern sources, including weak [9] and strong [10] lensing, the bullet cluster [11], large scale structure [12], as well as supernovae and the CMB, have confirmed earlier indications from rotation curves [13–15] that another 23% of the energy density of the Universe is in the form of weakly interacting matter called dark matter. (See Ref. [16] for a review of dark energy and [17] for a review of dark matter.)

While the existence of all of these components is reasonably well-established, the existence of other exotic fluids is not ruled out by the current data. For example, several models predict the existence of “dark radiation” either early or late in the history of the Universe (see e.g. [18] for an example of such a model, and references therein for others). Another exotic fluid which arises in various models is a “stiff fluid,” i.e., a fluid with an equation of state parameter $w_S \equiv p_S/\rho_S = 1$. This is the largest value of w consistent with causality, since the speed of sound of this fluid equals the speed of light. Such models were apparently first studied by Zeldovich [19]. The Friedman equation for such a fluid implies that its energy density ρ_S varies with the scale factor a as

$$\rho_S \propto a^{-6}. \quad (1)$$

In recent years, a variety of models have been proposed that produce a stiff cosmological fluid:

a. Kination: A kination field is a scalar field whose energy density is dominated by kinetic energy. A period of kination can follow a period of inflation and was first studied in the context of electroweak baryogenesis [20,21]. Its impact on reheating as well as on the freeze-out of dark matter particles has been studied in Refs. [22–26]. Kination fields have also been applied to unify inflation and dark energy by using the same scalar field for both inflation and quintessence. In these models inflation ends with a period of kination as the inflaton receives a “kick,” and the same field later on plays the role of quintessence [27].

b. Interacting dark matter: In models with a warm self-interacting dark matter component, the elastic self-interactions between the (scalar boson or fermionic) dark matter particles can be characterized by the exchange of vector mesons via minimal coupling. For these models, the self-interaction energy can be shown to behave like a stiff fluid [28].

c. Hořava-Lifshitz cosmologies: Stiff fluids also occur in certain cosmological models based on the recently proposed Hořava-Lifshitz gravity, a power-counting renormalizable and ultraviolet-complete field theoretic quantum gravity model based on anisotropic scaling of the space and time dimensions [29–32]. In the original formulation of this theory a “detailed balance” condition was imposed as a convenient simplification [31]. The validity and usefulness of the detailed balance condition have subsequently been discussed extensively (see e.g. [33,34]) as well as the consequences of relaxing it [35–38]. Stiff fluids arise in models in which the detailed balance condition has been relaxed. Cosmological models based on Hořava-Lifshitz gravity have been studied extensively (see e.g. [39–44]),

and observational constraints on such models, including the stiff fluid cases, were considered in [45–47]. However, it must also be noted that the theoretical foundations of Hořava-Lifshitz gravity are still under debate (see e.g. [48–56]).

d. Nonsingular cosmological models: Stiff fluids have been also found to show up as exact nonsingular solutions in inhomogeneous cosmological models [57–60].

Given the recent flurry of interest in such models, it is clearly useful to derive precise limits on the density of a stiff fluid in the early Universe. Because the density of a stiff fluid decays more rapidly than either radiation or matter, the effect of the stiff fluid on the expansion rate will be the largest at early times. Thus, the strongest limits on the density of such a fluid come from big bang nucleosynthesis (BBN), which remains the earliest cosmological process whose evolution can be determined with high precision. (Note that some attempts have been made to constrain the expansion rate of the Universe prior to BBN from the relic dark matter abundance [22,24,25,61–64]. While the freeze-out of the dark matter does occur at a much earlier time than BBN, the exact model for the dark matter is less precisely determined).

Previous discussions of stiff fluids have usually quoted BBN limits on the expansion rate at a fixed temperature (typically $T \sim 1$ MeV) and used these limits to constrain the density of the stiff fluid. Here, we numerically evolve the element abundances in the presence of the stiff fluid to derive the exact dependence of the element abundances on the stiff fluid density. The stiff fluid investigated here resembles a special case of the models examined by Masso and Rota [65] who investigated the effect of an arbitrary additional density of the form $\lambda(T/0.1 \text{ MeV})^\gamma$, although their $\gamma = 6$ case is not identical to the model considered here, since T does not scale as $1/a$ through the era of e^+e^- annihilation. Here, we examine the stiff fluid case in more detail, and we exploit the fact that WMAP now provides an independent determination of the baryon-photon ratio [2], effectively eliminating a degree of freedom from BBN and allowing for better constraints on the stiff fluid. (See also related work on a somewhat different variant model in Ref. [66].) We present our calculations in the next section, and discuss our limits in Sec. III.

II. EFFECT OF A STIFF FLUID ON BBN

BBN has long been used to constrain additional energy density in the early universe (for recent reviews, see Refs. [67–69]). The expansion rate H is given by

$$H^2 = \frac{8\pi G}{3}\rho, \quad (2)$$

where ρ is the total density, so any additional contribution to ρ increases the expansion rate and changes the resulting element abundances.

At high temperatures ($T \gtrsim 1$ MeV), the rates for the weak interactions which govern the interconversion of neutrons and protons,

$$\begin{aligned} n + \nu_e &\leftrightarrow p + e^-, & n + e^+ &\leftrightarrow p + \bar{\nu}_e, \\ n &\leftrightarrow p + e^- + \bar{\nu}_e, \end{aligned} \quad (3)$$

are larger than the expansion rate, H , and the neutron-to-proton ratio (n/p) tracks its equilibrium value. As the Universe expands and cools, the weak interaction rates drop below the expansion rate, and n/p freezes out at $T \sim 1$ MeV. Between $T \sim 1$ and 0.1 MeV, the neutrons undergo free decay, and then at $T \sim 0.1$ MeV, nucleosynthesis proceeds to fuse the remaining neutrons with the protons to produce heavier elements, primarily ^4He , but also trace amounts of ^2H , ^3He , and ^7Li . The final ^4He abundance is most sensitive to the expansion rate near $T \sim 1$ MeV, when the neutron/proton ratio freezes out, while the other element abundances are more sensitive to the expansion rate near $T \sim 0.1$ MeV, when fusion into heavier elements begins.

The dependence of the different element abundances on the expansion rate at various temperatures was explored quantitatively by Bambi *et al.* [70], who derived “response functions” that show the change in the abundance of each nuclide as a function of a change in the expansion rate at a given temperature. [Note that we use these response functions only to gain insight into the effects of the stiff fluid on the element abundances; our actual calculation of the element abundances utilizes a full numerical integration of the BBN equations, as discussed below]. Consider an additional source of energy density which changes the expansion rate, $H(T)$, by an amount $\Delta H(T)$. Then Bambi *et al.* argued that, for a given value of the baryon-photon ratio η , the change in a given nuclide abundance, ΔX_i , is given by

$$\Delta X_i = 2 \int \varrho_i(T) \frac{\Delta H(T)}{H(T)} \frac{dT}{T}, \quad (4)$$

where $\varrho_i(T)$ is the response function for a given nuclide, derived numerically for several elements of interest in Ref. [70]. As expected, the response functions for deuterium and ^7Li are strongly peaked near $T \sim 0.1$ MeV. In contrast, the response function for ^4He is broadly distributed between 1 and 0.1 MeV, with two peaks of roughly equal magnitude at these two temperatures [70]. The first corresponds to $n - p$ freeze-out, and the second to the onset of fusion. The latter affects the ^4He abundance primarily through the influence of free-neutron decay; the earlier fusion begins, the more undecayed neutrons remain to form ^4He .

Now consider a model, such as the one examined here, with some additional source of energy density, $\Delta\rho(T)$. As long as $\Delta\rho(T) \ll \rho(T)$, where $\rho(T)$ is the standard energy density, we can write Eq. (4) as

$$\Delta X_i = \int e_i(T) \frac{\Delta \rho(T)}{\rho(T)} \frac{dT}{T}. \quad (5)$$

Because of the sensitivity of the element abundances to the density at different epochs, the relative changes in the different element abundances will be different for different functional forms of $\rho(T)$. However, consider an arbitrary functional dependence of the form $\rho(T) = \rho(T_0)f(T/T_0)$, where $\rho(T_0)$ is the density at some fixed fiducial temperature T_0 , while $f(T/T_0)$ is an arbitrary function subject only to the constraint $f(1) = 1$. Thus, $f(T/T_0)$ parametrizes the dependence of ρ on T , while $\rho(T_0)$ fixes the overall amplitude of the density. The important point is that as long as Eq. (5) is a good approximation, and once $f(T/T_0)$ is fixed, the abundance of each nuclide will vary linearly with $\rho(T_0)$, regardless of the functional form of $f(T/T_0)$.

This linear dependence is seen in the case of additional relativistic energy density scaling as $\Delta \rho_R \propto a^{-4}$. Parametrizing this energy density in terms of the number of additional two-component neutrinos, ΔN_ν , the change in the primordial ${}^4\text{He}$ mass fraction, ΔY_p , is well approximated by [71]

$$\Delta Y_p = 0.013 \Delta N_\nu. \quad (6)$$

Given the different sensitivities of the element abundances to the expansion rate at different temperatures, it is clear that the change in the element abundances produced by a stiff fluid will differ from that produced by additional relativistic energy density. However, if we confine our attention to a single element (such as ${}^4\text{He}$), then we expect the overall abundance to scale linearly with the value of the stiff energy density at a fixed fiducial temperature.

We model our stiff fluid as a component with energy density ρ_S , given by

$$\rho_S = \rho_{S10} (a/a_{10})^{-6}, \quad (7)$$

where ρ_{S10} and a_{10} are the stiff fluid density and scale factor, respectively, at $T = 10$ MeV (well before e^+e^- annihilation). We use the Kawano [72] version of the Wagoner [73,74] nucleosynthesis code to derive the element abundances as a function of ρ_{S10} and of the baryon-photon ratio, η .

In Fig. 1 we compare the change in the element abundances produced by a stiff fluid with that from one extra neutrino species, for η in the range $5-7 \times 10^{-10}$. The blue (solid) curve gives the standard BBN model with no additional energy density. The black (dashed) curve denotes the element abundances due to one additional two-component neutrino, while the red (dotted) curve gives the abundances due to a stiff fluid for which ρ_{S10} is chosen to produce the same effect on Y_p as one extra neutrino (as can be seen in the top panel).

As expected, when the stiff fluid density is adjusted to give the same effect on the ${}^4\text{He}$ abundance as an extra neutrino, the stiff fluid produces a much smaller effect on

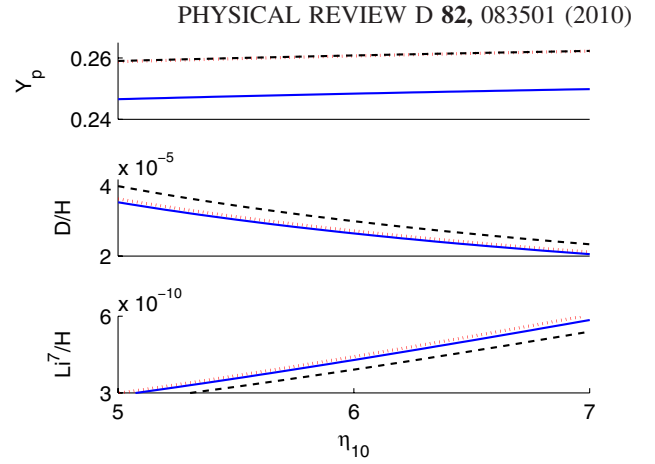


FIG. 1 (color online). Changes in primordial element abundances caused by a stiff fluid and by an extra neutrino, as a function of $\eta_{10} \equiv \eta \times 10^{10}$, where η is the baryon-photon ratio. The black (dashed) curve denotes the element abundances due to one extra two-component neutrino, the red (dotted) curve denotes the abundances for a stiff fluid, and the blue (solid) curve denotes the unmodified standard case of 3 neutrinos and no stiff fluid. The density of the stiff fluid has been adjusted to produce the same effect on Y_p as one extra neutrino, as can be seen from the top panel.

the deuterium and ${}^7\text{Li}$ abundances. This is because, as we have noted, the latter two element abundances are sensitive to the expansion rate at a much lower temperature than is the ${}^4\text{He}$ abundance, and the stiff fluid density decays with expansion rate much more rapidly than does the contribution of an extra neutrino.

The one minor surprise is that the stiff fluid increases the ${}^7\text{Li}$ abundance, while an additional neutrino decreases it. This can be understood in terms of the response function in Ref. [70]. The ${}^7\text{Li}$ response function has a sharp trough just below $T = 0.1$ MeV, but it also has a shallow positive plateau for $T > 0.1$ MeV. The physical reason for this behavior comes from the fact that the dominant reaction for ${}^7\text{Li}$ production is ${}^4\text{He} + {}^3\text{He} \rightarrow {}^7\text{Be} + \gamma$ [67]. An increase in the expansion rate at high temperatures ($T > 0.1$ MeV) gives a larger neutron abundance prior to nuclear fusion, enhancing the abundance of both ${}^4\text{He}$ and ${}^3\text{He}$ to make more ${}^7\text{Be}$. An increase in the expansion rate just below $T = 0.1$ MeV, when nuclear fusion is occurring, gives less time for the fusion reactions to build into heavier elements; thus, the decrease in the ${}^7\text{Li}$ abundance in this case is accompanied by increases in the abundances of deuterium and helium-3. The stiff fluid samples the positive plateau at high T more strongly than the trough at low T , while the reverse is true for an extra neutrino.

Once η is fixed by the CMB, the best constraint on the stiff fluid can be obtained from the ${}^4\text{He}$ abundance. WMAP7 [2] gives $\eta = 6.2 \times 10^{-10}$. For this value of η , we plot, in Fig. 2, the change in the primordial ${}^4\text{He}$ abundance, ΔY_p , as a function of ρ_{S10}/ρ_{R10} , where ρ_{S10} is the density of the stiff fluid at $T = 10$ MeV [as in Eq. (7)],

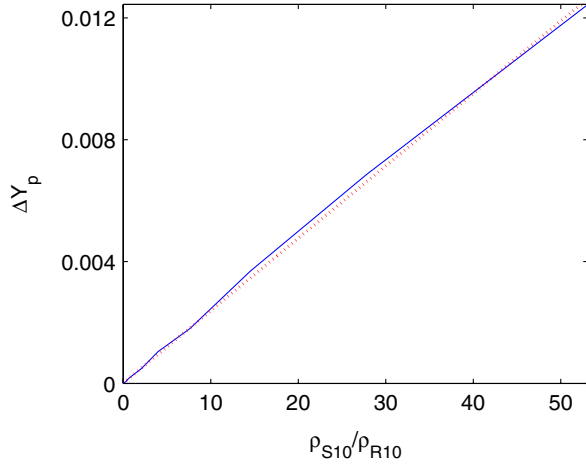


FIG. 2 (color online). The change in the primordial ${}^4\text{He}$ abundance, ΔY_p , produced by a stiff fluid with density $\rho_S = \rho_{S10}(a/a_{10})^{-6}$, where ρ_{S10} is the stiff fluid density at a temperature of 10 MeV, and a_{10} is the scale factor at this temperature. The horizontal axis shows ρ_{S10}/ρ_{R10} , where ρ_{R10} is the total energy density in relativistic particles (i.e., in the standard cosmological model without a stiff fluid) at 10 MeV. The blue (solid) curve is the numerical result, and the red (dotted) curve is the fit from Eq. (8). These results are for a baryon-photon ratio of $\eta = 6.2 \times 10^{-10}$.

while ρ_{R10} is the standard energy density in relativistic particles at $T = 10$ MeV. As expected, we find that ΔY_p is well-fit by a linear dependence on ρ_{S10}/ρ_{R10} , namely,

$$\Delta Y_p = 0.00024(\rho_{S10}/\rho_{R10}). \quad (8)$$

We find further that Eq. (8) is an excellent approximation when η lies in the range between 5×10^{-10} and 7×10^{-10} . Of course, higher-order corrections produce a slight deviation from exactly linear behavior, as is evident in Fig. 2 (see, e.g., Appendix F of Ref. [67] for a discussion of such corrections for ΔN_ν).

III. DISCUSSION

The strongest constraints on a stiff fluid clearly come from the primordial ${}^4\text{He}$ abundance. This abundance remains at present somewhat uncertain (for recent discussions, see, e.g., Refs. [75–77]). Recent analyses by Izotov and Thuan [75] and by Aver, Olive, and Skillman [76] are both consistent with a central value of $Y_p = 0.256$. Using

this limit with the WMAP7 value of $\eta = 6.2 \times 10^{-10}$, we obtain the bound

$$\rho_{S10}/\rho_{R10} < 30. \quad (9)$$

This result, however, should not be considered the main result of our paper, as the estimates of Y_p are likely to improve with time. Rather, our main conclusion is embodied in Eq. (8), a result that can be used to provide an upper bound on the stiff fluid density for any estimate of the primordial ${}^4\text{He}$ abundance.

We can exploit the fact that ΔY_p depends linearly on both ΔN_ν and ρ_{S10}/ρ_{R10} to derive a pivot temperature T_* at which equal contributions from relativistic energy density or from a stiff fluid will produce equal changes in ${}^4\text{He}$. In other words, suppose that we have a particular value of ΔY_p . This will correspond to a particular value of ΔN_ν in Eq. (6), or to a particular value of ρ_{S10}/ρ_{R10} in Eq. (8). In this case, the additional energy densities in radiation or in the stiff fluid will be equal at a single temperature T_* :

$$\Delta \rho_R(T_*) = \rho_S(T_*). \quad (10)$$

By comparing Eqs. (6) and (8), we find that

$$T_* = 0.55 \text{ MeV}. \quad (11)$$

We emphasize that this is a purely heuristic result. It is not based on the assumption that the helium abundance depends only on the expansion rate at a single temperature; we have seen that it does not. Indeed, it is not surprising that T_* lies in between the two peaks in the ${}^4\text{He}$ response function. Note further that T_* corresponds, strictly speaking, to the neutrino temperature, rather than the photon temperature. The neutrino temperature scales exactly as $1/a$, while the photon temperature, even near 0.55 MeV, has experienced a small increase due to e^+e^- annihilation. Since most bounds from BBN on additional energy density are expressed in terms of limits on additional radiation density (or equivalently, additional neutrinos) from the primordial helium abundance, our result for the pivot temperature can be used to convert these bounds into limits on an additional stiff fluid component.

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