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Acceleration of particles as a universal property of rotating black holes

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We argue that the possibility of having infinite energy in the center-of-mass frame of colliding particles is a generic property of rotating black holes. We suggest a general model–independent derivation valid for dirty black holes. The earlier observations for the Kerr or Kerr-Newman metrics are confirmed and generalized.

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I. INTRODUCTION

Quite recently, a series of works [1–6] appeared in which interesting observations were made about energetics of particles near rotating black holes. Namely, it was argued that under certain conditions, the energy in the center-of-mass frame can grow unbound, so a black hole acts as a supercollider. This opens a window into a new physics, including the possibility of unknown channels of reaction between elementary particles, with potential astrophysical applications such as elucidation of the nature of active nuclei in the Galaxy [7], etc. At the present, these results were obtained for the Kerr metric and extended to the Kerr-Newman one.

The aim of the present work is to show that this remarkable property of being an accelerator to infinitely high energies is the direct consequence of the general properties of the event horizon, provided one of the colliding particles approaches certain critical value of the angular momentum. We rely not on the particular properties of the Kerr or Kerr-Newman metric, but on the generic axially symmetric rotating black holes. This is especially important in the given context, since physical significance of the effect under discussion implies the presence of matter (say, an accretion disc) around the horizon, so the black hole, as is usual in astrophysics, is "dirty". Thus, our motivation is twofold: to elucidate the essence of the effect from general principles, and to give derivation valid for black holes surrounded by matter. The general approach we push forward enables us to give a natural explanation to some important features of black holes as particles accelerators, observed earlier in particular examples.

It was observed in [2] that the infinite acceleration can occur not only for extremal black holes (as was stated in [1]) but also for nonextremal ones, and the distinction between the two cases was traced in detail for the Kerr metric. This is important, since in [1,5,6] the effect under discussion was related to just extremal black holes; meanwhile, there are astrophysical limitations on the proximity of the angular momentum of a black hole to the extremal value [8]. In this sense, the aforementioned result of [2] enables us, in principle, to evade this restriction and consider the effect not only for extremal black holes. Therefore, it is desirable to trace whether this is retained for astrophysically relevant dirty black holes. Now, different kinds of limiting transitions and the role of the type of the horizon (nonextremal versus extremal) follow directly from this general approach.

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II. BASIC FORMULAS AND LIMITING TRANSITIONS

Consider the generic axially symmetrical metric. It can be written as

$$ds^2 = -N^2dt^2 + g_{\phi\phi}(d\phi - \omega dt)^2 + dl^2 + g_{zz}dz^2. \quad (1)$$

Here, the metric coefficients do not depend on t and ϕ. On the horizon, $N = 0$. Alternatively, one can use coordinates θ and r, similar to the Boyer-Lindquist ones for the Kerr metric, instead of l and z. In (1), we assume that the metric coefficients are even functions of z, so the equatorial plane $\theta = \frac{\pi}{2}$ ($z = 0$) is a symmetric one.

In the spacetime under discussion, there are two conserved quantities, $u_0 \equiv -E$ and $u_\phi \equiv L$, where $u^\mu = \frac{dx^\mu}{d\tau}$ is the four-velocity of a test particle, τ is the proper time, and $x^\mu = (t, \phi, l, z)$ are coordinates. The aforementioned conserved quantities have the physical meaning of the energy per unit mass (or frequency, for a lightlike particle) and azimuthal component of the angular momentum, respectively. It follows from the symmetry reasonings that there exist geodesics in such a background which lie entirely in the plane $\theta = \frac{\pi}{2}$. Then, the first integrals for such geodesics read (here, the dot denotes the derivative with respect to the proper time τ):

$$\dot{t} = u^0 = \frac{E - \omega L}{N^2}. \quad (2)$$

We assume that $\dot{t} > 0$, so that $E - \omega L > 0$.

$$\dot{\phi} = \frac{L}{g_{\phi\phi}} + \frac{(-\omega^2 L + E\omega)}{N^2}, \quad (3)$$

$$\dot{l}^2 = \frac{(E - \omega L)^2}{N^2} - \delta - \frac{L^2}{g_{\phi\phi}}. \quad (4)$$

Here, $\delta = 0$ for lightlike geodesics and $\delta = 1$ for timelike ones. For definiteness, we consider a pair of particles labeled by the subscript $i = 1, 2$ and having the equal rest masses $m_1 = m_2 = m$. We also assume that both particles are approaching the horizon, so $\dot{l} < 0$ for each of them.

The quantity that is relevant for us is the energy in the center-of-mass frame $E_{\text{c.m.}} = \sqrt{2}m\sqrt{1 - u_{\mu(1)}u^{\mu(2)}}$ [1–6]. After simple manipulations, one obtains from (2)–(4) that

$$\frac{E^2_{\text{c.m.}}}{2m^2} = c + 1 - Y, \qquad c = \frac{X}{N^2}, \quad (5)$$

where

$$X = X_1X_2 - Z_1Z_2, \quad (6)$$

$X_i \equiv E_i - \omega L_i$,

$$Z_i = \sqrt{(E_i - \omega L_i)^2 - N^2 b_i}, \qquad b_i = 1 + \frac{L_i^2}{g_{\phi\phi}}, \quad (7)$$

$$Y = \frac{L_1L_2}{g_{\phi\phi}}. \quad (8)$$

Here, the crucial role is played by the quantity c that determines whether the energy can grow unbound. Now, we will discuss different limiting transitions.

(1) For generic L_i, one approaches the horizon, so $N \to 0$. Expanding the radicals and retaining the first nonvanishing corrections in the numerator, one obtains (subscript H refers to the horizon value):

$$\left(\frac{E^2_{\text{c.m.}}}{2m^2}\right)_H = 1 + \frac{b_{1(H)}(L_{2(H)} - L_2)}{2(L_{1H} - L_1)} + \frac{b_{2(H)}(L_{(1)H} - L_1)}{2(L_{2(H)} - L_2)} - \frac{L_1L_2}{(g_{\phi\phi})_H}, \quad (9)$$

$$L_{i(H)} \equiv \frac{E_i}{\omega_H}.$$

By the very meaning of derivation, it is supposed in (9) that $L_1 \neq L_{2(H)}$, $L_2 \neq L_{2(H)}$.

Let us now specify the range of angular momenta in such a way that one of them is close to the critical value: $L_1 = L_{1(H)}(1 - \varepsilon)$, $\varepsilon \ll 1$, $L_2 \neq L_{2(H)}$. Then, we have that

$$\left(\frac{E^2_{\text{c.m.}}}{2m^2}\right)_H \approx \frac{b_{1(H)}(L_{2(H)} - L_2)}{2L_{1(H)}\varepsilon}. \quad (10)$$

This quantity can be made as large as one likes, due to $\varepsilon \to 0$. It follows from (5) and (9) that

$$\lim_{L_1 \to L_{1(H)}} \lim_{N \to 0} E_{\text{c.m.}} = \infty. \quad (11)$$

(2) Let us take $L_1 \to L_{1(H)}$ first and then consider the limit $N \to 0$. The previous formula (9) is valid both for nonextremal and extremal horizons. In contrast to it, the distinction between two types of the horizon now comes into play.

(i) First, consider the nonextremal case. We are interested in the immediate vicinity of the horizon where the effect under discussion is expected to show up. Near the horizon, we can infer the restriction that follows from the condition of positivity of the expression inside the square root in (7). To this end, let us use the general form of the asymptotic expansion for the metric coefficient ω that follows from the general requirement of regularity of the geometry near the nonextremal horizon [9]:

$$\omega = \omega_H + BN^2 + \ldots \quad (12)$$

Here, ω_H is constant and has the physical meaning of the angular velocity of the horizon itself, the coefficient $B = B(\theta)$. For the case $\theta = \frac{\pi}{2}$ under consideration, B is simply constant. Its exact value is model-dependent.

Thus, the condition of the positivity of (4) cannot be satisfied, since the first term has the order N^2, whereas the others have the order N^0 and are negative. It means that the horizon is unreachable (the admissible region adjacent to the horizon shrinks to the point, and there is some turning point situated on a finite distance from the horizon). Therefore, the present case should be rejected.

(ii) Now, consider the extremal horizon. Then, instead of (12), one has more general expansion:

$$\omega = \omega_H - B_1N + B_2N^2 + \ldots \quad (13)$$

The distinction between expansions for both horizons can be understood using the Kerr metric as an example. The first corrections have the order $r - r_H$, where r is the Boyer-Lindquist coordinate. However, for the nonextremal case $N^2 \sim r - r_H$, whereas for the extremal Kerr metric $N^2 \sim (r - r_H)^2$, $B_1 = M^{-1}$, where M is the mass.

In a more general case, one can just appeal to the definition of the nonextremal and extremal black holes using the proper length—namely, in the nonextremal case $N \approx \kappa l$ near the horizon, where κ is the surface gravity, and in the extremal one $N \approx N_0 \exp(-Al)$, with N_0, $A = \text{const} > 0$ and $l \to \infty$. In principle, so-called ultraextremal horizons with $N \sim l^{-s}$ with $s > 0$ are also possible and can contain fractional powers of $r - r_H$ (where r is the analogue of the Boyer-Lindquist coordinate), but we do not discuss them here. (For the spherically symmetric configurations, such horizons are classified in [4].)

After the substitution of (13) to (6) and (7), we obtain after simple manipulations that

$$\frac{E^2_{\text{c.m.}}}{2m^2} \approx \frac{(E_2 - \omega_H L_2)}{N}\left[B_1 \frac{E_1}{\omega_H} - \sqrt{\left(\frac{E_1^2}{\omega_H^2} B_1^2 - b_1\right)}\right]. \quad (14)$$

Here, it is implied that the condition of the positivity is fulfilled for the expression inside the radical (this cannot be worked out in more detail in a model-independent way). Thus,

$$\lim_{N\to 0} \lim_{L_1 \to L_{1(H)}} E_{\text{c.m.}} = \infty. \quad (15)$$

The extremal case has one more interesting feature. Namely, the proper time needed to reach the horizon tends to infinity. Indeed, it follows from (4) and (13) that, for the particle having $L = L_{(H)}$ and approaching the horizon,

$$\tau \sim \int \frac{dlN}{Z} \sim l \to \infty, \quad (16)$$

since the proper distance from any point to the extremal horizon is infinite. For the nonextremal horizon, the proper distance is finite, as well as the proper time. Also, one can easily find from (3) and (4) that the number of revolutions

$$\Delta\phi \approx \frac{EB_1}{\sqrt{(\frac{E^2}{\omega_H^2} B_1^2 - b_1)}} \int \frac{dl}{N}. \quad (17)$$

Using again the asymptotic form $N \approx N_0 \exp(-Al)$ in the extremal case, we see that $\Delta\phi \to \infty$.

(3) In two previous situations, the result was formally determined by a play between two small quantities ε and N and the order of taking the limits $\varepsilon \to 0$ and $N \to 0$. Meanwhile, it is of interest to trace what happens when both quantities are small but nonzero, and what are limitations on the possibility of collision with infinitely growing energies. For the Kerr metric, a particle with the critical value of the angular momentum $L_{(H)}$ cannot come from infinity because a potential barrier prevents this, so that the energy cannot grow unbound as a result of single scattering [1,6]. Meanwhile, as was demonstrated for the Kerr metric [2], this becomes possible if multiple scattering occurs, so one of the colliding particles does not come from infinity but receives the near-critical angular momentum as a result of collision near the horizon. As far as the generic spacetime is concerned, in principle it can happen that a particle with the critical angular momentum coming from infinity is able to reach the horizon. However, such a possibility is model-dependent and requires special conditions for the behavior of the metric. Meanwhile, we are interested in features that have general model-independent character. Therefore, we will not discuss such particular cases, and will assume that a particle has a near-critical angular momentum in the near-horizon region due simply to multiple collisions. Let us see the necessary condition for this.

From the condition $Z^2 \geq 0$, where Z is defined in (7), we obtain for the nonextremal horizon using (12) that the process under discussion can indeed occur, but only in the narrow strip near the horizon where

$$0 \leq N \leq \frac{E\varepsilon}{\sqrt{b_H}}. \quad (18)$$

Thus, the energy (10) can indeed be as large as one likes, but this happens provided a particle acquires the near-critical angular momentum in the region bounded by Eq. (18). If $\varepsilon = 0$ exactly, the permitted strip (18) shrinks to the point, and we return to Case 2(i) when the effect is impossible. For the extremal horizon, there is no limitation similar to (18) since one can put $\varepsilon = 0$ in accordance with case 2(ii), see also Eq. (13).

(4) For completeness, let us consider the case when $L_1 = L_{1(H)}$, $L_2 = L_{2(H)}$ simultaneously. In the nonextremal case, it follows from (12) that the radical cannot remain positive near the horizon and the horizon is unreachable, so this case is irrelevant for our analysis. For the extremal case, in the horizon limit, we obtain from (13) that the terms of the order N in the numerator cancel, so that the first nonvanishing term has the same order N^2 as the denominator. As a result, the quantity $E_{\text{c.m.}}$ is finite. However, the proper time needed to reach the horizon is still infinite.

If we compare the meaning of limits 1 and 2, we see that in the nonextremal case, the energy in the center-of-mass frame is finite but can be made as large as one likes if the angular momentum of one of two colliding particles is arbitrarily chosen close to the critical value. If this value is chosen exactly equal to the critical value from the very beginning, the energy can be made as large as one wishes when one approaches the horizon (it becomes possible in the extremal case only). This is direct generalization of observations made in [1,2] for the Kerr metric.

III. COMPARISON OF GENERAL RESULTS WITH CASE OF KERR METRIC

It is instructive to compare some general results obtained in our paper to those obtained earlier for the Kerr metric. Then, for the equatorial plane $\theta = \frac{\pi}{2}$,

$$g_{00} = -\left(1 - \frac{2M}{r}\right), \qquad g_{0\phi} = -\frac{2Ma}{r}, \qquad g_{\phi\phi} = r^2 + a^2 + \frac{2Ma^2}{r}, \qquad \omega = -\frac{g_{0\phi}}{g_{\phi\phi}}. \tag{19}$$

$$N^2 = \frac{(r - r_H)(r - r_C)}{r^2 + a^2 + \frac{2M}{r}a^2}, \tag{20}$$

where r is the Boyer-Lindquist coordinate, $r_H = M + \sqrt{M^2 - a^2}$, $r_C = M - \sqrt{M^2 - a^2}$, the horizon value of the coefficient omega is equal to $\omega_H = \frac{a}{2Mr_H}$. If we define $L = lEM$ and take $E = 1$ (that corresponds to a particle falling from infinity from the state of the rest), the critical value of the angular momentum is $l_{(H)} = \frac{2r_H}{a}$. Then, using (9)–(12), one can calculate the energy in the center of mass for a collision on the horizon:

$$\left(\frac{E_{\text{c.m.}}}{2m}\right)_H = \sqrt{1 + \frac{M(l_1 - l_2)^2}{2r_C(l_1 - l_H)(l_2 - l_H)}}, \tag{21}$$

which coincides exactly with Eq. (10) of [2], from which further analysis of collisions in the Kerr metric can be carried out.

Near the horizon, $b \approx \frac{2Mr_H}{a^2}$, and Eq. (18) turns into

$$0 \le r - r_H \le \frac{a^2\varepsilon^2}{r_H\sqrt{1 - \frac{a^2}{M^2}}}, \tag{22}$$

which is completely equivalent to Eq. (18) of [2], where instead of ε, the quantity $\delta = \varepsilon\frac{2r_H}{a}$ was used. A particle with $E = 1$ cannot penetrate from infinity to the horizon, but nonetheless, there is a narrow region between a horizon—a potential barrier where such motion can occur that can generate acceleration to arbitrarily large energies (see [2] for details). Equation (21) is valid for both nonextremal and extremal cases; Eq. (22) applies to the nonextremal metric. In the extremal case, Eq. (14) turns into a very simple formula:

$$\frac{E^2_{\text{c.m.}}}{2m^2} \approx \frac{(2 - l_2)}{2N}(2 - \sqrt{2}). \tag{23}$$

It can also be obtained from Eq. (17) of [10] by putting there $\varepsilon = 1$, $l_H = 2$.

In the case $E = 1$, substituting $B_1 = M^{-1}$, $\omega_H = (2M)^{-1}$, $r - M \approx M\exp(-\frac{l}{M})$, $N \approx \frac{r-M}{2M}$ into (17), we obtain that $\Delta\phi \approx \frac{M\sqrt{2}}{r-M} \to \infty$, in agreement with [2].

Thus, our general results from Sec. II correctly reproduce the basic formulas for the Kerr metric.

IV. DISCUSSION AND CONCLUSIONS

Thus, we suggested a very simple and direct derivation of the effect of growing energy from first principles and without using the explicit form of the black hole metric. This became possible due to the fact that the relevant region is the vicinity of the horizon only where universality of the black hole physics reveals itself. In particular, we generalized recent observations made for the Kerr metric in [2] and showed that, generically, for the nonextremal rotating black hole, the horizon value of the energy in the center of mass is finite but can be made as large as one likes if the angular momentum of one colliding particle approaches the critical value. In the extremal case, the energy for the critical value of the momentum grows unbound as a horizon is approached, but the proper time also does so. In this respect, the mechanism preventing infinite energies has an universal character.

It was stated that there are astrophysical limitations on the significance of the effect in question due to gravitational radiation, backreaction, etc. [5,6]. We did not consider here the role of such mechanisms, having restricted ourselves by the picture of geodesic motion. A separate important task beyond the scope of the present paper that needs further attention is to evaluate the relative role of such effects; this also includes studying some concrete models. There is one more obstacle to get infinitely large energies in the nonextremal case since, generically, one particle should acquire the critical angular momentum in the very narrow strip near the horizon due to multiple scattering only. Therefore, further investigation of these issues is needed. Nonetheless, bearing in mind that the main results described above have universal characteristics, potential acceleration to large (formally, infinite) energies should be taken seriously, both as a manifestation of general properties of black holes and as the relevant effect in astrophysics of high energies.

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