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TeV-scale gauged $B-L$ symmetry with inverse seesaw mechanism

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We propose a modified version of the TeV-scale $B-L$ extension of the standard model, where neutrino masses are generated through the inverse seesaw mechanism. We show that heavy neutrinos in this model can be accessible via clean signals at the LHC. The search for the extra gauge boson Z'_{B-L} through the decay into dileptons or two dileptons plus missing energy is studied. We also show that the $B-L$ extra Higgs boson can be directly probed at the LHC via a clean dilepton and missing energy signal.

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I. INTRODUCTION

The search for new physics at TeV scale is a major goal of the Large Hadron Collider (LHC). Nonvanishing neutrino masses represent a firm observational evidence of new physics beyond the standard model (SM). The TeV-scale Baryon minus Lepton ($B-L$) extension of the SM, which is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, has been recently proposed [1] as the simplest model beyond the SM that provides a viable and testable solution to the neutrino mass mystery of contemporary particle physics. There have been several attempts in the past to extend the gauge symmetry of the SM with $U(1)_{B-L}$; see, for example, [2].

In this model, three SM singlet fermions arise quite naturally due to the anomaly cancellation conditions. These three particles are accounted for right-handed neutrinos, and hence a natural explanation for the seesaw mechanism is obtained. In addition, the model also contains an extra gauge boson corresponding to $B-L$ gauge symmetry and an extra SM singlet scalar (heavy Higgs boson). If the scale of $B-L$ breaking is of order TeV, these new particles will lead to very interesting signatures at the LHC [3–8]. In general, the scale of $B-L$ symmetry breaking is unknown, ranging from TeV to much higher scales. However, it was proven [7] that in supersymmetric framework, the scale of $B-L$ is nicely correlated with the soft supersymmetry breaking scale, which is TeV.

In the TeV-scale $B-L$ extension of the SM, the Majorana neutrino Yukawa interaction $\lambda_{\nu_R}\chi\bar{\nu}^c_R\nu_R$ induces the following masses for the right-handed neutrinos after $U(1)_{B-L}$ symmetry breaking: $M_{\nu_R} = \lambda_{\nu_R} v'$, where $v' = \langle\chi\rangle$ is the vacuum expectation value (vev) of the $B-L$ symmetry breaking. Below the electroweak (EW) symmetry breaking, Dirac neutrino masses, $m_D = \lambda_\nu v$, are generated. Here v is the vev of the EW symmetry breaking and λ_ν are the Dirac neutrino Yukawa couplings. Therefore, the physical light neutrino masses are given by m_D^2/M_{ν_R}, which can account for the measured experimental results if $\lambda_\nu \lesssim 10^{-6}$. Such small couplings may be considered as unnatural fine-tuning. Nevertheless, they induce new interaction terms between the heavy neutrino, weak gauge boson W and Z, and the associated leptons. These couplings play an important role in the decay of the lightest heavy neutrino at the LHC [5,9]. This signal is one of the striking signatures of the TeV-scale $B-L$ extension of the SM.

It is very important to note that the above analysis, which led to severe constraints on the neutrino Yukawa couplings, were based on the canonical type-I seesaw mechanism. In this paper, we propose a new modification for our TeV-scale $B-L$ model [1], to prohibit type-I seesaw and allow another scenario for generating light neutrino masses, namely, the inverse seesaw mechanism [10,11]. Our modification is based on the following: (i) The SM singlet Higgs boson, which breaks the $B-L$ gauge symmetry, has $B-L$ unit charge. (ii) The SM singlet fermion sector includes two singlet fermions with $B-L$ charges ± 2 with opposite matter parity. In this case, we will show that small neutrino masses can be generated through the inverse seesaw mechanism, without any stringent constraints on the neutrino Yukawa couplings. Therefore, a significant enhancement of the verifiability of the TeV-scale $B-L$ extension of the SM is obtained.

The proposed TeV-scale $B-L$ extension of the SM is based on the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, where the $U(1)_{B-L}$ is spontaneously broken by a SM singlet scalar χ with $B-L$ charge $= +1$. As in the previous model, a gauge boson Z'_{B-L} and three SM singlet fermions ν_{R_i} with $B-L$ charge $= -1$ are introduced for the consistency of the model. Finally, three SM singlet fermions S_1 with $B-L$ charge $= +2$ and three singlet fermions S_2 with $B-L$ charge $= -2$ are considered to implement the inverse seesaw mechanism.

The Lagrangian of the leptonic sector in this model is given by

$$\begin{aligned}\mathcal{L}_{B-L} = {}& -\tfrac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + i\bar{L}D_\mu\gamma^\mu L + i\bar{e}_R D_\mu\gamma^\mu e_R \\ & + i\bar{\nu}_R D_\mu\gamma^\mu\nu_R + i\bar{S}_1 D_\mu\gamma^\mu S_1 + i\bar{S}_2 D_\mu\gamma^\mu S_2 \\ & + (D^\mu\phi)(D_\mu\phi) + (D^\mu\chi)(D_\mu\chi) - V(\phi,\chi) \\ & - (\lambda_e\bar{L}\phi e_R + \lambda_\nu\bar{L}\tilde{\phi}\nu_R + \lambda_S\nu^c_R\chi S_2 + \text{H.c.}), \quad (1)\end{aligned}$$

where $F'_{\mu\nu} = \partial_\mu Z'_\nu - \partial_\nu Z'_\mu$ is the field strength of the $U(1)_{B-L}$. The covariant derivative D_μ is generalized by adding the term $ig'' \; Y_{B-L} Z'_\mu$, where g'' is the $U(1)_{B-L}$ gauge coupling constant and Y_{B-L} is the $B-L$ quantum numbers of involved particles. Since $U(1)_{B-L}$ is not orthogonal to $U(1)_Y$, a mixing term between the two field strengths is expected. To prohibit a possible large mass term MS_1S_2 in the above Lagrangian, we assume that the SM particles, ν_R, χ, and S_2 are even under matter parity, while S_1 is an odd particle. $V(\phi, \chi)$ can be found in [1].

The nonvanishing vacuum expectation value of χ, $|\langle\chi\rangle| = v'/\sqrt{2}$, is assumed to be of order TeV, consistent with the result of radiative $B-L$ symmetry breaking found in the gauged $B-L$ model with supersymmetry [7]. After the $B-L$ gauge symmetry breaking, the gauge field Z' acquires the following mass: $M^2_{Z'_{B-L}} = 4g''^2 v'^2$. The bound on the $B-L$ gauge boson, due to a negative search at LEP II, implies that $M_{Z'_{B-L}}/g'' > 6$ TeV. This indicates that $v' \gtrsim O(\text{TeV})$. If the coupling $g'' < O(1)$, then one obtains $m_{Z'} \gtrsim O(600)$ GeV.

Now, we turn to neutrino masses in this model. As can be seen from Eq. (1), after $B-L$ and EW symmetry breaking, the neutrino Yukawa interaction terms lead to the following mass terms: $\mathcal{L}^\nu_m = m_D \bar{\nu}_L \nu_R + M_N \nu^c_R S_2$, where $m_D = \frac{1}{\sqrt{2}} \lambda_\nu v$ and $M_N = \frac{1}{\sqrt{2}} \lambda_{\nu_R} v'$. From this Lagrangian, one can easily observe that although the lepton number is broken through the spontaneous $B-L$ symmetry breaking, a remnant symmetry, $(-1)^{L+S}$, is survived, where L is the lepton number and S is the spin. After this global symmetry is broken at a much lower scale, a mass term for S_2 (and possibly for S_1 as well) is generated. Therefore, the Lagrangian of neutrino masses, in the flavor basis, is given by $\mathcal{L}^\nu_m = m_D \bar{\nu}_L \nu_R + M_N \nu^c_R S_2 + \mu_s S_2^2$. It is worth noting that in the limit $\mu_s \to 0$, which corresponds to the unbroken $(-1)^{L+S}$ symmetry, the light neutrinos remain massless. Therefore, a small nonvanishing μ_s can be considered as a slight breaking of a this global symmetry. Hence, according to 't Hooft criteria, the smallness of μ_s is natural. The possibility of generating small μ_s radiatively has been discussed in [12].

In the basis $\{\nu_L, \nu^c_R, S_2\}$, the 9×9 neutrino mass matrix takes the form

$$\begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_N \\ 0 & M_N^T & \mu_s \end{pmatrix}. \tag{2}$$

The diagonalization of this mass matrix leads to the following light and heavy neutrino masses, respectively: $m_{\nu_l} = m_D M_N^{-1} \mu_s (M_N^T)^{-1} m_D^T$, $m^2_{\nu_H} = m^2_{\nu_{H'}} = M_N^2 + m_D^2$. Thus, the light neutrino mass can be of order eV, as required by the oscillation data, for a TeV-scale M_N, provided μ_s is sufficiently small, $\mu_s \ll M_N$. In this case, the Yukawa coupling λ_ν is no longer restricted to a very small value and it can be of order 1. Therefore, the possibility of testing this type of model in LHC is quite feasible.

In general, the physical neutrino states are given in terms of ν_L, ν^c_R, and S_2 as follows:

$$\nu_l = \nu_L + a_1 \nu^c_R + a_2 S_2, \tag{3}$$

$$\nu_H = a_3 \nu_L + \alpha \nu^c_R - \alpha S_2, \tag{4}$$

$$\nu_{H'} = \alpha \nu^c_R + \alpha S_2. \tag{5}$$

For $m_D \simeq 100$ GeV, $M_N \simeq 1$ TeV, and $\mu_s \simeq 1$ KeV, one finds that $a_{1,2} \sim m_D/(M_N\sqrt{2 + 2m_D/M_N}) \sim \mathcal{O}(0.05)$, $a_3 \sim m_D/M_N \sim \mathcal{O}(0.1)$, and $\alpha \sim \sin\pi/4$. Therefore, one of the heavy neutrinos of this model can be accessible via a clean signal at LHC, as will be discussed below.

It is worth mentioning that the light neutrinos ν_l have suppressed mixing [of order $m_D\mu_s/(M_N^2 + m_D^2)$] with one type of the heavy neutrinos (say $\nu_{H'}$) and a large mixing (of order m_D/M_N) with the other type of heavy neutrinos (ν_H). The mixing between the heavy neutrino ν_H and ν'_H is maximal. The heavy neutrinos ν_H and ν'_H can mediate the lepton flavor processes, like $\mu \to e\gamma$. The $\mu \to e\gamma$ decay mediated by these heavy neutrinos have branching ratios [13]:

$$\text{BR}\,(\mu \to e\gamma) \simeq \frac{\alpha_W^3 \sin^2\theta_W m_\mu^5}{256\pi^2 M_W^4 \Gamma_\mu} \left| \sum_{i=1}^{3} (a_3)_{\mu i} (a_3^*)_{ei} I\left(\frac{m_{\nu^2_{H_i}}}{M_W^2}\right) \right|^2, \tag{6}$$

where Γ_μ is the total decay width of μ and the loop function $I(x)$ can be found in [13]. From the present experimental limit, $\text{BR}(\mu \to e\gamma)$, one finds $|(a_3)_{\mu\mu} \times (a_3^*)_{e\mu} I(\frac{m_{\nu^2_{H_2}}}{M_W^2})| < 10^{-4}$. Thus for $(a_3)_{\mu\mu} \simeq 0.1$, one obtains the following constraint on the off-diagonal element $(a_3)_{12}$: $(a_3)_{12} \simeq (m_D M_N^{-1})_{12} < 10^{-3}$.

The LHC discovery of Z'_{B-L} is considered as a smoking gun for the TeV-scale $B-L$ extension of the SM. In the minimal $B-L$ model, it was shown that $Z' \to l^+ l^-$ gives the dominant decay channel with $\text{BR}(Z' \to l^+ l^-) \simeq 20\%$. Therefore, the search for Z' can be accessible via a dilepton channel for 600 GeV $\leq M_{Z'} \leq 2$ TeV. In our new model of $B-L$ with inverse seesaw, the decay widths of Z' into the lightest heavy neutrinos ν_H and $\nu_{H'}$ are given by

$$\begin{aligned} \Gamma(Z' \to \nu_H \nu_H) &= \frac{(g'' Y^{\nu_H}_{B-L})^2}{48\pi} M_{Z'} \left(1 - 4\frac{m^2_{\nu_H}}{M^2_{Z'}}\right)^{3/2}, \\ \Gamma(Z' \to \nu_{H'} \nu_{H'}) &= \frac{(g'' Y^{\nu_{H'}}_{B-L})^2}{48\pi} M_{Z'} \left(1 - 4\frac{m^2_{\nu_{H'}}}{M^2_{Z'}}\right)^{3/2}. \end{aligned} \tag{7}$$

From Eqs. (4) and (5), the charges $Y^{\nu_H}_{B-L}$ and $Y^{\nu_{H'}}_{B-L}$ are given by $Y^{\nu_H}_{B-L} \simeq a_3^2 Y^{\nu_L}_{B-L} + \alpha(Y^{\nu^c_R}_{B-L} - Y^{S_2}_{B-L}) \simeq 3\alpha^2 \simeq \frac{3}{2}$, $Y^{\nu_{H'}}_{B-L} \simeq \alpha^2(Y^{\nu^c_R}_{B-L} + Y^{S_2}_{B-L}) = -\alpha^2 \simeq \frac{-1}{2}$.

Thus, for heavy Z'_{B-L} ($M_{Z'_{B-L}} \gg 2M_{\nu_H}$), the decay channel $Z'_{B-L} \to \nu_H \nu_H$ could be the dominant one. In Fig. 1 we present the decay branching ratios of $Z' \to f\bar{f}$ as a function of $M_{Z'}$ for $f = l^-$, ν_H, ν_l, $\nu_{H'}$, $q = u, c, d, s, b$, and $f = t$. As can be seen from this figure, the decay $Z' \to l^+ l^-$ is the

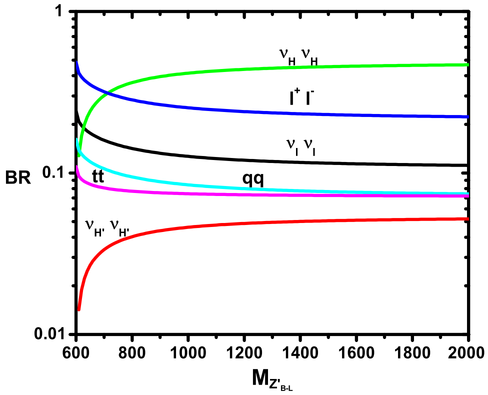

FIG. 1 (color online). Branching ratios of Z'_{B-L} as function of $M_{Z'_{B-L}}$.

dominant one if $M_{Z'_{B-L}} < 2M_{\nu_H}$. However, for $M_{Z'_{B-L}} \gg 2M_{\nu_H}$, the decay $Z'_{B-L} \to \nu_H\nu_H$ becomes dominant with branching ratio $\gtrsim 32\%$. Therefore, searching for Z'_{B-L} can be easily accessible at the LHC via: (i) A clean dilepton signal, which can be one of the first new physics signatures to be observed at the LHC, if Z'_{B-L} is lighter than twice the ν_H mass. As emphasized in [8], Z'_{B-L} can be discovered in this case, within a mass range [800, 1200] GeV and an integrated luminosity of 100 pb^{-1}. (ii) A signal of 2-dilepton plus missing energy, with a tiny SM background if $M_{Z'_{B-L}} \gg 2M_{\nu_H}$. In this case, one considers the Z'_{B-L} decay into two heavy neutrinos. This process could enhance the ν_H production cross section, due to the resonant contribution from Z'_{B-L} exchange in the s channel. Then, the ν_H mainly decays through the W gauge boson to lepton and neutrino, as shown in Fig. 2. As explained in [5], these decays are very clean with four hard leptons; therefore, they are distinctive LHC signals with nearly free background. Note that in this model, the coupling of $\nu_H W l$ is of order $0.05g_2$, which is not very suppressed as in the minimal $B-L$ model. Therefore, the decay width of $\nu_H \to W^+ l^-$ is not very small, and hence ν_H is no longer a long-lived particle. This could be a distinct difference between the two $B-L$ scenarios [14].

After the breakdown of the $B-L$ and EW symmetry, mixing between ϕ and χ is generated. The mixing between the neutral scalar components of Higgs multiplets, ϕ^0 and χ^0, leads to SM-like Higgs boson H and heavy Higgs boson H', with the following masses:

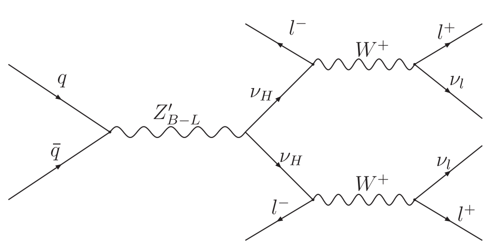

FIG. 2. Z'_{B-L} production and decay via 2-dilepton plus missing energy at LHC.

$$m^2_{H,H'} = \lambda_1 v^2 + \lambda_2 v'^2 \mp \sqrt{(\lambda_1 v^2 - \lambda_2 v'^2)^2 + \lambda_3^2 v^2 v'^2}.$$

From these expressions, it is clear that λ_3 is the measuring of the mixing between the SM Higgs and the $B-L$ extra Higgs.

As in the minimal $B-L$ model [3], the couplings among the SM-like Higgs, H, and the SM fermions and gauge bosons are modified by a factor of $\cos\theta$. It is interesting to note that a maximum mixing with $\theta = \pi/4$ can be obtained if $\lambda_1 v^2 - \lambda_2 v'^2 = 0$, which implies that $m_H \simeq m_{H'}$. However, the restriction from precision EW measurements, in particular, the fit of the parameters S, T, and U, impose the following constraint on the Higgs mixing angle [15]: For $m_H > 120$ GeV and $m_{H'} > 500$ GeV $\Rightarrow \cos\theta > 0.9$. Therefore, the cross sections of the SM-like Higgs production cross sections and decay branching ratios are slightly changed. Also, the decay widths of H' into SM fermions are suppressed by $\sin^2\theta$ factor. Because of a large mixing between light and heavy neutrinos in this model, the decay channels $H' \to \nu_l\nu_H$, $H' \to \nu_H\nu_H$, and $H' \to \nu_{H'}\nu_{H'}$ (in case of $m_{H'} > m_{\nu_H}$, $m_{H'} > 2m_{\nu_H}$, and $m_{H'} > 2m_{\nu_{H'}}$, respectively) are relevant and may lead to important effects. The decay widths of these channels are given by

$$\Gamma(H' \to \nu_l\nu_H) = \frac{|\lambda_S a_2|^2}{32\pi} m_{H'}\cos^2\theta\left[1 - \frac{m^2_{\nu_H}}{m^2_{H'}}\right]^2, \quad (9)$$

$$\Gamma(H' \to \nu_H\nu_H) \simeq \Gamma(H' \to \nu_{H'}\nu_{H'}) \simeq \Gamma(H' \to \nu_H\nu_{H'}) \simeq \frac{|\lambda_S|^2}{64\pi} m_{H'}\cos^2\theta\left[1 - \frac{4m^2_{\nu_H}}{m^2_{H'}}\right]^{3/2}, \quad (10)$$

where a_2 is the mixing between light and heavy neutrinos as defined in Eq. (3), which is of order 0.04. Thus, for $m_H \sim 1$ TeV, the decay width $\Gamma(H' \to \nu_l\nu_H) \sim 10^{-3}$. This should be compared with the dominant decay channel, $H' \to WW$, which has an order 1 decay width:

$$\Gamma(H' \to W^+W^-) = \frac{M^3_{H'}}{16\pi v^2}\sin^2\theta\left[1 - \frac{4m^2_W}{m^2_{H'}}\right]^{3/2}. \quad (11)$$

The decay branching ratios of H' into W^+W^-, ZZ, $\nu_l\nu_H$, $\nu_H\nu_H$, $t\bar{t}$, and $b\bar{b}$ are shown in Fig. 3 as function of $M_{H'}$. From this figure, it is clear that the decay of H' is dominated by the same channel of the SM-like Higgs boson. Therefore, these decay channels are experimentally challenged, due to a large background from the SM Higgs decays and cannot be considered for probing H' at the LHC. Furthermore, the H' decay into two heavy neutrinos gives the same signal of two dileptons and missing energy as in Z' decay, but with a smaller cross section. Therefore, the H' production and decay via $H' \to \nu_l\nu_H \to l^+l^- +$ missing energy, as shown in Fig. 4, remains as a distinctive signal at the LHC that is nearly background free.

The total cross section of this process, $\sigma_{2l} = \sigma(pp \to H' \to \nu_l\nu_H \to l^-l^- +$ missing energy) can be written as

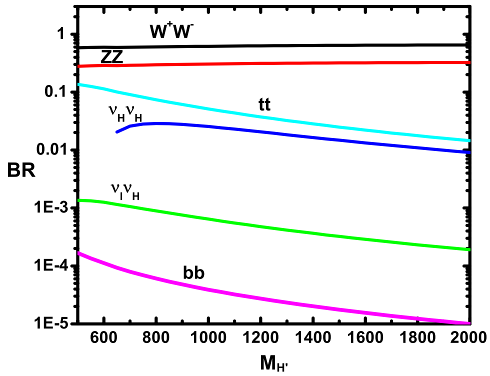

FIG. 3 (color online). Branching ratios of $H' \to f\bar{f}$ as function of $M_{H'}$.

$\sigma_{2l} \simeq \sigma(pp \to \tilde{\nu}_l \tilde{\nu}_H) \times \mathrm{BR}(\nu_H \to l^- W^+) \times \mathrm{BR}(W^+ \to l^+ \nu_l)$ where $\mathrm{BR}(W^+ \to l^+ \nu_l) \sim 0.1$ and $\mathrm{BR}(\nu_H \to l^- W^+) \sim \mathcal{O}(1)$, since $\nu_H \to l^- W^+$ is the dominant decay channel for the heavy neutrino to the SM particles. Finally the cross section $\sigma(pp \to H' \to \nu_l \nu_H)$ can be approximated as $\sigma(pp \to H') \times \mathrm{BR}(H' \to \nu_l \nu_H)$, where the H' production is dominated by a gluon-gluon fusion mechanism as shown in Fig. 4. In this case, $\sigma(pp \to H') \sim \mathcal{O}(0.01)$ as emphasized in [3]. Also from Fig. 3, one can notice that $\mathrm{BR}(H' \to \nu_l \nu_H) \sim 10^{-3}$. Therefore, $\sigma(pp \to H' \to \nu_l \nu_H) \sim 10^{-5}$. In this case, the total cross section of the two dilepton signal, which provides indisputable evidence for probing the $B - L$ extra Higgs H', is give by $\sigma_{2l} = \sigma(pp \to H' \to l^+ l^- + \text{missing energy}) \simeq 10^{-7}\ \mathrm{GeV}^{-2} \simeq \mathcal{O}(100)$ pb. For this value of cross section, the dilepton and missing energy signal can be probed at the LHC as a clear hint for the $B - L$ extra Higgs boson.

It is worth mentioning that if $m_{H'} > 2m_{\nu_{R_H}}$, then the decay width $\Gamma(H' \to \nu_H \nu_H)$ becomes relevant and may be dominant. However, as mentioned above, this process leads to a signal of two dileptons with missing energy similar to the decay of $Z' \to \nu_H \nu_H$ but with a smaller cross section.

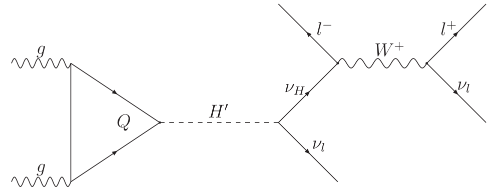

FIG. 4. H' production and decay into dilepton and missing energy at the LHC.

Therefore, this channel is not the best for probing H' at the LHC.

Finally, let us note that the above mentioned two dilptons and missing energy ($4l + \not{E}_T$) and dilpton plus missing energy ($2l + \not{E}_T$) final states are mediated by the heavy neutrinos ν_H; therefore, they are also clean signatures for probing ν_H at the LHC.

In conclusion, we have constructed a modified version of the minimal TeV-scale $B - L$ extension of the SM. In this model, the neutrino masses are generated through the inverse seesaw mechanism; therefore, the neutrino Yukawa coupling is no longer constrained to be less than 10^{-6}. Thus, the heavy neutrinos associated with this model can be quite feasible at the LHC. We have discussed the main phenomenological features of this class of models. We showed that searching for the Z'_{B-L} and heavy neutrinos is accessible via the $4l + \not{E}_T$ final state, while searching for the extra Higgs boson and also heavy neutrino can be accessible through the $2l + \not{E}_T$ final state. These final states are very clean signals at LHC, with a negligibly small SM background.

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