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Constraints from unrealistic vacua in the supersymmetric standard model with neutrino mass operators

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We analyze a scalar potential of the minimal supersymmetric standard model (MSSM) with neutrino mass operators along unbounded-from-below and color and/or charged breaking directions. We show necessary conditions to avoid the potential minima which can be deeper than the realistic vacuum. These conditions would constrain more strongly than conditions in the MSSM without taking into account neutrino mass operators and can improve the predictive power of supersymmetric models with neutrino mass operators.

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I. INTRODUCTION

The origin of neutrino masses is one of the unanswered questions in particle physics. It should be addressed by new physics that explains tininess of the neutrino masses because the masses of neutrinos are very small compared with other fermions. Small masses are, in general, realized by introducing heavy particles. After integrating out the heavy particles, an effective operator which is suppressed by the masses of the heavy particles, M, is obtained [1],

$$\frac{1}{M}(H \cdot L)(H \cdot L), \tag{1}$$

where H and L are the Higgs and the left-handed lepton doublets. Neutrinos acquire masses through the electroweak symmetry breaking (EWSB), and their masses can be very small if M is much larger than the vacuum expectation value (vev) of the Higgs scalar. The most famous mechanism in this regard is the so-called seesaw mechanism [2–6] in which heavy right-handed neutrinos are introduced. One obtains the same mass term, Eq. (1), after integrating out the right-handed neutrinos.

Supersymmetric extension of the standard model is one of the promising candidates for physics above the weak scale. In supersymmetric extension of the standard model with neutrino mass operator, Eq. (1) (which we call the νSSM), there is the following operator in the superpotential:

$$c(\hat{H}_2 \cdot \hat{L})(\hat{H}_2 \cdot \hat{L}), \tag{2}$$

and a dimension four operator in the soft supersymmetry (SUSY) breaking term,

$$c'(H_2 \cdot \tilde{L})(H_2 \cdot \tilde{L}), \tag{3}$$

where c and c' are coupling constants, and $\hat{H}_2(H_2)$ and $\hat{L}(\tilde{L})$ are the superfield (the scalar partner) of the up-type Higgs and the left-handed leptons. The coupling constants can be determined theoretically once a mechanism for the neutrino masses and the SUSY breaking is specified. The ratio c'/c would be comparable to (or larger than) SUSY breaking masses such as gaugino masses and soft scalar masses. For example, in the gravity mediation scenario, the ratio of the couplings, $|c'|/|c|$, would be on the order of the gravitino mass.

In SUSY models, each of the fermions has a scalar partner. The presence of the scalar partners generally leads color and/or charge breaking (CCB) directions and unbounded-from-below (UFB) directions [7–15]. Along the CCB directions, the scalar potential has minima on which color and/or charge symmetry is spontaneously broken. Along the UFB directions, the potential has no global minima and falls down to negative infinity. The existence of these dangerous directions makes the vacuum of the EWSB unstable, and hence these directions must be avoided. In the minimal supersymmetric extension of the standard model (MSSM), the scalar potential was analyzed systematically and necessary conditions to avoid the CCB and UFB directions were summarized in [16]. Recently, the scalar potential of the νSSM with Dirac neutrinos or Majorana neutrinos was analyzed when sneutrinos, scalar partners of the neutrinos, have nonvanishing vev's. It was found in [17] that the UFB directions disappear and turn to CCB directions due to the Yukawa coupling of neutrinos. Necessary conditions to avoid the CCB directions of the νSSM were also found in [17], which result in constraints on the soft SUSY breaking parameters. In addition to UFB and CCB directions, false EWSB minima appear in the νSSM. On such false EWSB minima, either color or charge symmetry is not broken, but Higgs scalars and

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sneutrinos develop their large vev's. Then, such minima would lead to too heavy gauge bosons and be excluded by precise electroweak measurements. Here, we refer to such false EWSB minima as CCB minima in view of incorrect vacua.

In this article, we consider the νSSM with the neutrino mass operators, Eqs. (2) and (3), and derive necessary conditions to avoid the CCB and UFB directions. We perform our analysis for tree-level potentials, although radiative corrections can modify the conditions as discussed in [18–20]. The conditions obtained in tree-level analysis, however, coincide with those from one-loop analysis if the analysis is performed at a scale that the radiative corrections is smaller enough than the tree-level potential. We assume that our analysis is performed at this scale. Also note that our potential is available below M, because effective operators (1)–(3) are induced below M.

The outline of this article is organized as follows. In Sec. II, we briefly review constraints of UFB and CCB directions in the MSSM. Then we analyze the scalar potential of the νSSM with the neutrino mass operators and derive necessary conditions to avoid UFB and CCB directions in Secs. III and IV, respectively. We show numerical results of constraints on the soft SUSY breaking parameters in Sec. V. Finally we summarize and discuss our analysis in the Sec. VI. The scalar potential of the νSSM with the neutrino mass operators and notations of fields and couplings are given in Appendix A. In Appendix B, we give a general form of the vacuum expectation values for CCB directions.

II. CONSTRAINTS FROM UFB AND CCB DIRECTIONS IN THE MSSM

We start our discussion by briefly reviewing constraints on soft SUSY breaking parameters from UFB and CCB directions in the MSSM. According to general properties given in [16], there are three types of the UFB and CCB directions, respectively, which we refer to as the "MSSM" UFB and CCB directions. We only show constraints from these directions for comparison with our results given in the following sections. Details of the derivation are found in [16]. Notations of couplings and scalar fields are summarized in Appendix A.

A. The MSSM UFB directions

The MSSM UFB directions appear when positive quartic terms in the scalar potential are vanishing or kept under control. Along these directions, the potential falls down to negative infinity in large values of fields, making the EWSB vacuum unstable.

The MSSM UFB-1 direction is a direction when H_1 and H_2 have an equal nonvanishing vev while other scalars are vanishing. The scalar potential along this direction becomes

$$V_{\mathrm{MSSM\,UFB-1}} = (m_1^2 + m_2^2 - 2|m_3^2|)|H_2|^2. \tag{4}$$

The potential is unbounded from below unless the coefficient in the right-hand side is positive. Thus, a necessary condition to be satisfied is

$$m_1^2 + m_2^2 - 2|m_3^2| \geq 0. \tag{5}$$

This is the well-known constraint on the soft SUSY breaking masses of Higgs scalars.

Another UFB direction, the so-called the MSSM UFB-2 direction, is along which

$$H_1, H_2, \tilde{L} \neq 0, \tag{6}$$

where $\tilde{L}$ is chosen along $\tilde{\nu}_L$. The vev's of H_1, H_2, and $\tilde{L}$ are chosen so that the D term potential is kept under control. Then, the potential becomes

$$V_{\mathrm{MSSM\,UFB-2}} = \left(m_2^2 + m_{\tilde{L}}^2 - \frac{|m_3^2|^2}{|m_1^2 - m_{\tilde{L}}^2|}\right)|H_2|^2 - \frac{2m_{\tilde{L}}^4}{g_1^2 + g_2^2}, \tag{7}$$

and a necessary condition to avoid a UFB potential is

$$m_2^2 + m_{\tilde{L}}^2 - \frac{|m_3^2|^2}{|m_1^2 - m_{\tilde{L}}^2|} \geq 0. \tag{8}$$

The last direction, the MSSM UFB-3, is along

$$H_2, \tilde{L}, \tilde{Q}, \tilde{d}_R \neq 0, \qquad \tilde{d}_L = \tilde{d}_R, \tag{9}$$

where $\tilde{Q}$ and $\tilde{L}$ are chosen along $\tilde{d}_L$ and $\tilde{\nu}_L$. The vev's of $\tilde{d}_L$ and $\tilde{d}_R$ are chosen so that the F term of H_1 vanishes. Then, the vev's of $\tilde{d}_L$ and $\tilde{d}_R$ are small compared to those of other scalars and can be neglected in the potential. A condition to avoid the MSSM UFB-3 direction is given by

$$m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 \geq 0. \tag{10}$$

The condition, (10), gives a stringent constraint since m_2^2 is negative in a large region of parameter space of the MSSM for the EWSB to occur.

As stressed in [17], the absence of the neutrino Yukawa coupling plays an essential role on the MSSM UFB directions, especially the UFB-2 and the UFB-3. It was shown in [17] that these directions become CCB directions when there exits the neutrino Yukawa coupling and the Majorana mass term.

B. The MSSM CCB directions

The MSSM CCB directions appear when a negative trilinear term dominates the potential against quartic terms in a certain value of fields.

As an example, we consider that $\tilde{Q}$, $\tilde{u}_R$, $\tilde{L}$ as well as H_1 and H_2 are nonvanishing. In order to show constraints from the MSSM CCB directions, it is helpful to express vev's of the scalars in terms of $|H_2|$,

$$|\tilde{Q}| = \alpha|H_2|, \qquad |\tilde{u}_R| = \beta|H_2|,$$
$$|H_1| = \gamma|H_2|, \qquad |\tilde{L}| = \gamma_L|H_2|. \tag{11}$$

In the following discussion, we consider that $\tilde{Q}$ is almost a vev along the $\tilde{u}_L$ direction. Then, the potential can be written

$$\begin{aligned} V_{\text{MSSM CCB}} = &Y_u^2\alpha^2\beta^2\hat{F}(\alpha, \beta, \gamma, \gamma_L)|H_2|^4 \\ &- 2Y_u\alpha\beta\hat{A}(\gamma)|H_2|^3 \\ &+ \hat{m}^2(\alpha, \beta, \gamma, \gamma_L)|H_2|^2, \end{aligned} \tag{12}$$

where

$$\hat{F}(\alpha, \beta, \gamma, \gamma_L) = 1 + \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{f(\alpha, \beta, \gamma, \gamma_L)}{\alpha^2\beta^2}, \tag{13a}$$

$$\begin{aligned} f(\alpha, \beta, \gamma, \gamma_L) = \frac{1}{Y_u^2}\Big[&\frac{1}{8}g_1^2\Big(1 + \frac{1}{3}\alpha^2 - \frac{4}{3}\beta^2 - \gamma^2 - \gamma_L^2\Big)^2 \\ &+ \frac{1}{8}g_2^2(1 - \alpha^2 - \gamma^2 - \gamma_L^2)^2 \\ &+ \frac{1}{6}g_3^2(\alpha^2 - \beta^2)^2\Big], \end{aligned} \tag{13b}$$

$$\hat{A}(\gamma) = |A_u| + |\mu|\gamma, \tag{13c}$$

$$\begin{aligned} \hat{m}^2(\alpha, \beta, \gamma, \gamma_L) = &m_1^2\gamma^2 + m_2^2 - 2|m_3^2|\gamma + m_{\tilde{Q}}^2\alpha^2 \\ &+ m_{\tilde{u}_R}^2\beta^2 + m_{\tilde{L}}^2\gamma_L^2. \end{aligned} \tag{13d}$$

Since the Yukawa couplings of quarks (except for the top) are smaller than the gauge couplings, the deepest direction appears along $f(\alpha, \beta, \gamma, \gamma_L) = 0$. The extremal value of the up-type Higgs scalar, $|H_2|_{\text{ext}}$, can be obtained by solving $\partial V_{\text{MSSM CCB}}/\partial|H_2| = 0$,

$$|H_2|_{\text{ext}} = \frac{3\hat{A}}{4Y_u\alpha\beta\hat{F}}\Bigg(1 + \sqrt{1 - \frac{8\hat{m}^2\hat{F}}{9\hat{A}^2}}\Bigg). \tag{14}$$

Inserting Eq. (14) into the potential, (12), the minimum of the potential is given by

$$V_{\text{MSSM CCB min}} = -\frac{1}{2}\alpha\beta|H_2|_{\text{ext}}^2\Big(\hat{A}Y_u|H_2|_{\text{ext}} - \frac{\hat{m}^2}{\alpha\beta}\Big). \tag{15}$$

The CCB constraints can be obtained by requiring that Eq. (15) is positive.

The MSSM CCB-1 direction is a direction along

$$H_2, \tilde{Q}, \tilde{u}_R \neq 0, \qquad |\tilde{d}_L|^2 = |\tilde{d}_R|^2, \tag{16}$$

where $\tilde{d}_L$ and $\tilde{d}_R$ are chosen such that the F term of H_1 cancels. Similar to the MSSM UFB-3 direction, the vev's of $\tilde{d}_L$ and $\tilde{d}_R$ are small and can be neglected in the potential. Then, the potential is given by setting $\gamma = 0$, and $\gamma_L^2 = 1 - \alpha^2$ with $\alpha = \beta$ for the D term potential to vanish. The most stringent constraint to avoid the CCB minimum is given, when $m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 > 0$ and $3m_{\tilde{L}}^2 - (m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2) + 2(m_2^2 - |\mu|^2) > 0$,

$$|A_u|^2 \leq 3(m_2^2 - |\mu|^2 + m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2), \tag{17}$$

with $\alpha = 1$, and when $m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 > 0$ and $3m_{\tilde{L}}^2 - (m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2) + 2(m_2^2 - |\mu|^2) < 0$

$$\begin{aligned} |A_u|^2 \leq \Big(1 + \frac{2}{\alpha^2}\Big) \times (&m_2^2 - |\mu|^2 + (m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2)\alpha^2 \\ &+ m_{\tilde{L}}^2(1 - \alpha^2)), \end{aligned} \tag{18}$$

with $\alpha^2 = \sqrt{2(m_{\tilde{L}}^2 + m_2^2 - |\mu|^2)/(m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2 - m_{\tilde{L}}^2)}$. When $m_2^2 - |\mu|^2 + m_{\tilde{L}}^2 < 0$ and $3m_{\tilde{L}}^2 - (m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2) + 2(m_2^2 - |\mu|^2) < 0$, the CCB constraint can not be satisfied and the MSSM CCB-1 direction becomes the MSSM UFB-3 direction.

The MSSM CCB-2 direction appears along

$$H_1, H_2, \tilde{Q}, \tilde{u}_R, \tilde{L} \neq 0, \tag{19a}$$
$$\text{sign}[A_u] = -\text{sign}[B], \tag{19b}$$

where $\tilde{Q}$ takes a vev along the $\tilde{u}_L$ direction. Similar to the MSSM CCB-1 constraint, the most stringent constraint is obtained as

$$\begin{aligned} (|A_u| + |\mu|\gamma)^2 \leq \Big(1 + \frac{2}{\alpha^2}\Big)(&m_2^2 + (m_{\tilde{Q}}^2 + m_{\tilde{u}_R}^2)\alpha^2 \\ &+ m_1^2\gamma^2 + m_{\tilde{L}}^2(1 - \alpha^2 - \gamma^2) - 2|m_3^2|\gamma), \end{aligned} \tag{20}$$

with $\alpha = \beta$ and $\gamma_L^2 = 1 - \alpha^2 - \gamma^2$. The minimum of the right-hand side of Eq. (20) can be found by numerical calculation by varying α and γ between 0 and 1.

The MSSM CCB-3 direction appears when the vev's are taken as the same as the MSSM CCB-2 direction but $\text{sign}[A_u] = \text{sign}[B]$. The constraint is given by Eq. (20) with the opposite sign of one of the three terms, $(|A_u|, |\mu|\gamma, -2|m_3^2|\gamma)$.

III. UFB CONSTRAINTS IN THE νSSM WITH NEUTRINO MASS OPERATORS

In this and the following sections, we analyze the scalar potential of the νSSM with neutrino mass operators. As was explained in Sec. I, the neutrino mass operators consist of the $(\hat{H}_2 \cdot \hat{L})(\hat{H}_2 \cdot \hat{L})$ operator (2) in the superpotential and a dimension four operator (3) in the soft SUSY breaking term. New terms of 6th, 5th, and 4th power of scalars arise from these mass operators and modify the structure of the potential. The MSSM UFB directions turn to CCB directions while the MSSM CCB directions have another minima under certain conditions that we will discuss later.

We focus our analysis on the new minima appearing due to these higher power terms and show conditions to avoid them. In the following, we assume that only one sneutrino has nonvanishing vev and we take a basis that the couplings of the neutrino mass operators, c and c', are diagonal. The full potential of the model is given in Appendix A.

A. Constraints from the MSSM UFB-2 direction

Let us first consider the MSSM UFB-2 direction which is given by Eq. (6). The effective scalar potential along the MSSM UFB-2 direction is given by

$$\begin{aligned} V_{\mathrm{UFB}-2} =& m_1^2|H_1|^2 + m_2^2|H_2|^2 - 2\mathrm{Re}(m_3^2 H_1 H_2) \\ &+ m_{\tilde{L}}^2|\tilde{\nu}_L|^2 + \frac{1}{8}(g_1^2+g_2^2)(|\tilde{\nu}_L|^2 + |H_1|^2 - |H_2|^2)^2 \\ &- 2\mathrm{Re}(c^*\mu H_1 H_2^*(\tilde{\nu}_L^*)^2) - \mathrm{Re}(c'(H_2)^2(\tilde{\nu}_L)^2) \\ &+ |c|^2|\tilde{\nu}_L|^2|H_2|^2(|\tilde{\nu}_L|^2 + |H_2|^2). \end{aligned} \tag{21}$$

It is easily understood that the potential is lifted up in large vev's since the term of 6th power is always positive. Thus, the MSSM UFB-2 direction becomes a CCB direction. Note that, along the MSSM UFB-2 direction, neither color nor electric charge symmetry is broken, but the Higgs scalars and sneutrinos acquire large vev's on the minima. Such minima result in too heavy masses of the gauge bosons and are excluded by precise electroweak measurements. Hence the EWSB does not occur correctly on such minima.

We refer to these false EWSB directions as CCB directions in view of incorrect vacuum.

Before we start a detailed analysis, it is helpful to parametrize vev's as

$$|\tilde{\nu}_L| = \alpha|H_2|, \qquad |H_1| = \gamma|H_2|, \tag{22}$$

and choose phases of vev's so that terms with undermined phases are negative. This choice of the phases is always possible. Then, the potential, (21), is expressed as

$$V_{\mathrm{UFB}-2} = \hat{C}(\alpha)|H_2|^6 - \hat{F}(\alpha,\gamma)|H_2|^4 + \hat{m}^2(\alpha,\gamma)|H_2|^2, \tag{23}$$

where

$$\hat{C}(\alpha) = \alpha^2(\alpha^2+1)|c|^2, \tag{24a}$$

$$\hat{F}(\alpha,\gamma) = 2\alpha^2\gamma|c||\mu| + \alpha^2|c'| - f(\alpha,\gamma), \tag{24b}$$

$$f(\alpha,\gamma) = \tfrac{1}{8}(g_1^2+g_2^2)(\alpha^2+\gamma^2-1)^2, \tag{24c}$$

$$\hat{m}^2(\alpha,\gamma) = m_1^2\gamma^2 + m_2^2 - 2\gamma|m_3^2| + \alpha^2 m_{\tilde{L}}^2. \tag{24d}$$

Differentiating the potential, (23), with respect to $|H_2|$, the extremal value of the up-type Higgs scalar is obtained

$$|H_2|^2_{\mathrm{ext}} = \frac{\hat{F}(\alpha,\gamma)}{3\hat{C}(\alpha)}\left(1+\sqrt{1-\frac{3\hat{C}(\alpha)\hat{m}^2(\alpha,\gamma)}{\hat{F}^2(\alpha,\gamma)}}\right), \tag{25}$$

and then the minimum of the potential becomes

$$V_{\mathrm{UFB}-2\,\mathrm{min}} = -\frac{1}{3}\hat{F}(\alpha,\gamma)|H_2|^2_{\mathrm{ext}}\left(|H_2|^2_{\mathrm{ext}} - \frac{2\hat{m}^2(\alpha,\gamma)}{\hat{F}(\alpha,\gamma)}\right). \tag{26}$$

The minimum becomes the deepest when $f(\alpha,\gamma)=0$, imposing $\alpha^2 = 1-\gamma^2$. The typical order of $|H_2|_{\mathrm{ext}}$ is

$$|H_2|_{\mathrm{ext}} \sim \sqrt{m_{\mathrm{SUSY}}M}, \tag{27}$$

where m_{SUSY} and M are a typical scale of the soft SUSY breaking masses and a cutoff scale of the neutrino mass operators. Thus, the potential could be deeper than that of the EWSB if M is larger than m_{SUSY}. A necessary condition to avoid this CCB minimum requires that the minimum of the potential, (26), becomes positive, i.e.

$$\left(\frac{|c'|}{|c|} + 2|\mu|\gamma\right)^2 \le \frac{4(2-\gamma^2)}{1-\gamma^2}\hat{m}^2(\gamma), \tag{28}$$

where $\hat{m}^2(\gamma)$ is the one inserting $\alpha^2 = 1-\gamma^2$. Then, the most stringent constraint is obtained by minimizing the following function $\eta(\gamma)$ with respect to γ,

$$\eta(\gamma) = 2\sqrt{\frac{2-\gamma^2}{1-\gamma^2}\hat{m}^2(\gamma)} - 2|\mu|\gamma. \tag{29}$$

That is, the extremal value, γ_{ext}, is given as a solution of the function,

$$\begin{aligned} \xi(\gamma) =& \gamma\hat{m}^2(\gamma) + (1-\gamma^2)(2-\gamma^2)((m_1^2 - m_{\tilde{L}}^2)\gamma - |m_3^2|) \\ &- (1-\gamma^2)\sqrt{(1-\gamma^2)(2-\gamma^2)\hat{m}^2(\gamma)}|\mu|, \end{aligned} \tag{30}$$

where $\xi(\gamma) \propto \partial\eta(\gamma)/\partial\gamma$. One can see that the constraint, (28), is similar to the constraint, (20).

B. Constraints from the MSSM UFB-3 direction

Next we consider the MSSM UFB-3 direction defined in Eq. (9). Parameterizing the vev's as

$$|\tilde{\nu}_L| = \alpha|H_2|, \tag{31}$$

the scalar potential along this direction is given in the same form as Eq. (23) by setting $\gamma = 0$ and making a replacement m_2^2 with $m_2^2 - |\mu|^2$. The minimum of the potential is obtained for $\alpha = 1$ and the most stringent constraint is given by using Eqs. (25) and (26),

$$\frac{|c'|^2}{|c|^2} \le 8(m_2^2 - |\mu|^2 + m_{\tilde{L}}^2). \tag{32}$$

Comparing the constraint, (32), with that in the MSSM, (10), we can see that Eq. (32) imposes a more severe bound on the soft SUSY breaking parameters if the ratio, $|c'|/|c|$, is of order the SUSY breaking scale.

IV. CCB CONSTRAINTS IN THE νSSM WITH NEUTRINO MASS OPERATORS

We analyze the potential along the MSSM CCB direction. We focus new minima which occur due to the higher order terms of 6th, 5th, and 4th power of fields. The scalar potential with the parametrization of Eq. (11) is expressed as

$$\begin{aligned} V = \hat{C}(\gamma_L)|H_2|^6 - \hat{D}(\alpha, \beta, \gamma_L)|H_2|^5 \\ + \hat{F}(\alpha, \beta, \gamma, \gamma_L)|H_2|^4 - \hat{A}(\alpha, \beta, \gamma)|H_2|^3 \\ + \hat{m}^2(\alpha, \beta, \gamma, \gamma_L)|H_2|^2, \end{aligned} \tag{33}$$

where

$$\hat{C}(\gamma_L) = \gamma_L^2(1 + \gamma_L^2)|c|^2, \tag{34a}$$

$$\hat{D}(\alpha, \beta, \gamma_L) = -2\epsilon_1 \alpha\beta\gamma_L^2|Y_u||c|, \tag{34b}$$

$$\begin{aligned} \hat{F}(\alpha, \beta, \gamma, \gamma_L) = |Y_u|^2(\alpha^2\beta^2 + \alpha^2 + \beta^2) \\ - \gamma_L^2(\epsilon_2|c'| + 2\epsilon_3\gamma|\mu||c|) \\ + |Y_u|^2 f(\alpha, \beta, \gamma, \gamma_L), \end{aligned} \tag{34c}$$

$$\hat{A}(\alpha, \beta, \gamma) = 2\alpha\beta|Y_u|(\epsilon_4|A_u| + \epsilon_5\gamma|\mu|), \tag{34d}$$

$$\begin{aligned} \hat{m}^2(\alpha, \beta, \gamma, \gamma_L) = \gamma^2 m_1^2 + m_2^2 - 2\epsilon_6\gamma|m_3^2| \\ + \alpha^2 m_{\tilde{Q}_L}^2 + \beta^2 m_{\tilde{u}_R}^2 + \gamma_L^2 m_{\tilde{L}}^2, \end{aligned} \tag{34e}$$

and $f(\alpha, \beta, \gamma, \gamma_L)$ is given in Eq. (13b). Here, ϵ_i ($i = 1$–6) denote the sign ($\pm$) and are defined as

$$\epsilon_1 = \mathrm{sign}[\mathrm{Re}(Y_u c^* \tilde{u}_L \tilde{u}_R^* H_2^* (\tilde{\nu}_L^*)^2)], \tag{35a}$$

$$\epsilon_2 = \mathrm{sign}[\mathrm{Re}(c'(H_2)^2(\tilde{\nu}_L)^2)], \tag{35b}$$

$$\epsilon_3 = \mathrm{sign}[\mathrm{Re}(\mu c^* H_1 H_2^* (\tilde{\nu}_L^*)^2)], \tag{35c}$$

$$\epsilon_4 = \mathrm{sign}[\mathrm{Re}(A_u Y_u H_2 \tilde{u}_L \tilde{u}_R^*)], \tag{35d}$$

$$\epsilon_5 = \mathrm{sign}[\mathrm{Re}(Y_u \mu^* H_1^* \tilde{u}_L \tilde{u}_R^*)], \tag{35e}$$

$$\epsilon_6 = \mathrm{sign}[\mathrm{Re}(m_3^2 H_1 H_2)]. \tag{35f}$$

In the following, we show possible choices of ϵ_i, ($i = 1$–6) for the MSSM CCB-1 and CCB-2 directions and derive constraints from the deepest directions.

A. Constraints from the MSSM CCB-1 directions

The MSSM CCB-1 direction is defined in Eq. (16) and the deepest minimum emerges when the D term potential is vanishing. The scalar potential along this direction is obtained from Eqs. (34) by setting $\gamma = 0$ and replacing m_2^2 with $m_2^2 - |\mu|^2$,

$$\hat{C}(\alpha) = (1 - \alpha^2)(2 - \alpha^2)|c|^2, \tag{36a}$$

$$\hat{D}(\alpha) = -2\epsilon_1\alpha^2(1 - \alpha^2)|Y_u||c|, \tag{36b}$$

$$\hat{F}(\alpha) = |Y_u|^2\alpha^2(2 + \alpha^2) - \epsilon_2(1 - \alpha^2)|c'|, \tag{36c}$$

$$\hat{A}(\alpha) = 2\epsilon_4\alpha^2|Y_u||A_u|, \tag{36d}$$

$$\begin{aligned} \hat{m}^2(\alpha) = m_2^2 - |\mu|^2 + \alpha^2(m_{\tilde{Q}_L}^2 + m_{\tilde{u}_R}^2) \\ + (1 - \alpha^2)m_{\tilde{L}}^2, \end{aligned} \tag{36e}$$

where $\alpha = \beta$ and $\gamma_L^2 = 1 - \alpha^2$ are used. By choosing an appropriate phase of fields, the signs, $\epsilon_{1,2,4}$, satisfy a relation

$$\epsilon_1\epsilon_2\epsilon_4 = \mathrm{sign}[c]\mathrm{sign}[c']\mathrm{sign}[A_u], \tag{37}$$

where we assumed that c, c' and Y_u, A_u are real numbers. We can find the properties from the relation according to the following:

(1) When the right-hand side of Eq. (37) is positive, all of or one of the three signs can be made positive.
(2) When the right-hand side of Eq. (37) is negative, all of or one of the three signs can be made negative.

The deepest direction corresponds to the choice of signs such that ϵ_1 is negative while ϵ_2 and ϵ_4 are positive.

Before we start our analysis, it is important to notice that the terms of $\hat{C}$, $\hat{D}$, and $\hat{F}$ except for $-(1 - \alpha^2)|c'|$ originate from the F term potential, and therefore the total contribution of the 6th, 5th, and 4th order terms is positive if $|c'|/|Y_u^2| \ll 1$. In this case, the condition to avoid the MSSM CCB-1 direction is the same as the one in [16]. The situation, however, changes when $|c'|/|Y_u^2| \gg 1$. The 4th order term is dominated by $-(1 - \alpha^2)|c'|$ and the new CCB minima emerge at large values of fields.

As shown in Appendix B, the leading terms of $|H_2|_{\mathrm{ext}}$ are independent of the Yukawa couplings in the case of $|c'|/|Y_u|^2 \gg 1$. Therefore, we can neglect the terms proportional to the Yukawa coupling. Then, the scalar potential is approximated as

$$V_{\mathrm{CCB-1}} \simeq \hat{C}(\alpha)|H_2|^6 + \hat{F}(\alpha)|H_2|^4 + \hat{m}^2(\alpha)|H_2|^2, \tag{38}$$

where

$$\hat{F}(\alpha) = -(1 - \alpha^2)|c'|. \tag{39}$$

The extremal value of the up-type Higgs scalar is obtained by differentiating Eq. (38) with respect to $|H_2|$,

$$|H_2|_{\mathrm{ext}}^2 = -\frac{\hat{F}(\alpha)}{3\hat{C}(\alpha)}\left(1 - \sqrt{1 - \frac{3\hat{C}(\alpha)\hat{m}^2(\alpha)}{\hat{F}^2(\alpha)}}\right), \tag{40}$$

where $\hat{F}(\alpha)$ must be negative for the potential to be minimum. The minimum of the potential is given by

$$V_{\mathrm{CCB-1\,min}} = \tfrac{1}{3}|H_2|_{\mathrm{ext}}^2(\hat{F}(\alpha)|H_2|_{\mathrm{ext}}^2 + 2\hat{m}^2(\alpha)), \tag{41}$$

and a necessary condition to avoid the MSSM CCB-1 minimum is

$$\hat{F}^2(\alpha) < 4\hat{C}(\alpha)\hat{m}^2(\alpha), \tag{42}$$

which gives the constraint

$$\frac{|c'|^2}{|c|^2} < 4\frac{2 - \alpha^2}{1 - \alpha^2}\hat{m}^2(\alpha). \tag{43}$$

The most stringent condition is obtained by minimizing the right-hand side of Eq. (43).

B. Constraints from the MSSM CCB-2 directions

The MSSM CCB-2 direction is defined in Eq. (19). Similar to the MSSM CCB-1 direction, the deepest direction emerges along a direction, $\alpha = \beta$ and $\gamma_L^2 = 1 - \alpha^2 - \gamma^2$. Along this direction, the signs, ϵ_i, $(i = 1 - 6)$, satisfy the relations,

$$\epsilon_1\epsilon_2\epsilon_4 = \mathrm{sign}[c]\mathrm{sign}[c']\mathrm{sign}[A_u], \tag{44a}$$

$$\epsilon_1\epsilon_3 = \epsilon_5, \tag{44b}$$

$$\epsilon_4\epsilon_5\epsilon_6 = \mathrm{sign}[A_u]\mathrm{sign}[m_3^2/\mu]. \tag{44c}$$

The minimum of the potential becomes the deepest when ϵ_1 is negative while the other ϵ's are positive. From Eqs. (44), we can find the following properties:

(1) ϵ_2 can be always set positive.

(2) When $\mathrm{sign}[A_u] = \mathrm{sign}[m_3^2/\mu]$, ϵ_4, ϵ_5, and ϵ_6 can be made positive simultaneously, and ϵ_1 and ϵ_3 must be positive (negative) if $\mathrm{sign}[c]\mathrm{sign}[c']\mathrm{sign}[A_u]$ is positive (negative).

(3) When $\mathrm{sign}[A_u] = -\mathrm{sign}[m_3^2/\mu]$, two of ϵ_4, ϵ_5, and ϵ_6 can be made positive and the other must be negative.

  (a) If ϵ_4 and ϵ_6 are positive, ϵ_1 and ϵ_3 must be the opposite signs of each other.

  (b) If either ϵ_4 or ϵ_6 is negative, ϵ_1 and ϵ_3 must be the same sign.

In the following, we consider the case that ϵ_i $(i = 1–6)$ are all positive. Then, the potential is given by

$$\hat{C}(\alpha, \gamma) = (1 - \alpha^2 - \gamma^2)(2 - \alpha^2 - \gamma^2)|c|^2, \tag{45a}$$

$$\hat{D}(\alpha, \gamma) = -2\alpha^2(1 - \alpha^2 - \gamma^2)|Y_u||c|, \tag{45b}$$

$$\hat{F}(\alpha, \gamma) = |Y_u|^2\alpha^2(\alpha^2 + 2) - (1 - \alpha^2 - \gamma^2)(|c'| + 2\gamma|\mu||c|), \tag{45c}$$

$$\hat{A}(\alpha, \gamma) = 2\alpha^2|Y_u|(|A_u| + \gamma|\mu|), \tag{45d}$$

$$\hat{m}^2(\alpha, \gamma) = \gamma^2 m_1^2 + m_2^2 - 2\gamma|m_3^2| + \alpha^2(m_{\tilde{Q}_L}^2 + m_{\tilde{u}_R}^2) + (1 - \alpha^2 - \gamma^2)m_{\tilde{L}}^2. \tag{45e}$$

Similar to the MSSM CCB-1 direction, a new CCB minimum appears when $|c'|/|Y_u|^2 \gg 1$. The extremal value of $|H_2|$ at the leading order is given by Eq. (40) with $\hat{F}(\alpha, \gamma) \simeq -(1 - \alpha^2 - \gamma^2)(|c'| + 2\gamma|\mu||c|)$, and a necessary condition to avoid the CCB minimum is

$$\left(\frac{|c'|}{|c|} + 2\gamma|\mu|\right)^2 < 4\frac{2 - \alpha^2 - \gamma^2}{1 - \alpha^2 - \gamma^2}\hat{m}^2(\alpha, \gamma). \tag{46}$$

In the end, we comment on other possibilities of choice of vev's. Since the terms proportional to the Yukawa coupling is irrelevant in the present analysis, similar results can be obtained when we take vev's of squarks as

$$\tilde{u}_L \to \tilde{d}_L, \qquad \tilde{u}_R \to \tilde{d}_R, \tag{47}$$

where $\tilde{d}_L$ and $\tilde{d}_R$ are different squarks from those to cancel the F term of H_1. Along this direction, the constraint for the MSSM CCB-1 is obtained by

$$\frac{|c'|^2}{|c|^2} < 4\frac{2 + \alpha^2}{1 + \alpha^2}\hat{m}^2(\alpha), \tag{48}$$

like Eq. (43), where $\hat{m}^2(\alpha)$ is replaced by

$$\hat{m}^2(\alpha) = m_2^2 - |\mu|^2 + \alpha^2(m_{\tilde{Q}_L}^2 + m_{\tilde{d}_R}^2) + (1 + \alpha^2)m_{\tilde{L}}^2. \tag{49}$$

Similarly, the constraint from the MSSM CCB-2 is obtained by

$$\left(\frac{|c'|}{|c|} + 2\gamma|\mu|\right)^2 < 4\frac{2 + \alpha^2 - \gamma^2}{1 + \alpha^2 - \gamma^2}\hat{m}^2(\alpha, \gamma), \tag{50}$$

like Eq. (46), where $\hat{m}^2(\alpha, \gamma)$ is replaced by

$$\hat{m}^2(\alpha, \gamma) = \gamma^2 m_1^2 + m_2^2 - 2\gamma|m_3^2| + \alpha^2(m_{\tilde{Q}_L}^2 + m_{\tilde{d}_R}^2) + (1 + \alpha^2 - \gamma^2)m_{\tilde{L}}^2. \tag{51}$$

V. NUMERICAL ANALYSIS

In this section, we show numerical results of the constraints of the MSSM UFB-2 and the MSSM UFB-3 derived in the previous sections. Similar analysis can be carried out for the MSSM CCB directions. As an illustrating example, we employ the constrained MSSM (CMSSM) to calculate the soft SUSY breaking parameters and the supersymmetric Higgs masses. For the couplings of the neutrino mass operators, we assume that c and c' are so small that these do not contribute to the renormalization group equations (RGEs) of other couplings and SUSY breaking parameters, significantly. Then, we treat the couplings, c and c', as input parameters and set values at a scale we perform the numerical calculation. As we mentioned in the Sec. I, the ratio of the couplings of the neutrino mass operators in the minimal supergravity SUSY breaking model is expected to be

$$\frac{|c'|}{|c|} = \mathcal{O}(m_{3/2}), \tag{52}$$

where $m_{3/2}$ is the gravitino mass. In the following, we consider the case that the ratio, $|c'|/|c|$, is between 100 and 1000 GeV. We will see that the constraint of the MSSM UFB-3 imposes more stringent bound on the soft SUSY parameters than those by the MSSM, Eq. (10) in this case.

The CMSSM is parametrized by four parameters and a sign,

$$M_{1/2}, m_0, A_0, \tan\beta, \mathrm{sign}[\mu], \tag{53}$$

where the first three parameters are the universal gaugino and scalar masses, and the universal trilinear couplings defined at the grand unified theory (GUT) scale. $\tan\beta$ is the ratio of the vev's of the Higgs scalars and μ is the supersymmetric Higgs mass. For simplicity, we fix the values of A_0 and $\tan\beta$,

$$A_0 = 0 \text{ GeV}, \qquad \tan\beta = 10, \tag{54}$$

and take the sign of μ positive. As we mentioned in Sec. I, radiative corrections to the scalar potential is minimized at a scale around the extremal value of the up-type Higgs scalar, $|H_2|_{\rm ext}$. Such a case would be an intermediate scale between the weak scale and the GUT scale, because $|H_2|_{\rm ext} = \mathcal{O}(\sqrt{m_{3/2}M})$. Some of our results change significantly around 10^6 GeV. Thus, we show numerical calculations for the RGE scale, Λ, around 10^5–10^7 GeV.

In Fig. 1(a), we plot the function, $\xi(\gamma)$, given in Eq. (30) with $m_0 = 300$ GeV and $M_{1/2} = 500$ GeV at $\Lambda = 10^6$ GeV. The function, $\xi(\gamma)$, is an increasing function of γ and is always negative at $\gamma = 0$ while it is positive at $\gamma = 1$ when the EWSB successfully occurs. Hence, $\xi(\gamma)$ has only one zero point for $0 \leq \gamma \leq 1$. The zero point, $\gamma_{\rm ext}$, is usually found around 0.5 unless $M_{1/2}$ is small. In the case of small $M_{1/2}$, $\gamma_{\rm ext}$ is found to be 0.4 or not found because the EWSB does not occur. The shape of $\xi(\gamma)$ is in general the same for other values of the CMSSM parameters and the RGE scale.

The constraint from the MSSM UFB-2 direction, Eq. (28), is shown in Fig. 1(b) for $\Lambda = 10^6$ GeV. We varied m_0 and $M_{1/2}$ and solved $\xi(\gamma)$ at each point. The solid (red), dashed (green), dotted (blue), and the dashed-dotted (pink) curves represent the constraint with $|c'|/|c| = 250$, 500, 750, and 1000 GeV, respectively. The inside of the curve are excluded by the constraint. It is seen that the excluded regions expand as $|c'|/|c|$ increases. The hatched (light blue) region is also excluded because the EWSB does not occur due to negative $|\mu|^2$. Results for other RGE scales Λ are also the same qualitatively.

The constraint from the MSSM UFB-3 direction is shown in Fig. 2(a) for $\Lambda = 10^5$ GeV and Fig. 2(b) for $\Lambda = 10^7$ GeV. The solid (red) curve represents the constraint with the ratio of the coefficients, $|c'|/|c| = 0$ GeV, corresponding to one in the MSSM, Eq. (10). The dashed (green), dotted (blue), and dashed-dotted (pink) curves correspond to the ratio of the coefficient, 400, 600, and 1000 GeV, respectively. In Fig. 2(a), the region below the curves is excluded by the constraint. It is seen that, due to

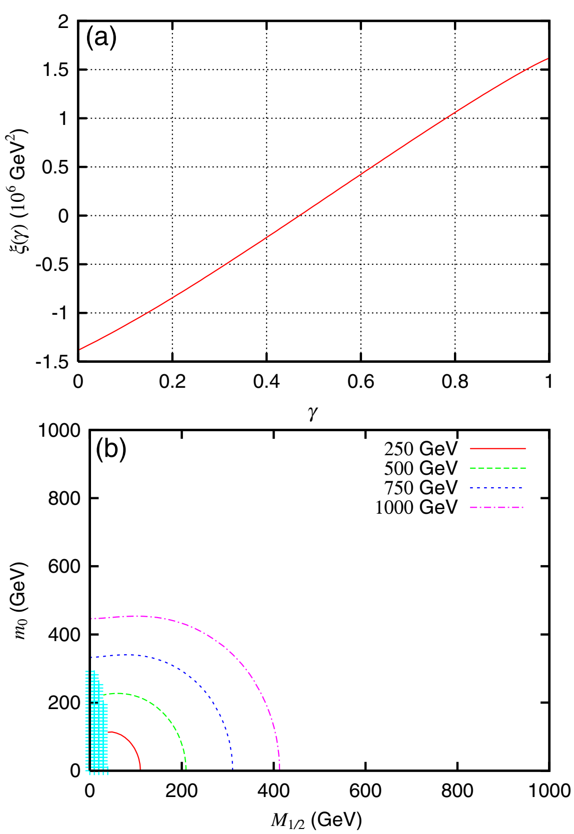

FIG. 1 (color online). (a) function, $\xi(\gamma)$ scaled by 10^{-6}. (b) constraint from the MSSM UFB-2. In (b), the solid (red), dashed (green), dotted (blue), and the dashed-dotted (pink) curves correspond to $|c'|/|c| = 250$, 500, 750, and 1000 GeV, respectively.

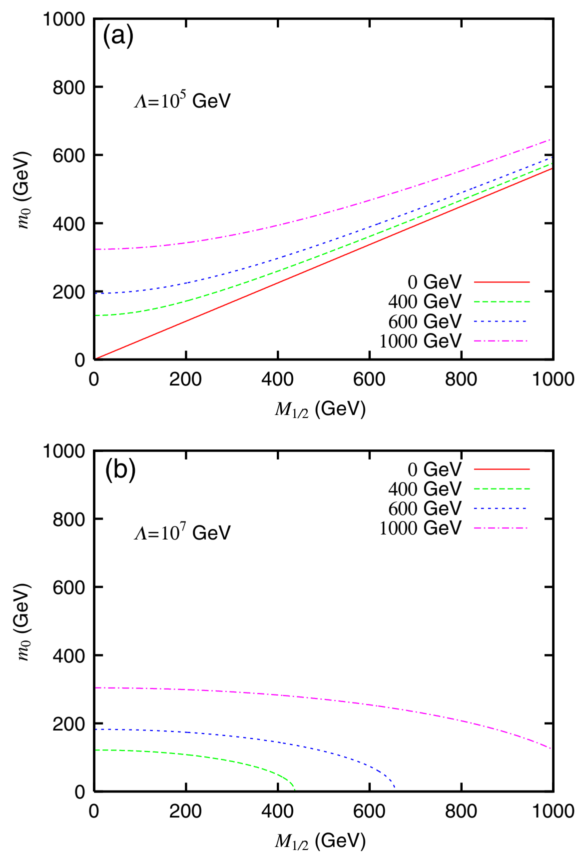

FIG. 2 (color online). Constraint from the MSSM UFB-3 direction is shown in (a) and (b) for $\Lambda = 10^5$ and 10^7, respectively. The solid (red), dashed (green), dotted (blue), and the dashed-dotted (pink) curves correspond to $|c'|/|c| = 0$, 400, 600, and 1000 GeV, respectively.

the presence of the neutrino mass operators, the constraint in the νSSM is tighter than that in the MSSM. In Fig. 2(b), the region inside the curves is excluded by the constraint. In the case of $|c'|/|c| = 0$ GeV, only the point, $M_{1/2} = m_0 = 0$ GeV, is excluded. It is seen that the shape of the excluded region is elliptic in Fig. 2(b) while it is hyperbolic in Fig. 2(a). The difference originates from the fact that when we write $m_{H_2}^2(\Lambda) = aM_{1/2}^2 + bm_0^2$, the coefficient a is negative for $\Lambda \leq \mathcal{O}(10^6)$ GeV but the coefficient a is positive for $\Lambda \geq \mathcal{O}(10^7)$ GeV. The CCB minimum along the MSSM UFB-3 direction can be negative even if the soft mass of the up-type Higgs scalar is positive since the potential is lowered by the term of order 5th power, which is proportional to $|c'|$. Results for other scales such as $\Lambda \leq \mathcal{O}(10^6)$ GeV and $\Lambda \geq \mathcal{O}(10^7)$ GeV are the same qualitatively as Figs. 2(a) and 2(b), respectively.

VI. SUMMARY AND DISCUSSION

We have considered the νSSM with neutrino mass operators where the $(\hat{H}_2 \cdot \hat{L})(\hat{H}_2 \cdot \hat{L})$ operator in the superpotential and the corresponding dimension four operator in the soft SUSY breaking terms are added to the MSSM. In this model, the scalar potential contains new terms of order 6th, 5th, and 4th power which are absent in the minimally supersymmetric extension of the SM. We have analyzed the scalar potential along the MSSM UFB, and CCB directions and found new unrealistic vacua which appear due to the higher order term in the scalar potential.

We have found that the MSSM UFB directions disappear and turn out to be CCB directions due to the presence of higher power terms in the scalar potential. The minima along these CCB directions are of $\mathcal{O}(m_{\rm SUSY}M)^2$ and can be deeper than that of the EWSB. We have derived necessary conditions to avoid the CCB minima along the MSSM UFB-2 and UFB-3 which impose constraints among the soft SUSY breaking parameters and the coefficients of the neutrino mass operators. The constraints are expressed in terms of the ratio of the coefficients, $|c'|/|c|$, thus these cannot be ignored even if c' and c are small. The most stringent constraint was obtained from the MSSM UFB-3 direction, which imposes bounds on the soft masses of the up-type Higgs scalar and the left-handed sleptons. The constraint holds even if the soft mass of the Higgs scalar is positive; therefore, it should be always taken into account at any scale.

We have also shown that there appears new CCB minima along the MSSM CCB directions in the case of $|c'|/|Y_u^2| \gg 1$. We showed that the extremal value of the Higgs scalar can be determined by neglecting the Yukawa coupling of quarks in this case and the potential can be deeper than that of the EWSB. Like the constraints along the UFB directions, necessary conditions to evade the CCB minima are expressed in terms of the ratio of the coefficients.

In Sec. V, we have applied our results to see differences from the constraints in the MSSM. We calculated the soft masses at scales $10^5 \leq \Lambda \leq 10^7$ GeV using the RGEs in the CMSSM. It was shown that the constraints we derived are more stringent than those of the MSSM. Thus, it is important to apply these constraints to several SUSY breaking models. We would study them elsewhere including detailed analysis on the CMSSM.

As we have mentioned in the introduction, radiative corrections must be included into analysis for our results to be applied at the electroweak scale. It is also needed to include finite temperature effects if one analyze the potential at high energy scale or for large vev's. We leave these for our future work. It would also be important to study our constraints from the viewpoint of the evolution of the Universe. (See e.g. [21].)

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APPENDIX A: SCALAR POTENTIAL

In this appendix, we give notations of scalars and the scalar potential with the neutrino mass operators. Throughout this article, flavor indexes are suppressed for simplicity.

The down-type and the up-type Higgs scalars are denoted as

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix}, \qquad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix}, \tag{A1}$$

where H_1^1 and H_2^2 are electrically neutral components. Throughout this article, we refer to H_1^1 and H_2^2 as H_1 and H_2. The left-handed squarks and the right-handed squarks are denoted as

$$\tilde{Q} = \begin{pmatrix} \tilde{u}_L \\ \tilde{d}_L \end{pmatrix}, \qquad \tilde{u}_R, \tilde{d}_R, \tag{A2}$$

and the left-handed sleptons and the right-handed sleptons are denoted as

$$\tilde{L} = \begin{pmatrix} \tilde{\nu}_L \\ \tilde{e}_L \end{pmatrix}, \qquad \tilde{e}_R. \tag{A3}$$

The superpotential is given

$$\mathcal{W} = \mathcal{W}_{\rm MSSM} - \tfrac{1}{2}c(\hat{H}_2 \cdot \hat{L})(\hat{H}_2 \cdot \hat{L}), \tag{A4}$$

where c is a coefficient. $\mathcal{W}_{\rm MSSM}$ is the superpotential of the MSSM,

$$\mathcal{W}_{\rm MSSM} = \mu \hat{H}_1 \cdot \hat{H}_2 + Y_d \hat{H}_1 \cdot \hat{Q} \hat{D}_R^c + Y_u \hat{H}_2 \cdot \hat{Q} \hat{U}_R^c + Y_e \hat{H}_1 \cdot \hat{L} \hat{E}_R^c, \tag{A5}$$

where μ is a supersymmetric Higgs mass, and $Y_{u,d,e}$ are the Yukawa couplings of up quarks, down quarks, and charged leptons, respectively.

The scalar potential, V, is divided into three parts which consist of F, D and the soft SUSY breaking terms,

$$V = V_F + V_D + V_{\rm soft}. \tag{A6}$$

The F term potential, V_F, is given by a sum of absolute square of all matter auxiliary fields,

$$V_F = \sum_{i={\rm matter}} |F_i|^2, \tag{A7}$$

where

$$F^*_{H_1^1} = \mu H_2^2 + Y_e \tilde{e}_L \tilde{e}_R^* + Y_d \tilde{d}_L \tilde{d}_R^*, \tag{A8a}$$
$$F^*_{H_1^2} = -\mu H_2^1 - Y_e \tilde{\nu}_L \tilde{e}_R^* - Y_d \tilde{u}_L \tilde{d}_R^*, \tag{A8b}$$
$$F^*_{H_2^1} = -\mu H_1^2 + Y_u \tilde{d}_L \tilde{u}_R^* - c \tilde{e}_L (H_2^1 \tilde{e}_L - H_2^2 \tilde{\nu}_L), \tag{A8c}$$
$$F^*_{H_2^2} = \mu H_1^1 - Y_u \tilde{u}_L \tilde{u}_R^* + c \tilde{\nu}_L (H_2^1 \tilde{e}_L - H_2^2 \tilde{\nu}_L), \tag{A8d}$$
$$F_{\tilde{e}_R} = Y_e (H_1^1 \tilde{e}_L - H_1^2 \tilde{\nu}_L), \tag{A8e}$$
$$F^*_{\tilde{e}_L} = Y_e H_1^1 \tilde{e}_R^* - c H_2^1 (H_2^1 \tilde{e}_L - H_2^2 \tilde{\nu}_L), \tag{A8f}$$
$$F^*_{\tilde{\nu}_L} = -Y_e H_1^2 \tilde{e}_R^* + c H_2^2 (H_2^1 \tilde{e}_L - H_2^2 \tilde{\nu}_L), \tag{A8g}$$
$$F_{\tilde{d}_R} = Y_d (H_1^1 \tilde{d}_L - H_1^2 \tilde{u}_L), \tag{A8h}$$
$$F^*_{\tilde{d}_L} = Y_d H_1^1 \tilde{d}_R^* + Y_u H_2^1 \tilde{u}_R^*, \tag{A8i}$$
$$F_{\tilde{u}_R} = Y_u (H_2^1 \tilde{d}_L - H_2^2 \tilde{u}_L), \tag{A8j}$$
$$F^*_{\tilde{u}_L} = -Y_d H_1^2 \tilde{d}_R^* - Y_u H_2^2 \tilde{u}_R^*. \tag{A8k}$$

The D term potential, V_D, is given by a sum of square of all gauge auxiliary fields,

$$V_D = \tfrac{1}{2}((D^a_{SU(3)})^2 + (D^a_{SU(2)})^2 + (D_{U(1)})^2), \tag{A9}$$

where a runs from 1 to 8(3) for $SU(3)(SU(2))$ and summation over a should be understood. The gauge auxiliary fields are given by

$$D^a_{SU(3)} = g_3\left(\tilde{Q}^\dagger \frac{\lambda^a}{2} \tilde{Q} - \tilde{u}_R^* \frac{\lambda^a}{2} \tilde{u}_R - \tilde{d}_R^* \frac{\lambda^a}{2} \tilde{d}_R\right), \tag{A10a}$$
$$D^a_{SU(2)} = g_2(\tilde{Q}^\dagger T^a \tilde{Q} + \tilde{L}^\dagger T^a \tilde{L} + H_1^\dagger T^a H_1 + H_2^\dagger T^a H_2), \tag{A10b}$$
$$D_{U(1)} = g_1(\tfrac{1}{6}\tilde{Q}^\dagger \tilde{Q} - \tfrac{2}{3}\tilde{u}_R^* \tilde{u}_R + \tfrac{1}{3}\tilde{d}_R^* \tilde{d}_R - \tfrac{1}{2}\tilde{L}^\dagger \tilde{L} + \tilde{e}_R^* \tilde{e}_R - \tfrac{1}{2}H_1^\dagger H_1 + \tfrac{1}{2}H_2^\dagger H_2), \tag{A10c}$$

where $g_i (i = 1, 2, 3)$ is a gauge coupling constant, and λ^a and T^a are the Gell-Mann and Pauli matrix, respectively.

The soft SUSY breaking term, $V_{\rm soft}$, is given as

$$\begin{aligned} V_{\rm soft} = {} & m_{H_1}^2 H_1^\dagger H_1 + m_{H_2}^2 H_2^\dagger H_2 + (B\mu H_1 \cdot H_2 + {\rm H.c.}) \\ & + m_{\tilde{Q}}^2 \tilde{Q}^\dagger \tilde{Q} + m_{\tilde{u}_R}^2 \tilde{u}_R^* \tilde{u}_R + m_{\tilde{d}_R}^2 \tilde{d}_R^* \tilde{d}_R + m_{\tilde{L}}^2 \tilde{L}^\dagger \tilde{L} \\ & + m_{\tilde{e}_R}^2 \tilde{e}_R^* \tilde{e}_R + (A_d Y_d H_1 \cdot \tilde{Q} \tilde{d}_R^* + A_u Y_u H_2 \cdot \tilde{Q} \tilde{u}_R^* \\ & + A_e Y_e H_1 \cdot \tilde{L} \tilde{e}_R^* + {\rm H.c.}) \\ & - \frac{1}{2}(c'(H_2 \cdot \tilde{L})(H_2 \cdot \tilde{L}) + {\rm H.c.}), \end{aligned} \tag{A11}$$

where $m_i (i = H_1, H_2, \tilde{Q}, \tilde{u}_R, \tilde{d}_R, \tilde{L}, \tilde{e}_R)$ are soft masses and $B\mu$ is a soft term for Higgs scalars. A symbol "dot" represents an inner product for $SU(2)$ doublets, $A \cdot B = A^1 B^2 - A^2 B^1$. The trilinear terms, $A_i (i = u, d, e)$, are defined to be proportional to the corresponding Yukawa coupling. We also use the following notations:

$$m_1^2 = m_{H_1}^2 + |\mu|^2, \tag{A12a}$$
$$m_2^2 = m_{H_2}^2 + |\mu|^2, \tag{A12b}$$
$$m_3^2 = -B\mu. \tag{A12c}$$

APPENDIX B: GENERAL FORM OF THE VACUUM EXPECTATION VALUES

We give the general form of the vacuum expectation value of H_2 and show that the extremal value of $|H_2|$ is independent of Y_u at the leading order when we assume that $|c'|/|Y_u|^2 \gg 1$.

First, we give the general form of the vev of $|H_2|$. Differentiating the potential, (33) with respect to $|H_2|$, the equation to be solved is obtained,

$$6\hat{C}|H_2|^4 - 5\hat{D}|H_2|^3 + 4\hat{F}|H_2|^2 - 3\hat{A}|H_2| + 2\hat{m}^2 = 0, \tag{B1}$$

where the dependence of coefficients on α, β, γ, and γ_L are omitted. We introduce a dimensionless parameter x which is defined by

$$x \equiv \frac{|H_2|}{M}, \tag{B2}$$

where M is the cutoff scale of the neutrino mass operators. Then, Eq. (B1) is written as

$$a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0 = 0, \tag{B3}$$

where

$$a_4 = 6\hat{C}M^2, \tag{B4a}$$
$$a_3 = -5\hat{D}M, \tag{B4b}$$
$$a_2 = 4\hat{F}, \tag{B4c}$$
$$a_1 = -3\hat{A}M^{-1}, \tag{B4d}$$
$$a_0 = 2\hat{m}^2 M^{-2}. \tag{B4e}$$

The equation, (B3), can be deformed to the following form by shifting $x = t + t_0$ with $t_0 = -a_3/(4a_4)$,

$$t^4 + b_2 t^2 + b_1 t + b_0 = 0, \tag{B5}$$

where

$$b_2 = 6t_0^2 + 3a_3 t_0/a_4 + a_2/a_4, \tag{B6a}$$
$$b_1 = 4t_0^3 + 3a_3 t_0^2/a_4 + 2a_2 t_0/a_4 + a_1/a_4, \tag{B6b}$$
$$b_0 = t_0^4 + a_3 t_0^3/a_4 + a_2 t_0^2/a_4 + a_1 t_0/a_4 + a_0/a_4. \tag{B6c}$$

We rewrite Eq. (B5) as

$$\left(t^2 + \frac{b_2}{2} + u\right)^2 = 2u\left(t - \frac{b_1}{4u}\right)^2 - \frac{b_1^2}{8u} + \left(\frac{b_2}{2} + u\right)^2 - b_0. \tag{B7}$$

The above equation has the solution

$$t = \frac{1}{2}\left(\mp\sqrt{2u} \pm \sqrt{2u - 2\left(b_2 + 2u \mp \frac{b_1}{\sqrt{2u}}\right)}\right), \tag{B8}$$

if u satisfies the following equation:

$$-\frac{b_1^2}{8u} + \left(\frac{b_2}{2} + u\right)^2 - b_0 = 0. \tag{B9}$$

The equation, (B9), is also rewritten as

$$u^3 + c_2 u^2 + c_1 u + c_0 = 0, \tag{B10}$$

where

$$c_2 = b_2, \tag{B11a}$$
$$c_1 = \frac{b_2^2}{4} - b_0, \tag{B11b}$$
$$c_0 = -\tfrac{1}{8} b_1^2, \tag{B11c}$$

and it is deformed by shifting $u = s + s_0$ with $s_0 = -c_2/3$,

$$s^3 + d_1 s + d_0 = 0, \tag{B12}$$

where

$$d_1 = 3s_0^2 + 2c_2 s_0 + c_1, \tag{B13a}$$
$$d_0 = s_0^3 + c_2 s_0^2 + c_1 s_0 + c_0. \tag{B13b}$$

The solutions of the equation, (B12), are well known and given by

$$s = p + q, \tag{B14}$$

where

$$(p, q) = (p_0, q_0), (p_0\omega, q_0\omega^2), (p_0\omega^2, q_0\omega), \tag{B15}$$

and

$$p_0 = \sqrt[3]{-\frac{d_0}{2} + \sqrt{\frac{d_0^2}{4} + \frac{d_1^3}{27}}}, \tag{B16a}$$
$$q_0 = \sqrt[3]{-\frac{d_0}{2} - \sqrt{\frac{d_0^2}{4} + \frac{d_1^3}{27}}}, \tag{B16b}$$
$$\omega = \tfrac{1}{2}(-1 + \sqrt{3}i). \tag{B16c}$$

Now we are in a position to show that the leading order of $|H_2|_{\rm ext}$ is independent of Y_u. We assume that

$$\frac{|c'|}{|Y_u|^2} \gg 1, \tag{B17}$$

$$|c'| \simeq \frac{m_{\rm SUSY}}{M}, \tag{B18}$$

where $m_{\rm SUSY}$ is the SUSY breaking scale and $|Y_u|$ is of order 10^{-5} which corresponds to the up quark mass. Then, the coefficients, a_i ($i = 0$–3) are of order

$$a_3 \sim |Y_u|, \tag{B19}$$

$$a_2 \sim \frac{m_{\rm SUSY}}{M}, \tag{B20}$$

$$a_1 \sim |Y_u| \frac{m_{\rm SUSY}}{M}, \tag{B21}$$

$$a_0 \sim \left(\frac{m_{\rm SUSY}}{M}\right)^2. \tag{B22}$$

Using this order estimation, we can estimate the u and t,

$$u \sim t \sim \frac{m_{\rm SUSY}}{M}, \tag{B23}$$

since b_1 and b_2 are of order

$$b_1 \sim |Y_u| \frac{m_{\rm SUSY}}{M}, \tag{B24}$$

$$b_2 \sim \left(\frac{m_{\rm SUSY}}{M}\right)^2. \tag{B25}$$

Thus, x can be estimated as

$$x \sim \sqrt{\frac{m_{\rm SUSY}}{M}}. \tag{B26}$$

The leading order of $|H_2|_{\rm ext}$ is determined by $m_{\rm SUSY}$ and M, and independent of $|Y_u|$. This result implies that we can neglect $\hat{D}$, $\hat{A}$ and the term proportional to $|Y_u|$ in $\hat{F}$ to obtain $|H_2|_{\rm ext}$.

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